Tree Unitarity, Gauge Invariance, and Coupling-Kinematics in On-Shell Amplitudes

Based on 2204.13119 with Da Liu, and ongoing work with Henrik Johansson

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Scattering amplitudes

The modern scattering amplitudes program

- Constructing amplitudes in purely on-shell ways
- Studying the properties of on-shell amplitudes

Why?

- Avoid redundancies of a local formulation, including EoM, field redefinitions, gauge invariance etc.
- Unveil properties of amplitudes obscured in a local formulation, e.g. the color-kinematics duality

The color-kinematics duality

Consider Yang-Mills

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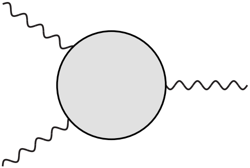
- Only controlled by a single coupling parameter (tensor): f^{abc}



$$\mathcal{A}(1^-2^-3^+) \;=\; f_{abc}\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle},$$

Spinor-helicity variables:

- Taking massless momentum p_μ:

$$p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}}$$

- $\det p = 0$, thus $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = (|\lambda\rangle[\tilde{\lambda}|)_{\alpha\dot{\alpha}}$
- $\langle 12\rangle = \lambda_{1\alpha}\lambda_{2\beta}\varepsilon^{\alpha\beta}$, $[12] = \tilde{\lambda}_{1\dot{\alpha}}\tilde{\lambda}_{2\dot{\beta}}\varepsilon^{\dot{\alpha}\dot{\beta}}$

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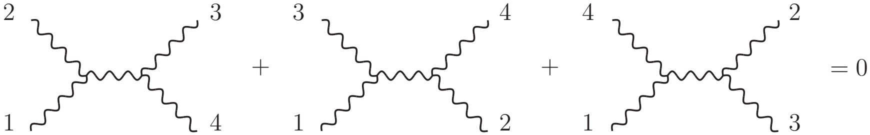
- At 4-pt, 3 factorization channels
- One can write

$$\mathcal{A}_4 = \sum_{I \in \{s,t,u\}} \frac{\mathsf{c}_I \; n_I}{d_I},$$

with

$$\mathsf{c}_s = f^{a_1 a_2 b} f^{b a_3 a_4}, \;\; \mathsf{c}_t = f^{a_1 a_4 b} f^{b a_2 a_3}, \;\; \mathsf{c}_u = f^{a_1 a_3 b} f^{b a_4 a_2},$$

and $c_s + c_t + c_u = 0$.



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and $c_s + c_t + c_u = 0$.

- $\exists \; n_I$, s.t. $n_s + n_t + n_u = 0$.

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Bern, Carrasco, Johansson, 0805.3993

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This is nice because

- Simplify computation for high-multiplicity/high-loop level
- Replacing c_g with n_g leads to gravity amplitudes:

$$\mathcal{M}_n = \sum_g \frac{n_g n_g}{d_g}.$$

Double copy: gravity $=$ (gauge theory)^2.

Review: Bern, Carrasco, Chiodaroli, Johansson, Roiban, 1909.01358

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- Works for all kinds of (S)YM
- Also works for NLSM
- EFTs?
  e.g. for YM, F^3 fine, but only a special combination of F^3 and F^4 works for dim-8

Broedel, Dixon, 1208.0876

EFT example: NLSM

Consider the Lagrangian:

$$\mathcal{L}^{(2)} = \frac{f^2}{2} \mathrm{tr}\left(d_\mu d^\mu\right),$$

where

$$d_\mu = \frac{1}{f}\left[\frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}\right]_{ab} \mathsf{X}^a \partial_\mu \pi^b, \ (\mathcal{T})_{ab} = \frac{1}{f^2} T^i_{ac} T^i_{db} \pi^c \pi^d.$$

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Here we have

$$\mathsf{c}_s = T^i_{a_1a_2}T^i_{a_3a_4},\ \mathsf{c}_t = T^i_{a_1a_4}T^i_{a_2a_3},\ \mathsf{c}_u = T^i_{a_1a_3}T^i_{a_4a_2}.$$

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No $\mathcal{O}(\partial^4)$ and only one $\mathcal{O}(\partial^6)$ operator, added to the above, satisfies color-kinematics duality.

Carrasco, Mafra, Schlotterer, 1608.02569
Elvang, Hadjiantonis, Jones, Paranjape, 1806.06079; Carrillo-Gonzalez, Penco, Trodden, 1908.07531

EFT example: NLSM

Relaxing the definition of the "color" / "flavor" numerators: e.g. at 4-pt we can have

$$\mathsf{c}'_s = \frac{1}{\Lambda^2}\left[\mathsf{c}_t(u-s) - \mathsf{c}_u(s-t)\right].$$

Carrasco, Rodina, Yin, Zekioglu, 1910.12850; Low, **ZY**, 1911.08490

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Still, of the following 4 $\mathcal{O}(\partial^4)$ operators:

$$O_1 = [\mathrm{tr}(d_\mu d^\mu)]^2, \qquad O_2 = [\mathrm{tr}(d_\mu d_\nu)]^2,$$
$$O_3 = \mathrm{tr}([d_\mu, d_\nu]^2), \qquad O_4 = \mathrm{tr}(\{d_\mu, d_\nu\}^2),$$

only the combination $O_1 - 2O_2$ satisfies color-kinematics duality.

Low, Rodina, **ZY**, 2009.00008

Question: Where does the Jacobi identity in YM come from?

Consider massless amplitudes. Locality, momentum conservation and little group scaling:

$$\mathcal{A}(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_1-h_3}, & h<0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_1+h_3-h_2}, & h>0 \end{cases}$$

where $h = \sum_i h_i$.

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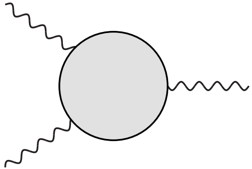

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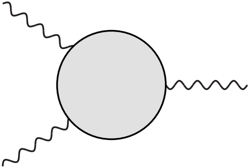

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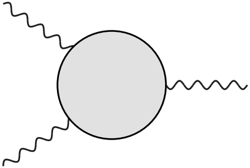

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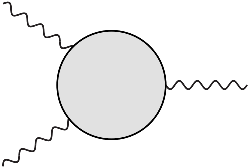

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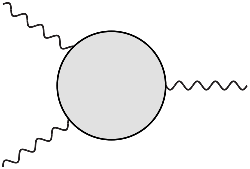

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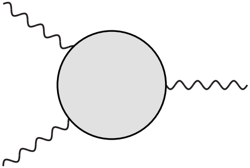

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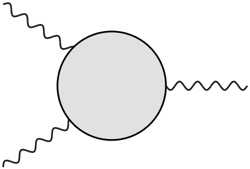

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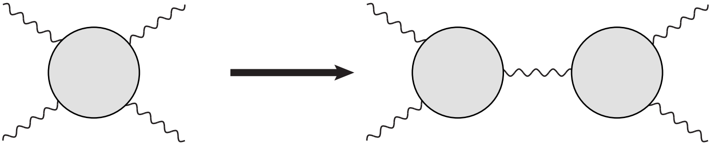

Emergence of the gauge group

$$\mathcal{A}(1^{a-}2^{b-}3^{c+}4^{d+}) = \langle 12\rangle^2[34]^2\left(\frac{c_{st}}{st}+\frac{c_{tu}}{tu}+\frac{c_{us}}{su}\right)$$

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Factorization leads to

$$\begin{aligned} c_{st}-c_{us} &= f_{abe}f_{cde}, \\ c_{tu}-c_{st} &= f_{bce}f_{ade}, \\ c_{us}-c_{tu} &= f_{cae}f_{bde} \end{aligned}$$

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$$f_{abe}f_{cde}+f_{bce}f_{ade}+f_{cae}f_{bde}=0.$$

The Jacobi identity. The vector states furnish the adjoint representation of some Lie group G. Tree unitarity $\rightarrow$ symmetry!

Benincasa, Cachazo, 0705.4305

Symmetry from unitarity

Tree level unitarity means

- Factorization
- For the n-pt tree amplitude $\mathcal{A}_n$, when taking the high energy limit,

$$\mathcal{A}_n \sim \mathcal{O}(E^{4-n})$$

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For example,

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Gauge theory from unitarity

Question: what is the most general renormalizable QFT with a finite spectrum of spin-0, $1/2$ and 1 states?

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Answer:

PHYSICAL REVIEW D VOLUME 10, NUMBER 4 15 AUGUST 1974

Derivation of gauge invariance from high-energy unitarity bounds on the S matrix*

John M. Cornwall,[†] David N. Levin, and George Tiktopoulos
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
(Received 21 March 1974)

A systematic search is made for all renormalizable theories of heavy vector bosons. It is argued that in any renormalizable Lagrangian theory high-energy unitarity bounds should not be violated in perturbation theory (apart from logarithmic factors in the energy). This leads to the specific requirement of "tree unitarity": the N-particle S-matrix elements *in the tree approximation* must grow no more rapidly than E^{4-N} in the limit of high energy (E) at fixed, nonzero angles (i.e., at angles such that all invariants $p_i \cdot p_j$, $i \neq j$, grow like E^2). We have imposed this tree-unitarity criterion on the most general scalar, spinor, and vector Lagrangian with terms of mass dimension less than or equal to four; a certain class of nonpolynomial Lagrangians is also considered. It is proved that any such theory is tree-unitary if and only if it is equivalent under a point transformation to a spontaneously broken gauge theory, possibly modified by the addition of mass terms for vectors associated with invariant Abelian subgroups. Our result suggests that gauge theories are the only renormalizable theories of massive vector particles and that the existence of Lie groups of internal symmetries in particle physics can be traced to the requirement of renormalizability.

Gauge theory from unitarity

Question: what is the most general renormalizable QFT with a finite spectrum of spin-0, $1/2$ and 1 states?
Short answer: a (spontaneously broken) gauge theory

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Key observation: the Jacobi identity is just a special case of invariant tensor relations.

The unbroken phase

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- Relations dictated by the gauge group (emerging from the tree unitarity constraints):

$$
\begin{aligned}
& f^{abe} f^{cde} + f^{ace} f^{dbe} + f^{ade} f^{bce} = 0, \\
& [T^a, T^b] = i f^{abc} T^c, \\
& [L^a, L^b] = i f^{abc} L^c, \qquad [R^a, R^b] = i f^{abc} R^c, \\
& L^a Y_i - Y_i R^a - Y_j T^a_{ji} = 0, \\
& P_{ijl} T^a_{lk} + P_{jkl} T^a_{li} + P_{kil} T^a_{lj} = 0, \qquad K_{i_1 i_2 i_3 j} T^a_{j i_4} + \text{cycl} = 0.
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- Our claim: whenever there is a relation for the couplings, there are corresponding kinematic numerators that satisfy such a relation

Example: VSF^2

The Yukawa coupling needs to be an invariant tensor:

$$L^a_{AC}(Y_i)_{CB} - (Y_i)_{AC}R^a_{CB} - (Y_j)_{AB}T^a_{ji} = 0.$$

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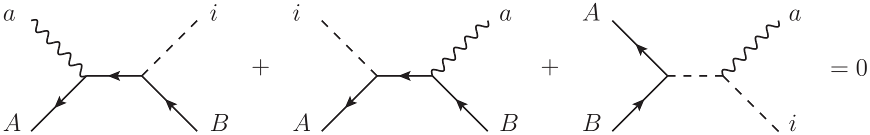
$$\mathcal{A}_4 = \sum_g \frac{\mathsf{c}_g \ n_g}{d_g}$$

- c_g satisfies the identity



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- c_g satisfies the identity
- $\exists \; n_g$ satisfying the identity:

$$n_s = -2ip_3 \cdot \varepsilon_4 \bar{v}_{2L} u_{1R}, \;\; n_t = i\bar{v}_{2L} \not{p}_3 \not{\varepsilon}_4 u_{1R}, \;\; n_u = i\bar{v}_{2L} \not{\varepsilon}_4 \not{p}_3 u_{1R}.$$

The broken phase

- The relations among couplings are more complicated because of the broken symmetry

Cornwall, Levin, Tiktopoulos, 1973; Llewellyn Smith, 1973

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- An on-shell bootstrap can be done to study these relations, similar to the unbroken phase

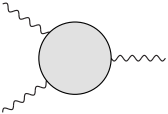
Liu, **ZY**, 2204.13119

Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652

s_1	s_2	s_3	$n^{\text{3-pt}}$	n_{rel}	spinor structures
0	0	0	1		constant
0	0	1	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle$
0	0	2	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2$
0	0	3	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^3$
0	1/2	1/2	2		$([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	3/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	5/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1	1	3		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	2	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	3	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	3/2	3/2	4		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	3/2	5/2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	2	2	5		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	2	3	5		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	5/2	5/2	6		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5)$
0	3	3	7		$([\mathbf{23}]^6, \langle\mathbf{23}\rangle[\mathbf{23}]^5, \langle\mathbf{23}\rangle^2[\mathbf{23}]^4, \langle\mathbf{23}\rangle^3[\mathbf{23}]^3, \langle\mathbf{23}\rangle^4[\mathbf{23}]^2, \langle\mathbf{23}\rangle^5[\mathbf{23}], \langle\mathbf{23}\rangle^6)$
1/2	1/2	1	4		$([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	3	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	3/2	6		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	5/2	6		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	2	8		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	3	8		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	2	5/2	10		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	5/2	3	12		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	1	1	7	1	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$

Example: V^3

V^3:



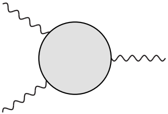
$$
\begin{aligned}
& C_{W^3,1}\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle + C_{W^3,2}\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + C_{W^3,3}\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle \\
+ \quad & C_{W^3,6}[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + C_{W^3,7}[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle + C_{W^3,8}[\mathbf{12}][\mathbf{13}][\mathbf{23}] \\
+ \quad & C_{W^3,4}[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle.
\end{aligned}
$$

where e.g. $\mathbf{1} = 1^I$.

Arkani-Hamed, Huang, Huang, 1709.04891

Example: V^3

V^3:



$$\begin{aligned} & C_{W^3,1}\langle \mathbf{12}\rangle\langle \mathbf{13}\rangle\langle \mathbf{23}\rangle + C_{W^3,2}\langle \mathbf{12}\rangle\langle \mathbf{13}\rangle[\mathbf{23}] + C_{W^3,3}\langle \mathbf{12}\rangle[\mathbf{13}]\langle \mathbf{23}\rangle \\ + \quad & C_{W^3,6}[\mathbf{12}]\langle \mathbf{13}\rangle[\mathbf{23}] + C_{W^3,7}[\mathbf{12}][\mathbf{13}]\langle \mathbf{23}\rangle + C_{W^3,8}[\mathbf{12}][\mathbf{13}][\mathbf{23}] \\ + \quad & C_{W^3,4}[\mathbf{12}]\langle \mathbf{13}\rangle\langle \mathbf{23}\rangle. \end{aligned}$$

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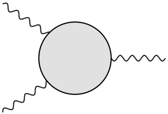
Arkani-Hamed, Huang, Huang, 1709.04891

In the HE limit, e.g.

- $\mathcal{M}(---) \to C_{W^3,1}\langle 12\rangle\langle 13\rangle\langle 23\rangle = \mathcal{O}(E^3)$.

Example: V^3

V^3:



$$
\begin{aligned}
& C_{W^3,1}\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle + C_{W^3,2}\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + C_{W^3,3}\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle \\
+ \;& C_{W^3,6}[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + C_{W^3,7}[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle + C_{W^3,8}[\mathbf{12}][\mathbf{13}][\mathbf{23}] \\
+ \;& C_{W^3,4}[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle.
\end{aligned}
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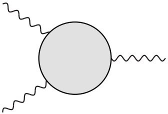
Arkani-Hamed, Huang, Huang, 1709.04891

In the HE limit, e.g.

- $\mathcal{M}(---) \to C_{W^3,1}\langle 12\rangle\langle 13\rangle\langle 23\rangle = \mathcal{O}(E^3)$.
- $\mathcal{M}(--0) \to (C_{W^3,2} - C_{W^3,3})\langle 12\rangle^2 = \mathcal{O}(E^2)$.

Tree unitary 3-pt amplitudes: V^3

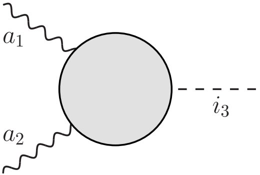
V^3:



$$\frac{i\sqrt{2}C_{a_1a_2a_3}}{m_{a_1}m_{a_2}m_{a_3}}\left(m_{a_2}\langle\mathbf{12}\rangle\langle\mathbf{23}\rangle[\mathbf{31}]+\text{cycl}\right),$$

where C_{abc} has to be totally antisymmetric.

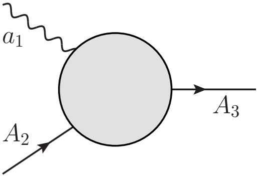
V^2S: $3 \to 1$



$$2F_{a_1a_2i_3}\frac{[\mathbf{12}]\langle\mathbf{21}\rangle}{m_{a_1}m_{a_2}},$$

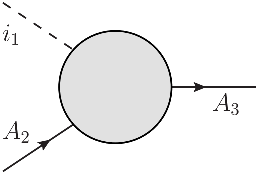
where $F_{abi} = F_{bai}$.

VF^2: $4 \to 2$



$$\frac{\sqrt{2}}{m_{a_1}}\left(R^{a_1}_{A_3A_2}[\mathbf{12}]\langle\mathbf{13}\rangle + L^{a_1}_{A_3A_2}\langle\mathbf{12}\rangle[\mathbf{13}]\right).$$

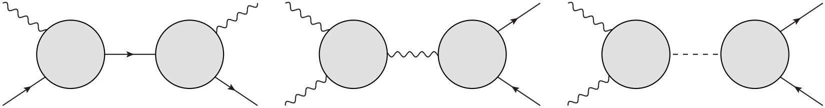
SF^2:



$$(Y_{i_1})_{A_3A_2}[\mathbf{23}] + (Y^\dagger_{i_1})_{A_3A_2}\langle\mathbf{23}\rangle.$$

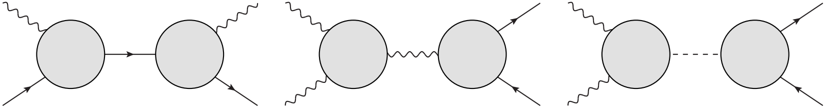
4-pt example: V^2F^2

Possible factorizations:



4-pt example: V^2F^2

Possible factorizations:



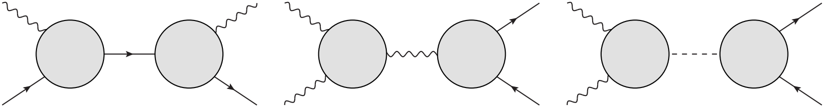
Possible contact terms:

$vvff$	$46 \to 38$	36	$(00{+}{+})$	$\langle \mathbf{12} \rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
			$(00{+}{-})$	$\langle \mathbf{14} \rangle \langle \mathbf{231}][\mathbf{23}], \langle \mathbf{24} \rangle \langle \mathbf{132}][\mathbf{13}]$	2	6
			$(0{-}{+}{+})$	$\langle \mathbf{12} \rangle [\mathbf{34}] \langle \mathbf{241}] \to \langle \mathbf{12} \rangle [\mathbf{34}] (\langle \mathbf{241}]/m_1 - \langle \mathbf{142}]/m_2)$	$\not 4 \to 2$	7
			$(0{+}{+}{+})$	$\langle \mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			$(0{+}{+}{-})$	$\langle \mathbf{14} \rangle [\mathbf{12}][\mathbf{23}]$	8	6
			$({+}{+}{+}{-})$	$[\mathbf{12}]^2 [\mathbf{314} \rangle$	4	8
			$({+}{+}{+}{+})$	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			$({-}{+}{+}{+})$	$\langle \mathbf{1231} \rangle [\mathbf{23}][\mathbf{24}] \to \varnothing$	$\not 4 \to 0$	9
			$({+}{+}{-}{-})$	$[\mathbf{12}]^2 \langle \mathbf{34} \rangle$	2	7
			$({+}{-}{-}{+})$	$[\mathbf{14}][\mathbf{132} \rangle \langle \mathbf{23} \rangle \to [\mathbf{14}][\mathbf{132} \rangle \langle \mathbf{23} \rangle - [\mathbf{24}][\mathbf{231} \rangle \langle \mathbf{13} \rangle$	$\not 4 \to 2$	8

Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652

4-pt example: V^2F^2

Possible factorizations:



Possible contact terms:

$vvff$	$46 \to 38$	36	$(00{+}{+})$	$\langle \mathbf{12}\rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
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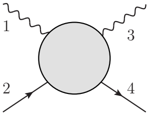
			$(0{+}{+}{+})$	$\langle \mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			$(0{+}{+}{-})$	$\langle \mathbf{14}\rangle[\mathbf{12}][\mathbf{23}]$	8	6
			$({+}{+}{+}{-})$	$[\mathbf{12}]^2[\mathbf{314}\rangle$	4	8
			$({+}{+}{+}{+})$	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			$({-}{+}{+}{+})$	$\langle \mathbf{1231}\rangle[\mathbf{23}][\mathbf{24}] \to \emptyset$	$\not{4} \to 0$	9
			$({+}{+}{-}{-})$	$[\mathbf{12}]^2\langle \mathbf{34}\rangle$	2	7
			$({+}{-}{-}{+})$	$[\mathbf{14}][\mathbf{132}\rangle\langle \mathbf{23}\rangle \to [\mathbf{14}][\mathbf{132}\rangle\langle \mathbf{23}\rangle - [\mathbf{24}][\mathbf{231}\rangle\langle \mathbf{13}\rangle$	$\not{4} \to 2$	8

Durieux, Kitahara, Machado, Shadmi, Weiss, 2008.09652

$\mathcal{M}_{4,\mathrm{f}} = \mathcal{O}(E^2)$, eliminating all contact terms.

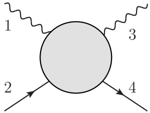
4-pt example: V^2F^2

V^2F^2:



$\mathcal{M}_{4,\mathsf{f}} = \mathcal{O}(E^2)$, eliminating all contact terms.

$V^2 F^2$:



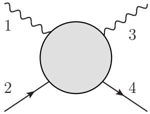
$\mathcal{M}_{4,\mathsf{f}} = \mathcal{O}(E^2)$, eliminating all contact terms.

- $\mathcal{O}(E^2)$ for $(0{-}0{+})$ and $(0{+}0{-})$, giving the relations:

$$iC_{a_1 a_3 b} L^b = [L^{a_1}, L^{a_3}],\ iC_{a_1 a_3 b} R^b = [R^{a_1}, R^{a_3}].$$

4-pt example: V^2F^2

V^2F^2:



$\mathcal{M}_{4,\text{f}} = \mathcal{O}(E^2)$, eliminating all contact terms.

- $\mathcal{O}(E^2)$ for $(0{-}0{+})$ and $(0{+}0{-})$, giving the relations:

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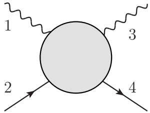
- $\mathcal{O}(E)$ for $(0{+}0{+})^*$, giving a relation:

$$\begin{aligned} & 2F_{a_1 a_3 i}(Y_i)_{A_4 A_2} - m_{A_2}\left\{L^{a_1}, L^{a_3}\right\}_{A_4 A_2} - m_{A_4}\left\{R^{a_1}, R^{a_3}\right\}_{A_4 A_2} \\ & + \sum_B 2m_B\left(L^{a_1}_{A_4 B} R^{a_3}_{B A_2} + L^{a_3}_{A_4 B} R^{a_1}_{B A_2}\right) \\ = \ \ & \sum_b iC^{a_1 a_3 b}\frac{\left(m^2_{a_1} - m^2_{a_3}\right)}{m_b^2}\left(m_{A_2} L^b_{A_4 A_2} - m_{A_4} R^b_{A_4 A_2}\right). \end{aligned}$$

* $(0{-}0{-})$ gives the conjugate of the above.

V^2F^2:



$\mathcal{M}_{4,\mathrm{f}} = \mathcal{O}(E^2)$, eliminating all contact terms.

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- $\mathcal{O}(E)$ for $(0{+}0{+})^*$, giving a relation:

$$L^aY_b - Y_bR^a - Y_{\tilde{i}}T^a_{\tilde{i}b} = 0,$$

if we recognise

$$T^a_{ib} = -T^a_{bi} = \frac{i}{m_b}F_{abi},\ T^a_{bc} = iC_{abc}\frac{m_a^2 - m_b^2 - m_c^2}{2m_bm_c},$$

$$(Y_a)_{AB} = \frac{i}{m_a}(m_BL^a - m_AR^a)_{AB}\,.$$

* $(0{-}0{-})$ gives the conjugate of the above.

Goldstone boson equivalence

$$(Y_a)_{AB} \quad = \quad \frac{i}{m_a}\left(m_B L^a - m_A R^a\right)_{AB}.$$

Why such redefinitions? Consider the (0++) components of the following amplitudes in the HE limit:

Goldstone boson equivalence

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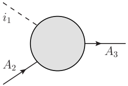

Goldstone boson equivalence

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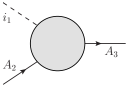

$$\Downarrow$$

$$(Y_{i_1})_{A_2 A_3}[23]$$

Goldstone boson equivalence

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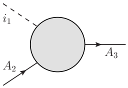
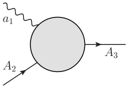


$\Downarrow$

$(Y_{i_1})_{A_2 A_3}[23]$



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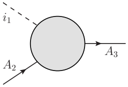
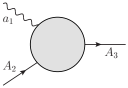


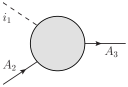


$$\Downarrow \qquad\qquad \Downarrow$$

$$(Y_{i_1})_{A_2 A_3}[23] \qquad\qquad \frac{i}{m_{a_1}}\left(m_{A_2} L^{a_1} - m_{A_3} R^{a_1}\right)_{A_3 A_2}[23]$$

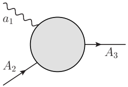
$$(Y_a)_{AB} \;=\; \frac{i}{m_a}(m_B L^a - m_A R^a)_{AB}\,.$$

Why such redefinitions? Consider the (0++) components of the following amplitudes in the HE limit:



$\Downarrow$

$$(Y_{i_1})_{A_2A_3}[23]$$



$\Downarrow$

$$(Y_{a_1})_{A_2A_3}[23]$$

In the HE limit, the longitudinal component of the vectors are equivalent to (Goldstone) scalars, which together with (Higgs) scalars furnish some representation of G

$WFWF$:

- $\mathcal{O}(E^2)$ for $(0{-}0{+})$ and $(0{+}0{-})$, giving the relations:

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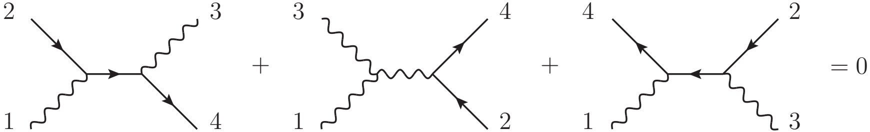

Coupling-kinematics at the broken phase

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- $\mathcal{O}(E)$ for $(0{+}0{+})$ gives the following, $(0{-}0{-})$ giving the conjugate:

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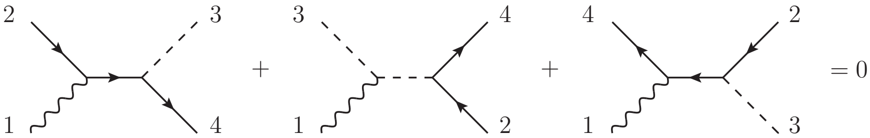

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4 sectors: e.g. in the s channel,

$$\sum_B \frac{c^B_{L,s} n_{L,s} + c^B_{R,s} n_{R,s} + \mathsf{f}^B_{L,s} n^{\mathsf{f}}_{L,s} + \mathsf{f}^B_{R,s} n^{\mathsf{f}}_{R,s}}{s - m^2_B},$$

with

$$c^B_{L,s} = L^{a_3}_{A_4B} L^{a_1}_{BA_2}, \qquad c^B_{R,s} = R^{a_3}_{A_4B} R^{a_1}_{BA_2},$$
$$\mathsf{f}^B_{L,s} = \left(Y_{a_3}\right)_{A_4B} L^{a_1}_{BA_2}, \qquad \mathsf{f}^B_{R,s} = \left(Y^\dagger_{a_3}\right)_{A_4B} R^{a_1}_{BA_2}.$$

Summary and outlook

- The coupling-kinematics duality: invariant tensor relations in renormalizable gauge theories correspond to kinematics numerators
- In spontaneously broken gauge theories, an amplitude of a massive gauge boson may involve multiple sets of numerators
- To explore: applications to SM, generalization to EFTs, double copy theories, higher spin

Discussion: minimal requirement

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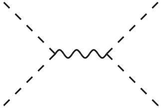

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If accidentally, $T^a_{i_1 i_2} T^a_{i_3 i_4} + T^a_{i_1 i_3} T^a_{i_4 i_2} + T^a_{i_1 i_4} T^a_{i_2 i_3} = 0$, no corresponding kinematic numerators guaranteed.

Discussion: the Standard Model

- The gauge group: $\mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)$.
- Couplings: f^{abc}, T^a_{ij}, L^a_{AB}, R^a_{AB}, K_{ijkl}, $(Y_i)_{AB}$.

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- Generators in general do not group according to invariant subgroups
- Generators in general are not block-diagonalized according to irreducible representations
  For example,

$$L^{a_1}_{A_4 B}\,(Y_{a_3})_{BA_2} - (Y_{a_3})_{A_4 B}\,R^{a_1}_{BA_2} - \left(Y_{\tilde{i}}\right)_{A_4 A_2} T^{a_1}_{\tilde{i} a_3} = 0.$$

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  For example, including dim-6 operators in YM $\leftrightarrow$ relaxing the high energy limit to satisfy

$$\mathcal{A}_n \sim \mathcal{O}(E^{6-n})$$

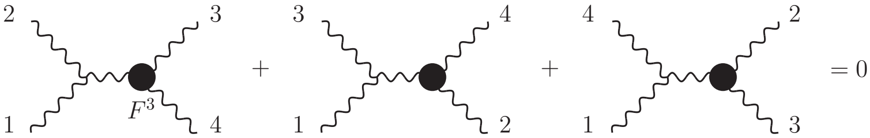
- Wilson coefficients of EFT operators need to be invariant tensors

Example: F^3 in YM

- The coupling of the F^3 operator, f'^{abc}, needs to be an invariant tensor:

$$f'^{a_1a_2b}f^{ba_3a_4} + f'^{a_1a_4b}f^{ba_2a_3} + f'^{a_1a_3b}f^{ba_4a_2} = 0,$$

thus may have corresponding kinematic numerators.



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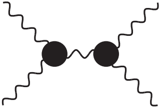

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  thus may have corresponding kinematic numerators.
- However, no relation for $f'^{a_1a_2b}f'^{ba_3a_4}$ from (relaxed) tree unitarity
- Indeed, only a special choice of F^4 operator, together with F^3, can make the color-kinematics duality also working for $f'^{a_1a_2b}f'^{ba_3a_4}$

Broedel, Dixon, 1208.0876