中国科学技术大学 交叉学科理论研究中心
Interdisciplinary Center for Theoretical Study

Reconstruction of primordial power spectrum of curvature perturbation on small scales from the merger rate of primordial black hole binaries

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Tongji University

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Mainly based on:

R. Kimura, T. Suyama, M. Yamaguchi and YZ, JCAP 04 (2021) 031 [arXiv: 2102.05280]

Why Primordial Black Hole (PBH)?

- BHs exist in the universe
- No need for new physics
- PBHs may dominate Dark Matter
- Detected GW events from LIGO may originate from the merger of PBH binaries

M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, PRL 117, no. 6, 061101 (2016)

Idea

PBHs from large curvature perturbations

\+ LIGO data from mergers of PBH binaries

↓

Primordial Power Spectrum of
curvature perturbations?
(on small scales)

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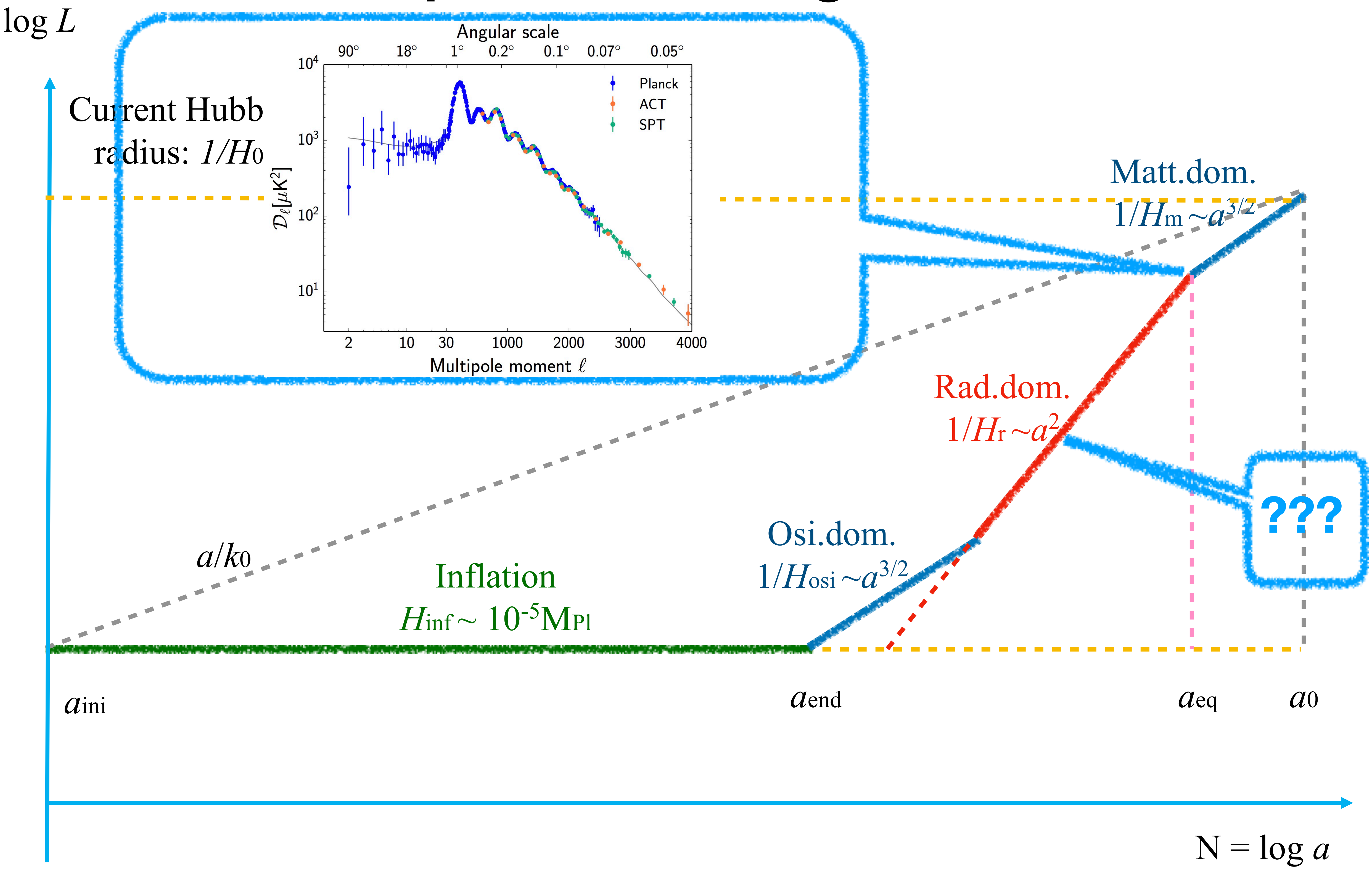
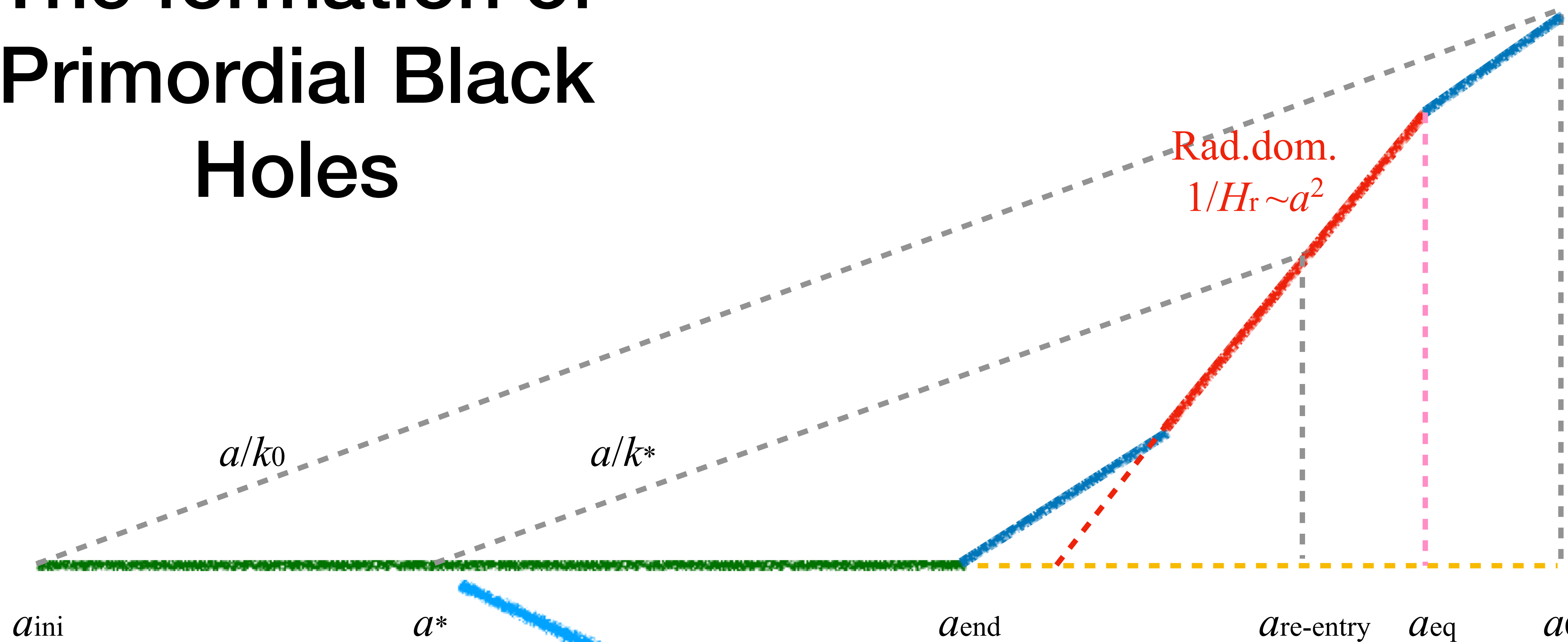
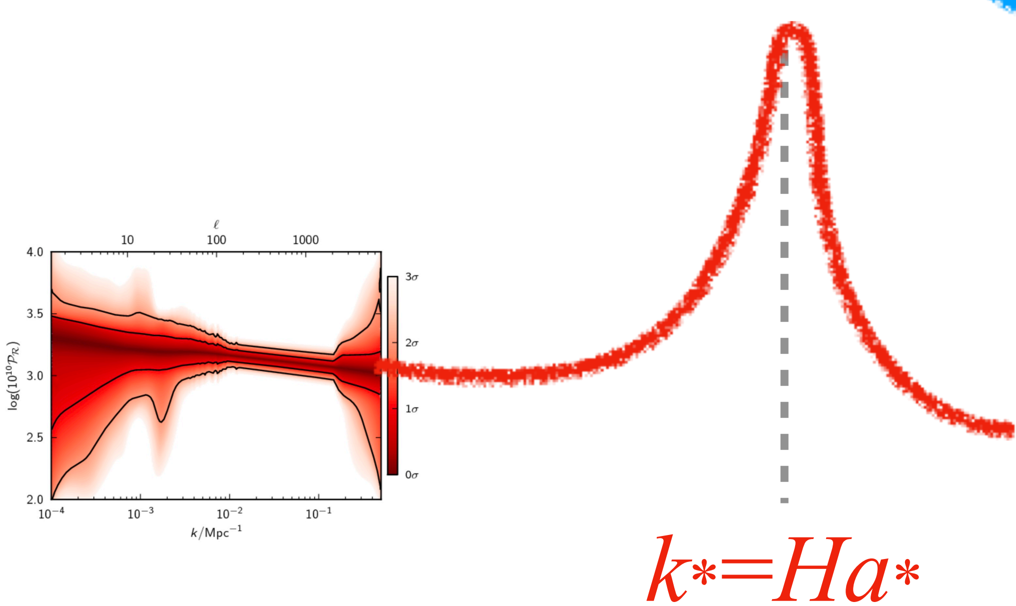
  M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, PRL 117, no. 6, 061101 (2016)
- A possible way to probe the primordial power spectrum of curvature perturbation on small scales
- …

Spacetime diagram



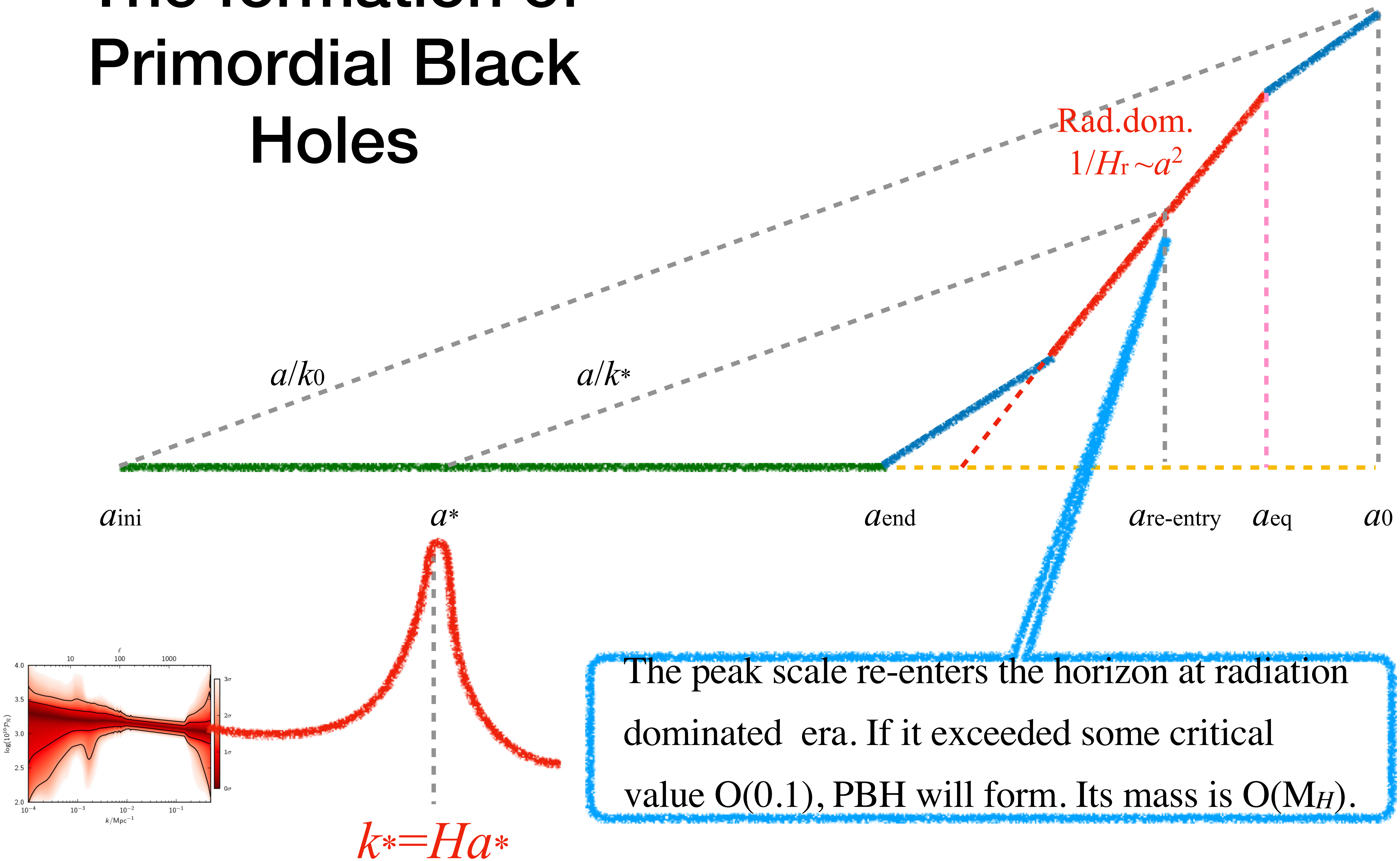
The formation of Primordial Black Holes





There is a peak on the primordial density perturbation, which leaves horizon and gets frozen at a_*.

The formation of Primordial Black Holes



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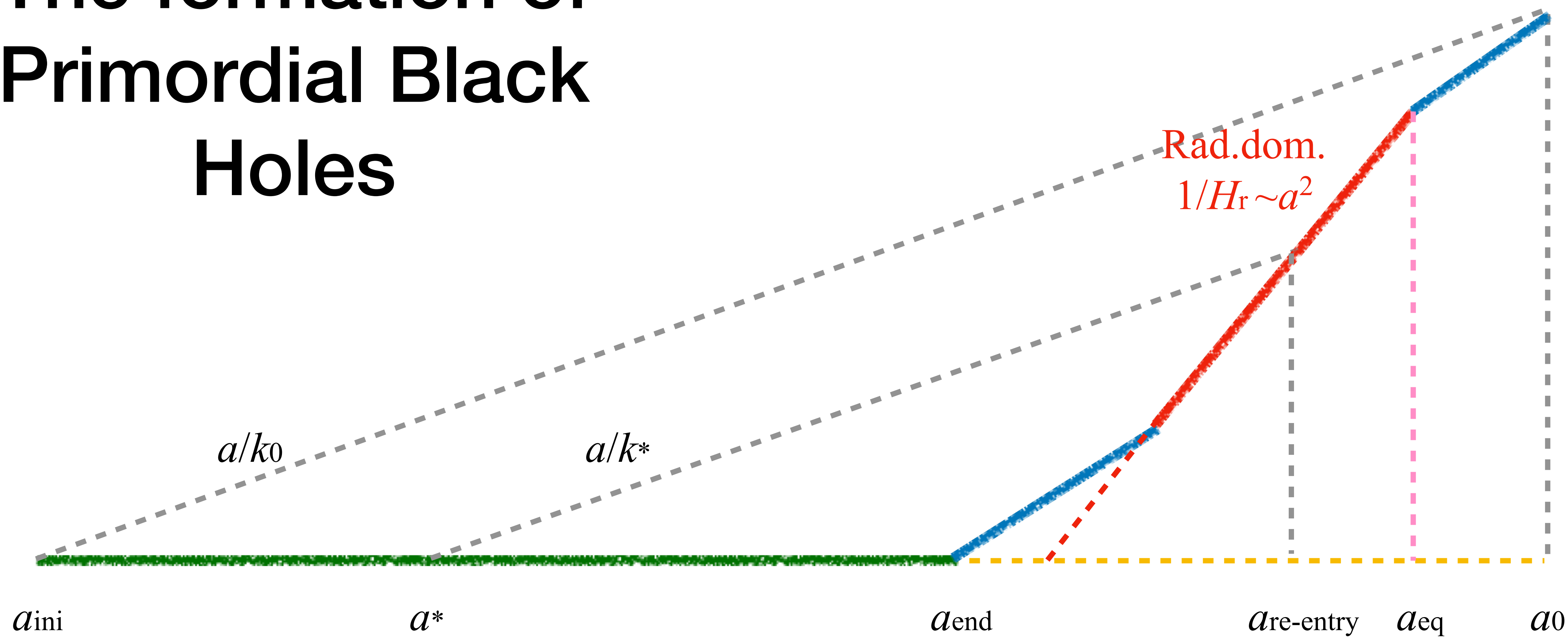

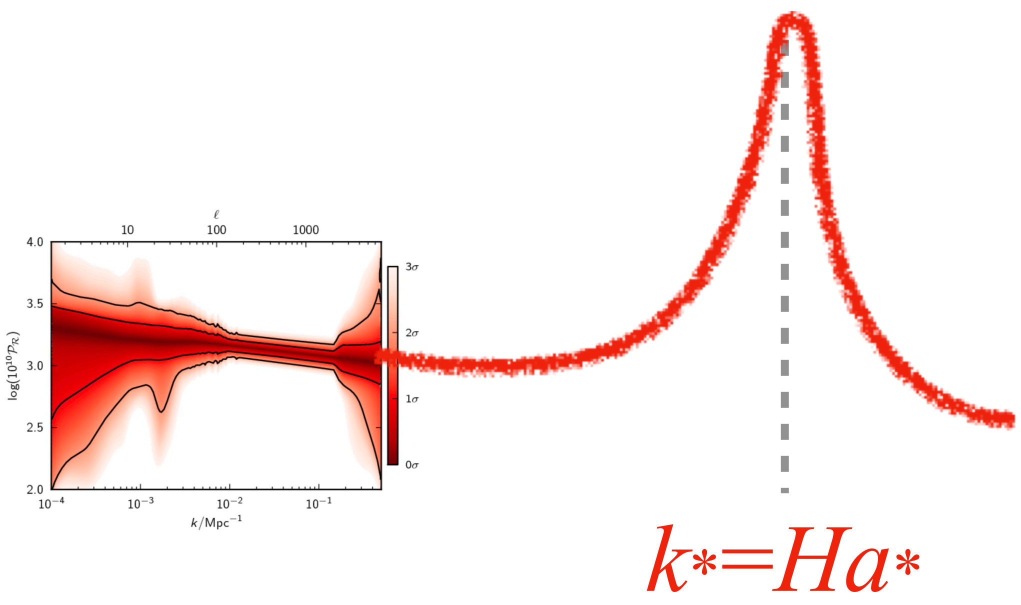

Shi Pi, YZ, Qing-Guo Huang, Misao Sasaki, JCAP 1805, 42, 2018 [arXiv:1712.09896]

PBHs from large curvature perturbations on small scales

\+ LIGO data from mergers of PBH binaries

↓

Primordial Power Spectrum of
curvature perturbations
(on small scales)

Questions:

1. Correct? (Uniquely determined?)
2. How to reconstruct?

Assumptions

0. At least some of the BBH LIGO events are PBHs

   merger rate from observations

1. PBHs formed out of high peaks of curvature perturbations

   the simplest case

2. Window function: top-hat form

   semi-analytic expression for calculation of merger rate

3. Gaussian distribution of density perturbation

   simple relation between between power spectrum and the variance

The logic

$$\mathcal{P}_{\mathcal{R}}(k) \xleftarrow{\quad ? \quad} \mathcal{R}(m_1, m_2, t)$$

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$$\mathcal{P}_{\mathcal{R}}(k) \xleftarrow{\quad ? \quad} \mathcal{R}(m_1, m_2, t)$$

$$\mathcal{P}_{\mathcal{R}}(k) \overset{\text{(i)}}{\Longleftrightarrow} \sigma^2(R) \overset{\text{(ii)}}{\Longleftrightarrow} f(m) \overset{\text{(iii)}}{\Longleftrightarrow} \mathcal{R}(m_1, m_2, t)$$

Question: can we uniquely determine $\mathcal{P}_{\mathcal{R}}(k)$?

Step (i) $\mathcal{P}_{\mathcal{R}}(k) \overset{\text{(i)}}{\Longleftrightarrow} \sigma^2(R)$

$$P(\Delta) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\Delta^2}{2\sigma^2}}$$

$$\mathcal{P}_\Delta(k) \longrightarrow \sigma^2(R) = \int_0^\infty W^2(kR)\mathcal{P}_\Delta(k)\, \mathrm{d}(\ln k)$$

$$W(kR) = \begin{cases} 1 & ;\ 0 < k < 1/R\,, \\ 0 & ;\quad \text{otherwise}\,, \end{cases}$$

$$\mathcal{P}_\Delta(t,k) = \frac{16}{81}\left(\frac{k}{aH}\right)^4 \mathcal{P}_{\mathcal{R}}(k)$$

$$\frac{81}{16} R^4 \frac{\mathrm{d}}{\mathrm{d}R}\left(\frac{\sigma^2(R)}{R^4}\right)\Bigg|_{1/R=k} = -k\mathcal{P}_{\mathcal{R}}(k)$$

Step (ii) $\sigma^2(R) \overset{\text{(ii)}}{\Longleftrightarrow} f(m)$

The mass function $f(m)$ is expressed in terms of PBH abundance

$$f(m) \propto \beta \equiv \int_{\Delta_{\rm th}}^{1} P(\Delta)\mathrm{d}\Delta = \frac{1}{2}\mathrm{erfc}\left(\frac{\Delta_{\rm th}}{\sqrt{2}\sigma}\right)$$

Step (iii) $f(m) \overset{\text{(iii)}}{\Longleftrightarrow} \mathcal{R}(m_1, m_2, t)$

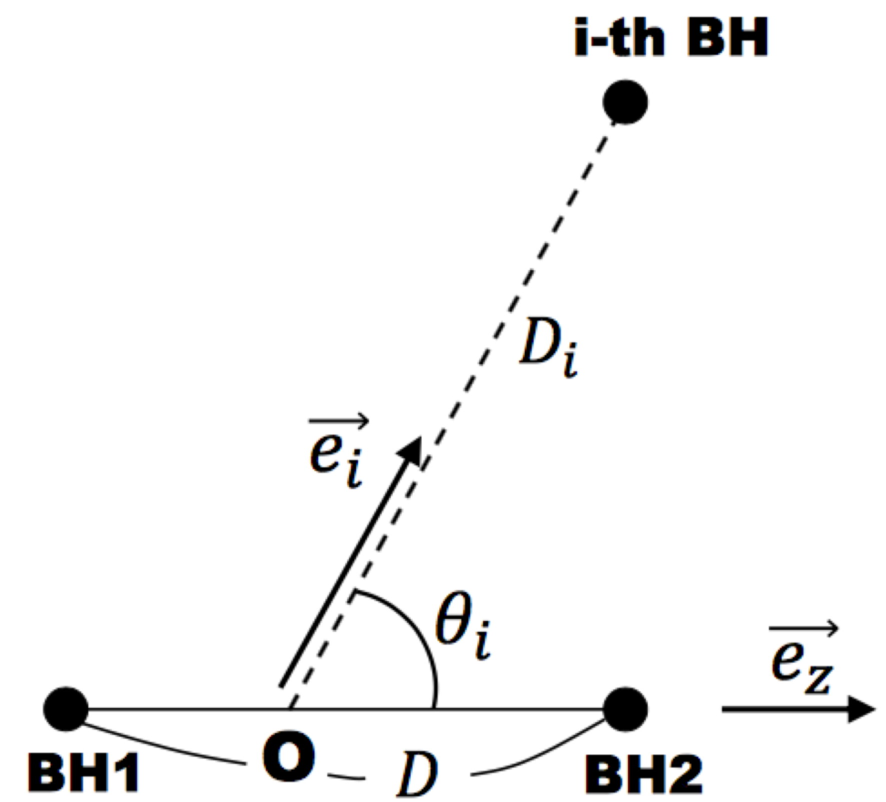
Mainly 2 possible process for PBH merger events: PBH binaries formed at

(a). early (radiation-dominated) epoch;

Sasaki et al., PRL 117(2016)6, 061101

(b). late (matter-dominated) epoch.

Bird et al., PRL 116(2016)20, 201301



B. Kocsis, T. Suyama, T. Tanaka and S. Yokoyama, Astrophys. J. 854, no. 1, 41 (2018)

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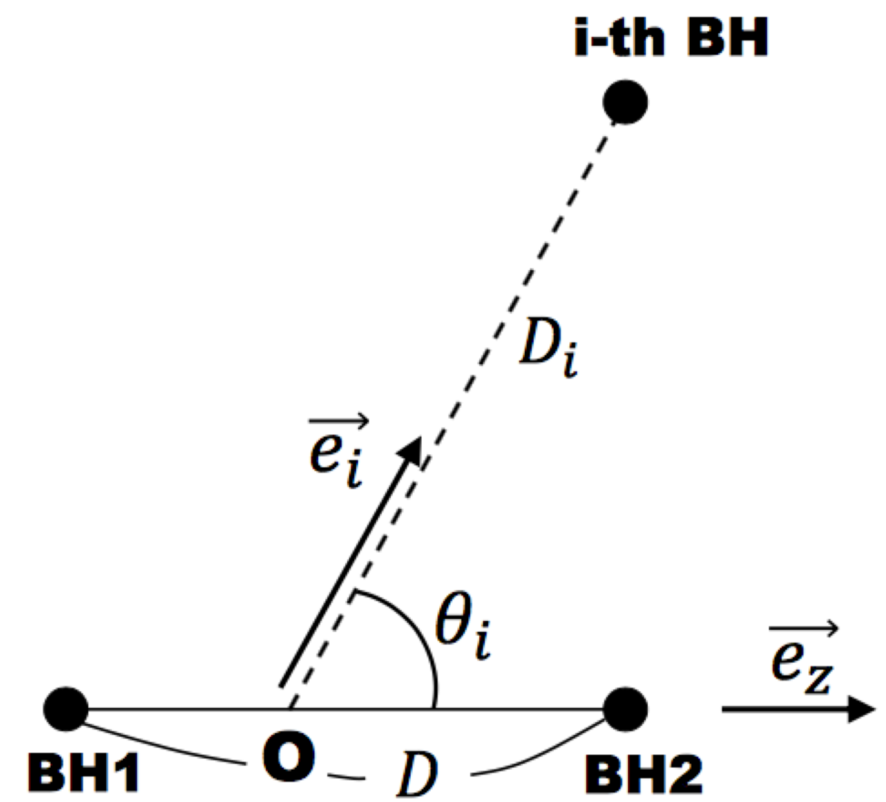

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$$\mathcal{R}(m_1, m_2, t) \propto \left(\frac{m_t}{M_\odot}\right)^{-\frac{32}{37}} \left(\frac{m_t^2}{m_1 m_2}\right)^{\frac{34}{37}} S[f|f_{\rm PBH}, m_t] m_1 m_2 f(m_1) f(m_2)$$

suppression factor

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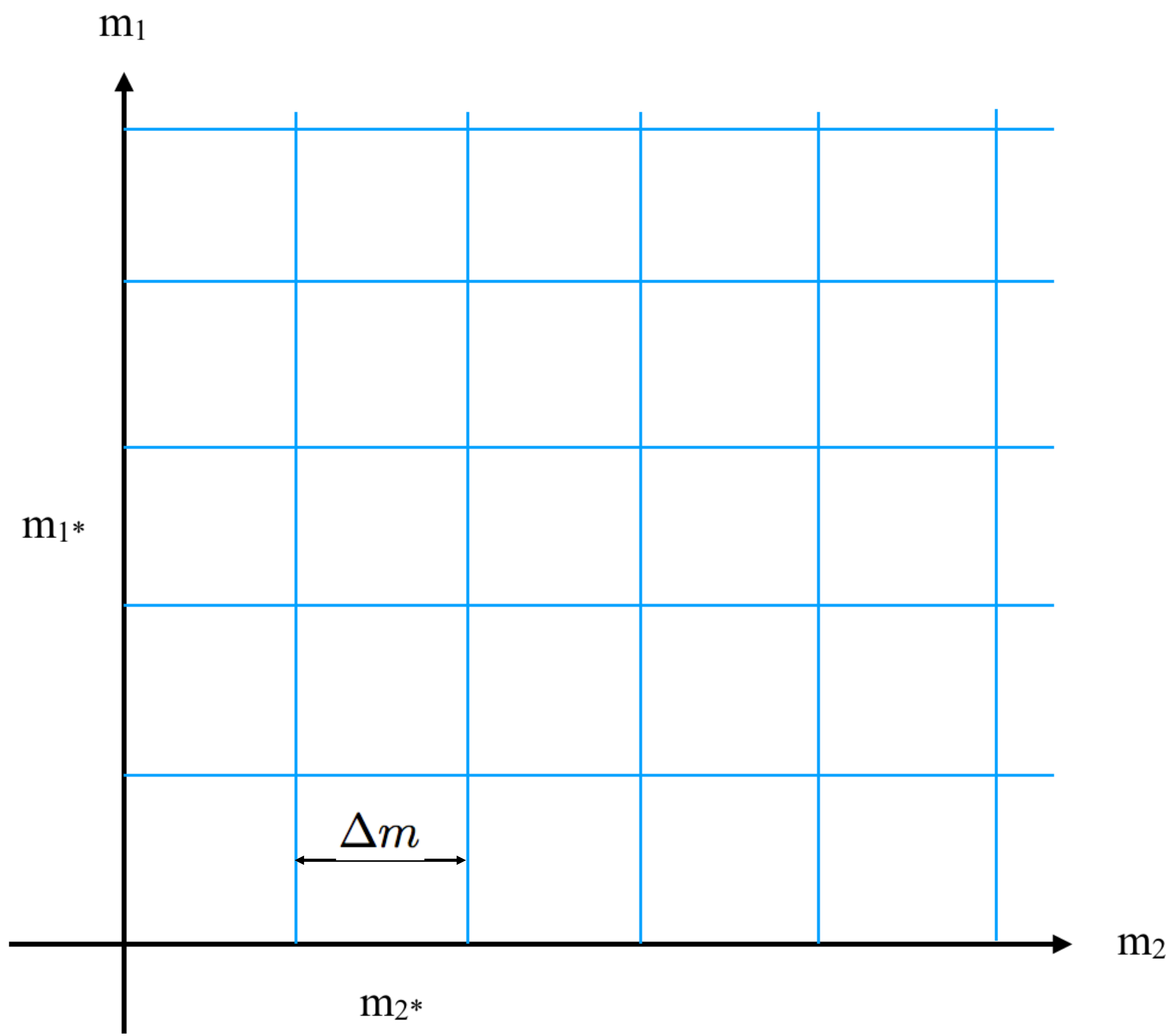
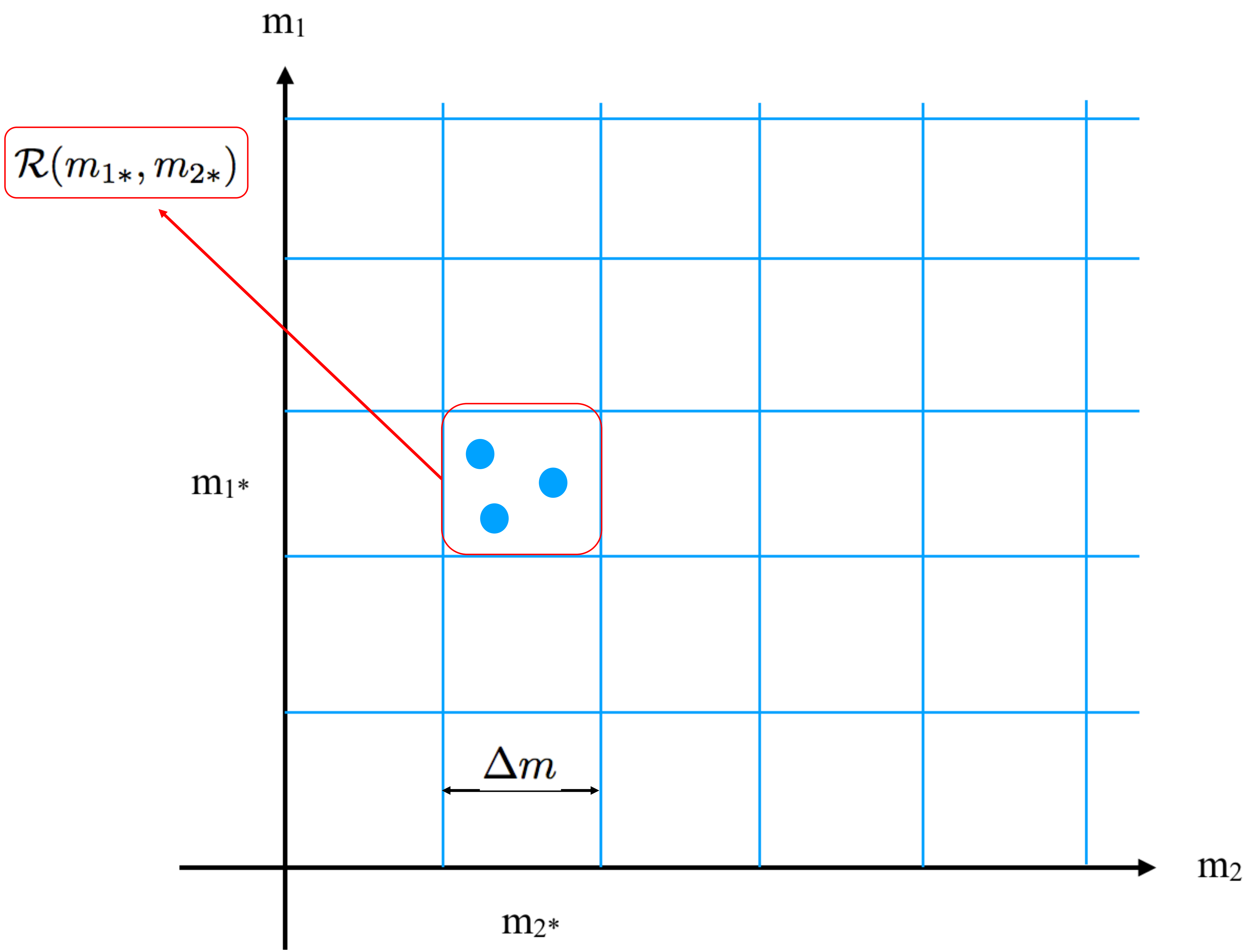
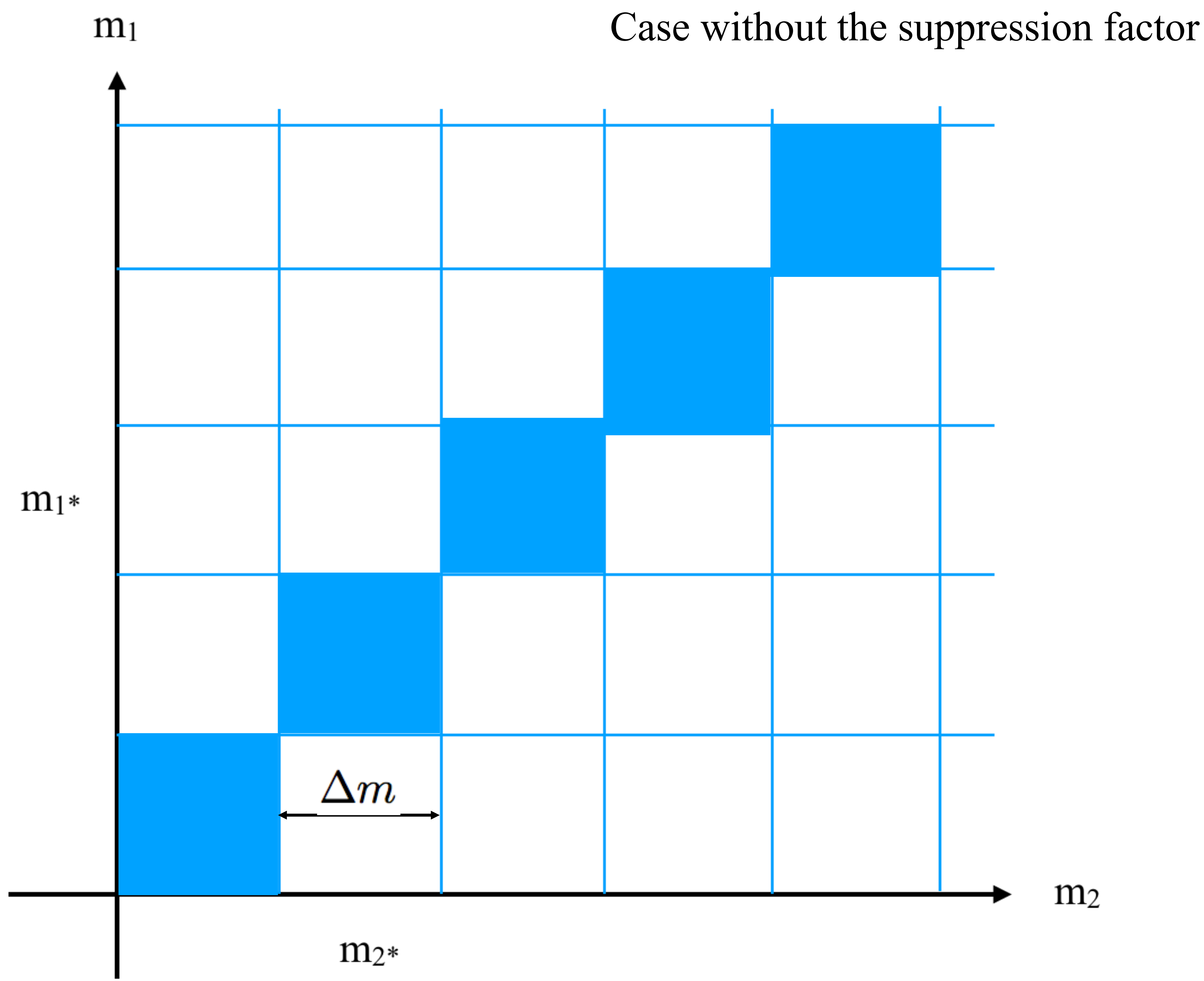
$$\Downarrow$$

$$\frac{f(m_1)}{f(m_2)} = \left(\frac{m_2}{m_1}\right)^{\frac{21}{37}} \sqrt{\frac{\mathcal{R}(m_1, m_1, t)}{\mathcal{R}(m_2, m_2, t)}}$$

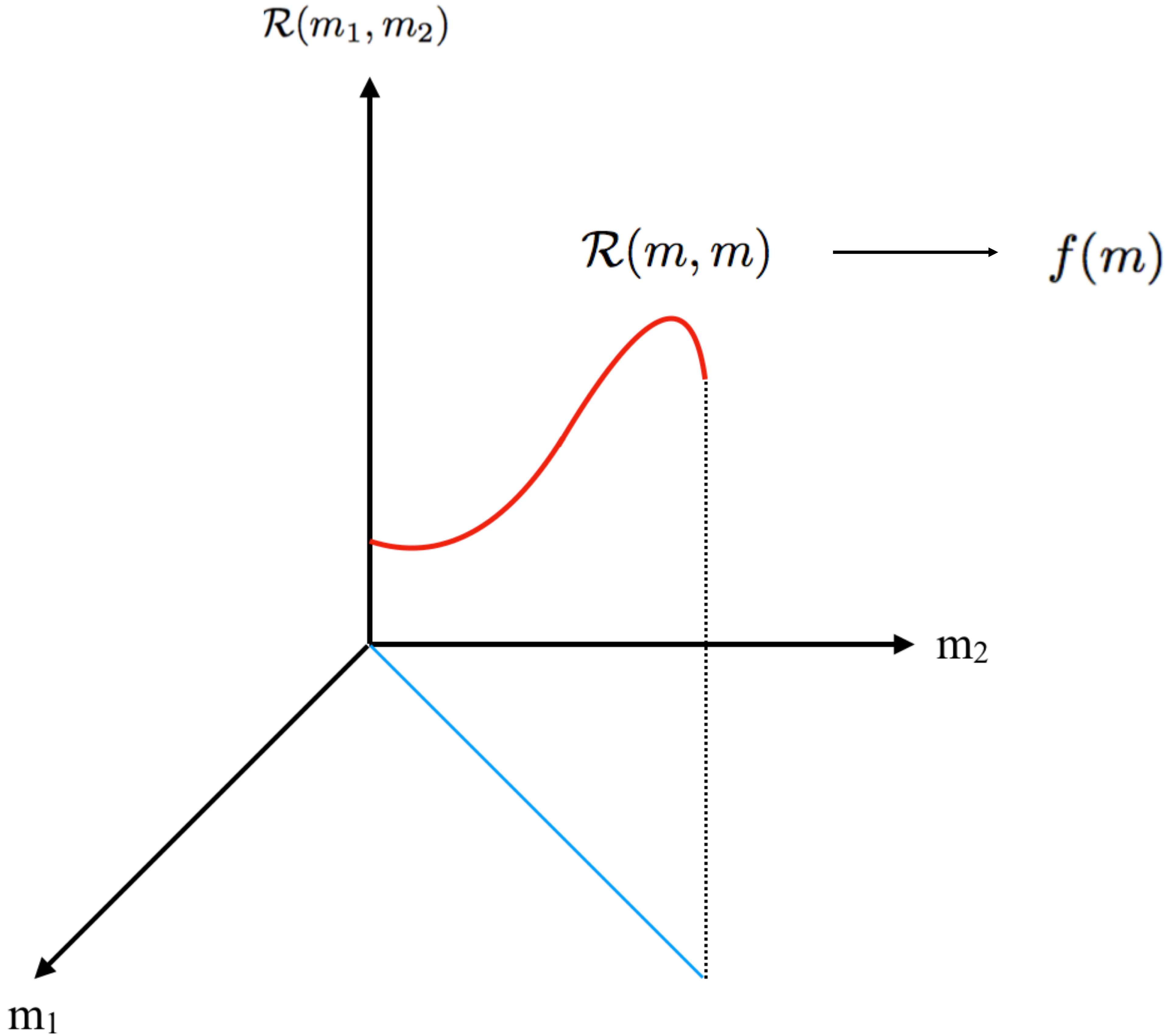

m1
m1*
Δm
m2
m2*



m1
$\mathcal{R}(m_{1*}, m_{2*})$
m1*
Δm
m2
m2*



Case without the suppression factor
m1
m1*
Δm
m2
m2*



$\mathcal{R}(m_1, m_2)$
$\mathcal{R}(m, m)$
$f(m)$
m_2
m_1

A simple example: no suppression factor

Step (iii) ⟹ $$f(m) = g(t) m^{-\frac{21}{37}} \sqrt{\mathcal{R}(m, m, t)}$$

$$g(t) \equiv f(m_*) m_*^{\frac{21}{37}} \left[\mathcal{R}(m_*, m_*, t)\right]^{-\frac{1}{2}}$$

Step (ii) ⟹ $$\sigma(m(R)) = \frac{\Delta_{\rm th}}{\sqrt{2}} \left[\mathrm{erfc}^{-1}\left(\frac{2 f_{\rm PBH} \Omega_{\rm CDM}}{(M_{\rm eq} K^3)^{\frac{1}{2}}} m^{\frac{3}{2}} f(m)\right)\right]^{-1}$$

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$$m(k) = \left(\frac{k_{\rm eq}}{k}\right)^2 M_{\rm eq} K \left(\frac{g_{*,\rm eq}}{g_*}\right)^{\frac{1}{3}}$$

Step (i) ⟹ $$\mathcal{P}_{\mathcal{R}}(k)$$

A simple example: no suppression factor

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{81}{8}\sigma^2\left[2-\sqrt{\pi}A(m)\left(\frac{69}{74}+\frac{m}{2\mathcal{R}(m,m,t)}\frac{\partial\mathcal{R}(m,m,t)}{\partial m}\right)\right.$$
$$\left.\times\left[\mathrm{erfc}^{-1}\left(A(m)\right)\right]^{-1}\exp\left\{\left[\mathrm{erfc}^{-1}\left(A(m)\right)\right]^2\right\}\right]_{m=m(k)}$$

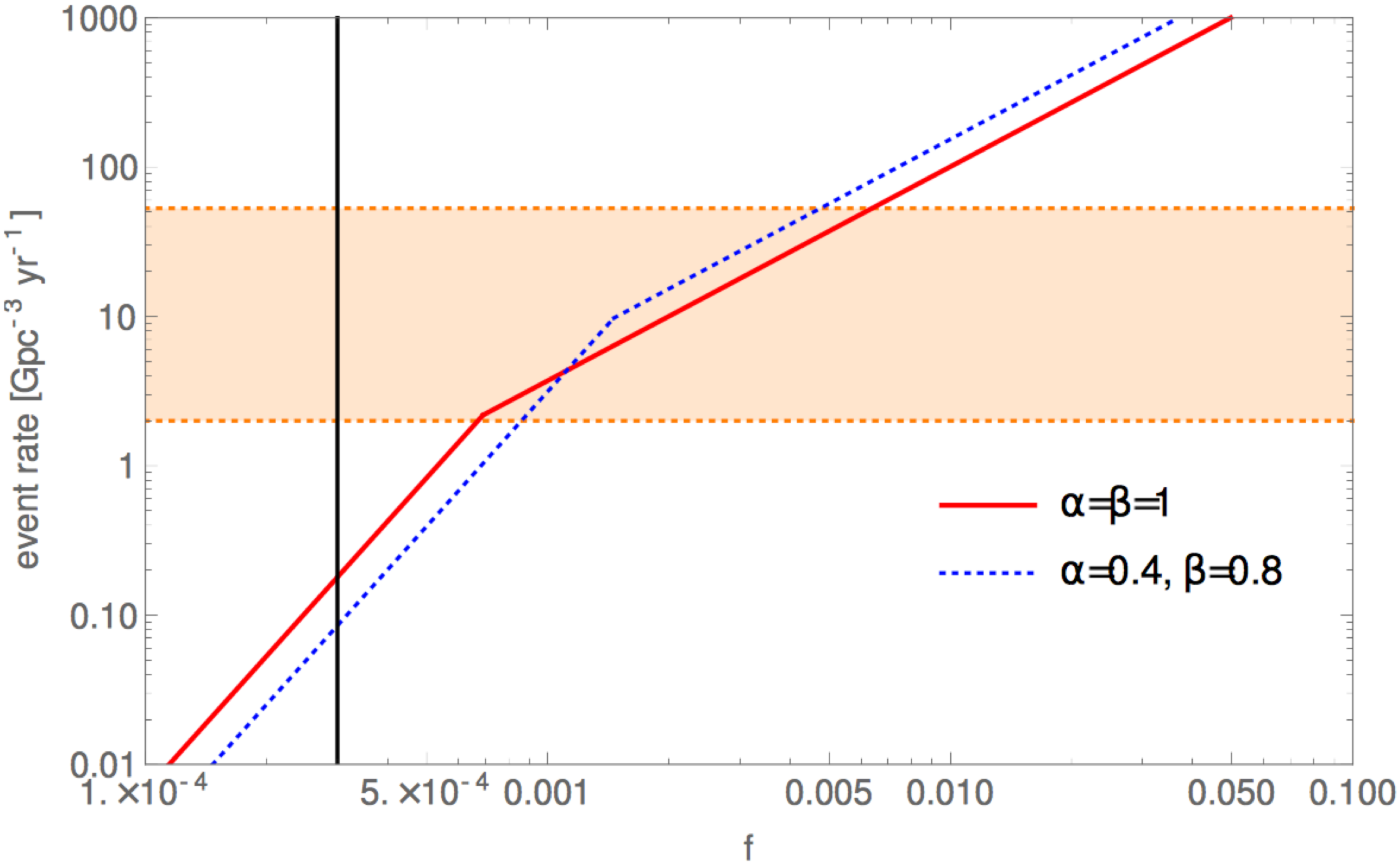
$$A(m) \equiv \frac{2f_{\mathrm{PBH}}\Omega_{\mathrm{CDM}}}{(M_{\mathrm{eq}}K^3)^{\frac{1}{2}}}\left(g(t)m^{\frac{69}{74}}\sqrt{\mathcal{R}(m,m,t)}\right)$$

Only the merger rate with equal mass is relevant!

Conclusion

We proposed the method to reconstruct the primordial power spectrum of curvature perturbation from the merger rate of PBH binaries

We need more data from LIGO, or mock data for the real reconstruction process



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