Mirror Symmetry of Bad Theories

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Based on

- S. Giacomelli, CH, F. Marino, S. Pasquetti, M. Sacchi,

  “**Probing bad theories with the dualization algorithm I,**” [arXiv:2309.05326],

  “**Probing bad theories with the dualization algorithm II,**” to appear.

Also see

- R. Comi, CH, F. Marino, S. Pasquetti, M. Sacchi, “**The SL(2, $\mathbb{Z}$) dualization algorithm at work,**” JHEP 06 (2023) 119, [arXiv:2212.10571],

- CH, S. Pasquetti, M. Sacchi, “**Rethinking mirror symmetry as a local duality on fields,**” Phys.Rev.D 106 (2022) 10, 105014, [arXiv:2110.11362],

- L. E. Bottini, CH, S. Pasquetti, M. Sacchi, “**4d S-duality wall and SL(2, $\mathbb{Z}$) relations,**” JHEP 03 (2022) 035, [arXiv:2110.08001].

Outline

- Introduction: building blocks of dualities
- 3d mirror symmetry as the piecewise dualization
- Application: mirror symmetry of bad theories
- Remarks

Building blocks of dualities

Example I: 3D Gauge-Scalar Duality

- **The 3d Maxwell theory**

$$Z_{Maxwell} = \int \mathcal{D}A_\mu \exp\left[\int d^3x \frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu}\right]$$

- The field-strength $F_{\mu\nu}$ satisfies the Bianchi identity:

$$\epsilon^{\mu\nu\rho}\,\partial_\mu F_{\nu\rho} = 0 \quad \Rightarrow \quad \textbf{A conserved current} \quad J^\mu_{top} = \frac{1}{4\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}$$

- Rewriting the integration measure in terms of gauge invariant $F_{\mu\nu}$,

$$Z_{Maxwell} = \int \mathcal{D}F_{\mu\nu}\,\mathcal{D}\gamma\, \exp\left[\int d^3x \left(\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{i}{4\pi}\gamma\,\epsilon^{\mu\nu\rho}\,\partial_\mu F_{\nu\rho}\right)\right]$$

$$= \int \mathcal{D}\gamma\, \exp\left[\int d^3x \frac{e^2}{8\pi^2}\partial_\mu\gamma\,\partial^\mu\gamma\right] \qquad J^\mu_{top} = \frac{ie^2}{(2\pi)^2}\partial^\mu\gamma \quad \Rightarrow \quad \gamma + \alpha$$

- A simple duality example between ***the 3d Maxwell theory*** and ***a scalar theory***, both of which are free.

Example II: 3D Vortex-Particle Duality

Peskin 78, Dasgupta-Halperin 81, Chen-Fisher-Wu 93, Barkeshli-McGreevy 14, Aharony 15, Karch-Tong 16, Seiberg-Senthil-Wang-Witten 16

- Generalization to **interacting** theories: **the 3d bosonization**

$$i\bar{\chi}\gamma_\mu D_a^\mu \chi - \frac{1}{2\pi}Bda - \frac{1}{4\pi}BdB \quad \longleftrightarrow \quad |D_B\phi|^2 - |\phi|^4$$

- Don't know how to prove it; nevertheless, a number of new dualities can be derived from it; e.g., **the vortex-particle duality**

$$|D_b\phi|^2 - |\phi|^4 + \frac{1}{2\pi}bdC \quad \longleftrightarrow \quad i\bar{\chi}\gamma_\mu D_a^\mu \chi - \frac{1}{2\pi}bda - \frac{1}{4\pi}bdb + \frac{1}{2\pi}bdC \quad \longleftrightarrow \quad |D_C\hat{\phi}|^2 - |\hat{\phi}|^4$$

- Adding $\frac{1}{2\pi}BdC$ + gauging $B \to b$ + the (time-reversed) bosonization

- The bosonization —> the vortex-particle duality;
  The vortex-particle duality -/-> the bosonization

- Namely, the bosonization is more fundamental than the vortex-particle duality.

What are the building blocks of 3d mirror symmetry?

3d mirror symmetry as the piecewise dualization

3D Mirror Symmetry

- A duality typically relating two 3d $\mathcal{N} = 4$ theories whose Higgs and Coulomb branches are exchanged (Intriligator-Seiberg 96, …)
- The quantum corrected Coulomb branch from the classically exact Higgs branch of a mirror dual
- Non-perturbative monopoles <-> perturbative mesons
- Lagrangian mirror duals of non-Lagrangian theories

Duality of Theories vs Duality of Fields

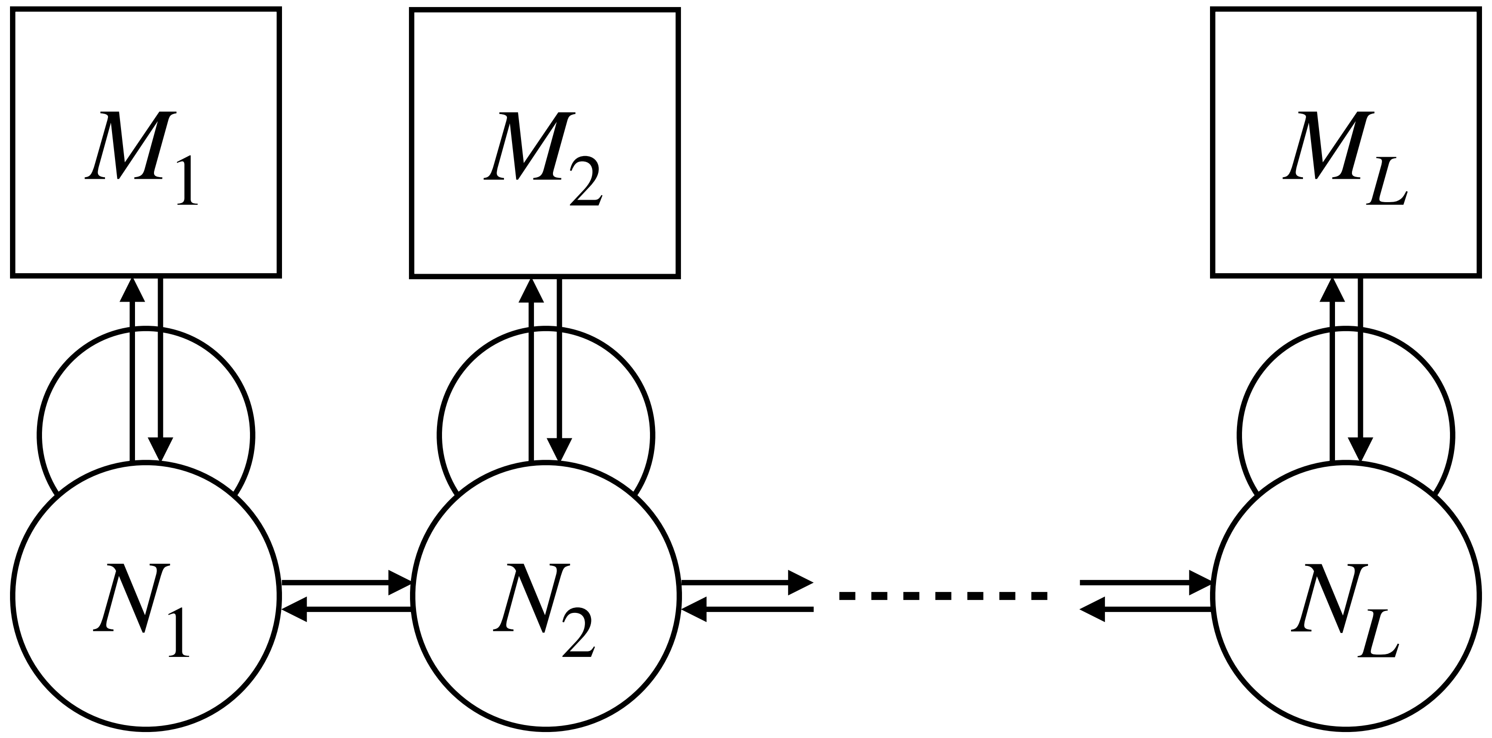
Building blocks of 3d mirror symmetry?

- **Idea:** dualize a “field” rather than a theory!

- If we know how to dualize each field, we may also be able to dualize the entire theory by gauging symmetries (and adding interactions if necessary) as in the previous example.

Today, focus on mirror symmetry of 3d $\mathcal{N} = 4$ $U(N)$ *linear quiver* gauge theories,

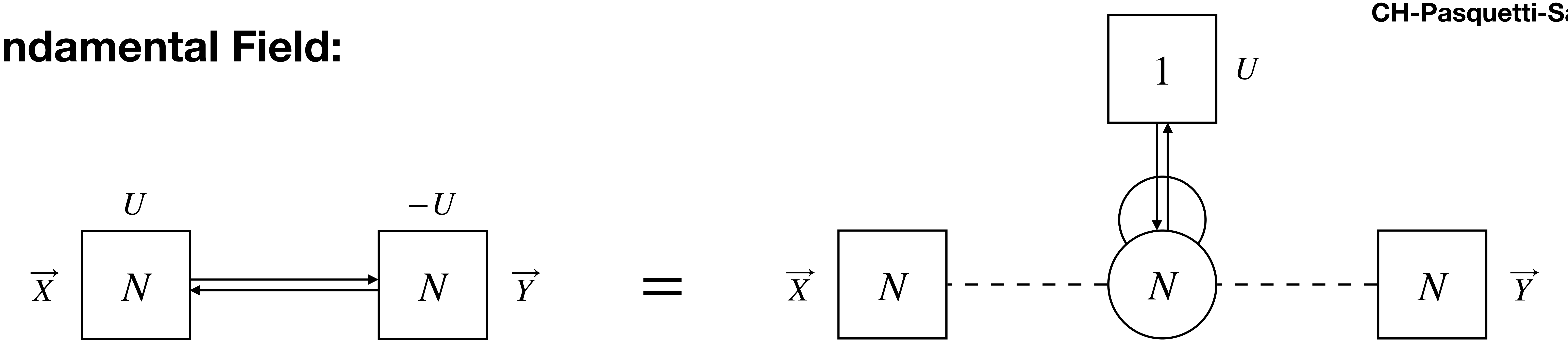
- having interactions fixed by the $\mathcal{N} = 4$ supersymmetry
- consisting of (bi-)fundamental matter fields



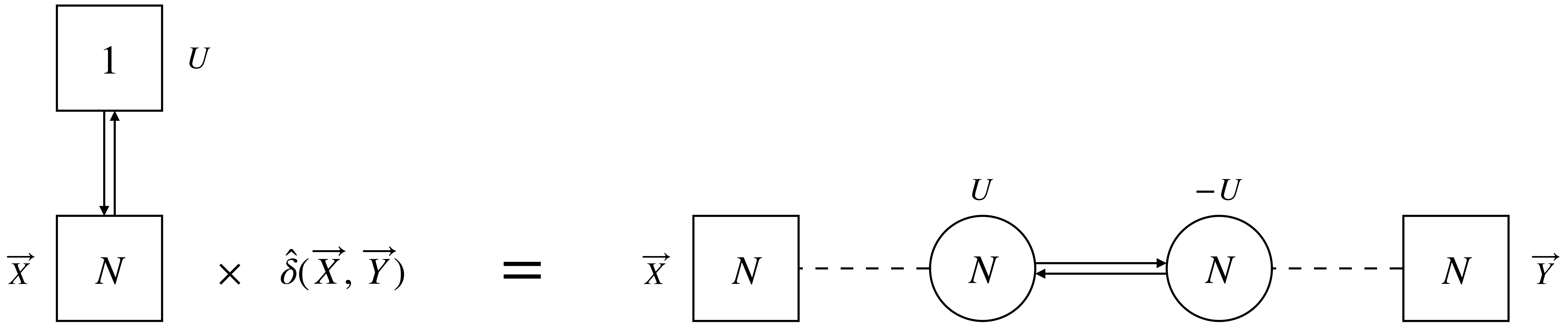
Building Blocks of 3D Mirror

Bottini-CH-Pasquetti-Sacchi 21
CH-Pasquetti-Sacchi 21

Bifundamental Field:

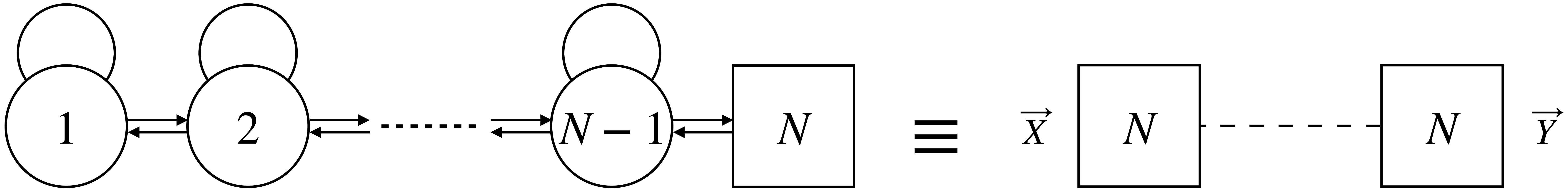


Fundamental Field:

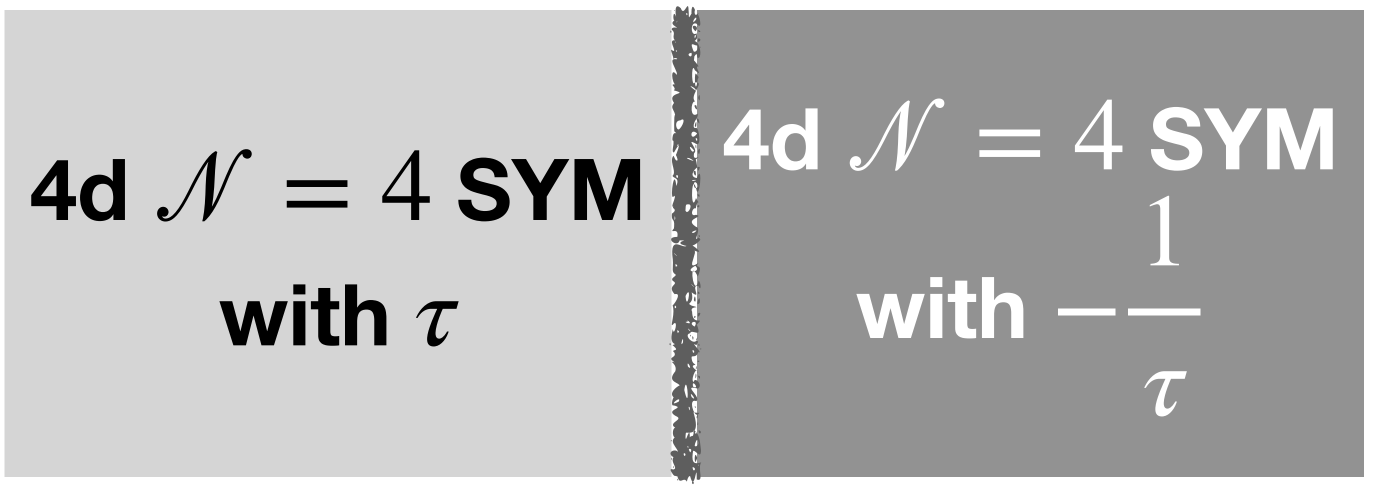


Basic Ingredients: S-Wall

- The S-wall theory: $T[U(N)]$



- The S-duality domain-wall theory of the 4d $\mathcal{N} = 4$ SYM (Giaotto-Witten 08)

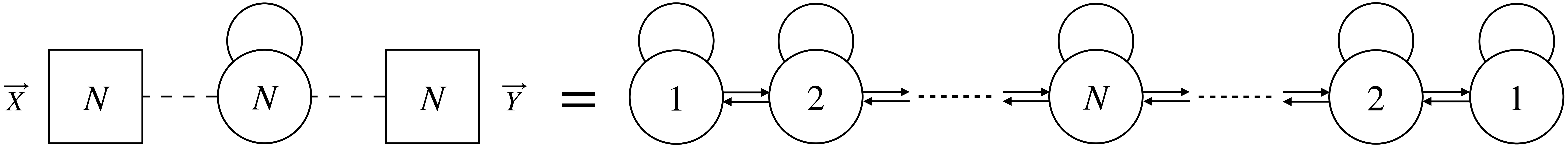


- $U(1)^{N-1}$ topological symmetry + background $U(1)$ coupled via mixed CS + $U(N)_Y$ flavor symmetry

- Enhanced $U(N)_X \times U(N)_Y$ symmetry in the IR

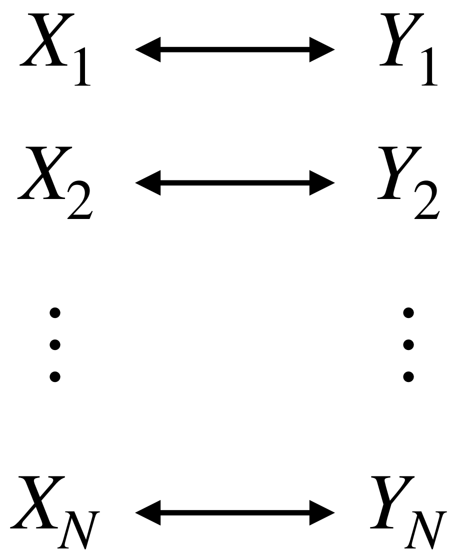
Basic Ingredients: I-Wall

- The identity-wall theory: two S-walls glued by gauging common $U(N)$



- The partition function proportional to the delta function

$$\sim \sum_{\sigma \in S_N} \prod_{j=1}^{N} \delta\left(X_j - Y_{\sigma(j)}\right)$$



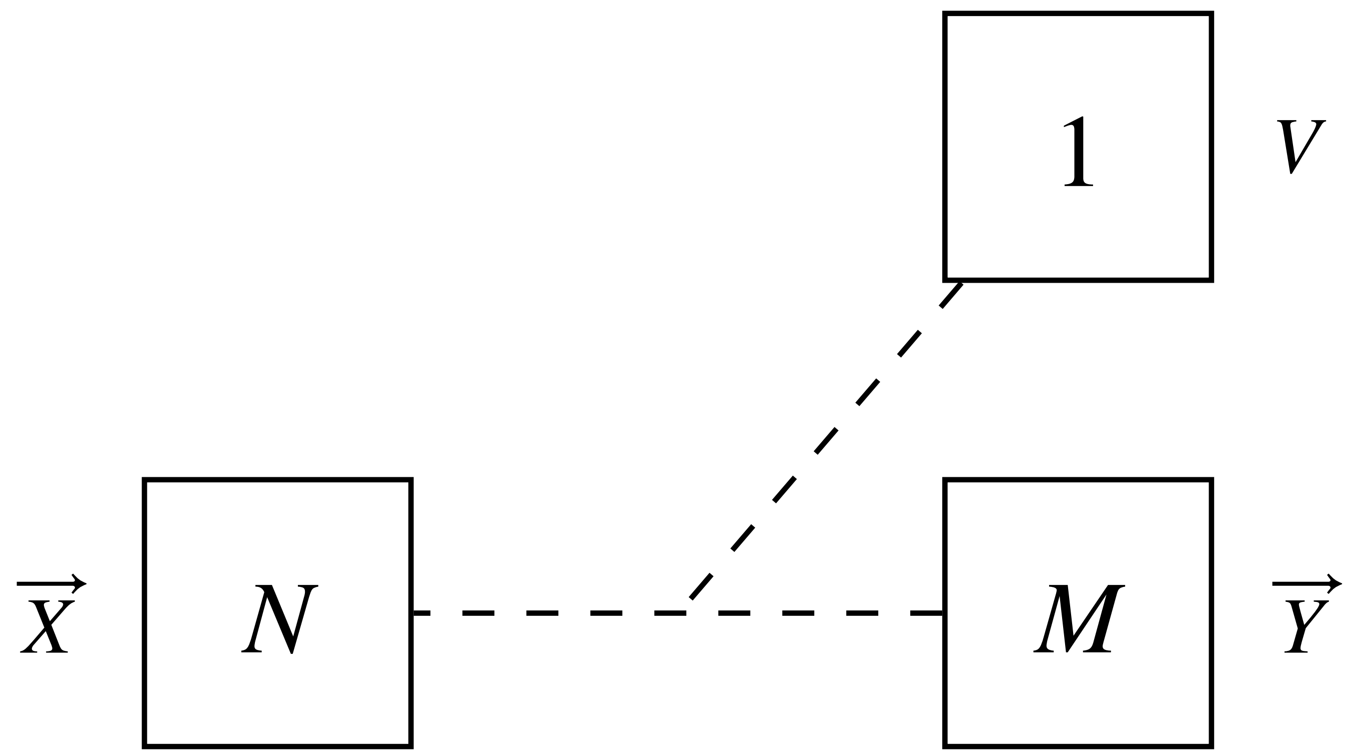
+ permutations

- $\langle \mathfrak{M} \rangle \neq 0$, where $\mathfrak{M}$ is a (monopole) operator in the $U(N)_X \times U(N)_Y$ bifundamental representation, breaking

$$U(N)_X \times U(N)_Y \to U(N)_D$$

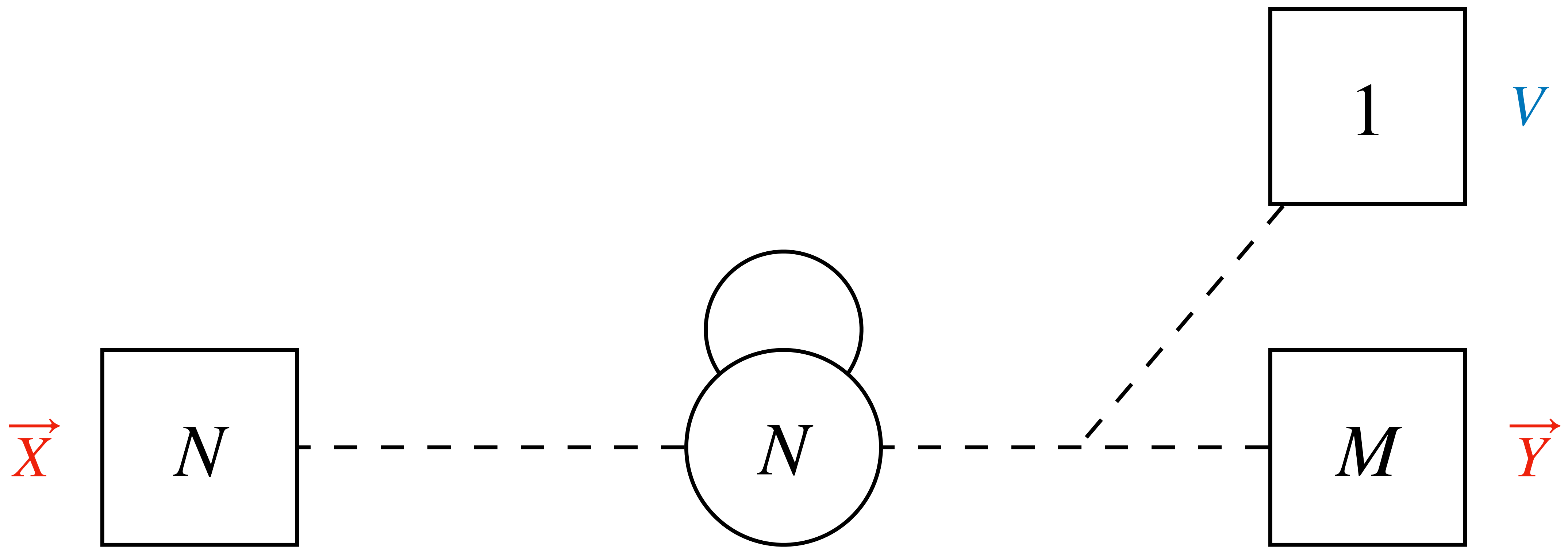
Generalization: Mass-Deformed S- & I-Walls

- Mass-deformed S-wall: S-wall + mass terms breaking $U(N)_Y \to U(M) \times U(1)$



$$Y_{M+j} = V + \frac{N - M + 1 - 2j}{2}(iQ - 2m_A)\,, \qquad j = 1, \ldots, N - M$$

- Mass-deformed I-wall: S-wall + mass-deformed S-wall



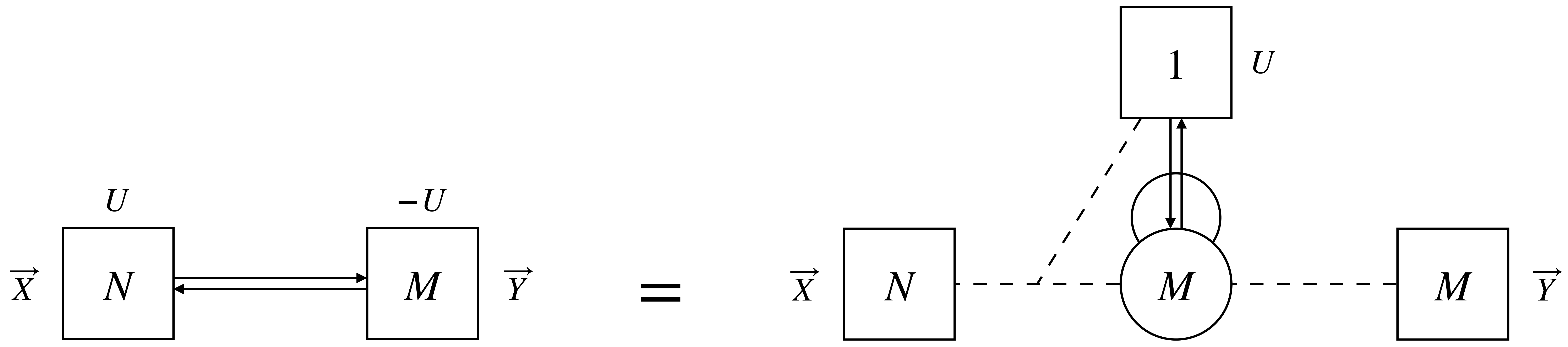
$U(N)_X \times U(M)_Y \times U(1)$
$\to U(M) \times U(1)$

$$\begin{aligned}
X_1 &\longleftrightarrow Y_1 \\
&\vdots \\
X_M &\longleftrightarrow Y_M \\
X_{M+1} &\longleftrightarrow V + (N - M - 1)(iQ - 2m_A)/2 \\
&\vdots \\
X_N &\longleftrightarrow V - (N - M - 1)(iQ - 2m_A)/2
\end{aligned}$$

+ permutations

(Generalized) Building Blocks of 3D Mirror

- Dualization of a $U(N) \times U(M)$ **bifundamental** hypermultiplet



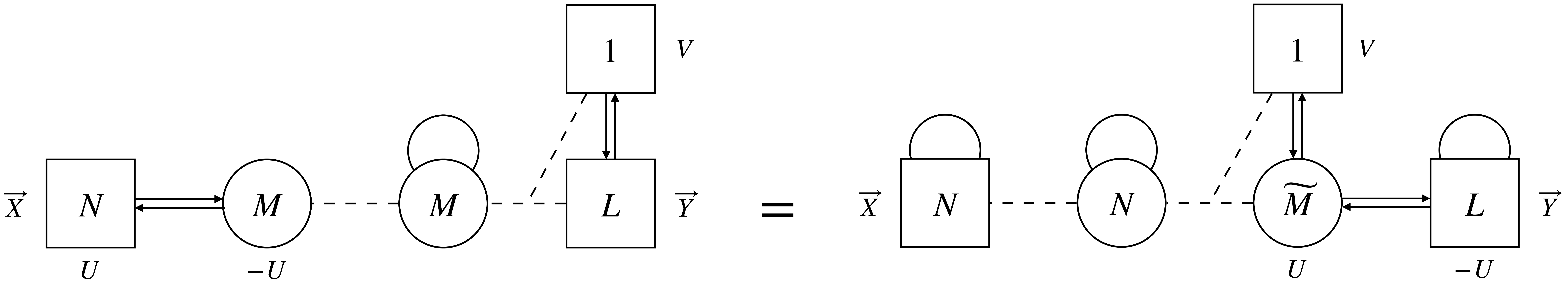
- A bifundamental hypermultiplet = a fundamental (twisted) hypermultiplet dualized by the ***mass-deformed S-wall***

- Similarly for the fundamental field

The Local Hanany-Witten Move in QFT

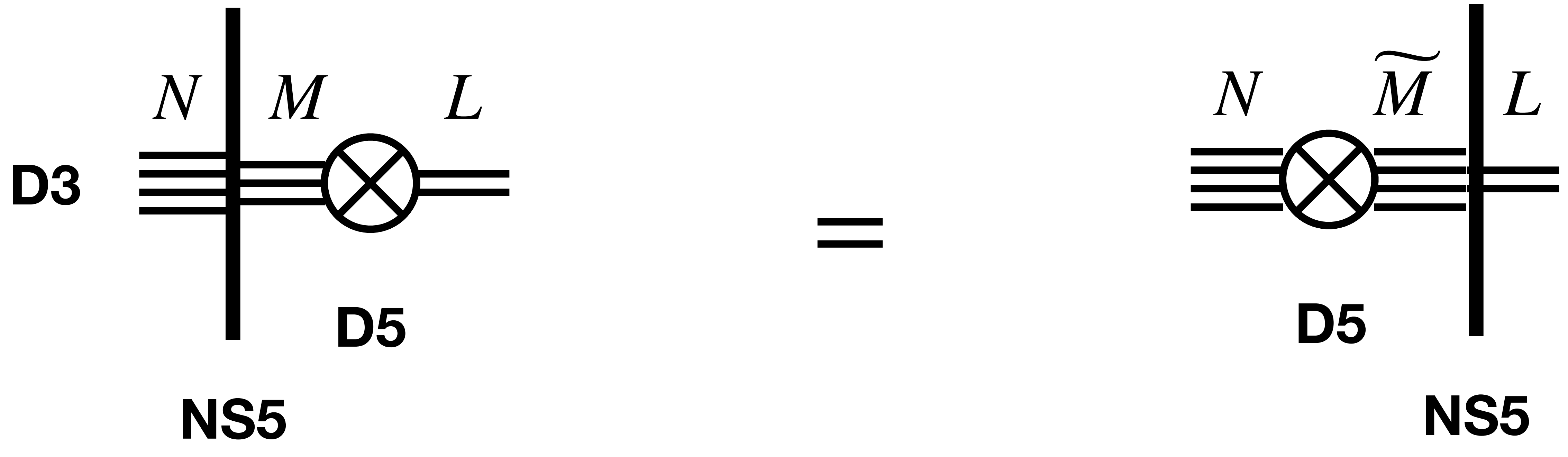
- The mass-deformed I-wall satisfies an interesting property resembling the Hanany-Witten brane move in IIB string theory (Comi-CH-Marino-Pasquetti-Sacchi 22).

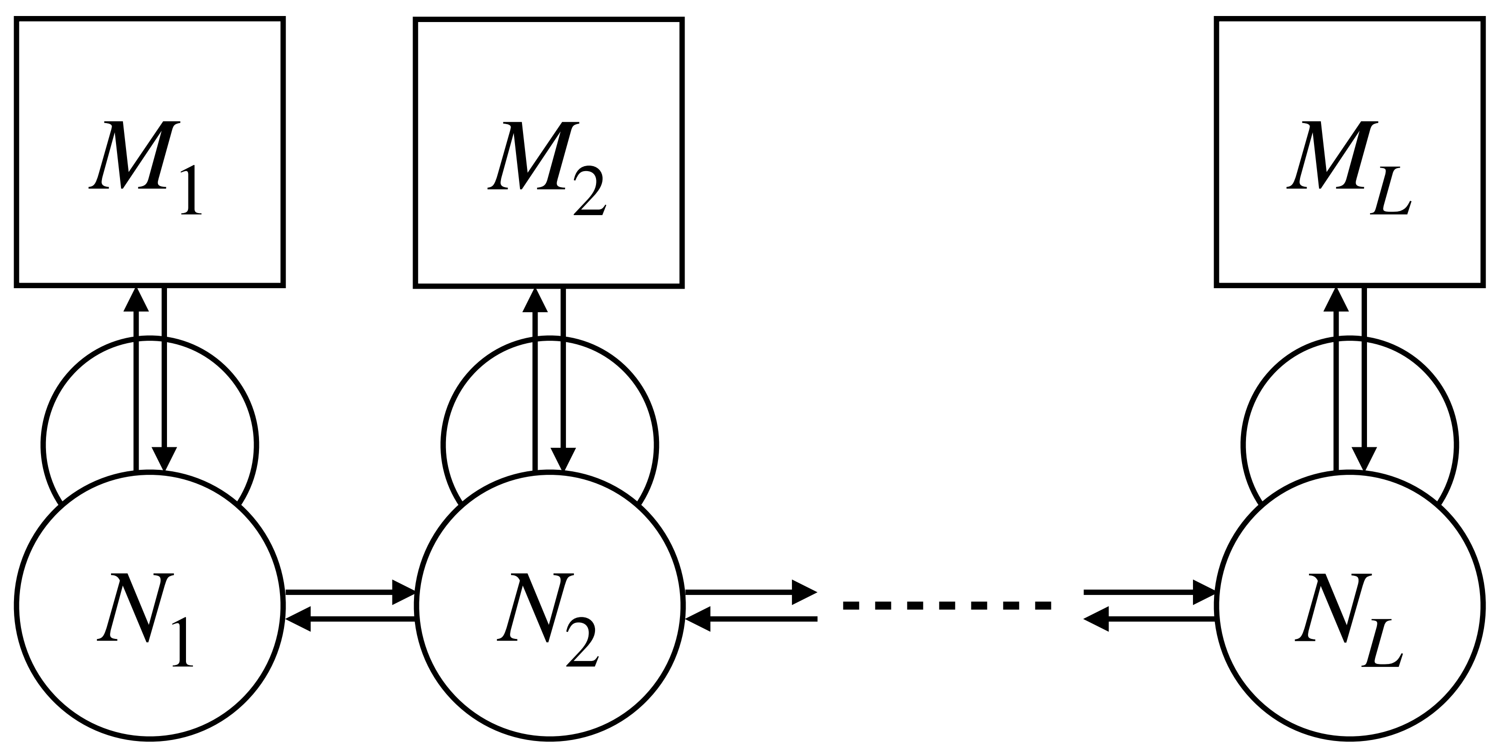
- Nothing but Higgs mechanism



$$\widetilde{M} = N + L - M + 1$$

IIB String Theory





$$M_{L-i} = k_i\,,$$
$$N_{L-i} = \sum_{j=i+1}^{L} \rho_j - \sum_{j=i+1}^{N} (j-i)k_j$$

- The general 3d linear quivers can be labeled by two sets of integers

$$\rho = [\rho_1, \ldots, \rho_L],$$
$$\sigma = [\sigma_1, \ldots, \sigma_K] = [N^{k_N}, \ldots, 1^{k_1}]\,.$$

- If ρ & σ are partitions satisfying $\sigma^T > \rho$, the theory, named as $T^\sigma_\rho[U(N)]$, is called *good*

- **Mirror symmetry** of $T^\sigma_\rho[U(N)]$ is well known; it corresponds to swapping partitions $\rho \leftrightarrow \sigma$.

All such dual pairs can be derived from the building blocks!

More applications:

- Generalization to SL(2,Z) dualities
- Generalization to 4d
- ***Generalization to bad theories***
- …

Mirror symmetry of bad theories

What Are Bad Theories?

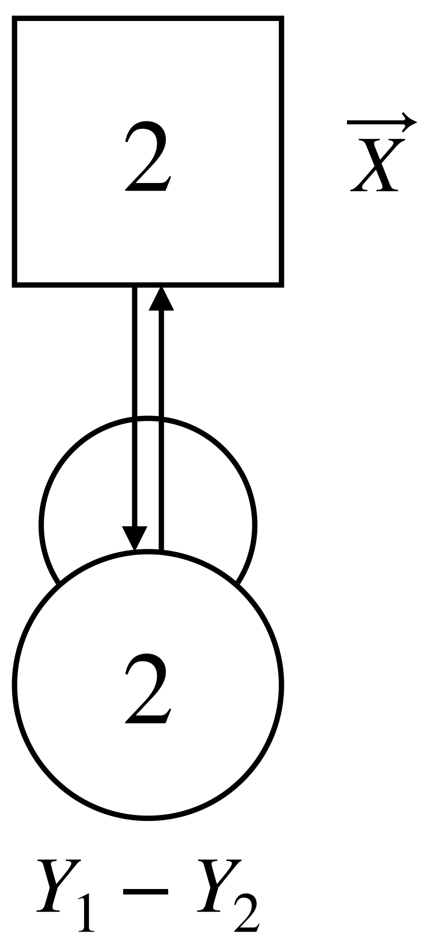
- Classification of 3d $\mathcal{N} = 4$ $U(N_c)$ SQCDs with N_f flavors (Gaiotto-Witten 08)
  - Good theories: $N_f \geq 2N_c$
  - Ugly theory: $N_f = 2N_c - 1$
  - **Bad theories:** $N_f \leq 2N_c - 2$
- Quiver gauge theories generally include bad nodes.

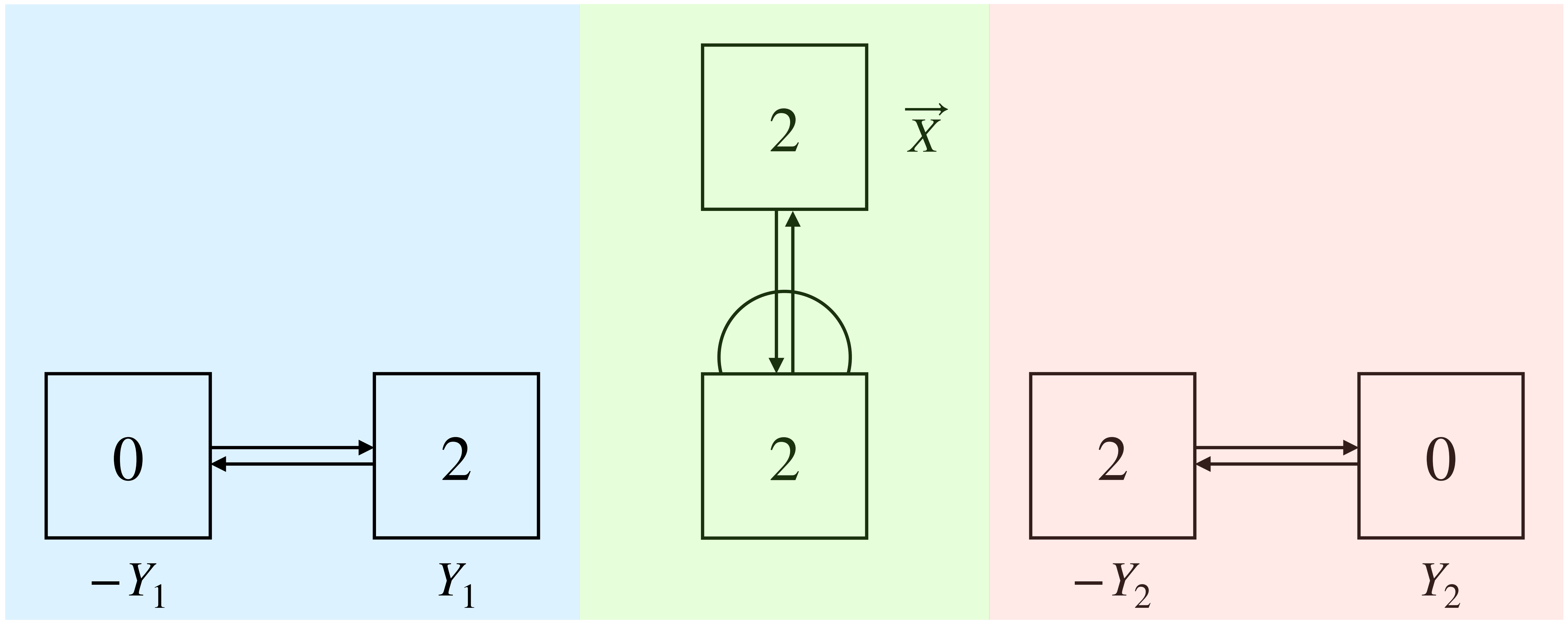
Why Bad?

- **IR R-symmetry $\neq$ UV R-symmetry**
- Divergent partition functions when calculated with the UV R-symmetry
- Due to decoupled monopole operators in the IR
- The R-symmetry mixed with emergent symmetries rotating the decoupled monopoles

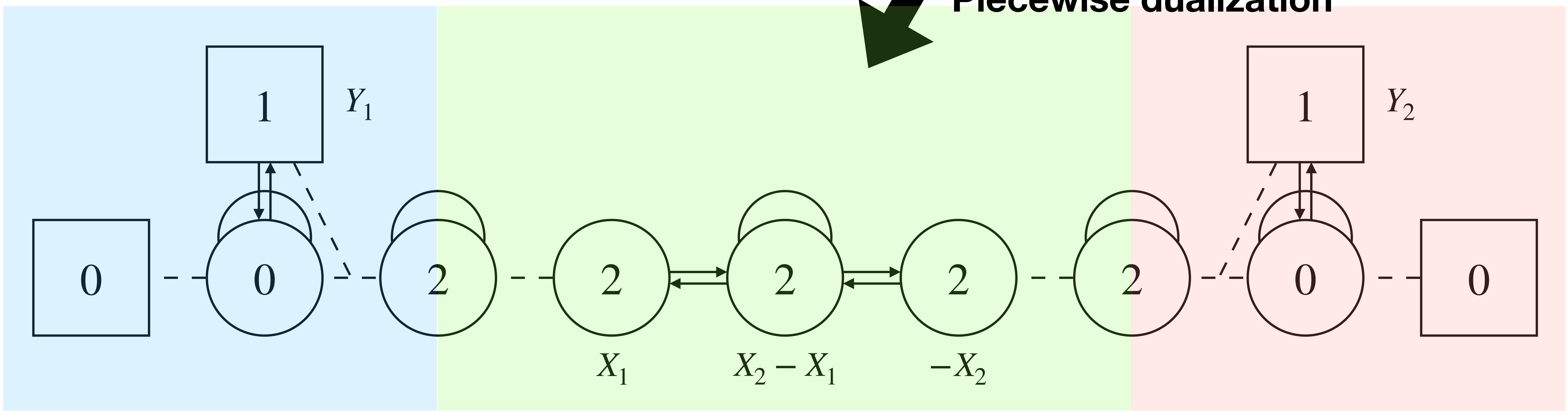
Example: Bad SQCD

$N_c = 2, \quad N_f = 2$

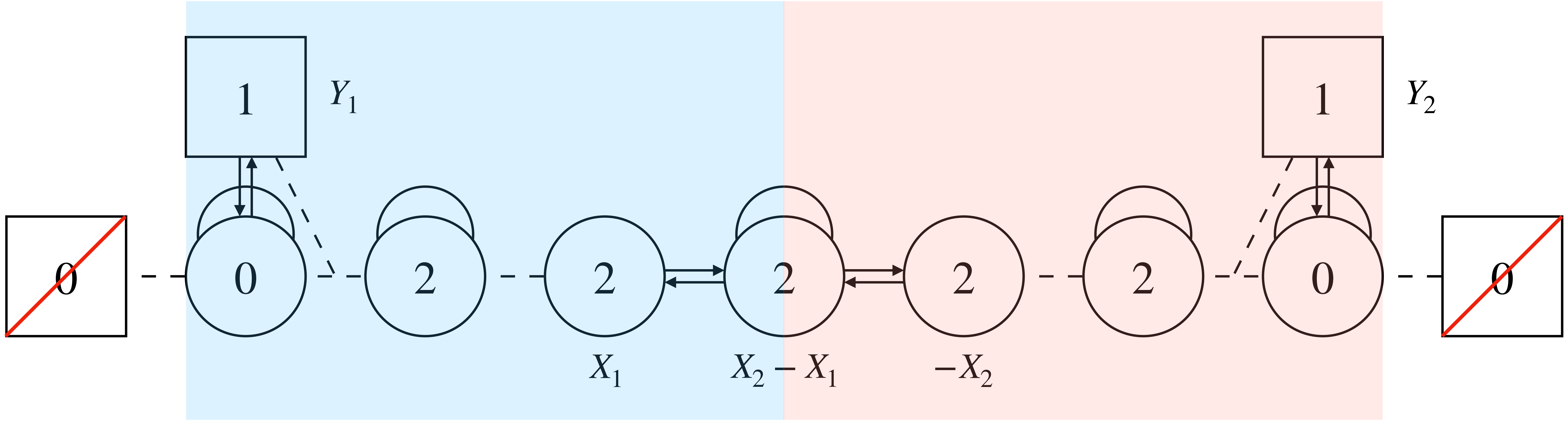




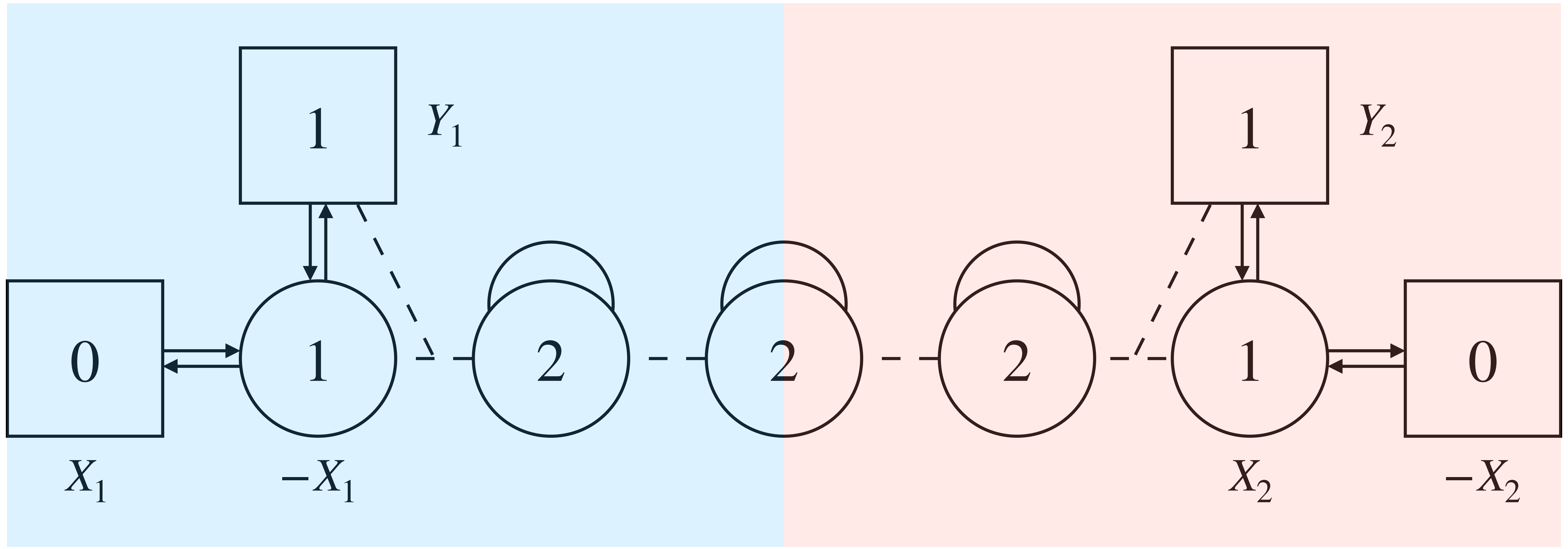
Piecewise dualization



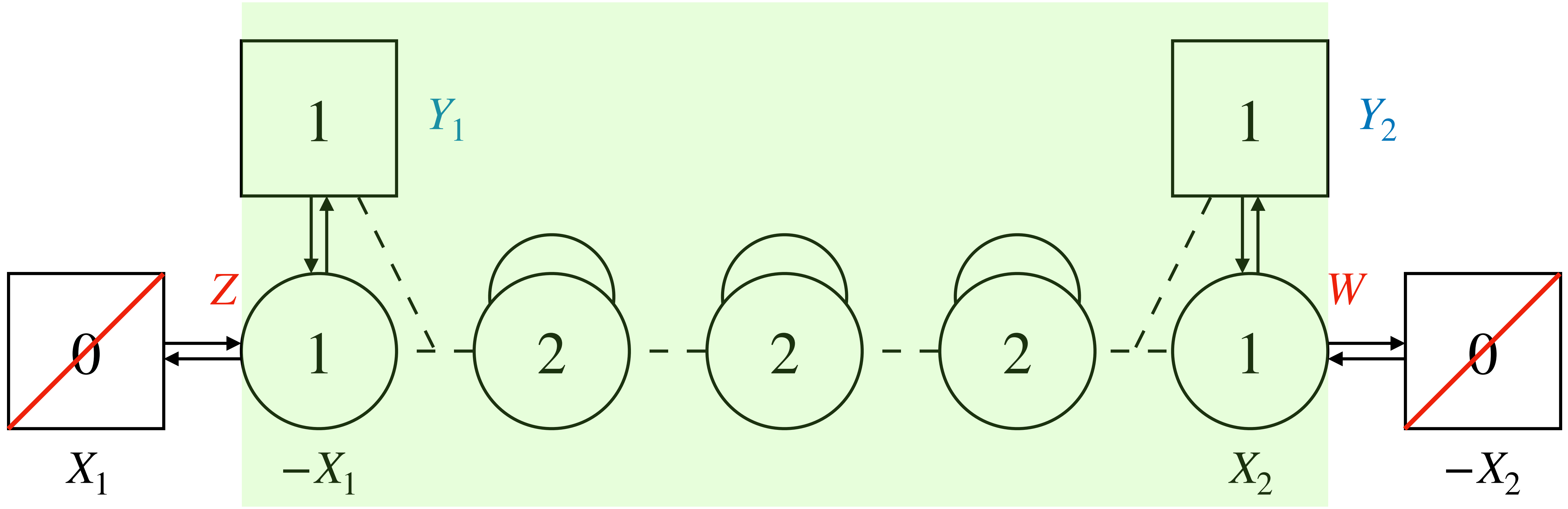
Example: Bad SQCD



The HW moves

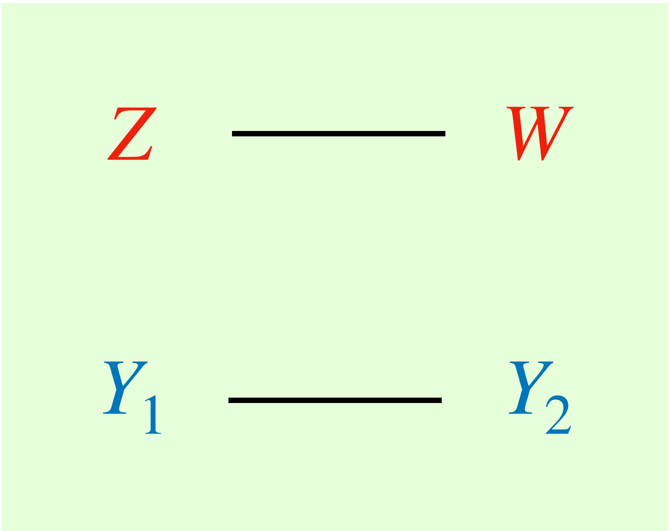


Example: Bad SQCD



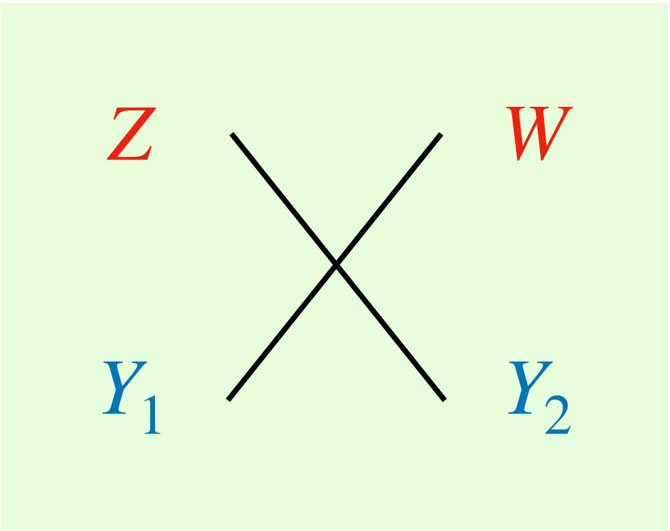
Colliding mass-deformed I-walls

Two ways of Higgsing



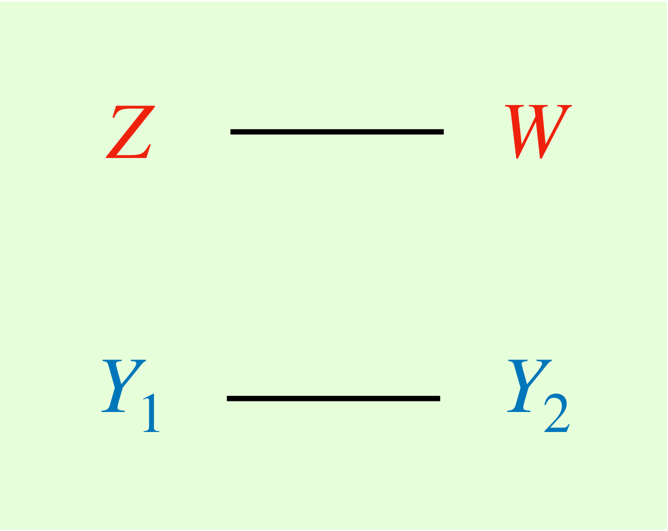
$U(1)$ **unbroken**

$\delta(Y_1 - Y_2)$

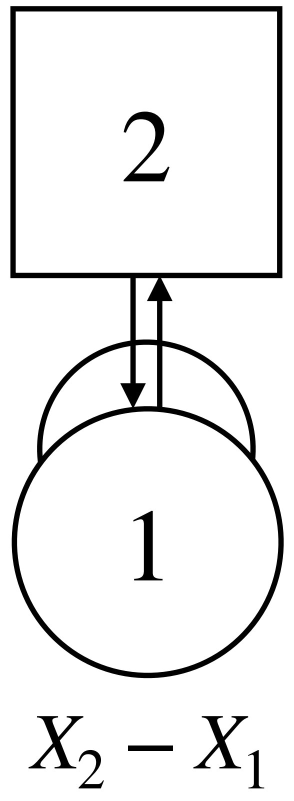


$U(1)$ **Higgsed**

Example: Bad SQCD



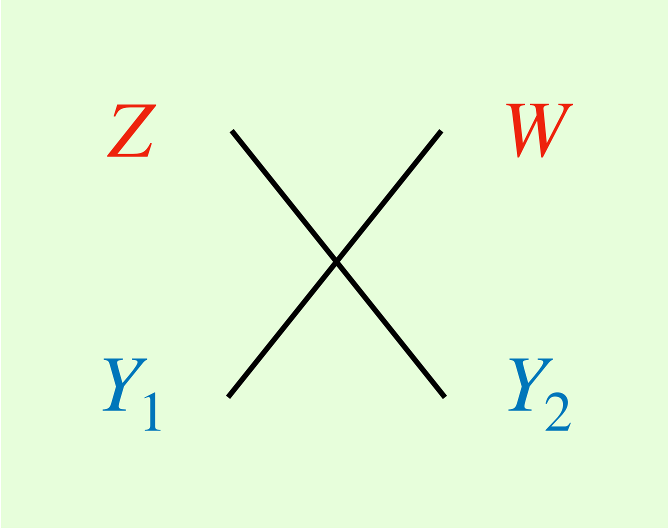
$U(1)$ **unbroken**



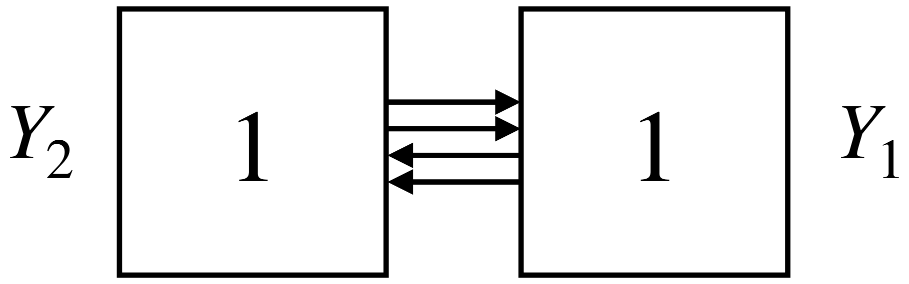
$$\vec{W} = \vec{Y} \pm \frac{1}{2}(iQ - 2m_A)$$

$$\times\ \delta(Y_1 - Y_2)$$

$$Y_1 - Y_2 = 0$$



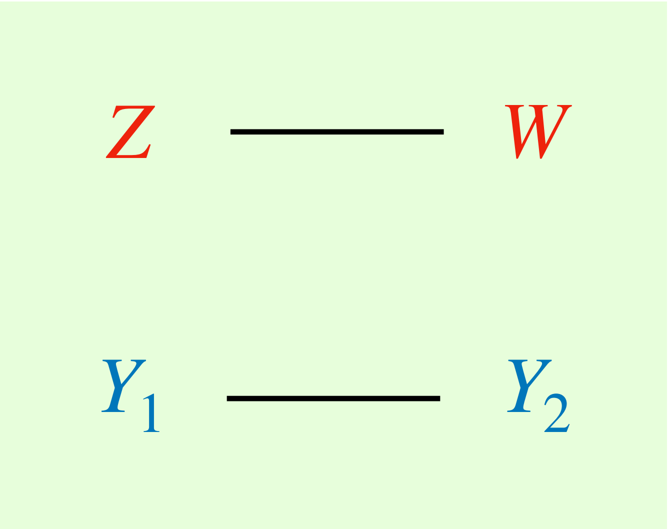
$U(1)$ **Higgsed**



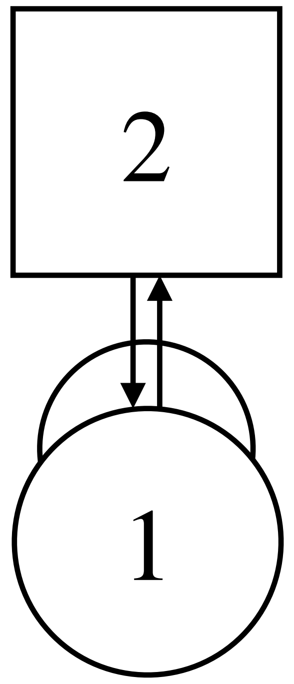
Generic FI parameter

We get two different duals depending on the FI!

Example: Bad SQCD



$U(1)$ **unbroken**



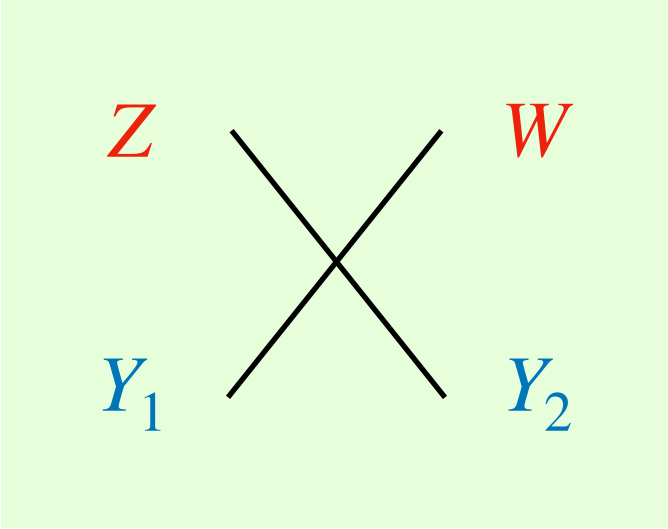
$\vec{W} = \vec{Y} \pm \frac{1}{2}(iQ - 2m_A)$

$X_2 - X_1$

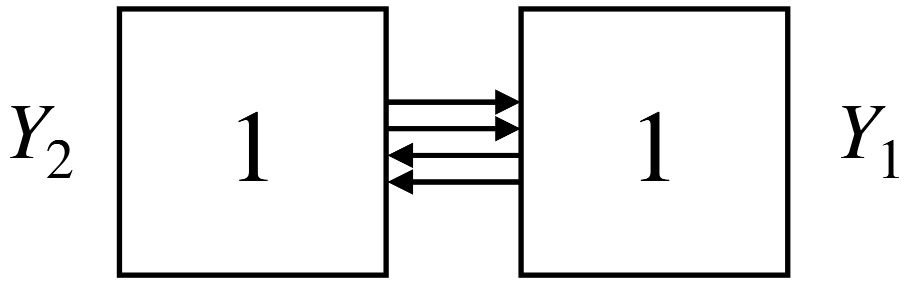
$\times\ \delta(Y_1 - Y_2)$

$Y_1 - Y_2 = 0$

$\delta(0) \sim s_b\left(\frac{iQ}{2}\right)$ **monopole with nonzero VEV**



$U(1)$ **Higgsed**



Generic FI parameter

We get two different duals depending on the FI!

Mirror Symmetry of Bad Theories

- The piecewise dualization algorithm can be applied to the 3d $\mathcal{N}=4$ $U(N)$ SQCD regardless of whether it is good or bad (Giacomelli-CH-Marino-Pasquetti-Sacchi 23).
- For a bad theory, the algorithm stops ***when two mass-deformed I-walls collide.***
- Different Higgs flows, resulting in multiple dual frames depending on the FI:

$$
\begin{aligned}
&\mathrm{SQCD}(N_c, N_f, Y_1 - Y_2) \\
&= \sum_{r=N_f-N_c+1}^{\lfloor \frac{N_2}{2} \rfloor} \delta(Y_1 - Y_2 + \ldots) \times \mathrm{mSQCD}(r, N_f, Y_1 - Y_2 + \ldots) \times \left(\text{extra decoupled fields}\right) \\
&\quad + \mathrm{mSQCD}(N_f - N_c, N_f, Y_1 - Y_2) \times \left(\text{extra decoupled fields}\right), \qquad N_c \leq N_f < 2N_c
\end{aligned}
$$

- Applicable to any linear quivers—each bad node can be dualized using the SQCD result (Giacomelli-CH-Marino-Pasquetti-Sacchi WIP)

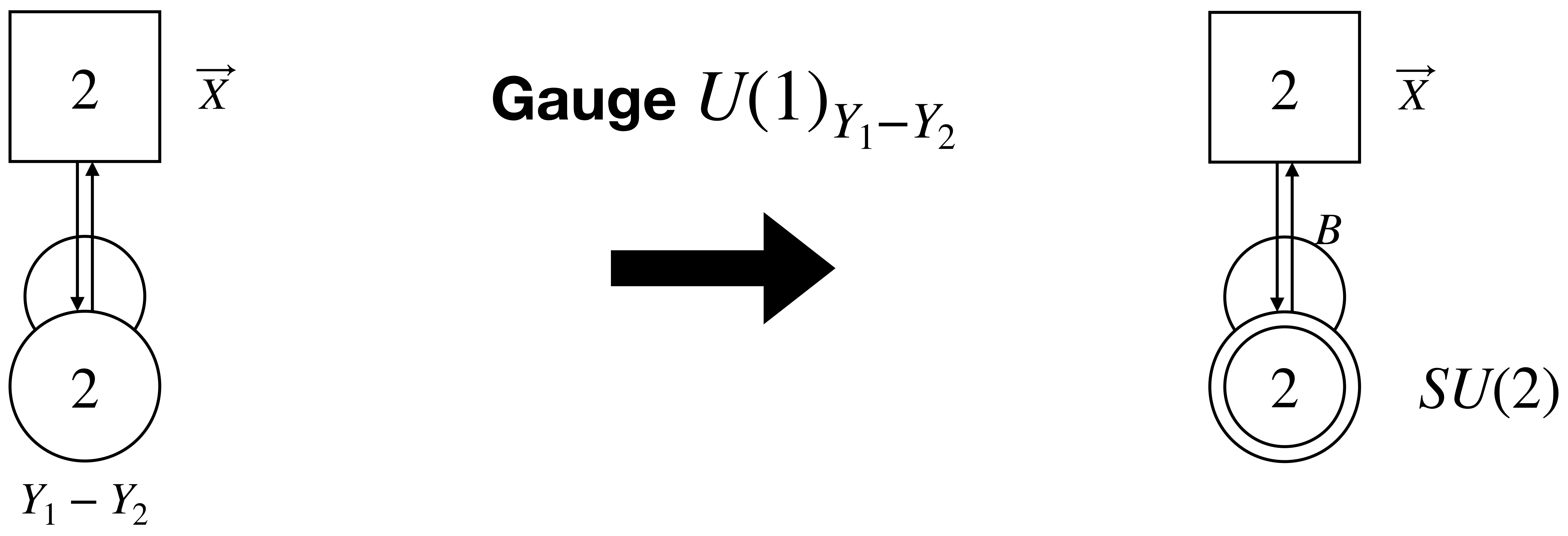
A couple more things…

I. Mirror Symmetry of SU(N)

- Recall that each $U(N)$ gauge node implies the topological $U(1)$ global symmetry coupled to the diagonal $U(1)$ via mixed CS coupling.

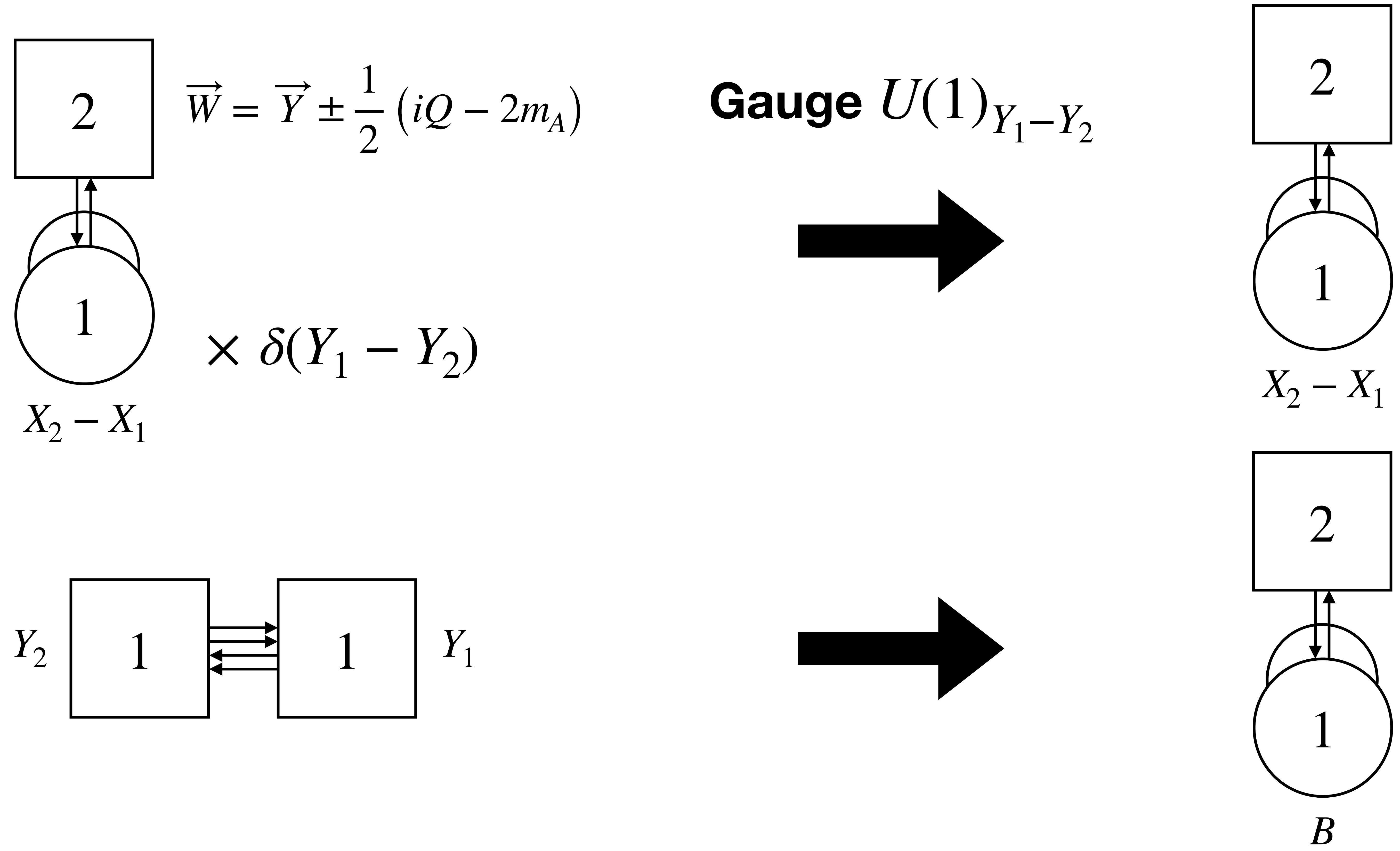
- An $SU(N)$ node can be obtained by gauging such topological $U(1)$.

$N_c = 2, \quad N_f = 2$



I. Mirror Symmetry of SU(N)

- The same operation on the dual side leads to the mirror dual(s) of the $SU(N)$ theory (Giacomelli-CH-Marino-Pasquetti-Sacchi WIP).



Two copies of SQED with 2 flavors (with different FI)

I. Mirror Symmetry of SU(N)

- Comparison with the moduli space?
- The moduli space of the 3d $\mathcal{N} = 4$ $USp(2N)$ theory (Assel-Cremonesi 18):

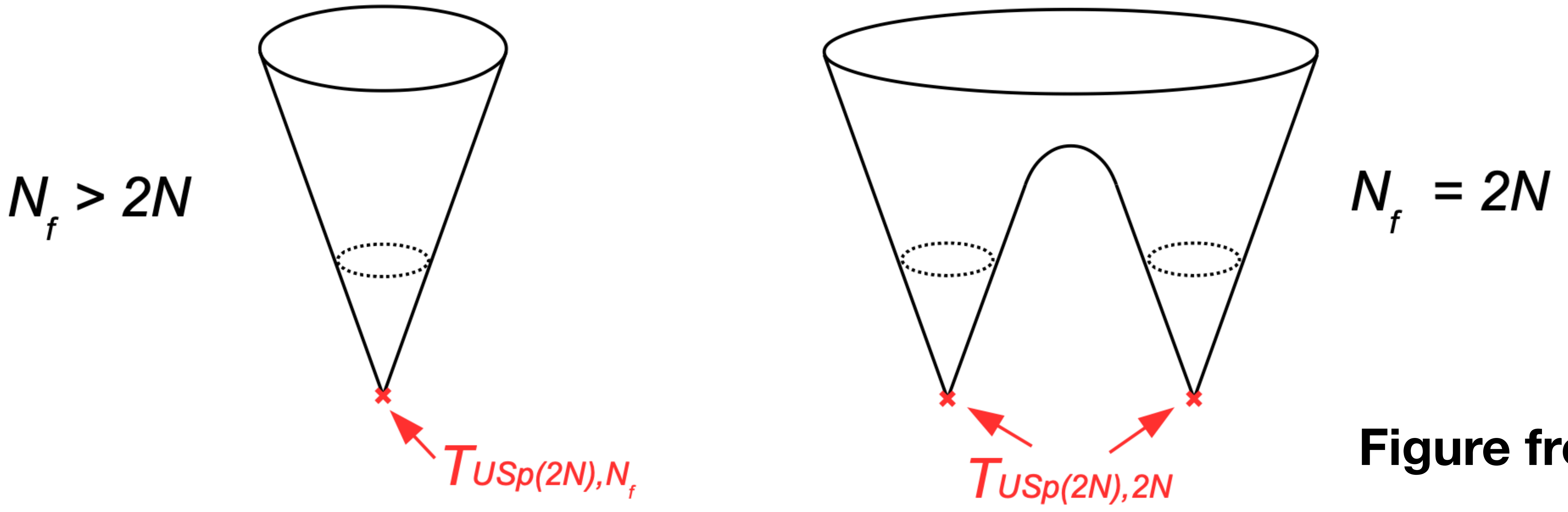
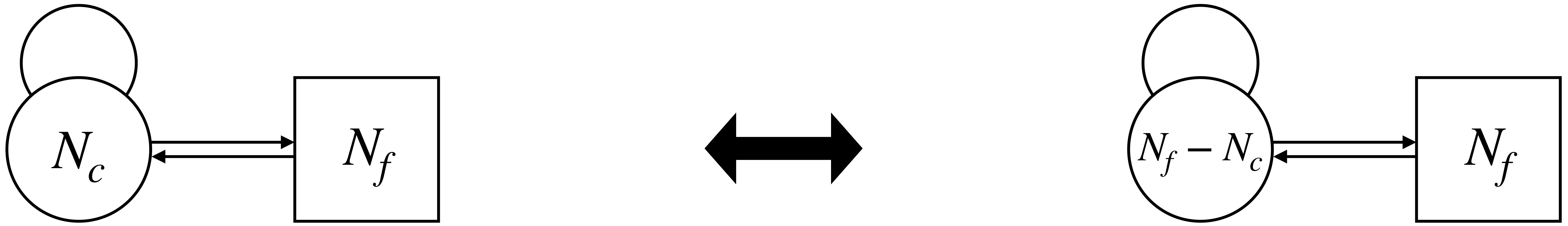


Figure from Assel-Cremonesi 18

- Two interacting fixed points for $SU(2)$ with 2 flavors, consistent with our mirror!

II. 3D $\mathcal{N} = 4$ Seiberg-Like Duality

- Duality relating a bad/ugly $U(N_c)$ theory to a good $U(N_f - N_c)$ theory with the same number of flavors (Kim[3]-Lee 12, Yaakov 13)
- Turns out not exact for bad theories—the bigger moduli space of the bad theory (Assel-Cremonesi 17)
- *Needs a modification*



+ decoupled hypers

II. 3D $\mathcal{N} = 4$ Seiberg-Like Duality

- **Solution:** a 3d monopole is a chiral operator and can participate in the superpotential (Giacomelli-CH-Marino-Pasquetti-Sacchi 23).

$$W = \sum_{i=0}^{2N_c - N_f - 1} X_i^{\pm} \, \mathfrak{m}_i^{\mp}$$

- The EOM of the extra singlets $X_i^{\pm}$ forbids monopole VEVs which led to multiple duals

$$\text{SQCD}(N_c, N_f, Y_1 - Y_2) \; + \; W$$

$$= \sum_{r=N_f-N_c+1}^{\lfloor \frac{N_2}{2} \rfloor} \delta(Y_1 - Y_2 + \ldots) \times \text{mSQCD}(r, N_f, Y_2 - Y_1 + \ldots) \times \big(\text{extra decoupled fields}\big) \times \big(\cancel{X_i^{\pm}}\big) \to 0^2$$

$$+ \text{mSQCD}(N_f - N_c, N_f, Y_2 - Y_1) \times \cancel{\big(\text{extra decoupled fields}\big)} \times \big(\cancel{X_i^{\pm}}\big)$$

$$= \text{SQCD}(N_f - N_c, N_f, Y_2 - Y_1)$$

Remarks

- We have found building blocks of mirror symmetry of 3d linear quiver gauge theories.
  -> ***The fundamental mechanism of these dualities is universal.***

- We have found multiple mirror duals of bad theories determined by the FI.

- The mirror building blocks can be extended to SL(2,Z)
  (Comi-CH-Marino-Pasquetti-Sacchi 22).

- 4d versions of mirror & SL(2,Z) dualities are also found
  (CH-Pasquetti-Sacchi 20, Comi-CH-Marino-Pasquetti-Sacchi 22).

Remarks (Continued)

- Other types of matters? Other types of quivers? E.g., the adjoint representation -> a special case of circular quivers (WIP).
- Seiberg-like dualities? (WIP)
- Connections to higher-dimensional SCFTs?
  - The 4d S-wall from the compactification of the 6d E-string (Pasquetti-Razamat-Sacchi-Zafrir 19, CH-Razamat-Sabag-Sacchi 21)
  - Wenbin's talk
  - Higgs branches via magnetic quivers
- Application to holographic models, e.g., ABJ(M)?
  - Cf. the holographic dual of the 3d S-wall (Assel-Bachas-Estes-Gomis 11)

Thank you