The Story Of Neutrinos And The Secretes They Carry
中微子的故事和它们携带的秘密

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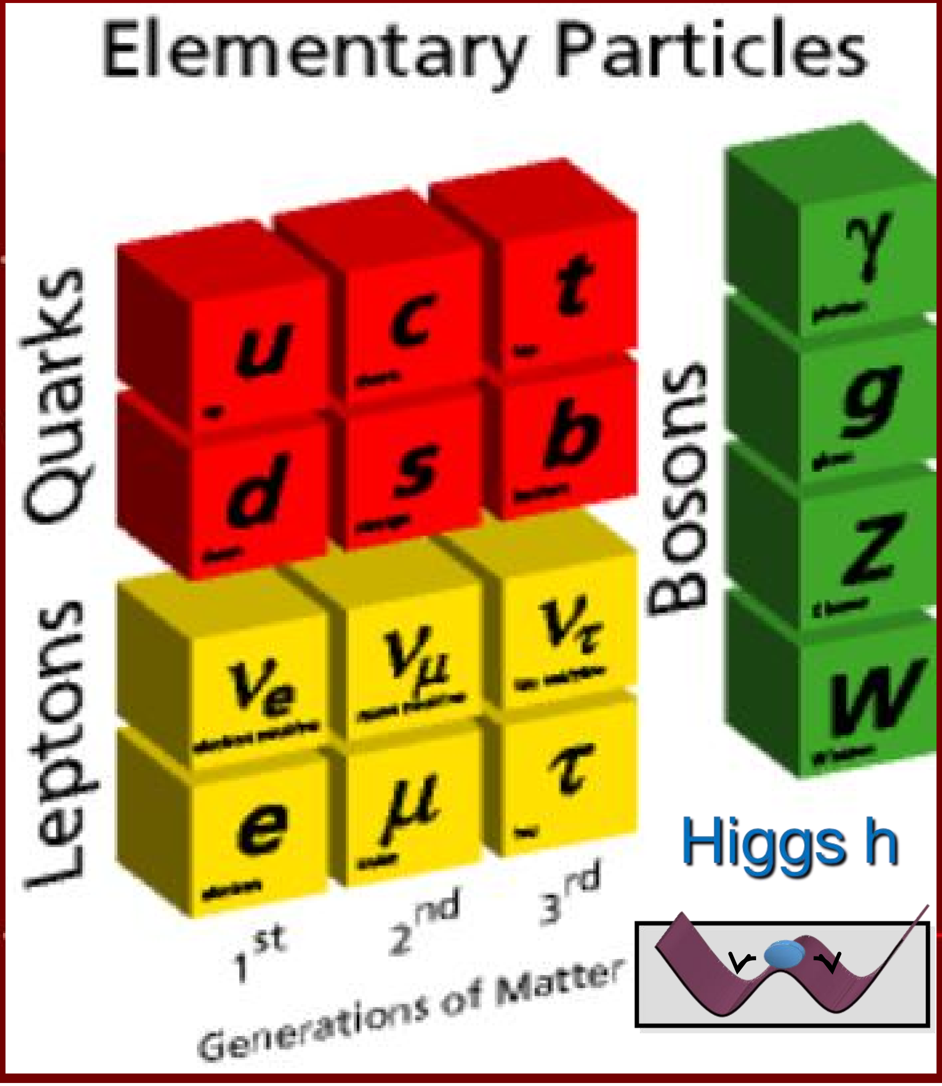
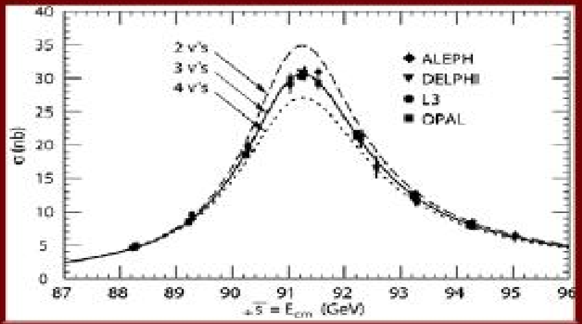
Colloquium at the Cross Interdisplenary Center of USTC
中科大交叉学科理论研究中心&彭桓武高能基础理论研究中心学术报告
10, November, 2023

1. A Brief History Of Neutrino
2. Oscillation, Mass, Mixing, Dirac Or Majorana
3. Neutrinos In Cosmology And Astrophysics

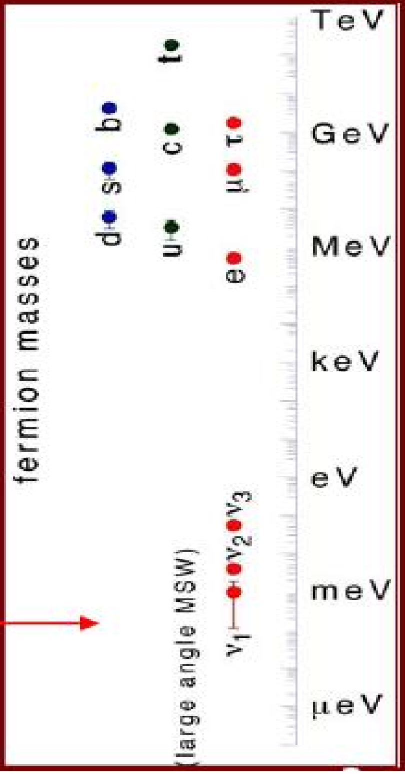
1. A Brief History Of Neutrino

Nature's building blocks





Three flavors or generations, and no more, and we do not know why.



Some mass, but curiously little.

The Surprising Discovery Of Radioactivity

November 1895 :William Roentgen discovers X-rays from Cathode Ray Tubes (Wuerzburg)

- January 1896: Henri Becquerel (Paris) hears about the X-rays in a lecture by Poincare

- February 1896: H.B. discovers Radioactivity(in Uranium ore) while trying to find natural sources for X-rays!

Beta decay, the birth of moerden weak interaction

In 1899, Ernest Rutherford separated radioactive emissions into two types: alpha and beta (now beta minus), based on penetration of objects and ability to cause ionization.

In 1900, Paul Villard identified a still more penetrating type of radiation, which Rutherford identified as a fundamentally new type in 1903 and termed gamma rays.

In 1900, Becquerel measured the mass-to-charge ratio (*m*/*e*) for beta particles by the method of J.J. Thomson used to study cathode rays and identify the electron. He found that *m*/*e* for a beta particle is the same as for Thomson's electron, and therefore suggested that the beta particle is in fact an electron.

The Surprising Energy Conservation Crisis

Puzzle with Beta Spectrum

- Three-types of radioactivity: α, β, γ
- Both α, γ discrete spectrum because $E_{\alpha,\gamma} = E_i - E_f$
- But β spectrum continuous

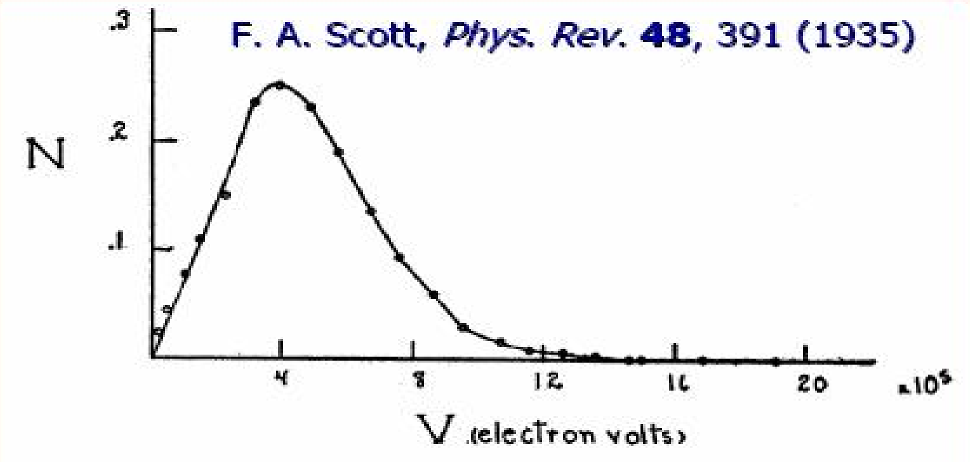
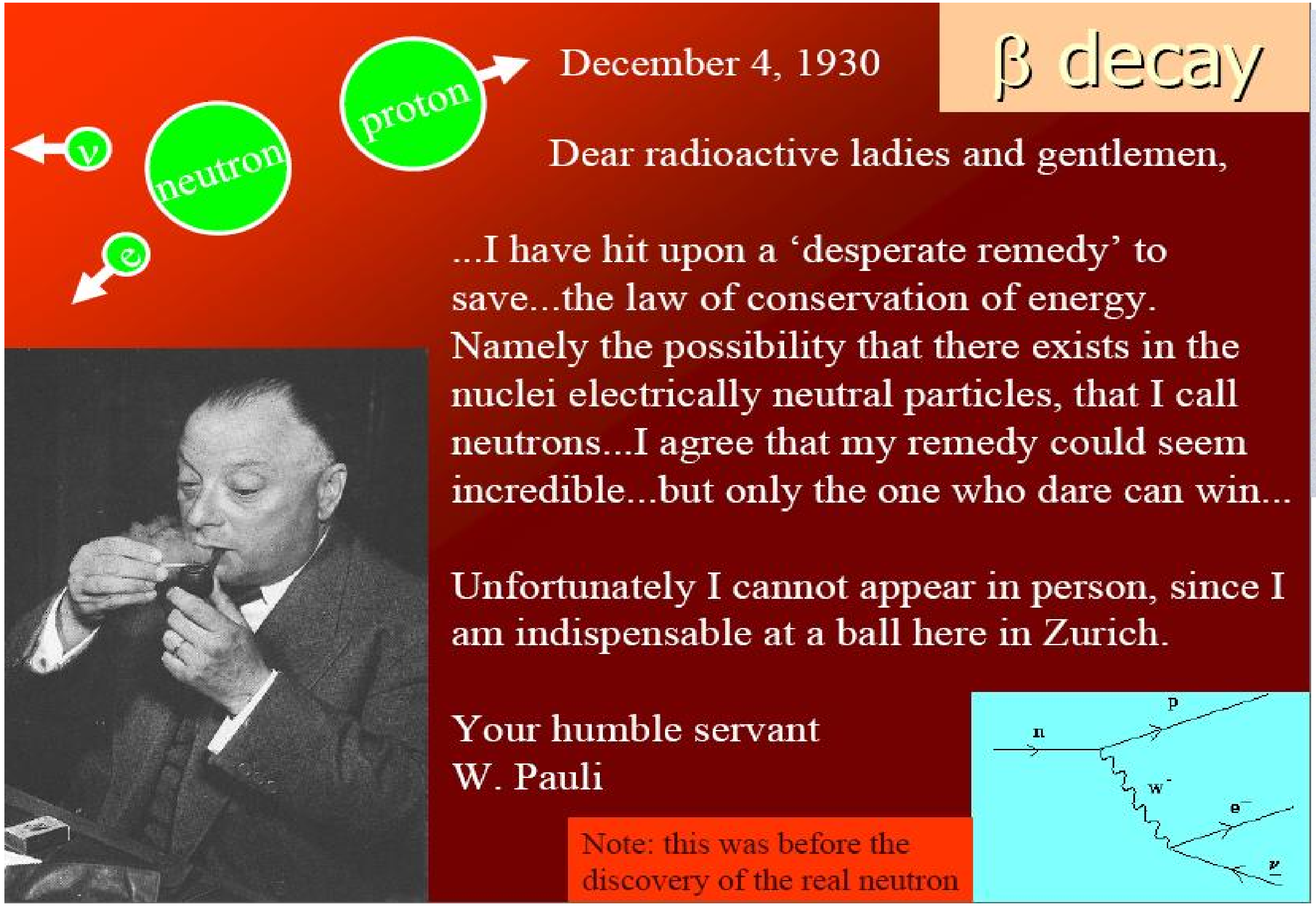


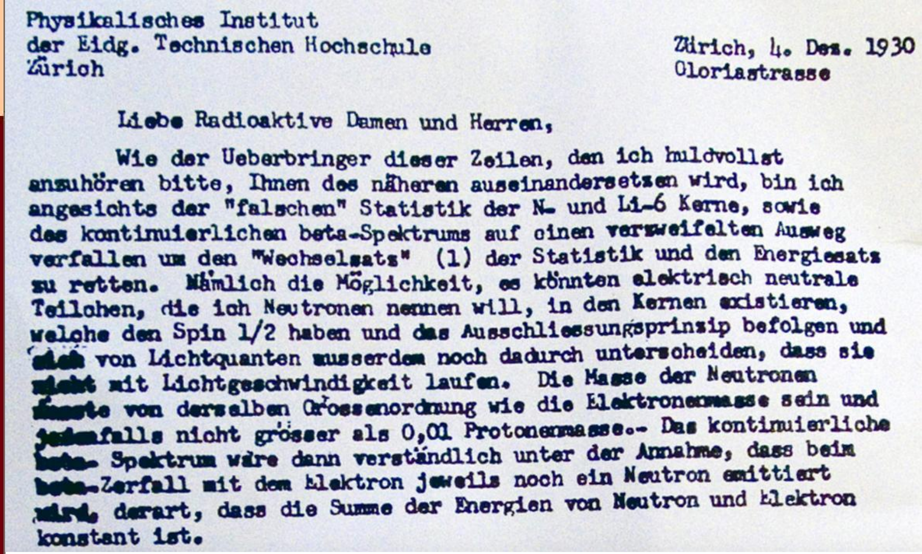
FIG. 5. Energy distribution curve of the beta-rays.

Bohr: *At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β-ray disintegrations*

Neutrino Hypothesis

A weakly interacting particle in weak interaction, no EM and strong interactions



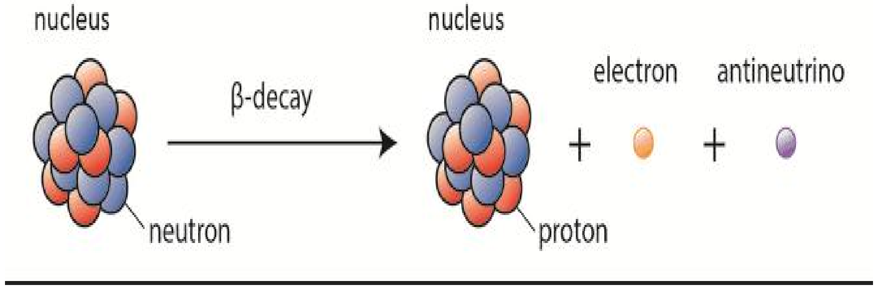


Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.



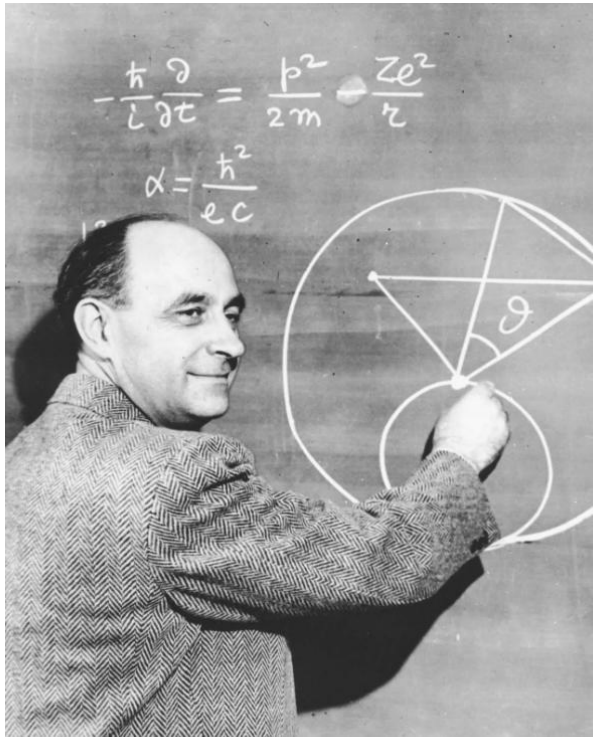
Theory Of Neutrino Interaction

December 1933: Enrico Fermi

submits a paper to Nature: "Tentativo di Una Teoria Della Emissione di raggi Beta"

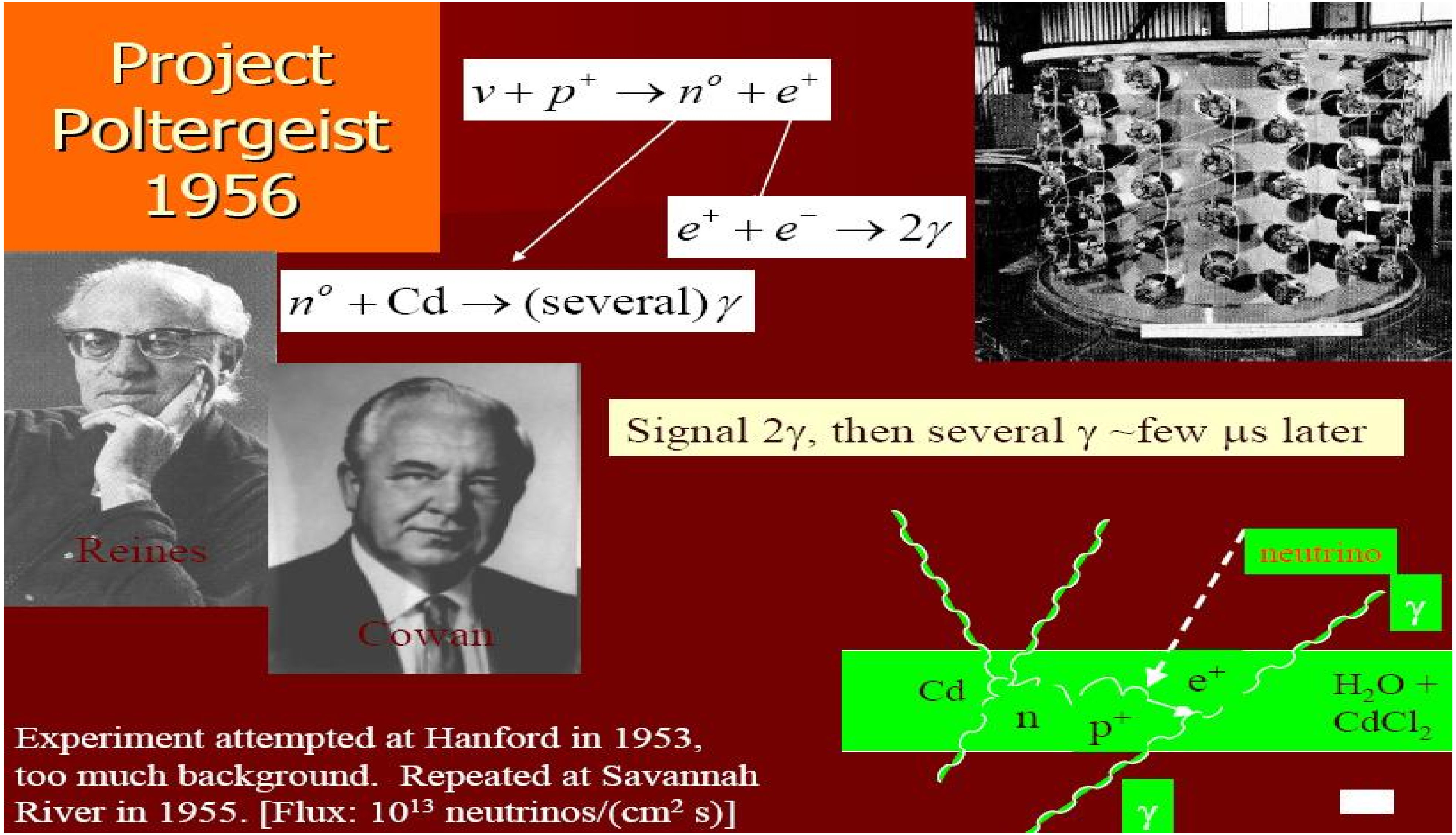
It was rejected:"speculations too remote from reality to be of interest to the readers".

Eventually published in Nuovo Cimento This paper laid out the essential theory of beta decay that has survived almost unchanged until now, with "small" modifications. It predicted the spectrum, obtained the correct value for the coupling etc………



$$G(\bar{\psi}_p \gamma_\mu \psi_n)(\bar{\psi}_e \gamma^\mu \gamma_5 \psi_\nu)$$

Three Types Of Neutrinos



Universality And Left-handedness

Interaction of Mesons with Nucleons and Light Particles

T. D. Lee, M. Rosenbluth, and C. N. Yang
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
January 7, 1949

We have been making a phenomenological study of the various experiments which have been done in recent years on the interaction between the various types of particles. In the course of this investigation two interesting points have come to light.

First, we found that if the decay of the μ-mesons and the capture of the μ^--mesons by nuclei are described by the reactions[1]

$$\mu \rightarrow e + \nu + \nu \qquad (e = \text{electron},\ \nu = \text{neutrino})$$
$$\mu^- + P \rightarrow N + \nu \qquad (P = \text{proton},\ N = \text{neutron}),$$

and that the Fermi type interactions are assumed to be responsible for these processes, the coupling constants would have the values

$$g_{\mu\nu} \sim 3 \times 10^{-49} \text{ erg cm}^3$$

and

$$g_{\mu P} \sim 2 \times 10^{-49} \text{ erg cm}^3,$$

respectively. These values are so determined as to fit the experimental lifetime[2] of the μ-mesons and the capture probability of the μ^--mesons by nuclei.[3] It is remarkable that the three independent experiments: the β-decay of the nucleons and the μ-mesons and the interaction of the nucleons with the μ-mesons lead to coupling constants of the same order of magnitude.

One can perhaps attempt to explain the equality of these interactions in a manner analogous to that used for the Coulomb interactions, i.e. by assuming these interactions to be transmitted through an intermediate field with respect to which all particles have the same "charge." The "quanta" of such a field would have a very short lifetime and would have escaped detection.

Second, if we assume the π-mesons to have integral spin and assume direct couplings for the processes

$$\pi \rightarrow \mu + \text{anti } \nu$$
$$N \rightarrow P + \pi^-$$

with coupling constants determined from the lifetime of the π-mesons[4] and the strength of nuclear forces,[5] the interaction between the μ-mesons and the nucleons can be *quantitatively* explained as a second-order interaction through the virtual creation and annihilation of π-mesons.

After the completion of our work Mr. A. Ore has kindly informed us that similar considerations have been carried out by J. A. Wheeler and J. Tiomno.

[1] The masses of the π- and μ-mesons are taken to be

$$m_\pi = 286 m_e, \qquad m_\mu = 212 m_e.$$

[2] B. Rossi, Rev. Mod. Phys. 20, 537 (1948).
[3] B. Rossi, Rev. Mod. Phys. 20, 537 (1948). In the calculation for the capture process the Fermi model for the nucleus is assumed and only single particle excitations are considered. See M. Rosenbluth, Phys. Rev. 75, 532 (1949).
[4] J. R. Richardson, Phys. Rev. 74, 1720 (1948).
[5] H. Bethe, Phys. Rev. 57, 390 (1940).

PHYSICAL REVIEW VOLUME 105, NUMBER 5 MARCH 1, 1957

Parity Nonconservation and a Two-Component Theory of the Neutrino

T. D. Lee, *Columbia University, New York, New York*
AND
C. N. Yang, *Institute for Advanced Study, Princeton, New Jersey*
(Received January 10, 1957; revised manuscript received January 17, 1957)

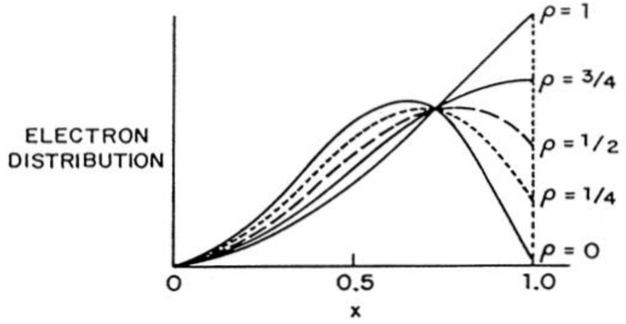
A two-component theory of the neutrino is discussed. The theory is possible only if parity is not conserved in interactions involving the neutrino. Various experimental implications are analyzed. Some general remarks concerning nonconservation are made.

Neutrinos are Left-handed

Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar
Brookhaven National Laboratory, Upton, New York
(Received December 11, 1957)

A combined analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] $0-$, we find that the neutrino is "left-handed," i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).



Universality properties: T.-D Lee, Rosenbruth and C.-N Yang ;

Independently by O. Klein;

G. Puppi, J. Tiomno and J. Wheeler.

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Active neutrinos are left-handed particles.

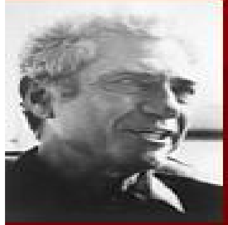
Use of Neutrino As Probes for New Physcis

FEASIBILITY OF USING HIGH-ENERGY NEUTRINOS TO STUDY THE WEAK INTERACTIONS

M. Schwartz*
Columbia University, New York, New York
(Received February 23, 1960)

THEORETICAL DISCUSSIONS ON POSSIBLE HIGH-ENERGY NEUTRINO EXPERIMENTS*

T. D. Lee
Columbia University, New York, New York
and
C. N. Yang
Institute for Advanced Study, Princeton, New Jersey
(Received February 23, 1960)



More than one kind of neutrino?

Date: 1962

Intent: Measure weak force at high energies

Expectation: Since neutrinos are created with muons and electrons, the neutrino beam should create both electrons and muons in the detector.

Result: No electrons produced, only muons

Conclusion: There must be two kinds of neutrinos.

Observation of the third neutrino FermiLab 2000

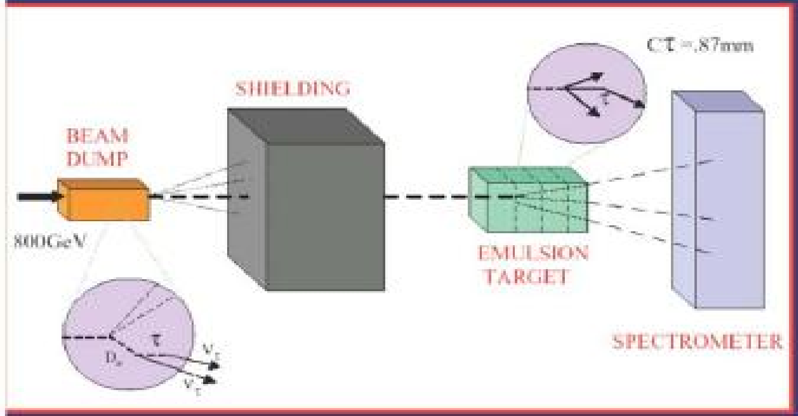
ν_τ discovery: year 2000

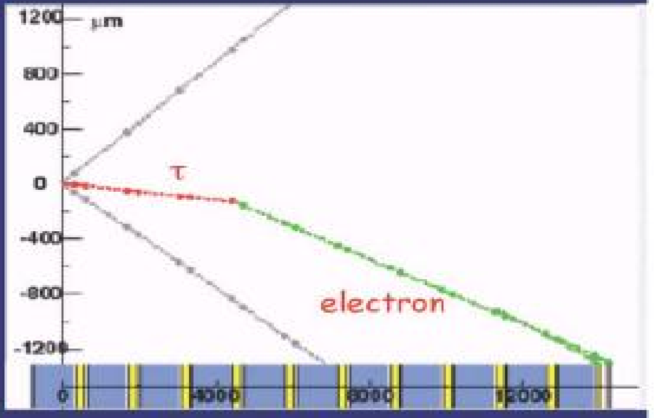
DONUT experiment at Fermilab

Protons → target → X + D_s

↳ τ + ν_τ → detector → τ

↳ X + ν_τ → detector → τ





There are three types of neutrinos!

Neutrino Mixing

B. Pontecorvo (1957). "Mesonium and anti-mesonium". *Zh. Eksp. Teor. Fiz.* 33: 549–551.

Oscillation between neutrino and anti-neutrino.

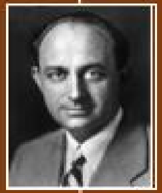
Z. Maki, M. Nakagawa, and S. Sakata (1962). "Remarks on the Unified Model of Elementary Particles". *Progress of Theoretical Physics* 28 (5): 870.
B. Pontecorvo (1967). "Neutrino Experiments and the Problem of Conservation of Leptonic Charge". *Zh. Eksp. Teor. Fiz.* 53: 1717.

Oscillations between different flavors.

1930

1930, Pauli postulated light neutral particle to save energy conservation in beta decay

1933



1933, Fermi developed theory of beta decay. Christened light neutral particles “Neutrino”

1953-59

1953-59 Reines and Cowan discovered anti-electron neutrino, Nobel Prize 1995

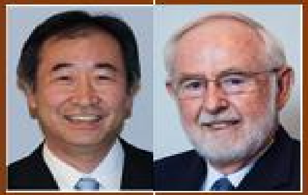
1962

1962, Lederman, Steinberger, Schwatz discovered muon neutrino, 1988 Nobel Prize

1960-90

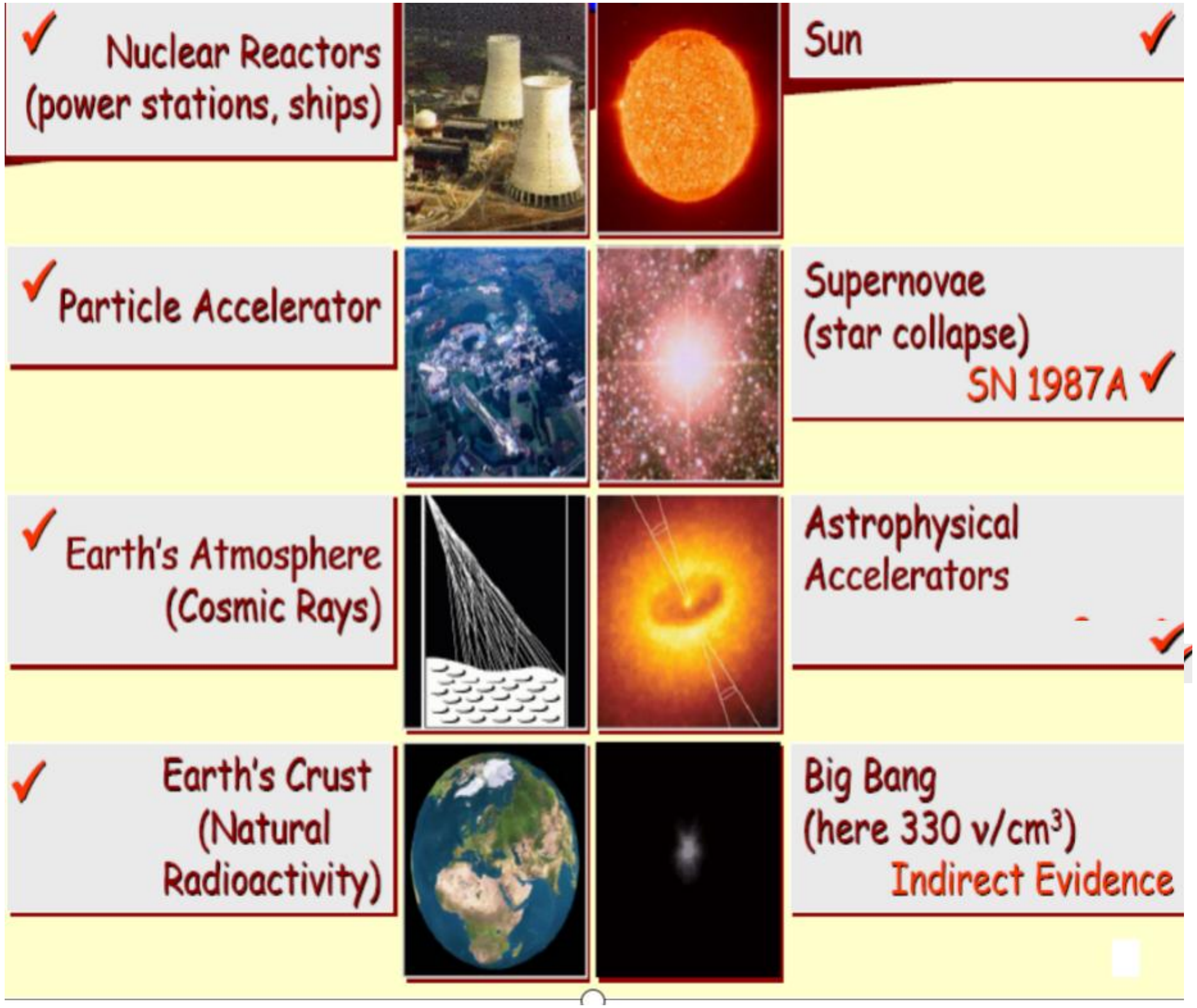
1960-90 Davis, 小柴昌俊 discovered cosmic neutrino, Nobel Prize 2002

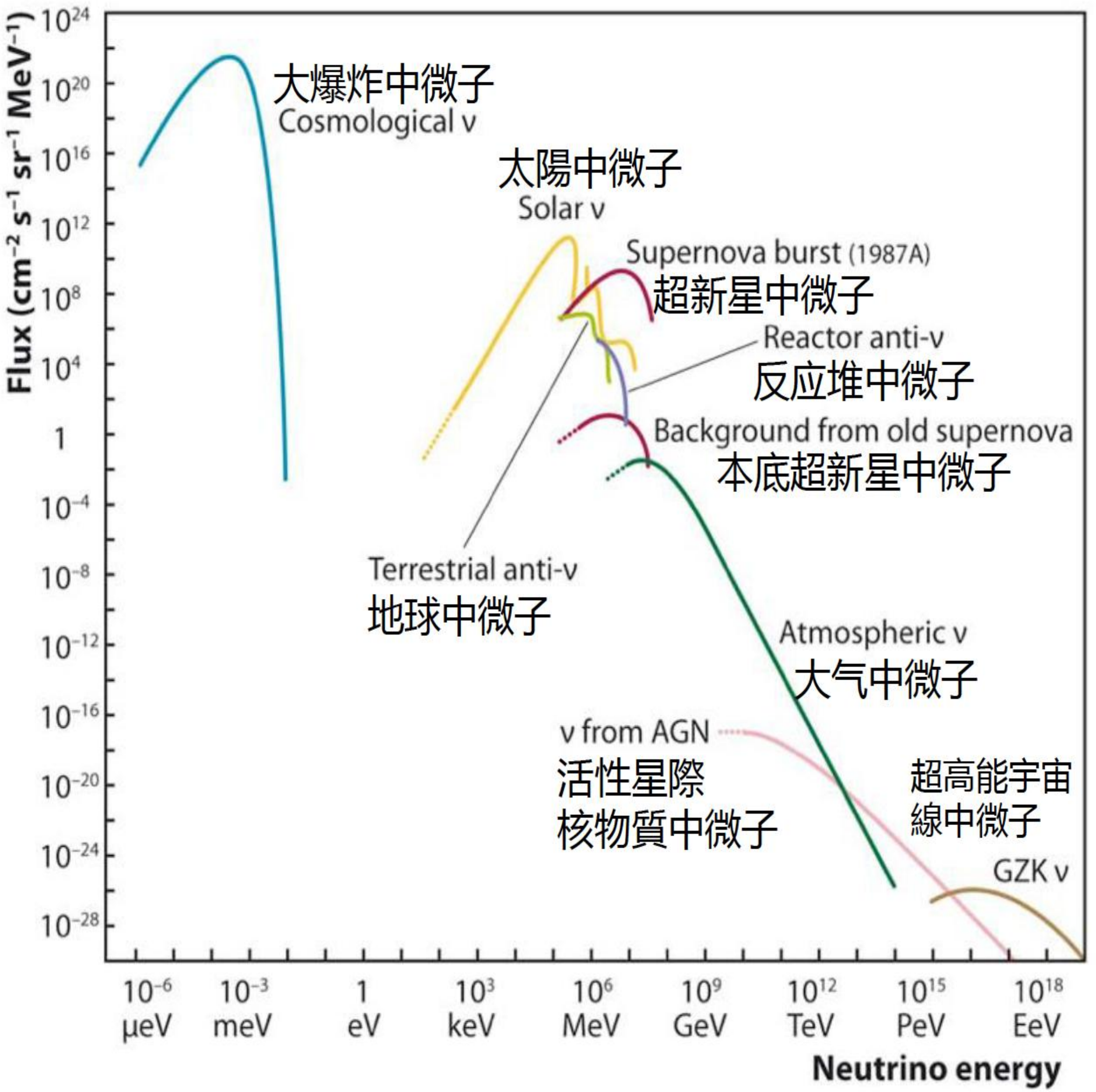
1998-2001



1998-2001,梶田隆章, McDonald discovered neutrino oscillation, 2015 Nobel Prize

Where neutrinos come from ?





Although weakly interacting, can be seen almost everywhere!

2. Oscillation, Masses, Mixing, Dirac Or Majorana

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
& G(8,1)\ (0), W(1,3)\ (0), B\ (1,1)(0)\ , \\
& Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} (3,2)(1/6)\ ,\ \ U_R\,(3,1)(2/3)\ ,\ \ D_R\,(3,1)(-1/3)\ , \\
& L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (1,2)(-1/2)\ ,\ \ E_R\,(1,1)(-1)\ , \\
& H = \begin{pmatrix} h^+ \\ (v+h^0)/\sqrt{2} \end{pmatrix} (1,2,1/2)\ ,\ \ \text{v - vev of Higgs}\ .
\end{aligned}
$$

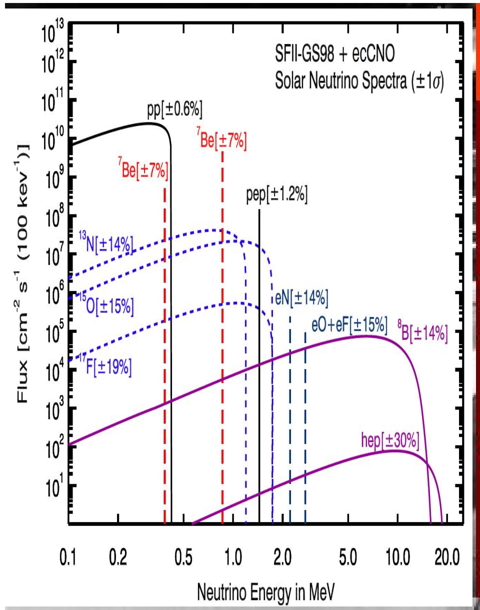
Quark and charged lepton masses are from the following Yukawa coupolings

$$\bar{Q}_L \tilde{H} U_R\ ,\ \ \bar{Q}_L H D_R\ ,\ \ \bar{L}_L H E_R\ .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

Experimental Discovery Of Nuetrino Mixing



Homestake Gold Mine

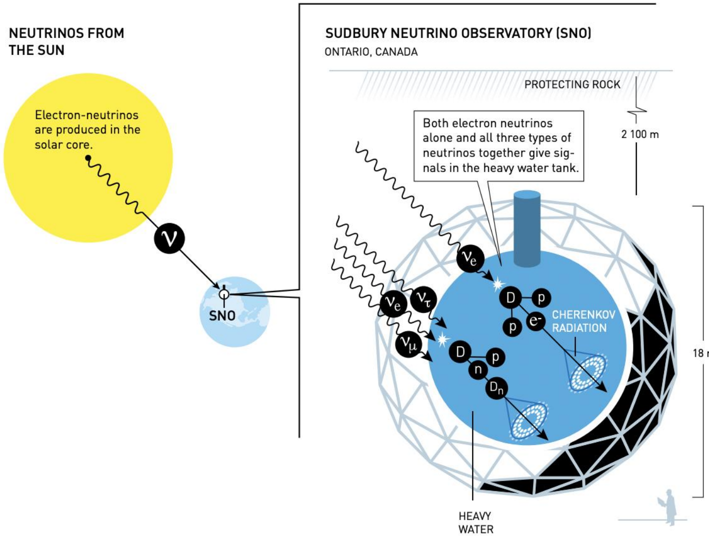
100,000 gallons of cleaning fluid C_2Cl_4

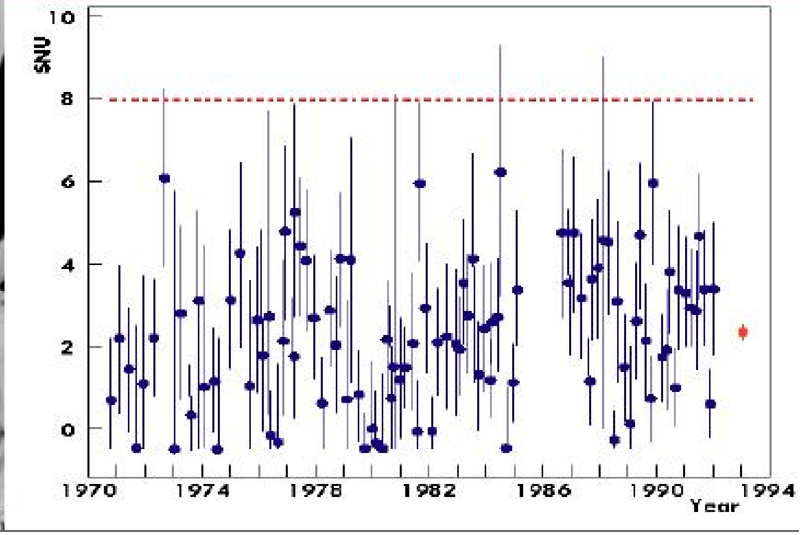
Expected 1.5 interactions per day
Measured 0.5 interactions per day

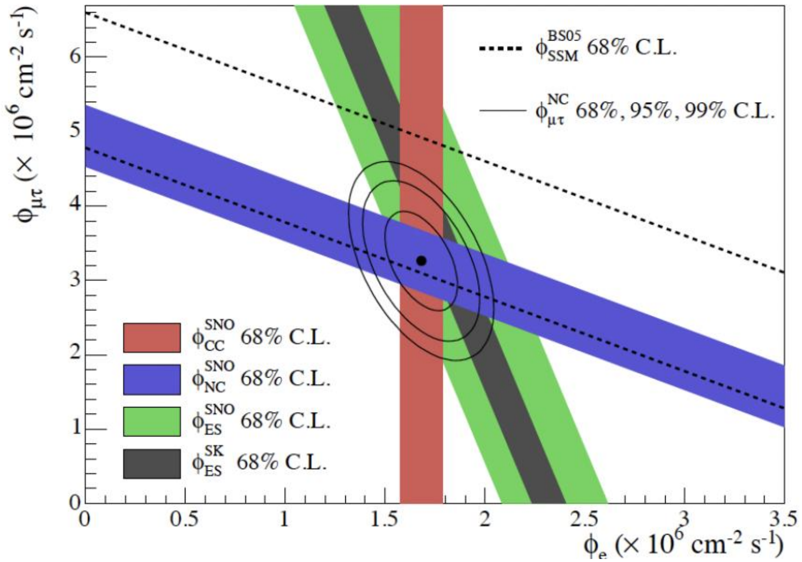
Sensitive to ^8B solar neutrinos only

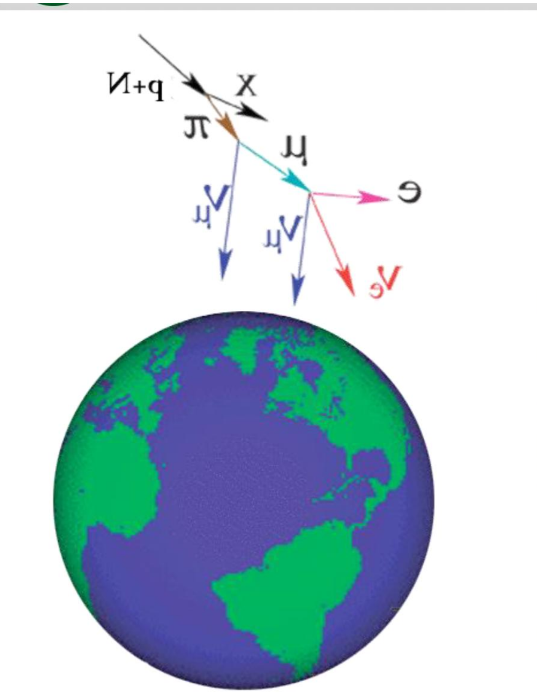
1968!

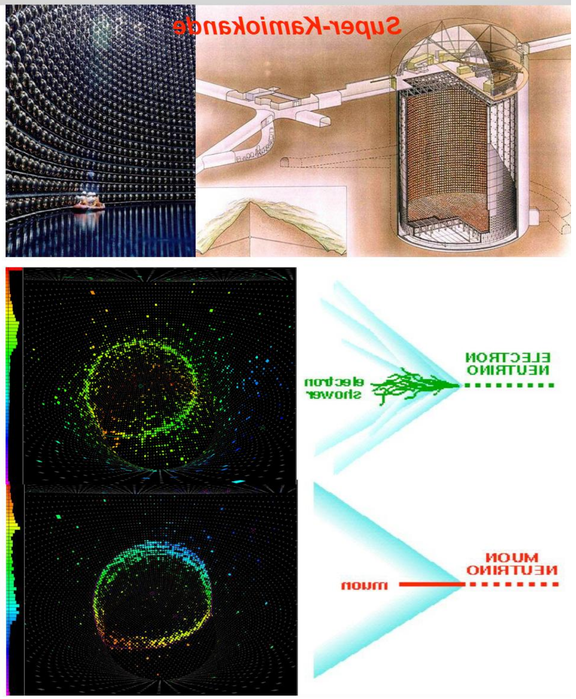
$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$

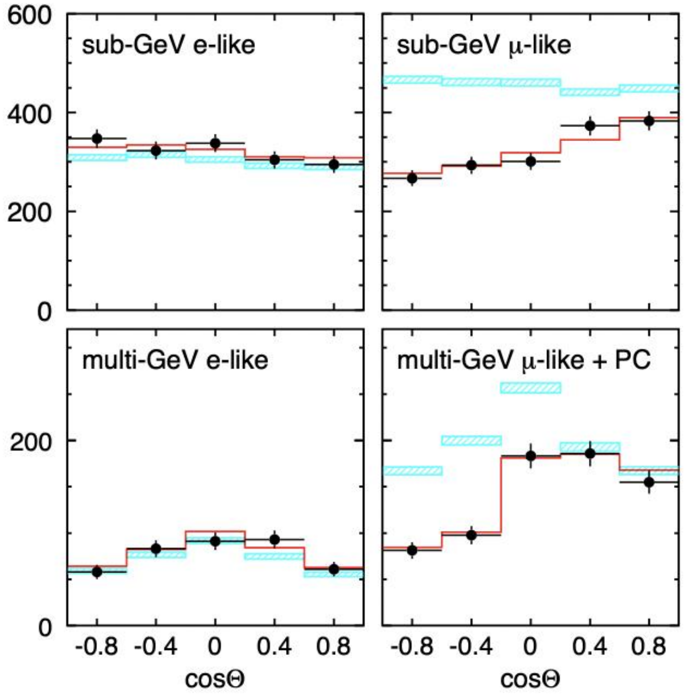












Evidence for Oscillation of Atmospheric Neutrinos

Y. Fukuda *et al.* (Super-Kamiokande Collaboration)
Phys. Rev. Lett. 81, 1562 —Published 24 August 1998

Measurement of the Rate of $\nu_e + d \rightarrow p + p + e^-$ Interactions Produced by ^8B Solar Neutrinos at the Sudbury Neutrino Observatory

Q. R. Ahmad *et al.* (SNO Collaboration)
Phys. Rev. Lett. **87**, 071301 – Published 25 July 2001

Abundant data show that neutrinos have non-zero masses and mix

Solar neutrino oscillation: Homestake, Sage+Gallex/GNO, Super-K, SNO,Borexino …

Atmospherical neutrino oscillation: Super-Kamokande, …

Accelerator neutrino source: K2K, Minos , Nova …

Reactor neutrino source: Kamland, T2K, Chooz, Daya-Bay, Reno…

have observed neutrino oscillation phenomenon.

LSND and Miniboon…?

Mixing and Non-zero Neutrino Masses

$$\begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

At t=0, neutrino $\nu_1(0)$ produced, at t, the state becomes

$$|\nu_1(0)> = \cos\theta e^{-i(Et-p_1 L)}|\nu_{m1}> - \sin\theta e^{-i(Et-p_2 L)}|\nu_{m2}>.$$

Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_1^2/2E$,
the probability amplitude of find $\nu_2(0)$ at time t is given by

$$<\nu_2(0)|\nu_1(t)> = \cos\theta \sin\theta (e^{-im_1^2 L/2E} - e^{-im_2^2 L/2E})$$

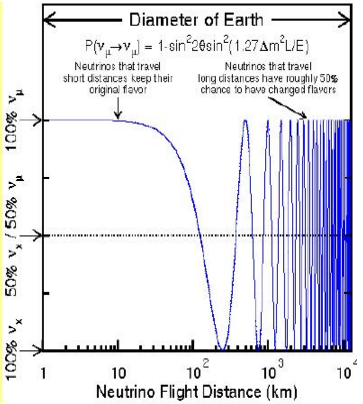
which leads to the probability of finding $\nu_2(0)$ is

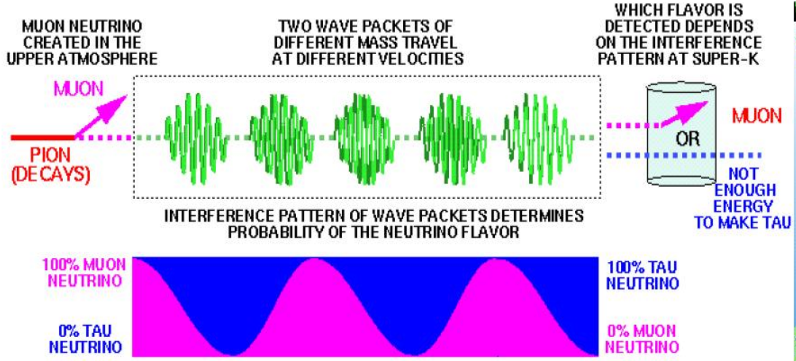
$$P(\nu_1 \to \nu_2) = |<\nu_1(0)|\nu_2(t)>|^2 = \sin^2(2\theta)\sin^2(\Delta m_{21}^2 L/4E),$$

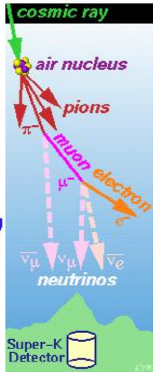
$$\Delta m_{21}^2 = m_2^2 - m_1^2.$$

$$P(\nu_e \to \nu_\mu) \cong \sin^2(2\theta)\sin^2[\frac{\Delta m^2}{4}\frac{L}{E}]$$

$$m_1 \neq m_2$$







Need mixing and non-zero mass to have oscillation!

Active neutrinos mix with each other and have non-zero masses!
Must go beyond SM to have neutrino masses!

How To Generate Masses For Neutrinos?

Neutrinos have small mass, they mix with each other

There may be additional light sterile neutrinos

Electrically neutral, has the possibility of being its own anti-particle, Majorana particle. Dirac or Majorana particle?

To have Dirac mass, need to introduce right handed neutrinos v_R: (1,1)(0)

Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C \;, \rightarrow \quad -\bar{\nu}_L m_\nu \nu_R \;, \rightarrow m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$m_{\nu_e} < 0.3$ eV, $\rightarrow \quad Y_{\nu_e}/Y_e < 10^{-5}$, very much fine tuned!

Majorana Masses For Neutrinos?

Neutrino is its own anti-particle, particle and anti-particle to pair up for mass term

The Seesaw Mechanism

$$L = \bar{\nu}_L (Y_\nu v/\sqrt{2}) \nu_R + \bar{\nu}_R^c M_R \nu_R / 2$$

The Seesaw mechanism refers to the neutrino mass matrix of the form

$$L_m = -\frac{1}{2} (\nu_L^c, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} .$$

For one generation, if $M_R >> m_D$, the eigenmasses are

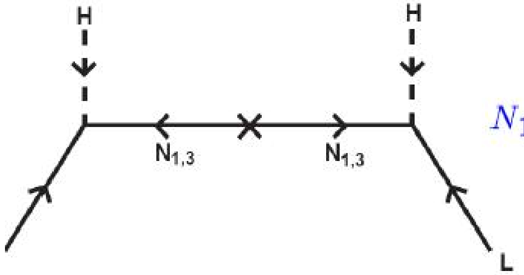
$$m_\nu \approx -m_D M_R^{-1} m_D^T, \quad m_N \approx M_R$$

A very nice way to explain why light neutrino masses are so much lighter than their charged lepton partners.

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

Majorana Neutrinos and Seesaw Mechanism

Type (I,III) seesaw

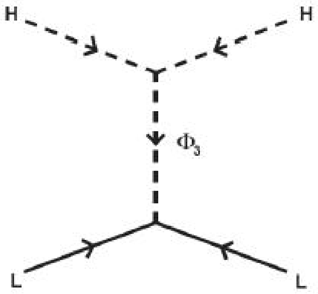


$N_1 : (1, 1, 0)$, $N_3 : (1, 3, 0)$

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, & Slansky (1980)
Mohapatra & Senjanovic (1980)

Foot, Lew, He, & Joshi (1989)

Type II seesaw

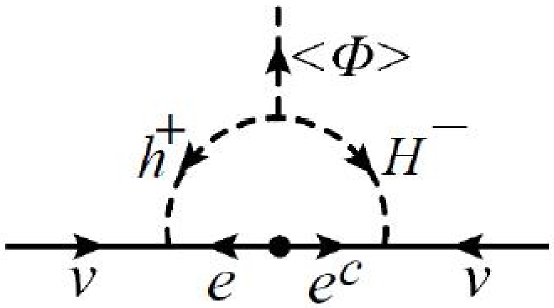


$\Phi_3 : (1, 3, +1)$

Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)

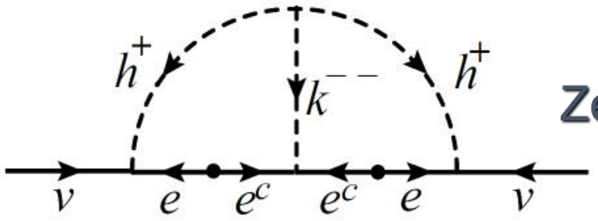
$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \Rightarrow m_\nu \sim \frac{v^2}{M}$$

Loop generation neutrino masses



Zee, 1980

Simplest model ruled out
XG He, EPJC34(2004)371



Zee, 1985, Babu 1988

Provide understand why neutrino masses are small
but no information about mixing!

Three Generations and Their Mixing

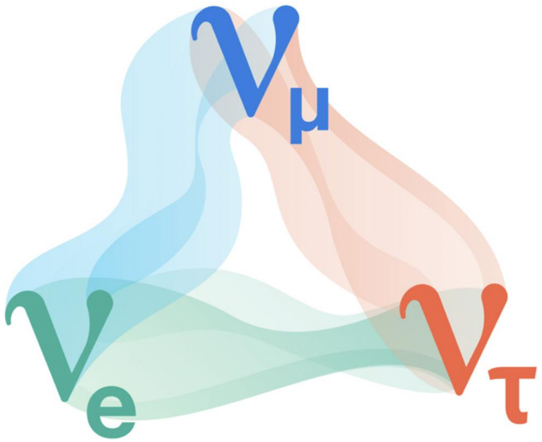
Quark mixing the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo-Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$

$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\,,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n \times n$ unitary matrix.



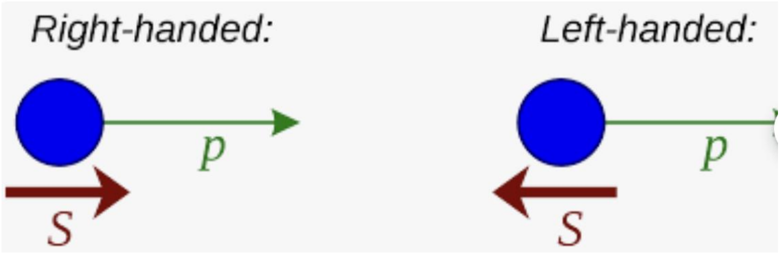
A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Neutrino Chiral Oscillation



Equation of motion for a neutrino in free space

$$(i\partial\!\!\!/ - m)\psi = 0\,, \quad i\partial\!\!\!/\psi_L - m\psi_R = 0\,, \quad i\partial\!\!\!/\psi_R - m\psi_L = 0\,,$$

$$\psi(t, \mathbf{x}) = U(t)\psi(0)e^{i\mathbf{p}\cdot\mathbf{x}}$$

$$\psi_L = \frac{1-\gamma_5}{2}\psi\,, \quad \psi_R = \frac{1+\gamma_5}{2}\psi\,, \quad \psi = \psi_L + \psi_R\,.$$

$$U = e^{-iHt}\,, \quad H = \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0 = \boldsymbol{\alpha}\cdot\mathbf{p} + m\beta$$

How left-handed and right-handed are entangeled in free space?

In chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \; \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \; \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

$$\psi^h(t, \mathbf{x}) = U(t)\psi^h(0)e^{i\mathbf{p}\cdot\mathbf{x}} = \psi^h(0)e^{-i(Et - \mathbf{p}\cdot\mathbf{x})}\,, \quad \psi^h(0) = \frac{1}{\sqrt{2E}}\begin{pmatrix} \sqrt{E - h\cdot p}\; u^h \\ \sqrt{E + h\cdot p}\; u^h \end{pmatrix}$$

$\psi(0)^\dagger\psi(0) = 1.$ $h = \pm 1$ - helicity, $\mathbf{p}\cdot\boldsymbol{\sigma}u^h = (h\cdot p)u^h$, $\mathbf{p} = (p_x, p_y, p_z) = p(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$.

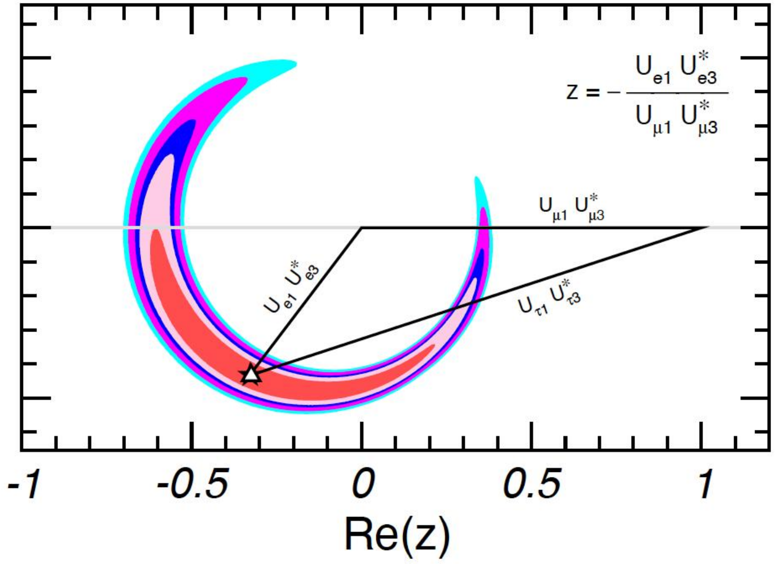
$$u^{h=+1} = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix}, \quad u^{h=-1} = \begin{pmatrix} -\sin(\theta/2)e^{-i\phi} \\ \cos(\theta/2) \end{pmatrix},$$

Oscillation probability from i to k for Dirac neutrinos:

$$P(\psi_i \to \psi_k) = |<\psi(0)_k|\psi(t)_i>|^2 = |\sum_j V_{ij}V^*_{kj}e^{-i(E_j t - \mathbf{p}\cdot\mathbf{x})}|^2$$

Neutrion Oscillation With Three Generations

$$\begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{CP}} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{i\delta_{CP}} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{i\delta_{CP}} & c_{13}\,s_{23} \\ s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{i\delta_{CP}} & -c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{i\delta_{CP}} & c_{13}\,c_{23} \end{pmatrix}.$$



PDG 2023

Table 14.7: 3ν oscillation parameters obtained from different global analyses of neutrino data. In all cases, the numbers labeled as NO (IO) are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum. SK-ATM makes reference to the tabulated χ^2 map from the Super-Kamiokande analysis of their data in Ref. [97].

	Ref. [185] w/o SK-ATM		Ref. [185] w SK-ATM		Ref. [186] w SK-ATM		Ref. [187] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.04^{+0.14}_{-0.13}$	$2.65 \to 3.46$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.46^{+0.87}_{-0.88}$	$30.98 \to 36.03$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.58^{+0.20}_{-0.33}$	$4.27 \to 6.09$	$5.63^{+0.18}_{-0.24}$	$4.33 \to 6.09$	$5.51^{+0.19}_{-0.80}$	$4.30 \to 6.02$	$5.47^{+0.20}_{-0.30}$	$4.45 \to 5.99$
$\theta_{23}/^\circ$	$48.3^{+1.2}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$47.9^{+1.1}_{-4.0}$	$41.0 \to 50.9$	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.241^{+0.066}_{-0.065}$	$2.046 \to 2.440$	$2.237^{+0.066}_{-0.065}$	$2.044 \to 2.435$	$2.14^{+0.09}_{-0.07}$	$1.90 \to 2.39$	$2.160^{+0.083}_{-0.069}$	$1.96 \to 2.41$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.41^{+0.18}_{-0.14}$	$7.9 \to 8.9$	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	221^{+39}_{-28}	$144 \to 357$	238^{+41}_{-33}	$149 \to 358$	218^{+38}_{-27}	$157 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$2.449^{+0.032}_{-0.030}$	$2.358 \to 2.544$	$2.454^{+0.029}_{-0.031}$	$2.362 \to 2.544$	$2.419^{+0.035}_{-0.032}$	$2.319 \to 2.521$	2.424 ± 0.03	$2.334 \to 2.524$
IO	$\Delta\chi^2 = 6.2$		$\Delta\chi^2 = 10.4$		$\Delta\chi^2 = 9.5$		$\Delta\chi^2 = 11.7$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.03^{+0.14}_{-0.13}$	$2.64 \to 3.45$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$	$33.40^{+0.87}_{-0.81}$	$30.92 \to 35.97$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.63^{+0.19}_{-0.26}$	$4.30 \to 6.12$	$5.65^{+0.17}_{-0.22}$	$4.36 \to 6.10$	$5.57^{+0.17}_{-0.24}$	$4.44 \to 6.03$	$5.51^{+0.18}_{-0.30}$	$4.53 \to 5.98$
$\theta_{23}/^\circ$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$	$48.2^{+1.0}_{-1.4}$	$41.8 \to 50.9$	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.261^{+0.067}_{-0.064}$	$2.066 \to 2.461$	$2.259^{+0.065}_{-0.065}$	$2.064 \to 2.457$	$2.18^{+0.08}_{-0.07}$	$1.95 \to 2.43$	$2.220^{+0.074}_{-0.076}$	$1.99 \to 2.44$
$\theta_{13}/^\circ$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$	$8.49^{+0.15}_{-0.14}$	$8.0 \to 9.0$	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
$\delta_{CP}/^\circ$	285^{+24}_{-26}	$205 \to 354$	282^{+23}_{-25}	$205 \to 348$	247^{+26}_{-27}	$193 \to 346$	281^{+23}_{-27}	$202 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$-2.509^{+0.032}_{-0.032}$	$-2.603 \to -2.416$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$	$-2.478^{+0.035}_{-0.033}$	$-2.577 \to -2.375$	$-2.50\pm^{+0.04}_{-0.03}$	$-2.59 \to -2.39$

Neutrino Oscillation Something new?

Chiral oscillation, Majorana phses appear in oscillation... profund implication for cosmic background neutrinos....
M-W Li, Z-L Huang and XG He, arXiv: 2307.12561

In the SM neutrinos are produced by W and/or Z interactions.

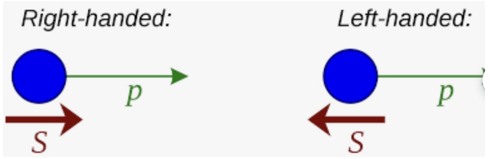
At production $t=0$ point, they are left-handed and normalized, $\psi^h_L(0) = \sqrt{\frac{2E}{E-h\cdot p}}\frac{1-\gamma_5}{2}\psi^h(0)$.

$$\psi^h_L(t) = \sqrt{\frac{2E}{E-h\cdot p}}e^{-iHt}\frac{1-\gamma^5}{2}\psi^h(0) = \psi^h_L(t) = \sqrt{\frac{2E}{E-h\cdot p}}\left(e^{-iEt}\frac{1-\gamma_5}{2}\psi^h(0) - i\frac{m}{E}\sin(Et)\left[\beta, \frac{1-\gamma_5}{2}\right]\psi^h(0)\right).$$

Used $U(t) = e^{-iHt} = \cos(Et) - i\frac{\boldsymbol{\alpha}\cdot\mathbf{p}+m\beta}{E}\sin(Et)$ $\quad \psi^{h\dagger}_L\psi^h_L(t) = \left(\cos(Et) + i\frac{h\cdot p}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}\ , \quad \psi^{h\dagger}_R\psi^h_L(t) = \left(-i\frac{m}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}$

$$P(\nu^h_L \to \nu^h_L) = |\psi^{h\dagger}_L\psi^h_L(t)|^2 = 1 - \frac{m^2}{E^2}\sin^2(Et)\ ,\quad P(\nu^h_L \to \nu^h_R) = |\psi^{h\dagger}_R\psi^h_L(t)|^2 = \frac{m^2}{E^2}\sin^2(Et)$$

Left-handed neutrinos oscillated into right-handed ones!



$$P(\nu^h_{Li} \to \nu^h_{Lk}) = |V_{ij}V^*_{kj}(\cos(E_jt) + i\frac{h\cdot p}{E_j}\sin(E_jt))|^2\ ,\quad P(\nu^h_{Li} \to \nu^h_{Rk}) = |-iV_{ij}V^*_{kj}\frac{m_j}{E_j}\sin(E_jt)|^2\psi^{h\dagger}_R\psi^h_L(t)|^2$$

S-F Ge & P Pasquini, PLB811(2020)135961; V Bittencourt, A. Bernardini & M. Blasone, EPJC81 (2021)411.

Chiral oscillation probability extremely small for relativistic neutrinos because the suppresion factor: m^2/E^2.
Numerically vailid to use Dirac neutrinos to describe neutrino oscillations. Important cosmological implkications!

Neutrino Oscillation In Matter

When neutrinos travel in matter, due to interaction of neutrions with matter mediated by W and Z, the Lagrangian is modified

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - j^\mu \bar{\psi}\gamma_\mu \frac{1-\gamma_5}{2}\psi \ .$$

j^μ is the matter current which neutrino can interact.
In the rest frame of the homogeneous, isotropic, unpolarized electrical neutrality medium, $j^\mu = (\rho, \vec{0})$
$\rho = \sqrt{2} G_F \left(N_e \delta_{\alpha e} - \frac{1}{2} N_n\right)$. N_e, N_n number density of electron and neutron, $\delta_{\alpha e}$ are zero for ν_μ and ν_τ.

$$H = \mathbf{p}\cdot\boldsymbol{\alpha} + m\beta + \rho\frac{1-\gamma_5}{2} = \begin{pmatrix} \rho - \mathbf{p}\cdot\boldsymbol{\sigma} & m \\ m & \mathbf{p}\cdot\boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} \rho - h\cdot p & m \\ m & h\cdot p \end{pmatrix}$$

The evolution for a given wave function entering the media with momentum $\vec{p}$ at $t=0$ is

$$\psi_L^h(t) = e^{-iHt}\psi_L^h(0) = e^{-\frac{i}{2}\rho t}\begin{pmatrix} \cos(E_h t) + i\frac{h\cdot p - \frac{\rho}{2}}{E_h}\sin(E_h t) & -i\frac{m}{E_h}\sin(E_h t) \\ -i\frac{m}{E_h}\sin(E_h t) & \cos(E_h t) - i\frac{h\cdot p - \frac{\rho}{2}}{E_h}\sin(E_h t) \end{pmatrix}\begin{pmatrix} u^h \\ 0 \end{pmatrix} ,$$

$$P(\psi_L^h \to \psi_L^h) = 1 - \frac{m^2}{E_h^2}\sin^2(E_h t) \ , \quad P(\psi_L^h \to \psi_R^h) = \frac{m^2}{E_h^2}\sin^2(E_h t).$$

Similar as that for free space, but dependent on matter density via $E_h = \sqrt{m^2 + (h\cdot p - \rho/2)^2}$.

There is a resonant enhanced chiral oscillation at $h\cdot p - \rho/2 = 0$!

Majorana And Seesaw Neutrinos In Matter

The general seesaw neutrino Lagrangian in matter propagation

$$\begin{aligned}\mathcal{L} &= \bar{\nu}_L i\partial\!\!\!/ \nu_L + \bar{N}_R i\partial\!\!\!/ N_R - \frac{1}{2}\left(\begin{pmatrix}\bar{\nu}_L^c & \bar{N}_R\end{pmatrix}\begin{pmatrix}M_L & M_D^T \\ M_D & M_R\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R^c\end{pmatrix} + \text{h.c.}\right) - \begin{pmatrix}\bar{\nu}_L & \bar{N}_R^c\end{pmatrix}\begin{pmatrix}j_L^\mu & j_{RL}^\mu \\ j_{RL}^{\mu\dagger} & j_R\end{pmatrix}\gamma_\mu\begin{pmatrix}\nu_L \\ N_R^c\end{pmatrix} \\ &= \bar{\psi}_L i\partial\!\!\!/ \psi_L - \frac{1}{2}\left(\bar{\psi}_L^c \mathcal{M}\psi_L + \text{h.c.}\right) - \bar{\psi}_L J^\mu \gamma_\mu \psi_L\end{aligned}$$

For homogeneous, isotropic, unpolarized electrical neutrality matter medium at rest, only j_L^0 is non-zero

$$j_L^0 = \begin{pmatrix}\rho_e & 0 & 0 \\ 0 & \rho_\mu & 0 \\ 0 & 0 & \rho_\tau\end{pmatrix} = \begin{pmatrix}\sqrt{2}G_F\left(N_e - \frac{1}{2}N_n\right) & 0 & 0 \\ 0 & -\frac{G_F}{\sqrt{2}}N_n & 0 \\ 0 & 0 & -\frac{G_F}{\sqrt{2}}N_n\end{pmatrix} .$$

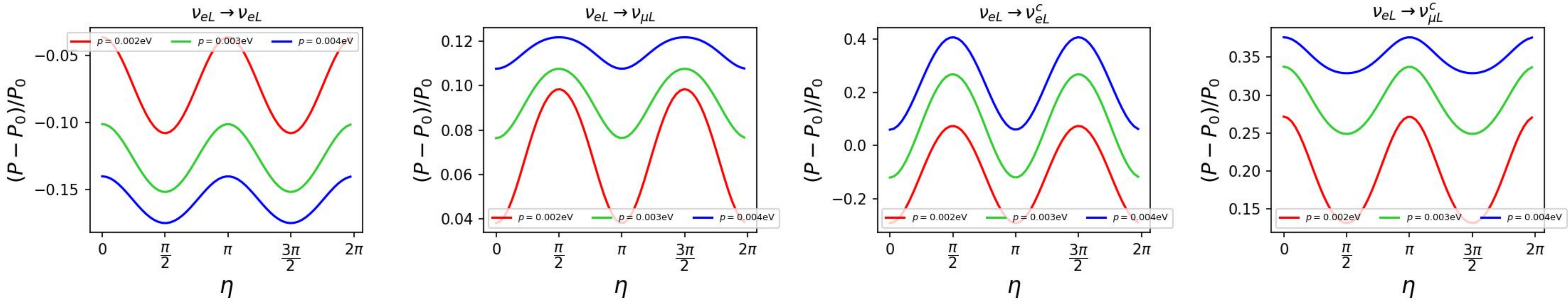
In terms of the mass eigenstate $\psi_L = V\psi_L^m$, the Lagrangian is

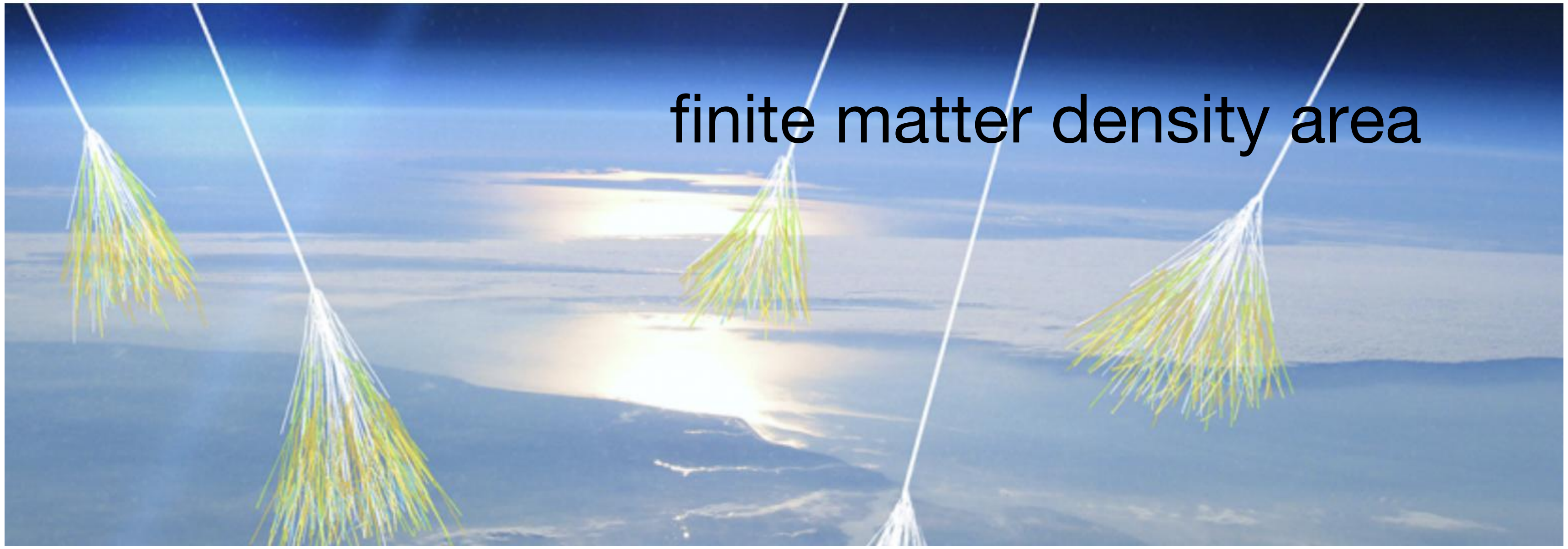
$$\mathcal{L} = \frac{1}{2}\left(\bar{\psi}^m(i\partial\!\!\!/ - \widehat{M})\psi^m\right) - \bar{\psi}^m \widetilde{J}^\mu \gamma_\mu \frac{1-\gamma_5}{2}\psi^m , \quad \psi^m = \psi_L^m + (\psi_L^m)^c , \;\; \widetilde{J}^\mu = V^\dagger J^\mu V.$$

$$(i\partial\!\!\!/ - \widehat{M})\psi^m - \widetilde{J}^\mu\gamma_\mu\frac{1-\gamma_5}{2}\psi^m + (\widetilde{J}^\mu)^*\gamma_\mu\frac{1+\gamma_5}{2}\psi^m = 0 .$$

$$H = \begin{pmatrix}\frac{\rho}{2}(1+\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & m & \frac{\rho}{2}e^{i\eta}\sin 2\theta & 0 \\ m & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1+\cos 2\theta) & 0 & -\frac{\rho}{2}e^{-i\eta}\sin 2\theta \\ \frac{\rho}{2}e^{-i\eta}\sin 2\theta & 0 & \frac{\rho}{2}(1-\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & M \\ 0 & -\frac{\rho}{2}e^{i\eta}\sin 2\theta & M & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1-\cos 2\theta)\end{pmatrix}$$

$h = -1$, $\rho_W = 0.0026\text{eV}$, $\rho_Z = -0.0013\text{eV}$, $\theta = 33.41^\circ$, $m = 10^{-3}\text{eV}$, $M^2 - m^2 = 7.41 \times 10^{-5}\text{eV}^2$.





Compare neutrino with different momentum detection to extract information about eta.

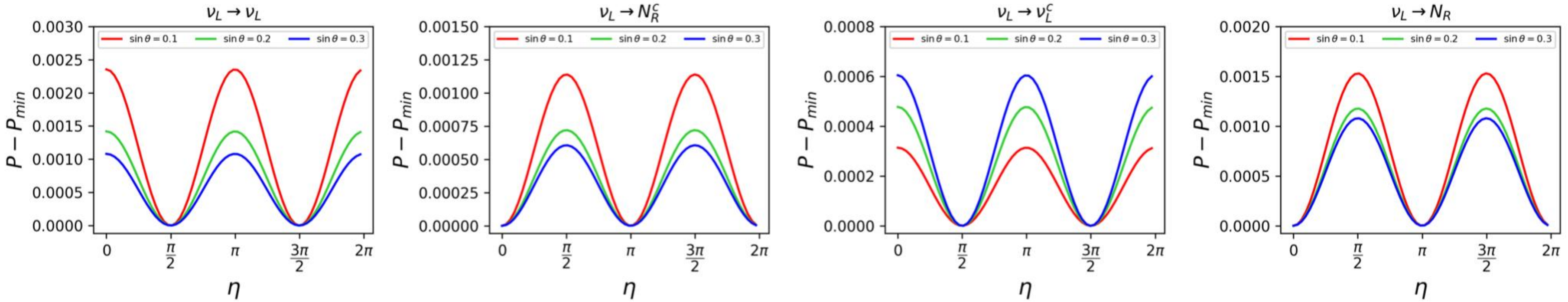


FIG. 3: The time averaged Majorana phase η effects on probabilities. The vertical axis is the difference between time averaged probabilities and their minimum values. The oscillating probabilities depend on the mixing angle θ. Other parameters are chosen from a point in matter effect significant region in Fig. 2(a).

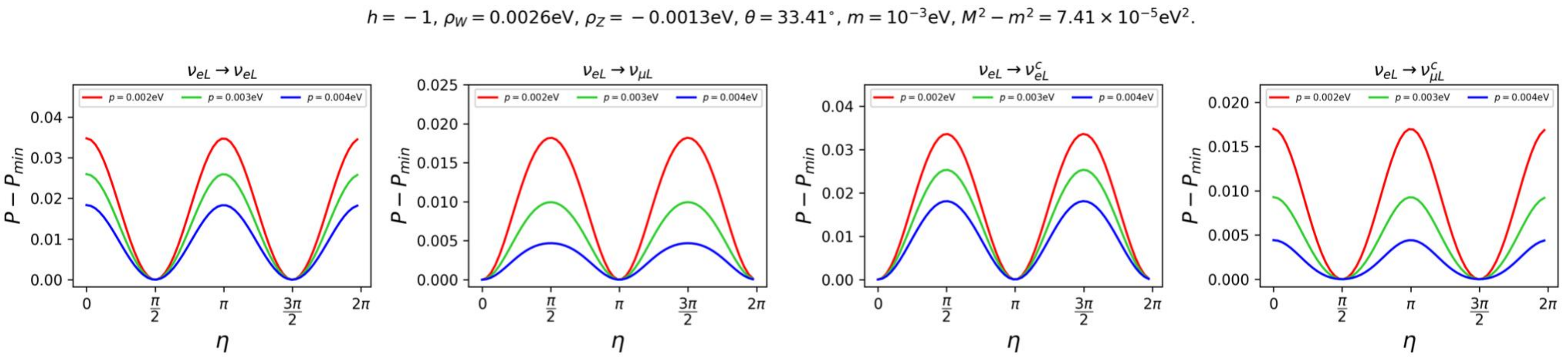


FIG. 4: The time averaged Majorana phase η effects on probabilities in two active Majorana neutrinos case. The vertical axis is the difference between time averaged probabilities and their minimum values. The red, green and blue lines are corresponding to $p = 0.002\text{eV}$, 0.003eV and 0.004eV, respectively. The mixing angle $\theta = \theta_{12}$ and mass square difference $M^2 - m^2 = \Delta m_{21}^2$, whose values are from [1]. $\rho_W = 0.0026\text{eV}$ and $\rho_Z = -0.0013\text{eV}$ are corresponding the mass density $a_p = 3.4 \times 10^{10}\text{g/cm}^3$ and $a_n = 3.4 \times 10^{10}\text{g/cm}^3$, respectively, so the total mass density $a \approx 6.8 \times 10^{10}\text{g/cm}^3$, which could be found in neutron star cluster [12].

Neutrino Mixing Pattern

Early days, expecting neutrino mixing might be following a similar pattern as quarks, mixing angles are small.

For example, 1992 people are

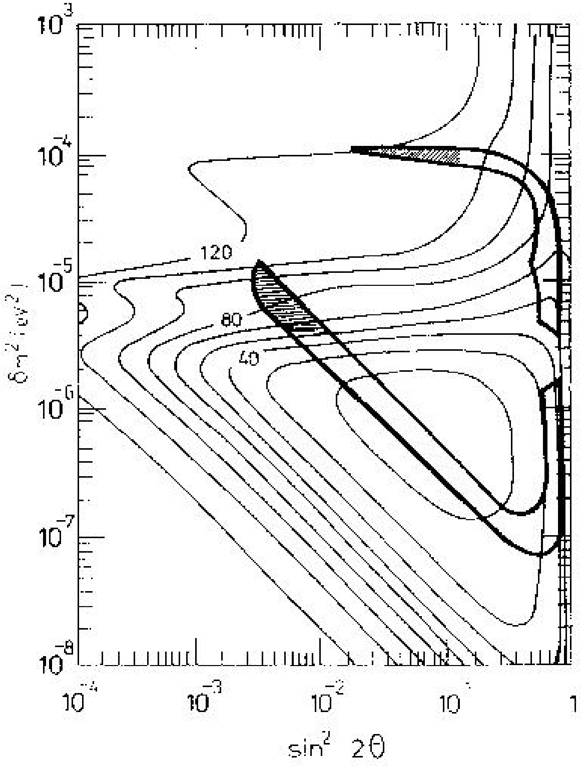
trying to produce mixing on the right

10^{-3} to 0.3 eV^2 [4], and the allowed parameter

sin² 2θ

FIG. 1. The figure shows (boldly outlined) the two reg $\sin^2 2\theta_{12} - \delta m^2_{12}$ space allowed to solve the solar neutrin lem. The solid contour lines are the ν_e flux from the Su

(Davies and He, PRD46, 3208)



But both solar and atmospheric show large mixing angles!

Very different quark and lepton mixing patterns

Mixing pattern in quark sector

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

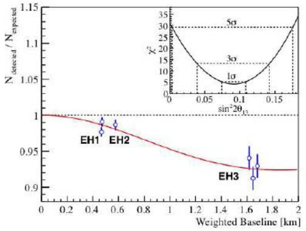
$$V_{CKM} \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Mixing pattern in lepton sector

$$\theta_{12}^Q = 13.021^\circ \pm 0.039^\circ, \quad \theta_{23}^Q = 2.350^\circ \pm 0.052^\circ,$$

$$= 0.199^\circ \pm 0.008^\circ. \quad \delta^Q = 68.9^\circ$$

$$|U| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix},$$



ArXiv:1203.1669 (hep-ex)

$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$

Read as $\theta_{13} = 8.8^\circ$ with significance 5.2σ

More precise data needed to determine mixing pattern and neutrino mass hierachies:
T2K, NOvA, JUNO, DUNE ... Stay tuned!

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing

Harrison, Perkins, Scott (2002) , Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ, with the tri-bimaximal mixing

$$V_{tri-bi} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

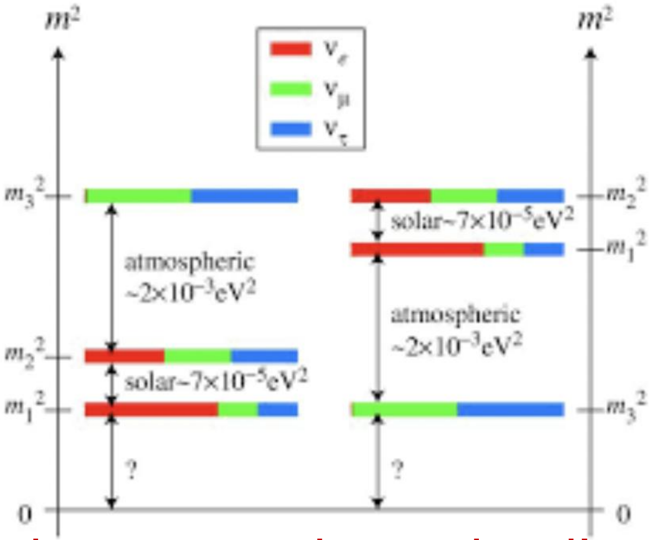
A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, Δ(27, 96), $PSL_2(7)$ … discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthanpa&M-C. Chen; Frampton&Kephart; Y-L Wu, ….

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A_4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix}.$$

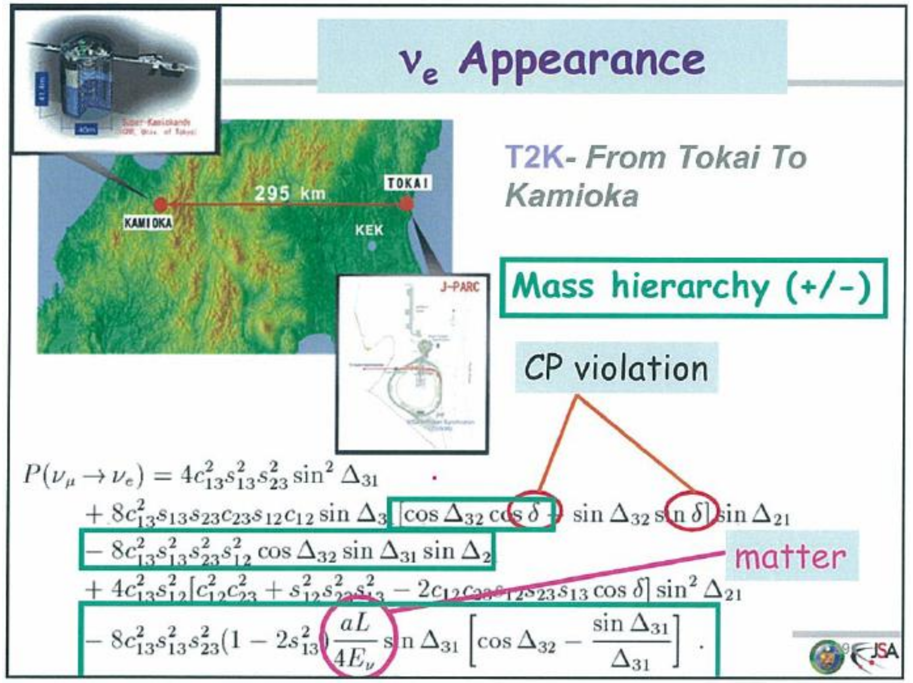


Mixing pattern very differnt than their quark conter parts. Mass hierarchy not yet determined!

CP Violation In Neutrino Oscillation

Table 14.6: Experiments contributing to the present determination of the oscillation parameters.

Experiment	Dominant	Important
Solar Experiments	θ_{12}	Δm^2_{21} , θ_{13}
Reactor LBL (KamLAND)	Δm^2_{21}	θ_{12} , θ_{13}
Reactor MBL (Daya-Bay, Reno, D-Chooz)	θ_{13}, $\lvert\Delta m^2_{31,32}\rvert$	
Atmospheric Experiments (SK, IC-DC)		θ_{23},$\lvert\Delta m^2_{31,32}\rvert$, θ_{13},δ_{CP}
Accel LBL ν_μ,$\bar{\nu}_\mu$, Disapp (K2K, MINOS, T2K, NOνA)	$\lvert\Delta m^2_{31,32}\rvert$, θ_{23}	
Accel LBL ν_e,$\bar{\nu}_e$ App (MINOS, T2K, NOνA)	δ_{CP}	θ_{13} , θ_{23}



$$P_{\nu_\mu\to\nu_e,(\bar{\nu}_\mu\to\bar{\nu}_e)} \approx 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2\Delta}{(1-A)^2} + \alpha^2\sin^2 2\theta_{12}\cos^2\theta_{23}\frac{\sin^2 A\Delta}{A^2} + 8\,\alpha\,J_{CP}^{\max}\cos(\Delta\pm\delta_{CP})\frac{\sin\Delta A}{A}\frac{\sin\Delta(1-A)}{1-A},$$

with

$$J_{CP}^{\max} = \cos\theta_{12}\sin\theta_{12}\cos\theta_{23}\sin\theta_{23}\cos^2\theta_{13}\sin\theta_{13},$$

and

$$\Delta \equiv \frac{\Delta m^2_{31}L}{4E_\nu}, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}},$$

V is the effective matter potential in the Earth's crust

$$J_{CP}^{\max} = 0.03359 \pm 0.0006\,(\pm 0.0019)$$

There maybe CP violation in neutrino scillation. Need more confirmation!

Model Building with $\theta_{12}=\pi/4$ and $\delta_{CP} = 3\pi/(-\pi/2)$

Structure of the mass matrix (charged lepton is already diagonal)

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}$$

Grimus and Lavora; Xing et al and others

A4 model realizations!
(X-G He, Chin. J. Phys 53, 100101(2015);
X-G He and G-N Li, Phys. Lett. B750,620(2015);
E Ma, Phys. Rev. D92, 051301(2015).
Group, Modular theory, Gui-Jun Ding ....

Grand Unification Theory SO(10) Predictions

$$16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$$

Minimal SO(10) Model without 120

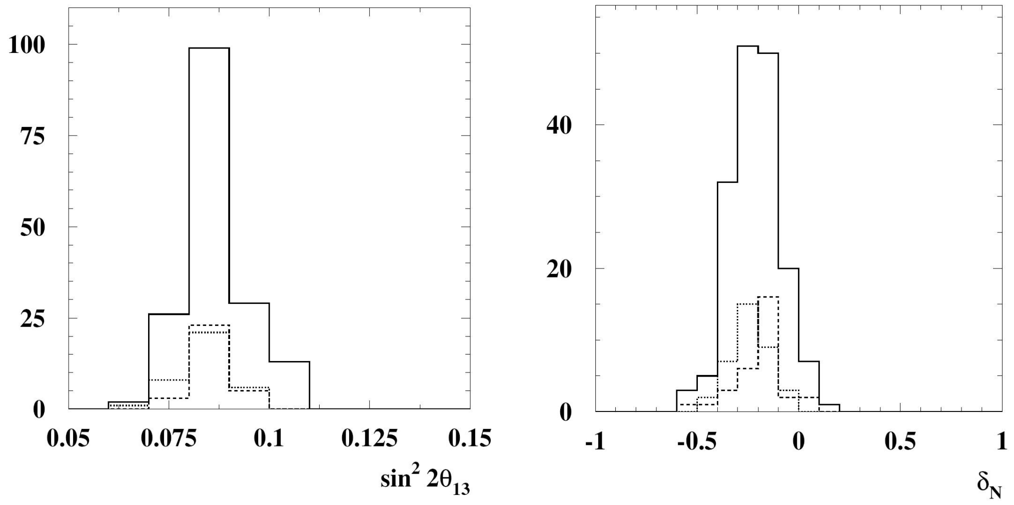
$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

$$\begin{aligned} M_{\nu R} &= \langle\Delta_R\rangle Y_{126} \\ M_{\nu L} &= \langle\Delta_L\rangle Y_{126} \end{aligned}$$

Model has only 11 real parameters plus 7 phases

Good prediction for $\theta_{13}\delta$ Away from $-\pi/2$!



Loop generated Dirac mass models also work!
Eax:Babu, He, Su &Thapa, JHEP 08 (2022) 140

More data will help to finally determine mixing pattern, whether there is CPV!

Absolute Neutrino masses and Neutrinos Are Dirac Or Majorana Particle?

Tritium decay spectrum

Katrin, Mainz, Troits

$$^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e$$

$$\frac{dN}{dE} = C\,p\,E\,(Q-T)\sqrt{(Q-T)^2 - (m^{\text{eff}}_{\nu_e})^2}\,F(E) \equiv R(E)\sqrt{(E_0-E)^2 - (m^{\text{eff}}_{\nu_e})^2}$$

$$1.1\text{ eV} \geq m^{\text{eff}}_{\nu_e} = \sqrt{\sum_i m_i^2 |U_{ei}|^2} = \begin{cases} \sqrt{m_0^2 + \Delta m^2_{21}(1 - c^2_{13}c^2_{12}) + \Delta m^2_{32}s^2_{13}} & \text{in NO}, \\ \sqrt{m_0^2 + \Delta m^2_{21}c^2_{13}c^2_{12} - \Delta m^2_{32}c^2_{13}} & \text{in IO}, \end{cases}$$

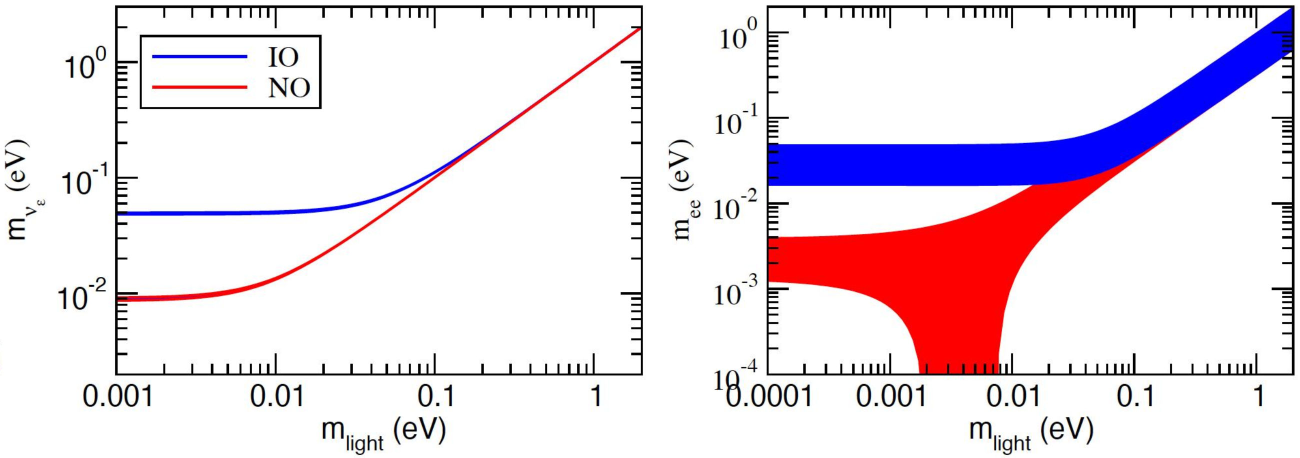


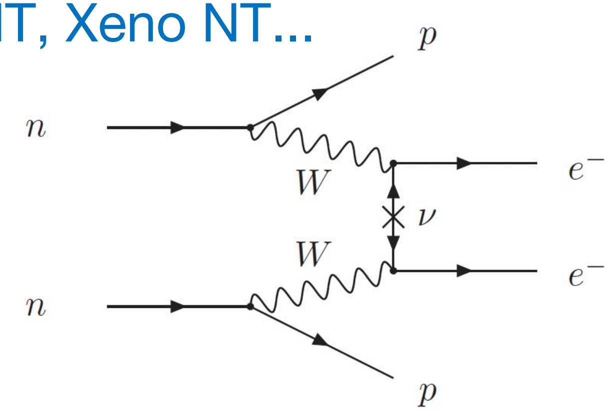
Figure 14.11: Allowed 95% CL ranges (1 dof) for the neutrino mass observable determined in ^{3}H beta decay (left panel) and in $0\nu\beta\beta$ (right panel) in the framework of 3ν mixing as a function of the lightest neutrino mass. The ranges are obtained by projecting the results of the global analysis of oscillation data (w/o SK-atm) in Ref. [184]. The region for each ordering is defined with respect to its local minimum.

Neutrinoless double beta decay

KamLAND-Zen, LEGEND, CUORE, GERDA...
and more to come, PandaX NT, Xeno NT...

$$m_{ee} = \left|\sum_i m_i U^2_{ei}\right|$$

$$= \begin{cases} \left| m_0 c^2_{12}c^2_{13} + \sqrt{\Delta m^2_{21} + m_0^2}\, s^2_{12}c^2_{13}e^{2i(\eta_2-\eta_1)} + \sqrt{\Delta m^2_{32} + \Delta m^2_{21} + m_0^2}\, s^2_{13}e^{-2i(\delta_{\text{CP}}+\eta_1)} \right| & \text{in NO}, \\ \left| m_0 s^2_{13} + \sqrt{m_0^2 - \Delta m^2_{32}}\, s^2_{12}c^2_{13}e^{2i(\eta_2+\delta_{\text{CP}})} + \sqrt{m_0^2 - \Delta m^2_{32} - \Delta m^2_{21}}\, c^2_{12}c^2_{13}e^{2i(\eta_1+\delta_{\text{CP}})} \right| & \text{in IO}, \end{cases}$$



If neutrinoless double beta decay observed, neutrinos are Majhorana particles!

3. Neutrinos In Cosmology And Astrophysics

In early universe, all energy forms existed in form of elementary particles, or ….
Temperature is high and were in thermal equilibrium

Criteria for thermal equilibrium: particle interaction length $1/\Gamma$ (Γ interaction rate) is smaller than Hubble length $1/H_0$

???Planck mass T ~ 10^{19}GeV

Inflation
Big Bang ~ T> 10^{16} GeV

(Not in thermal equillibrium by SM for particle physics)

Grand Unification ~ 10^{16} GeV

EW symmetry breaking 300GeV

Color confinement ~300 MeV

BBN ~ 1 MeV

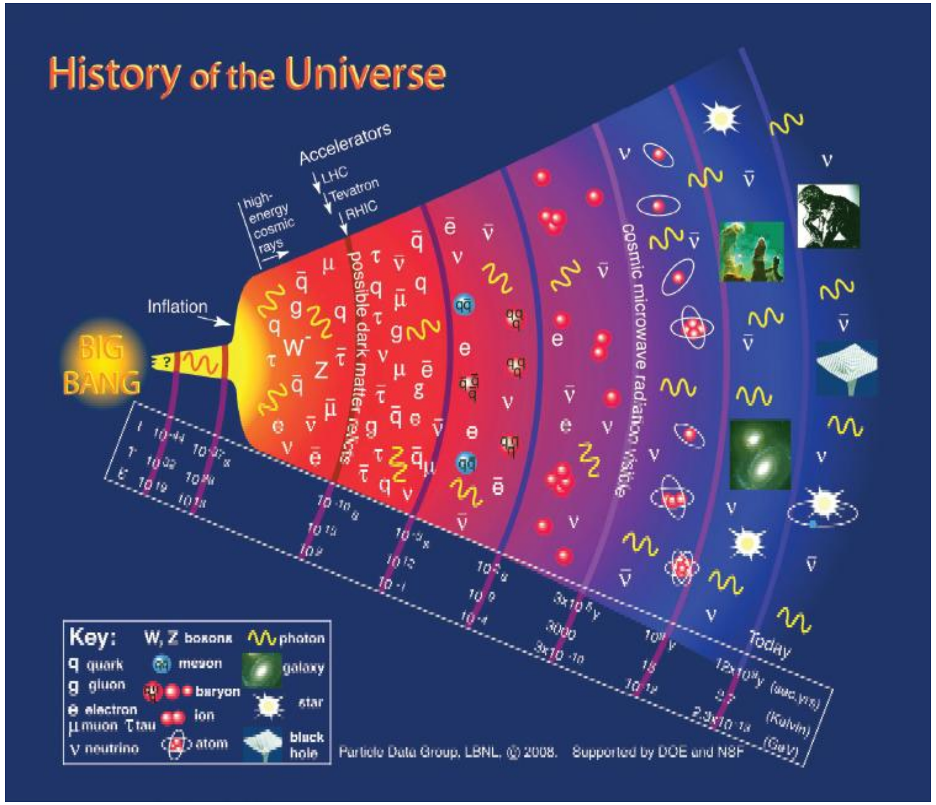
CνB ~ 1 MeV

CMB ~0.3 eV

Large structure formation

…

Today ~2.7K



Relic neutrino density: $$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\rm eff} \left(\frac{4}{11} \right)^{4/3}$$

Big-Bang Nucleothynsis: $N_{\rm eff}$ ~ 3 light neutrinos, consistent with Z decay witdth.

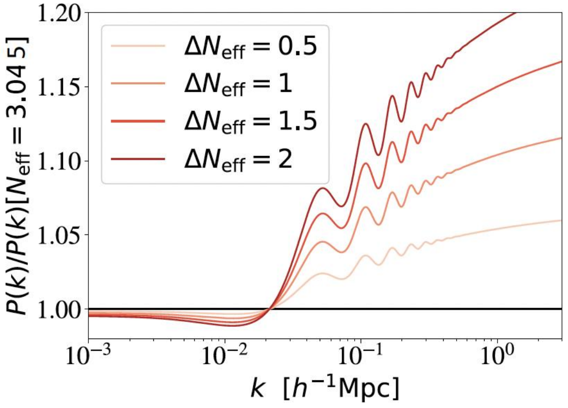


Table 26.1: Summary of $N_{\rm eff}$ constraints.

	Model	95%CL	Ref.
CMB alone			
Pl18[TT,TE,EE+lowE]	ΛCDM+$N_{\rm eff}$	$2.92^{+0.36}_{-0.37}$	[22]
CMB + background evolution + LSS			
Pl18[TT,TE,EE+lowE+lensing] + BAO	ΛCDM+$N_{\rm eff}$	$2.99^{+0.34}_{-0.33}$	[22]
" + BAO + R21	ΛCDM+$N_{\rm eff}$	3.34 ± 0.14 (68%CL)	[11]
"	" +5-params.	2.85 ± 0.23 (68%CL)	[23]

Neutrino back ground temperature: $T_\nu/T_\gamma = (4/11)^{1/3} = 1.95$ K

$n_\gamma = 440/\mathrm{cm}^3$, $n_\nu = 339/\mathrm{cm}^3$

Energy density from neutrinos: $$\Omega_\nu = \frac{\rho_\nu^0}{\rho_{\rm crit}^0} = \frac{\sum m_\nu}{93.14 h^2 \,\mathrm{eV}}$$

Neutrino Mass On CMB And Matter Spectrum

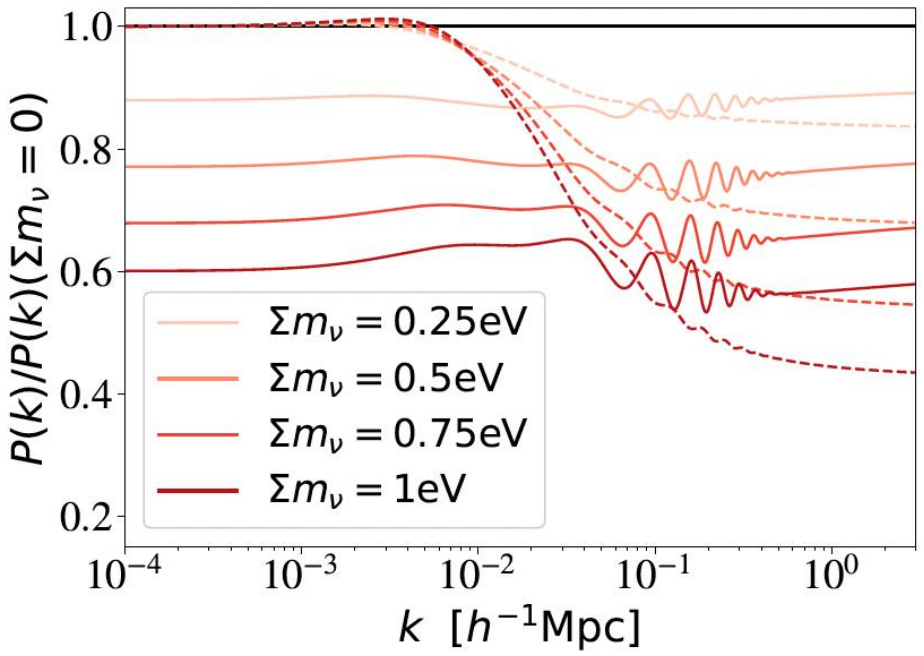


Table 26.2: Summary of $\sum m_\nu$ constraints.

	Model	95% CL (eV)
CMB alone		
Pl18[TT+lowE]	ΛCDM+$\sum m_\nu$	< 0.54
Pl18[TT,TE,EE+lowE]	ΛCDM+$\sum m_\nu$	< 0.26
CMB + probes of background evolution		
Pl18[TT+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.13
Pl18[TT,TE,EE+lowE]+BAO	ΛCDM+$\sum m_\nu$+5 params.	< 0.515
CMB + LSS		
Pl18[TT+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.44
Pl18[TT,TE,EE+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.24
CMB + probes of background evolution + LSS		
Pl18[TT,TE,EE+lowE] + BAO + RSD	ΛCDM+$\sum m_\nu$	< 0.10
Pl18[TT+lowE+lensing] + BAO + Lyman-α	ΛCDM+$\sum m_\nu$	< 0.087
Pl18[TT,TE,EE+lowE] + BAO + RSD + Pantheon + DES	ΛCDM+$\sum m_\nu$	< 0.13

The sum of the neutrino masses should be less than 0.6 eV.

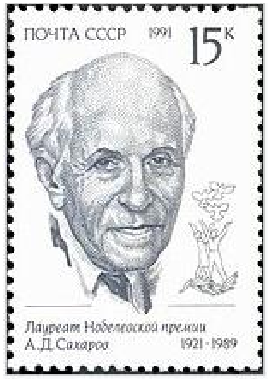
Future, reach 0.06 eV reaching the allowed range by neutrino oscillation!
DESI, Euclid, LSST, SPHEREx, SKA.

Neutrinos play important role in BBN!
Cosmology constraint their masses compareable to neutrino oscillation data!

Neutrino And Our Matter Universe

In our Universe, matter dominates over anti-matter

- Why this is so is the problem of Baryon Asymmetry of our Universe (BAU)



Andrei Sakharov on *Soviet Nobel Peace Prize winners*, the USSR stamp issued on 14 May 1991

In cosmological terms, the problem is as follows

If initially, the universe is matter and anti-matter symmetric

$n_B/n_\gamma = n_{\bar{B}})/n_\gamma \sim 10^{-20}$

$n_B(n_{\bar{B}})$ - baryon (anti-baryon) number density, n_γ - photon number density

However observation, BBN and CMB, show that

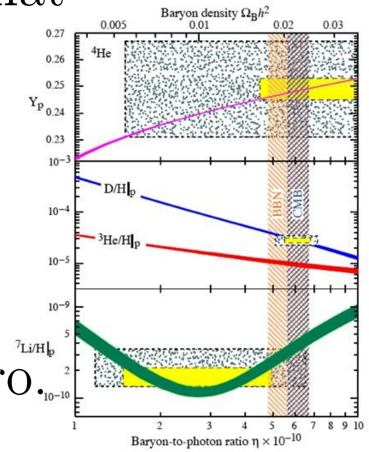
$\eta = (n_B - n_{\bar{B}})/n_\gamma \sim 6 \times 10^{-10}$

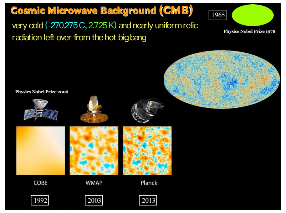
There is a 10^{10} order of magnitude difference.

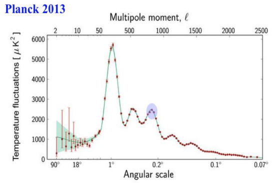
Initially, there is a baryon asymmetry?

But inflation will dilute any asymmetry to zero.

Possible to generate a η which fits observtion from an, initially, matter anti-matter symmetry universe?







Sakharov Conditions (1967)

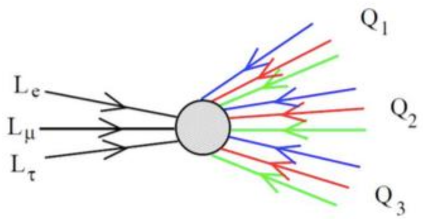
Baryon Number B Violation

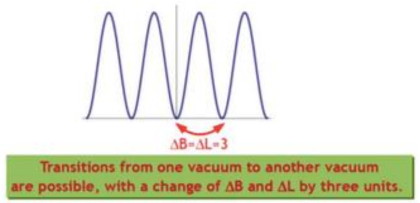
C and CP Violation

Interactions Out Of Thermal Equilibrium

Standard Model Has All Ingradients, But Too Small

Baryon number violation: Sphelaron effects-tunneling effects from different vacuum states with non-zero baryon number differences. Violated B+L, but conserves B-L.





C and CP violation: Electroweak interaction violates C, and phase in Kobayashi-Maskawa mixing matrix violates CP.

Out of thermal equilibrium: Electroweak symmetry breaking

But, CP violation rate too small, out of thermal equilibrium too weak. Not enough to generate a large enough Baryon Asymmetry.

If Higgs mass is less than 70 GeV, second order phase transition at electroweak symmetry breaking, too weak.

$\eta \sim 10^{-20}$ Too small. Needs to go beyond SM!

Electroweak baryogenesis, Leptogenesis, Gut baryogenesis….

Leptogenesis

Fukugita and Yanagida, PLB174, 45(1986)

Translate lepton number asymmetry generated in the early universe to baryon number asymmetry!

Requires lepton asymmetry generated before Sphelaron effects to be in effective ($T \sim 10^{12}$ – a few TeV). Initial $a_L(i)=a$, $a_B(i)=0$.

Sphelaron effect: Conserve B-L, but violates B+L

After: $a_L(f)+a_B(f) = 0$, $a_L(i) - a_B(i) = a_L(f) - a_B(f)$

$$a_L(f) = a/2; \quad a_B(f) = -a/2$$

half of initial lepton asymmetry will be translated into baryon asymmetry if complete.

SM Sphelaron effect: $a_B = -(28/79)a_L$

Seesaw Model Plays The Right Role

$$L_M = -\bar{L}_L Y_e \tilde{H} E_R - \bar{L}_L Y_\nu H \nu_R - \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^{\prime c}) M^\nu \begin{pmatrix} \nu_L^c \\ \nu_R^{\prime} \end{pmatrix} + H.C.$$

The last term violates lepton number L by two units!

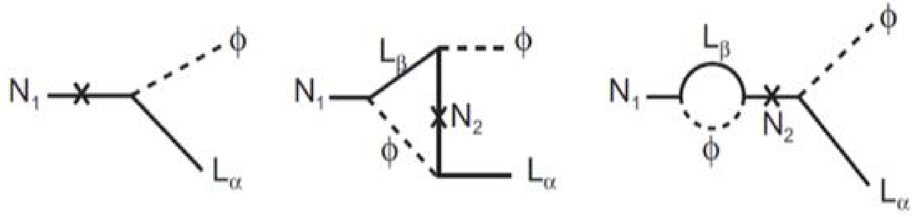
Out of thermal equilibrium decay, new CP violation in N -> L h(ϕ)

N decays into L and anti-L differently

$$a_i = \frac{\Gamma(N_i \to L\phi) - \bar{\Gamma}(N_i \to \bar{L}\phi)}{\Gamma(N_i \to L\phi) + \bar{\Gamma}(N_i \to \bar{L}\phi)}$$

$$a_i \approx -\frac{1}{8\pi}\frac{1}{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ii}} \sum_j Im\{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ij}^2\} f\left(\frac{M_j^2}{M_i^2}\right)$$

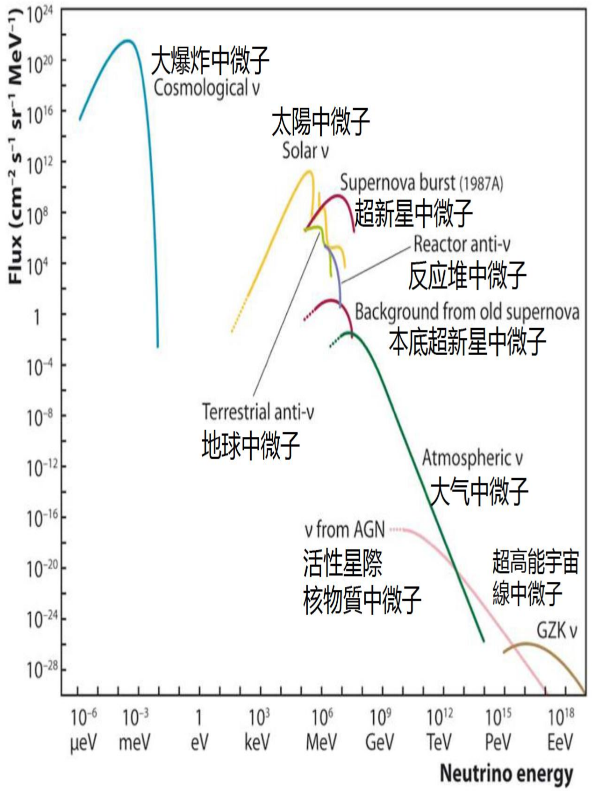
$$f(x) = \sqrt{x}\left(\frac{2}{x-1} + \ln\frac{1+x}{x}\right).$$



M_N and m_ν masses are correlated to obtain the right number for η,
m_ν of order 0.05 eV, M_N ~ 1000 GeV.

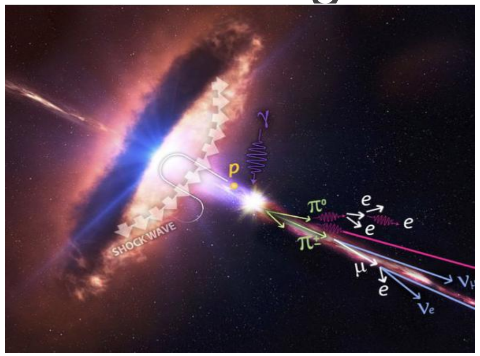
Neutrino Seesaw model is a viable model for Baryon Asymmetry of our Universe

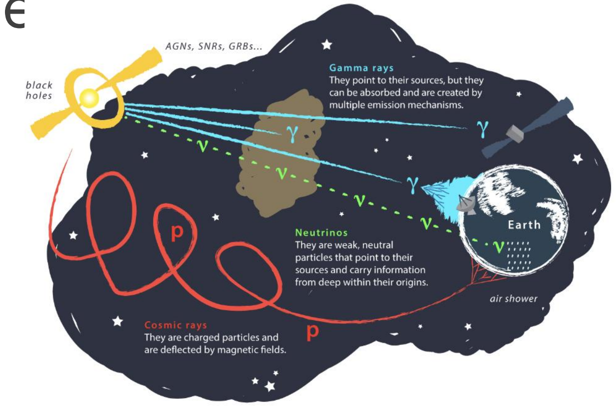
Cosmic Neutrino Ray As Messenger For Our Universe



Where the very high energy neutrino ray come from? How they are acceleratted to have such high energie

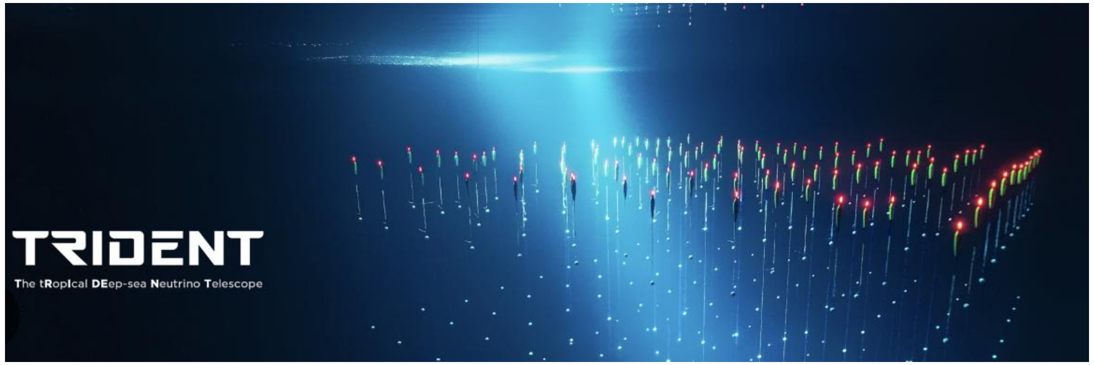
Any use of the neutrino cosmic ray?

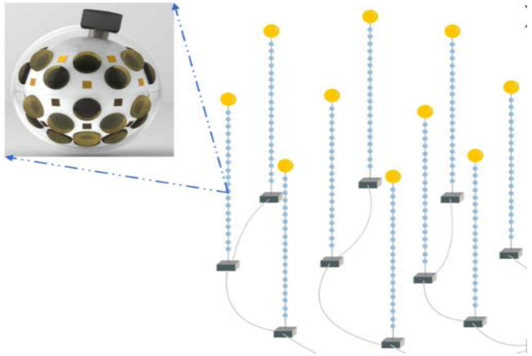




Like we use neutrino beam on earth to probe new interactions, human being can use cosmic neutrino beams as messengers to probe our universe with much less disturbances compared with hadrons, photons!

Extremely low energy: CνB and super high energy neutrino rays.





Messenger for our universe! TDLI trident project is an initiative for this!

Thank you for your attentions!