Wilson-loop One-point Functions in ABJM Theory

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Introduction

- AdS/CFT correspondence plays an important role in the study of theoretical physics in the since 1997. [Maldacena, 97][Gubser, Klebanov, Polyakov, 98][Witten, 98]
- In many cases, this correspondence is a strong-weak duality.
- So we can use weakly coupled gravity/string theory to compute quantities in strongly coupled gauge theory in the large N limit.
- The quantities include amplitudes, correlation functions of local operators, vacuum expectation values of loop operators, entanglement entropy, etc.

Introduction

- However, this also makes it hard to confirm this correspondences, since we need to compute quantities in the gauge theory side non-perturbatively.
- The non-perturbative tools in the field theory side of gauge/gravity correspondence include integrability, supersymmetric localization, conformal bootstrap...
- Integrability makes people be able to compute many quantities in the large N limit, even beyond the BPS sectors.

Integrability in AdS_5/CFT_4

- Minahan and Zarembo (02) found that the planar one-loop anomalous dimension matrix in the $SO(6)$ sector of $\mathcal{N}=4$ SYM can be mapped to an integrable Hamiltonian on a spin chain!
- Then the eigenvalues of this anomalous dimension matrix can be computed using intergrability.
- This was later generalized to the full sector at planar all-loop level (in the asymptotic sense). [Beisert, Kistjansen, Staudacher, 03][Beisert, Staudacher, 04] ...
- Benna, Polchinski and Roiban (03) found that the worldsheet theory of IIB superstring on $AdS_5 \times S^5$ in the free limit is a two-dimensional integrable field theory.
- Integrability is an important non-pertubative tool in AdS_5/CFT_4 correspondence.

Integrability in AdS_4/CFT_3

- Three dimensional $\mathcal{N} = 6$ $U(N)_k \times U(N)_{-k}$ super-Chern-Simons theory is dual to IIA string theory on $AdS_4 \times \mathbf{CP}^3$.
- The integrable structure was also found in this AdS_4/CFT_3 correspondence. [Minahan, Zarembo, 08][Bak, Rey, 08][Gromov, Vieira, 08]
- Almost every aspect of integrability in this case is more complicated and difficult.

Integrable boundary state (IBS)

- Integrable boundary states play important role in both quantum quench dynamics and AdS/CFT correspondence. [Piroli, Pozsgay, Vernier, 17]
- IBS appears in the one-point functions of a single-trace operator with a domain wall [de Leeuw, Kristjansen, Zarembo, 15]/Wilson loop [Jiang, Komatsu, Vescovi, to appear]/'t Hooft loop [Kristjansen, Zarembo, 23], and three point functions of two BPS determinant operators and one single-trace operator in $\mathcal{N} = 4$ SYM theory [Jiang, Komatsu, Vescovi, 19].

IBS in ABJM theory

- In ABJM theory, IBS also appears in similar three-point functions [Yang, Jiang, Komatsu, **JW**, 21] and domain wall one-point functions [Kristjansen, Vu, Zarembo, 21].
- One aim of this talk is to show that IBS also appears in some BPS Wilson-loop one-point functions in ABJM theory.

Heisenberg XXX spin chain

- The Hilbert space of a closed XXX spin chain,

$$\mathcal{H} = \otimes_{i=1}^{L} \mathcal{H}_i, \ \mathcal{H}_i \cong \mathbf{C}^2 \,. \tag{1}$$

- We consider the Hamiltonian

$$H = J \sum_{j=1}^{L} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + S_j^z S_{j+1}^z \right) , \tag{2}$$

with periodic boundary condition,

$$S_{L+1}^{\alpha} = S_1^{\alpha}, \, \alpha = x, y, z. \tag{3}$$

Conserved charges of Heisenberg XXX spin chain

- This Hamiltonian is integrable.
- It has infinity many conserved charges, $Q_j, j = 1, 2, \cdots$
- The generating function of these Q's can be chosen as

$$T(u) = U \exp\left(\sum_{n=1}^{\infty} \frac{u^n}{n!} Q_{n+1}\right) , \tag{4}$$

- Here $U = T(0) = Q_1$ is a shift operator.

IBS for XXX chain

- The definition of IBS [Piroli, Pozsgay, Vernier, 17] for XXX chain is that the state $|B\rangle$ satisfying

$$Q_{2l+1}|B\rangle = 0,\ l = 1, 2, \cdots \tag{5}$$

- This is equivalent to

$$\Pi T(u)\Pi|B\rangle = T(u)|B\rangle\ . \tag{6}$$

where Π is the reflection operator:

$$\Pi|i_1, i_2, \cdots, i_L\rangle = |i_L, i_{L-1}, \cdots, i_1\rangle. \tag{7}$$

- For ABJM theory, since there are two sets of conserved charges, the definition of the integrable boundary states is a bit different. (More on this later.)

Properties of IBS

- Eigenstates of integrable Hamiltonian can be labelled by Bethe roots, solutions to certain Bethe ansatz equations (BAEs).
- There exists a selection rule for the overlap between an integrable boundary state and a Bethe state. The overlap is nonzero only when the Bethe roots satisfy certain pairing conditions.
- When this selection rule is satisfied, the overlap can often be expressed as a product of a Gaudin determinant $\det G_+$ and a prefactor. Great simplification!

ABJM theory

- Aharony-Bergman-Jafferis-Maldacena (ABJM) theory is a $3d$ $\mathcal{N}=6$ Chern-Simons-matter theory.
- The gauge group is $U(N)\times U(N)$ with CS levels $(k,-k)$.
- The gauge fields are denoted by A_μ and $\hat{A}_\mu$, respectively.
- The matter fields include complex scalars Y^A and spinors ψ_A $(A=1,\cdots,4)$ in the bi-fundamental representation of the gauge group.
- This theory should be low energy effective theory of N M2-branes putting at the tip of $\mathbf{C}^4/\mathbf{Z}_k$.

Holographic duals

- When $N \gg k^5$, this theory is dual to M-theory on $AdS_4 \times S^7/Z_k$.
- When $k \ll N \ll k^5$, a better description is in terms of IIA superstring theory on $AdS_4 \times \mathbf{CP}^3$.

Bosonic $1/6$-BPS circular WLs

- We consider the Wilson loops (WLs) along $x^\mu(\tau) = (R\cos\tau, R\sin\tau, 0), \tau \in [0, 2\pi]$.
- The construction is the following,

$$W^B_{1/6} = \mathrm{Tr}\mathcal{P}\,\exp\left(-i\oint d\tau \mathcal{A}^B_{1/6}(\tau)\right)\,, \tag{8}$$

$$\hat{W}^B_{1/6} = \mathrm{Tr}\mathcal{P}\,\exp\left(-i\oint d\tau \hat{\mathcal{A}}^B_{1/6}(\tau)\right)\,, \tag{9}$$

$$\mathcal{A}^B_{1/6} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} R_I{}^J Y^I Y^\dagger_J |\dot{x}|\,, \tag{10}$$

$$\hat{\mathcal{A}}^B_{1/6} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} R^J{}_I Y^\dagger_J Y^I |\dot{x}|\,, \tag{11}$$

with $R^I{}_J = \mathrm{diag}(i, i, -i, -i)$. [Drukker, Plefka, Young, 08][Chen, **JW**, 08][Rey, Suyama, Yamaguchi, 08]

Half-BPS WLs

- Drukker and Trancanelli found the half-BPS WLs in 2009 by including the fermions in the construction.

$$W_{1/2} = \mathrm{Tr}\mathcal{P}\,\exp\left(-i\oint d\tau L_{1/2}(\tau)\right), \quad L_{1/2} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix},$$

$$\mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} U_I{}^J Y^I Y_J^\dagger |\dot{x}|, \qquad \bar{f}_1 = \sqrt{\frac{2\pi}{k}}\bar{\alpha}\bar{\zeta}\psi_1|\dot{x}|, \tag{12}$$

$$\hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} U_I{}^J Y_J^\dagger Y^I |\dot{x}|, \qquad f_2 = \sqrt{\frac{2\pi}{k}}\psi^{\dagger 1}\eta\beta|\dot{x}|, \tag{13}$$

with $\bar{\alpha}\beta = i$, and $U_I{}^J = \mathrm{diag}(i, -i, -i, -i)$.

Fermionic $1/6$-BPS WL

- We found various fermionic $1/6$-BPS WLs along a circle [Ouyang, **JW**, Zhang, 15], [My talk @ EAJW16].
- Here we focus on a class of fermionic $1/6$-BPS WLs:

$$W^F_{1/6} = \mathrm{Tr}\mathcal{P}\exp\left(-i\oint d\tau L^F_{1/6}(\tau)\right), \quad L^F_{1/6} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix},$$

$$\mathcal{A} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} U_I{}^J Y^I Y^\dagger_J |\dot{x}|, \quad \bar{f}_1 = \sqrt{\frac{2\pi}{k}}\bar{\alpha}\bar{\zeta}\psi_1|\dot{x}|, \tag{14}$$

$$\hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} U_I{}^J Y^\dagger_J Y^I |\dot{x}|, \quad f_2 = \sqrt{\frac{2\pi}{k}}\psi^{\dagger 1}\eta\beta|\dot{x}|, \tag{15}$$

with $U_I{}^J = \mathrm{diag}(i, i - 2\bar{\alpha}\beta, -i, -i)$.

Fermionic $1/6$-BPS WLs

- When $\bar{\alpha} = \beta = 0$, these fermionic $1/6$-BPS WLs become (essentially) the bosonic $1/6$-BPS WLs.
- When $\bar{\alpha}\beta = i$, these fermionic $1/6$-BPS WLs become half-BPS WLs.

Local operators

- We are interested in the tree-level correlation function of a BPS WL along $x^\mu(\tau) = (R\cos\tau, R\sin\tau, 0)$ and a local operator $\mathcal{O}_C$ at the origin.
- The definition of $\mathcal{O}_C$ is $\mathcal{O}_C = C^{J_1\cdots J_L}_{I_1\cdots I_L}\mathrm{tr}(Y^{I_1}Y^\dagger_{J_1}\cdots Y^{I_L}Y^\dagger_{J_L})$.
- When C is symmetric and traceless, $\mathcal{O}_C$ is a chiral primary operator.
- Here we take $\mathcal{O}_C$ to be a generic local operator which is eigen-operator of the planar two-loop anomalous dimension matrix.

ABJM spin chain

- The operator $\mathcal{O}_C = C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \mathrm{Tr}(Y^{I_1} Y^\dagger_{J_1} \cdots Y^{I_L} Y^\dagger_{J_L})$ can be mapped to a state $|C\rangle := C^{J_1 \cdots J_L}_{I_1 \cdots I_L} |I_1 \bar{J}_1 \cdots I_L \bar{J}_L\rangle$ on an alternating closed $SU(4)$ spin chain with length $2L$.
- The Hilbert space of this chain is $\mathbf{C}^{8L} = \otimes_{i=1}^{2L} \mathbf{C}^4$.
- The odd site of the chain is in the $\mathbf{4}$ representation of $SU(4)$, while the even site is in the $\bar{4}$ representation.

The spin chain Hamiltonian

- The planar two-loop anomalous dimension matrix can be map to the following Hamiltonian on the above chain ([Minahan, Zarembo, 08][Bak, Rey, 08]),

$$\mathbb{H} = \frac{\lambda^2}{2} \sum_{l=1}^{2L} \left(2 - 2\mathrm{P}_{l,l+2} + \mathrm{P}_{l,l+2}\mathrm{K}_{l,l+1} + \mathrm{K}_{l,l+1}\mathrm{P}_{l,l+2}\right) , \qquad (16)$$

where P_{ab} and K_{ab} are permutation and trace operators acting on the a-th and b-th sites. We denote the set of orthonormal basis of the Hilbert space at each site by $|i\rangle$, $i = 1, \cdots, 4$. These two operators act as

$$\mathrm{P}|i\rangle \otimes |j\rangle = |j\rangle \otimes |i\rangle, \qquad \mathrm{K}|i\rangle \otimes |j\rangle = \delta_{ij} \sum_{k=1}^{4} |k\rangle \otimes |k\rangle . \qquad (17)$$

Integrability

- Using algebraic Bethe ansatz method one can constructed two transfer matrices $\tau(u)$ and $\bar{\tau}(u)$, satisfying

$$[\tau(u), \tau(v)] = [\tau(u), \bar{\tau}(v)] = [\bar{\tau}(u), \bar{\tau}(v)] = 0\,. \qquad (18)$$

- They are generating functions of commuting conserved charges, among whom there is the Hamiltonian.
- This proves the integrability of two-loop ABJM spin chain. [Minahan, Zarembo, 08][Bak, Rey, 08]

Bethe roots

- Eigenstates of $\mathbb{H}$ can be constructed using algebraic Bethe ansatz and the states are parameterized by three set of Bethe roots,

$$u_1, \cdots, u_{K_{\mathbf{u}}}, \tag{19}$$

$$v_1, \cdots, v_{K_{\mathbf{v}}}, \tag{20}$$

$$w_1, \cdots, w_{K_{\mathbf{w}}}. \tag{21}$$

Bethe ansatz equations

- These Bethe roots should satisfy the following Bethe ansatz equations,

$$1=\left(\frac{u_j+\frac{i}{2}}{u_j-\frac{i}{2}}\right)^L\prod_{\substack{k=1\\k\neq j}}^{K_\mathrm{u}}S\left(u_j,u_k\right)\prod_{k=1}^{K_\mathrm{w}}\tilde{S}\left(u_j,w_k\right), \tag{22}$$

$$1=\prod_{\substack{k=1\\k\neq j}}^{K_\mathrm{w}}S\left(w_j,w_k\right)\prod_{k=1}^{K_\mathrm{u}}\tilde{S}\left(w_j,u_k\right)\prod_{k=1}^{K_\mathrm{v}}\tilde{S}\left(w_j,v_k\right), \tag{23}$$

$$1=\left(\frac{v_j+\frac{i}{2}}{v_j-\frac{i}{2}}\right)^L\prod_{\substack{k=1\\k\neq j}}^{K_\mathrm{v}}S\left(v_j,v_k\right)\prod_{k=1}^{K_\mathrm{w}}\tilde{S}\left(v_j,w_k\right), \tag{24}$$

Bethe ansatz equations

- In the previous page, the S-matrices $S(u,v)$ and $\tilde{S}(u,v)$ are given by

$$S(u,v) \equiv \frac{u-v-i}{u-v+i}, \quad \tilde{S}(u,v) \equiv \frac{u-v+\frac{i}{2}}{u-v-\frac{i}{2}}. \tag{25}$$

- The cyclicity property of the single trace operator is equivalent to the zero momentum condition

$$1 = \prod_{j=1}^{K_{\mathbf{u}}} \frac{u_j+\frac{i}{2}}{u_j-\frac{i}{2}} \prod_{j=1}^{K_{\mathbf{v}}} \frac{v_j+\frac{i}{2}}{v_j-\frac{i}{2}}. \tag{26}$$

- The eigenvalues of $\tau(u), \bar{\tau}(u), \mathbb{H}$ on the Bethe state $|\mathbf{u},\mathbf{v},\mathbf{w}\rangle$ can be expressed in terms of the Bethe roots, $\mathbf{u},\mathbf{v},\mathbf{w}$.

Numerical solution

- The BAEs and zero momentum condition can be solved using rational Q-system. [Marboe, Volin, 16][Gu, Jiang, Sperling, 22].
- The Bethe states can be constructed using the algorithm in [Yang, Jiang, **JW**, Komatsu, 21] based on coordinate Bethe ansatz.

Wick contractions

- We plan to study the tree-level correlation function of a BPS WL and a local operator. We take $W(\mathcal{C})^B_{1/6}$ as an example. At tree-level, the correlator $\langle W(\mathcal{C})^B_{1/6}\mathcal{O}_C(0)\rangle$ only gets contributions from

$$\oint\cdots\oint d\tau_{1>2>\cdots>L}\left(\frac{2\pi}{k}\right)^L\langle\mathrm{tr}(R^{\tilde{J}_1}{}_{\tilde{I}_1}Y^{\tilde{I}_1}(x_1)Y^\dagger_{\tilde{J}_1}(x_1)\cdots R^{\tilde{J}_L}{}_{\tilde{I}_L}Y^{\tilde{I}_L}(x_L)Y^\dagger_{\tilde{J}_L}(x_L))C^{J_1\cdots J_L}_{I_1\cdots I_L}\mathrm{tr}(Y^{I_1}(0)Y^\dagger_{J_1}(0)\cdots Y^{I_L}(0)Y^\dagger_{J_L}(0))\rangle\,, \tag{27}$$

- where $x_i=(R\cos\tau_i,R\sin\tau_i,0)$, $i=1,\cdots,L$, and

$$\oint\cdots\oint d\tau_{1>2>\cdots>L}=\int_0^{2\pi}d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{L-1}}d\tau_L\,. \tag{28}$$

Wick contractions

- One can easily obtain

$$\langle W(\mathcal{C})^B_{1/6}\mathcal{O}_C(0)\rangle = \frac{\lambda^{2L}k^L}{(L-1)!(2R)^{2L}} C^{J_1\cdots J_L}_{I_1\cdots I_L} R^{I_L}{}_{J_L}\cdots R^{I_1}{}_{J_1}\,. \qquad (29)$$

- Here $\lambda \equiv \frac{N}{k}$ is the 't Hooft coupling of ABJM theory and we have used the tree-level propagators of the scalar fields

$$\langle Y^{I\alpha}{}_{\bar\beta}(x) Y^{\dagger\ \bar\gamma}_{J\ \ \rho}(y)\rangle = \frac{\delta^I_J \delta^\alpha_\rho \delta^{\bar\gamma}_{\bar\beta}}{4\pi|x-y|}\,. \qquad (30)$$

Boundary state

- In the spin chain language, we can introduce the following boundary state

$$|\mathcal{B}^B_{1/6}\rangle = |\mathcal{B}_R\rangle\,, \tag{31}$$

where, for a four-dimensional matrix M, we define the boundary state $|\mathcal{B}_M\rangle$ through

$$\langle\mathcal{B}_M| \equiv M^{I_1}{}_{J_1} M^{I_2}{}_{J_2} \cdots M^{I_L}{}_{J_L} \langle I_1, J_1, \cdots, I_L, J_L| = \left(M^I{}_J \langle I, J|\right)^{\otimes L}\,, \tag{32}$$

which is a two-site state.

The overlap

Then the above correlation function can be expressed as

$$\langle W(\mathcal{C})^B_{1/6}\mathcal{O}_C(0)\rangle = \frac{\lambda^{2L}k^L}{(L-1)!(2R)^{2L}}\langle\mathcal{B}^B_{1/6}|\mathcal{O}_C\rangle\,, \quad (33)$$

where $|\mathcal{O}_C\rangle$ is the spin chain state corresponding to the operator $\mathcal{O}_C$.

The norm

- Let us define the normalization factor $\mathcal{N}_{\mathcal{O}}$ using the two-point function of $\mathcal{O}$ and $\mathcal{O}^\dagger$ as

$$\langle \mathcal{O}(x)\mathcal{O}^\dagger(y)\rangle = \frac{\mathcal{N}_{\mathcal{O}}}{|x-y|^{2\Delta_{\mathcal{O}}}}\,, \tag{34}$$

  where $\Delta_{\mathcal{O}}$ is the conformal dimension of $\mathcal{O}$.
- At tree level and the planar limit, we have

$$\mathcal{N}_{\mathcal{O}} = \left(\frac{N}{4\pi}\right)^{2L} L\langle \mathcal{O}|\mathcal{O}\rangle\,. \tag{35}$$

WL one-point function

- We define the Wilson-loop one-point function as

$$\langle\langle \mathcal{O} \rangle\rangle_{W(\mathcal{C})} \equiv \frac{\langle W(\mathcal{C}) \mathcal{O} \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}} . \tag{36}$$

- Then for $W^B_{1/6}$ we have

$$\langle\langle \mathcal{O} \rangle\rangle_{W(\mathcal{C})^B_{1/6}} = \frac{\pi^L \lambda^L}{R^{2L}(L-1)!\sqrt{L}} \frac{\langle \mathcal{B}^B_{1/6} | \mathcal{O} \rangle}{\sqrt{\langle \mathcal{O} | \mathcal{O} \rangle}} . \tag{37}$$

- The computation of the Wilson loop one-point function thus amounts to the calculation of

$$\frac{\langle \mathcal{B}^B_{1/6} | \mathcal{O} \rangle}{\sqrt{\langle \mathcal{O} | \mathcal{O} \rangle}} , \tag{38}$$

which will be performed by integrability in some cases.

Boundary states from other WLs

- For $\hat{W}(\mathcal{C})^B_{1/6}$, the boundary state is

$$\langle \hat{\mathcal{B}}^B_{1/6}| = R^{I_1}{}_{J_L} R^{I_2}{}_{J_1} \cdots R^{I_L}{}_{J_{L-1}} \langle I_1, J_1, \cdots, I_L, J_L| \,. \tag{39}$$

- We can rewrite $|\hat{\mathcal{B}}^B_{1/6}\rangle$ as

$$|\hat{\mathcal{B}}^B_{1/6}\rangle = U_{\text{even}} |\mathcal{B}^B_{1/6}\rangle \tag{40}$$

where U_{even} is the shift operator which shifts all even site to the left by two units and leave the odd sites untouched.

Boundary states from other WLs

- The boundary state from $W^F_{1/6}$ is

$$|\mathcal{B}^F_{1/6}\rangle = (1 + U_{\text{even}})|\mathcal{B}_U\rangle\,, \tag{41}$$

  with $U = \text{diag}(i, i - 2\bar{\alpha}^1\beta_1, -i, -i)$.
- The boundary state from $W_{1/2}$ is

$$|\mathcal{B}_{1/2}\rangle = |\mathcal{B}^F_{1/6}\rangle|_{\bar{\alpha}^1\beta_1 = i} \tag{42}$$

IBS from WLs

- Partly based on [Piroli, Pozsgay, Vernier, 17], we proved that the boundary state $|\mathcal{B}\rangle$ from a bosonic $1/6$-BPS WL or a half-BPS WL statisfies the following twisted integrable condition,

$$\tau(-u-2)|\mathcal{B}\rangle = \tau(u)|\mathcal{B}\rangle. \tag{43}$$

- This leads to the pairing condition which states that $\langle\mathcal{B}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle$ is non-zero only when the selection rule

$$\mathbf{u} = -\mathbf{v}\,, \qquad \mathbf{w} = -\mathbf{w} \tag{44}$$

is satisfied.

- Here $\mathbf{u},\mathbf{v},\mathbf{w}$ are three sets of Bethe roots.
- Another selection rule for $\langle\mathcal{B}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle$ being nonzero is that $K_{\mathbf{u}} = K_{\mathbf{v}} = K_{\mathbf{w}} = L$.

Non-integrable boundary states

- In our paper, we also showed that the boundary state from a generic(*) fermionic $1/6$-BPS WL is not integrable.
- $*$ By 'generic', we mean that this WL is neither half-BPS nor essentially bosonic $1/6$-BPS.

Overlaps

- We obtained the following formula for the normalized overlap between $|\mathcal{B}^R_{1/6}\rangle$ and a Bethe state,

$$\frac{|\langle \mathcal{B}^R_{1/6}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle|^2}{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle} = \prod_{i=1}^{K_{\mathbf{w}}/2} \frac{w_i^2}{w_i^2+1/4} \times \frac{\det G^+}{\det G^-}. \tag{45}$$

- Here the Bethe roots satisfy the pairing condition, G^+ and G^- are Gaudin matrices depending on $\mathbf{u},\mathbf{v},\mathbf{w}$. The definition of these matrices can be found in [Yang, Jiang, Komatsu, **JW**, 21]
- This result was obtained using [Gombor, Bajnok, 20][Gombor, Kristjansen, 22] and passed non-trivial checks based on numerical computations.

Overlaps

- For another bosonic $1/6$-BPS WL, we have

$$\frac{\langle \widehat{\mathcal{B}}^R_{1/6}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}{\sqrt{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}} = \prod_{j=1}^{K_\mathbf{u}} \frac{u_j + i/2}{u_j - i/2} \frac{\langle \mathcal{B}_R|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}{\sqrt{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle}}. \tag{46}$$

- Hence there is a relative phase between these two boundary state.

Overlaps

For half-BPS WLs, we have

$$\frac{|\langle \mathcal{B}_{1/2}|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle|^2}{\langle \mathbf{u},-\mathbf{u},\mathbf{w}|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle} = \left|1+\prod_{j=1}^{K_{\mathbf{u}}}\left(\frac{u_j+i/2}{u_j-i/2}\right)^2\right|^2 \frac{|\langle \mathcal{B}_U|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle|^2}{\langle \mathbf{u},-\mathbf{u},\mathbf{w}|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle}. \tag{47}$$

$$\frac{|\langle \mathcal{B}_U|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle|^2}{\langle \mathbf{u},-\mathbf{u},\mathbf{w}|\mathbf{u},-\mathbf{u},\mathbf{w}\rangle} = (-1)^L \prod_{i=1}^{K_{\mathbf{u}}}(u_i^2+\frac{1}{4}) \prod_{j=1}^{[K_{\mathbf{w}}/2]} \frac{1}{w_i^2(w_i^2+1/4)} \frac{\det G_+}{\det G_-}. \tag{48}$$

Summary

- By studying WL one-point function at tree level, we found that bosonic $1/6$-BPS and half-BPS WLs lead to integrable boundary states (in the scalar sector).
- For generic fermionic $1/6$-BPS WLs, the corresponding boundary states are not integrable.
- We computed the norm of the overlap of the integrable boundary states from WLs and the Bethe states.

Outlook

- Compute the phases of the overlaps when the boundary states from WLs are integrable.
- Are boundary states from bosonic $1/6$-BPS and half-BPS WLs integrable in the full sector and at higher loop level?
- Possible all loop overlaps in the asymptotic sense.
- And finite size effects from TBA.
- Integrable boundary states from circular WLs in higher dimensional representations of a suitable (super-)group? How about the case of more complicated WLs?
- Correlators of BPS WLs and CPOs from localization and/or holography.

Thanks for Your Attention !