BOOTSTRAPPING PERTURBATIVE STRING AMPLITUDES

Yu-tin Huang
National Taiwan University

Li-Yuan Chiang (Yale U), Y-t H, He-Chen Weng (Brown U) 2310.10710

some overlap with J. Berman, H. Elvang, A. Herderschee 2310.10729

JEAW Xi An China

Any UV completion of quantum gravity has a low energy EFT description

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R+\hat{\alpha}_1\frac{R^2}{M^2}+\hat{\alpha}_2\frac{R^3}{M^4}+\hat{\alpha}_4\frac{R^4}{M^6}\cdots\right)$$

Can we confine the space for allowed $\hat{\alpha}_i$ → Can we carve out the landscape of perturbative strings ?

Given a finite set of $\hat{\alpha}_i$ → Can we recover the information of the UV spectrum ?

Given the specific string theory $\hat{\alpha}_i$ → Is it a unique answer to a set of bootstrap equations?

$$-\frac{1}{st(s+t)}\exp\left[\sum_{k=2}^{\infty}\frac{2\zeta(2k+1)}{2k+1}(s^{2k+1}+t^{2k+1}-(s+t)^{2k+1})\right]$$

In recent years there's been tremendous progress in studying quantum gravity by implementing the S-matrix bootstrap program for the gravitational EFT

. arXiv:2102.08951 [pdf, other] hep-th doi 10.1007/JHEP07(2021)110

Sharp Boundaries for the Swampland

Authors: Simon Caron-Huot, Dalimil Mazac, Leonardo Rastelli, David Simmons-Duffin

5. arXiv:2205.01495 [pdf, other] hep-th gr-qc

Graviton partial waves and causality in higher dimensions

Authors: Simon Caron-Huot, Yue-Zhou Li, Julio Parra-Martinez, David Simmons-Duffin

. arXiv:2103.12728 [pdf, other] hep-th gr-qc hep-ph doi 10.1088/1751-8121/ac0e51

Gravitational Effective Field Theory Islands, Low-Spin Dominance, and the Four-Graviton Amplitude

Authors: Zvi Bern, Dimitrios Kosmopoulos, Alexander Zhiboedov

. arXiv:2102.02847 [pdf, other] hep-th doi 10.1103/PhysRevLett.127.081601

Where is String Theory?

Authors: Andrea Guerrieri, Joao Penedones, Pedro Vieira

arXiv:2201.07177 [pdf, other] hep-th

(Non)-projective bounds on gravitational EFT

Authors: Li-Yuan Chiang, Yu-tin Huang, Wei Li, Laurentiu Rodina, He-Chen Weng

. arXiv:2104.09682 [pdf, other] hep-th gr-qc hep-ph doi 10.1103/PhysRevLett.127.091602

Is the Standard Model in the Swampland? Consistency Requirements from Gravitational Scattering

Authors: Katsuki Aoki, Tran Quang Loc, Toshifumi Noumi, Junsei Tokuda

The Gravitation EFT

A ``bottom up" approach to constrain QG where the EFT operators serve as IR parameterization of UV completions

$$\mathcal{L} = \int dx^D \sqrt{-g}\left(M_{\rm pl}^{D-2} R + \alpha_1 R^2 + \alpha_2 R^3 + \alpha_4 R^4 \cdots\right)$$

Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots\right)$$

$$M \ll M_{\rm pl}$$

From non-perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat{\alpha}_1 \frac{R^2}{M_{\rm pl}^2} + \hat{\alpha}_2 \frac{R^3}{M_{\rm pl}^4} + \hat{\alpha}_4 \frac{R^4}{M_{\rm pl}^6} \cdots\right)$$

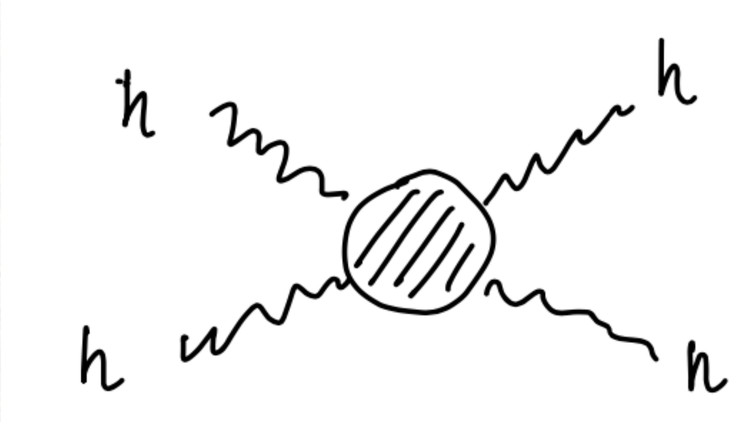
String theory provides solutions to both scenarios

$$M_{\rm s} \propto (g_s)^{\frac{1}{4}} M_{\rm p}$$

The Gravitation S-matrix

The four graviton amplitude, being a physical observable, is subject to a set of consistency conditions

$$M(s,t)$$



$$s = (p_1+p_2)^2 = E_c^2$$

$$t = (p_1-p_4)^2 = -\frac{E_c^2}{2}(1-\cos\theta)$$

Unitarity: the imaginary part of the partial wave has to be positive

$$M(s,t) = \sum_\ell \rho_\ell(s) P_\ell(1+2t/s), \quad \mathrm{Im}[\rho_\ell] \geq 0$$

Analyticity: the amplitude for t < m^2 is analytic on the complex s-plane away from real axes

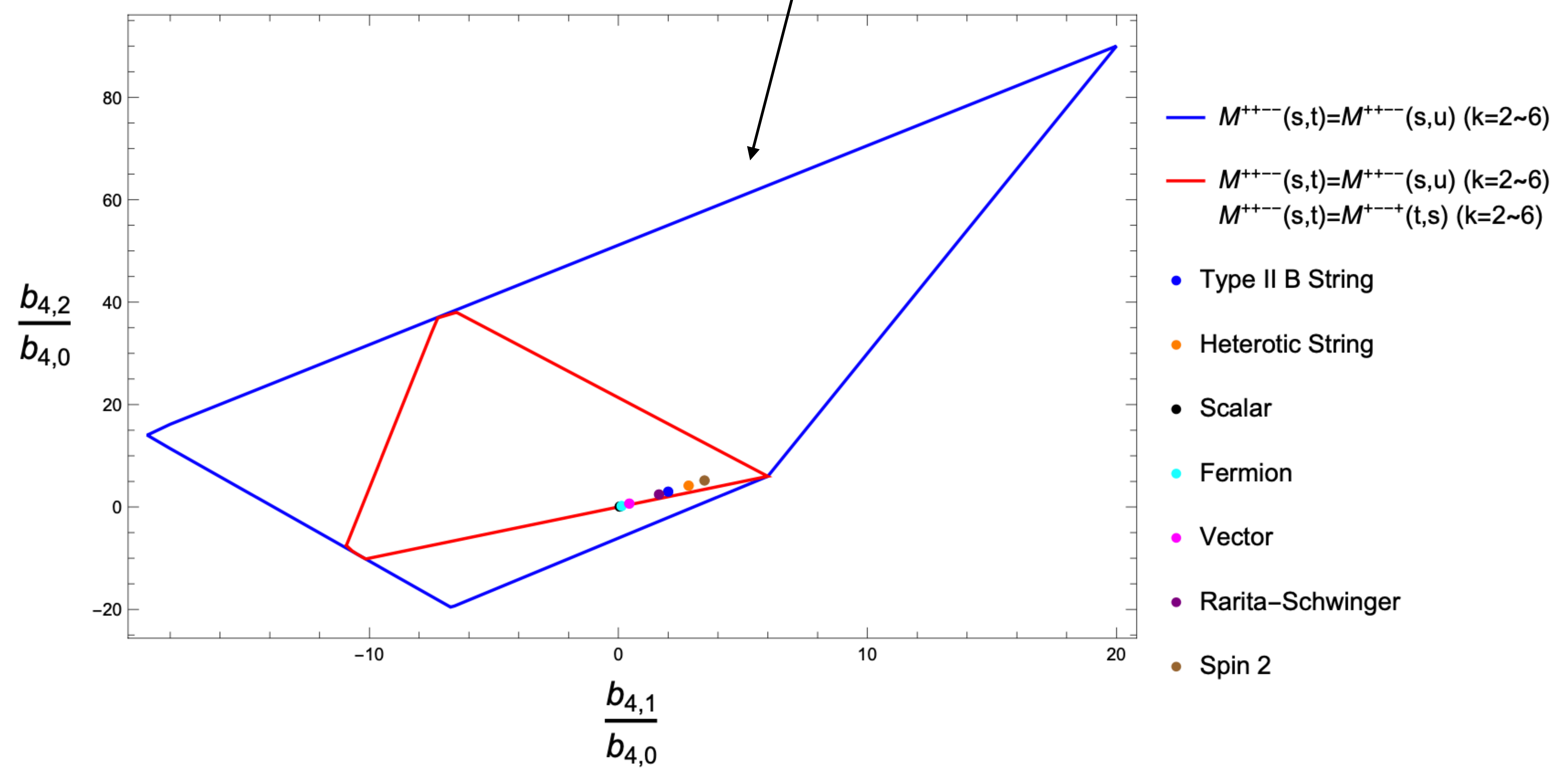
Boundedness: the amplitude satisfies $$\lim_{|s|\to\infty} \frac{M(s,t)}{|s|^2} = 0$$

Crossing: the amplitude in (s>0 t<0) can be analytically continued to (u>0, t>0)

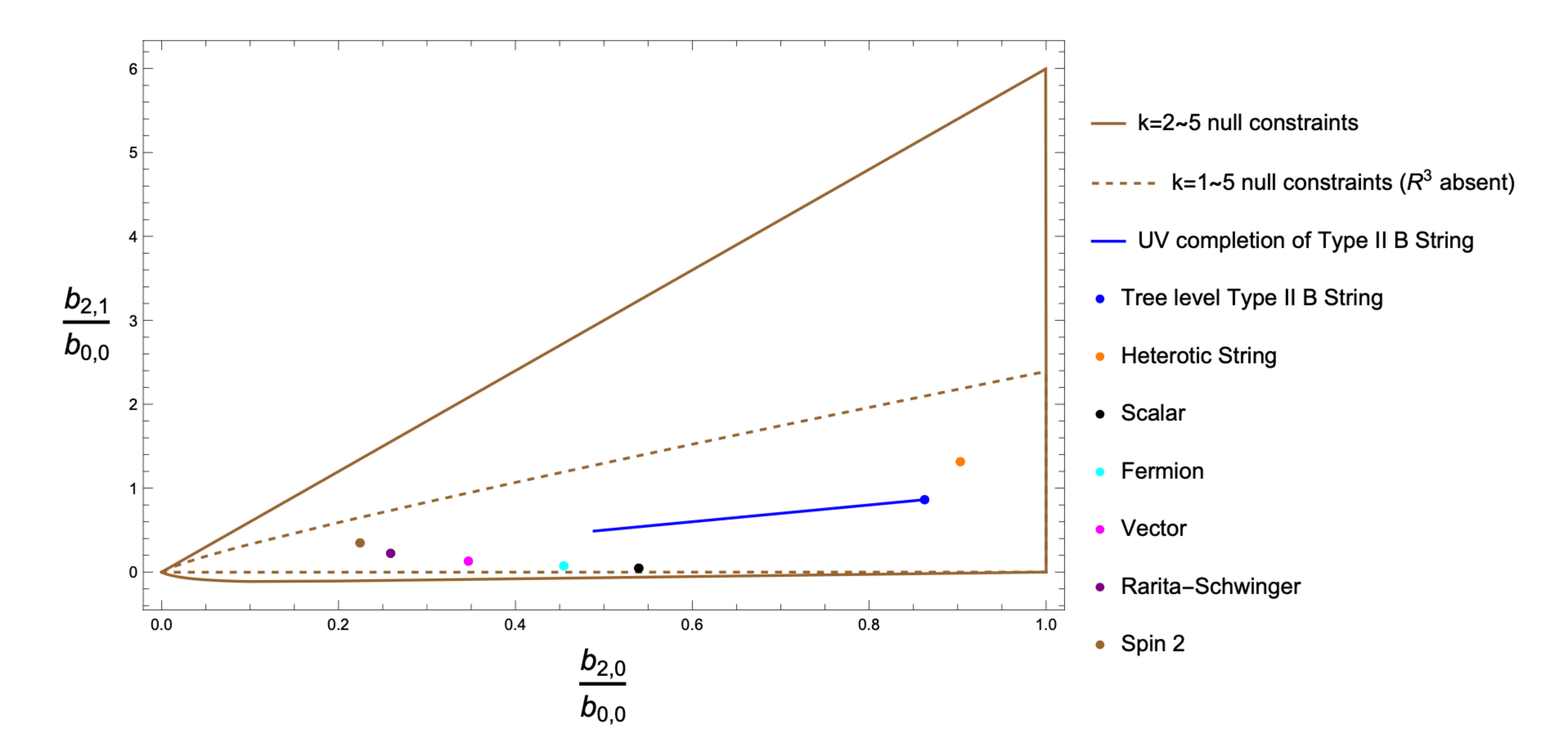
For fixed derivative couplings, with SDPB Li-Yuan Chiang, Y-t H, Wei Li, Laurentiu Rodina, He-Chen Weng 2105.02862

$D^8 R^4$

Z. Bern D. Kosmopoulos, A Zhiboedov 2103.12729



Indeed, when we consider ratios of couplings of different dimensions, the known theories start to span larger regions



D^4R^4 operators normalized by R^4 coefficients.

Impose further constraint?
Top-down ??

Perturbative sting amplitude

Being a theory of quantum gravity, string amplitudes are known to exhibit exponential suppression at high energies fixed angle scattering

Gross, Mende, PLB 303 issue 3,4 1988

$$A^{(4-\text{tachyon})}_{\text{closed}}(s,t,u) \simeq \frac{\sin(\pi t/2)\sin(\pi u/2)}{\sin(\pi s/2)}(stu)^{-3}\exp\left(-\frac{s\ln s + t\ln t + u\ln u}{4}\right)$$

This originated from the saddle point approximation to the world sheet integral

$$\int d^2z\,(1-z)^s(1-\bar{z})^s z^t \bar{z}^t\;\; f(z,\bar{z}) = \int d^2z\; e^{s\ln|1-z|^2 + t\ln|z|^2}\;\; f(z,\bar{z})$$

$$s\to\infty, \quad \frac{t}{s} = \text{fixed} \quad \longrightarrow \quad z = \frac{t}{u}$$

how do we incorporate the presence of a world sheet in our bootstrap?

Perturbative sting amplitude

For perturbative string, the amplitude is given by a world sheet integral of a 2D CFT

$$M(s,t) = \int \prod_i dz_i^2 \, \frac{1}{J} \prod_{i\neq j} |z_{ij}|^{-k_{ij}} \tilde{f}^{\text{sphere}}(z_i, \bar{z}_i, k_i, \epsilon_i)$$

The singularities of f occurs only when two insertion points coincide, writing $z_i = z_j + \tau e^i \theta$

$$\int d\tau \tau^{-k_{ij}+\alpha_{ij}} \sim \frac{1}{-k_{ij}+\alpha_{ij}+1}\,.$$

Thus in order for there to be massless poles, we expect that $\tilde{f}^{\text{sphere}}(z_i, \bar{z}_i, k_i, \epsilon_i)$ have trivial monodromy

Then one can trivially apply the Kawai-Lewellen-Tye contour deformation to obtain

$$M(s,t) = \sin \pi s A(s,t) A(s,u)$$

$$A(s,t) = \int_{\mathbf{I}} dz |z|^{k_1\cdot k_2} |z-1|^{k_2\cdot k_3} f^{\text{disk}}(z, \epsilon_i, k_i)$$

Perturbative sting amplitude

Comment: one can attempt to generalize the exponents

$$\int_0^1 dz|z|^{v(s)-1}|1-z|^{v(t)-1}f(\epsilon_i,k_i,z)$$

For the amplitude to have massless poles, $v(0)$ must be a non-positive integer.

Near z=1, 0, we have poles of the form

$$\frac{1}{v(s)-N}$$

This suggest $v(s)=-s+a$, where a is an integer

One can get around this if we allow for none-single valued $v_\pm(s)=\pm\sqrt{s}+a$

$$\int_0^1 dz\left(\sum_\pm |z|^{v_\pm(s)-1}\right)\left(\sum_\pm |1-z|^{v_\pm(t)-1}\right)f(\epsilon_i,k_i,z) \longrightarrow \frac{1}{\sqrt{s}+a}+\frac{1}{-\sqrt{s}+a}=-\frac{2a}{s-a^2}$$

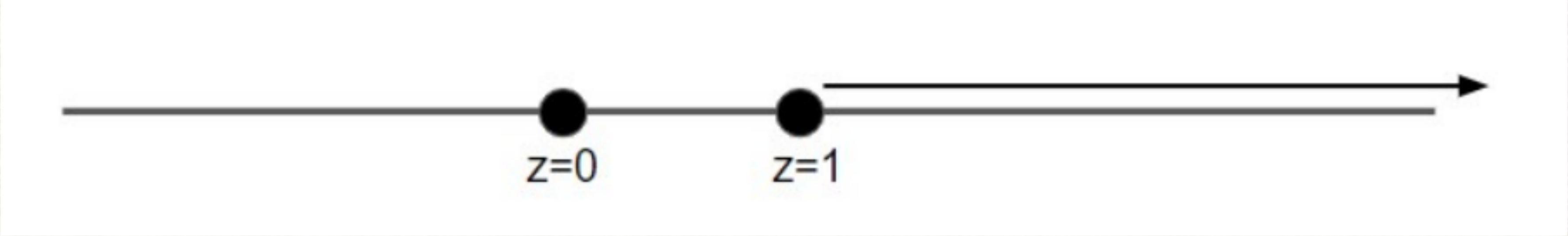
Cheung, Remmen, “bespoke dual resonance” 2308.03833

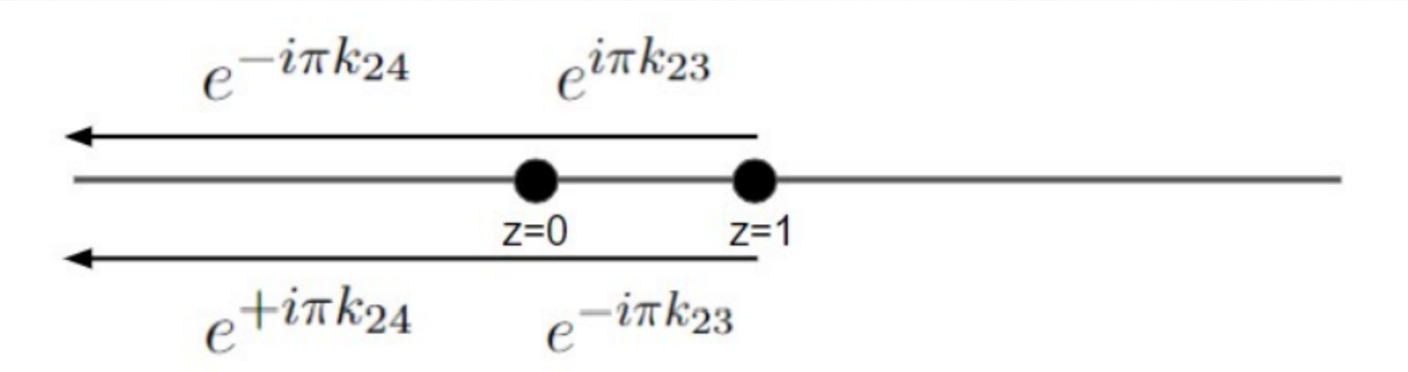
Perturbative sting amplitude

For the open string amplitude, we have three distinct ordering depending on the integration region

$$A(1234) \sim \int_0^1 dz\, z^{k_{12}}(1-z)^{k_{23}} f^{\text{disk}}$$
$$A(1324) \sim \int_1^\infty dz\, z^{k_{12}}(z-1)^{k_{23}} f^{\text{disk}}$$
$$A(2134) \sim \int_{-\infty}^0 dz\, (-z)^{k_{12}}(1-z)^{k_{23}} f^{\text{disk}}$$

Distinct ordering are related by monodromies picked up from contour deformations. If we assume f(z) does not generate any non-trivial monodromy at z=0, 1





$$z^{k_{12}}(z-1)^{k_{23}} \to \begin{matrix} z^{k_{12}}(1-z)^{k_{23}}e^{i\pi k_{23}} \\ z^{k_{12}}(1-z)^{k_{23}}e^{-i\pi k_{23}} \end{matrix} \to \begin{matrix} (-z)^{k_{12}}(1-z)^{k_{23}}e^{-i\pi k_{24}} \\ (-z)^{k_{12}}(1-z)^{k_{23}}e^{+i\pi k_{24}} \end{matrix}.$$

$$A(u,t) = -\mathbf{Re}\left[e^{i\pi k_{23}}A(s,t)+e^{-i\pi k_{24}}A(s,u)\right]$$
$$0 = \mathbf{Im}\left[e^{i\pi k_{23}}A(s,t)+e^{-i\pi k_{24}}A(s,u)\right].$$

Monodromy relations!

Stieberger, S. 0907.2211
Bjerrum-Bohr, Damgaard, Vanhove, 0907.1425

Perturbative sting amplitude

For amplitudes with the massless pole we begin with trivial monodromy

$$A\left(s,u\right)+e^{i\pi s}A\left(s,t\right)+e^{-i\pi u}A\left(t,u\right)=0$$

Let's first consider maximal supersymmetry, $A(s,t)=\delta^8(Q)\ f(s,t)$

$$f(s,t)=-\frac{1}{st}+\left(\frac{b}{s}+\frac{b}{t}\right)+\left(c\frac{t}{s}+c\frac{s}{t}\right)+g_{00}+(g_{1,0}s+g_{1,1}t)+\sum_{k\geq q\geq 0}g_{k,q}s^{k-q}t^q$$

Crossing symmetry + monodromy relations imply b, c=0 and

$$\begin{pmatrix} g_{00} & & & & \\ g_{1,0} & g_{1,1} & & & \\ g_{2,0} & g_{2,1} & g_{2,2} & & \\ g_{3,0} & g_{3,1} & g_{3,2} & g_{3,3} & \\ g_{4,0} & g_{4,1} & g_{4,2} & g_{4.3} & g_{4,4} \end{pmatrix} = \begin{pmatrix} \frac{\pi^2}{6} & & & & \\ g_{1,0} & g_{1,0} & & & \\ \frac{\pi^4}{90} & \frac{\pi^4}{360} & \frac{\pi^4}{90} & & \\ g_{3,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & g_{3,0} & \\ \frac{\pi^6}{945} & g_{4,1} & -\frac{\pi^6}{15120}+2g_{4,1} & g_{4,1} & \frac{\pi^6}{945} \end{pmatrix}$$

Now impose unitarity

Perturbative sting amplitude

Unitarity :

The amplitude admits a single channel dispersion relation

$$A(s,t) = -\int_{M^2}^{\infty} ds' \, \frac{\rho_\ell(s') G_\ell^D(1+2t/s')}{s-s'}$$

Thus the Wilson coefficients can be represented as

$$g_{k,q} = \frac{1}{q!}\frac{\partial^q}{\partial t^q} \int \frac{ds'}{s'^{k-q+1}} \sum_\ell \rho_\ell(s') G_\ell^D(1+2t/s') \Big|_{t=0}$$

Unitarity implies positivity

The monodromy relations constitute "null constraints"

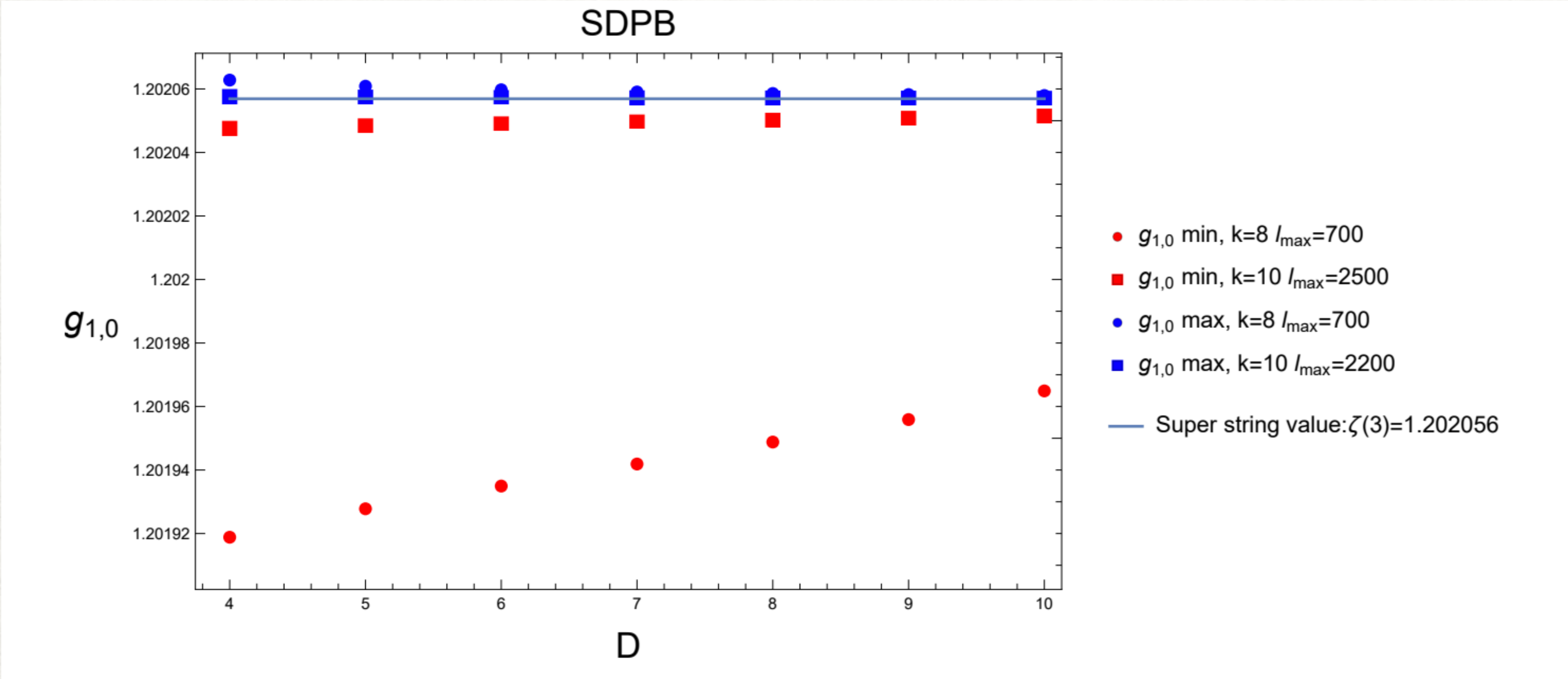
$$\begin{pmatrix} g_{00} & & & & \\ g_{1,0} & g_{1,1} & & & \\ g_{2,0} & g_{2,1} & g_{2,2} & & \\ g_{3,0} & g_{3,1} & g_{3,2} & g_{3,3} & \\ g_{4,0} & g_{4,1} & g_{4,2} & g_{4.3} & g_{4,4} \end{pmatrix} = \begin{pmatrix} \frac{\pi^2}{6} & & & & \\ g_{1,0} & g_{1,0} & & & \\ \frac{\pi^4}{90} & \frac{\pi^4}{360} & \frac{\pi^4}{90} & & \\ g_{3,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & g_{3,0} & \\ \frac{\pi^6}{945} & g_{4,1} & -\frac{\pi^6}{15120}+2g_{4,1} & g_{4,1} & \frac{\pi^6}{945} \end{pmatrix}$$

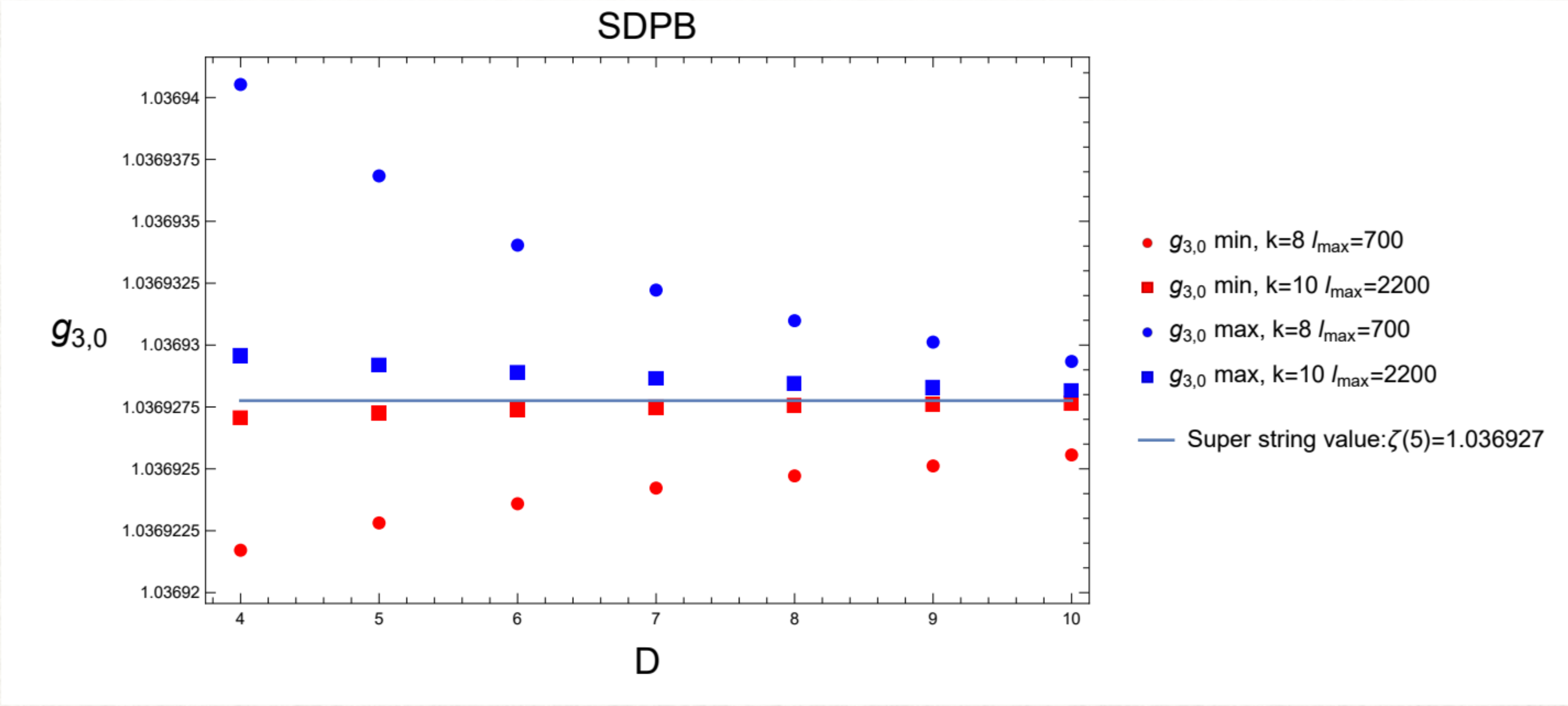
Null constraints

EFT for super string theory

$$A(s,t) = \delta^8(Q)\ f(s,t)$$

$$f(s,t) = -\frac{1}{st} + \left(\frac{b}{s}+\frac{b}{t}\right) + \left(c\frac{t}{s}+c\frac{s}{t}\right) + g_{00} + (g_{1,0}s + g_{1,1}t) + \sum_{k\geq q\geq 0} g_{k,q} s^{k-q} t^q$$





D	$\tilde{g}_{k,q}$	Two-sided bound	Superstring value	Relative error
4	$\tilde{g}_{1,0}$	$1.20204774 < \tilde{g}_{1,0} < 1.20205755$	1.20205690	8.1×10^{-6}
4	$\tilde{g}_{3,0}$	$1.03692704 < \tilde{g}_{3,0} < 1.03692956$	1.03692775	2.4×10^{-6}
4	$\tilde{g}_{4,1}$	$0.0405367063 < \tilde{g}_{4,1} < 0.0405469176$	0.0405368972	2.5×10^{-4}
10	$\tilde{g}_{1,0}$	$1.20205185 < \tilde{g}_{1,0} < 1.20205700$	1.20205690	4.3×10^{-6}
10	$\tilde{g}_{3,0}$	$1.03692764 < \tilde{g}_{3,0} < 1.03692814$	1.03692775	4.8×10^{-7}
10	$\tilde{g}_{4,1}$	$0.0405368583 < \tilde{g}_{4,1} < 0.0405426553$	0.0405368972	1.4×10^{-4}

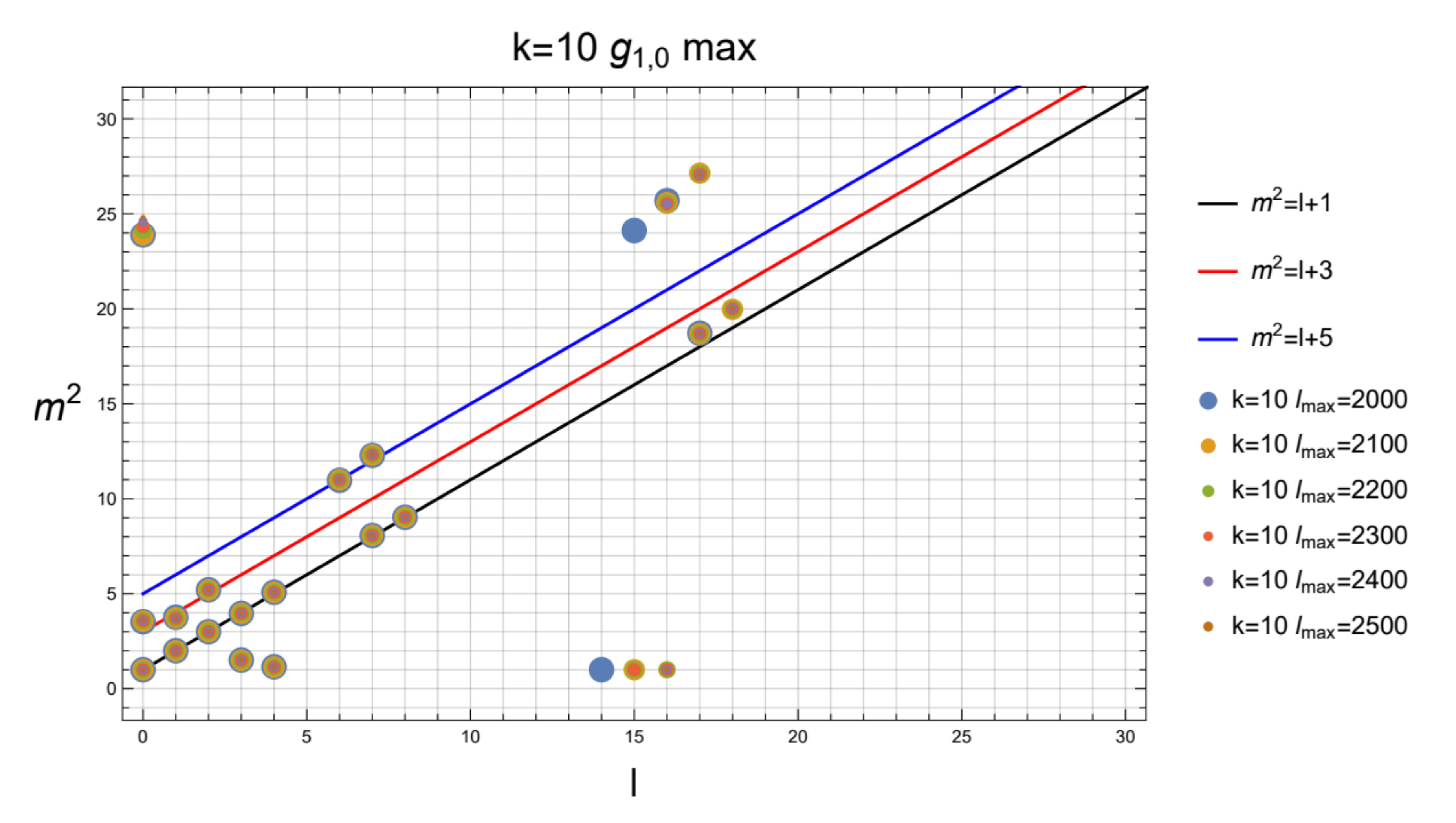
EFT for super string theory

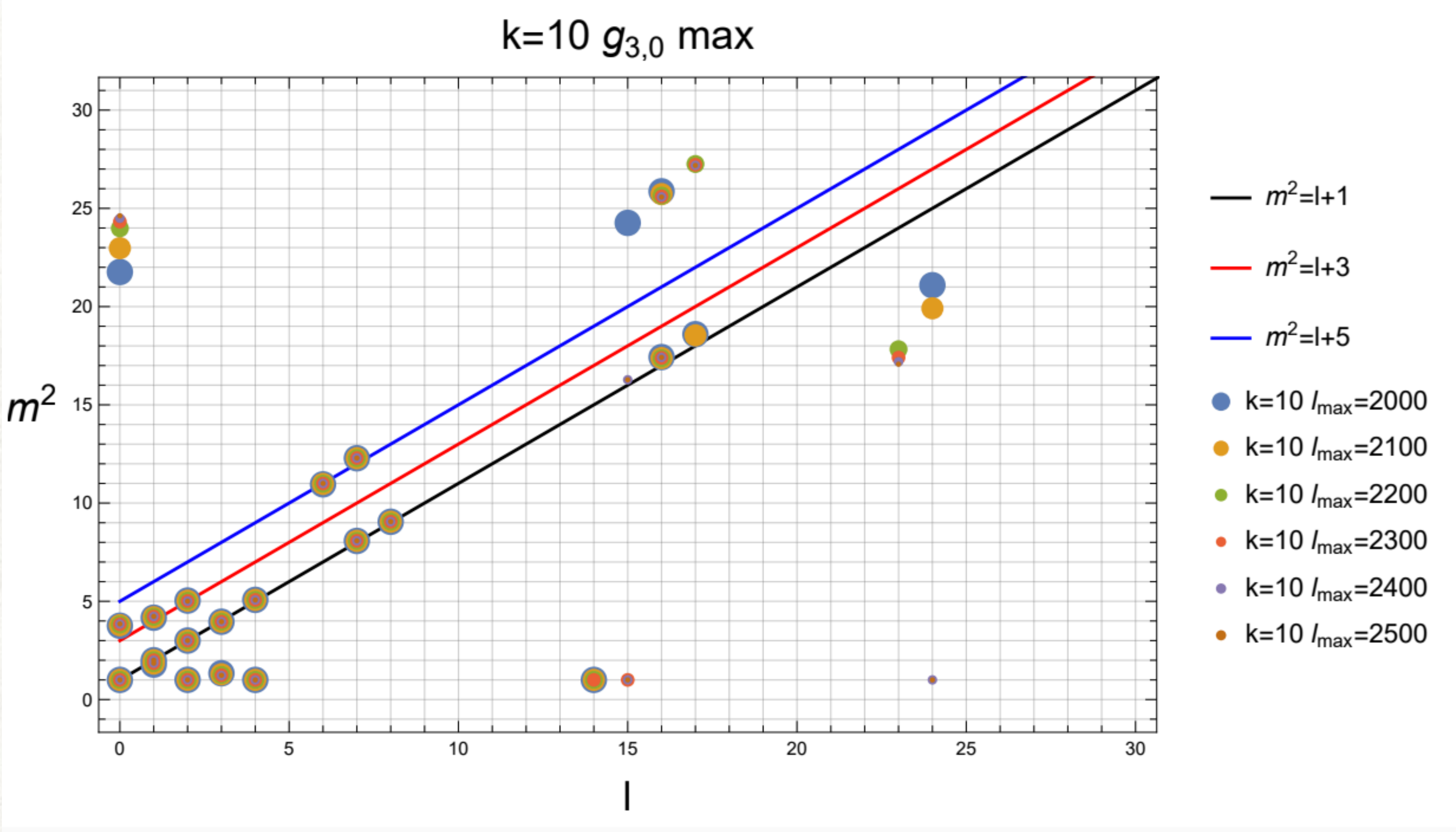
Geometric approach allows us to derive critical dimension

$$
\begin{aligned}
\text{Min} \quad & t \\
\text{s.t.} \quad & \sum_{i=1}^{m} x_i A_i^{(l)} - C^{(l)} + tI \succeq 0, \quad l = 1, \ldots, L, \\
& B^T \mathbf{x} = \mathbf{b},
\end{aligned}
$$

	$D=10$	$D=11$	$D=12$	$D=13$
$k_{\max}=6$	$t^* < -1.35 \times 10^{-4}$	$t^* < -8.45 \times 10^{-5}$	$t^* < -7.05 \times 10^{-6}$	$t^* < -5.51 \times 10^{-5}$
$k_{\max}=8$	$t^* < -1.32 \times 10^{-7}$	$t^* < -2.64 \times 10^{-7}$	$t^* > 3.70 \times 10^{-5}$	$t^* > 3.19 \times 10^{-4}$
$k_{\max}=10$	$t^* < -5.53 \times 10^{-9}$	$t^* > 8.62 \times 10^{-5}$	$t^* > 3.43 \times 10^{-4}$	$t^* > 1.50 \times 10^{-3}$

SDPB gives the spectrum of the string





EFT for scalar string theory

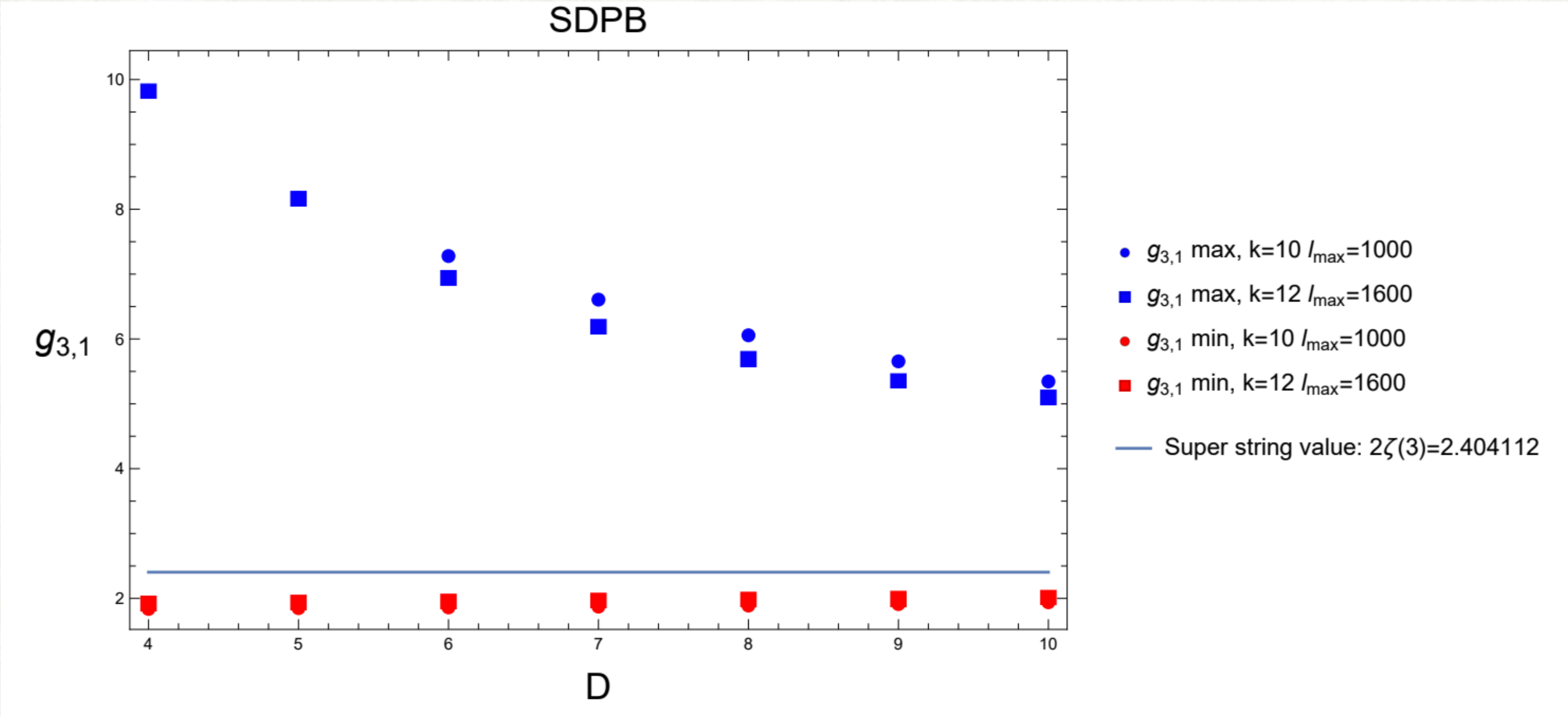
Let's consider none-supersymmetric scalar EFT

$$A(s,t) = -\left(\frac{s}{t}+\frac{t}{s}\right) + b\left(\frac{1}{s}+\frac{1}{t}\right) + \sum_{k,q\geq 0} g_{k,q} s^{k-q} t^q.$$

The monodromy relation implies

$$\begin{pmatrix} g_{0,0} & & & \\ g_{1,0} & g_{1,1} & & \\ g_{2,0} & g_{2,1} & g_{2,2} & \\ g_{3,0} & g_{3,1} & g_{3,2} & g_{3,3} \end{pmatrix} = \begin{pmatrix} -1 & & & \\ g_{1,0} & g_{1,0} & & \\ \frac{\pi^2}{6} & \frac{\pi^2}{6} & \frac{\pi^2}{6} & \\ g_{3,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & g_{3,0} \end{pmatrix}$$

The EFT space is no longer an island



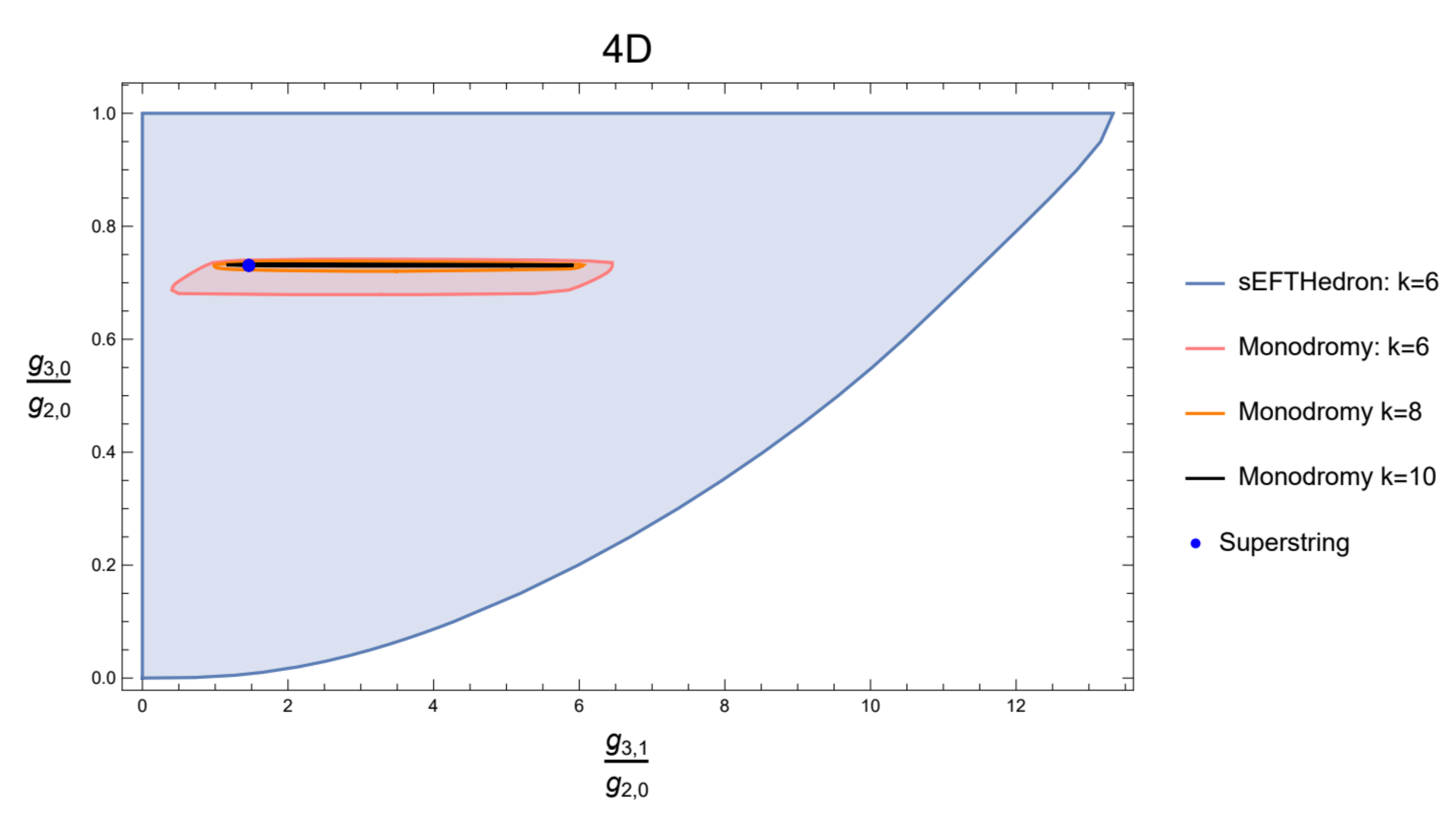
D	$g_{k,q}$	Two-sided bound	Superstring value	Relative error
4	$g_{3,0}$	$1.2012 < g_{3,0} < 1.20369$	1.202056	2.0×10^{-3}
4	$g_{3,1}$	$1.91 < g_{3,1} < 9.8125$	2.404	3.2
10	$g_{3,0}$	$1.20186 < g_{3,0} < 1.20266$	1.202056	6.6×10^{-4}
10	$g_{3,1}$	$1.99 < g_{3,1} < 5.09$	2.404	1.2

EFT for scalar string theory

Let's consider none-supersymmetric scalar EFT

$$A(s,t) = -\left(\frac{s}{t}+\frac{t}{s}\right) + b\left(\frac{1}{s}+\frac{1}{t}\right) + \sum_{k,q\geq 0} g_{k,q} s^{k-q} t^q.$$

The monodromy relation implies



Critical dimension <13

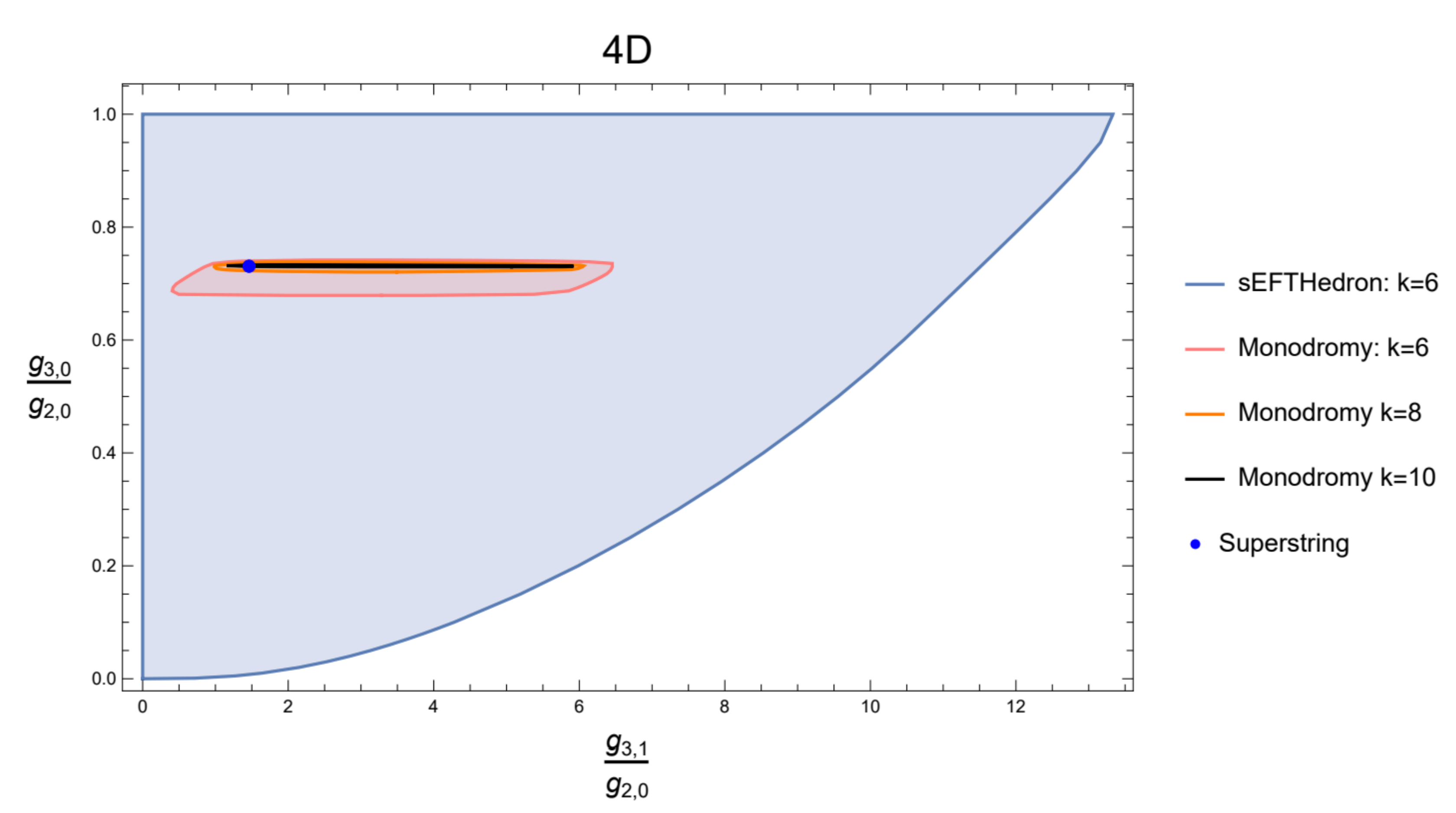
	$D=12$	$D=13$	$D=14$	$D=15$
$k_{\max}=20$	$\cdots$	$\cdots$	$t^*=2.81\times10^{-15}$	$t^*=7.37\times10^{-10}$
$k_{\max}=22$	$\cdots$	$t^*=-7.62\times10^{-10}$	$t^*=1.64\times10^{-10}$	$\cdots$
$k_{\max}=26$	$t^*=-8.998\times10^{-25}$	$t^*=2.276\times10^{-14}$	$\cdots$	$\cdots$

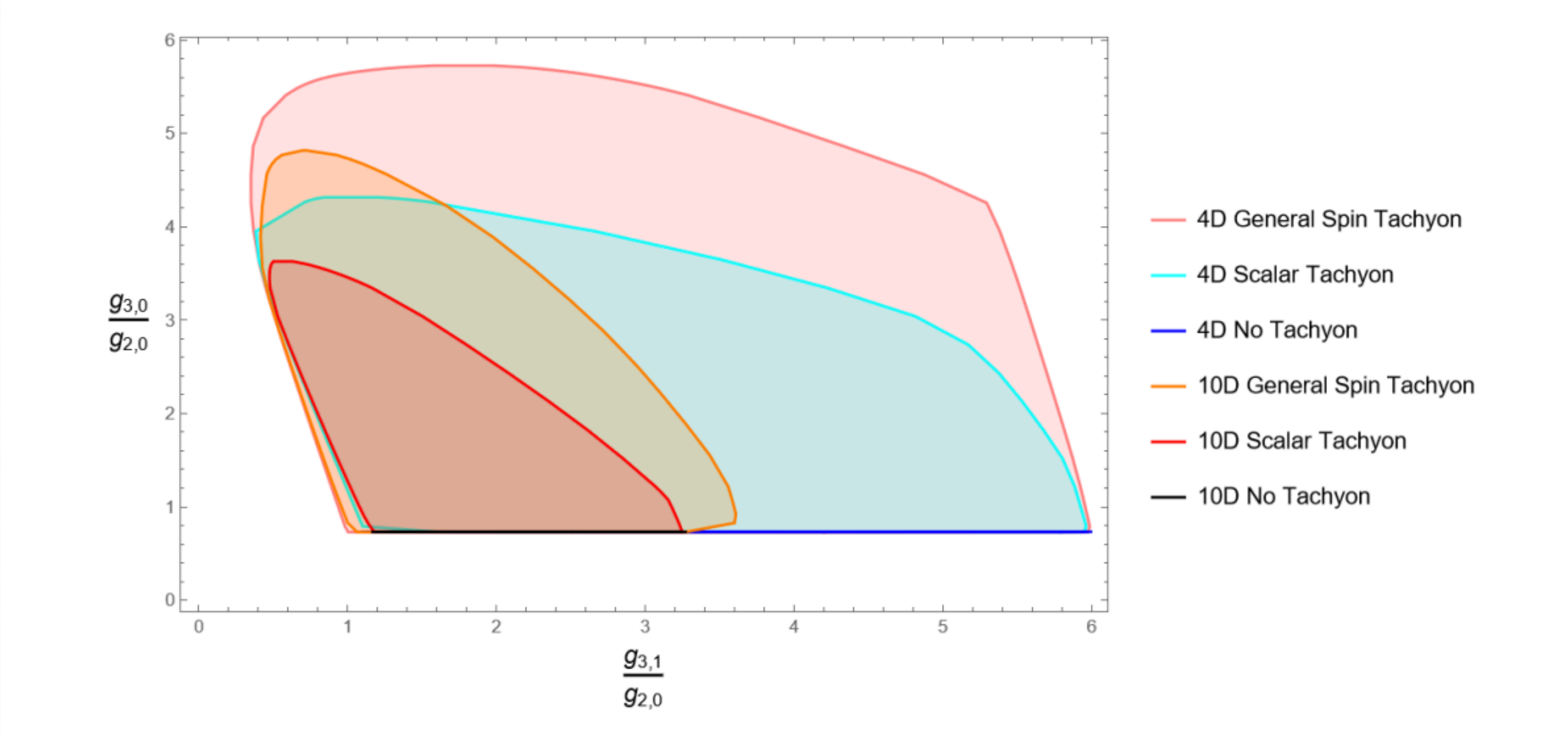
EFT for scalar string theory

We can include Tachyons

$$A(s,t) = -\sum_{\ell} \int dm^2 \left(\delta(m^2 + M^2)\rho_\ell \frac{G_\ell^D(\theta)}{s - m^2} + \rho_\ell(m^2) \frac{G_\ell^D(\theta)}{s - m^2} \right)$$

The monodromy relation implies





Gluon EFT

Is the island a consequence of maximal SUSY ? No !

Let's consider 4D vector EFT

$$A(1^-2^+3^-4^+) = \langle 13\rangle^2[24]^2\left(\frac{-1}{st}+\sum_{k,q} b_{k,q}s^{k-q}t^q\right) \qquad = -u^2\sum_l\int dm^2\frac{d^{\ell\geq 2}_{-2,-2}(\theta)}{\cos^4\frac{\theta}{2}m^4}\frac{\rho_\ell^{+-}(m^2)}{s-m^2}$$

$$A(1^-2^-3^+4^+) = \langle 12\rangle^2[34]^2\left(\frac{-1}{st}+\frac{\tilde{a}}{s}+\tilde{c}\frac{t}{s}+\sum_{k,q}\tilde{b}_{k,q}s^{k-q}t^q\right). \quad = -s^2\sum_l\int dm^2\frac{d^{\ell\geq 0}_{0,0}(\theta)}{m^4}\frac{\rho_\ell^{++}(m^2)}{s-m^2}$$

Monodromy relations

$$A(1^-3^+2^-4^+) = -\mathbf{Re}\left[e^{i\pi(k_2\cdot k_3)}A(1^-2^-3^+4^+)+e^{i\pi(-k_2\cdot k_4)}A(2^-1^-3^+4^+)\right]$$

$$0 = \mathbf{Im}\left[e^{i\pi(k_2\cdot k_3)}A(1^-2^-3^+4^+)+e^{i\pi(-k_2\cdot k_4)}A(2^-1^-3^+4^+)\right]$$

$$\begin{pmatrix} b_{0,0} & & & \\ b_{1,0} & b_{1,1} & & \\ b_{2,0} & b_{2,1} & b_{2,2} & \\ b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \end{pmatrix} = \begin{pmatrix} -\tilde{c}+\frac{\pi^2}{6} & & & \\ \tilde{b}_{1,0} & \tilde{b}_{1,0} & & \\ \frac{\pi^4}{45}-\tilde{b}_{2,0} & \frac{\tilde{c}\pi^2}{6}+\frac{\pi^4}{40}-2\tilde{b}_{2,0} & \frac{\pi^4}{45}-\tilde{b}_{2,0} & \\ \tilde{b}_{3,0} & -\frac{\pi^2}{4}\tilde{b}_{1,0}+3\tilde{b}_{3,0}-\frac{1}{2}\tilde{b}_{3,2} & -\frac{\pi^2}{4}\tilde{b}_{1,0}+3\tilde{b}_{3,0}-\frac{1}{2}\tilde{b}_{3,2} & \tilde{b}_{3,0} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{b}_{0,0} & & & \\ \tilde{b}_{1,0} & \tilde{b}_{1,1} & & \\ \tilde{b}_{2,0} & \tilde{b}_{2,1} & \tilde{b}_{2,2} & \\ \tilde{b}_{3,0} & \tilde{b}_{3,1} & \tilde{b}_{3,2} & \tilde{b}_{3,3} \end{pmatrix} = \begin{pmatrix} \tilde{c}+\frac{\pi^2}{6} & & & \\ \tilde{b}_{1,0} & \tilde{b}_{1,0} & & \\ \tilde{b}_{2,0} & -\frac{\tilde{c}\pi^2}{6}-\frac{\pi^4}{120}+\tilde{b}_{2,0} & -\frac{\tilde{c}\pi^2}{6}+\frac{\pi^4}{90} & \\ \tilde{b}_{3,0} & -\frac{\pi^2}{12}\tilde{b}_{1,0}+\tilde{b}_{3,0}+\frac{1}{2}\tilde{b}_{3,2} & \tilde{b}_{3,2} & \frac{\pi^2}{12}\tilde{b}_{1,0}+\frac{1}{2}\tilde{b}_{3,2} \end{pmatrix}$$

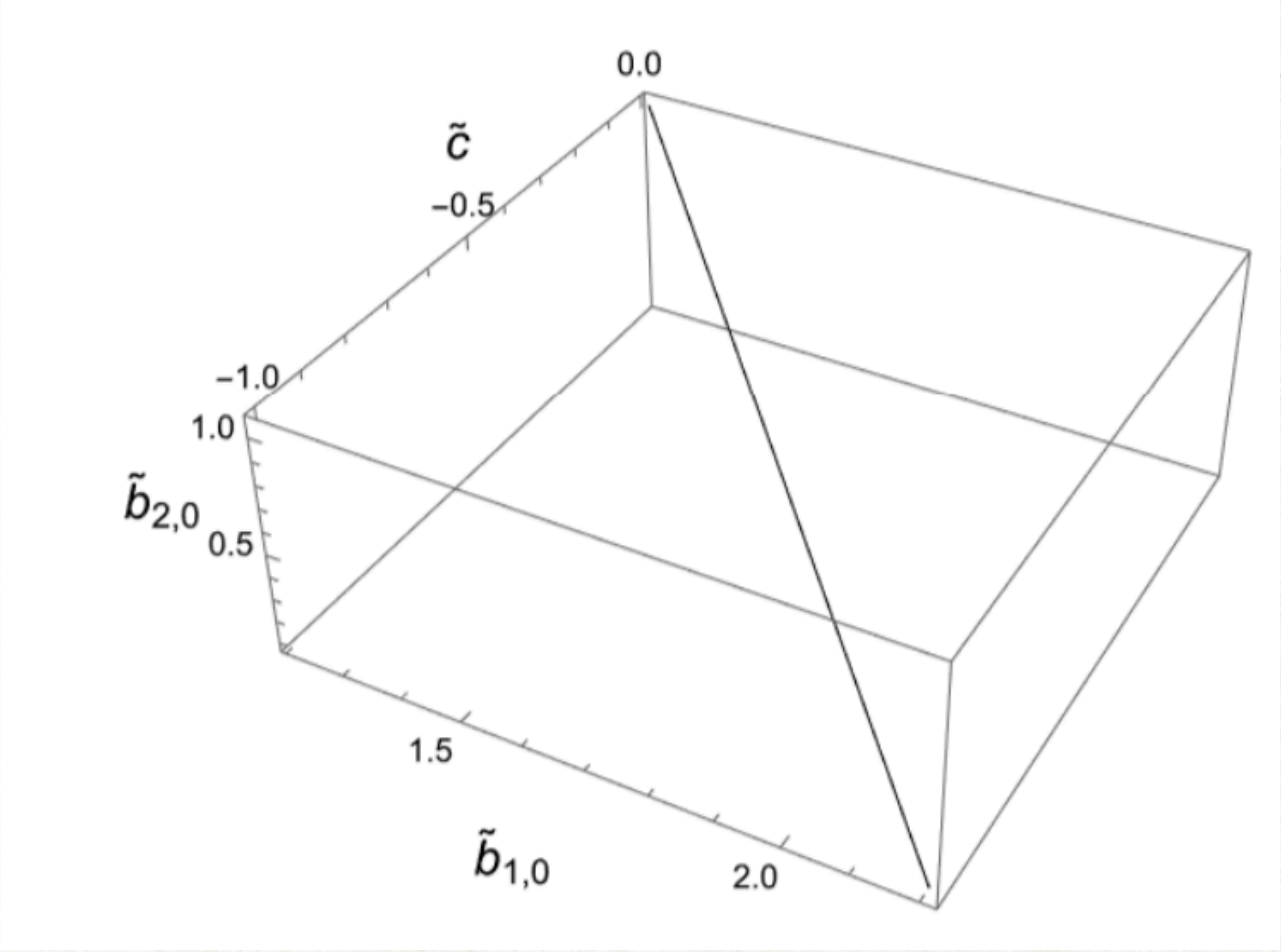
Gluon EFT

Gluon EFT also leads to an island

Wilson coefficients	Two-sided bound	Superstring value	Relative error
$\tilde{b}_{1,0}$	$1.20203 < \tilde{b}_{1,0} < 1.202059$	1.202056	2.4×10^{-5}
$\tilde{b}_{2,0}$	$1.08231 < \tilde{b}_{2,0} < 1.08233$	1.082323	1.8×10^{-5}
$\tilde{b}_{3,2}$	$0.09653 < \tilde{b}_{3,2} < 0.0966$	0.09655	7.2×10^{-4}
$\tilde{c}$	$-1\times10^{-6} < \tilde{c} < 2\times10^{-5}$	0	N/A

=

If we include Tachyons, we get a space spanned by boson and superstring !



Wilson coefficients	Error of bosonic string	Error of Superstring
$\tilde{b}_{1,0}$	(Max) 8.8×10^{-4}	(Min) 1.6×10^{-5}
$\tilde{b}_{2,0}$	(Min) 2.5×10^{-2}	(Max) 9.2×10^{-6}
$\tilde{b}_{3,2}$	(Max) 1.8×10^{-3}	(Min) 1.5×10^{-3}
$\tilde{c}$	(Min) 2×10^{-3}	(Max) N/A

Gravitation EFT

With trivial monodromy, we can easily KLT to obtain closed string EFT

$$M_{\text{closed string}}(s,t) = A(s,t)\sin(\pi s)\,A(s,-s-t) = \frac{-\pi}{st(s+t)} + \sum_{i,j=0}^{\infty} G_{i,j} s^{i-j} t^{i}$$

$$\begin{aligned}
&G_{0,0} = 2\pi g_{1,0}, \quad G_{2,0} = G_{2,1} = G_{2,2} = 2\pi g_{3,0}, \\
&G_{3,1} = G_{3,2} = -\pi\left(\frac{\pi^6}{630} + g_{1,0}^2 - 2g_{4,1}\right), \quad G_{3,0} = G_{3,3} = 0 \\
&G_{4,0} = G_{4,4} = 2\pi g_{5,0}, \quad G_{4,1} = G_{4,4} = 4\pi g_{5,0}, \quad G_{4,2} = 6\pi g_{5,0}
\end{aligned}$$

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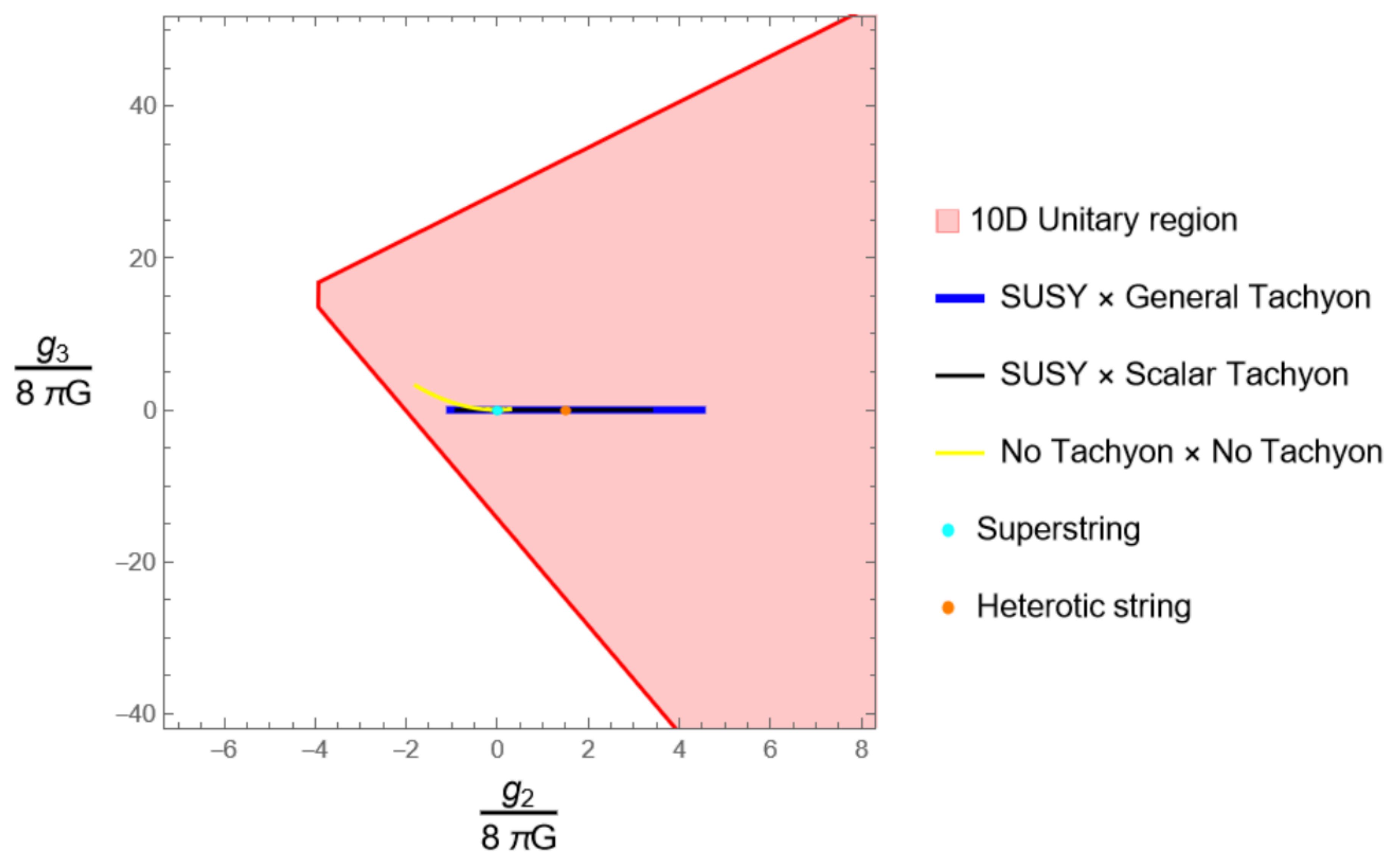
\end{aligned}$$

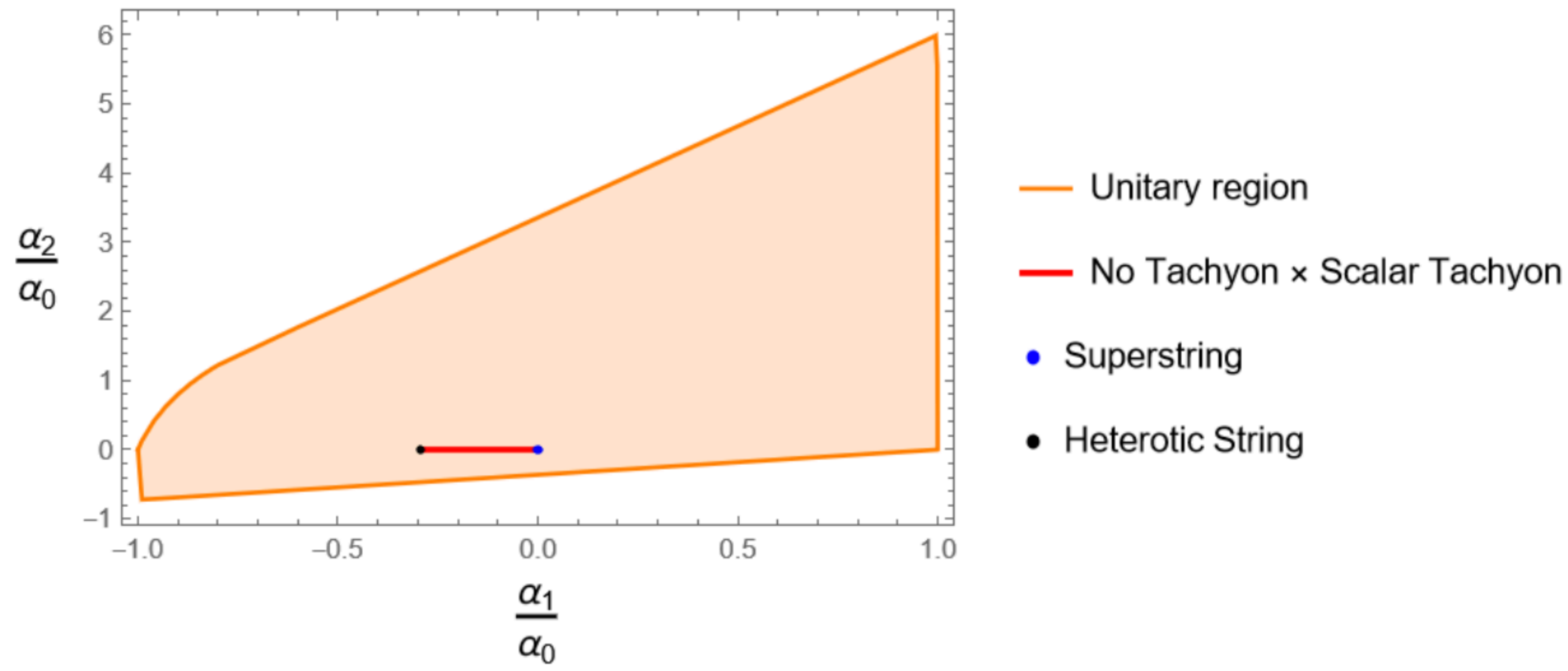
Gravitation EFT

Dilaton EFT



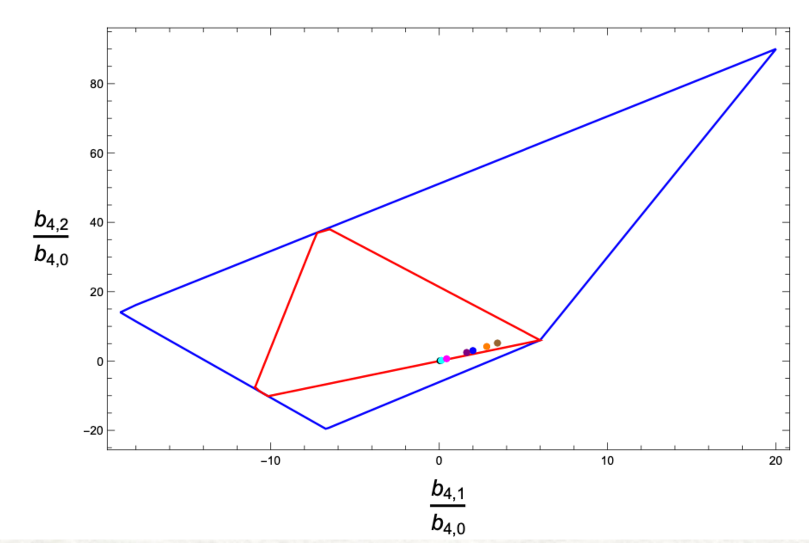
(c) 10D unitary region and the three scenarios of KLT double copy.

Gravitation EFT

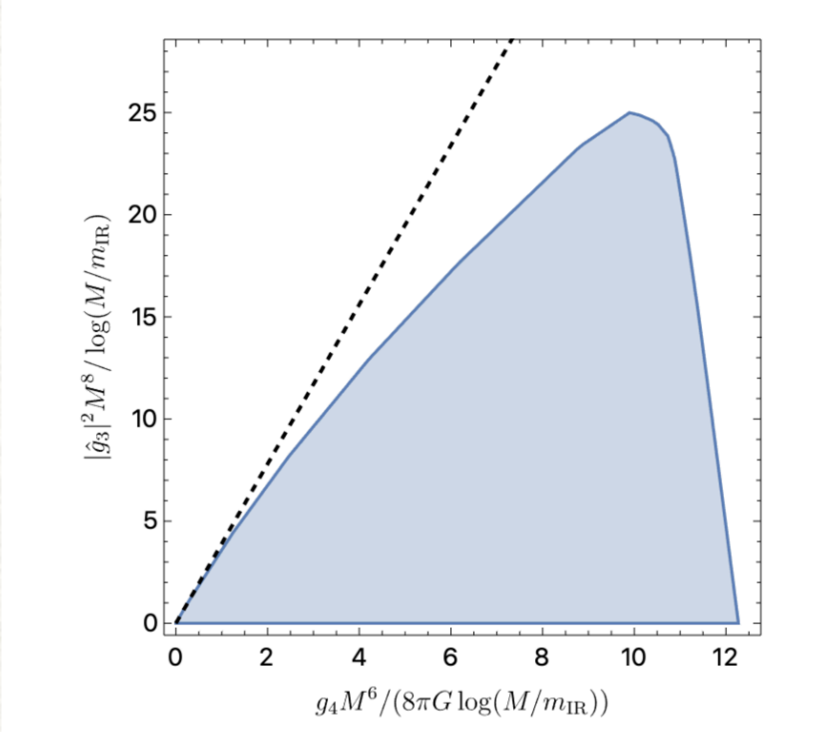


Summary

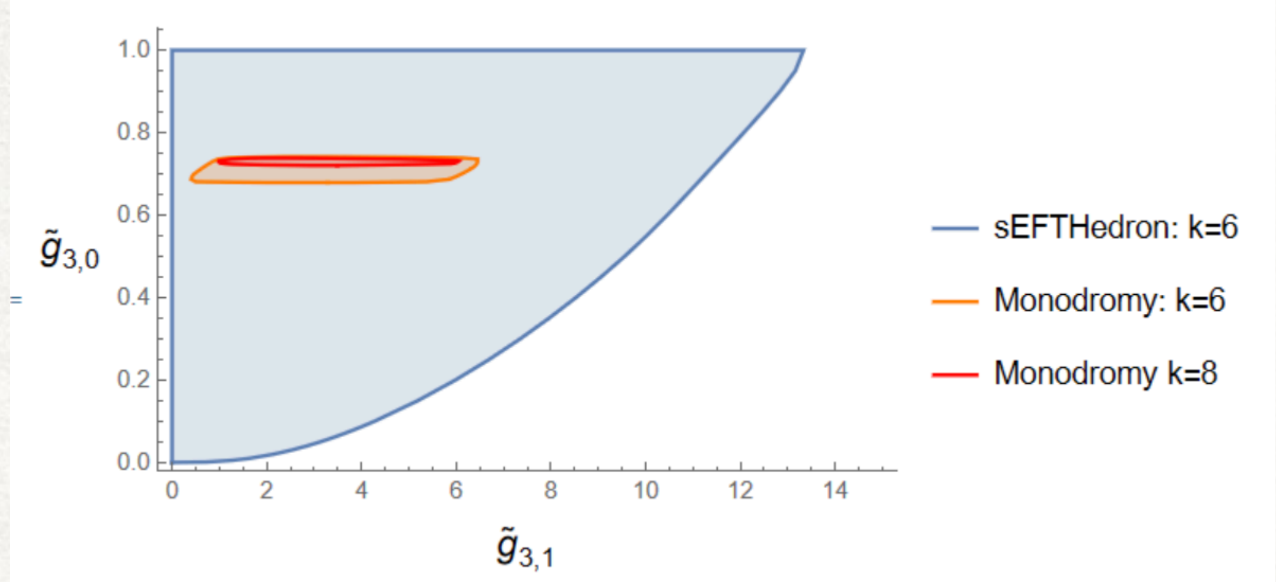
- Imposing unitarity and crossing constrains the space of consistent gravitational EFTs
- Forward limit bounds ratios of EFT coupling



- Finite impact parameters bounds couplings with respect to G_N



- Perturbative string EFTs can be carved out by monodromy relations



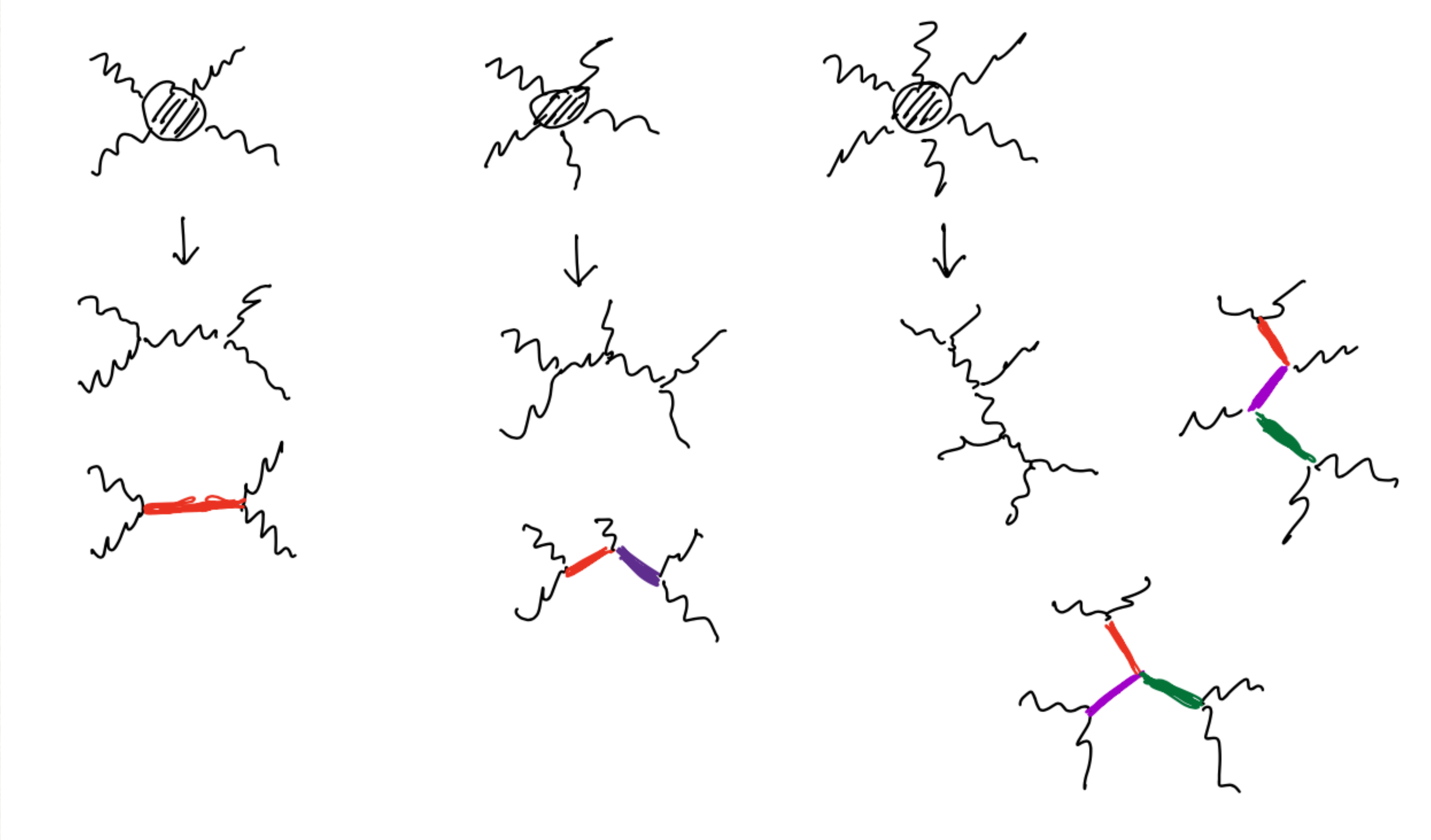
- Are there more constraints one can impose on EFT?

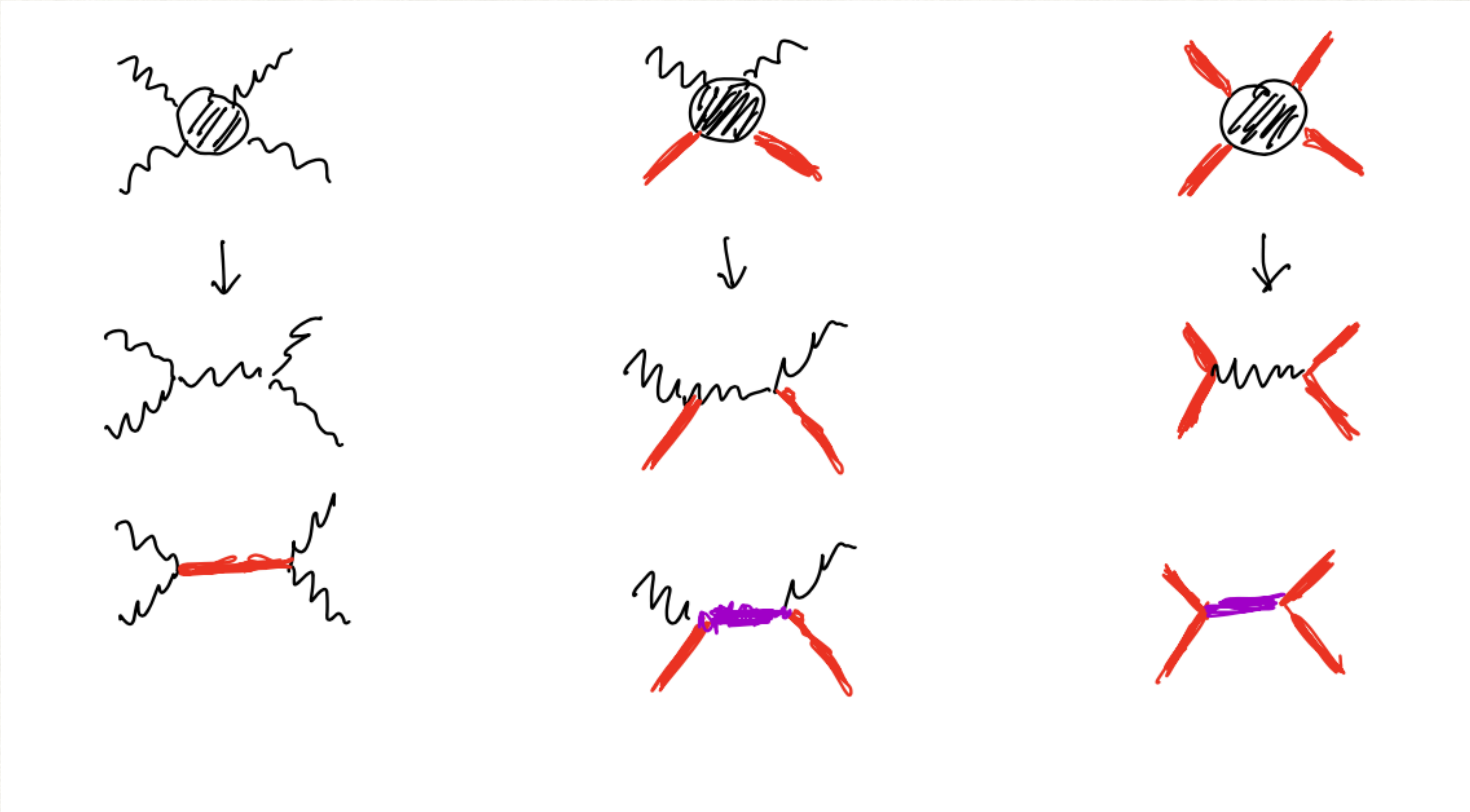
low spin dominance, consistent higher spin scattering.

- Is there a systematic way to explore consistent monodromy relations, enlarging the string EFT ?

Future Directions: mixed amplitudes

Consistent higher point generalizations ⟶ Consistent mixed amplitudes





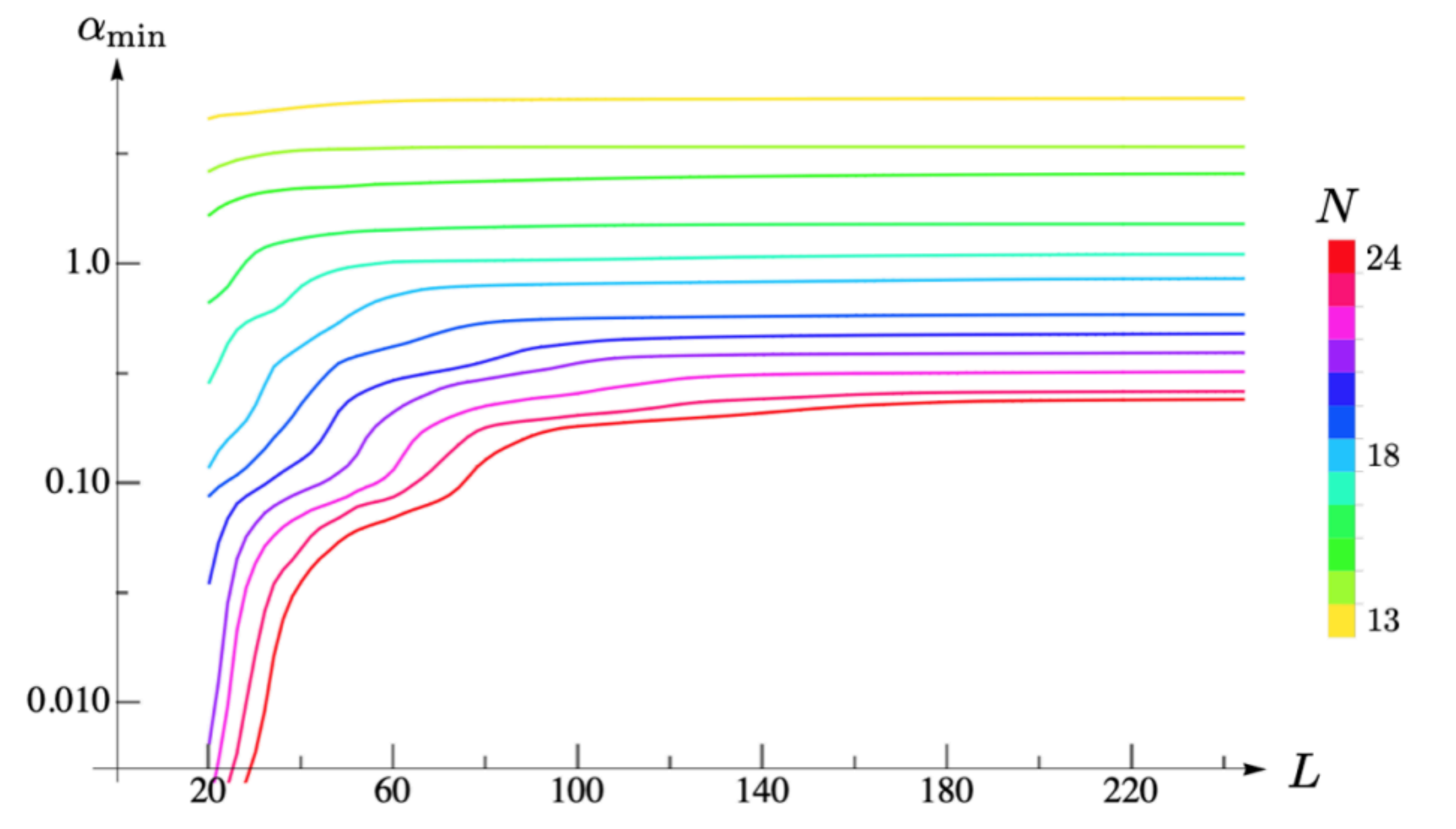
ℓ	2	3	4	5	6	7	...
$h=1, \tilde{\omega}_1^u(\ell)$	-	+	+	+	+	+	...
$h=2, \tilde{\omega}_1^u(\ell)$			-	+	+	+	...
$h=1, \tilde{\omega}_2^u(\ell)$	-	-	+	+	+	+	...
$h=2, \tilde{\omega}_1^u(\ell)$			-	-	+	+	...

Consistent mixed (spin-4) amplitudes

For maximal SUSY in 10D

Andrea Guerrieri, João Penedones, and Pedro Vieira
Phys. Rev. Lett. 127, 081601, 2021

R^4 operator are non-renormalized, their coefficients are well defined



$$\frac{T(s,t,u)}{8\pi G_N = 64\pi^7 \ell_P^8} = s^4\left(\frac{1}{stu} + \alpha \ell_P^6 + O(s)\right)$$

$$\frac{T}{8\pi G_N} = s^4\Bigg(\underbrace{\frac{1}{stu}}_{\text{SUGRA}} + \underbrace{\prod_{A=s,t,u}(\rho_A+1)^2 \sum_{a+b+c\le N}{}' \alpha_{(abc)}\rho_s^a\rho_t^b\rho_u^c}_{\text{UV completion}}\Bigg)$$

$$\rho_s \equiv \frac{\sqrt{s_0}-\sqrt{-s}}{\sqrt{s_0}+\sqrt{-s}}$$

$$\alpha_{\min}^{\text{Boot}} \equiv \lim_{N\to\frac{\infty}{L}\to\infty} \alpha_{\min}(N,L) \approx 0.13 \pm 0.02.$$

$$\alpha^{\text{IIA}} = \frac{\zeta(3)}{32 g_s^{3/2}} + g_s^{1/2}\frac{\pi^2}{96} \ge \frac{\pi^{3/2}(\zeta(3))^{1/4}}{24\sqrt{3}} \approx 0.1403,$$

$$\alpha^{\text{IIB}} = \frac{1}{2^6}E_{\frac{3}{2}}(\tau,\bar\tau) \ge \frac{1}{2^6}E_{\frac{3}{2}}(e^{i\pi/3}, e^{-i\pi/3}) \approx 0.1389,$$

Geometry from unitarity

At fixed-t the open string only s-channel thresholds are present, unitarity implies

$$g^s_{k,q} = \sum_a p_a \frac{v_{\ell_a,q}}{m_a^{2(k+1)}} \qquad G_\ell(1+2\delta) = v_{\ell,0}+v_{\ell,1}\delta+\cdots v_{\ell,\ell}\delta^\ell = \sum_{q=0}^{\ell} v_{\ell,q}\delta^q$$

We have a product geometry

$$\begin{pmatrix} g_{0,0} & & & \\ g_{1,0} & g_{1,1} & & \\ g_{2,0} & g_{2,1} & g_{2,2} & \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \sum_a p_a \begin{pmatrix} \frac{1}{m_a^2} \\ \frac{1}{m_a^4} \\ \frac{1}{m_a^6} \\ \frac{1}{m_a^8} \\ \vdots \end{pmatrix} \otimes (v_{\ell_a,0}, v_{\ell_a,1}, v_{\ell_a,2}, \cdots)$$

Note that v is a simple polynomial in J

$$v_{\ell,q} = \frac{2^q}{q!(2-q)!}\frac{(\alpha)_{\ell+q}}{\prod_{a=1}^{q}(\alpha+2a-1)} = \frac{\prod_{a=0}^{q}(J-a(a-1))}{(q!)^2} \qquad J=\ell(\ell+1)$$

After a linear transformation $\vec{a}^T = \vec{g}^T\mathbf{G}$ we have

at the core, the couplings are governed by the hull of product moments →

$$\begin{pmatrix} a_{0,0} & & & \\ a_{1,0} & a_{1,1} & & \\ a_{2,0} & a_{2,1} & a_{2,2} & \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \sum_a p_a \begin{pmatrix} \frac{1}{m_a^2} \\ \frac{1}{m_a^4} \\ \frac{1}{m_a^6} \\ \frac{1}{m_a^8} \\ \vdots \end{pmatrix} \otimes (1, J, J^2, J^3, \cdots)$$

Li-Yuan Chiang, Wei Li, He-Chen Wen, Laurentiu Rodina, Y-T H 2105.02862

B. Bellazzini, J. Elias Miró, R. Rattazzi, M. Riembau and F. Riva, Phys. Rev. D **104** (2021) no.3, 036006 doi:10.1103/PhysRevD.104.036006 [arXiv:2011.00037 [hep-th]].

Geometry from unitarity

The fact that the couplings are given by the convex hull of double moments imply an infinite number of positivity constraints

$$[\mathbf{Y}]_{d+1\times d+1} = \begin{pmatrix} y^{(0,0)} & y^{(0,1)} & \dots & y^{(0,d)} \\ y^{(1,0)} & y^{(1,1)} & \dots & y^{(1,d)} \\ \vdots & \vdots & \vdots & \vdots \\ y^{(d,0)} & y^{(d,1)} & \dots & y^{(d,d)} \end{pmatrix} = \int_I \rho(x,\tilde{x})\vec{x}\vec{\tilde{x}}^T dx d\tilde{x},$$

$$\rho(x,\tilde{x}) \geq 0 \text{ for } x,\tilde{x} \in I$$

Then the ``generalized Hankel matrix" is a positive definite matrix

$$\mathbf{K}[\mathbf{Y}] \equiv \begin{pmatrix} y^{(0,0)} & y^{(0,1)} & y^{(1,0)} & y^{(0,2)} & \dots \\ y^{(0,1)} & y^{(0,2)} & y^{(1,1)} & y^{(0,3)} & \dots \\ y^{(1,0)} & y^{(1,1)} & y^{(2,0)} & y^{(1,2)} & \dots \\ y^{(0,2)} & y^{(0,3)} & y^{(1,2)} & y^{(0,4)} & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \end{pmatrix} = \int_I \rho(x,\tilde{x})\vec{\mathbf{X}}(\vec{\mathbf{X}})^T\, dx d\tilde{x},$$

where $(\vec{\mathbf{X}})^T = (1, x, \tilde{x}, x^2, \tilde{x}x, \tilde{x}^2, \cdots)$.

Geometry from unitarity

The fact that the couplings are given by the convex hull of double moments imply an infinite number of positivity constraints

$$[\mathbf{Y}]_{d+1\times d+1} = \begin{pmatrix} y^{(0,0)} & y^{(0,1)} & \dots & y^{(0,d)} \\ y^{(1,0)} & y^{(1,1)} & \dots & y^{(1,d)} \\ \vdots & \vdots & \vdots & \vdots \\ y^{(d,0)} & y^{(d,1)} & \dots & y^{(d,d)} \end{pmatrix} = \int_I \rho(x,\tilde{x})\vec{x}\vec{\tilde{x}}^T dx d\tilde{x}\,,$$

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where $(\vec{\mathbf{X}})^T = (1, x, \tilde{x}, x^2, \tilde{x}x, \tilde{x}^2, \cdots)$.