Bootstrapping integrable systems from 5D gauge theories

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Based on

- Hee-Cheol Kim, Minsung Kim, Sung-Soo Kim, Kimyeong Lee, Yuji Sugimoto, XW: to appear + in progress

Introduction

- Since Seiberg and Witten, the SUSY gauge theory and integrable system correspondence are brought to the table
- This correspondence appears in various dimensions and with various numbers of supercharges
- I want to explore the (quantum) integrable models related to the 5D supersymmetric gauge theories with eight supercharges.

$$H = e^{p} + e^{-p} + e^{x} + e^{-x}$$

Introduction

- We consider 5D SUSY gauge theories with 8 supercharges on $\mathbb{R}^4 \times S^1$, in the (extended) Coulomb branch

$$\phi_i, m_i$$

- Omega deformation: Turning on the holonomies of

$$SO(4) \sim SU(2)_L \times SU(2)_R$$

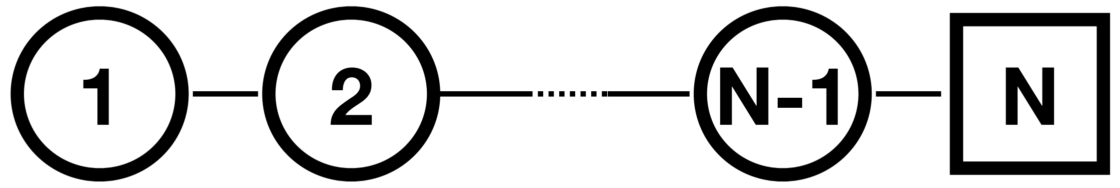
$$(z_1, z_2) \mapsto (e^{2\pi i \epsilon_1} z_1, e^{-2\pi i \epsilon_2} z_2), \quad (z_1, z_2) \in \mathbb{C}^2$$

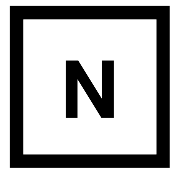
- Instanton partition functions can be calculated from ADHM.

$$Z(\epsilon_1, \epsilon_2, m, \phi)$$

Introduction

- One can consider the insertion of a codimension-two defect, along $R^2_{\epsilon_2} \times S^1$
  - which can be realized by starting with a 3D N=2 quiver gauge theory, and coupling it with the 5D theory





$$\Psi(\epsilon_1, \epsilon_2, m, \phi, x)$$

- Or a codimension-four defect, along S^1

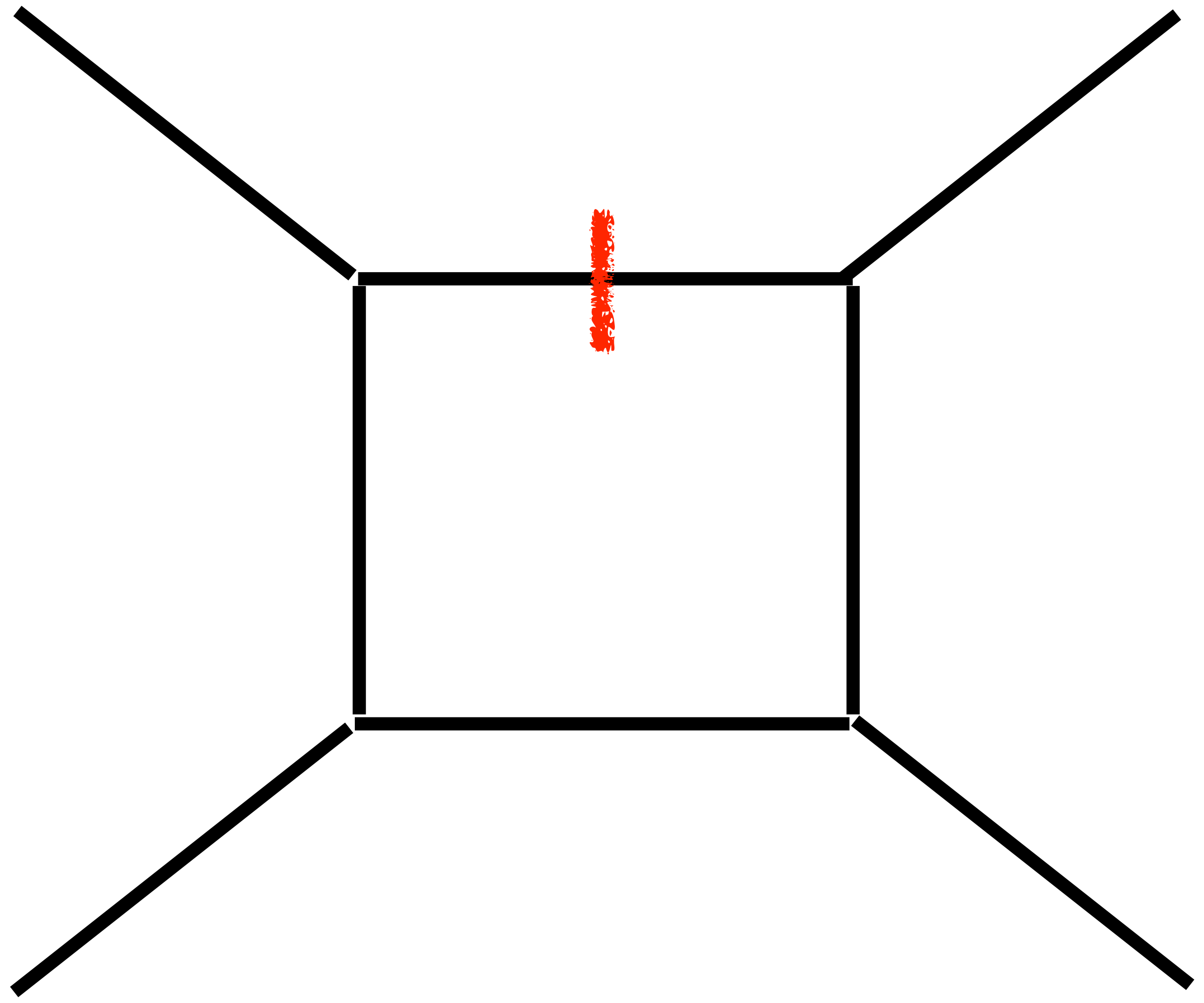
M-theory & Topological string theory relations

- Compactification of M-theory on non-compact Calabi-Yau threefolds X

with compact 4-cycles

  - Closed topological string partition function
    - M2-branes on holomorphic two-cycles
- Inserting M5 brane on the Lagrangian submanifold
  - Codimension-two defect
  - Open topological string partition function / topological branes
    - Refined Chern-Simons theory

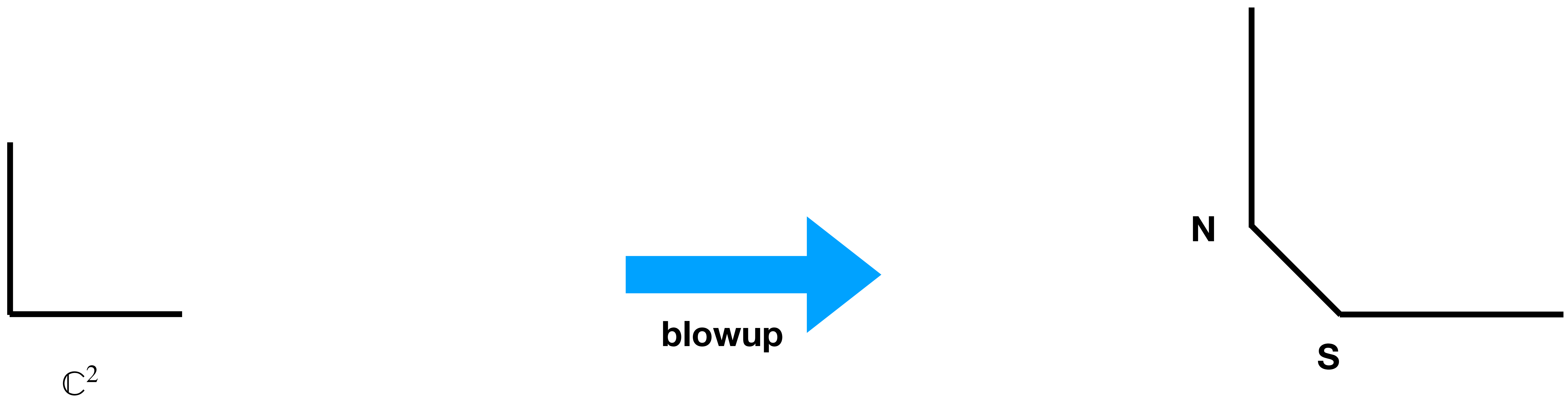
IIB or topological branes



	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			
D3	×	×	×					×		

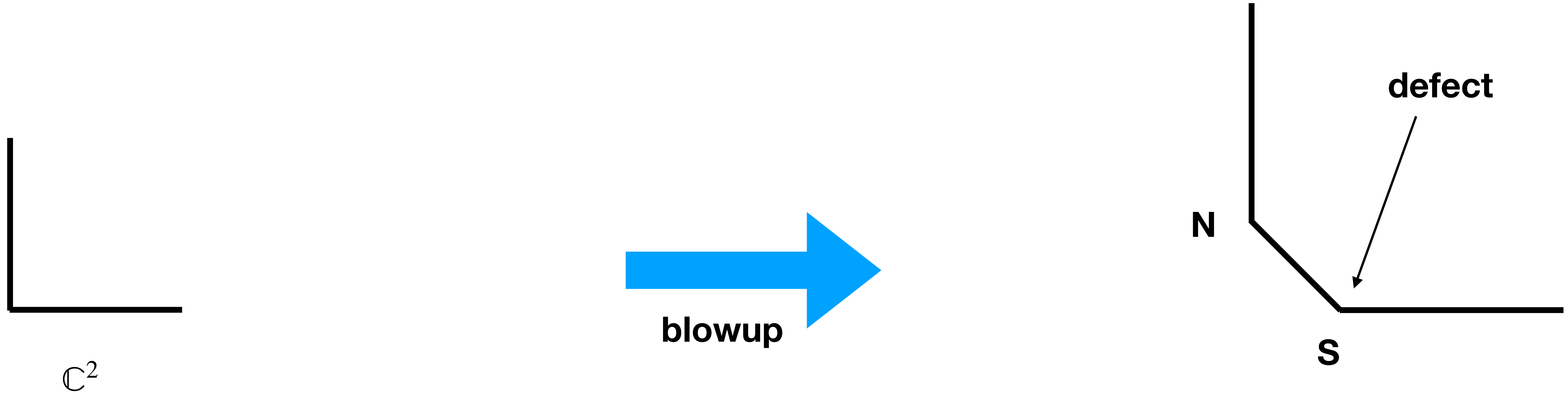
Blowup equation

[Nakajima, Yoshioka ’02, ’03, ’09]



$$Z = \sum_{flux} Z^{S} Z^{N}$$

Our proposal



$$\Psi = \sum_{flux} \Psi^{S} Z^{N}$$

Algorithm

- Input: Gauge theory(gauge groups, matters, Chern-Simons levels, 3d quiver gauge theories)
- Output: partition functions with and without defect

$$\Psi(\epsilon_1, \epsilon_2, m, \phi, x) \qquad Z(\epsilon_1, \epsilon_2, m, \phi) \qquad \langle W_i \rangle(\epsilon_1, \epsilon_2, m, \phi)$$

- Bootstrapping H with a proper ansatz

$$H_k(x_i, \partial x_i, \epsilon_1, m) \lim_{\epsilon_2 \to 0} \Psi(\epsilon_1, \epsilon_2, m, \phi, x) = \lim_{\epsilon_2 \to 0} \langle W_k \rangle(\epsilon_1, \epsilon_2, m, \phi) \lim_{\epsilon_2 \to 0} \Psi(\epsilon_1, \epsilon_2, m, \phi, x)$$

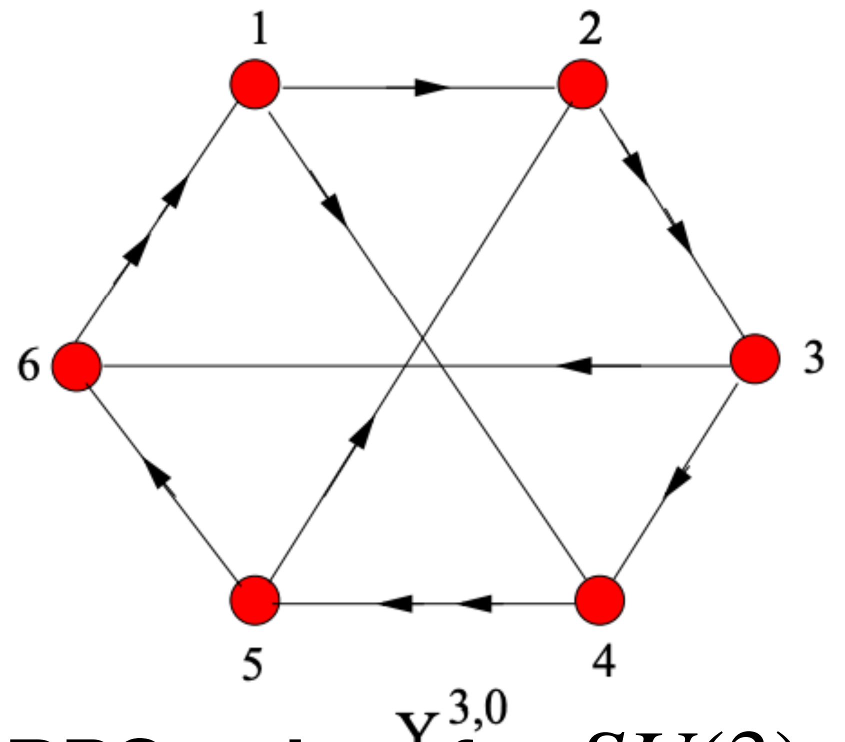
Results

- Selected Hamiltonians for $SU(3)_0$:

$$H_1 = p_{\tau_1} + p_{\tau_2} + p_{\tau_3} - \frac{\tau_2}{\tau_1} p_{\tau_2} - \frac{\tau_3}{\tau_2} p_{\tau_3} + q_1^{\frac{3}{2}} R \frac{\tau_1}{\tau_3} p_{\tau_1},$$

$$H_2 = p_{\tau_1} p_{\tau_3} + p_{\tau_2} p_{\tau_3} + p_{\tau_1} p_{\tau_2} - \frac{\tau_2}{\tau_1} p_{\tau_2} p_{\tau_3} - \frac{\tau_3}{\tau_2} p_{\tau_1} p_{\tau_3} + q_1^{\frac{3}{2}} R \frac{\tau_1}{\tau_3} p_{\tau_1} p_{\tau_2},$$

- $\sim$ Quantum Hamiltonians of the U(3) relativistic Toda chain.



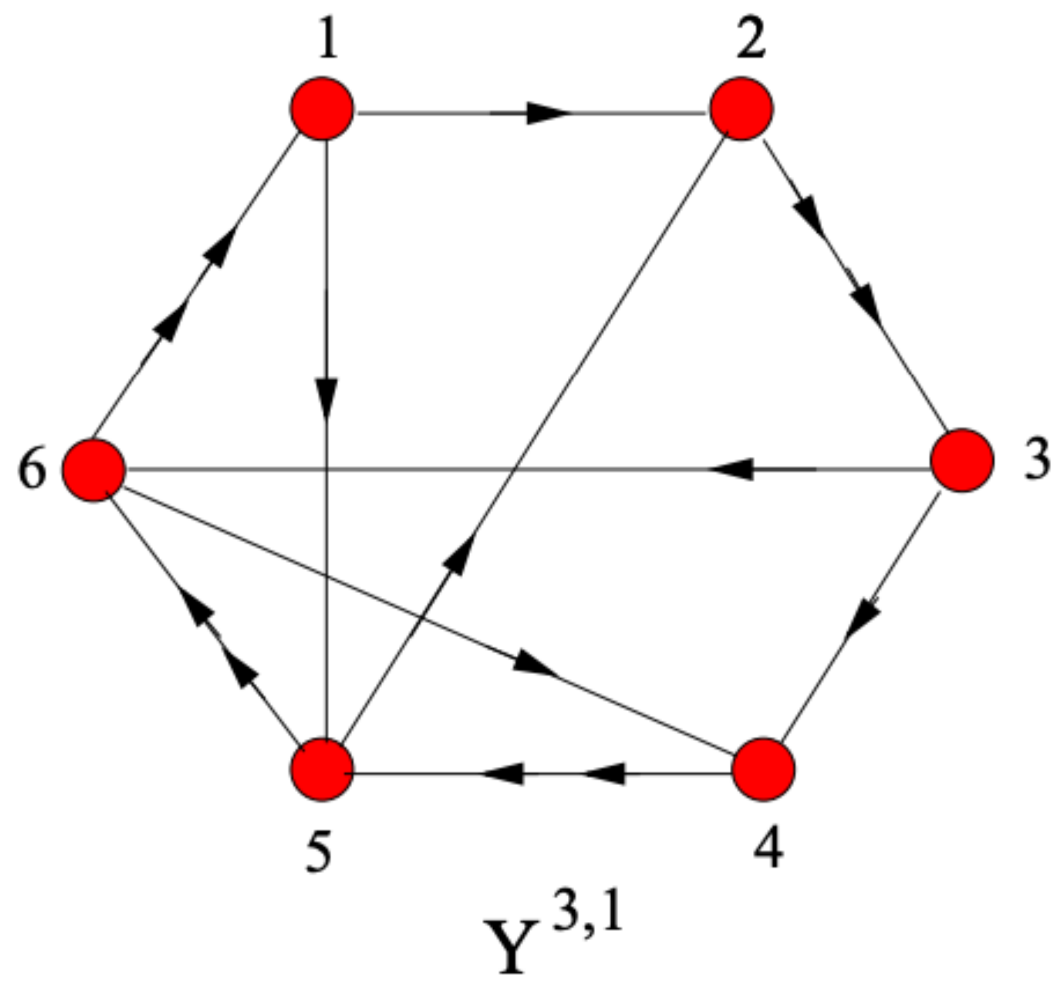
BPS quiver for $SU(3)_0$

Results

- Selected Hamiltonians for $SU(3)_1$:

$$H_1 = p_{\tau_1} + p_{\tau_2} + p_{\tau_3} - \frac{\tau_2}{\tau_1} p_{\tau_2} - \frac{\tau_3}{\tau_2} p_{\tau_3} + q_1 R \frac{\tau_1}{\tau_3},$$

$$H_2 = p_{\tau_1} p_{\tau_3} + p_{\tau_2} p_{\tau_3} + p_{\tau_1} p_{\tau_2} - \frac{\tau_2}{\tau_1} p_{\tau_2} p_{\tau_3} - \frac{\tau_3}{\tau_2} p_{\tau_1} p_{\tau_3} + q_1 R \left(\frac{\tau_1}{\tau_3} - \frac{\tau_2}{\tau_3} \right) p_{\tau_2},$$



BPS quiver for $SU(3)_1$

Results

- Selected Hamiltonians for $SU(3)_2$:

$$H_1 = p_{\tau_1} + p_{\tau_2} + p_{\tau_3} - \frac{\tau_2}{\tau_1} p_{\tau_2} - \frac{\tau_3}{\tau_2} p_{\tau_3} + q_1^{\frac{1}{2}} R \left(\frac{\tau_1}{\tau_3} - \frac{\tau_2}{\tau_3} \right) p_{\tau_2} p_{\tau_3},$$

$$H_2 = p_{\tau_1} p_{\tau_3} + p_{\tau_2} p_{\tau_3} + p_{\tau_1} p_{\tau_2} - \frac{\tau_2}{\tau_1} p_{\tau_2} p_{\tau_3} - \frac{\tau_3}{\tau_2} p_{\tau_1} p_{\tau_3} + q_1^{\frac{1}{2}} R \left(1 - \frac{\tau_2}{\tau_1} \right) \left(1 - q_1 \frac{\tau_2}{\tau_1} \right) \cdot \frac{\tau_1}{\tau_3} p_{\tau_2}^2 p_{\tau_3} - q_1^{\frac{1}{2}} R \frac{\tau_2}{\tau_3},$$

Comment on Integrability

- Here we do not impose any boundary conditions. The solution here is a formal solution, which is not necessarily square integrable.

- By proper phase shifts of the parameters, we usually can find a bounded phase space of the system, that we can impose the (non-perturbative) WKB quantization condition

- Then the square-integrable wave function can be constructed from the linear combination of (non-perturbative) Ψ^{np}

Thank you!