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Towards Naturalness (Hierarchy Problem) of Higgs from String theory

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Hierarchy problem is a key concept to go beyond SM in particular physics, cosmology, string theory

My previous works since 2009 (in collaborations with)

Phenomenology: Yuta Orikasa, Nobu Okada, Michio Hashimoto
Cosmology: Kengo Shimada, Pasquale Serpico, Kazu Kohri
String: Nori Kitazawa, Hikaru Ohta, Takao Suyama
Hierarchy problem: Hajime Aoki, Kiyoharu Kawana

Today talk: based on a paper
in collaboration with T. Suyama and N. Kitazawa
2303.16608 (PRD 2023) and papers in 2019, 2020

Hierarchy problem = Dynamics of EWSB and its Stability

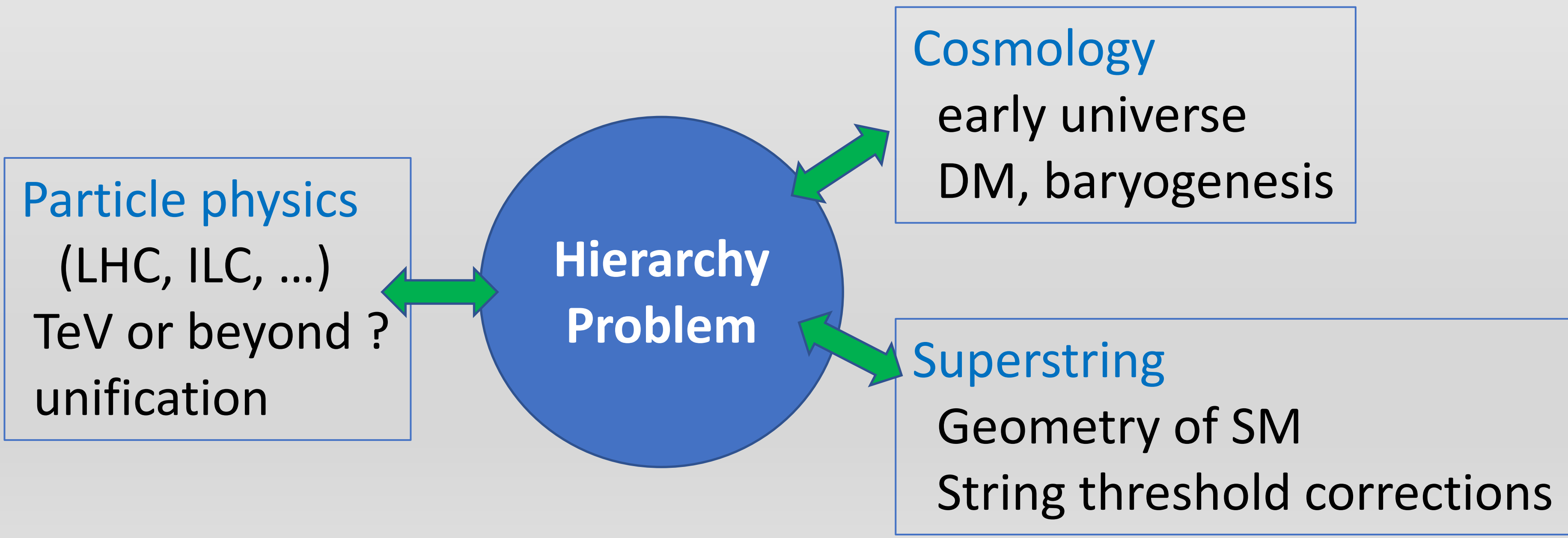
How EWSB occurs dynamically ?

Who ordered the Higgs potential ?

Why Higgs VEV <H> = 256 GeV , Higgs mass m_H = 125 GeV ?

Why are they stable against (if exists) higher energy scales?

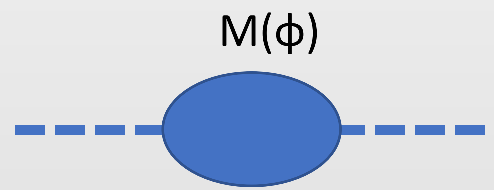
e.g. GUT 10^{16} GeV, Planck 10^{19} GeV, or SUSY? RHν?



Hierarchy problem of Higgs

$$V = -\mu^2|H|^2 + \lambda(|H|^2)^2$$

Radiative corrections to the Higgs potential



$$\delta V(\phi) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\, \mathrm{Str}\log(k^2 + M^2(\phi))$$

$$= \frac{\Lambda^2}{32\pi^2}\mathrm{STr}M^2(\phi) + \mathrm{STr}\frac{M^4(\phi)}{64\pi^2}(\ln(M^2/\Lambda^2) - 1/2) + \Lambda^4\, Str\, 1$$

Quadratic divergence

$\mathrm{STr}M^2(\phi) \neq 0$ Quadratic divergence in Higgs mass term

$\mathrm{STr}M^2(\phi) = 0$ Cancellation of Quadratic divergence (supersymmetry etc.)

My approach to the Hierarchy problem

is based on an assumption of "classical conformality" (CC),

Higgs mass may not receive threshold corrections (power "divergence") from UV physics.

In conventional EFT calculations,
Higgs mass in EFT receive large threshold corrections from UV physics.

Difference between logarithmic and power terms

- Logarithmic part is physical and affects the low energy phenomenology (such as accelerator experiments or dynamics of early universe).

- Quadratic divergence gives a boundary condition of RGE, but it does not affect the EFT once EFT is renormalized. (Thus it is also called threshold corrections.)

I have been working on various physics based on the idea of CC.

Suppose that our universe is described by

low energy EFT with "classical conformality"

Particle physics models:

B-L model with CC (2009 --)

Connection with Planck scale physics (2012)

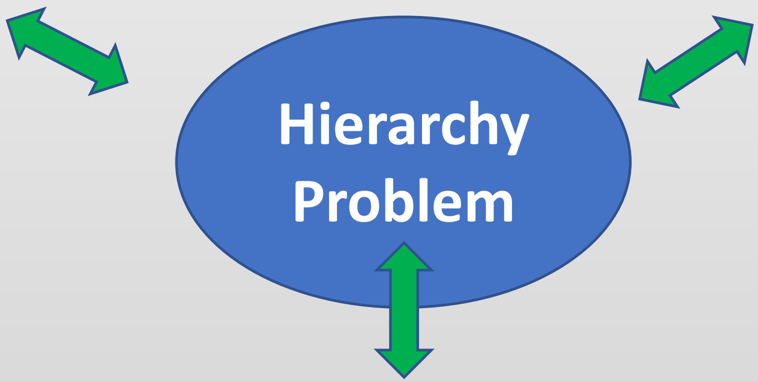
Leptogenesis and DM (2011, 2013 -)

Cosmology

Early universe of CC model (2014 --)

QCD-induced EW phase transition

supercooling (2017)



Can String theory provide the classical conformality to EFT?

(1) Revolving D-branes (2015-)

(2) Stringy Threshold corrections (2023)

Can string theory have a chance to provide CC in EFT ?
Can string theory control threshold corrections ?

Sting theory is a collection of infinite fields, but more than that.
The most prominent feature is absence of UV divergences
owing to modular invariance.

Field theory calculation

$$\begin{aligned}\Lambda_{\text{field}} &= \frac{1}{2}\sum_i (-1)^F \int^{\Lambda} \frac{d^D p}{(2\pi)^D} \log(p^2 + M_i^2) \\ &= -\frac{1}{2}\sum_i (-1)^F \int \frac{d^D p}{(2\pi)^D} \int_0^{\Lambda} \frac{dt}{t} e^{-(p^2+M_i^2)t} \\ &= -\frac{1}{2}\frac{1}{(4\pi)^{D/2}} \sum_i (-1)^F \int_0^{\Lambda} \frac{dt}{t^{1+D/2}} e^{-M_i^2 t} .\end{aligned}$$

$t \sim \infty$ IR
$t \sim 0$ UV

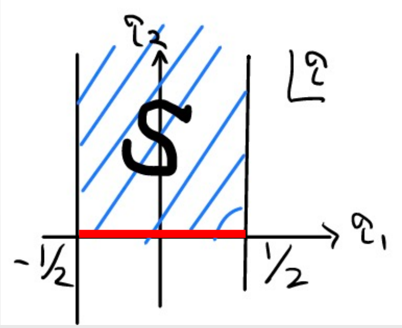
(1) IR-finite, if there are no tachyons.

(2) UV-finite, if $t \sim 0$ behavior is given by $\sum(-1)^F e^{-M_i^2 t} \sim t^{\alpha}$ for α > D/2 .

The condition is equivalent to : Str $M^{2\beta}$ = 0 for 2β = 0,2,..., D-3, D-2.

If the mass spectrum is given by $M_n{}^2 \propto n\mu$ as in string theory,

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{4\pi}\right)^D \int_S \frac{d^2\tau}{\tau_2{}^2} Z(\tau)$$



$$\tau_2 = \frac{1}{4\pi}\mu^2 t$$

UV

degeneracy in n-sector

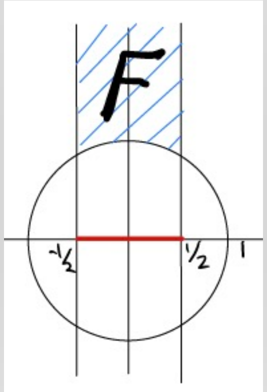
$$Z(\tau) = \tau_2{}^{1-D/2} \sum_{m,n} g_{mn}\, \overline{q}^{M_m^2/\mu^2} q^{M_n^2/\mu^2}$$

$$g_{nn} = (-1)^{F_n} g_n$$

Closed string theory is UV finite due to "modular invariance"

$$\tau \rightarrow \tau + 1$$
$$\tau \rightarrow 1/\tau$$

Cut off scale Λ is replaced by M_{str},
and the theory is finite irrespective of vanishing of $Str\ M^{2\beta}$.



What are $Str\ M^{2\beta}$ for $\beta <$ D/2
in "non-supersymmetric" string theory ?

Cribiori et.al. (2021)

In non-supersymmetric closed/open sting theories,
most of the terms $Str\ M^{2\beta}$ vanish:

- called misaligned SUSY by Dienes
- reminiscent of mass-formulae in non-linear realized SUSY

(1) Closed string: e.g. Non-SUSY heterotic SO(16) × SO(16)

$$\mathrm{Str}\ e^{-M_i^2 t} \sim t^{\frac{D}{2}-1} \quad \Longleftrightarrow \quad \mathrm{Str}\ M^{2\beta} = 0 \quad \text{for } 2\beta = 0, 2, \ldots, D-3$$

But $\mathrm{Str}\ M^{D-2} \neq 0$

This gives string scale cosmological constant.

(2) Open string: (no modular inv, but it changes open to closed)

e.g. Sugimoto model of Usp(32) gravitino → brane SUSY breaking

32 Dp on Op (in particular, p=9)

or Bosonic type 0'B with SU(32)

$$\Lambda_{\mathrm{D}p} = -\frac{1}{2\pi} T_{\mathrm{D}p} \int_0^\infty \frac{\mathrm{d}t}{2t} M_{\mathrm{D}p}(t),$$

$$M_{\mathrm{D}p}(t) = \frac{1}{(2t)^{\frac{1}{2}(p+1)}} \sum_n (-1)^{F_n} g_n\, \mathrm{e}^{-2\pi t n}. \sim t^{-(p+1)/2}\ Str\ e^{-t\,M^2}$$

Yoneya (2000)

$$\mathrm{Str}\, M^0 = \mathrm{Str}\, M^2 = \mathrm{Str}\, M^4 = \mathrm{Str}\, M^6 = 0, \qquad \mathrm{Str}\, M^8 \neq 0.$$

String theory seems to satisfy non-trivial mass relation

$\mathrm{Str}\, M^{2\beta} = 0$ for 2β=0,2,…, D-3

But $\mathrm{Str}\, M^{D-2} \neq 0$

even without explicit supersymmetry (misaligned or non-linear?).

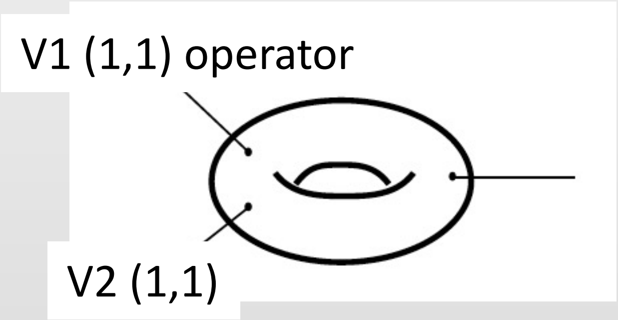
Does it help to solve hierarchy problem of Higgs in string theory?

Controlling threshold corrections by $\mathrm{Str}\, M^{2\beta} = 0$ is our goal.

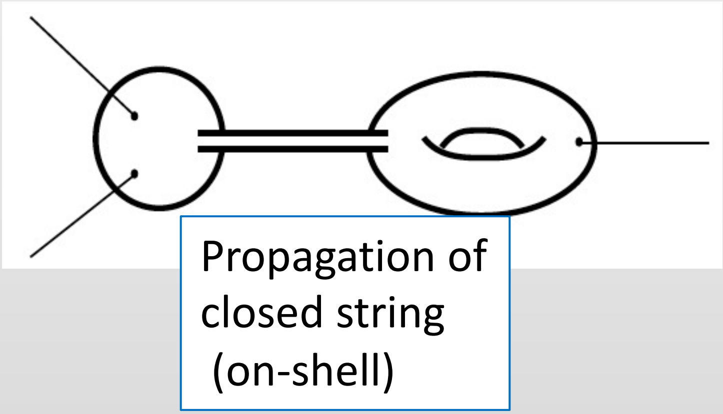
So far, we have considered vacuum energy.
For mass & w.f. renormalization in string theory,
we need to calculate off-shell string amplitude
and
separate the mass shift into two parts:
(1) low energy EFT correction (logarithmic)
(2) stringy threshold corrections $\propto (M_{str})^2$.

(A) Calculation of off-shell string amplitude

Mass shift changes the conformal dimension of vertex operator and violate conformal invariance. Then off-shell amplitudes may depend on the conformal factor of the world sheet coordinates.



Limit of v1 → v2



This divergence in the corner of moduli integration, which gives mass shift, is interpreted as an anomalous dimension of vertex operator (Weinberg 85, Seiberg 87)

Furthermore, there is a development in calculating off-shell amplitude (Pius, Rudra, Sen 2014). Naïve extension of on-shell amplitude to off-shell is valid, as far as "gluing condition" of world sheet coordinates holds.
Coordinate dependence of mass shift is cancelled by that of w.f. renormalization.

(B) How to separate mass shift into
(1) EFT log correction (2) String threshold correction
→ Partial modular transformation (PMT) SI et.al. 2020

$$\mathcal{A} = \int_0^\infty dt\, I(t), \quad \Rightarrow \quad \mathcal{A} = \int_{\Lambda^{-2}}^\infty dt\, I(t) + \int_{\Lambda^2}^\infty ds\, \tilde{I}(s) \qquad s := 1/t$$

t: Schwinger parameter

(1) IR (2) UV

Sum of IR and UV contributions

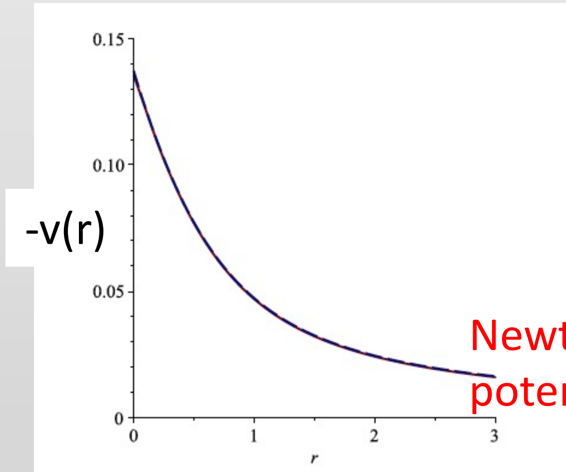
(a) In D-brane cases, a choice of $\Lambda = 1$ gives a very good approximation for the effective action for all region of r (distance) interpolating $r \sim 0$ and ∞.
Effective action can be written as
a sum of contributions from gauge theories on D-branes and supergravity

(b) For the purpose of separating amplitude into (1) and (2), we take $\Lambda \to 0_+$.
Then PMT gives Wilsonian effective action (or amplitude) with
(1) logarithmic EFT corrections
(2) string threshold corrections: power of M_{str} + $1/M_{str}$ corrections

(cf.) How good is the approximation by SYM + SUGRA ?

Example: D3-branes at angle. For simplicity, we set $\phi_1 = \phi_2 = \phi_3 = \phi, \;\; \phi_4 = 0$

$$V(R) \;=\; -\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}t} \frac{i\prod_{a=1}^4 \vartheta_{11}(\frac{i}{\pi}\phi'_a t, it)}{\eta(it)^3 \prod_{a=1}^3 \vartheta_{11}(\frac{i}{\pi}\phi_a t, it)},$$



Newton potential

red line = exact 1-loop potential

blue broken line = SYM + SUGRA
(only the lowest modes are taken into account)

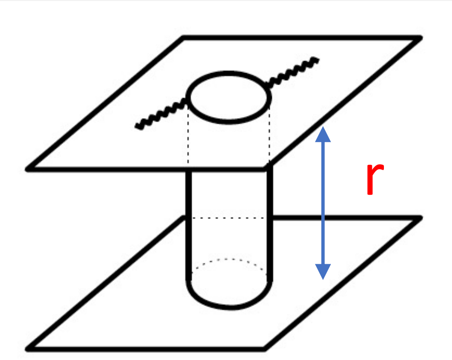
for the simple bosonic case

$$e^{-2\pi} = 0.001867$$

$$\sum_{n=0}^{\infty} d_n e^{-2\pi n} \;=\; \eta(i)^{-24} - e^{2\pi} \;=\; 1.026\, d_0.$$

Error is 2.6 %.

A simple example: Dp - Dp (→ technique is more general)



Mass shifts (w.f. renorm.) of
open-string massless states on Dp brane
from 2 point string amplitude

Results of Mass shifts
(a) Gauge field: no corrections due to gauge invariance
(b) Scalar field: some corrections depending on r
(c) Fermion field: no corrections (even w/o chiral symmetry)

For illustrative purpose, I will present the bosonic case in this page.

$$\mathcal{A} = (2\pi)^2 \int_0^\infty dt_1 \int_0^\infty dt_2 \, \mathrm{Tr}_X \left(e^{-2\pi t_1 (L_0^X - 1)} V_1 \, e^{-2\pi t_2 (L_0^X - 1)} V_2 \right)$$

$$V_s := c_t \exp\left(i k_{s,\mu} X^\mu + i c_\epsilon \epsilon_{s,\mu} \partial_\tau X^\mu + c_\zeta \zeta_{s,I} \partial_\sigma X^I \right)\Big|_{\tau=\sigma=0}.$$

$$\mathcal{A} \propto (2\pi)^2 \int_0^\infty dt \int_0^t d\nu \, (8\pi^2 \alpha' t)^{-(p+1)/2} e^{-2\pi r^2 t} \eta(it)^{-24} \exp\left[-2\alpha' k^2 \left(\pi \frac{\nu(t-\nu)}{t} + G_B(\nu, t) \right) \right]$$
$$\times \begin{cases} 1, & \text{(tachyon)} \\ 2\alpha' \epsilon_1 \cdot \epsilon_2 \left(\frac{1}{2\pi t} - \frac{1}{(2\pi)^2} \partial_\nu^2 G_B(\nu, t) \right), & \text{(gauge boson)} \\ (\zeta_1 \cdot w)(\zeta_2 \cdot w) - \frac{2\alpha'}{(2\pi)^2} \zeta_1 \cdot \zeta_2 \partial_\nu^2 G_B(\nu, t), & \text{(scalar)} \end{cases} \quad \text{(A.11)}$$

Now we apply the PMT to the amplitude $\mathcal{A}$.

$$\mathcal{A} = \mathcal{A}_{\mathrm{IR}} + \mathcal{A}_{\mathrm{UV}}$$ and take $\lambda \to 0_+$

$$\mathcal{A}_{\mathrm{IR}} = C' \int_{\lambda^{-2}}^\infty dt \int_0^t d\nu \, (\cdots)$$

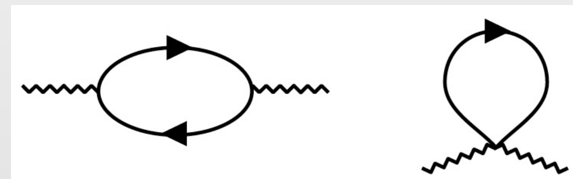
$$\mathcal{A}_{\mathrm{UV}} = C' \int_{\lambda^2}^\infty ds \int_0^{1/s} (\cdots)$$

(1) IR part of 2 point amplitude in superstring case ($\lambda \to 0_+$ limit)

(a) Gauge field

$$\mathcal{A}_{\rm IR} = C' \int_{\lambda^{-2}}^{\infty} dt \int_0^t d\nu \; (\cdot\;\cdot\;\cdot)$$

consists of 2 terms corresponding to

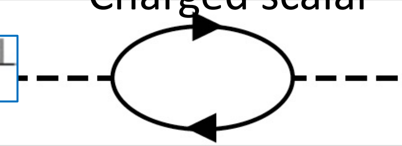


(b) Scalar field

$$\frac{C''}{\alpha'} \zeta_1 \cdot \zeta_2 \int_{\Lambda^{-2}}^{\infty} d\tau \, (8\pi^2\tau)^{-(p+1)/2} e^{-2\pi m^2 \tau} (-2\pi).$$

Charged scalar

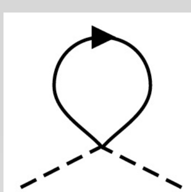
$\phi^{\perp}$



$$v^2|T|^2 + 2v(\varphi - \bar{\varphi})|T|^2 + (\varphi - \bar{\varphi})^2|T|^2$$

$$v^2 = \frac{r^2}{\alpha'} = \left(\frac{l}{2\pi\alpha'}\right)^2$$

also



(2) UV part: stringy threshold corrections in type II superstring

(b) Mass shift for scalar field:

$$\Delta m^2(r) = -\frac{\xi}{\alpha'}\int_{\lambda^2}^{\infty} ds\, s^{(p-11)/2} e^{-2\pi r^2/s} \frac{\vartheta_{10}(0,is)^4}{\eta(is)^{12}}\left(2(2\pi)^2 r^2 s^{-1} - 2\pi\right)$$

⬇

$\phi^{||}$ $\quad -32\pi(8-p)\Gamma(\frac{9-p}{2})(2\pi r^2)^{(p-9)/2}\frac{\xi}{\alpha'},$ + exp(-r) terms

massless closed string states massive closed string states

(cf.) It can be obtained from effective potential V(r) as V''(r). But the method of PMT can be applied to any scalar field and its mass shift can be similarly obtained from the moduli dependent vacuum energy V(φ) by V''(φ).

(c) Mass shift for fermions:

2pt amplitude with fermion vertex operators seems to give vanishing mass shift:

$$\delta\, m^2 = 0$$

On Dp (p<9), there is no chiral symmetry to protect fermion mass.

Summary

"Question"
What low energy EFT does the string theory provide?
SM does not have any dimensionful parameter except for the Higgs mass.
Is there any chance that string theory provides "classical conformality" and solves the hierarchy problem of Higgs ?

We showed that we can systematically separate radiative corrections in string theory into

(1) low energy logarithmic corrections

(2) stringy threshold corrections

by using the partial modular transformation.

We applied the method to a simple case of Dp – $\underline{\mathrm{Dp}}$
and calculated mass shift (and w.f. renorm.) of massless scalars and fermions.
But the method can be applied to any scalar field.

The issue in future is
how to control the threshold corrections in non-supersymmetric string.
A hint is the property of (misaligned or non-linear realized SUSY)

$$\mathrm{Str}\, M^0 = \mathrm{Str}\, M^2 = \mathrm{Str}\, M^4 = \mathrm{Str}\, M^6 = 0, \qquad \mathrm{Str}\, M^8 \neq 0.$$