Ghost problem and constraints on brane-localized higher derivative gravity

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based on [arXiv:2310.16297 [hep-th]] and JHEP 03 (2022) 145

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East Asia Joint Workshop on Fields and Strings 2023, Xi'an

Outline

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Motivations for HD gravity

Higher derivative (HD) gravity is renormalizable but suffers ghost problem.

- Motivation of HD: renormalization, string theory prediction, effective theory, general AdS/CFT, modified gravity,...
- Ghost problem of HD:

$$I \sim \int d^d y \sqrt{|h|}\,\bar{h}^{ij}\Box(\Box - m^2)\bar{h}_{ij}, \tag{1}$$

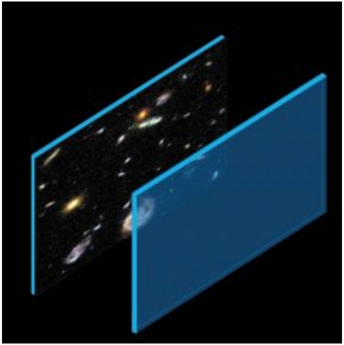
where $\bar{h}^{ij}$ obeys transverse traceless gauge and the propagator always includes one ghost

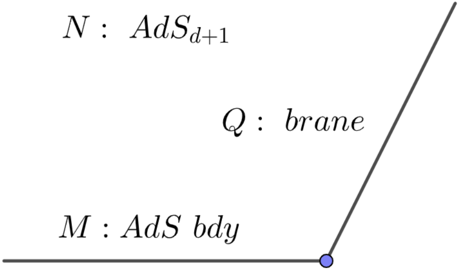
$$G \sim \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{m^2}\left(\frac{1}{p^2} - \frac{1}{p^2+m^2}\right). \tag{2}$$

Outline

Motivations for braneworld

- Braneworld plays an important role in cosmology, particle physics, string theory...
- Double holography is vital in recent breakthroughs in resolving the black hole information paradox.
- AdS/BCFT is a powerful tool to study the quantum boundary effects, such as Casimir effect, anomalous transport...





Outline

Toy model: HD scalar on branes

- HD scalar in a flat strip

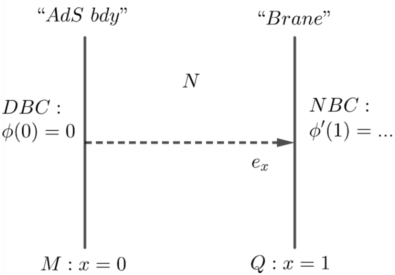
$$I = \int_N dx d^d y \left(\frac{1}{2} \phi \Box_N \phi \right) + \int_Q d^d y \left(\frac{\lambda}{2} \phi \Box \phi + \frac{\alpha}{2} \phi \Box^2 \phi \right), \quad (3)$$

- boundary condition

$$\text{DBC on M} : \phi|_{x=0} = 0 \quad (4)$$

$$\text{NBC on Q} : \partial_x \phi|_{x=1} = (\lambda \Box + \alpha \Box^2) \phi|_{x=1} \quad (5)$$

- geometry



Toy model: mass spectrum

- expand bulk scalar in KK modes: $\phi = \sum_m \sin(mx) Y_m(y)$
- Neumann boundary condition (NBC)

$$m \sin(m) \left(\lambda + \alpha m^2\right) - \cos(m) = 0. \tag{6}$$

- mass spectrum in large α limit

$$m^2 : \frac{-1}{\sqrt{\alpha}}\left(1 + \frac{1+3\lambda}{6\sqrt{\alpha}} + ...\right),\ \frac{1}{\sqrt{\alpha}}\left(1 - \frac{1+3\lambda}{6\sqrt{\alpha}} + ...\right),\ \pi^2 \mathbb{Z}_+^2, \tag{7}$$

- mass spectrum for finite α

m^2 for $\alpha = 0.1$	-4.722	1.786	11.508	39.977
m^2 for $\alpha = -0.1$	$\times$	$2.089 \pm 3.157i$	7.008	38.964

- $m^2 < 0$ for $\alpha > 0$, m^2 complex for $\alpha < 0$, both unstable!

Toy model: effective action

- expand bulk scalar in KK modes: $\phi = \sum_m X_m(x) Y_m(y)$
- EOM: $X_m(x) = \sin(mx)$, $(\Box - m^2) Y_m = 0$
- effective action on brane Q

$$
\begin{aligned}
I &= \frac{1}{2} \sum_{m,m'} \int_Q d^d y Y_m \Big[c_m \delta_{m,m'} (\Box - m^2) \\
&\qquad + \alpha X_m(1) X_{m'}(1) (\Box - m^2)(\Box - m'^2) \Big] Y_{m'}, \qquad (8)
\end{aligned}
$$

- ghost-free condition

$$
\begin{aligned}
c_m &= \langle X_m, X_m \rangle \geq 0 \\
&= \int_0^1 dx X_m^2(x) + \lambda X_m^2(1) + 2\alpha m^2 X_m^2(1), \qquad (9)
\end{aligned}
$$

which imposes constraints on parameters.

Toy model: ghosts

- For $\alpha > 0$, we have $m^2 < 0$. This mode $X_m = \sinh(|m|x)$ yields negative inner product

$$c_m = \langle X_m, X_m \rangle = -\frac{1}{2} - \frac{3\sinh(2\,|m|)}{4\,|m|} < 0, \tag{10}$$

- For $\alpha < 0$, we have a pair of complex conjugate m^2, yielding a negative inner product

$$\langle \frac{X_m + X_{m*}}{2}, \frac{X_m + X_{m*}}{2} \rangle = \frac{c_m + c_{m*}}{4}, \tag{11}$$

$$\langle \frac{X_m - X_{m*}}{2i}, \frac{X_m - X_{m*}}{2i} \rangle = -\frac{c_m + c_{m*}}{4}. \tag{12}$$

- Brane-localized HD scalar always yields a ghost.

Toy model: novel relations

- Effective action $I \sim \int d^d y Y^T M Y$ summing over non-ghost modes implies a ghost mode
  $M = \left[c_m \delta_{m,m'} (\Box - m^2) + \alpha X_m(1) X_{m'}(1) (\Box - m^2)(\Box - m'^2) \right]$

$$|M| \sim (\Box - m_g^2) \prod_{m \neq m_g} (\Box - m^2), \quad (13)$$

- Ghost mass lies in mass spectrum fixed by NBC

$$m_g^2 = \frac{-\frac{1}{\alpha} + \sum_{m \neq m_g} \frac{X_m^2(1) m^2}{c_m}}{\sum_{m \neq m_g} \frac{X_m^2(1)}{c_m}} \quad (14)$$

- It implies two novel relations, physical meaning?

$$\sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = 0, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} = \frac{1}{\alpha} \quad (15)$$

Toy model: summary

- There is always a complex-mass state for $\alpha \neq 0$, implying unstable.
- There is always a ghost for brane-localized HD scalar with $\alpha \neq 0$.
- The theory is well-defined for DGP-like scalar with $\lambda > 0, \alpha = 0$.
- We find novel relations between ghost and non-ghost modes.

$$\sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = 0, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} = \frac{1}{\alpha} \tag{16}$$

- The relations hold if and only if there are ghosts.
- We have checked (16) numerically and perturbatively for large α.
- The case of gravity is similar.

HD gravity on branes

- Action in AdS/BCFT

$$
\begin{aligned}
I &= I_{\text{Einstein}} + 2\int_Q d^d y \sqrt{|h|}\Big[K - T + \lambda R \\
&\qquad \alpha_1 L_{\text{GB}}(\hat{R}) + \alpha_2\big(\hat{R}^{ij}\hat{R}_{ij} - \frac{d}{4(d-1)}\hat{R}^2\big) + \alpha_3 \hat{R}^2\Big], \quad (17)
\end{aligned}
$$

- Effective action implies a ghost on brane $Q : (r = \rho)$

$$
\begin{aligned}
I &= \frac{1}{4}\sum_{m,m'}\int_Q d^d y \sqrt{|\bar{h}^{(0)}|}\cosh^{d-2}(\rho)\bar{h}^{(m)}_{ij}\Big[c_m \delta_{m,m'}(\bar{\Box} + 2 - m^2) \\
&+ 2\alpha_2 \text{sech}^2(\rho) H_m(\rho) H_{m'}(\rho)(\bar{\Box} + 2 - m^2)(\bar{\Box} + 2 - m'^2)\Big]\bar{h}^{(m')ij},
\end{aligned}
$$

- Like toy model, there is always a ghost for $\alpha_2 \neq 0$.
- Unlike toy model, there are no negative or complex m^2 for $\alpha_2 < 0$ and large ρ.

Outline

Various Constraints from AdS/BCFT

- Tachyon-free and ghost-free conditions require positive DGP and brane-localized GB gravity

$$\lambda + 4(d-3)\alpha_1 \operatorname{sech}^2(\rho) \geq 0, \ \textit{for} \ \alpha_2 = 0 \tag{18}$$

- Ghost-free condition rules out brane-localized HD gravity

$$\alpha_2 = 0, \ \textit{for} \ \lambda + 4(d-3)\alpha_1 \operatorname{sech}^2(\rho) = 0 \tag{19}$$

- We also analyze constraints from brane-bending mode, boundary central charge and holographic entanglement entropy (HEE), which are weaker.

Various Constraints from AdS/BCFT

Tachyon-free $m^2 \geq 0$	$\lambda + 4(d-3)\alpha_1 \text{ sech}^2(\rho) \geq 0, \text{ for } \alpha_2 = 0,$ $\alpha_2 \leq 0, \text{ for } \lambda + 4(d-3)\alpha_1 \text{ sech}^2(\rho) = 0,$
KK mode	$c_m = 2\Big(\lambda + 4(d-3)\alpha_1 \text{sech}^2(\rho) + 2\alpha_2 \text{sech}^2(\rho) m^2\Big) H_m^2(\rho)$ $+ \int_{-\infty}^{\rho} \frac{\cosh(r)^{d-2}}{\cosh^{d-2}(\rho)} H_m^2(r) dr \geq 0,$
Bending mode	$\hat{\lambda} \geq -\frac{\int_0^{\rho} dr \text{ sech}^{d-2}(r)}{2\left(\text{sech}^{d-2}(\rho) + (d-2)\tanh(\rho) \int_0^{\rho} dr \text{ sech}^{d-2}(r)\right)},$
A-type charge	$\lambda \geq -\int_0^{\rho} \frac{\cosh^{d-2}(r)}{2\cosh^{d-2}(\rho)} dr,$
B-type charge	$\hat{\lambda} \geq -\frac{\coth(\rho)}{2(d-2)},$
HEE	$\bar{\lambda} \geq -\frac{1}{2(d-2)},$
Notations	$\hat{\lambda} = \lambda + 4\alpha_1(d-3)\text{sech}^2(\rho) - \alpha_2(d-2)\text{sech}^2(\rho),$ $\bar{\lambda} = \lambda + 2\alpha_1(d-2)(d-3)\text{sech}^2(\rho) - \frac{\alpha_2}{2}(d-2)^2\text{sech}^2(\rho).$

Summary and outlook

Summary:

- The brane-localized HD gravity has a ghost problem generally.
- The tachyon-free and ghost-free conditions require a positive DGP and GB gravity.
- We find novel relations for the mass spectrum on branes.

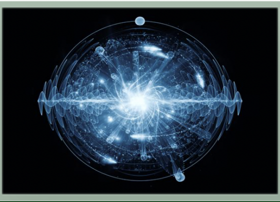
$$\text{HD} \quad \alpha \neq 0 : \sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = 0, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} = \frac{1}{\alpha} \quad (20)$$

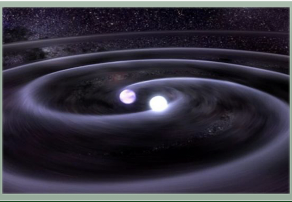
$$\text{DGP} \; \alpha = 0 : \sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = \frac{1}{\lambda}, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} \to \infty (21)$$

Outlook:

- Ghost-free HD gravity by considering infinite derivatives
- Applications of HD brane to cosmology and black holes

Thanks!





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Novel identities of mass spectrum in braneworld

- bulk scalar in KK modes: $\phi = \sum_m \sin(mx) Y_m(y)$, $X_m(x) = \sin(mx)$
- Neumann boundary condition (NBC)

$$m \sin(m) \left(\lambda + \alpha m^2\right) - \cos(m) = 0. \tag{22}$$

- inner product

$$\langle X_m, X_m \rangle = \int_0^1 dx X_m^2(x) + \lambda X_m^2(1) + 2\alpha m^2 X_m^2(1), \tag{23}$$

- identities of mass spectrum

$$\text{HD} \quad \alpha \neq 0: \ \sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = 0, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} = \frac{1}{\alpha} \tag{24}$$

$$\text{DGP} \ \alpha = 0: \ \sum_m \frac{X_m^2(1)}{\langle X_m, X_m \rangle} = \frac{1}{\lambda}, \quad \sum_m \frac{X_m^2(1) m^2}{\langle X_m, X_m \rangle} \to \infty \tag{25}$$

- It works for general braneworld.