EAST ASIA JOINT WORKSHOP ON FIELDS AND STRINGS 2023

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New aspects in orientifold planes

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2023-11-14

with Hirotaka Hayashi, Kimyeong Lee, Futoshi Yagi [2306.11631] ← Today

Hee-Cheol Kim, Minsung Kim, Gabi Zafrir [2307.03231]

Xiaobin Li, Satoshi Nawata, Futoshi Yagi [in progress]

I will talk about 5d $\mathcal{N} = 1$ supersymmetric theories.

Yesterday, we had two interesting talks [Piljin, Yi-Nan]
with a very well summarized introduction on 5d SCFTs.

This talk will be a continuation of 5d SCFT story from Type IIB branes

I would like to report an interesting observation,
two different classes of gauge theories can be connected,
not as a duality nor as a Higgsing.

This observation is based on Type IIB 5-brane descriptions
involving an orientifold 7-plane ($O7^-$ or $O7^+$).

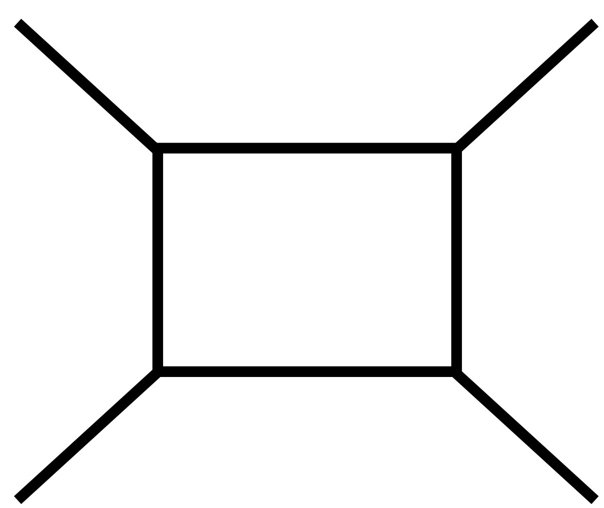
M-theory constructions

M-theory on non-compact Calabi-Yau 3 fold (CY3) [Yi-Nan's talk]

M2 wrapping compact 2-cycles <==> BPS particle mass
= vol(2-cycles)

M5 wrapping compact 4-cycles <==> monopole string tension
= vol(4-cycles)

e.g., $\mathbb{P}^1 \times \mathbb{P}^1$



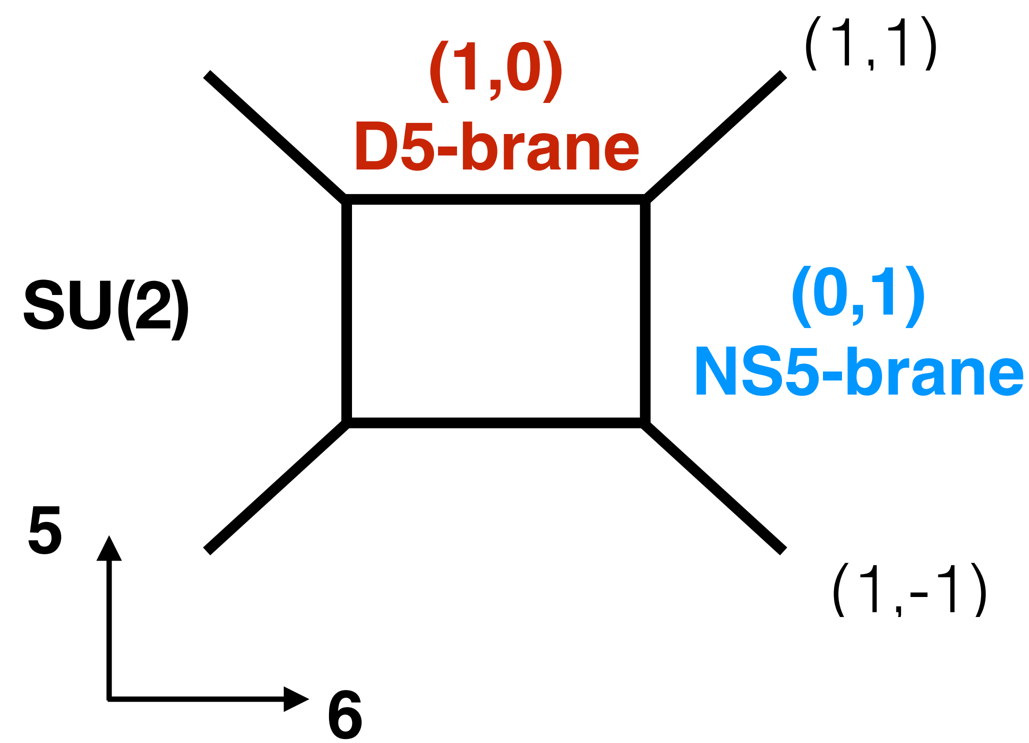
Type IIB brane (p,q) 5-brane

Theories with 8 supercharges are described by D-branes and NS5-branes.

	0	1	2	3	4	5	6	7	8	9
NS5	x	x	x	x	x	x				
D5	x	x	x	x	x		x			
(1,1)	x	x	x	x	x	-	-			
D7	x	x	x	x	x			x	x	x

5d world vol.

e.g., **SU(2)**



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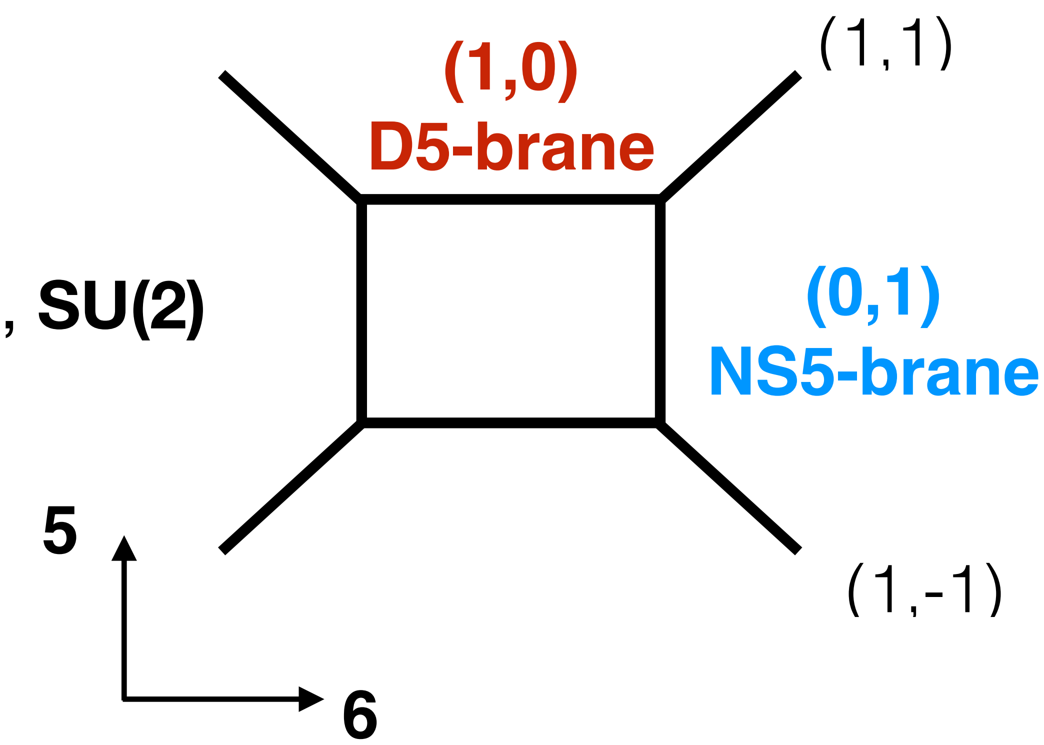
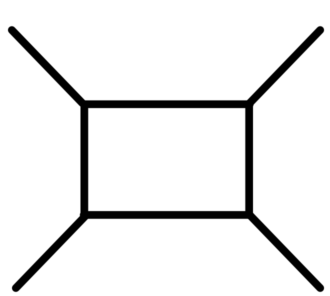
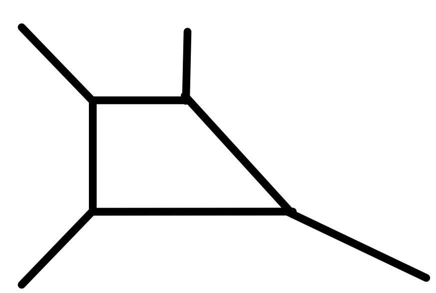


$\pi_4(\mathrm{SU}(2)) = \mathbb{Z}_2$ [Piljin's talk]



$\mathrm{SU}(2)_0$



$\mathrm{SU}(2)_\pi$

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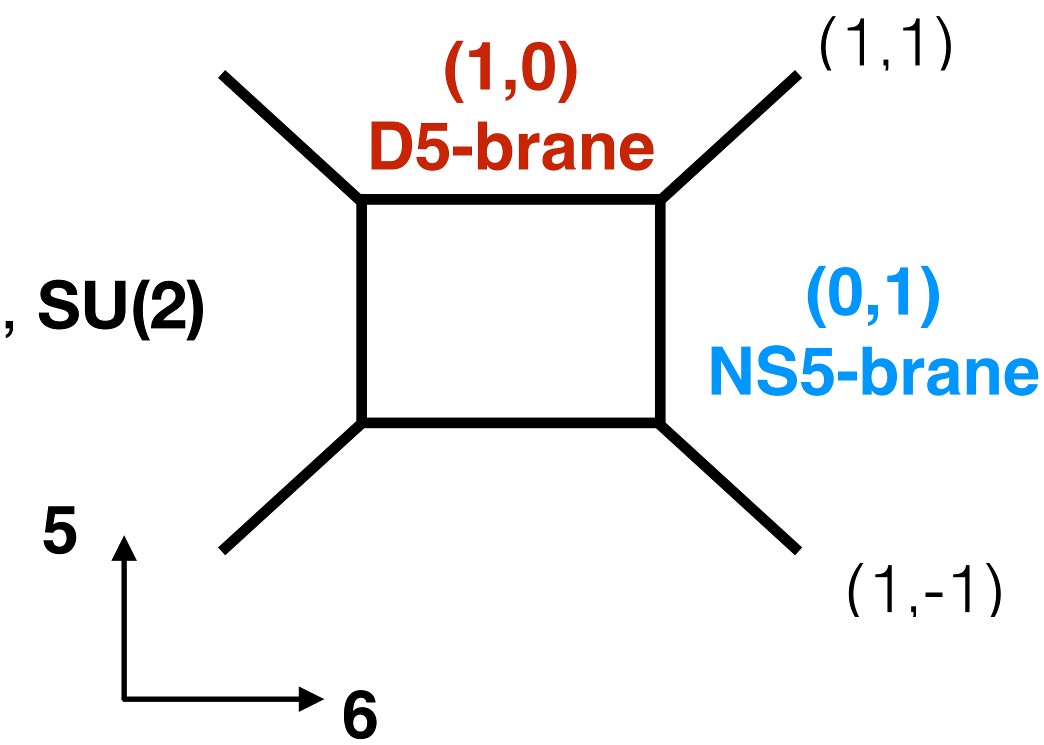

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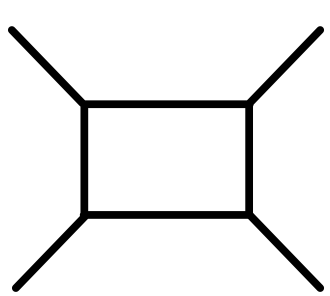
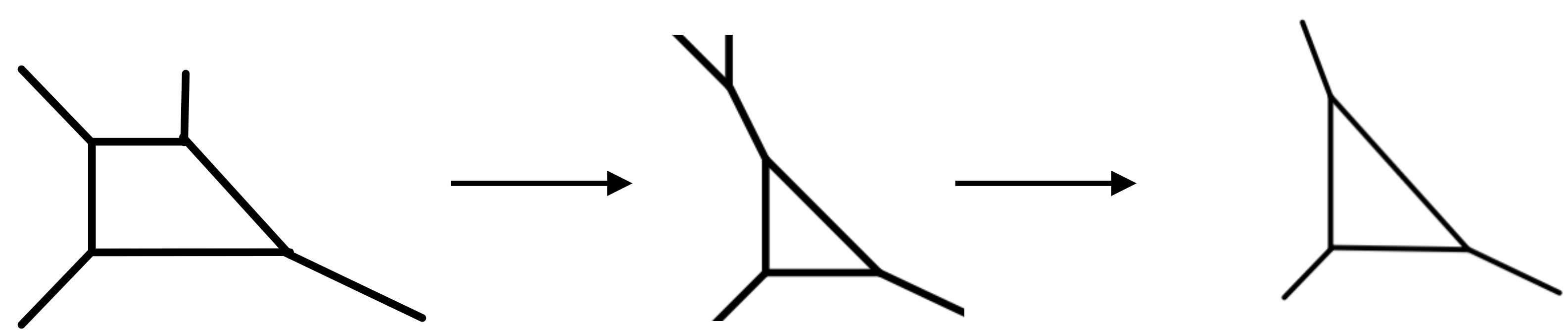


$\mathrm{SU}(2)_0$

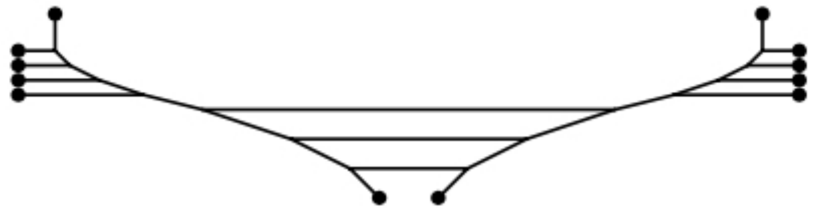


$\mathrm{SU}(2)_\pi$ E_0

Matters

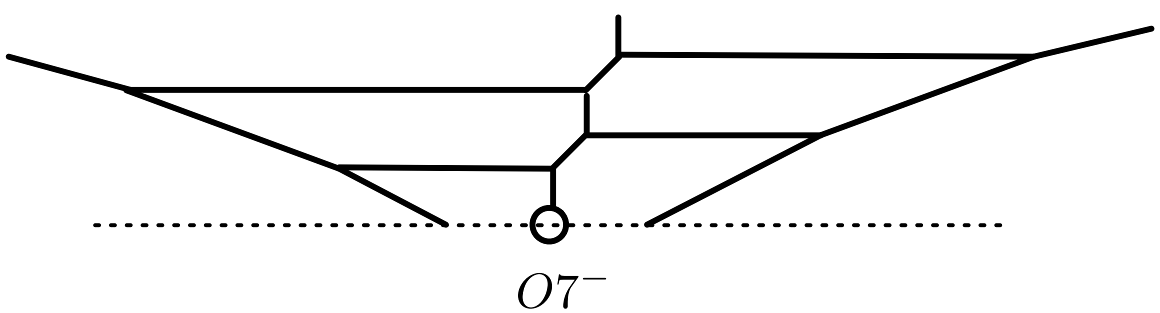
Matter (hypermultiplets)

- **Fundamental** hypers (**F**) can be described by introducing D7s

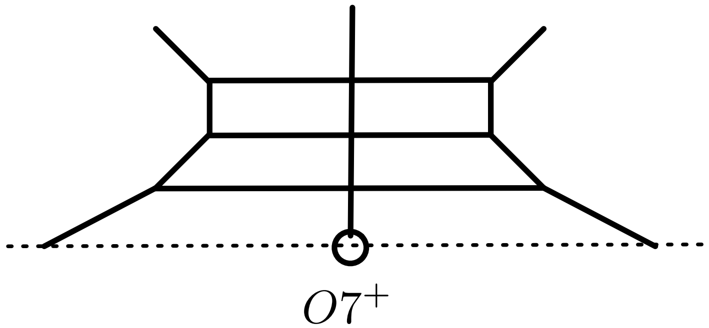


$SU(3)_1$ + 8**F**

- **Antisymmetric hypers** (**AS**) : half NS5 on an $O7^-$



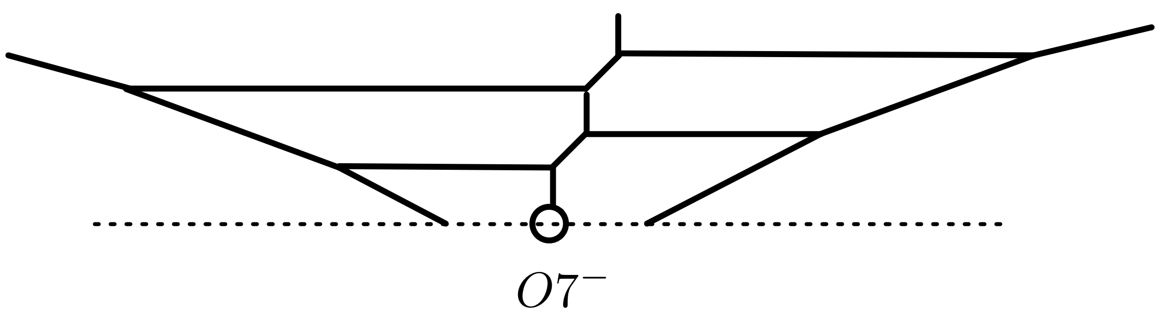
- **Symmetric hypers** (**Sym**) : half NS5 on an $O7^+$



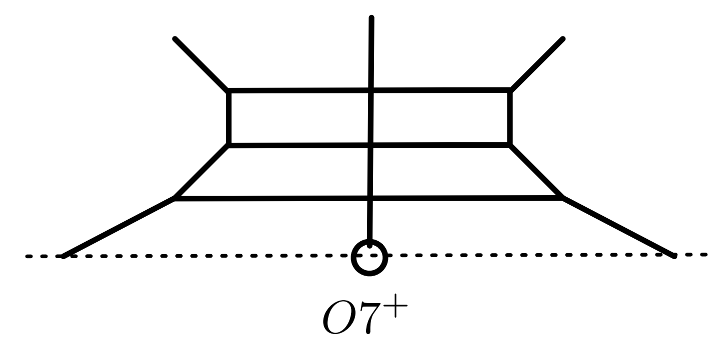
5-brane webs with O7-plane

With orientifold planes, 5-brane webs can describe
SU(*N*) + **AS** or **Sym** and also Sp / SO gauge theories

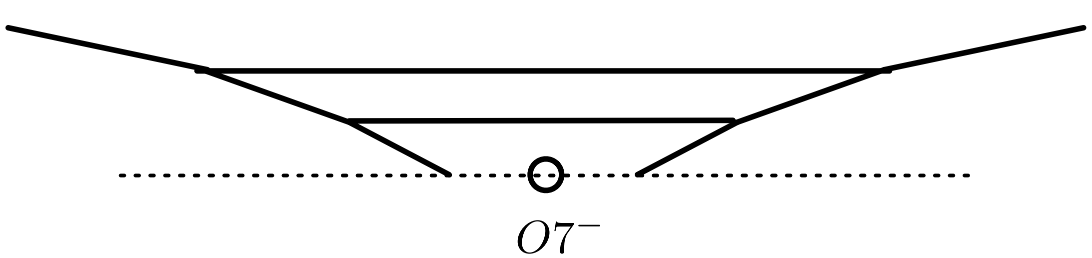
SU + 1**AS**



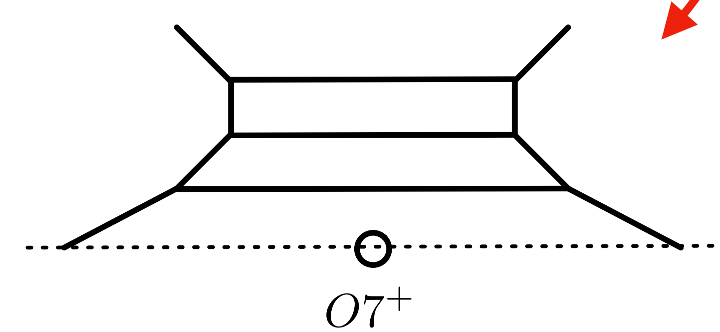
SU + 1**Sym**



Sp



SO



An observation

$O7$-plane

[2306.11631] Hirotaka Hayashi, SSK, Kimyeong Lee, Futoshi Yagi

$O7^-$ vs. $O7^+$

It is well known that the monodromy of $O7^- + 8D7$ is the same as $O7^+$

$$O7^- + 8D7 \quad \longleftrightarrow \quad O7^+$$

Monodromy: $\begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} -1 & -4 \\ 0 & -1 \end{pmatrix}$

It is also well known that their brane charges are the same

$$O7^- + 8D7 \quad \longleftrightarrow \quad O7^+$$

It is tempting to think that there may be some intriguing relations…

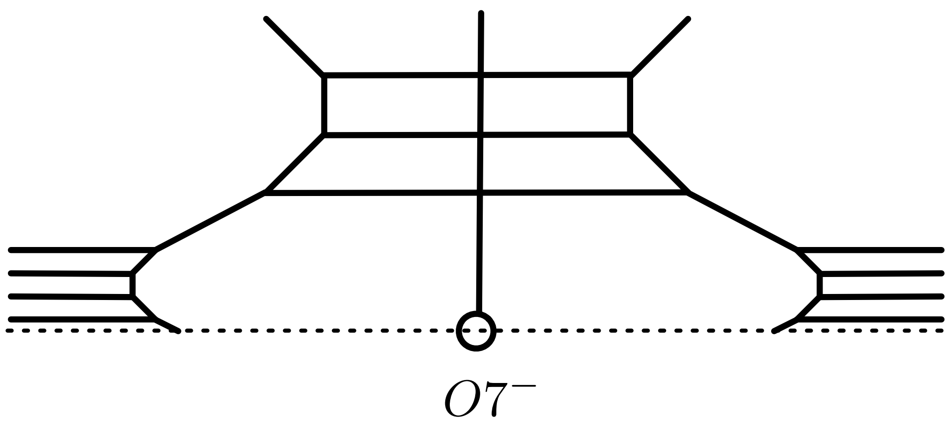
SU(N)+1**AS**+8**F** $\longleftrightarrow$ SU(N)+1**Sym**

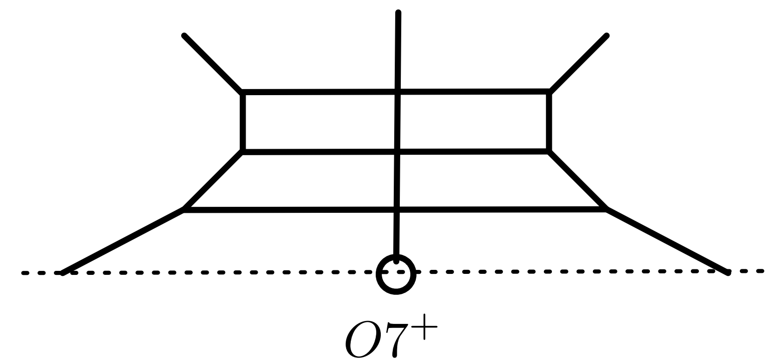
Sp(N)+8**F** $\longleftrightarrow$ SO(2N)

From 5-brane webs

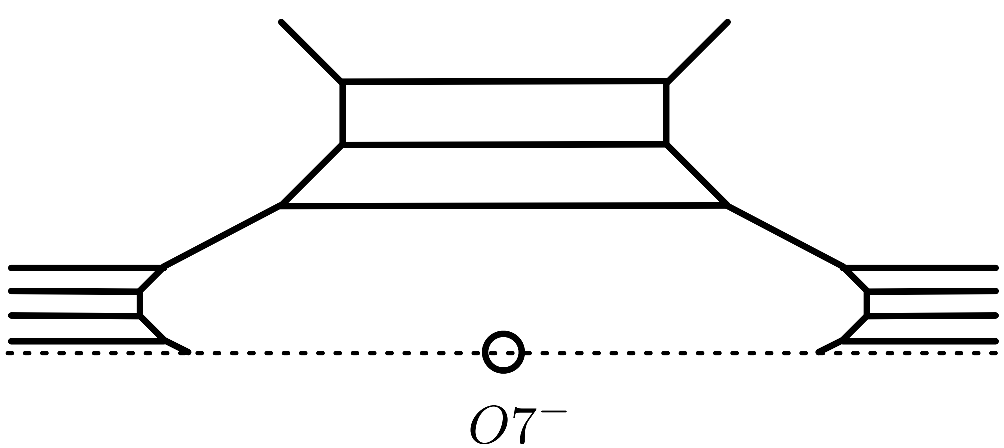
5-branes also suggest an interplay between $O7^+$ and $O7^- + 8D7$

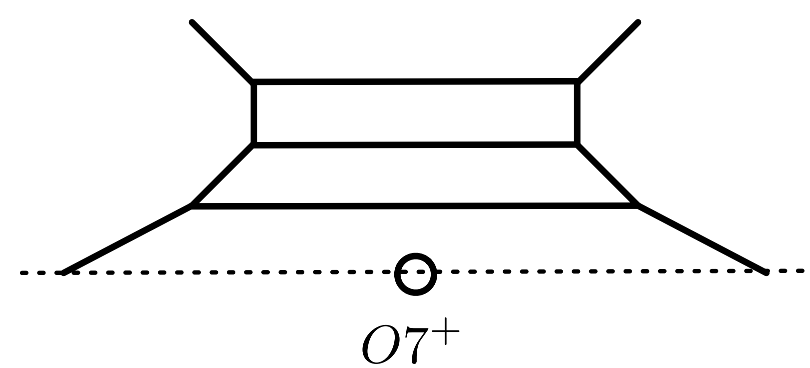
SU(2*N*) +1**AS** + 8**F** vs. SU(2*N*) +1**Sym**





Sp(*N*) + 8**F** vs. SO(2*N*)





Cubic prepotential

On Coulomb branch

- Prepotential characterizing the abelian low energy effective theory

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij}\phi^i\phi^j + \frac{\kappa}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\Big(\sum_{\text{Roots}} |R\cdot\phi|^3 - \sum_f \sum_{w\in W_f} |w\cdot\phi + m_f|^3 \Big)$$

$$h_{ij} = \text{Tr}(T_i T_j),\ d_{abc} = \frac{1}{2}\text{Tr}T_a(T_b T_c + T_c T_b), \quad W_f = \text{Weight of } G \text{ in the rep. } r_f$$

[96 Morrison-Seiberg]
[97 Intriligator-Morrison-Seiberg]

By specially tuning mass parameters (freezing)

$O7^+$	$O7^- + 8D7$
$\mathcal{F}_{\text{SU}(N)_\kappa + 1\mathbf{Sym}}$	$= \mathcal{F}_{\text{SU}(N)_\kappa + 8\mathbf{F} + 1\mathbf{AS}}$
$\mathcal{F}_{\text{SO}(2N)}$	$= \mathcal{F}_{\text{Sp}(N) + 8\mathbf{F}}$

Chern-Simons level shifts

Along RG flows or decouplings of hypermultiplets, the Chern-Simons level is shifted:

$$SU(3)_{\frac{1}{2}} + 1\mathbf{F}$$

$$\swarrow \qquad \searrow$$

$$SU(3)_0 \qquad\qquad SU(3)_1$$

CS level shifts:

$$\kappa \quad \to \quad \kappa \pm \frac{1}{2} I^{(3)}$$

Cubic Dynkin Index $I^{(3)}$

Hypermultiplet	$I^{(3)}$
$\mathbf{F}$	1
$\mathbf{AS}$	$N-4$
$\mathbf{Sym}$	$N+4$

$$I^{(3)}_{\mathbf{Sym}} = I^{(3)}_{\mathbf{AS}} + 8 I^{(3)}_{\mathbf{F}}$$

Observation

From the monodromy, prepotential, 5-brane webs, and CS level shifts,

$$O7^+ \leftarrow O7^- + 8D7s$$

It is done by tuning masses of flavors by special values.

So far, it seems to work, at least perturbatively or by accident(?).

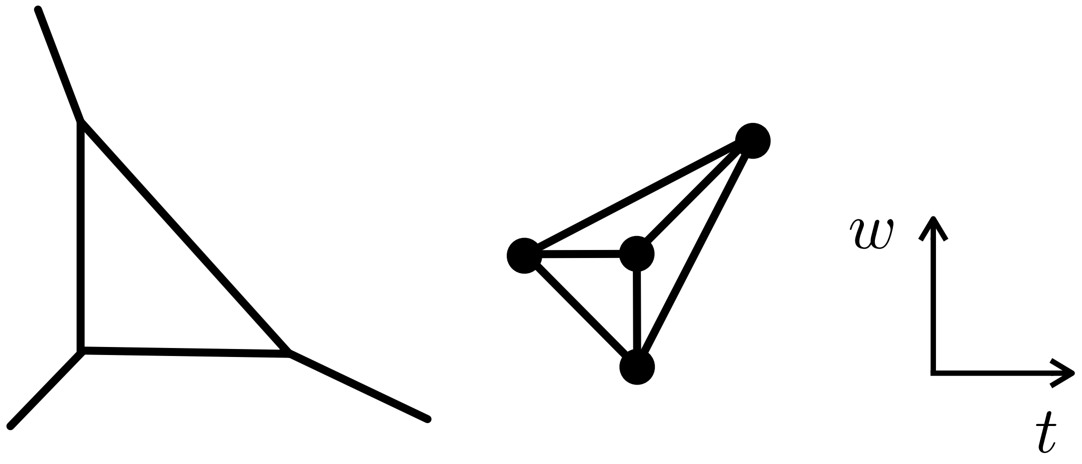
What about non-perturbatively?

Beyond perturbative level

- Seiberg-Witten (SW) curves

From dual diagram or generalized toric diagram, we can compute SW curves.

Simple example: E_0 theory (or local $\mathbb{P}^2$)



$$wt + u + w^{-1} + t^{-1} = 0$$

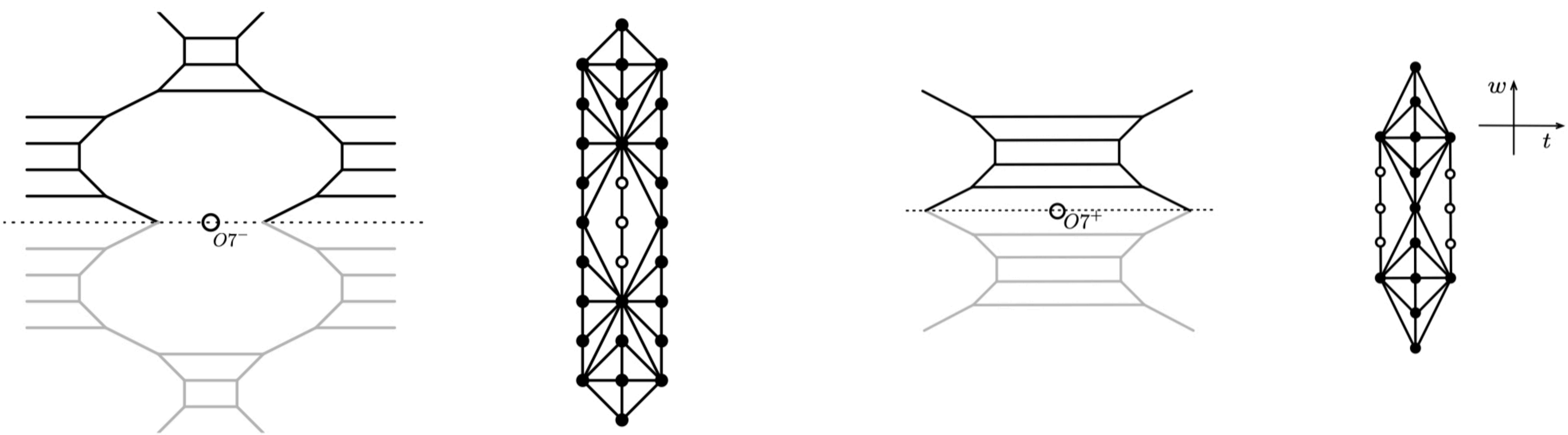
We developed a systematic procedure for obtaining SW curves of theories associated with O7-planes

[Hirotaka-SSK-Lee-Yagi '23]

$$O7^- + 8D7 \longrightarrow O7^+$$

$$SW_{SU+1\mathbf{AS}+8\mathbf{F}} = (w - w^{-1})^2 SW_{SU+1\mathbf{Sym}}$$

$$SW_{Sp(N)+8\mathbf{F}} = (w - w^{-1})^2 SW_{SO(2N)}$$



Other observables ?

We also checked the instanton partition functions also works.

[SSK-Li-Nawata-Yagi, in progres]

$$
\begin{aligned}
&Z^{\text{vec}}_{\text{SU}(N),k} Z^{\text{sym}}_{\text{SU}(N),k}(m) \\
=&Z^{\text{vec}}_{\text{SU}(N),k} Z^{\text{anti}}_{\text{SU}(N),k}(m) Z^{\text{fund}}_{\text{SU}(N),k}\left(\frac{\epsilon_+ \pm m}{2}, \frac{\epsilon_+ \pm m}{2} + \pi i, \frac{\epsilon_- \pm m}{2}, \frac{\epsilon_- \pm m}{2} + \pi i\right)
\end{aligned}
$$

Freezing

And superconformal index also works.

[Hee-Cheol Kim, Minsung Kim-SSK-Zafrir '23]

This is an intriguing relation…

but now.. what is good for?

Applying this relation to rank-1 case, SU(2) gauge theory,

SU(2)+1**AS** ($O7^-$) = SU(2), because **AS** is singlet,

SU(2)$_\theta$ +1**Sym** ($O7^+$) = SU(2)$_\theta$ +1**Adj**, because **Sym** of SU(2) is **Adj,**

M-string on a circle ($\theta = 0$)
A_2 theory with $\mathbb{Z}_2$ outer automorphism twist ($\theta = \pi$)

we get non-Lagrangian theory:

$$\mathbb{P}^2 + 1\mathbf{Adj} \ (\text{or } SU(2)_\pi + 1\mathbf{Adj} - 1\mathbf{F})$$

[Bhardwaj '19]

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$$\mathbb{P}^2 + 1\mathbf{Adj}\ (\text{or}\ SU(2)_\pi + 1\mathbf{Adj} - 1\mathbf{F})$$

[Bhardwaj '19]

by replacing $1\mathbf{Adj}$ (or $1\mathbf{Sym}$) with $1\mathbf{AS} + 8\mathbf{F}$

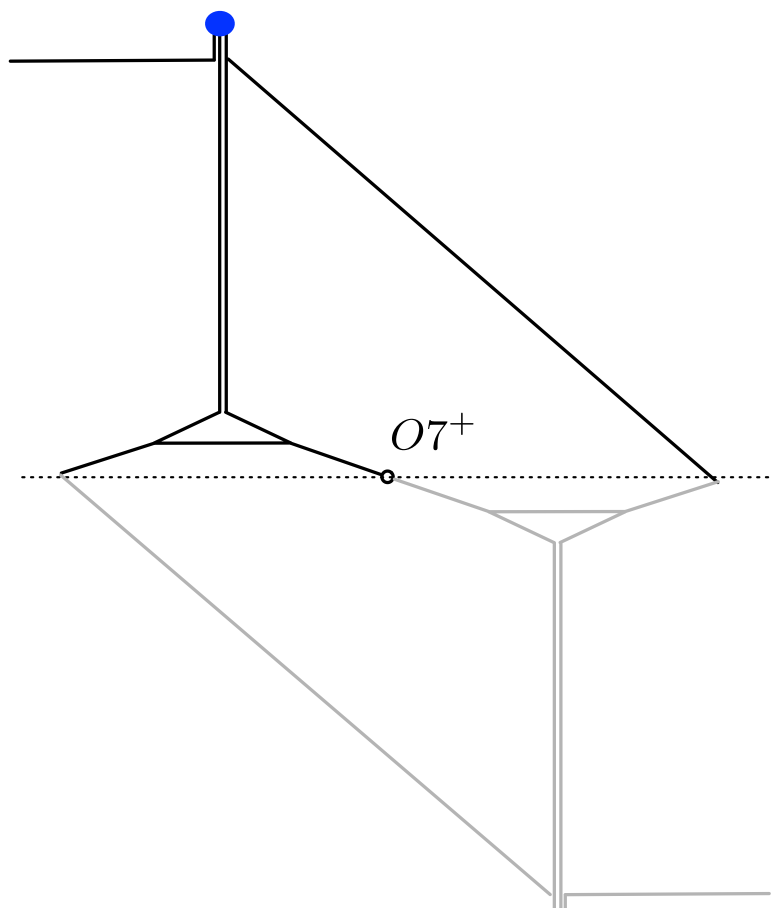
$$\begin{aligned} SU(2)_\pi + 1\mathbf{Adj} - 1\mathbf{F} \quad &\rightarrow \quad SU(2)_\pi + 1\mathbf{AS} + 8\mathbf{F} - \mathbf{F} \\ &= \quad SU(2) + 7\mathbf{F} \end{aligned}$$

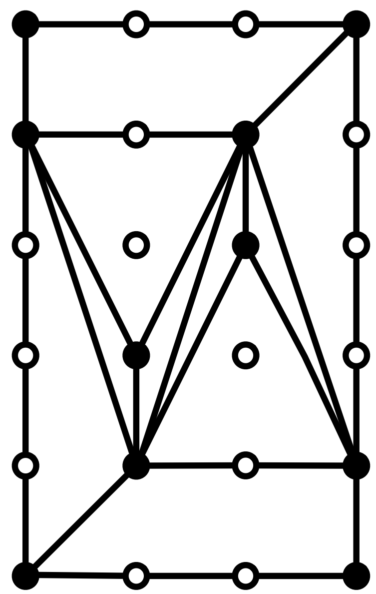
SW $_{\mathrm{SU}(2)+7\mathbf{F}}$ $\xrightarrow{?}$ SW $_{\mathbb{P}^2 + 1\mathbf{Adj}}$

[Eguchi-Sakai '02]

[Huang-Klemm-Poretschkin '13] $O7^- + 8D7$ $\qquad$ $O7^+$

Seiberg-Witten curve for $\mathbb{P}^2 + \mathbf{1Adj}$





$$
\begin{aligned}
&w^5(t-M)^3 \\
&- M^3w^4(t-M)^2(t-M^{-5}) \\
&- 2w^3(t-M)\Big(t^2+(M-U/2)t+M^2\Big) \\
&- 2w^2(1-Mt)\Big(1+(M-U/2)t+M^2t^2\Big) \\
&- M^3w(1-Mt)^2(1-M^{-5}t) \\
&+ (1-Mt)^3 = 0
\end{aligned}
$$

U : Coulomb branch parameter

M : mass of Symmetric hyper

SW $_{\mathrm{SU}(2)+7\mathbf{F}}$ **Yes!** $\Longrightarrow$ SW $_{\mathbb{P}^2+1\mathbf{Adj}}$

$O7^- + 8D7$ $\qquad$ $O7^+$

Mass tuning: $\mathsf{M}_0 = \widetilde{M}^{-1}, \quad \mathsf{M}_{1,2,3} = \widetilde{M}, \quad \mathsf{M}_{4,5,6,7} = -\widetilde{M}$

$$y^2 = 4x^3 - g_2 x - g_3 \ ,$$

where

$$\begin{aligned}
g_2 =& \frac{1}{12}u^4 - \frac{4}{3}\left(3\chi_1^2 + 52\chi_1 + 1164\right)u^2 \\
&- \left(2\chi_1^4 + 48\chi_1^3 - 336\chi_1^2 - 7488\chi_1 - 115168\right)u \\
&+ 16\chi_1^5 + 288\chi_1^4 + 3200\chi_1^3 - \frac{16640}{3}\chi_1^2 - 201472\chi_1 - 2401792, \\
g_3 =& \frac{1}{216}u^6 - 4u^5 - \frac{1}{9}\left(3\chi_1^2 - 92\chi_1 - 7764\right)u^4 \\
&- \frac{1}{6}\left(\chi_1^4 - 72\chi_1^3 - 744\chi_1^2 + 12000\chi_1 + 503824\right)u^3 \\
&+ \frac{4}{9}\left(3\chi_1^5 - 18\chi_1^4 - 4776\chi_1^3 - 33616\chi_1^2 + 323184\chi_1 + 9487968\right)u^2 \\
&- \frac{4}{3}\left(15\chi_1^6 + 236\chi_1^5 - 1212\chi_1^4 - 92256\chi_1^3 - 553968\chi_1^2 + 3339968\chi_1 + 80170944\right)u \\
&+ \chi_1^8 + 64\chi_1^7 + \frac{4912}{3}\chi_1^6 + 15488\chi_1^5 - \frac{95584}{3}\chi_1^4 - \frac{63023104}{27}\chi_1^3 \\
&- \frac{39014656}{3}\chi_1^2 + \frac{148920320}{3}\chi_1 + 1084823808.
\end{aligned}$$

Conclusion

- $O7^+ \leftarrow O7^- + 8D7s$

- Questions and Future directions
  - other dimensions?
    4d reductions or 6d uplifts
    3d S^3 partition functions: Sp(N)+(N_f+2)**F** vs. SO(2N)+N_f **F**

    [SSK-Li-Nawata-Yagi, in progres]

  - Other physical observables or quantum curves?