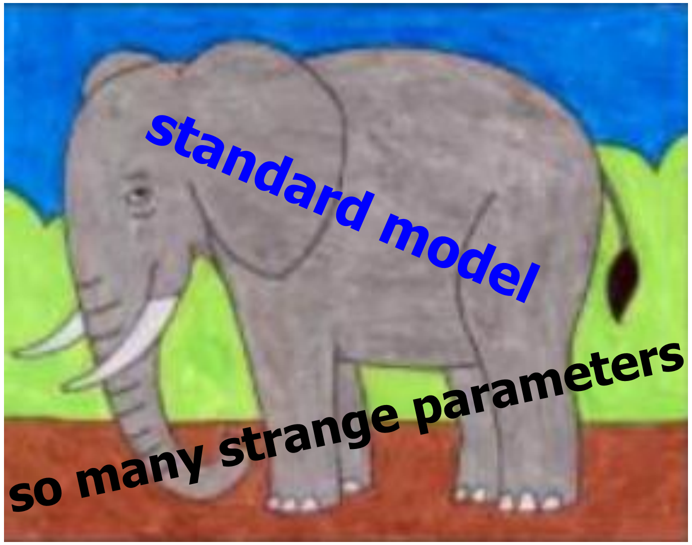
Fermion masses: why small and how small

邢志忠

xingzz@ihep.ac.cn



In the spring of 1953, Enrico Fermi told Freeman Dyson, "I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

中科大交叉学科理论研究中心与彭桓武高能基础理论研究中心，2021.03.26

OUTLINE

- ♣ **Weinberg's concerns and attempts**

- ♣ **If a fermion looks like a Goldstone**

Weinberg's paper in 1967

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

三无产品

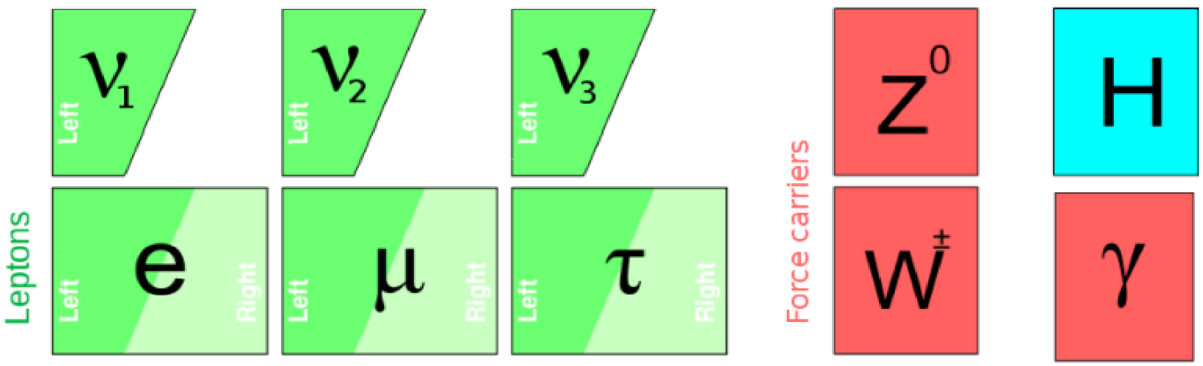
A MODEL OF LEPTONS*

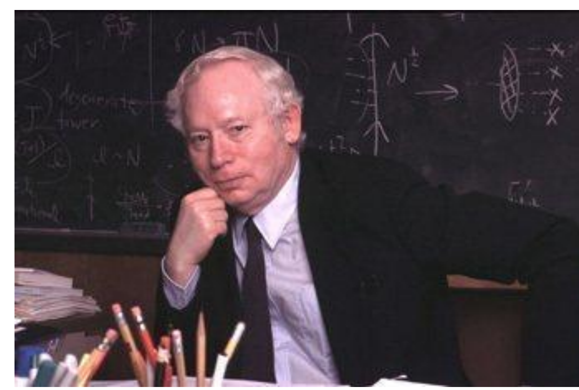
Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)

Theoretical ingredients: **it's got what it matters（五脏俱全）**

Particle content: **no neutrino mass, no quarks, no flavor mixing & CPV**





My style is usually not to propose specific models that will lead to specific experimental predictions, but rather to interpret in a broad way what is going on and make very general remarks, like with the development of the point of view associated with effective field theory ---- Weinberg 2021@CERN Courier

Objecting to Weinberg's razor

Albert Einstein: **Everything should be made as simple as possible, but not simpler!**

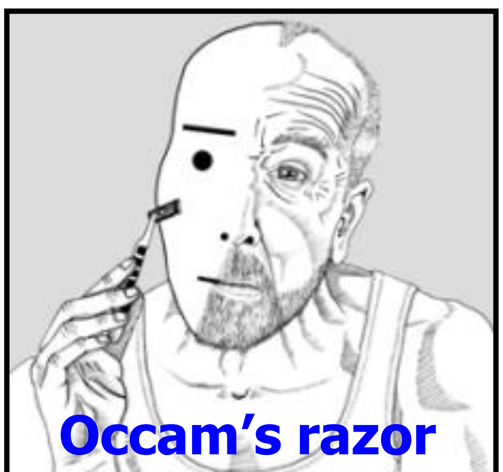
结构存在自然性问题

maximal P violation

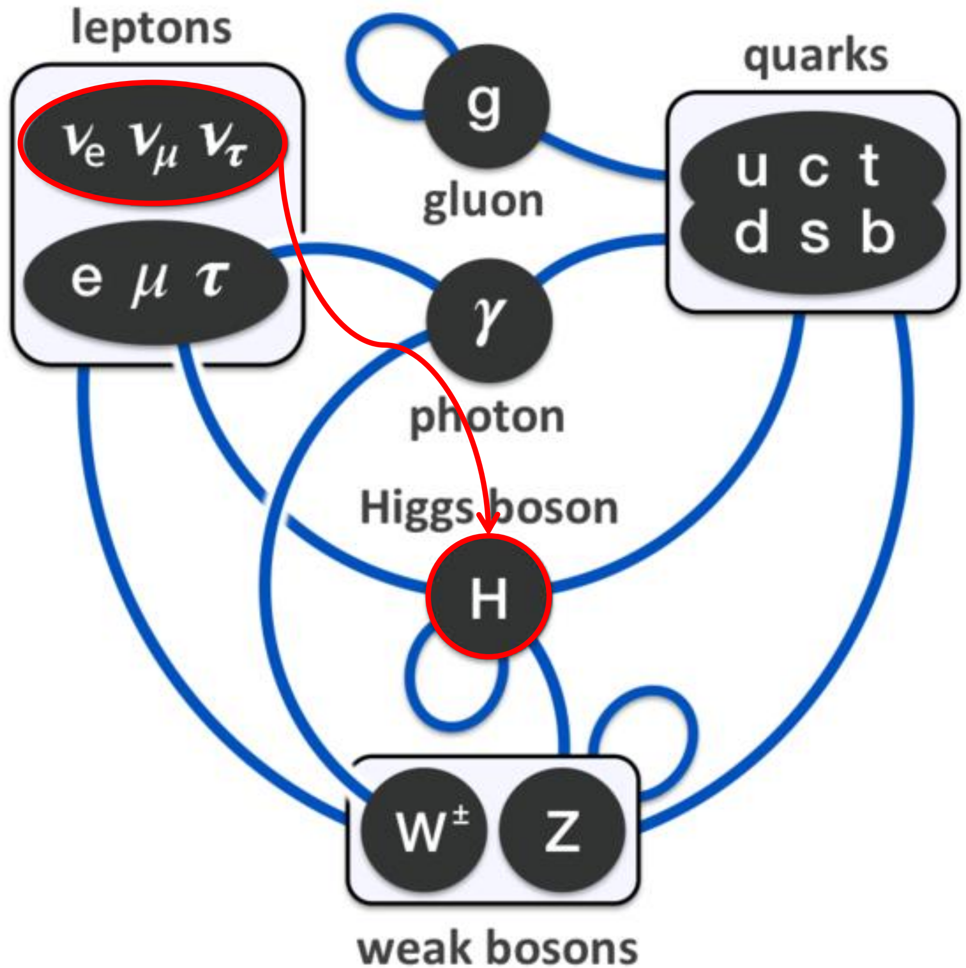
$$\begin{pmatrix} u_{\rm L} \\ d_{\rm L} \end{pmatrix} \quad \Longleftrightarrow \quad \begin{matrix} u_{\rm R} \\ d_{\rm R} \end{matrix}$$

$$\begin{pmatrix} \nu_{e\rm L} \\ e_{\rm L} \end{pmatrix} \quad \Longleftrightarrow \quad \begin{matrix} \quad \\ e_{\rm R} \end{matrix}$$

- **Theoretically unnatural**
- **Experimentally natural**



cut off 7 physical parameters



★ **The least cost to generate ν-mass is a Yukawa coupling**

★ **Use charge-conjugated fields of left-handed ν's? Pay with the scalar fields**

Majorana is more natural

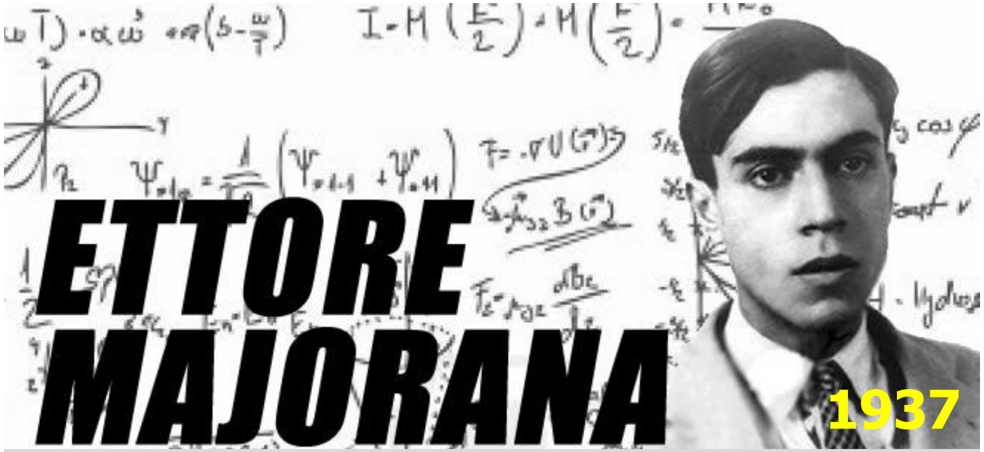
★ **The simplest way to extend the SM is to introduce the right-handed neutrino fields and write out a Dirac mass term.**

Dirac mass $\overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} \longrightarrow M_{\rm D} = Y_\nu \langle H \rangle$

Murray Gell-Mann: everything not forbidden is compulsory!

Majorana mass $\frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R}$

It is lepton-number-violating.



mass state: antineutrino=neutrino

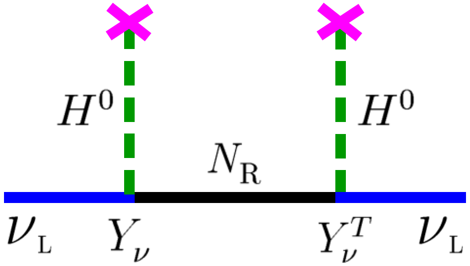
In the SM, L and B are violated by instantons, only B − L is conserved.

$$-\mathcal{L}_{\nu+N} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.} = \frac{1}{2}\overline{[\nu_{\rm L} \quad (N_{\rm R})^c]} \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.}$$

P. Minkowski 1977,
T. Yanagida 1979…

$$M_\nu \simeq -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = -\langle H \rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^T$$

★ **Such a seesaw picture is consistent with the unique operator proposed by Weinberg (1979)**



But the big shot did it in this way

VOLUME 29, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1972

1972

Electromagnetic and Weak Masses*

电子质量源于圈图量子效应

Steven Weinberg
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 19 June 1972)

In theories with spontaneously broken gauge symmetries, various masses, or mass differences may vanish in zeroth order as a consequence of the representation content of the fields appearing in the Lagrangian. These masses or mass differences can then be calculated as finite higher-order effects. The mechanism for cancelation of divergences in second-order fermion masses is described explicitly. The weak interactions play an essential role in canceling infinities in electromagnetic masses.

VOLUME 43, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 1979

1979

Baryon- and Lepton-Nonconserving Processes

马约拉纳中微子的质量源于五维有效算符

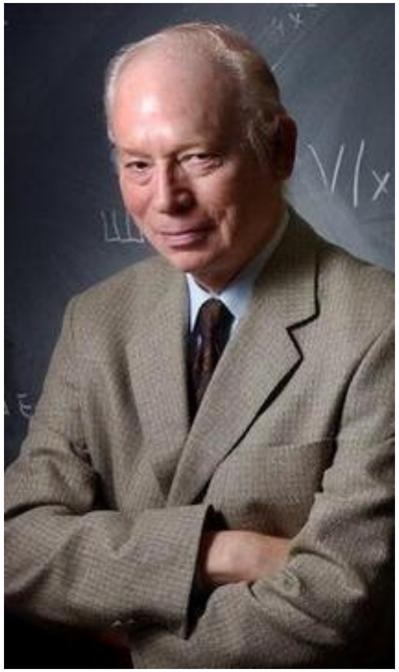
Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, and Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138
(Received 13 August 1979)

A number of prope ng processes are
shown to follow unde to the importance of
measuring μ^+ polari ans of discriminating
among specific mode

$$\mathcal{O}_{\rm Weinberg} = \frac{\kappa_{\alpha\beta}}{2}\left[\overline{\ell_{\alpha L}}\widetilde{H}\widetilde{H}^T\ell^c_{\beta L}\right]$$

From 2017 to 2020

Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn't have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses. In the summer of 1972, when the SM was coming together, he set himself the task of figuring it out but couldn't come up with anything. "It was the worst summer of my life! I mean, obviously there are broader questions such as: why is there something rather than nothing? But if you ask for a very specific question, that's the one. And I'm no closer now to answering it than I was in the summer of 1972," he says, still audibly irritated.



一代和二代费米子质量源于圈图量子效应

2020

87岁高龄

PHYSICAL REVIEW D **101,** 035020 (2020)

Models of lepton and quark masses

Steven Weinberg*

Theory Group, Department of Physics, University of Texas, Austin, Texas 78712, USA

(Received 15 December 2019; accepted 27 January 2020; published 19 February 2020)

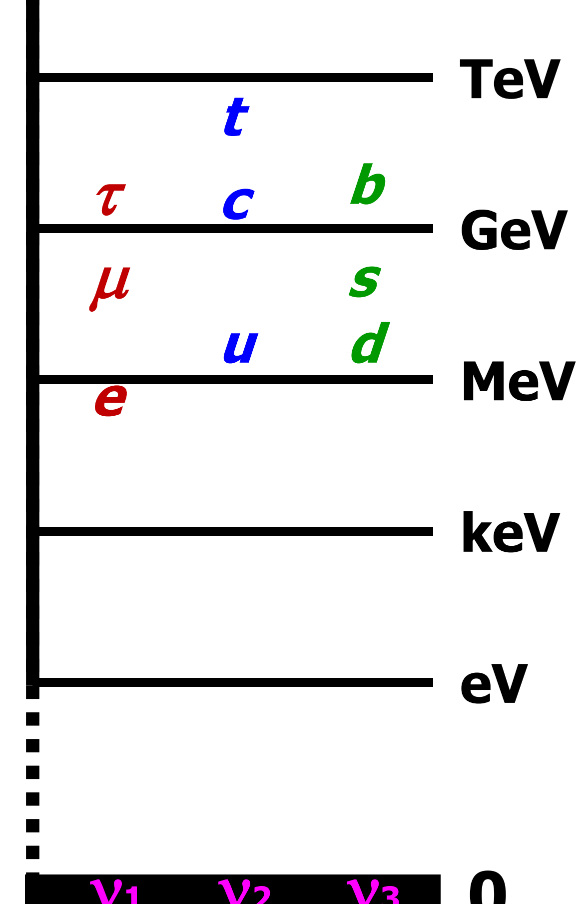
A class of models is considered in which the masses only of the third generation of quarks and leptons arise in the tree approximation, while masses for the second and first generations are produced respectively by one-loop and two-loop radiative corrections. So far, for various reasons, these models are not realistic.

Is **Weinberg** on the right track?

Fermion masses: the **Yukawa** interactions ***at the tree level*** in the SM.

Flavor mixing: both **Yukawa** and charged-current gauge interactions.

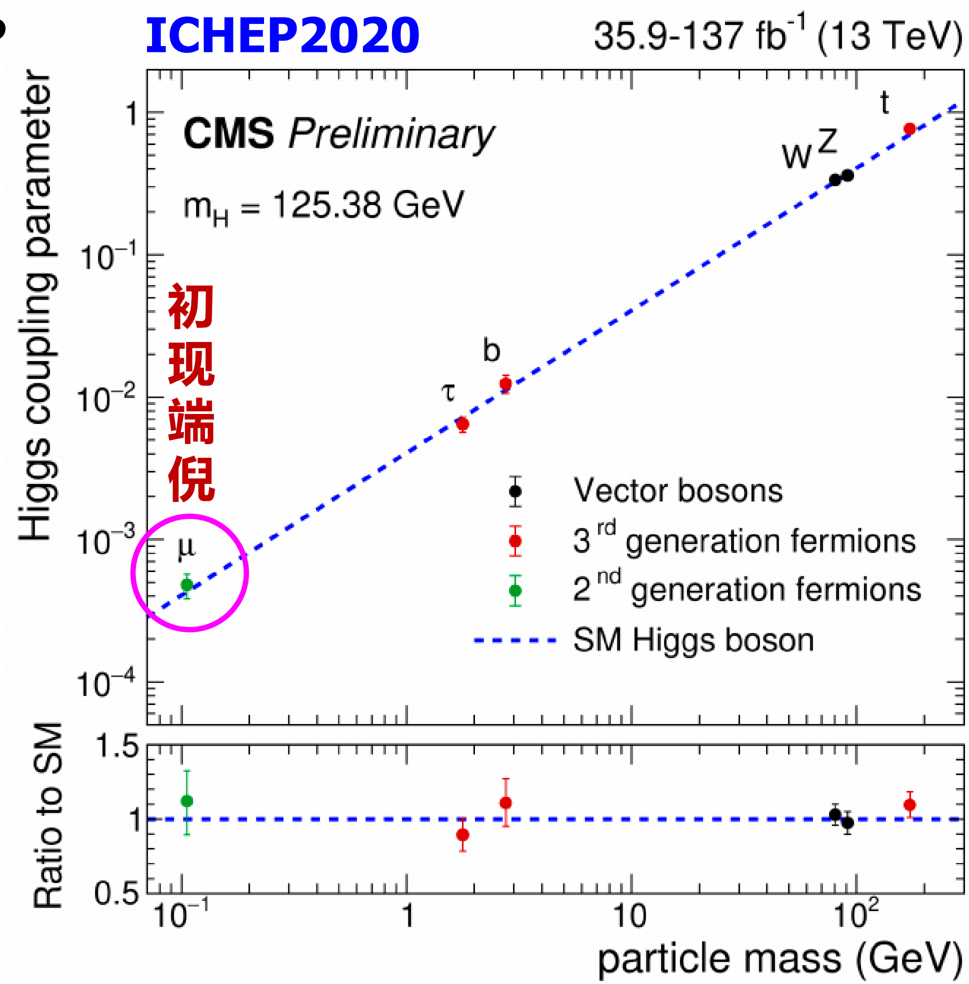
Small masses from tree or loop?



Problem 1
flavor hierarchy

Problem 2
flavor desert

Problem 3
ν-mass origin



Richard Feynman (**1959**): there is plenty of room at the bottom. True!

Two examples

Example 1: tree-level nearest-neighbor interactions to generate mass

$$M_f = \begin{pmatrix} 0 & B_f \\ B_f^* & A_f \end{pmatrix}$$

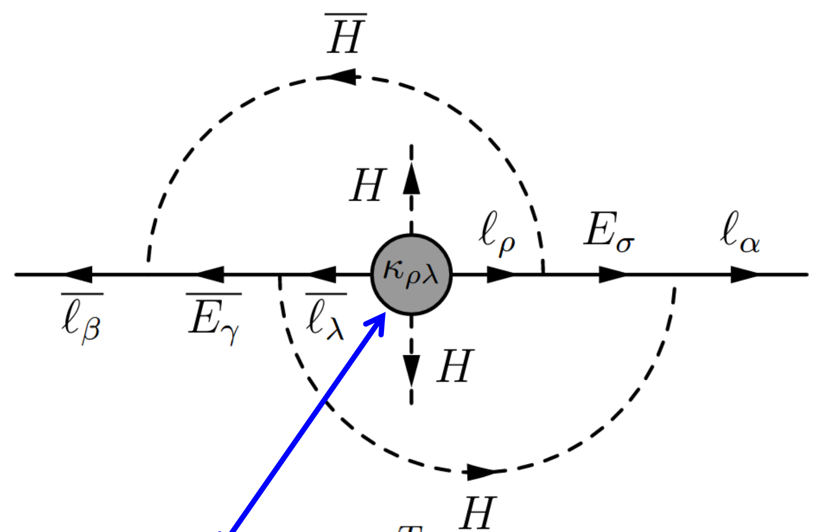
S. Weinberg
H. Fritzsch
F. Wilczek + A. Zee
1977

with $|B_f| \ll A_f$ **.**

A seesaw-like mass relation: $m_f \sim |B_f|^2/A_f$ **for the lightest fermion.**

The Fritzsch texture (double seesaw):

$$M_f = \begin{pmatrix} 0 & C_f & 0 \\ C_f^* & 0 & B_f \\ 0 & B_f^* & A_f \end{pmatrix} \Rightarrow m_f \sim A_f \frac{|C_f|^2}{|B_f|^2}$$



Example 2: two-loop renormalization-group running for zero ν-mass.

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha_\kappa \kappa - \frac{3}{2}\left[\left(Y_l Y_l^\dagger\right)\kappa + \kappa\left(Y_l Y_l^\dagger\right)^T\right] + \frac{1}{8\pi^2}\left(Y_l Y_l^\dagger\right)\kappa\left(Y_l Y_l^\dagger\right)^T$$

D. Zhang, ZZX 2005.05171

$$m_1 = 0 \text{ at } \Lambda \simeq 10^{14}\,\text{GeV} \Rightarrow m_1 \sim \mathcal{O}(10^{-13})\,\text{eV at } \Lambda \simeq 10^2\,\text{GeV}$$

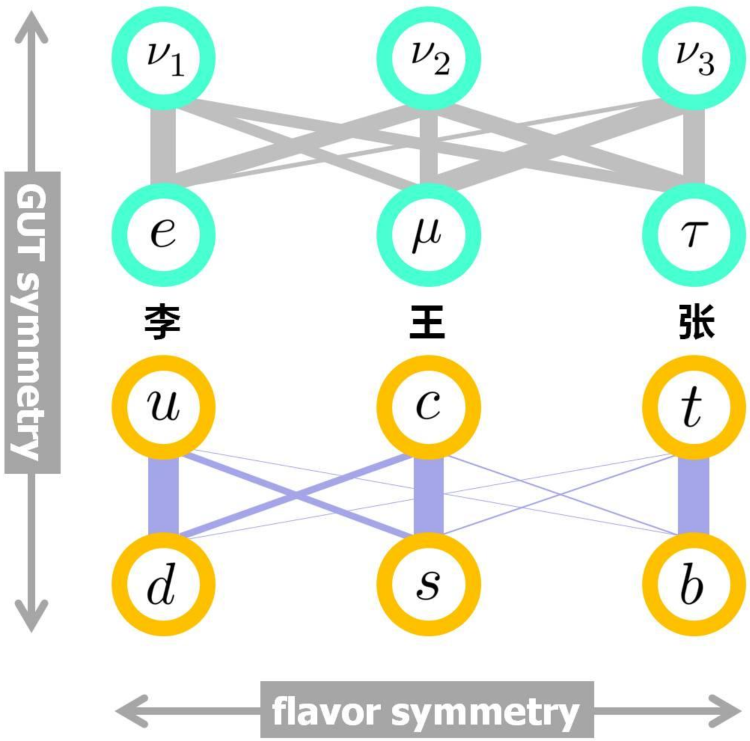
Y. Cai, et al: 1706.08524

From the trees to the forest: a review of radiative ν-mass models

Challenge from flavor mixing

Weinberg's approach doesn't help much in interpreting *flavor mixing*, which cannot be well understood unless the flavor structure is known.

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\left[\overline{\begin{pmatrix} u & c & t\end{pmatrix}_{\rm L}}\ \gamma^\mu\ V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + \overline{\begin{pmatrix} e & \mu & \tau\end{pmatrix}_{\rm L}}\ \gamma^\mu\ U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu\right] + {\rm h.c.}$$



♣ **I. Esteban et al (2007.14792):**

$$U = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.155 \\ 0.234 \to 0.500 & 0.471 \to 0.689 & 0.637 \to 0.776 \\ 0.271 \to 0.525 & 0.477 \to 0.694 & 0.613 \to 0.756 \end{pmatrix}$$

Model: a constant matrix + corrections.

♣ **Particle Data Group (2020):**

$$V = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

Model: the identity matrix + correction.

ZZX, Phys. Rept. 854 (2020) 1—147

No success !

Flavor structures of charged fermions and massive neutrinos

OUTLINE

- ♣ **Weinberg's concerns and attempts**

- ♣ **If a fermion looks like a Goldstone**

Translation of a neutrino field

Volume 46B, number 1 PHYSICS LETTERS 3 September 1973

1973

citations > 1900

IS THE NEUTRINO A GOLDSTONE PARTICLE?

D.V. VOLKOV and V.P. AKULOV

Physico-Technical Institute, Academy of Sciences of the Ukrainian SSR, Kharkov 108, USSR

Received 5 March 1973

Using the hypotheses, that the neutrino is a goldstone particle, a phenomenological Lagrangian is constructed, which describes an interaction of the neutrino with itself and with other particles.

Recently much attention has been paid in the elementary particle physics to the problem of spontaneously broken symmetries and the related degeneracy of the vacuum state. An immediate consequence of the vacuum degeneracy is that it gives rise to a possible existence of zero mass particles, the so-called Goldstone particles [1].

Among known elementary particles only the neutrino, the photon and the graviton have zero masses. However, the last two correspond to the gauge fields and do not require the vacuum degeneracy for their existence. Therefore the neutrino is the only elementary particle the existence of which may be immediately related to the vacuum degeneracy.

For the determination of the type of spontaneously broken symmetry that causes the degeneracy of the vacuum and the corresponding properties of the neutrino as a Goldstone particle, let us consider the equation for a free neutrino

$$i\sigma_\mu \, \partial\psi/\partial x_\mu = 0 \qquad (1)$$

Eq. (1) is invariant under transformations of the Poincaré group and the chiral transformations as well as under translations in the spinor space, i.e. under the transformations of the type

$$\psi \to \psi' = \psi + \zeta \qquad x_\mu \to x'_\mu = x_\mu , \qquad (2)$$

如何理解？

where ζ is a constant spinor, anticommuting with ψ.

Translation of a scalar field

Proceedings of the NATO Advanced Study Institute on Recent Developments in Gauge Theories, held in Cargèse, Corsica, August 26–September 8, 1979.

1979

Gerard 't Hooft: Naturalness, chiral symmetry & spontaneous chiral symmetry breaking

– at any energy scale μ, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = o$ would increase the symmetry of the system.

A renormalizable scalar field theory is described by the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 \,. \qquad \text{(III3)}$$

There are two parameters, λ and m. Of these, λ may be small because $\lambda = o$ would correspond to a non-interacting theory with total number of ϕ particles conserved. But is small m allowed? If we put m = o in the Lagrangian (III3) then the symmetry is not enhanced*). However we can take both m and λ to be small, because if $\lambda = m = o$ we have invariance under

$$\phi(x) \rightarrow \phi(x) + \Lambda \,. \qquad \text{(III5)}$$

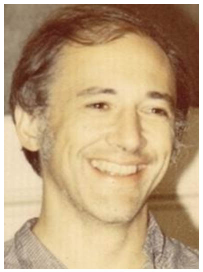
如何理解？

This would be an approximate symmetry of a new underlying theory

Translation of 3 neutrino fields

第 30 卷 第 7 期
2006 年 7 月
高 能 物 理 与 核 物 理
HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS
Vol. 30, No. 7
Jul., 2006

A Possible Relation between the Neutrino Mass Matrix and the Neutrino Mapping Matrix



R. Friedberg[1] T. D. Lee[1,2]

1 (Physics Department, Columbia University, New York, NY 10027, U.S.A.)

2 (China Center of Advanced Science and Technology(CCAST/World Lab.), Beijing 100080, China)

Abstract We explore the consequences of assuming a simple 3-parameter form, first without T-violation, for the neutrino mass matrix M in the basis ν_e, ν_μ, ν_τ with a new symmetry. This matrix determines the three neutrino masses m_1, m_2, m_3, as well as the mapping matrix U that diagonalizes M. Since U, without T-violation, yields three measurable parameters s_{12}, s_{23}, s_{13}, our form expresses six measurable quantities in terms of three parameters, with results in agreement with the experimental data. More precise measurements can give stringent tests of the model as well as determining the values of its three parameters. An extension incorporating T-violation is also discussed.

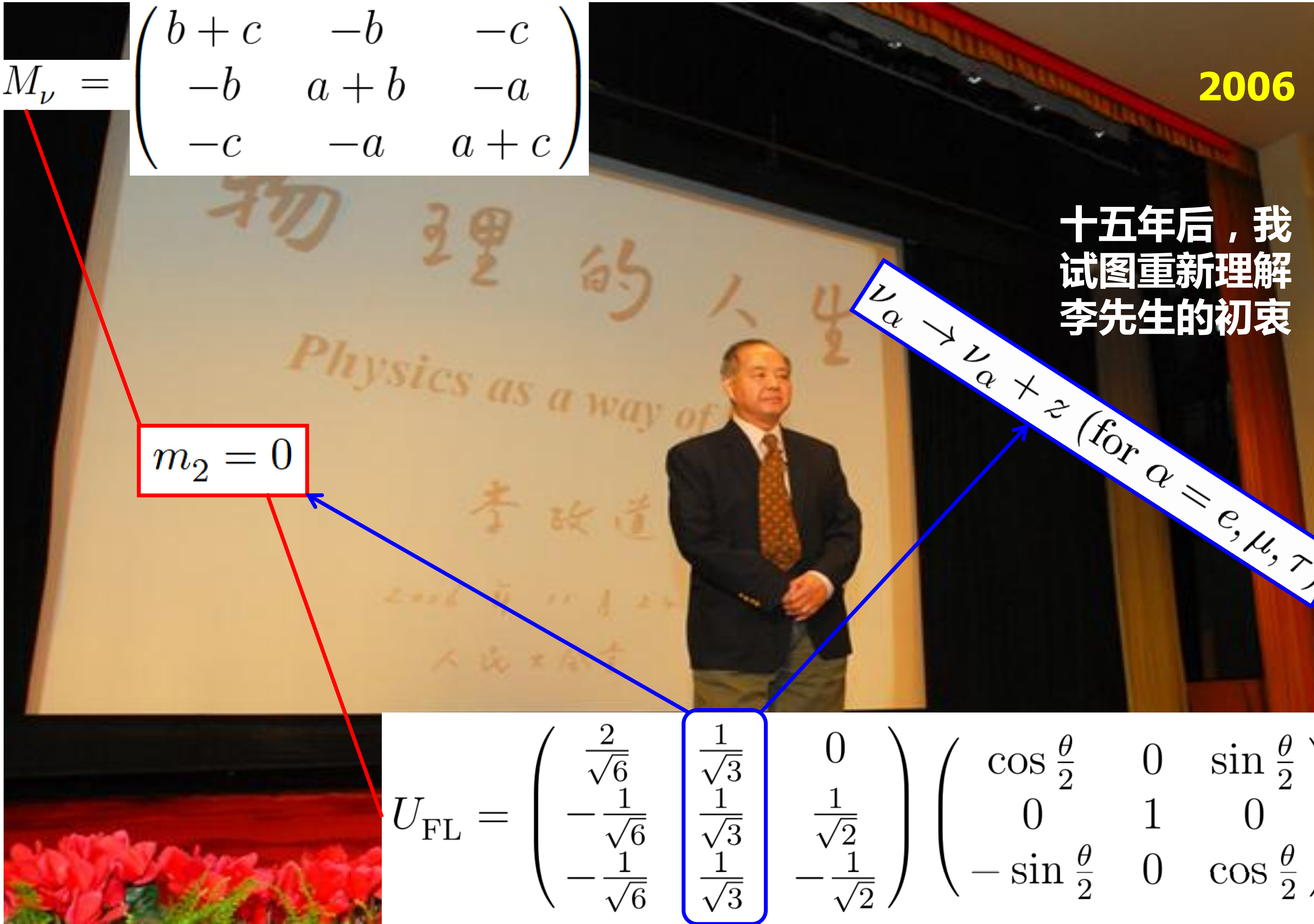
$$\mathcal{L}_{\nu-\text{mass}} = a\left(\overline{\nu}_\tau - \overline{\nu}_\mu\right)\left(\nu_\tau - \nu_\mu\right) + b\left(\overline{\nu}_\mu - \overline{\nu}_e\right)\left(\nu_\mu - \nu_e\right) + c\left(\overline{\nu}_e - \overline{\nu}_\tau\right)\left(\nu_e - \nu_\tau\right)$$

keeps invariant under the transformation: $\nu_\alpha \rightarrow \nu_\alpha + z \ (\text{for } \alpha = e, \mu, \tau)$

T.D. Lee's 80th birthday

$$M_\nu = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}$$

$$m_2 = 0$$



2006

十五年后，我
试图重新理解
李先生的初衷

$$\nu_\alpha \to \nu_\alpha + z \ (\text{for } \alpha = e, \mu, \tau)$$

$$U_{\rm FL} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & 0 & \sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix}$$

A zero mass limit is OK?

★ **A global fit on neutrino masses: I. Esteban et al (arXiv:2007.14792)**

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

★ **Neutrinos: the lightest neutrino can be exactly massless at the tree level**

★ **Charged leptons: the electron mass is so small that it approximates to zero**

★ **Quarks: u- and d-quark are light; $m_u=0$ would solve the strong CP problem**

My exercise: Majorana neutrinos

★ **The effective Majorana neutrino mass term:**

$$-\mathcal{L}_{\rm M} = \frac{1}{2}\sum_{\alpha}\sum_{\beta}\left[\overline{\nu_{\alpha\rm L}}\,\langle m\rangle_{\alpha\beta}\,(\nu_{\beta\rm L})^{c}\right] + {\rm h.c.}$$

$$U^{\dagger} M_{\nu} U^{*} \; = \; {\rm diag}\{m_1, m_2, m_3\}$$

$$\langle m\rangle_{\alpha\beta} = \sum_{i}\left(m_i U_{\alpha i} U_{\beta i}\right)$$

$$\sum_{\alpha}\left[U^{*}_{\alpha j}\langle m\rangle_{\alpha\beta}\right] = m_j U_{\beta j} \; ,$$

$$\sum_{\beta}\left[\langle m\rangle_{\alpha\beta} U^{*}_{\beta j}\right] = m_j U_{\alpha j} \; ,$$

$$\sum_{\alpha}\sum_{\beta}\left[U^{*}_{\alpha j}\langle m\rangle_{\alpha\beta} U^{*}_{\beta j}\right] = m_j$$

★ **Make a translational transformation for the left-handed neutrino fields in the flavor space**

$$\nu_{\alpha\rm L} \to \nu_{\alpha\rm L} + U_{\alpha j} z_{\nu}$$

spacetime- and flavor-independent element of the Grassmann algebra

★ **Then the Majorana mass term will keep invariant if** $m_j = 0$ **holds.**

$$-\mathcal{L}^{\prime}_{\rm M} = -\mathcal{L}_{\rm M} + \frac{1}{2}m_j\left[\overline{z_{\nu}}\,z^{c}_{\nu} + \sum_{\alpha}\left[U_{\alpha j}\overline{\nu_{\alpha\rm L}}\right] z^{c}_{\nu} + \overline{z_{\nu}}\sum_{\beta}\left[U_{\beta j}\left(\nu_{\beta\rm L}\right)^{c}\right]\right]$$

★ **So we arrive at a general correlation between a zero neutrino mass and a proper column of the neutrino mixing matrix. It is now possible to have** $m_1 = 0$ **or** $m_3 = 0$ **for viable flavor mixing (ZZX, 2102.03050).**

My exercise: Dirac fermions

★ **The Dirac fermion mass term (in the Hermitian basis):**

$$-\mathcal{L}_{\rm D} = \sum_\alpha \sum_\beta \left[\overline{\psi_{\alpha \rm L}} \, \langle m \rangle_{\alpha\beta} \, \psi_{\beta \rm R}\right] + \text{h.c.}$$

$$V^\dagger M_\psi V = \text{diag}\{\lambda_1, \lambda_2, \lambda_3\}$$

$$\langle m \rangle_{\alpha\beta} = \langle m \rangle^*_{\beta\alpha} = \sum_i \left(\lambda_i V_{\alpha i} V^*_{\beta i}\right)$$

★ **Make a translational transformation for left- & right-handed fermion fields:**

$$\psi_{\alpha \rm L(R)} \to \psi_{\alpha \rm L(R)} + n_\alpha z_\psi$$

★ **The Dirac mass term will be invariant under the condition:**

$$\left\{ \begin{array}{l} \sum_\alpha \left[n^*_\alpha \langle m \rangle_{\alpha\beta}\right] = 0 \, , \\ \sum_\beta \left[\langle m \rangle_{\alpha\beta} \, n_\beta\right] = 0 \, , \\ \sum_\alpha \sum_\beta \left[n^*_\alpha \langle m \rangle_{\alpha\beta} \, n_\beta\right] = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \sum_i \left[\lambda_i \sum_\alpha \left(n^*_\alpha V_{\alpha i}\right) V^*_{\beta i}\right] = 0 \, , \\ \sum_i \left[\lambda_i V_{\alpha i} \sum_\beta \left(V^*_{\beta i} \, n_\beta\right)\right] = 0 \, , \\ \sum_i \left[\lambda_i \sum_\alpha \left(n^*_\alpha V_{\alpha i}\right) \sum_\beta \left(V^*_{\beta i} \, n_\beta\right)\right] = 0 \end{array} \right.$$

★ **The nontrivial solution is**

$$\sum_\alpha \left(n^*_\alpha V_{\alpha i}\right) = \sum_\beta \left(V^*_{\beta i} \, n_\beta\right) = 0 \Rightarrow \left\{ \begin{array}{l} n_\beta \propto V_{\beta j} \\ \lambda_j = 0 \end{array} \right.$$

★ **So we arrive at the correlation thanks to the symmetry.**

It remains very preliminary

★ **Big shots have considered a possible translational symmetry in the neutrino or scalar sector, my attempts are to highlight this possibility and extend it to all the fundamental fermions.**

★ **Two obvious obstacles in this connection: 1) this symmetry is only imposed on the effective mass term instead of the whole Lagrangian; 2) symmetry breaking is unclear and maybe arbitrary (like others).**

★ **Excuse: new and even seemingly exotic ideas are always called for in order to pin down the true flavor dynamics, in view of the fact that those popular approaches do not help much either.**

★ **Michael Duff theorem (1993): some cynic said in order for physicists to accept a new idea, they must first pass through the following three stages:**

It's wrong	It's trivial	I thought of it first

THANKS A LOT