3d $\mathcal{N} = 2$ SCFTs from M-theory on CY4

Work in progress w/ Marwan Najjar and Jiahua Tian

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Conformal Field Theories

• Classification of CFTs is an interesting but hard question

(1) 2d CFT: Virasoro algebra provides strong constraints, rational CFT

(2) For higher dimensional CFTs (e. g. $d \geq 3$), the full operator spectrum, OPEs ... are not known

• Today we will focus on SCFTs with 8 or 4 supercharges

• Partial classification comes from geometric constructions, in the framework of a higher-dimensional theory

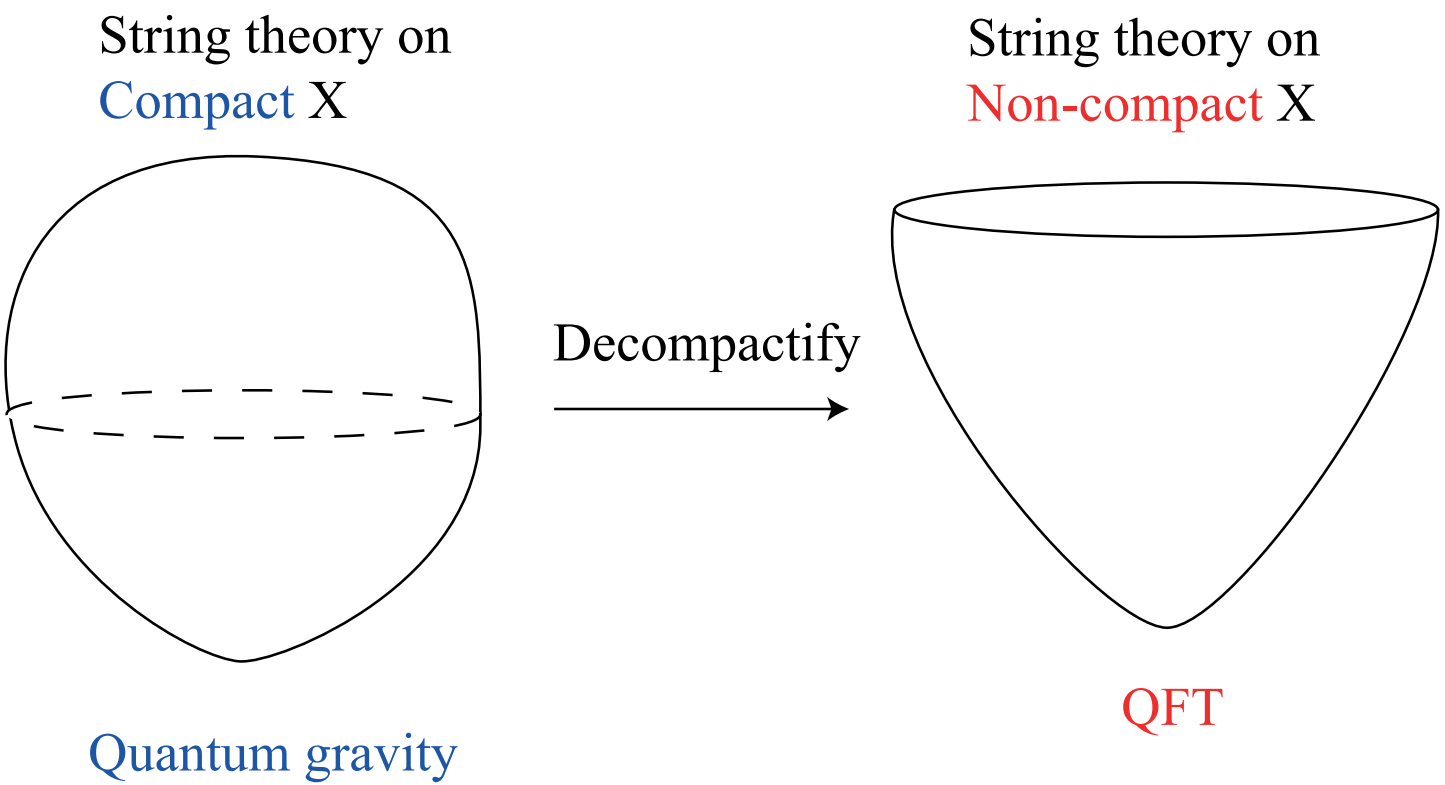
(1) Superstring/M/F-theory on a non-compact space

(2) Dimensional reduction of 6d SCFTs on a compact space

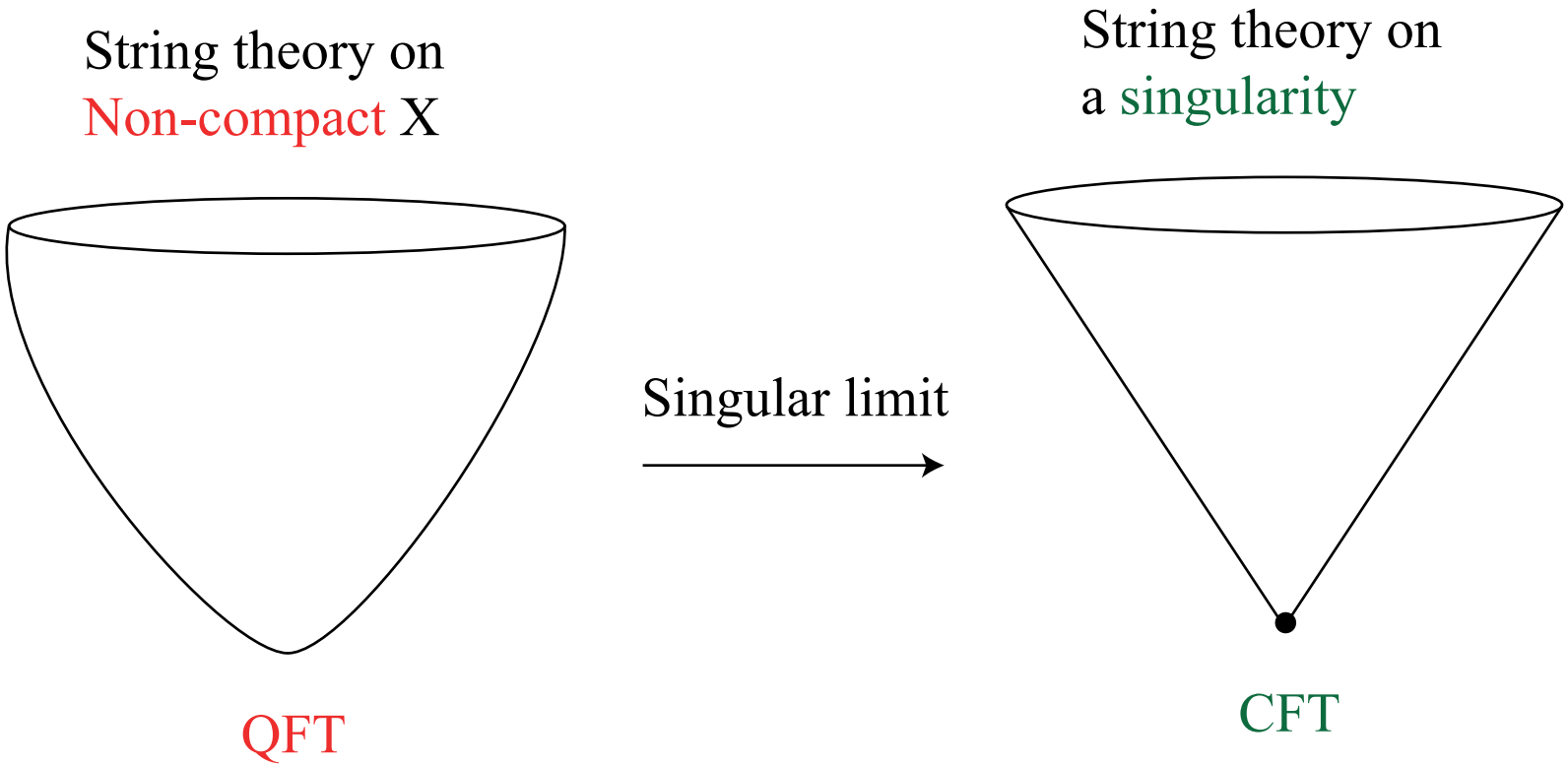
(3) Worldvolume theory of brane objects in superstring/M/F-theory (AdS/CFT)

Geometric Engineering

- Superstring/M/F-theory on a non-compact space, decouple gravity



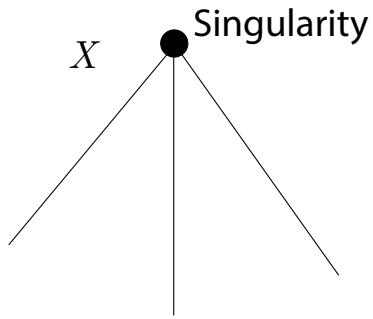
Geometric Engineering



- The CFT degree of freedoms are localized around the origin

5d SCFTs

(1) 11d M-theory on canonical threefold singularity



(Xie, Yau 15')(Apruzzi, Bhardwaj, Closset, Collinucci, De Marco, Del Zotto, Eckhard, Giacomelli, Heckman, Hubner, Jefferson, Katz, Kim, Lawrie, Lin, Morrison, Mu, Sangiovanni, Saxena, Schafer-Nameki, Tarazi, Tian, Vafa, Valandro, YNW, Zafrir, Zhang. . .).

(2) Brane web constructions in IIB superstring

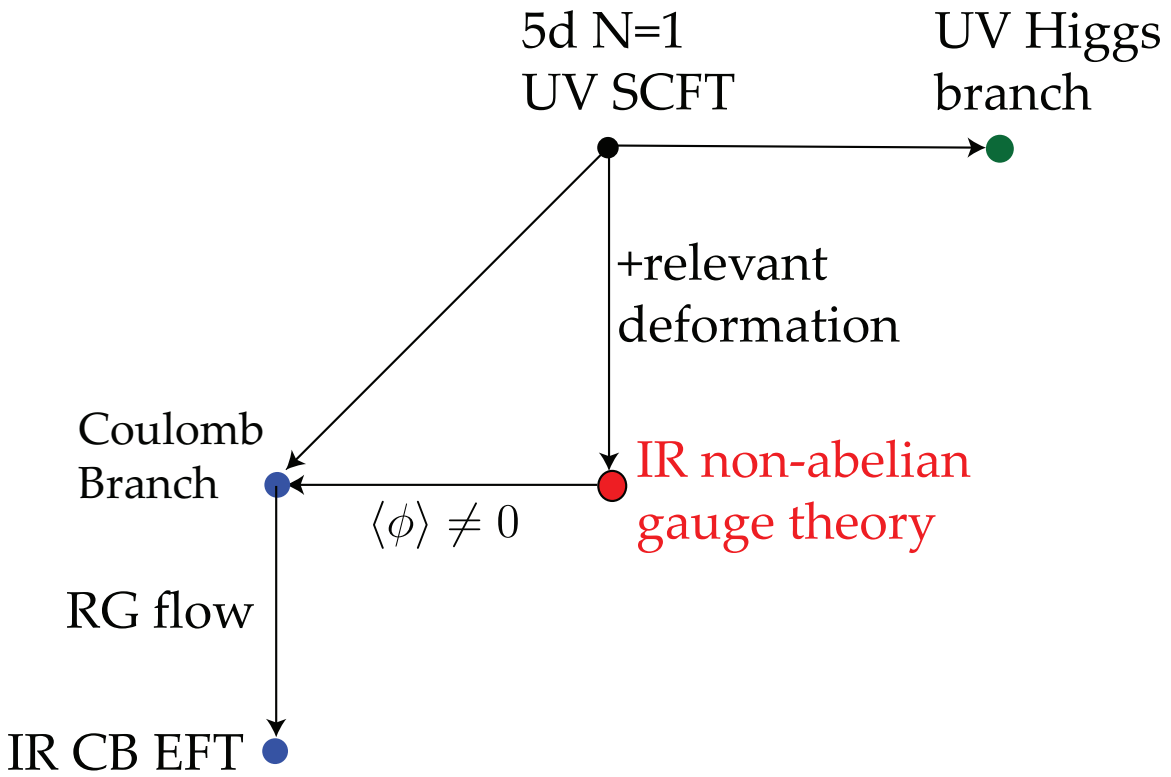
(Akhond, van Beest, Bergman, Bourget, Cabrera, Carta, Dwivedi, Eckhard, Ferlito, Giacomelli, Grimminger, Hanany, Hayashi, He, Kalveks, Kim, Kim, Kim, Lee, Mekareeya, Ohmori, Schafer-Nameki, Shimizu, Sperling, Tachikawa, Taki, Uhlemann, Yagi, Zafrir, Zajac, Zoccarato, Zhong . . .).

Deformations of SCFTs

- Directly study the operator spectrum/ OPE etc. Hard!

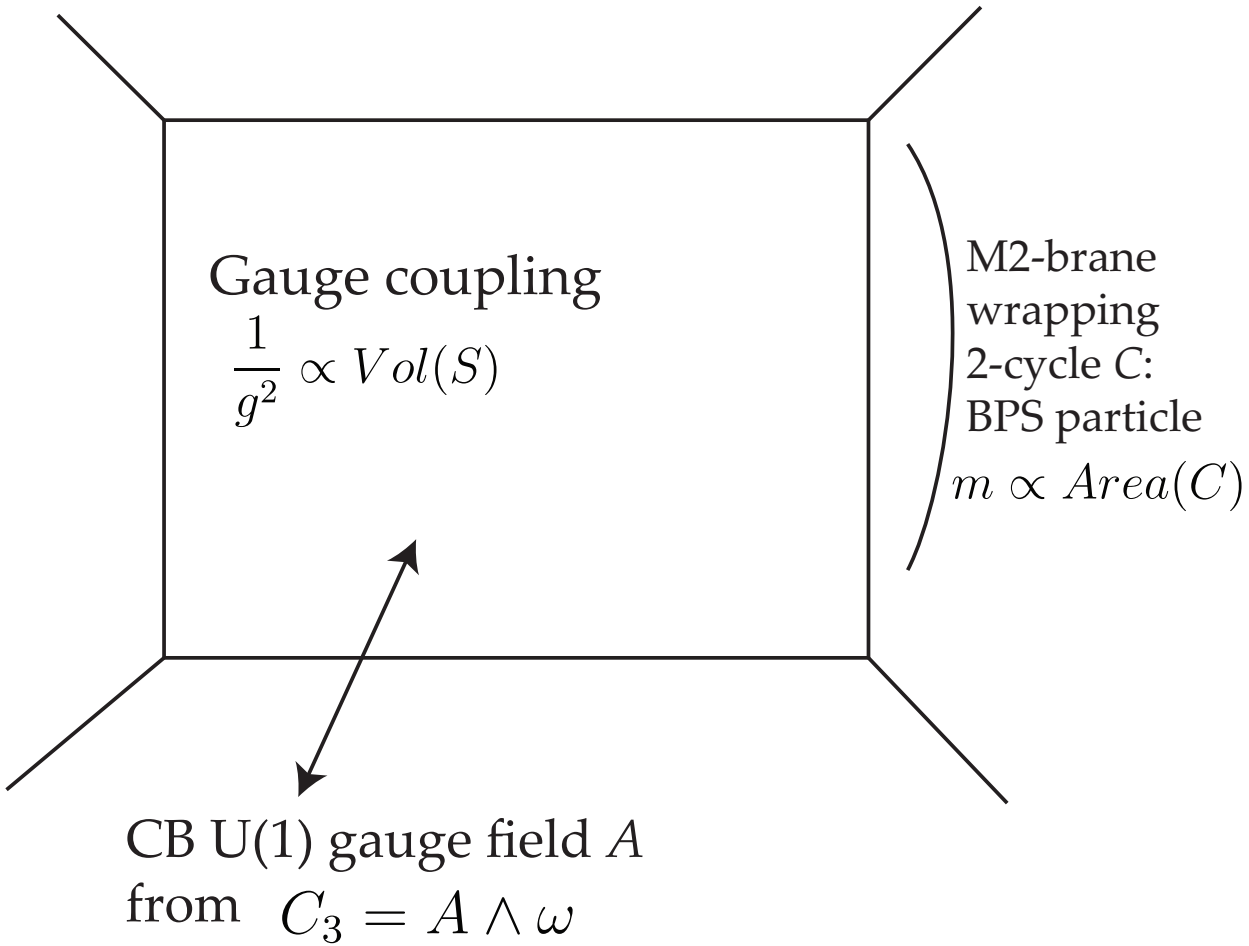
(1) Coulomb branch: scalars ϕ^i in the vector multiplets have non-zero vev.

(2) Higgs branch: scalars in the hypermultiplets have non-zero vev.



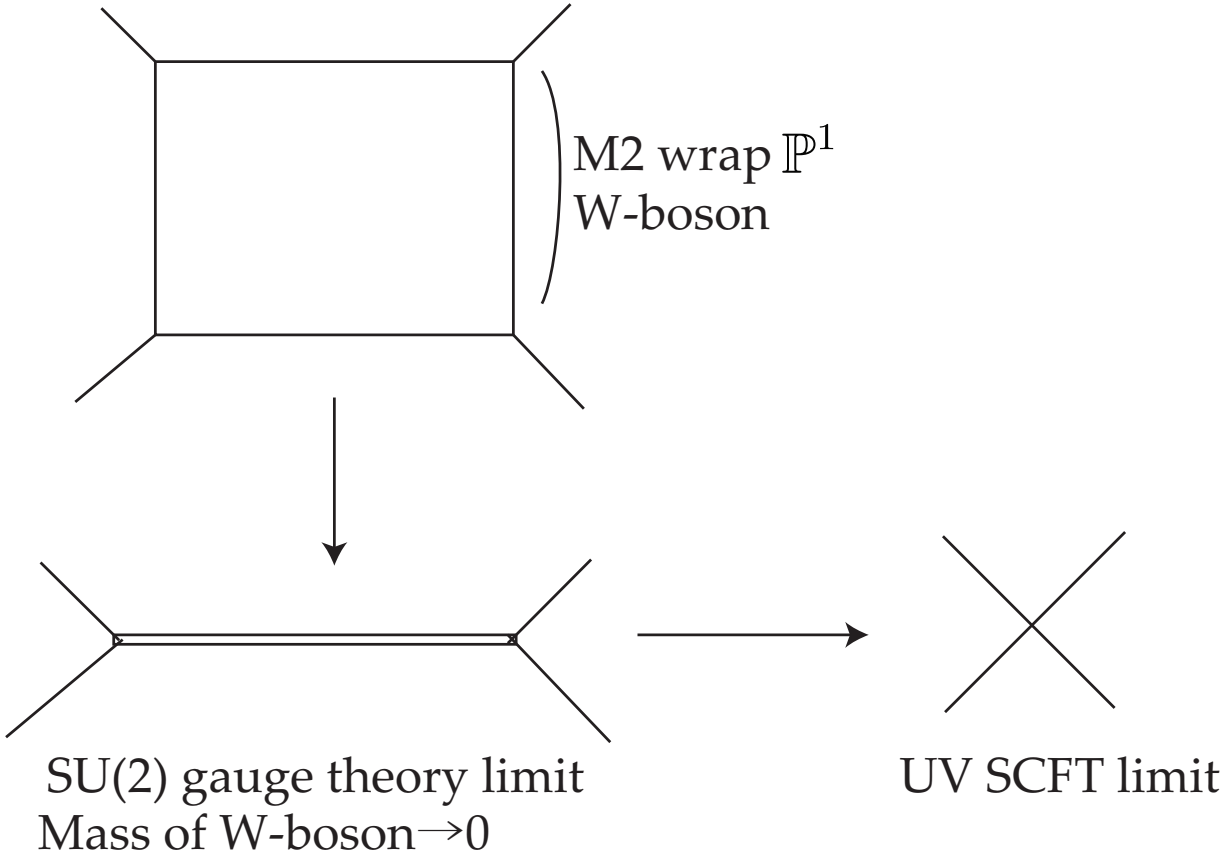
5d CB and M-theory on resolved CY3

- M-theory on a resolved CY3 $\rightarrow$ CB physics, $U(1)^r+$ massive charged matter



Non-abelian and SCFT limit

- Non-abelian gauge theory description exists when the CY3 has a $\mathbb{P}^1$-fibration structure



- Similar picture in the IIB (p, q) 5-brane web constructions!

Partial Classifications in 5d

(1) Classifying Contractible surfaces (Jefferson, Katz, Vafa, Zafrir 17')(Jefferson, Katz, Kim, Vafa 18')...

(2) 5d KK theories as twisted reductions of 6d (1,0) SCFTs (Bhardwaj, Jefferson, Kim, Tarazi, Vafa, 19')...

(3) Non-flat resolution of non-isolated elliptic threefold singularities, with non-compact 4-cycles, capture the flavor symmetry G_F.

- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "5d Superconformal Field Theories and Graphs", Physics Letters B 800, (2020) 135077
- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "Fibers add Flavor, Part I: Classification of 5d SCFTs, Flavor Symmetries and BPS States", JHEP 11 (2019) 068
- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "Fibers add Flavor, Part II: 5d SCFTs, Gauge Theories, and Dualities", JHEP 03 (2020) 052
- Apruzzi, Schafer-Nameki, YNW, "5d SCFTs from Decoupling and Gluing", JHEP 08 (2020) 153

Partial Classifications in 5d

(4) Toric singularities (Xie, Yau 17')...

- Eckhard, YNW, Schafer-Nameki, "Trifectas for T_N in 5d", JHEP 07 (2020) 07, 199.

(5) Isolated hypersurface singularities (IHS)

- Closset, Schafer-Nameki, YNW, "Coulomb and Higgs Branches from Canonical Singularities: Part 0", JHEP 02 (2021) 003
- Closset, Giacomelli, Schafer-Nameki, YNW, "5d and 4d SCFTs: Canonical Singularities, Trinions and S-Dualities", JHEP 05 (2021) 274
- Closset, Schafer-Nameki, YNW, "Coulomb and Higgs branches from canonical singularities. Part I. Hypersurfaces with smooth Calabi-Yau resolutions", JHEP 04 (2022) 061.

- Relation between 5d Higgs branch and IIB on the same singularity X

Partial Classifications in 5d

(6) $\mathbb{C}^3$ orbifold singularities

- Tian, YNW, "5d and 6d SCFTs from $\mathbb{C}^3$ orbifolds", SciPost Phys. 12 (2022) 4, 127.

(Acharya, Lambert, Najjar, Svanes, Tian 21')(Kim, Kim, Lee 22')(Del Zotto, Heckman, Meynet, Moscrop, Zhang 22')...

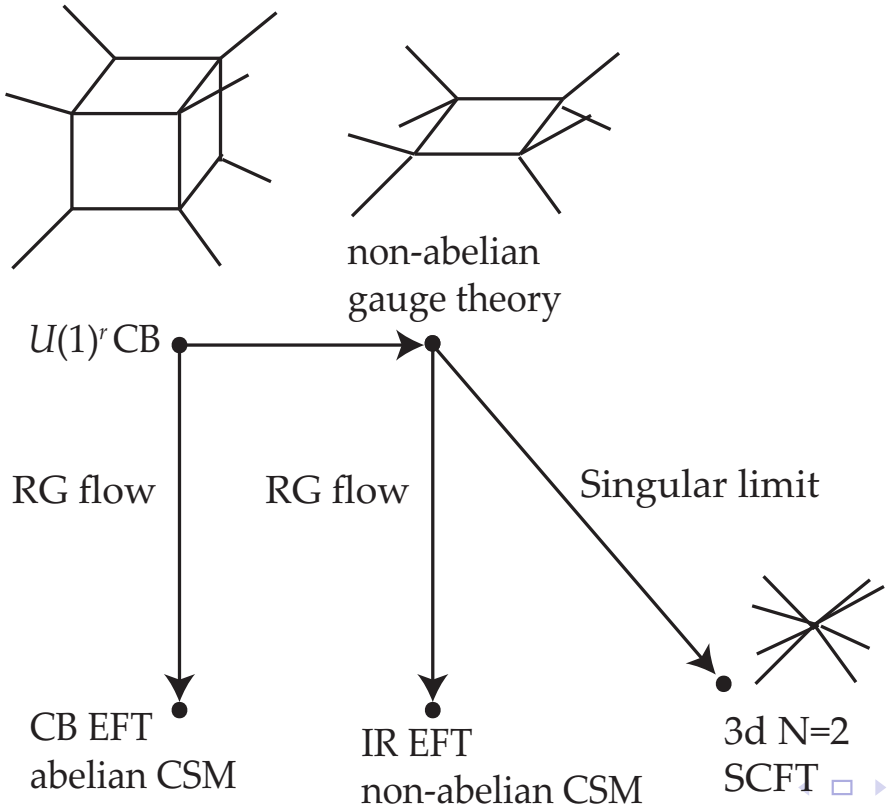
• Read off physical information, 1-form symmetry from McKay correspondence

(7) Isolated Complete Intersection Singularities (ICIS)

- Mu, YNW, Zhang, "5d SCFTs from Isolated Complete Intersection Singularities", arXiv: 2311.05441.

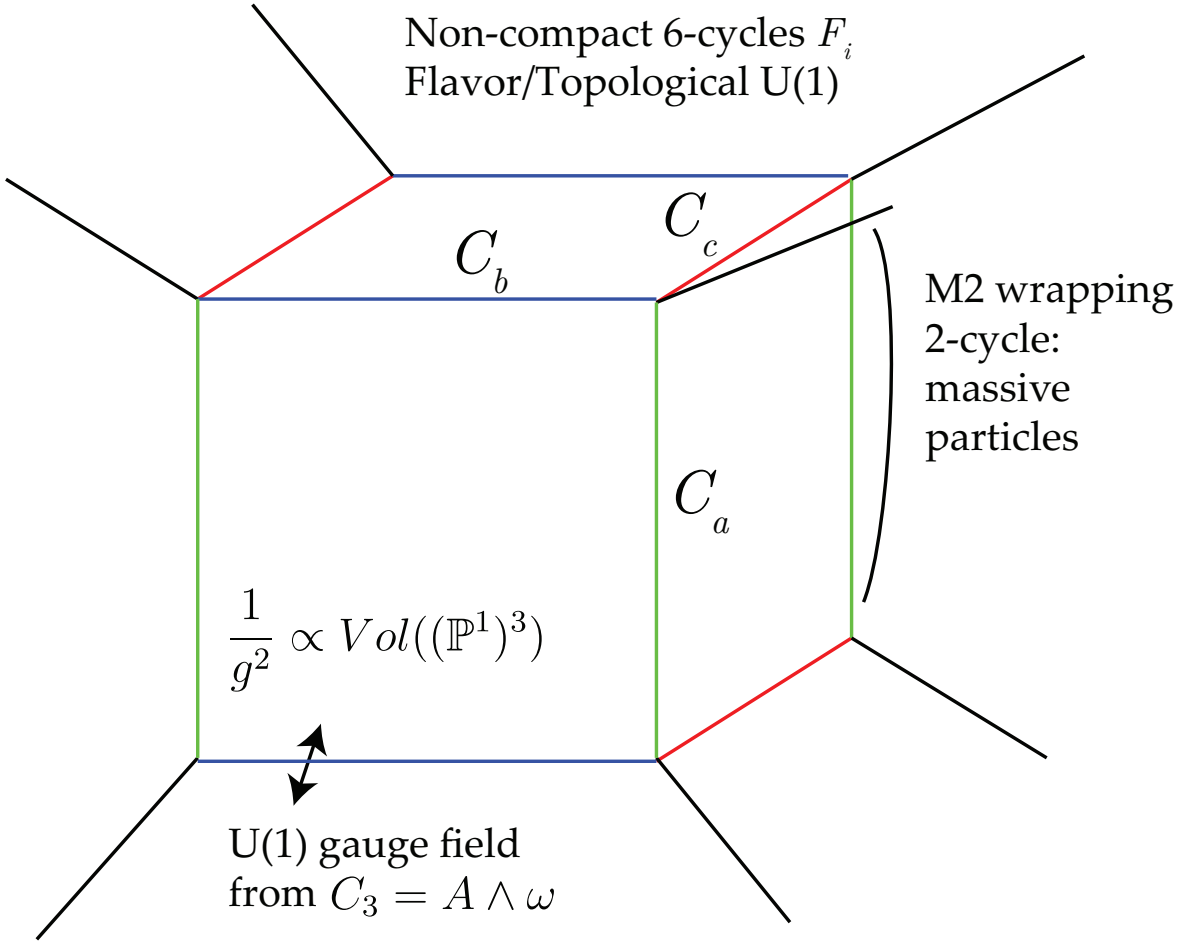
What about 3d $\mathcal{N} = 2$?

- Naturally, M-theory on local CY4 singularity $\to$ 3d $\mathcal{N} = 2$ SCFT, because of the absence of geometric scale
- Originally explored in (Gukov Vafa Witten 99'), but almost no development.
- Build up geometric dictionary, investigate 3d $\mathcal{N} = 2$ physics from M-theory on CY4



Resolved CY4

- M-theory on resolved local CY4 X_4, e. g. local $D = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \rightarrow$ 3d $\mathcal{N} = 2$ U(1) gauge theory+ massive matter fields



Resolved CY4

- BPS states from M2-brane wrapping $\mathbb{P}^1$ curves C. Hint from 4d/3d F/M-duality.

(1) $N_{C|X_4} = \mathcal{O} \oplus \mathcal{O} \oplus \mathcal{O}(-2)$, C is locally a $\mathbb{P}^1$ fiber (over a base 4-cycle).
Interpreted as 3d $\mathcal{N} = 2$ massive vector multiplet

(2) $N_{C|X_4} = \mathcal{O} \oplus \mathcal{O}(-1) \oplus \mathcal{O}(-1)$, C is locally a $\mathbb{P}^1$ fiber (over a base Σ_g).
Interpreted as 3d $\mathcal{N} = 2$ massive chiral multiplets.

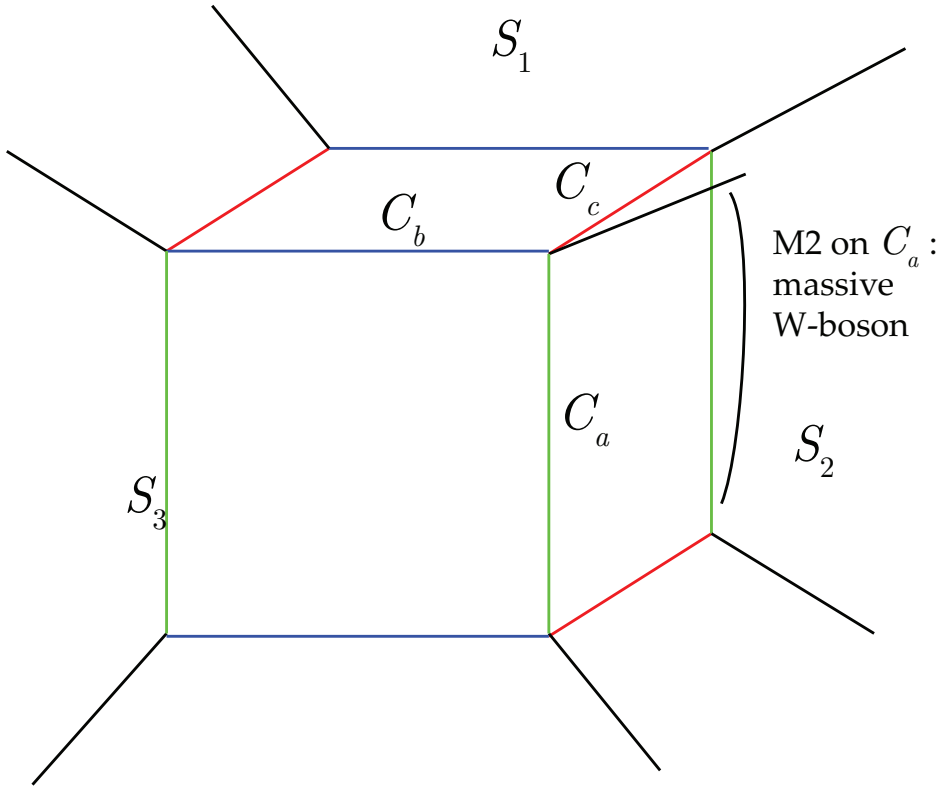
- Mass of the BPS particle $m \propto Area(C)$
- Charge under Cartan: $q = C \cdot D$
- Charge under flavor Cartan $q_i^F = C \cdot F_i$

Resolved CY4

- Denote the non-compact divisors to be S_1, S_2, S_3, compact divisor is D

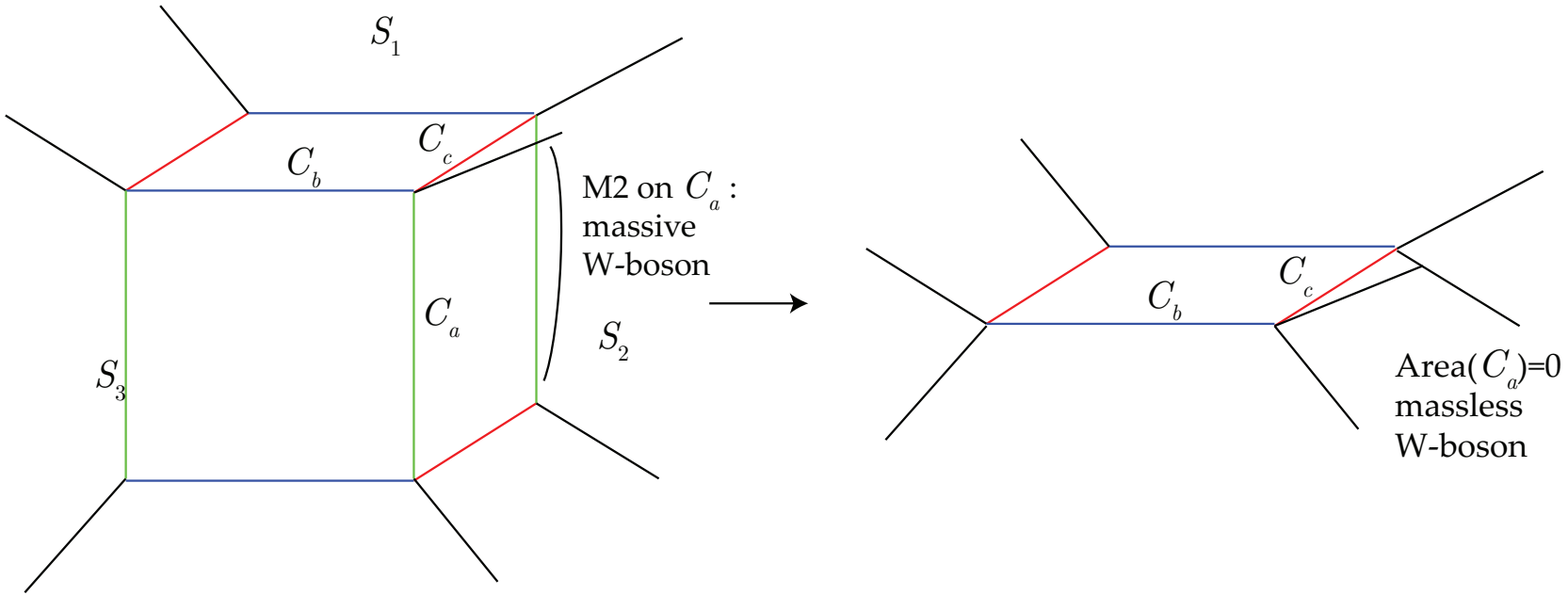
$$C_a = D \cdot S_2 \cdot S_3 \ , \ C_b = D \cdot S_1 \cdot S_3 \ , \ C_c = D \cdot S_1 \cdot S_2 \tag{1}$$

- $C_a \cdot D = C_b \cdot D = C_c \cdot D = -2$, hence one can choose C_a, C_b or C_c as gauge W-boson.



SU(2) limit

• In the limit of e. g. $Area(C_a) \to 0$, $SU(2)$ gauge theory+massive charged matter



Flavor symmetry enhancement

• In the singular limit of X_4, 3d $\mathcal{N} = 2$ SCFT with non-abelian flavor symmetry enhancement G_F

• Read off from the CB picture from M-theory on resolved CY4

• Identify non-compact 6-cycles F_i generating flavor Cartan $U(1)^f$

• Identify flavor W-bosons as M2 wrapping C_i.

(1) Vector multiplet: $N_{C_i|X_4} = \mathcal{O} \oplus \mathcal{O} \oplus \mathcal{O}(-2)$

(2) Charge under $U(1)^f$ forming the Cartan matrix of G_F

(3) Neutral under $U(1)^r$ gauge symmetry

Flavor symmetry enhancement

- In the example of local $(\mathbb{P}^1)^3$, flavor Cartans

$$F_1 = S_1 - S_2 \ , \ F_2 = S_2 - S_3 \, . \tag{2}$$

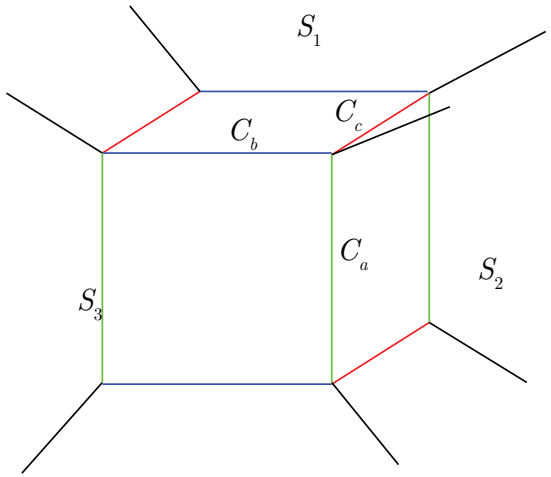
- Flavor W-bosons

$$C_1 = D \cdot (S_1 - S_2) \cdot S_3 \ , \ C_2 = D \cdot (S_2 - S_3) \cdot S_1 \, . \tag{3}$$

	F_1	F_2
C_1	-2	1
C_2	1	-2

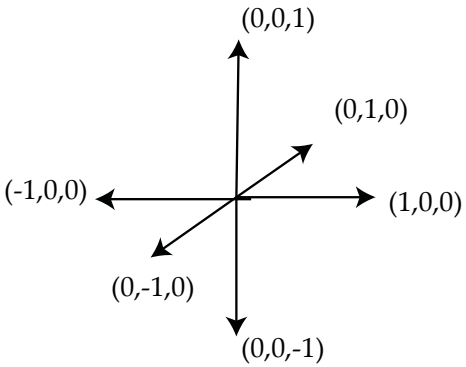
, Exactly the Cartan matrix for $SU(3)$! (4)

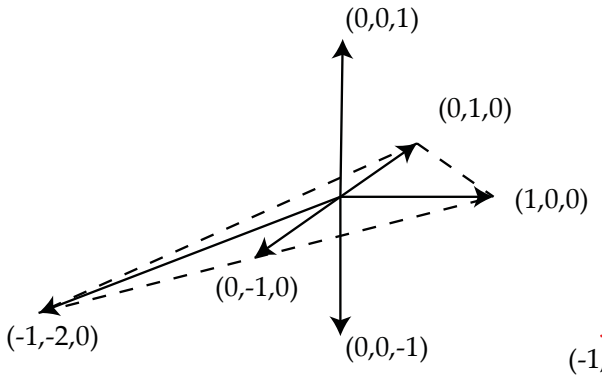
- C_a, C_b and C_c form the **3** rep. of $SU(3)$!

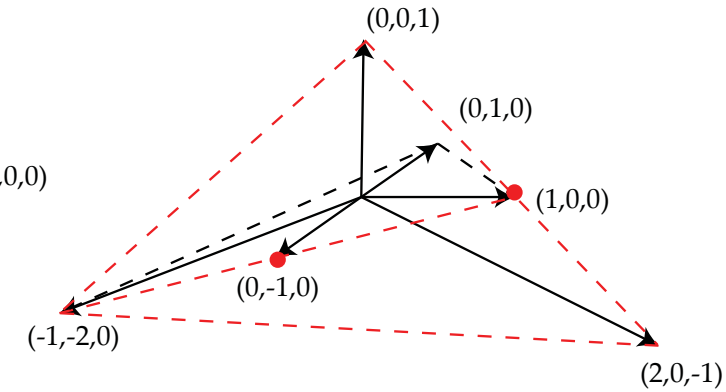


Flavor symmetry enhancement

- Flavor W-boson being non-effective?
- Similar to the 5d case, local $\mathbb{F}_0 \approx$ local $\mathbb{F}_2$ (Seiberg rank-1 E_1 theory with $G_F = SU(2)$)
- Deformation $\mathbb{F}_0 \to \mathbb{F}_2$ gives the same SCFT!
- CY4 case, toric diagram from local $(\mathbb{P}^1)^3$:







IR EFT, CS terms

• Now let us go back to the CB picture: $U(1)^r$+massive matter fields

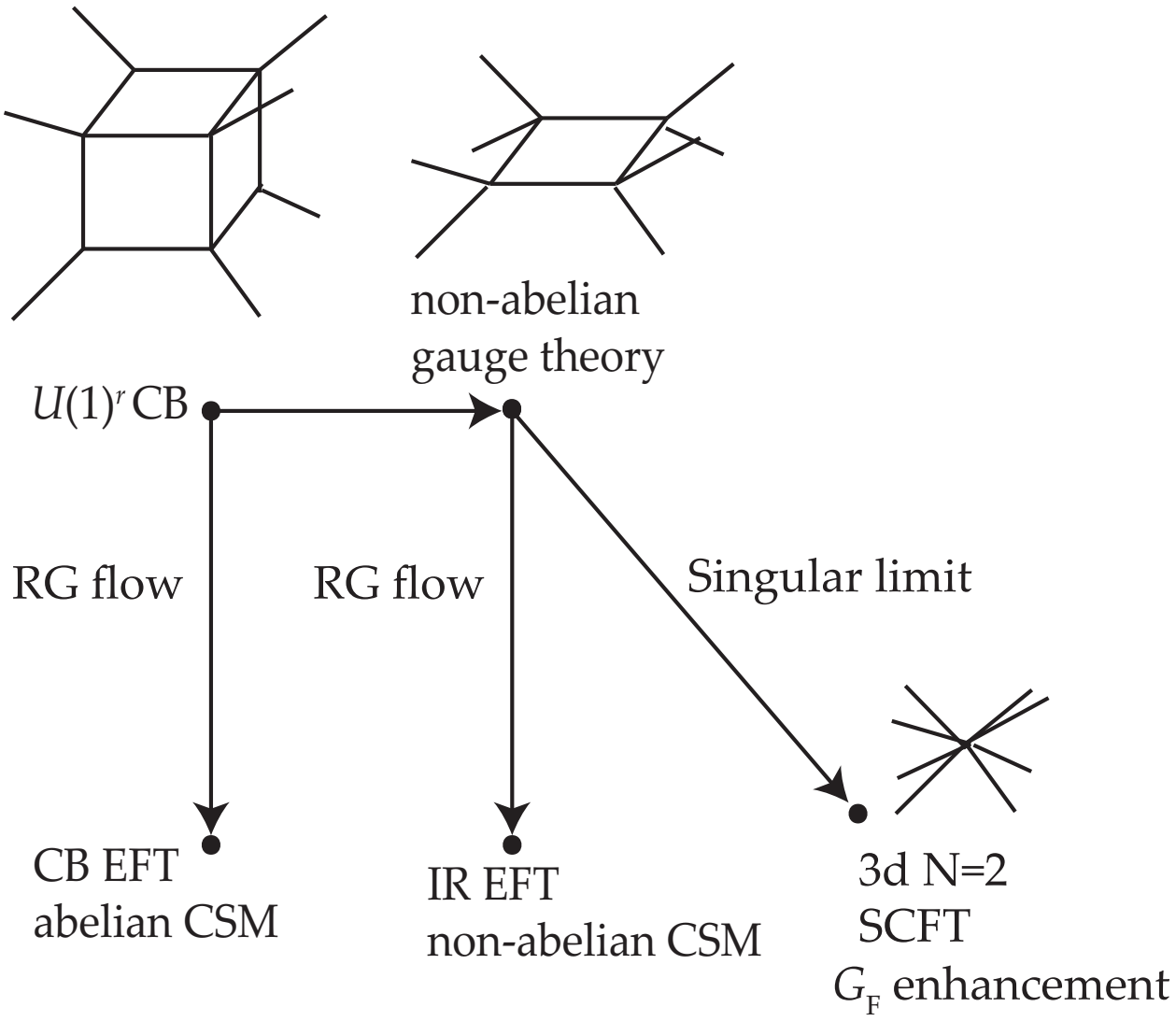
• IR Effective action: integrate out massive fermions $f \to$ Chern-Simons terms from 1-loop effects

$$k_{ij} = \frac{1}{2} \sum_f q_i q_j \text{sign}((q_f)_k \xi^k + m_f) \tag{5}$$

• Here i, j can be either gauge $U(1)$ or flavor (topological) $U(1)$, ξ^k are FI parameters

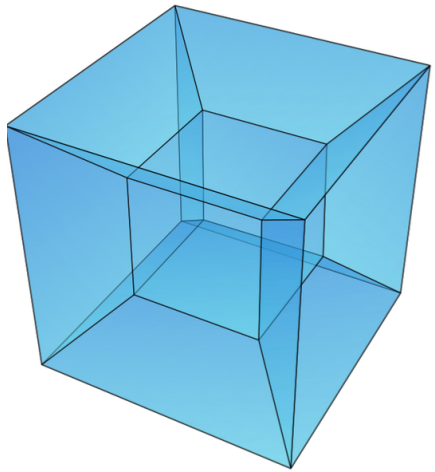
• In the IR, the kinetic terms for gauge $U(1)$s are unimportant, it is in general a 3d $\mathcal{N} = 2$ Chern-Simons-Matter theory.

IR EFT, CS terms



Brane web picture in IIB

- In the case of toric CY4, a dual brane web description in IIB! (Leung, Vafa 97')



- First consider M-theory on T^3 $(3, 7, 10)$ directions
- The toric CY4 is equivalent to the system of $(6+1)$-dim. KK7M monopoles

	0	1	2	3	4	5	6	7	8	9	10
KK7M$^{(p)}$	✓	✓	✓	✓	✓	✓	•	✓	•	•	TN
KK7M$^{(q)}$	✓	✓	✓	✓	✓	•	✓	TN	•	•	✓
KK7M$^{(r)}$	✓	✓	✓	TN	•	✓	✓	✓	•	•	✓

Brane web picture in IIB

- M-theory on $S^1_{(10)} \to$ IIA, T-duality along $S^1_{(7)} \to$ IIB

	0	1	2	3	4	5	6	7	8	9
$D_5 = (1,0,0)$	✓	✓	✓	✓	✓	✓	•	•	•	•
$NS_5 = (0,1,0)$	✓	✓	✓	✓	✓	•	✓	•	•	•
KK6B $= (0,0,1)$	✓	✓	✓	TN	•	✓	✓	✓	•	•

- Can be viewed as a web of (p, q, r) 4-branes in 8d SUGRA (remove 3, 7 directions)! (Leung, Vafa 97')(Lu, Roy 98')
- (p, q, r) transforms under $SL(3, \mathbb{Z})$ (part of 8d U-duality)

Brane web picture in IIB

- (p, q, r)-strings connecting to branes:

	0	1	2	3	4	5	6	7	8	9
F_1 (1,0,0)	✓	•	•	•	•	•	✓	•	•	•
D_1 (0,1,0)	✓	•	•	•	•	✓	•	•	•	•
D_3 (0,0,1)	✓	•	•	✓	✓	•	•	✓	•	•

- 4-string junctions built out of (p, q, r)-strings.
- M2-brane wrapping 2-cycle in M-theory $\leftrightarrow$ open string modes on 4-string junction!

What's next?

- Detailed study of 3d $\mathcal{N} = 2$ IR EFT
- Adding G_4 flux $\rightarrow$ additional chiral matter and CS term
- Higher-form symmetries
- $\mathbb{C}^4/\Gamma$ orbifolds, 4d McKay correspondence
- Superpotential from geometry? Hard even for $SU(2) + N_f\mathbf{F}$!
- Realize known 3d $\mathcal{N} = 2$ dualities
- Higgs branch?

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Thank you for your attention!