Emergence of near-threshold structures in heavy hadron spectrum

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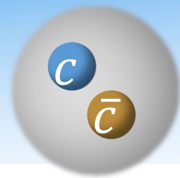
General model-independent analysis on threshold structures:

X.-K. Dong, FKG, B.-S. Zou, Phys. Rev. Lett. 126 (2021) 152001 [arXiv:2011.14517]

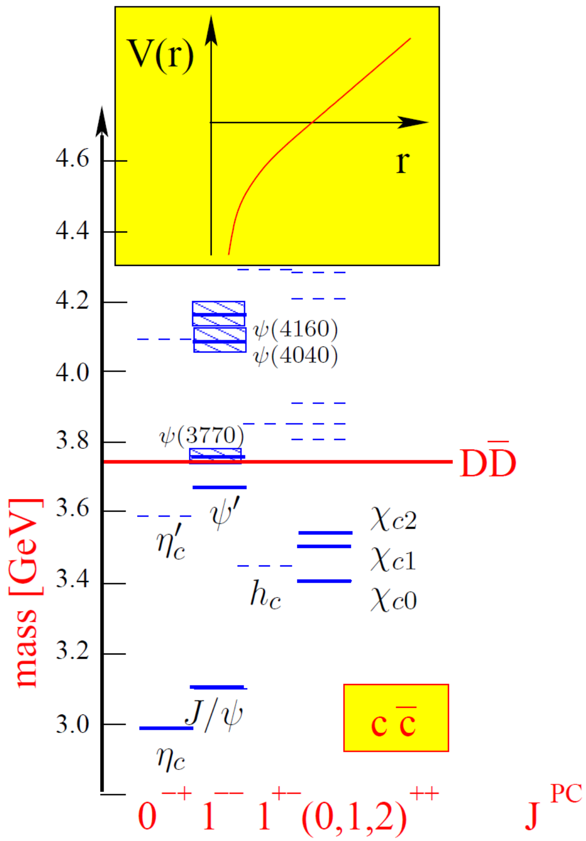
Systematic spectrum of hadronic molecules:

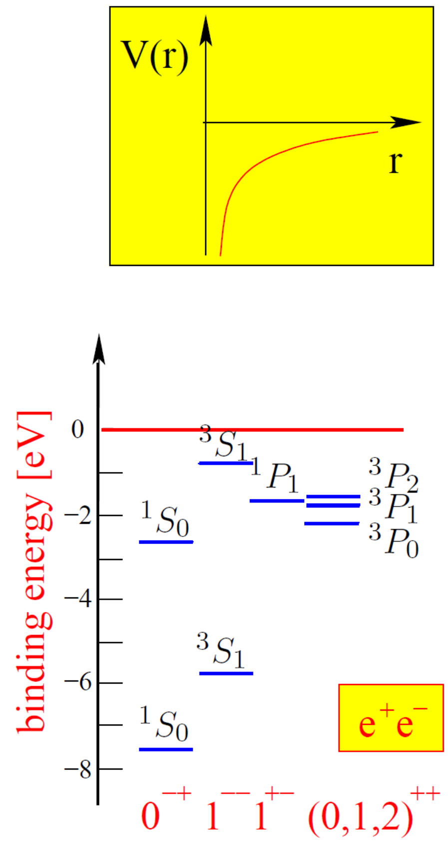
X.-K. Dong, FKG, B.-S. Zou, 物理学进展 41 (2021) 65 [arXiv:2101.01021]

Charmonium



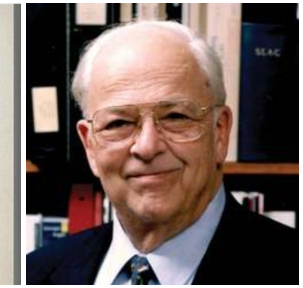
- Meson consisting of a charm quark and an anticharm quark
  - ➢ The first charmonium: J/ψ
  - ➢ Probing both perturbative and nonperturbative QCD

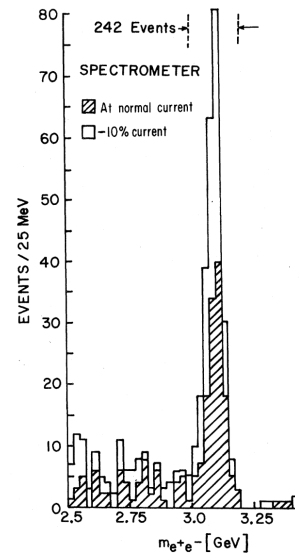




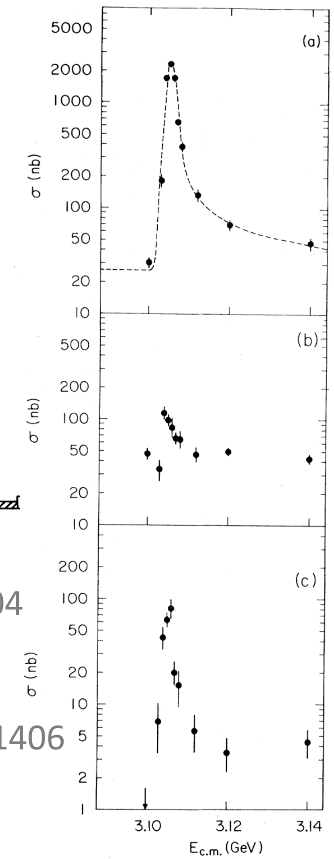
Cornell potential model:
Eichten et al., PRD17(1978)3090

From talk by Hanhart at APS2018



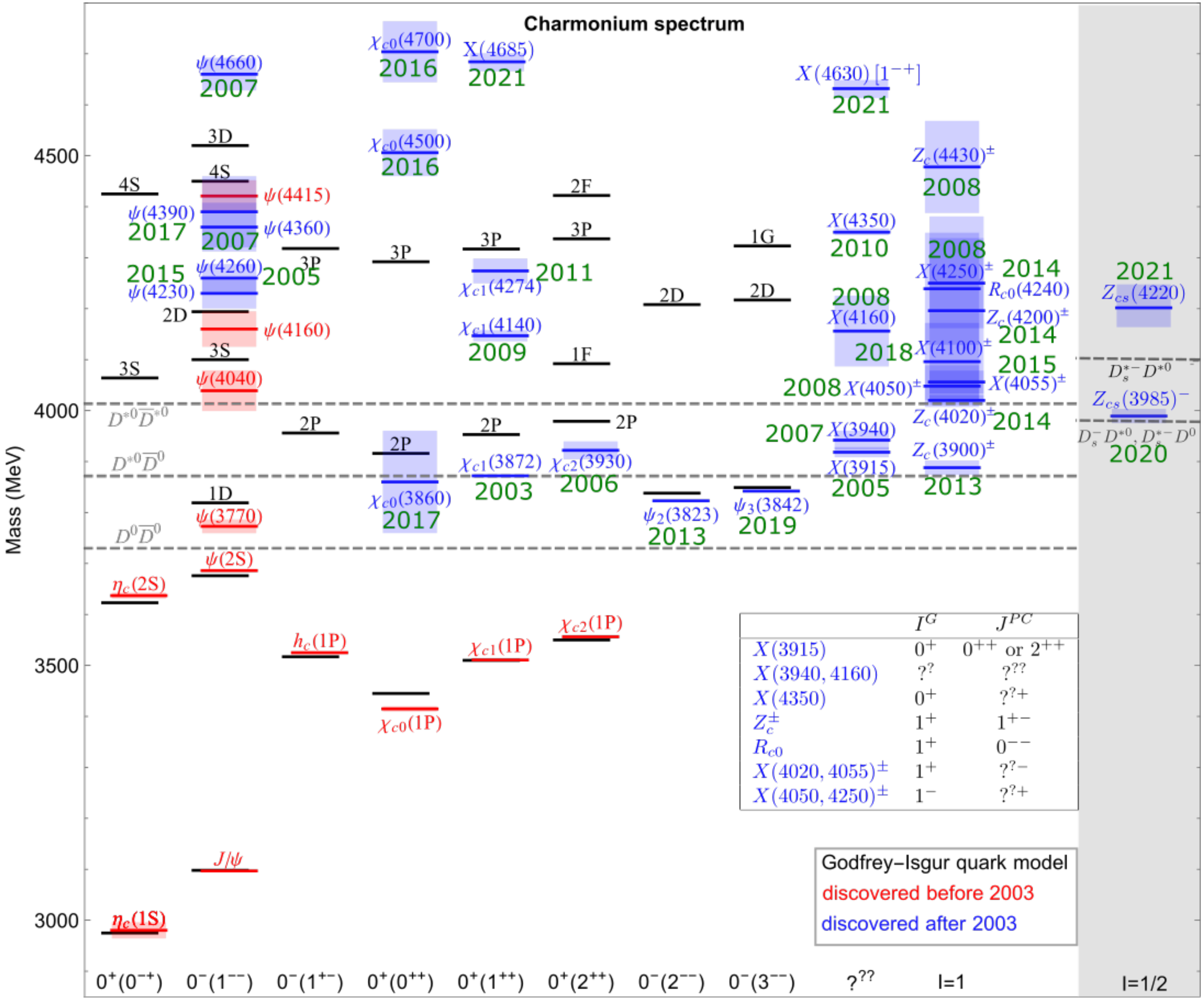


PRL33(1974)1404



PRL33(1974)1406

Charmonium(-like) structures

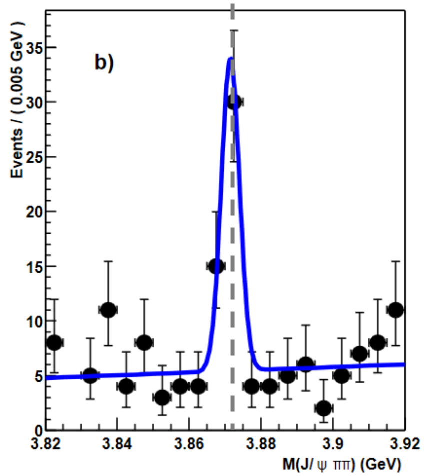


Charmonium-like structures

- X(3872)

  Belle, PRL91(2003)262001

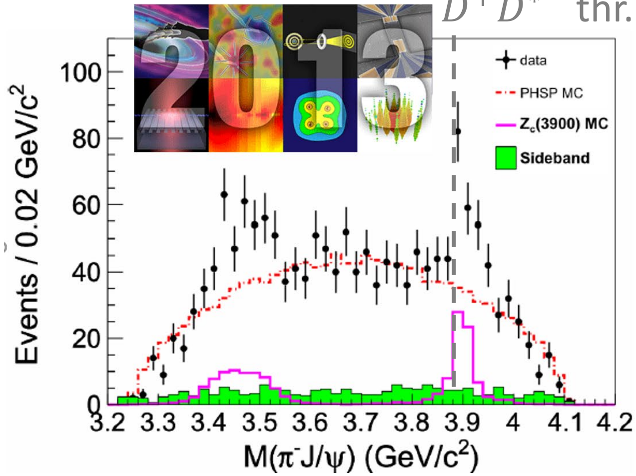
$D^0\overline{D}^{*0}$ thr.

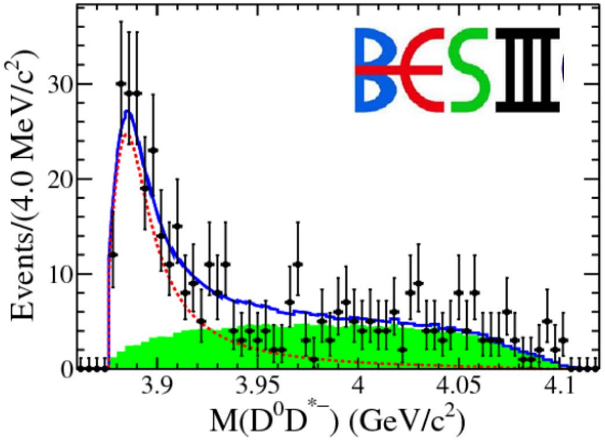


- $Z_c(3900)^{\pm}$

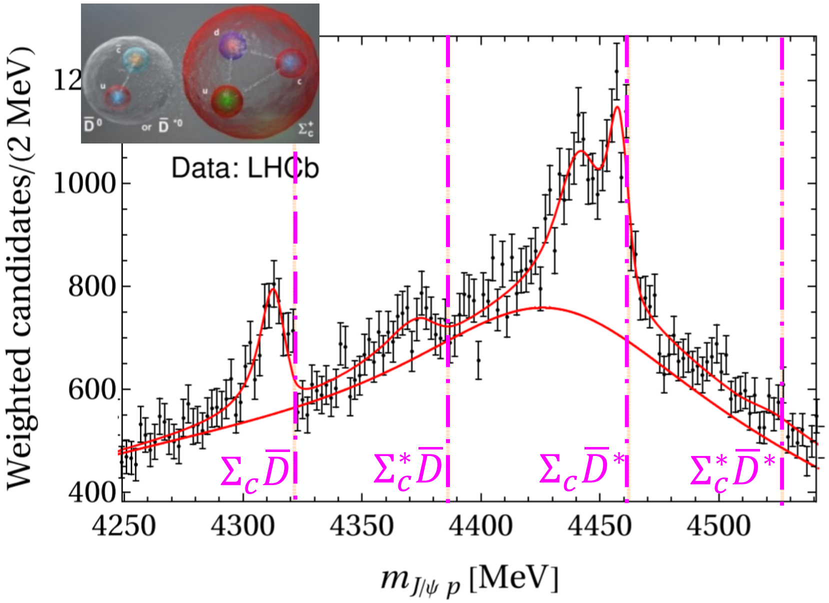
  BESIII, PRL110(2013)252001; Belle, PRL110(2013)252002

D^+D^{*-} thr.

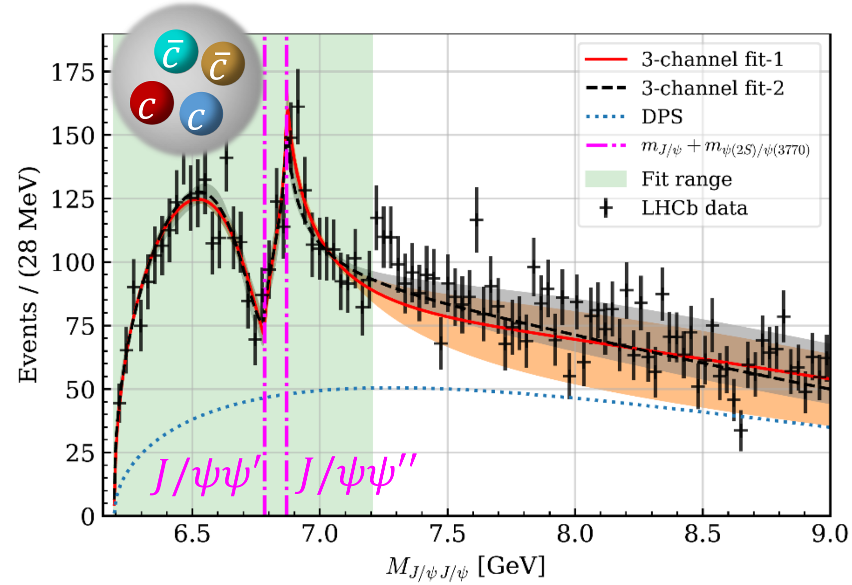




P_c and double-J/ψ structures



data from LHCb, PRL122 (2019) 222001;
fit from
M.-L. Du, Baru, FKG, Hanhart, Meißner, Oller, Q. Wang,
PRL124 (2020) 072001



data from LHCb, Sci.Bull.65 (2020) 1983;
fit from
X.-K. Dong, Baru, FKG, Hanhart, Nedefiev,
PRL126 (2021) 132001

Many new structures are near thresholds of a pair of hadron hadrons.

Why are there so many (near-)threshold structures in heavy-hadron spectrum?
Is there any rule?

Threshold structures

X.-K. Dong, FKG, B.-S. Zou, *Explaining the Many Threshold Structures in the Heavy Quark Hadron Spectrum*, Phys.Rev.Lett.126 (2021) 152001 [arXiv:2011.14517]

Threshold cusp

- Unitarity of the S-matrix: $S\,S^\dagger = S^\dagger S = \mathbb{1}$, $S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)T_{fi}$

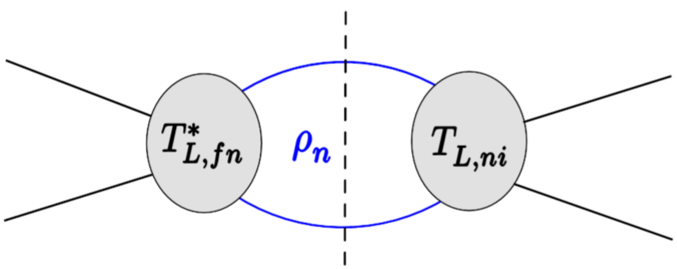
  T-matrix: $$T_{fi} - T_{fi}^\dagger = -i(2\pi)^4 \underbrace{\sum_n}_{\text{all physically accesible states}} \delta(p_n - p_i) T_{fn}^\dagger T_{ni}$$

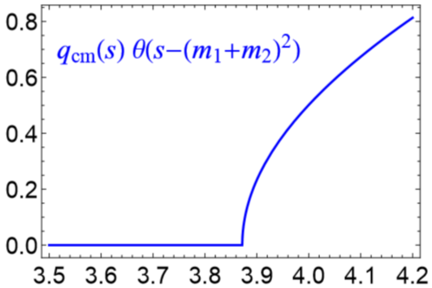
  assuming all intermediate states are two-body, partial-wave unitarity relation:

  $$\boxed{\mathrm{Im}\, T_{L,fi}(s) = -\sum\nolimits_n T_{L,fn}^*\,\rho_n(s)\,T_{L,ni}}$$

  2-body phase space factor: $\rho_n(s) = q_{\text{cm},n}(s)/(2\sqrt{s})\theta(\sqrt{s} - m_{n1} - m_{n2})$,

  $$q_{\text{cm},n}(s) = \sqrt{[s-(m_{n1}+m_{n2})^2][s-(m_{n1}-m_{n2})^2]}/(2\sqrt{s})$$





- There is always a cusp at an S-wave threshold

Effective range expansion

$$f_0^{-1}(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}\left(\frac{k^4}{\beta^4}\right)$$

a_0: S-wave scattering length; negative for repulsion or attraction w/ a bound state

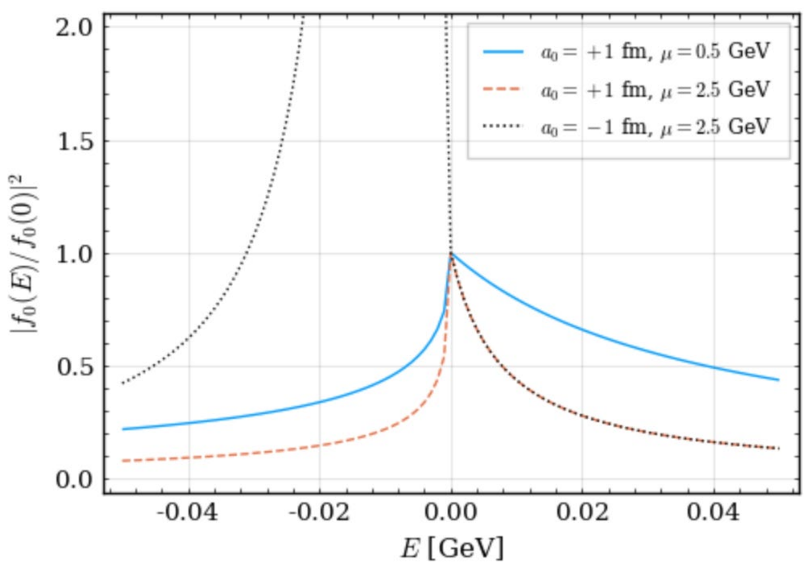
positive for attraction w/o bound state

Very close to threshold, then scattering length approximation: $f_0^{-1}(E) = \frac{1}{a_0} - i\sqrt{2\mu E}$

$$|f_0(E)|^2 = \begin{cases} \dfrac{1}{1/a_0^2 + 2\mu E} & \text{for } E \geq 0 \\ \dfrac{1}{\left(1/a_0 + \sqrt{-2\mu E}\right)^2} & \text{for } E < 0 \end{cases}$$

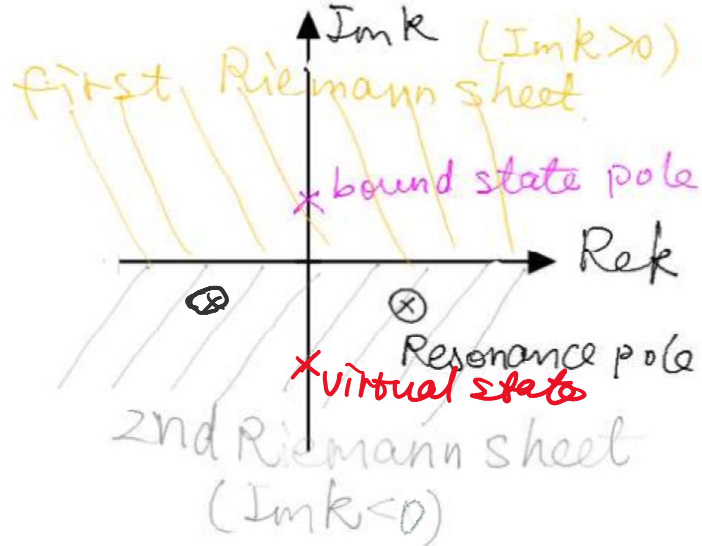
- Cusp at threshold (E=0)
- Maximal at threshold for positive a_0 (attraction)
- Half-maximum width: $\frac{2}{\mu a_0^2}$;

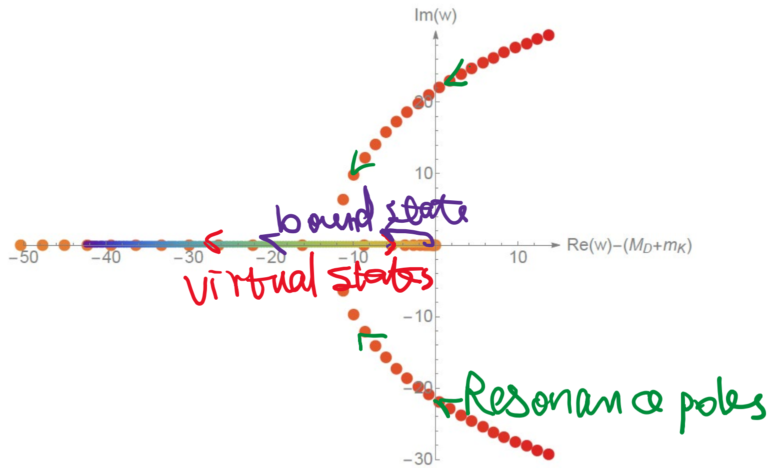
  virtual state pole at $E_{\text{virtual}} = -1/(2\mu a_0^2)$
- Strong interaction, a_0 becomes negative, pole below threshold, peak below threshold



Bound state, virtual state and resonance

- Bound state: pole below threshold on real axis of the first Riemann sheet of complex energy plane
- Virtual state: pole below threshold on real axis of the second Riemann sheet
- Resonance: pole in the complex plane on the second Riemann sheet



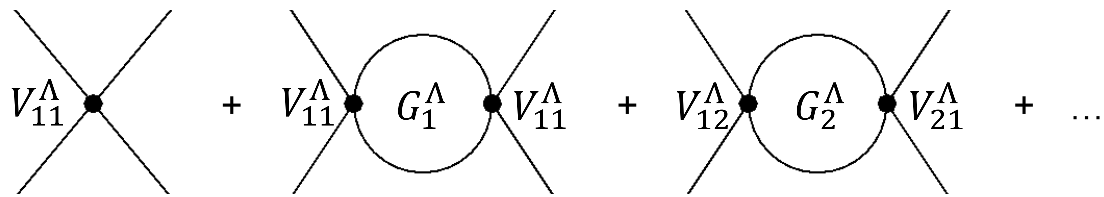


Plot from Matuschek, Baru, FKG, Hanhart, 2007.05329

For $\dfrac{1}{1/a_0 - i\,k}$, only bound or virtual state poles are possible

Coupled channels

- Full threshold structure needs to be measured in a lower channel ⇨ coupled channels
- Consider a two-channel system, construct a nonrelativistic effective field theory (NREFT)
  - ➢ Energy region around the higher threshold, Σ_2
  - ➢ Expansion in powers of $E = \sqrt{s} - \Sigma_2$
  - ➢ Momentum in the lower channel can also be expanded



$$T(E) = V + VG(E)V + VG(E)VG(E)V + \cdots = \frac{1}{V^{-1} - G(E)}$$

$$G_1^{\Lambda}(E) = i\int^{\Lambda_1} \frac{d^4q}{(2\pi)^2} \frac{1}{(q^2 - m_{1,1}^2 + i\epsilon)[(P-q)^2 - m_{1,2}^2 + i\epsilon]} = R(\Lambda_1) - i\frac{k_1}{8\pi\sqrt{s}}$$

$$G_2^{\Lambda}(E) = -\frac{1}{4m_{2,1}m_{2,2}} \int^{\Lambda_2} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{2\mu_2}{\mathbf{q}^2 - 2\mu_2 E - i\epsilon} = \frac{1}{8\pi\Sigma_2}\left[-\frac{2\Lambda_2}{\pi} + \boxed{\sqrt{-2\mu_2 E - i\epsilon}} + \mathcal{O}\left(\frac{k_2^2}{\Lambda_2}\right)\right]$$

- ➢ Λ dependence absorbed by V^{-1}

Nonanalyticity only from here

NREFT at LO

- Very close to the higher threshold, LO:

$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1}$$
$$= -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix},$$

$$\det = \left(\frac{1}{a_{11}} - i\,k_1\right)\left(\frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}\right) - \frac{1}{a_{12}^2}$$

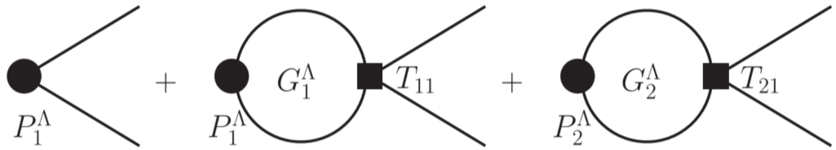
Effective scattering length with open-channel effects becomes complex, $\mathrm{Im}\,\frac{1}{a_{22,\mathrm{eff}}} \leq 0$

$$T_{22}(E) = -\frac{8\pi}{\Sigma_2}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}$$

$$\frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1+a_{11}^2k_1^2)} - i\frac{a_{11}^2 k_1}{a_{12}^2(1+a_{11}^2k_1^2)}.$$

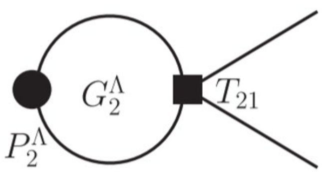
NREFT at LO

- Consider a production process, must go through final-state interaction (unitarity)



$$
\begin{aligned}
&P_1^\Lambda[1+G_1^\Lambda T_{11}(E)]+P_2^\Lambda G_2^\Lambda(E)T_{21}(E)\\
&=P_1^\Lambda(V_{11}^\Lambda)^{-1}T_{11}(E)+\left[P_1^\Lambda(V_{11}^\Lambda)^{-1}V_{12}^\Lambda+P_2^\Lambda\right]G_2^\Lambda T_{21}(E)\\
&\equiv P_1 T_{11}(E)+P_2 T_{21}(E)
\end{aligned}
$$

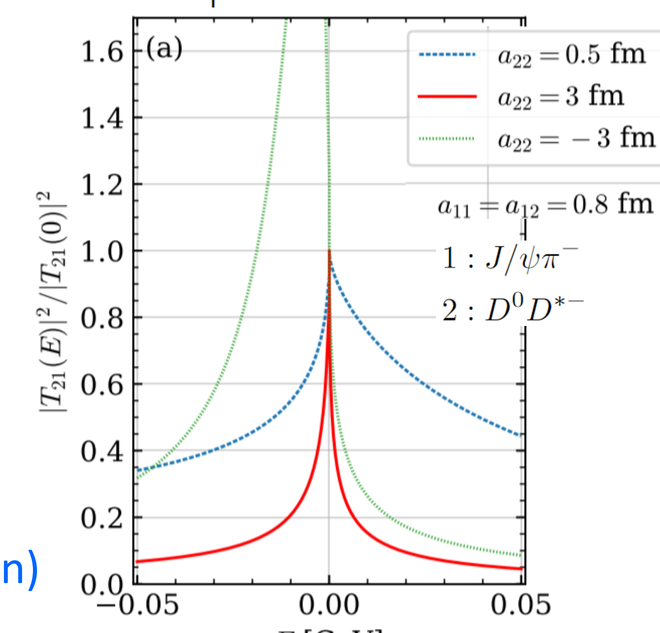
- All nontrivial energy dependence are contained in $T_{11}(E)$ and $T_{21}(E)$
- Case-1: dominated by $T_{21}(E)$,



$$
T_{21}(E)=\frac{-8\pi\Sigma_2}{a_{12}(1/a_{11}-ik_1)}\left[\frac{1}{a_{22,\text{eff}}}-i\sqrt{2\mu_2 E}+\mathcal{O}(E)\right]^{-1}.
$$

$$
|T_{21}(E)|^2\propto|T_{22}(E)|^2\propto
$$

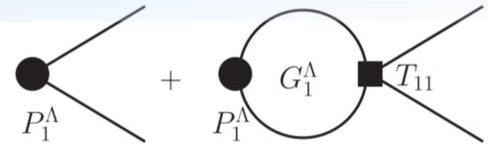
$$
\begin{cases}
\left[\left(\mathrm{Re}\,\frac{1}{a_{22,\text{eff}}}\right)^2+\left(\mathrm{Im}\,\frac{1}{a_{22,\text{eff}}}-\sqrt{2\mu E}\right)^2\right]^{-1} & \text{for } E\geq 0\\
\left[\left(\mathrm{Im}\,\frac{1}{a_{22,\text{eff}}}\right)^2+\left(\mathrm{Re}\,\frac{1}{a_{22,\text{eff}}}+\sqrt{-2\mu E}\right)^2\right]^{-1} & \text{for } E<0
\end{cases}
$$



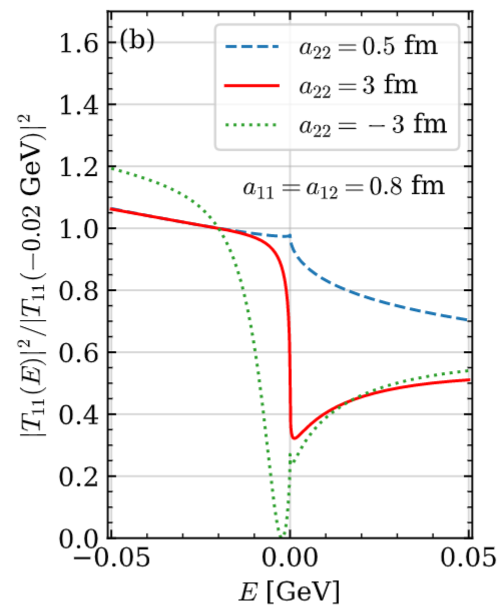
- ➢ Cusp at threshold (E=0)
- ➢ Maximal at threshold for positive $\mathrm{Re}(a_{22,\text{eff}})$ (attraction)
- ➢ Peaking at pole for negative $\mathrm{Re}(a_{22,\text{eff}})$

NREFT at LO

- Case-2: dominated by $T_{11}(E)$



$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$



Poles in complex momentum plane:

$(0.37 - i0.08)$GeV

$(0.04 - i0.08)$GeV

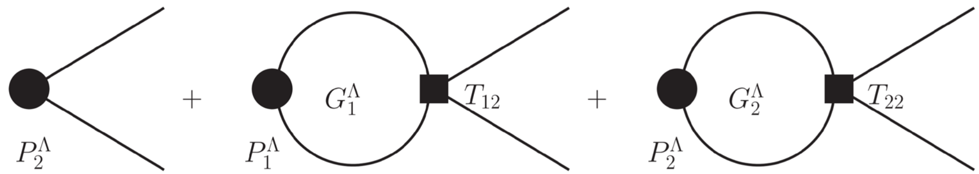
$(-0.09\ - i0.08)$GeV

- Cusp at threshold (E=0)
- One pole and one zero
- For strongly interacting channel-2 (large a_{22}), there must be a dip around threshold
- Abrupt drop if there is a nearby pole

- More complicated line shape if both channels are important for the production

NREFT at LO

- Case-3: final states in channel-2



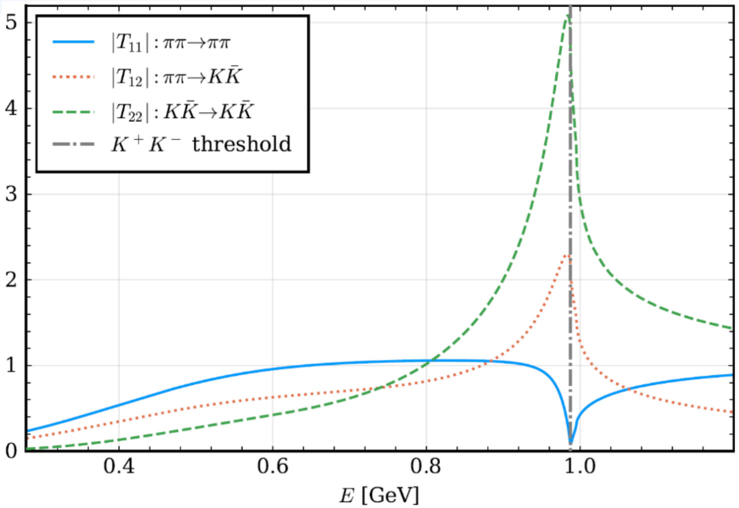
$$P_1 T_{12}(E) + P_2 T_{22}(E) \propto \left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}$$

- ➢ Suppression due to phase space
- ➢ Peak just above threshold would require the pole to be nearby

Phenomenology

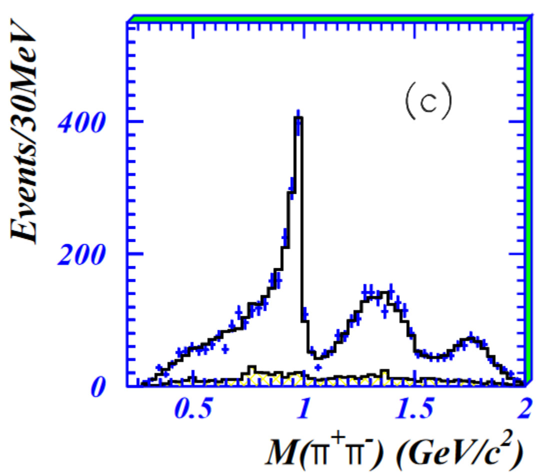
- T-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

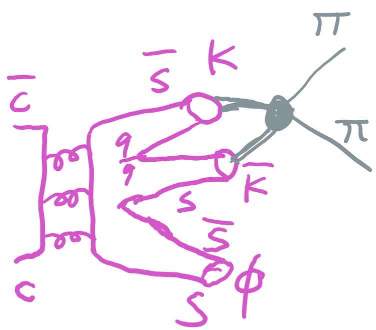
  with the T-matrix from
  L.-Y. Dai, M. R. Pennington, PRD90(2014)036004



- $f_0(980)$ in $J/\psi \to \phi\pi^+\pi^-$ and $J/\psi \to \omega\pi^+\pi^-$

Channels: $\pi\pi$ and $K\bar{K}$

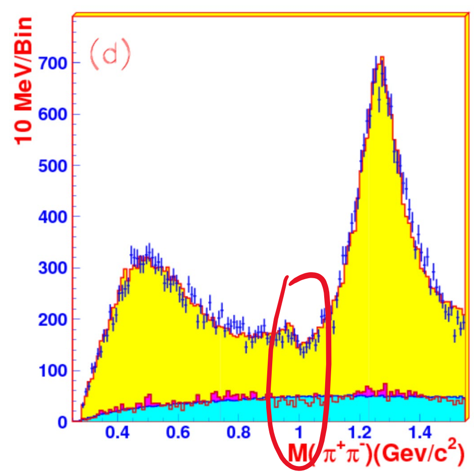


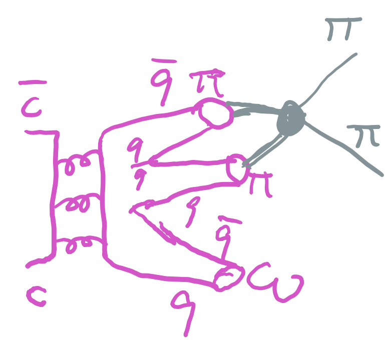


BES, PLB607(2005)243

Driving channel: $K\bar{K}$

$J/\psi \to \phi K\bar{K} \to \phi\pi^+\pi^-$





BES, PLB598(2004)149

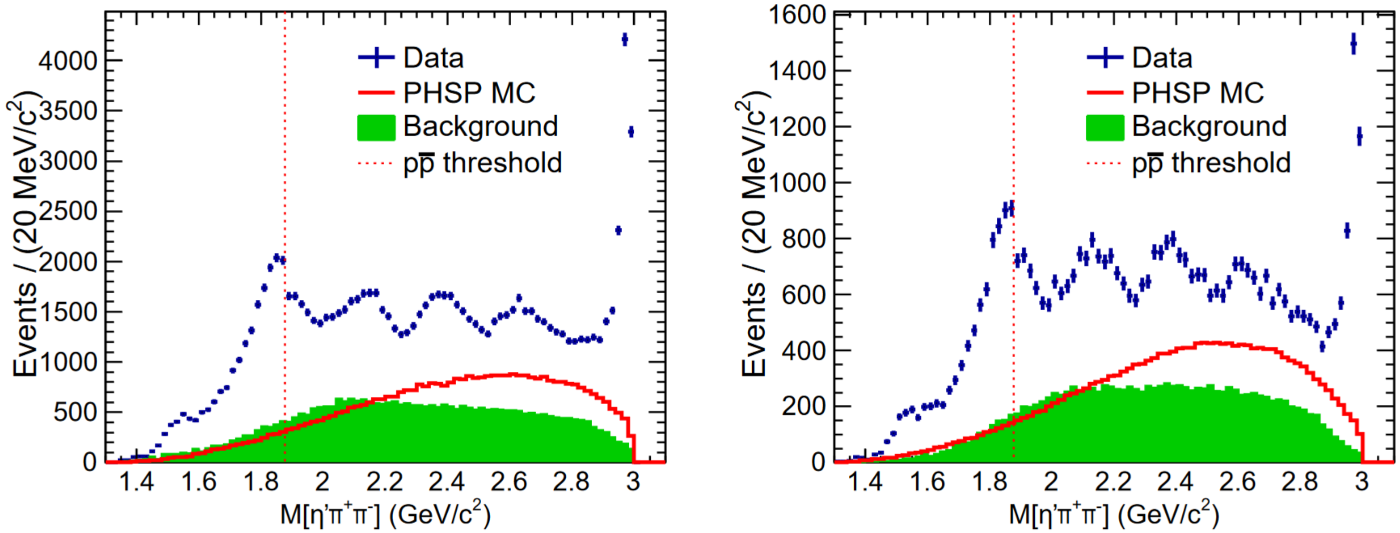
Driving channel: $\pi\pi$

$J/\psi \to \omega\pi\pi \to \omega\pi^+\pi^-$

Phenomenology

- $p\bar{p}$ threshold in $J/\psi \to \gamma\eta'\pi^+\pi^-$

BESIII, PRL117(2016)042002



Drastic drop:

- ➢ there should be a pole near the $p\bar{p}$ threshold
- ➢ $p\bar{p}$ is not the driving channel

- $D^{(*)}\bar{D}^{(*)}$ should be the driving channels for $X(3872), Z_c(3900), Z_c(4020)$

Phenomenology

- Production of states with hidden-charm and hidden-bottom: open-flavor much easier than $Q\bar{Q}$ + light hadrons, generally peaks around threshold of a pair of open-flavor hadrons for attractive interaction
- Complications due to more channels
- Threshold structures should be more pronounced in bottom than in charm
  - Either threshold cusp or below-threshold peak
  - peak width $\propto 1/m_Q$ for fixed a
  - Perturbative estimate of scattering length: $a \propto m_Q$ [potential independent of m_Q]; nonperturbative for strong attraction, near-threshold pole

Model estimate of near-th. interactions

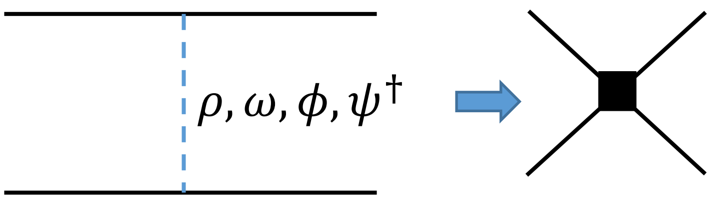
- Constant contact terms saturated by light-vector-meson exchange, similar to VMD in the resonance saturation of the low-energy constants in CHPT

V, A, S, S_1 and η_1 contributions to the coupling constants L_i^r in units of 10^{-3}.

	$L_i^r(M_\rho)$	V	A	S	S_1	η_1	Total
L_1^r	0.7 ± 0.3	0.6	0	-0.2	$0.2^{b)}$	0	0.6
L_2^r	1.3 ± 0.7	1.2	0	0	0	0	1.2
L_3	-4.4 ± 2.5	-3.6	0	0.6	0	0	-3.0
L_4^r	-0.3 ± 0.5	0	0	-0.5	$0.5^{b)}$	0	0.0
L_5^r	1.4 ± 0.5	0	0	$1.4^{a)}$	0	0	1.4
L_6^r	-0.2 ± 0.3	0	0	-0.3	$0.3^{b)}$	0	0.0
L_7	-0.4 ± 0.15	0	0	0	0	-0.3	-0.3
L_8^r	0.9 ± 0.3	0	0	$0.9^{a)}$	0	0	0.9
L_9^r	6.9 ± 0.7	$6.9^{a)}$	0	0	0	0	6.9
L_{10}^r	-5.2 ± 0.3	-10.0	4.0	0	0	0	-6.0

a) Input.
b) Large-N_C estimate.

Ecker, Gasser, Pich, de Rafael, NPB321(1989)311



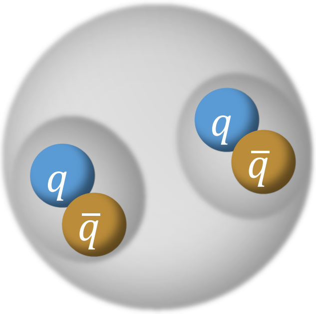
- List of attractive pairs

$H\bar{H}$	$D^{(*)}\bar{D}^{(*)}[0, 1^\dagger]$; $X(3872)$, $Z_c(3900, 4020)$ $Z_b(10610, 10650)$	$D_s^{(*)}\bar{D}^{(*)}$ $[\frac{1}{2}^\dagger]$; $Z_{cs}(3985)$	$D_s^{(*)}\bar{D}_s^{(*)}$ $[0]$ $X(4140)$
$\bar{H}T$	$\bar{D}^{(*)}\Xi_c$ $[0]$; $P_{cs}(4459)$	$\bar{D}_s^{(*)}\Lambda_c$ $[0^\dagger]$	
$\bar{H}S$	$\bar{D}^{(*)}\Sigma_c^{(*)}$ $[\frac{1}{2}]$; $P_c(4312, 4440, 4457)$ $\bar{D}^{(*)}\Omega_c^{(*)}$ $[\frac{1}{2}^\dagger]$	$\bar{D}_s^{(*)}\Sigma_c^{(*)}$ $[1^\dagger]$;	$\bar{D}^{(*)}\Xi_c'^{(*)}$ $[0]$;
$T\bar{T}$	$\Lambda_c\bar{\Lambda}_c$ $[0]$;	$\Lambda_c\bar{\Xi}_c$ $[\frac{1}{2}]$;	$\Xi_c\bar{\Xi}_c$ $[0, 1]$
$T\bar{S}$	$\Lambda_c\bar{\Sigma}_c^{(*)}$ $[1]$; $\Xi_c\bar{\Sigma}_c^{(*)}$ $[\frac{3}{2}^\dagger, \frac{1}{2}]$;	$\Lambda_c\bar{\Xi}_c'^{(*)}$ $[\frac{1}{2}]$; $\Xi_c\bar{\Xi}_c'^{(*)}$ $[1, 0]$;	$\Lambda_c\bar{\Omega}_c^{(*)}$ $[0^\dagger]$; $\Xi_c\bar{\Omega}_c^{(*)}$ $[\frac{1}{2}]$
$S\bar{S}$	$\Sigma_c^{(*)}\bar{\Sigma}_c^{(*)}$ $[2^\dagger, 1, 0]$; $\Xi_c'^{(*)}\bar{\Xi}_c'^{(*)}$ $[1, 0]$;	$\Sigma_c^{(*)}\bar{\Xi}_c'^{(*)}$ $[\frac{3}{2}^\dagger, \frac{1}{2}]$; $\Xi_c'^{(*)}\bar{\Omega}_c^{(*)}$ $[\frac{1}{2}]$;	$\Sigma_c^{(*)}\bar{\Omega}_c^{(*)}$ $[0^\dagger]$; $\Omega_c^{(*)}\bar{\Omega}_c^{(*)}$ $[0]$

Hadronic molecules

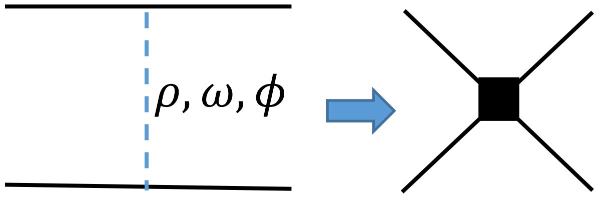
X.-K. Dong, FKG, B.-S. Zou, *A Survey of Heavy-Antiheavy Hadronic Molecules*,

物理学进展 41 (2021) 65 [arXiv:2101.01021]



Method

- Approximations:
  - Constant contact terms (V) saturated by light-vector-meson exchange, similar to the vector-meson dominance in the resonance saturation of the low-energy constants in CHPT G. Ecker, J. Gasser, A. Pich, E. de Rafael, NPB321(1989)311
  - Single channels
  - Neglecting mixing with normal charmonia



- The T-matrix:

$$T = \frac{V}{1 - VG}$$

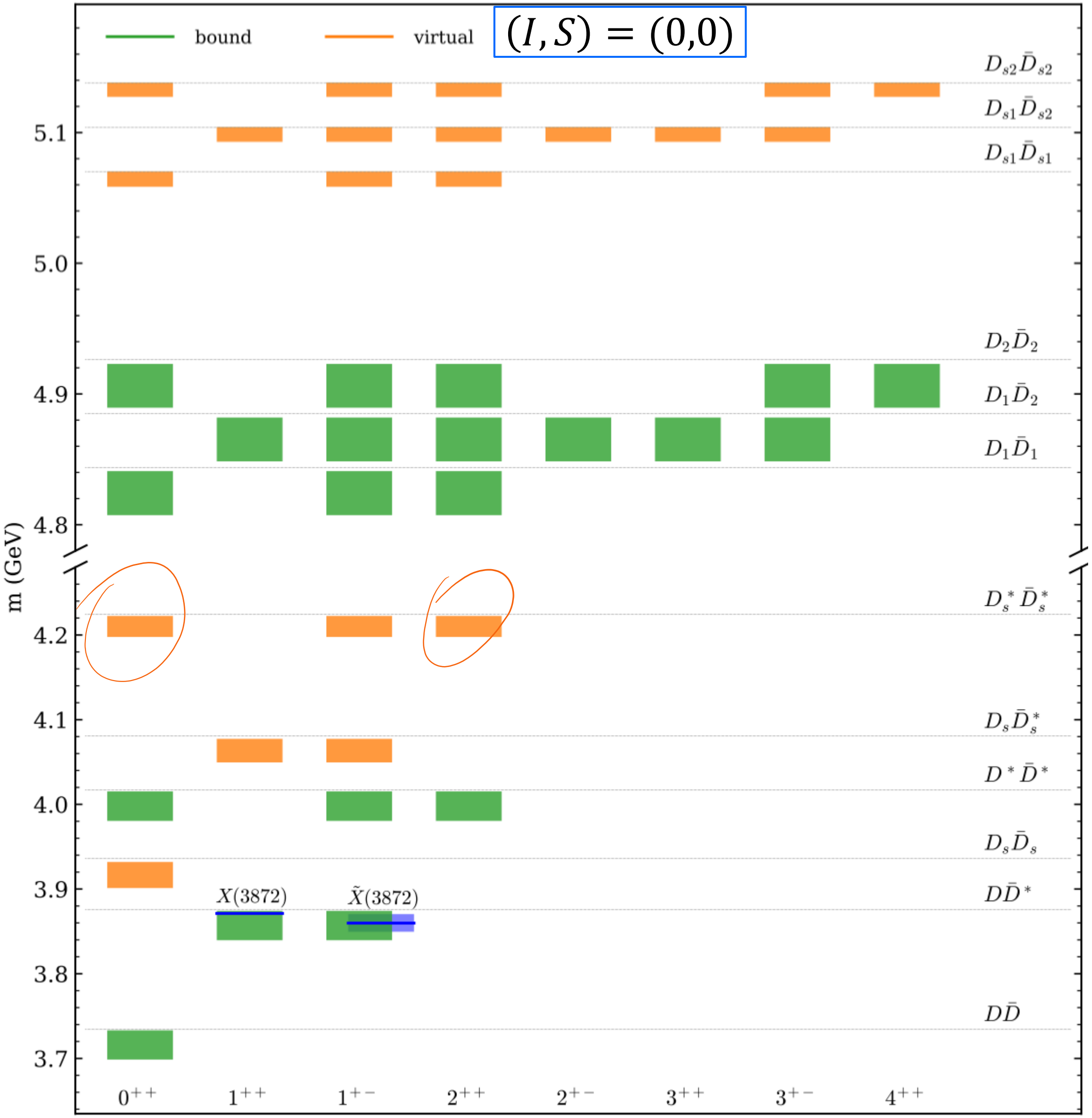
G: two-point scalar loop integral regularized using dim.reg. with a subtraction constant matched to a Gaussian regularized G at threshold

$$G(E) = \frac{1}{16\pi^2}\left\{a(\mu) + \log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\log\frac{m_2^2}{m_1^2} + \frac{k}{E}\log\frac{(2kE+s)^2 - m_1^2 + m_2^2}{(2kE-s)^2 - m_1^2 + m_2^2}\right\}$$

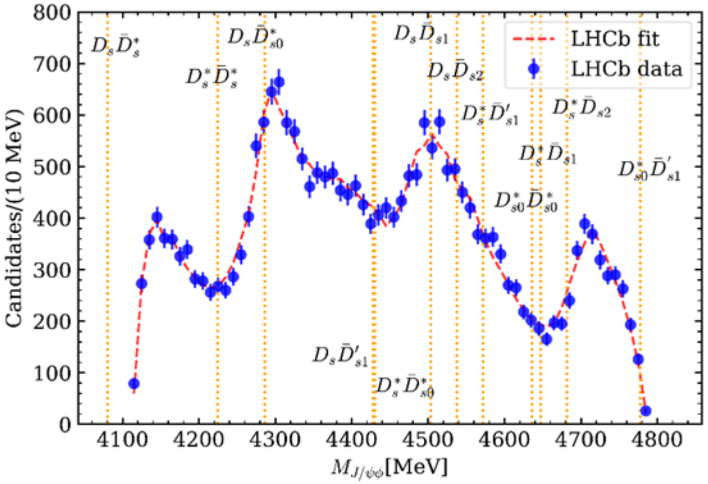
$$G(E) = \int \frac{l^2 dl}{4\pi^2}\frac{\omega_1+\omega_2}{\omega_1\omega_2}\frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1+\omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

- Hadronic molecules appear as bound or virtual state poles of the T matrix

X(3872) and related states

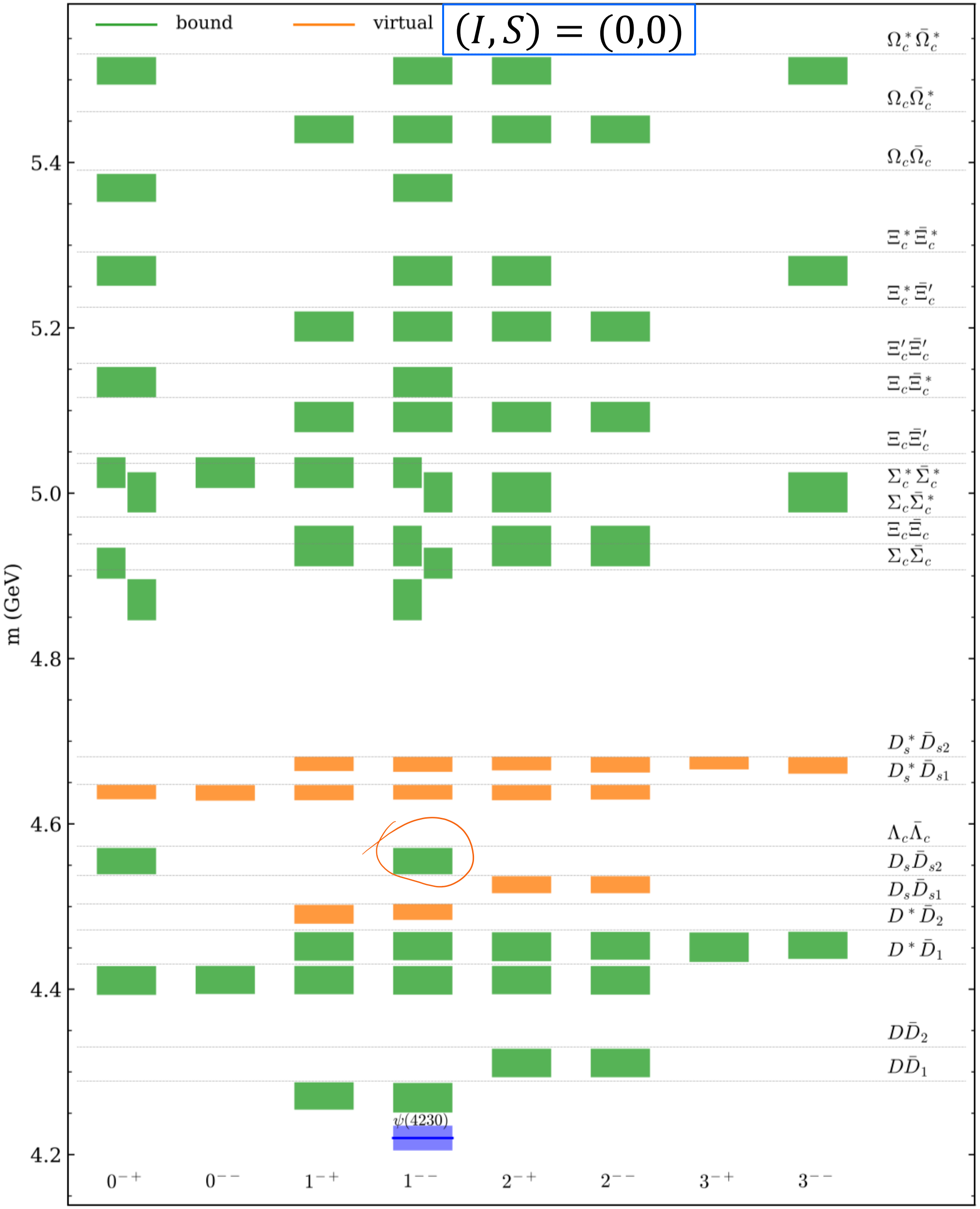


- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ Negative-C parity partner observed by COMPASS PLB783(2018)334
- ✓ $\bar{D}D$ bound state predicted with lattice Prelovsek et al., 2011.02542
- ✓ Evidence for a $D_s^*\bar{D}_s^*$ virtual state in LHCb data?

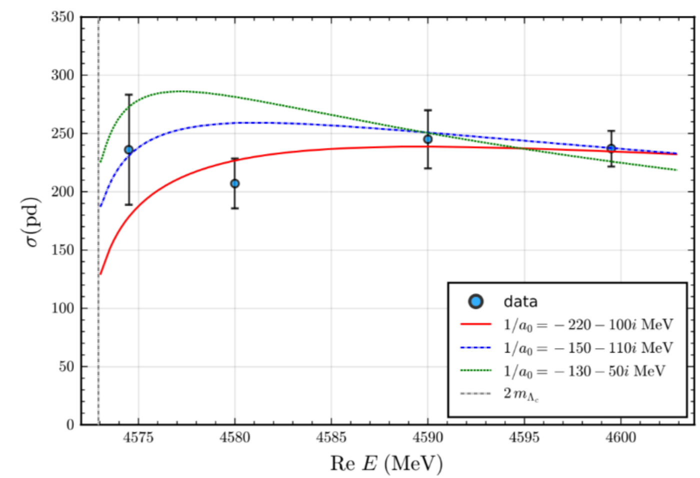


LHCb data: arXiv:2103.01803

Isoscalar vectors and related states



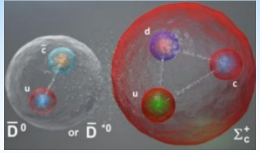
- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ Vector charmonia around 4.4 GeV unclear
- ✓ Evidence for 1^{--} $\Lambda_c\bar{\Lambda}_c$ bound state in BESIII data
  - Sommerfeld factor
  - Near-threshold pole
  - Different from $Y(4630/4660)$



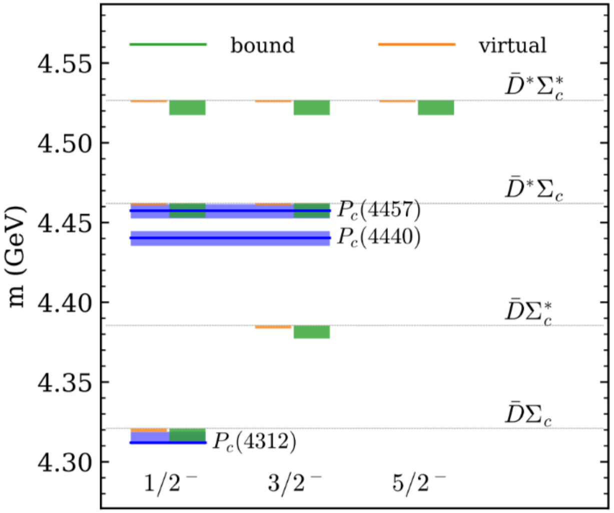
Data taken from BESIII, PRL120(2018)132001

- ✓ Many 1^{--} states above 4.8 GeV: Belle-II, BEPC-III, STCF

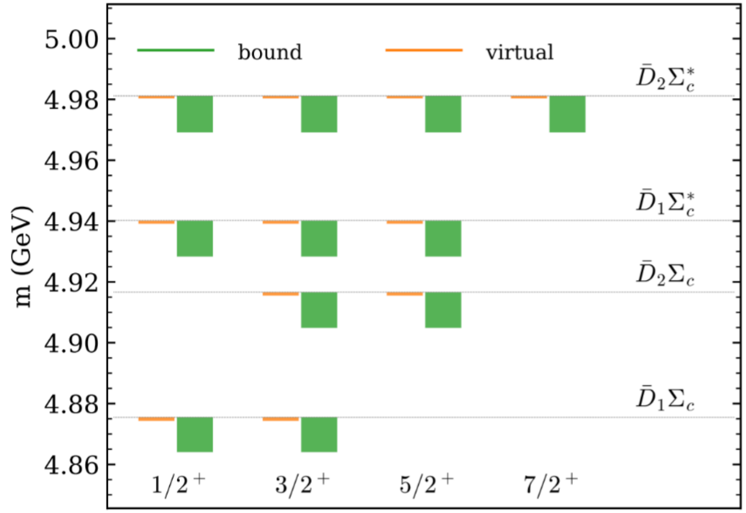
Hidden-charm pentaquarks



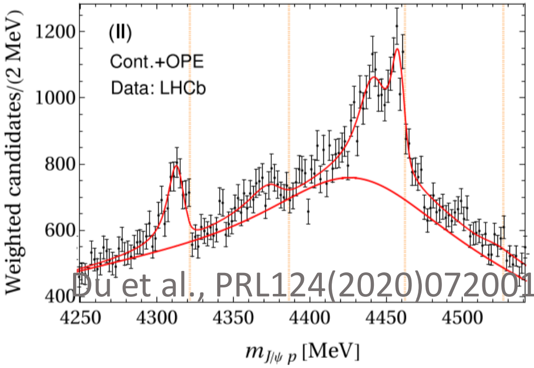
$(I, S) = (1/2, 0)$



$(I, S) = (1/2, 0)$

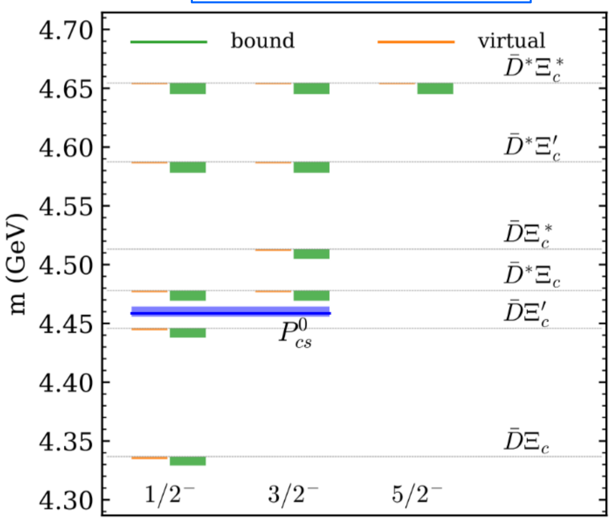


- ✓ The LHCb P_c states as $\bar{D}^{(*)}\Sigma_c$ molecules
- ✓ $\bar{D}\Sigma_c^*$ molecule: hint in the LHCb data

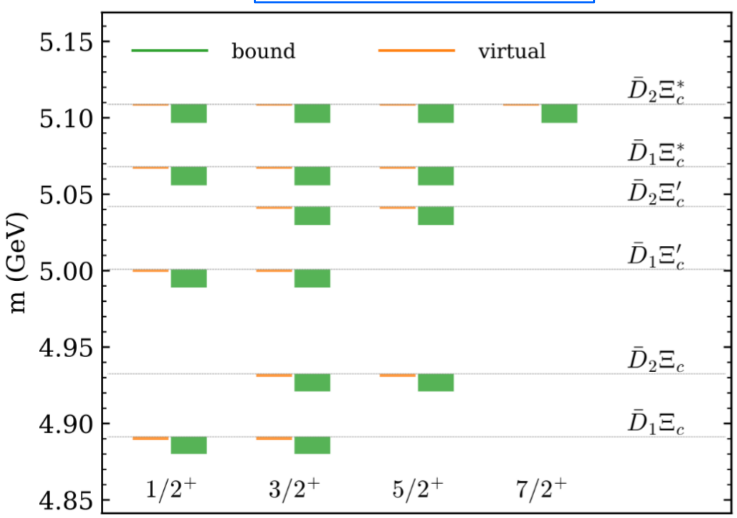


Du et al., PRL124(2020)072001

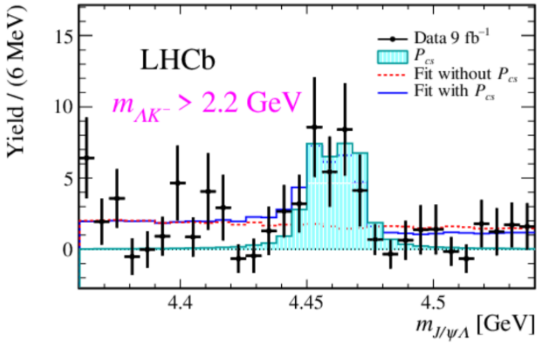
$(I, S) = (0, 1)$



$(I, S) = (0, 1)$



- ✓ The $P_{cs}(4459)$ could be two $\bar{D}^*\Xi_c$ molecules



LHCb, 2012.10380

- ✓ Many more baryon-antibaryon molecular states above 4.7 GeV

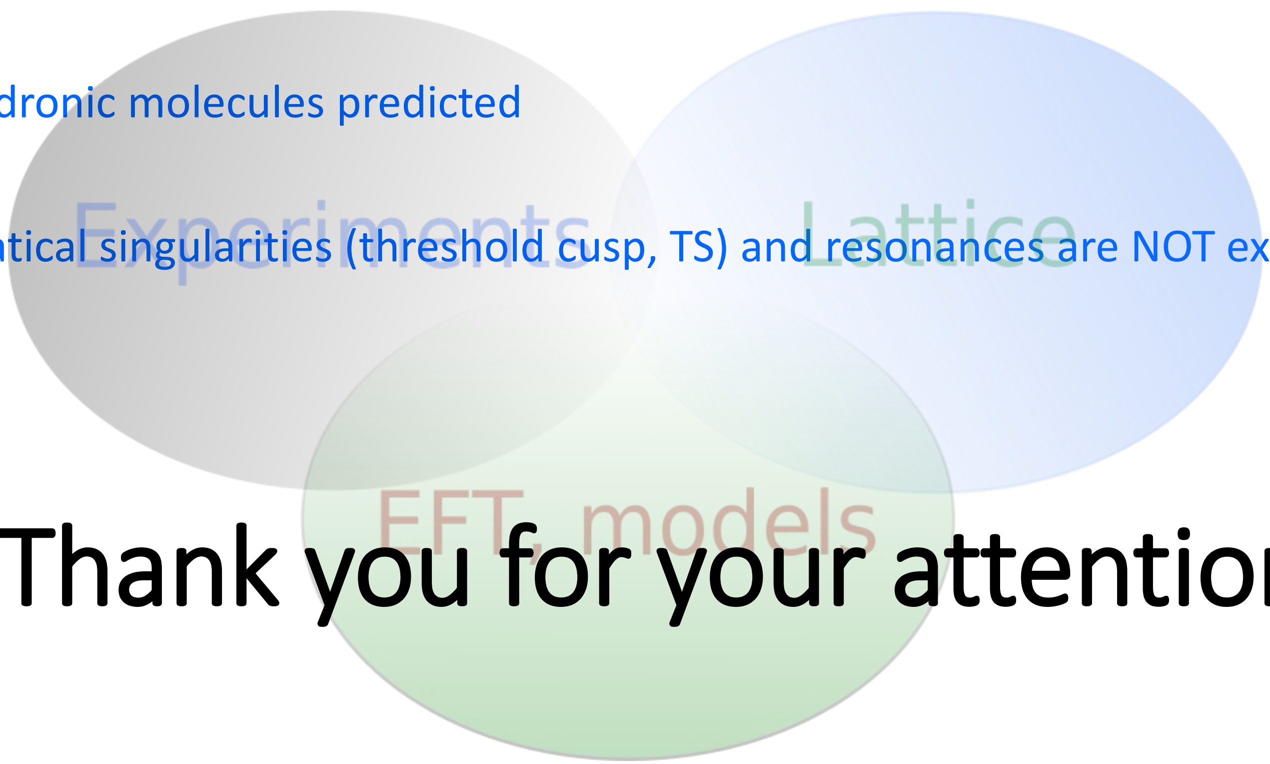
Conclusion

- General rule for (near-)threshold structures: S-wave attraction, more prominent for heavier particles and stronger attraction
- Strong attraction, then hadronic molecules below threshold, otherwise threshold cusps

- Structures should be more prominent in bottom than in charm

- 229 hadronic molecules predicted

- Kinematical singularities (threshold cusp, TS) and resonances are NOT exclusive

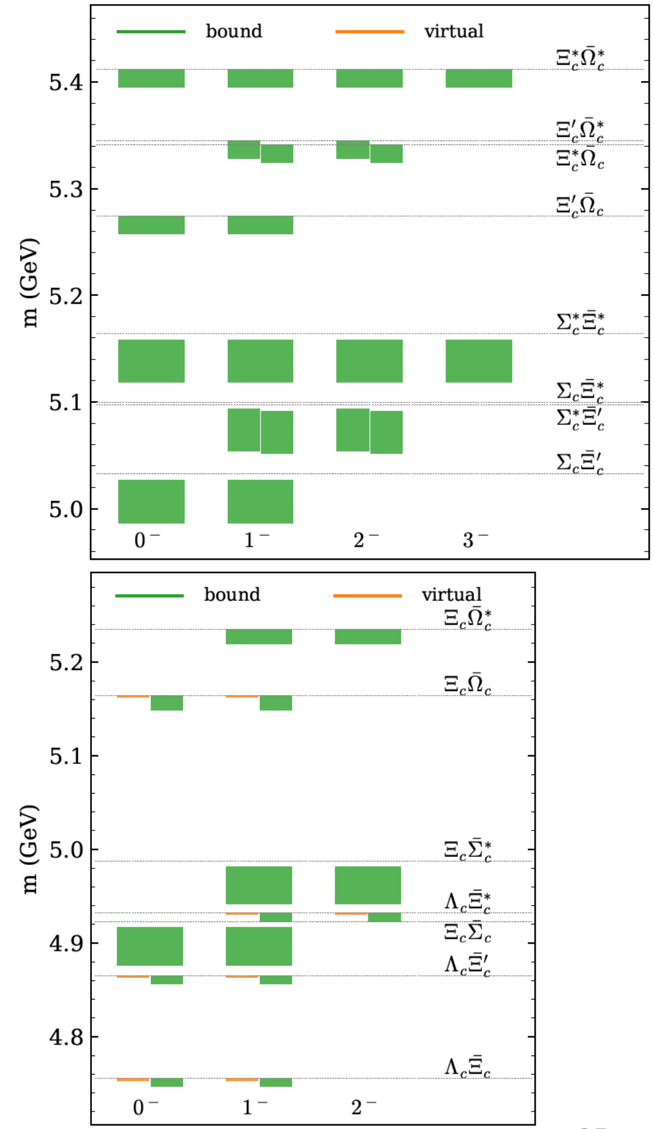


Thank you for your attention!

More states with exotic quantum numbers

$(I, S) = (1,0)$

$(I, S) = (1/2,1)$



✓ Many baryon-antibaryon molecular states above 4.7 GeV, beyond the current exp. region

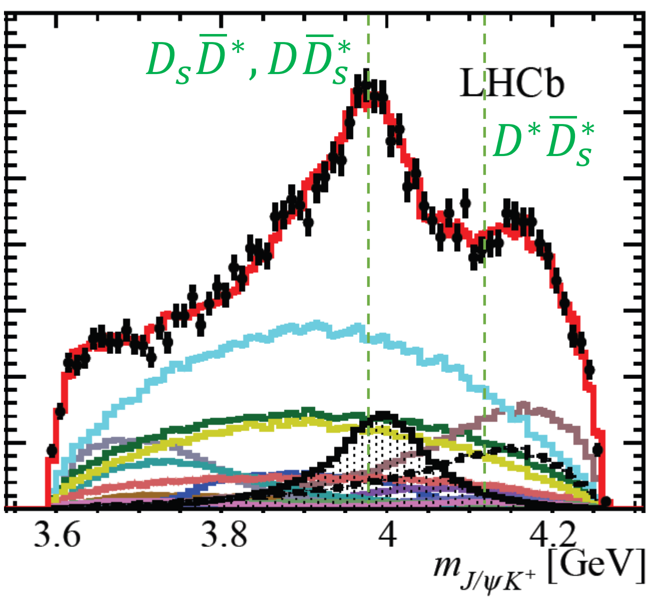
Comments on Z_c and Z_{cs}

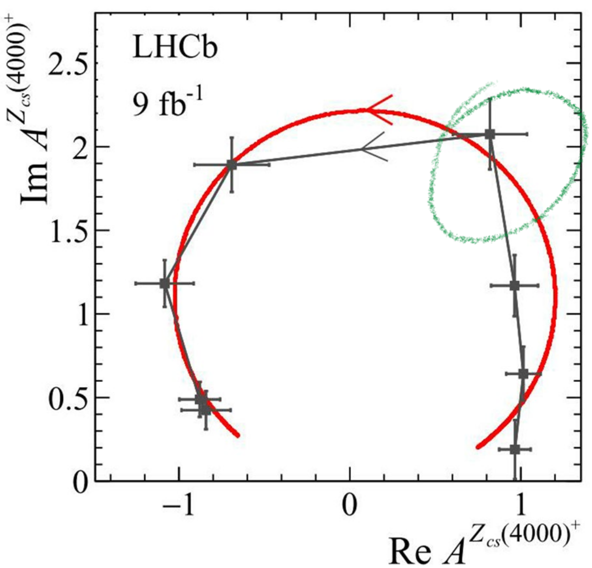
- ✓ Isovector interaction between $D^{(*)}\bar{D}^{(*)}$ from light vector exchange vanishes
- ✓ Charmonia exchange might be important here: F.Aceti, M.Bayar, E.Oset et al., PRD90(2014)016003

  no mass hierarchy, a series of charmonia can be exchanged Dong, FKG, Zou, arXiv:2101.01021

  axial-vector meson exchange considered in Yan, Peng, Sanchez Sanchez, Pavon Valderrama 2102.13058
- ✓ $Z_c(3900{,}4020)$ as $\bar{D}^{(*)}D^*$ virtual states
- ✓ $Z_{cs}(3985)$ as $D_s\bar{D}^*, D\bar{D}_s^*$ virtual state; there should also be a $D^*\bar{D}_s^*$ state around 4.1 GeV

  Z. Yang, X. Cao, FKG, J. Nieves, M. Pavon Valderrama, PRD103(2021)074029





Threshold behavior with a strong coupling

LHCb, arXiv: 2103.01803

Comments on Z_c and Z_{cs}

✓ Simultaneous fit to the BESIII and LHCb Z_{cs} data: Z_{cs} as virtual states

Ortega, Entem, Fernandez, 2103.07781