Weak Cosmic Censorship and Second Law of Black Hole Thermodynamics

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Based on Baoyi Chen, Feng-Li Lin, BN and Yanbei Chen, 2006.08663, 2211.17225

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Second Law $\implies$ Weak Cosmic Censorship

for higher derivative gravity, and beyond

Cosmic Censorship Conjecture

proposed to save the predictability of GR Penrose 1969

- Weak CCC:
  Singularity should be hidden behind the future event horizon, so as not to influent the spacetime outside the black hole
- Strong CCC:
  Cauchy horizon should be destroyed by perturbations, otherwise wormhole travelers would be influenced by the timelike singularity

WCCC: Evidence

Gedanken experiment: trying to destroy horizon by throwing matter into black hole, overcharging or overspinning are found impossible

- extremal black hole: dropping test particles into black hole, geodesics into horizon are forbidden in linear order perturbation Wald 1974
- near extremal black hole: an inequality based on Wald formalism is developed to connect falling matter and conserved charges of black hole, taking care of second order perturbation effect Sorce-Wald 2017

Basically a first law point of view.

WCCC: Evidence

Our approach: physical constraints on infalling matter could come from the second law of black hole thermodynamics

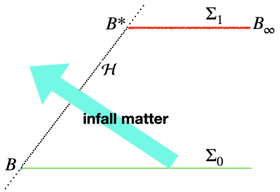
Picture

For vacuum solution family parametrized by (m, q), denote condition for black hole by

$$W(m, q) \geq 0$$

denote resultant changes of parameters due to infalling matter by δm and δq, WCCC is satisfied iff

$$W(m + \delta m,\, q + \delta q) \geq 0$$



Higher derivative gravity

Consider general quartic order corrections to Einstein-Maxwell theory:

$$
\begin{aligned}
L \;=\;& \frac{1}{2\kappa}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\
& + c_4 \kappa R F_{\mu\nu}F^{\mu\nu} + c_5 \kappa R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 \kappa R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \\
& + c_7 \kappa^2 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 \kappa^2 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}
\end{aligned}
$$

- Higher derivative theories can arise naturally from quantum corrections from the point of view of effective field theory
- WCCC should apply to generic effective field theories of gravity if it were a fundamental principle for protecting the predictive power of theory

Higher derivative gravity

2nd order perturbed solution (e.g. for c_4 only), solved by generalizing the perturbative method in Kats-Motl-Padi 2007 :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega$$

$$\begin{aligned}
f(r) &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_4\left(\frac{4\kappa^2 q^2}{r^4} - \frac{6\kappa^3 m q^2}{r^5} + \frac{4\kappa^3 q^4}{r^6}\right) + c_4^2\left(-\frac{32\kappa^4 q^4}{7r^8} - \frac{6\kappa^5 m q^4}{r^9} + \frac{32\kappa^5 q^6}{3r^{10}}\right) \\
g(r) &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_4\left(-\frac{16\kappa^2 q^2}{r^4} + \frac{14\kappa^3 m q^2}{r^5} - \frac{6\kappa^3 q^4}{r^6}\right) + c_4^2\left(\frac{1088\kappa^4 q^4}{7r^8} - \frac{126\kappa^5 m q^4}{r^9} + \frac{152\kappa^5 q^6}{3r^{10}}\right) \\
A_t &= -\frac{q}{r} - c_4\frac{2\kappa^2 q^3}{r^5} + c_4^2\left(\frac{576\kappa^3 q^3}{7r^7} - \frac{96\kappa^4 m q^3}{r^8} + \frac{50\kappa^4 q^5}{r^9}\right)
\end{aligned}$$

Criterion function:

$$W(m,q) = m^2 - q^2\left(1 + \frac{128c_4^2}{21q^4} + \cdots\right)^2$$

WCCC and 2nd Law

Consider initial nearly extremal black hole (for c_4) characterized by ϵ :

$$q = \sqrt{1-\epsilon^2}\left(m - \frac{128c_4^2}{21m^3}\right)$$

perturbed by a one-parameter family of infalling matter, finally settling down to a new solution with parameter

$$m(\lambda) = m + \lambda\delta m + \frac{\lambda^2\delta^2 m}{2}\,, \quad q(\lambda) = q + \lambda\delta q + \frac{\lambda^2\delta^2 q}{2}$$

- If constraints on δm, δq, $\delta^2 m$, $\delta^2 q$ arising from $S(m(\lambda), q(\lambda)) \geq S(m, q)$ will guarantee $W(m+\delta m, q+\delta q) \geq 0$?

WCCC and 2nd Law

Assuming first order variation optimally done: 2nd law satisfied marginally

$$\delta S = \frac{\partial S}{\partial m}\delta m + \frac{\partial S}{\partial q}\delta q = 0$$

gives (for c_4)

$$\delta m = \left(1 - \epsilon - \frac{64(2+1098\epsilon)c_4^2}{7m^4}\right)\delta q + \mathcal{O}(\epsilon^2)$$

- For extremal black holes $\epsilon = 0$, up to linear order $\delta S \geq 0$ guarantees WCCC always satisfied (with all $c_i{}'s$ present), consistent with Sorce-Wald approach Chen-Lin-BN-Chen 2021

WCCC and 2nd Law

Second order variations due to infalling matter should satisfy $\delta^2 S \geq 0$, giving

$$\delta^2 m \geq \left(\frac{1-\epsilon}{m} + \frac{256(1655 - 17372\epsilon + 33099\epsilon^2)c_4^2}{21m^5}\right)(\delta q)^2 + \left(1 - \epsilon + \frac{\epsilon^2}{2} - \frac{64(2 + 1098\epsilon - 8815\epsilon^2)c_4^2}{7m^4}\right)\delta^2 q$$

$$W(\lambda) \geq \left(\epsilon\Big(\frac{256c_4^2}{21m^3} - m\Big) + \lambda\Big(1 + \frac{211072c_4^2}{21m^4}\Big)\delta q\right)^2 + \mathcal{O}(c_4^3, \epsilon^3, \lambda^3)$$

positive definite, hence WCCC satisfied!

- all c_i cases and $c_2 + c_4$ case checked, successfully
- Kerr-Newman BHs with spin also consistent with Sorce-Wald 2017

WCCC and 1st Law

Sorce-Wald formalism based on 1st law

$$\begin{aligned} & \delta^n m_{\mathrm{ADM}} - \Phi_{\mathrm{H}}(\delta^n q_{\mathrm{H}} + \delta^n q_B) - T_{\mathrm{H}} \delta^n S_B \\ = \quad & \delta_{n,2}\, \mathcal{E}_\Sigma(\phi; \delta\phi, \mathcal{L}_\xi \phi) - \int_{\mathcal{H}} \xi^a \epsilon_{ebcd}\, \delta^n T_a{}^e \ \geq\ \delta_{n,2}\, \mathcal{E}_\Sigma(\phi; \delta\phi, \mathcal{L}_\xi \phi) \end{aligned}$$

fails to yield WCCC in higher derivative gravity:

$$W(\lambda) \ \geq\ \left(\epsilon\Big(\frac{161024 c_4^2}{21 m^3} + m\Big) - \lambda\Big(1 - \frac{165248 c_4^2}{21 m^4}\Big)\delta q\right)^2 - \frac{15360 \epsilon^2 c_4^2}{m^2}$$

NOT positive definite.

General proof of WCCC from 1st law

Condition for extremal solution not become singular:

$$\delta M - \left(\frac{dM}{dQ}\right)_{\text{ext}} \geq 0\,.$$

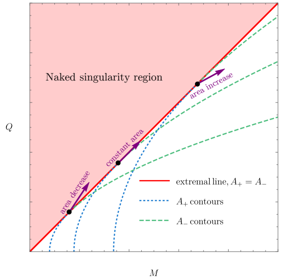
On the other hand, 1st law $\delta M = T\delta S + \Phi_{\text{H}}\delta Q$ shows for $T \to 0$

$$\left(\frac{dM}{dQ}\right)_{\text{ext}} = \Phi_{\text{H}}$$

then Wald's linear order variational identity

$$\delta M - \Phi_{\text{H}}\delta Q \geq 0$$

gives rise to non-singular condition hence WCC.



General proof of WCCC from 2nd law

For $\mu = m - m_{\text{ex}}(q_j)$ in an open neighborhood of 0, horizon radius should behave like

$$r_h(\mu, q_j) \;=\; R(q_j) + \sqrt{\mu}\,\rho(q_j, \sqrt{\mu})$$

start from configuration $(\mu, q_j) \;=\; (\epsilon^2, q_{j\,0})$ and deviation

$$\mu = \epsilon^2 + \delta\mu\lambda + \delta^2\mu\frac{\lambda^2}{2}\;, \qquad q_j = q_{j\,0} + \delta q_j\lambda + \delta^2 q_j\frac{\lambda^2}{2}$$

up to leading order

$$\left.\frac{dS}{d\lambda}\right|_{\lambda=0} \;=\; \frac{\partial S}{\partial r_h}\,\frac{\rho}{2\epsilon}\,\delta\mu$$

finiteness requires $\delta\mu \sim \epsilon$,

$$\left.\frac{d^2S}{d\lambda^2}\right|_{\lambda=0} \;=\; \frac{\rho}{2\epsilon^3}\left(\epsilon^2\delta^2\mu - \frac{1}{2}\delta\mu^2\right)\frac{\partial S}{\partial r_h}$$

finiteness requires $\delta^2\mu \;=\; \frac{\delta\mu^2}{2\epsilon^2}$, hence

$$\mu \;=\; \epsilon^2 + \delta\mu\lambda + \frac{\lambda^2\delta\mu^2}{4\epsilon^2} \;=\; \left(\epsilon + \frac{\delta\mu\lambda}{2\epsilon}\right)^2\;,$$

ensures μ positive and WCCC holds.

Conclusion

The second law of black hole thermodynamics ensures WCCC in general higher derivative gravity.

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The second law of black hole thermodynamics ensures WCCC in general higher derivative gravity.

Thanks for Your Attention!