All Gauged Curvature Squared Supergravities in Five Dimensions

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Based on work with Ozkan, Tartaglino-Mazzucchelli and others over the last 10 years

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Contents

Introduction

- Higher curvature terms encode information about new physics.
- In weakly coupled string theory $\Lambda_{\rm new} \ll M_p$

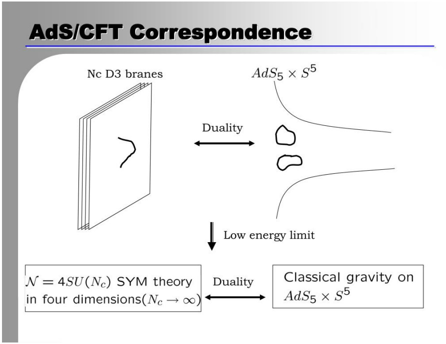
$$
\begin{aligned}
S_g &= M_p^2 \int dx^D \left[R + \frac{1}{\Lambda_{\rm new}^2} R^2 + \frac{1}{\Lambda_{\rm new}^3} R^3 + \cdots \right] \\
&= M_p^2 \int dx^D \left[R + \frac{a_2}{M_p^2} R^2 + \frac{a_3}{M_p^3} R^3 + \cdots \right], \quad a_2, a_3 \propto \left(\frac{M_p}{\Lambda_{\rm new}}\right)^n \gg 1
\end{aligned}
$$

- In AdS/CFT Correspondence,

$$
R \sim 1/L_{AdS}^2, \quad M_p^2 L^2 \sim N^\alpha, \quad a_i \sim f_i(\lambda) \gg 1
$$

Introduction

- Right time to carry out precision tests in holography and resolve certain discrepancy in literature.
- Non-perturbative approaches to compute certain BPS observables in SCFTs beyond the large N limit.
- In the past, $\mathcal{O}(N^0)$ contributions in the gravity side were computed by summing over KK spectrum.



- Curvature squared invariants in 5D, $N = 2$ supergravity, with eight supercharges.

- M-theory on Calabi-Yau/ IIB on or S^5/Z_n.

- The R-symmetry anomaly in the dual CFT is captured by $A \wedge F \wedge F$. The gauge-gravitational mixed anomaly is captured by $A \wedge R \wedge R$ whose supersymmetrization leads to curvature squared terms.

- Supersymmetric counter terms in computing the five-sphere partition functions. The fact that supersymmetric completion of CS terms breaks conformal symmetry signals a new anomaly [Chang, Fluder, Lin, Wang, 17'].

Methods

Challenge

- Noether procedure is too complicated.
- Dimensional reduction does not work: In IIB or M-theory, the leading higher derivative corrections are R^4 whose supersymmetric completion are not known.

Tools

- When $D \leq 6$, $\#$ of Qs ≤ 8, off-shell supersymmetry is available. There exist powerful tools such as superconformal tensor calculus, conformal superspace$\cdots$.

Advantages

- One can add up arbitrarily many off-shell invariant actions without modifying the supersymmetry transformation rules.
- Gauging the R-symmetry becomes relatively easy.

Superconformal formalism

$N=2$, $D=5$ superconformal algebra is $F^2(4)$
[Bergshoeff et al.hep/th0104113]

Generators and Gauge fields

Translation	P_a	$e_\mu{}^a$
Rotation	M_{ab}	$\omega_\mu{}^{ab}$
Dilatation	b_μ	D
Conformal boosts	K_a	f_a^μ
SU(2) R-symmetry	Λ^{ij}	$V_\mu{}^{ij}$
Local-susy	Q^i	ψ_μ^i
Special-local-susy	S^i	ϕ_μ^i

$i,j=1,2$ label the indices of fundamental irreps of SU(2)

- $\chi^i_\alpha = \varepsilon^{ij}\varepsilon_{\alpha\beta}\bar{\chi}^\beta$, symplectic Majorana $\Rightarrow$ 8 independent real components

Weyl multiplets

- Impose constraints and add matter contents so that at the off-shell level

$$\# \quad \text{of bosons} = \# \quad \text{of fermions}$$

- Standard Weyl multiplet

$$e_\mu{}^a(10) \quad b_\mu(-1) \quad \psi^i_\mu(24) \quad V_\mu{}^{ij}(12) \quad T_{\mu\nu}(10) \quad D(1) \quad \chi^i(8)$$

- Dilaton Weyl multiplet

$$e_\mu{}^a \quad b_\mu \quad \psi^i_\mu \quad V_\mu{}^{ij} \quad B_{\mu\nu}(6) \quad C_\mu(4) \quad \sigma(1) \quad \psi^i(8)$$

$T_{\mu\nu}$, D, χ^i turn into composite fields and enter the susy transformations.

Weyl multiplets

$$
\begin{aligned}
T_{\mu\nu} &= \frac{1}{8}\sigma^{-1}G_{\mu\nu} + \frac{1}{48}\sigma^{-2}\varepsilon_{\mu\nu\rho\sigma\lambda}H^{\rho\sigma\lambda} + \ldots \\
D &= -\frac{1}{32}R + \frac{1}{8}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{16}\sigma^{-2}G_{\mu\nu}G^{\mu\nu} + \ldots \\
\chi^i &= \frac{1}{8}i\sigma^{-1}\not{D}\psi^i + \frac{1}{16}i\sigma^{-2}\not{D}\sigma\psi^i - \frac{1}{32}\sigma^{-2}\gamma\cdot G\psi^i + \cdots
\end{aligned}
$$

where $G_{\mu\nu} = 2\partial_{[\mu}C_{\nu]}$

$$H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + \tfrac{3}{2}C_{[\mu}G_{\nu\rho]}$$

Off-shell Matter multiplets

- Vector multiplet

Field	Weyl weight
A_μ	0
ρ	1
Y^{ij}	2
λ^i	$\frac{3}{2}$

- Linear multiplet

Field	Weyl weight
L_{ij}	3
φ^i	$\frac{7}{2}$
E_a	4
N	4

The closure of susy algebra requires that

$$\mathscr{D}^a E_a = 0 \Rightarrow E^a = -\frac{1}{12} e_\mu{}^a e^{-1} \varepsilon^{\mu\nu\rho\sigma\lambda} \mathscr{D}_\nu E_{\rho\sigma\lambda}$$

Steps

- Construct an action invariant under local superconformal symmetry with certain compensating matter multiplets.
- Fix the redundant local symmetry
  $\Rightarrow$ Ordinary Poincaré supergravity.
- Eliminate auxiliary fields and obtain on-shell supergravity models. (The same on-shell supergravity admits different off-shell formulations.)

A 2-derivative example

(un)gauged minimal supergravity in $D = 5$

conformal scalar action

$$\mathscr{L} = -\frac{1}{2}\sqrt{g}\phi\Box^c\phi = -\frac{1}{2}\sqrt{g}\phi\Box\phi + \frac{1}{12}\sqrt{g}R\phi^2$$

Using dilatation symmetry

$$\phi \to e^{\omega}\phi, \qquad g_{\mu\nu} \Rightarrow e^{2\omega}g_{\mu\nu}$$

one fixes the dilatation symmetry by choosing

$$\phi = \sqrt{6}/\kappa \Rightarrow \mathscr{L} = \frac{1}{2\kappa^2}\sqrt{g}R$$

A 2-derivative example

Coupling Dilaton Weyl multiplet

$$(e_\mu{}^a \quad b_\mu \quad \psi^i_\mu \quad V_\mu{}^{ij} \quad B_{\mu\nu} \quad C_\mu \quad \sigma \quad \psi^i)$$

to a linear multiplet

$$(L_{ij} \quad \varphi^i \quad E_{\mu\nu\rho} \quad N)$$

leads to the superconformal invariant action

$$\begin{aligned} e^{-1}\mathscr{L}_L = {} & L^{-1}L_{ij}\Box^c L^{ij} - L^{ij}\mathscr{D}_\mu L_{k(i}\mathscr{D}^\mu L_{j)m}L^{km}L^{-3} - N^2L^{-1} \\ & -E_\mu E^\mu L^{-1} + \tfrac{8}{3}LT^2 + 4DL - \tfrac{1}{2}L^{-3}E^{\mu\nu}L^l_k\partial_\mu L^{kp}\partial_\nu L_{pl} \\ & +2E^{\mu\nu}\partial_\mu(L^{-1}E_\nu + V^{ij}_\nu L_{ij}L^{-1}) + \text{fermions} \end{aligned}$$

The Einstein Hilbert term comes from

$$\begin{aligned} & L_{ij}\Box^c L^{ij} = L_{ij}\Box L^{ij} - \tfrac{3}{8}L^2R + \cdots \\ & D = -\tfrac{1}{32}R + \cdots \end{aligned}$$

We choose the gauge

- $L_{ij} = \frac{1}{\sqrt{2}}\delta_{ij}L$, breaks SU(2) to U(1), $V_\mu^{ij} = V_\mu^{\prime ij} + \frac{1}{2}\delta^{ij}V_\mu$
- $\sigma = 1$, fixes dilatation
- $\psi^i = 0$, fixes special supersymmetry transformations
- $b_\mu = 0$, fixes conformal boosts

To maintain the gauge, the needed compensating transformations are

$$\lambda^{ij} = -\frac{1}{\sqrt{2}L}\Big(S^{k(i}\delta^{j)l}\varepsilon_{kl}\Big), \quad S^{ij} = \bar{\varepsilon}^{(i}\varphi^{j)} - \frac{1}{2}\delta^{ij}\bar{\varepsilon}^k\varphi^l\delta_{kl},$$
$$\eta^i = \Big(-\gamma\cdot T + \tfrac{1}{4}\gamma\cdot\widehat{G}\Big)\varepsilon^i, \quad \Lambda_{K\mu} = -\tfrac{1}{4}\mathrm{i}\bar{\varepsilon}\phi_\mu - \tfrac{1}{4}\mathrm{i}\bar{\eta}\psi_\mu + \bar{\varepsilon}\gamma_\mu\chi$$

where $\hat{G}$ is the supercovariant field strength of C_μ.

$$\begin{aligned} e^{-1}\mathscr{L}_{LR} = {} & \tfrac{1}{2}LR + \tfrac{1}{2}L^{-1}\partial_\mu L\partial^\mu L - \tfrac{1}{4}LG_{\mu\nu}G^{\mu\nu} - \tfrac{1}{6}LH_{\mu\nu\rho}H^{\mu\nu\rho} - L^{-1}N^2 \\ & + \tfrac{1}{6}L^{-1}\partial_{[\mu}E_{\nu\rho\sigma]}\partial^\mu E^{\nu\rho\sigma} + \frac{1}{6\sqrt{2}}\varepsilon^{\mu\nu\rho\sigma\lambda}V_\mu\partial_\nu E_{\rho\sigma\lambda} + LV_\mu^{\prime ij}V_{ij}^{\prime\mu} \\ & + \text{fermions} \end{aligned}$$

Dualizing the two form to a one form

$$B_{\mu\nu} \to \widetilde{C}_{\mu}$$

results in a Lagrangian which is an off-shell formulation of the ungauged $D=5, N=2$ Einstein-Maxwell supergravity. Eliminate the auxiliary fields

$$N=0, \quad V'^{ij}_{\mu}=0, \quad V_{\mu}=0, \quad E_{\mu\nu\rho}=0$$

and perform consistent truncation by setting

$$L=1, \qquad \widetilde{C}_{\mu}=C_{\mu}$$

we obtain the ungauged minimal supergravity in 5D

$$e^{-1}\mathscr{L}^{\rm min}_{\rm EM}=\frac{1}{2}R-\frac{3}{8}G_{\mu\nu}G^{\mu\nu}+\frac{1}{8}\varepsilon^{\mu\nu\rho\sigma\lambda}C_{\mu}G_{\nu\rho}G_{\sigma\lambda}\,.$$

Since $V_{\mu}=0$, U(1) R-symmetry is ungauged on-shell.

In order to obtain the gauged supergravity, one needs to couple the supergravity multiplet with an Abelian vector multiplet

$$(A_\mu, \rho, Y^{ij}, \lambda^i)$$

via adding the vector-linear action

$$\mathscr{L}_{\rm gEM} = \mathscr{L}_L + g\mathscr{L}_{VL}$$

where g is the U(1) coupling

$$\begin{aligned} e^{-1}\mathscr{L}_{VL} &= Y^{ij}L_{ij} + i\bar{\lambda}\varphi - \tfrac{1}{2}\bar{\psi}^i_\mu\gamma^\mu\lambda^j L_{ij} - \tfrac{1}{12}\varepsilon^{\mu\nu\rho\sigma\lambda}A_\mu\partial_\nu E_{\rho\sigma\lambda} \\ &\quad +\rho(N + \tfrac{1}{2}\bar{\psi}_\mu\gamma^\mu\varphi + \tfrac{1}{4}i\bar{\psi}^i_\mu\gamma^{\mu\nu}\psi^j_\nu L_{ij}). \end{aligned}$$

Gauging also deforms the composite map. This fact was omitted by [J. Liu, Saskowski, 22']

$$
\begin{aligned}
T_{ab} &= \tfrac{1}{8}\sigma^{-1}G_{ab} + \tfrac{1}{48}\sigma^{-2}\varepsilon_{abcde}H^{cde} + \text{f.t.}\,, \\
D &= \tfrac{1}{4}\sigma^{-1}\nabla^a\nabla_a\sigma + \tfrac{1}{8}\sigma^{-2}(\nabla^a\sigma)\nabla_a\sigma - \tfrac{1}{32}R \\
&\quad -\tfrac{1}{16}\sigma^{-2}G^{ab}G_{ab} - (\tfrac{26}{3}T^{ab} - 2\sigma^{-1}G^{ab})T_{ab} \\
&\quad +\tfrac{g}{4}\sigma^{-2}N + \tfrac{g^2}{16}\sigma^{-4}L^2 + \text{f.t.}\,, \\
\chi^i &= \tfrac{1}{8}i\sigma^{-1}\not{D}\psi^i + \tfrac{1}{16}i\sigma^{-2}\not{D}\sigma\psi^i - \frac{1}{32}\sigma^{-2}\gamma\cdot G\psi^i \\
&\quad +\tfrac{1}{8}g\sigma^{-2}\varphi^i + \cdots
\end{aligned}
$$

in which

$$
H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}C_{[\mu}G_{\nu\rho]} + \frac{1}{2}gE_{\mu\nu\rho}
$$

Imposing the same gauge fixing as before

$$
\begin{aligned}
e^{-1}\mathscr{L} = & L(R - \tfrac{1}{2}G_{\mu\nu}G^{\mu\nu} - \tfrac{1}{3}H_{\mu\nu\rho}H^{\mu\nu\rho} + 2V_\mu^{\prime ij}V_{ij}^{\prime\mu}) \\
& + L^{-1}\partial_\mu L\partial^\mu L + \tfrac{1}{3}L^{-1}\partial_{[\mu}E_{\nu\rho\sigma]}\partial^\mu E^{\nu\rho\sigma} - 2\sqrt{2}E_\mu V^\mu \\
& - 2N^2L^{-1} - 4gC_\mu E^\mu - 2gNL - 4gN - \tfrac{1}{2}g^2L^3 + 2g^2L^2 \ .
\end{aligned}
$$

Dualizing the two form into a one form

$$
B_{\mu\nu} \to \widetilde{C}_\mu
$$

Eliminate the auxiliary fields

$$
N = -\tfrac{1}{2}gL(L+2), \quad V_\mu^{\prime ij} = 0, \quad V_\mu = -\tfrac{3}{\sqrt{2}}C_\mu, \quad E_{\mu\nu\rho} = 0 \ ,
$$

and perform consistent truncation by setting

$$
L = 1, \qquad \widetilde{C}_\mu = C_\mu \ ,
$$

After $C_\mu \to \frac{1}{\sqrt{3}}C_\mu$, $g \to \sqrt{2}g$, we get

$$
e^{-1}\mathscr{L}_{EM} = R + 12g^2 - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \tfrac{1}{12\sqrt{3}}\varepsilon^{\mu\nu\rho\sigma\lambda}C_\mu G_{\nu\rho}G_{\sigma\lambda} \ .
$$

Composite linear multiplet from vector and dilaton Weyl multiplet [Ozkan, Pang 13']

$$
\begin{aligned}
L^{ij} &= \tfrac{1}{4}\mathrm{i}\bar{\widehat{R}}^{(i}_{ab}(Q)\widehat{R}^{j)ab}(Q) + \tfrac{256}{3}\mathrm{i}\bar{\chi}^{(i}\chi^{j)} + \tfrac{16}{3}\widehat{R}_{ab}{}^{ij}(V)T^{ab} \\
\varphi^{i} &= -\tfrac{1}{8}\gamma_{cd}\widehat{R}^{i}_{ab}(Q)\widehat{R}^{abcd}(M) - 4\mathrm{i}\gamma_{c}\widehat{R}^{i}_{ab}(Q)\mathscr{D}^{a}T^{bc} + \tfrac{128}{3}\chi^{i}D \\
&\quad +8\mathrm{i}\gamma_{c}\mathscr{D}^{c}\widehat{R}^{i}_{ab}(Q)T^{ab} + 8\mathrm{i}\gamma_{a}\mathscr{D}^{c}\widehat{R}^{i}_{bc}(Q)T^{ab} - \tfrac{64}{3}\mathrm{i}\gamma^{ab}\gamma^{c}\mathscr{D}^{a}T_{bc}\chi^{i} \\
&\quad +\tfrac{1024}{9}T^{2}\chi^{i} + \tfrac{128}{3}\mathrm{i}\gamma_{a}\mathscr{D}_{b}\chi^{i}T^{ab} + \tfrac{16}{3}\gamma_{ab}\widehat{R}^{i}_{cd}(Q)T^{ab}T^{cd} \\
&\quad +\tfrac{1}{2}\widehat{R}^{ab\,i}{}_{j}(V)\widehat{R}^{j}_{ab}(Q) - \tfrac{8}{3}\widehat{R}^{ab\,i}{}_{j}(V)\gamma_{ab}\chi^{j}, \\
E_{a} &= \tfrac{1}{16}\varepsilon_{abcde}C^{bcfg}C^{de}{}_{fg} - \tfrac{1}{12}\varepsilon_{abcde}V^{bc}{}_{ij}V^{de\,ij} \\
&\quad +\mathscr{D}^{b}\Big(4C_{abcd}T^{cd} - \tfrac{64}{3}D\,T_{ab} - \tfrac{128}{9}T_{ab}T^{2} - \tfrac{512}{3}T_{ac}T^{cd}T_{bd}\Big) \\
&\quad -32\varepsilon_{abcde}\mathscr{D}^{b}\Big(\tfrac{2}{3}T^{cf}\mathscr{D}_{f}T^{de} + T^{c}{}_{f}\mathscr{D}^{d}T^{ef}\Big), \\
N &= \tfrac{1}{8}C^{abcd}C_{abcd} + \tfrac{64}{3}D^{2} + \tfrac{1024}{9}T^{2}D - \tfrac{16}{3}C_{abcd}T^{ab}T^{cd} - \tfrac{1}{3}V_{ab}{}^{ij}V^{ab}{}_{ij} \\
&\quad -\tfrac{64}{3}\mathscr{D}_{a}T_{bc}\mathscr{D}^{a}T^{bc} + \tfrac{64}{3}\mathscr{D}_{b}T_{ac}\mathscr{D}^{a}T^{bc} - \tfrac{128}{3}T_{ab}\mathscr{D}^{b}\mathscr{D}_{c}T^{ac}
\end{aligned}
$$

- To obtain the results above, we write down ansatz for composite L^{ij} which is SU(2)-triplet and has conformal weight 3.

$$
\begin{aligned}
\delta L^{ij} &= i\bar{\varepsilon}^{(i}\varphi^{j)} + 0\eta\,, \\
\delta\varphi^{i} &= -\tfrac{1}{2}i\not{\mathscr{D}}L^{ij}\varepsilon_j - \tfrac{1}{2}i\gamma^{a}E_a\varepsilon^{i} + \tfrac{1}{2}N\varepsilon^{i} - \gamma\cdot TL^{ij}\varepsilon_j + 3L^{ij}\eta_j\,, \\
\delta E_a &= -\tfrac{1}{2}i\bar{\varepsilon}\gamma_{ab}\mathscr{D}^{b}\varphi - 2\bar{\varepsilon}\gamma^{b}\varphi T_{ba} - 2\bar{\eta}\gamma_a\varphi\,, \\
\delta N &= \tfrac{1}{2}\bar{\varepsilon}\not{\mathscr{D}}\varphi + \tfrac{3}{2}i\bar{\varepsilon}\gamma\cdot T\varphi + 4i\bar{\varepsilon}^{i}\chi^{j}L_{ij} + \tfrac{3}{2}i\bar{\eta}\varphi\,,
\end{aligned}
$$

- Fix L^{ij} using its invariance under special supersymmetry parametrized by η. Then the Q-supersymmetry transformation yields other composite fields in the linear multiplet.

Substituting the composite linear multiplet into the superconformal vector-linear action

$$e^{-1}\mathscr{L}_{VL} = Y^{ij}L_{ij} + i\bar{\lambda}\varphi - \tfrac{1}{2}\bar{\psi}^i_a\gamma^a\lambda^j L_{ij} - \tfrac{1}{12}\varepsilon^{\mu\nu\rho\sigma\lambda}A_\mu\partial_\nu E_{\rho\sigma\lambda}$$
$$+\rho(N + \tfrac{1}{2}\bar{\psi}_a\gamma^a\varphi + \tfrac{1}{4}i\bar{\psi}^i_a\gamma^{ab}\psi^j_b L_{ij}).$$

where the vector multiplet is made from Dilaton Weyl multiplet

$$(\rho\,, A_\mu\,, \lambda^i\,, T^{ij}) \to (\sigma\,, C_\mu\,, \psi^i\,, \frac{1}{4}\mathrm{i}\sigma^{-1}\bar{\psi}^i\psi^j - \frac{g}{2}\sigma^{-1}L^{ij})$$

Applying the same gauge fixing, we obtain the off-shell super-Poincaré invariant Weyl squared action

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{Weyl}^2} = & -\frac{1}{8}\epsilon^{abcde}C_a R_{bcfg}R_{de}{}^{fg} + \frac{1}{6}\epsilon^{abcde}C_a V_{bc}{}^{ij}V_{deij} + \frac{2}{3}V^{abij}V_{abij} \\
& -\frac{1}{4}R_{abcd}R^{abcd} + \frac{1}{3}R_{ab}R^{ab} - \frac{1}{12}R^2 + \frac{1}{3}R_{abcd}(G^{ab}G^{cd} - 2H^{ab}H^{cd} - 3H^{ab}G^{cd}) \\
& -\frac{4}{3}R_{ab}H^{ac}G^{b}{}_{c} + \frac{16}{3}R^{ab}H^2_{ab} + \frac{1}{3}RH_{ab}G^{ab} - \frac{4}{3}RH^2 - 4(H^2)^2 - 8H^4 \\
& -\frac{16}{3}H^2H_{cd}G^{cd} - \frac{40}{3}H^2_{ad}H^a{}_cG^{cd} + \frac{8}{3}H^2G^2 + \frac{2}{3}H_{ab}H_{cd}G^{ab}G^{cd} + \frac{1}{12}(G^2)^2 \\
& -\frac{16}{3}H^2_{ab}G^{2ab} - \frac{4}{3}H_{ab}H_{cd}G^{ac}G^{bd} - \frac{1}{3}H_{ab}G^{ab}G^2 + 2G^{2ab}G^c{}_bH_{ca} \\
& -\frac{1}{3}(\nabla^aG_{bc})\nabla_aG^{bc} + \frac{8}{3}(\nabla^aH_{bc})\nabla_aH^{bc} - \frac{1}{2}G^4 + \frac{4}{3}\epsilon^{abcde}H_{ab}H_{cd}\nabla^fG_{ef} \\
& -2\epsilon^{abcde}H_{bf}(\nabla_aH_c{}^f)G_{de} - \frac{2}{3}\epsilon^{abcde}H_{ab}(\nabla^fG_{cf})G_{de} - \frac{1}{24}\epsilon^{abcde}(\nabla^fG_{af})G_{bc}G_{de} \\
& -g\left(-\frac{4}{3}NH_{ab}G^{ab} + \frac{4}{3}NG^2 - 8NH^2 + \frac{4}{3}V_{ab}{}^{ij}L_{ij}H^{ab} - \frac{2}{3}V_{ab}{}^{ij}L_{ij}G^{ab} - \frac{2}{3}RN\right) \\
& +g^2\left(\frac{1}{3}L^2H^{ab}G_{ab} - \frac{1}{3}L^2G^2 + 2L^2H^2 - \frac{8}{3}N^2 + \frac{1}{6}RL^2\right) \\
& -\frac{4}{3}g^3NL^2 - \frac{1}{6}g^4L^4\ ,
\end{aligned}
$$

Supersymmetric Weyl squared action based on the standard Weyl multiplet was obtained in [Hanaki, Ohashi, Tachikawa, 06’] .

Supersymmetric Ricci scalar squared

Supergravity coupled to n Abelian vector multiplets[Ozkan, Pang 13']

$$e^{-1}\mathscr{L}_V^S = C_{IJK}\Big(-\tfrac{1}{4}\rho^I F^J_{\mu\nu}F^{K\,\mu\nu} + \tfrac{1}{3}\rho^I\rho^J\Box^C\rho^K + \tfrac{1}{6}\rho^I\mathscr{D}_\mu\rho^J\mathscr{D}^\mu\rho^K + \rho^I Y^{J\,ij}Y^K_{ij} - \tfrac{4}{3}\rho^I\rho^J\rho^K\big(D+\tfrac{26}{3}T_{\mu\nu}T^{\mu\nu}\big) + 4\rho^I\rho^J F^K_{\mu\nu}T^{\mu\nu} - \tfrac{1}{24}\varepsilon^{\mu\nu\rho\sigma\lambda}A^I_\mu F^J_{\nu\rho}F^K_{\sigma\lambda}\Big)$$

Consider n=2 case with $C_{122}=1$ (otherwise 0), replacing one vector multiplet by

$$(\rho^{(1)}, A^{(1)}_\mu, \lambda^{(1)i}, Y^{(1)ij}) \rightarrow (\sigma, C_\mu, \psi^i, \frac{1}{4}\mathrm{i}\sigma^{-1}\bar{\psi}^i\psi^j - \frac{g}{2}\sigma^{-1}L^{ij})$$

and the other vector multiplet by

$$
\begin{aligned}
\rho^{(2)} &= 2L^{-1}N + \mathrm{i}L^{-3}\bar{\varphi}^i\varphi^j L_{ij}\,, \\
\lambda_i^{(2)} &= -2\mathrm{i}\not{\mathscr{D}}\varphi_i L^{-1} + (16L_{ij}\chi^j + 4\gamma\cdot T\varphi_i)L^{-1} - 2NL_{ij}\varphi^j L^{-3} \\
&\quad +2\mathrm{i}(\not{\mathscr{D}}L_{ij}L^{jk}\varphi_k - \not{E}L_{ij}\varphi^j)L^{-3} + 2\mathrm{i}\varphi^j\bar{\varphi}_i\varphi_j L^{-3} - 6\mathrm{i}\varphi^j\bar{\varphi}_k\varphi_l L^{kl}L_{ij}L^{-5}, \\
Y_{ij}^{(2)} &= L^{-1}\Box^C L_{ij} - \mathscr{D}_a L_{k(i}\mathscr{D}^a L_{j)m}L^{km}L^{-3} - N^2 L_{ij}L^{-3} - E_\mu E^\mu L_{ij}L^{-3} \\
&\quad +\tfrac{8}{3}L^{-1}T^2 L_{ij} + 4L^{-1}DL_{ij} + 2E_\mu L_{k(i}\mathscr{D}^\mu L_{j)}{}^k L^{-3} - \tfrac{1}{2}\mathrm{i}NL^{-3}\bar{\varphi}_{(i}\varphi_{j)} \\
&\quad -\tfrac{4}{3}L^{-5}NL_{k(i}L_{j)m}\bar{\varphi}^k\varphi^m - \tfrac{2}{3}L^{-3}\bar{\varphi}_{(i}\not{E}\varphi_{j)} - \tfrac{1}{3}L^{-5}L_{k(i}L_{j)m}\bar{\varphi}^k\not{E}\varphi^m \\
&\quad -8\mathrm{i}L^{-1}\bar{\chi}_{(i}\varphi_{j)} + 16\mathrm{i}L^{-3}L_{k(i}L_{j)m}\bar{\chi}^k\varphi^m + 2L^{-3}L_{k(i}\bar{\varphi}^k\not{\mathscr{D}}\varphi_{j)} \\
&\quad +2\mathrm{i}L^{-3}L_{ij}\bar{\varphi}\gamma\cdot T\varphi - \tfrac{2}{3}L^{-3}\bar{\varphi}_{(i}\not{\mathscr{D}}L_{j)k}\varphi^k - L^{-5}L_{mn}L^k{}_{(i}\bar{\varphi}^m\not{\mathscr{D}}L_{j)k}\varphi^n \\
&\quad -\tfrac{1}{6}L^{-5}L_{km}\bar{\varphi}_i\gamma^a\varphi_j\bar{\varphi}^k\gamma_a\varphi^m + \tfrac{1}{12}L^{-7}L_{ij}L_{km}L^{pq}\bar{\varphi}^k\gamma_a\varphi^m\bar{\varphi}_p\gamma^a\varphi_q, \\
G_{\mu\nu}^{(2)} &= 4\mathscr{D}_{[\mu}(L^{-1}E_{\nu]}) + 2L^{-1}\widehat{R}_{\mu\nu}{}^{ij}(V)L_{ij} - 2L^{-3}L^l_k\mathscr{D}_{[\mu}L^{kp}\mathscr{D}_{\nu]}L_{lp} \\
&\quad -2\mathscr{D}_{[\mu}(L^{-3}\bar{\varphi}^i\gamma_{\nu]}\varphi^j L_{ij}) - \mathrm{i}L^{-1}\bar{\varphi}\widehat{R}_{\mu\nu}(Q)\,.
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathscr{L}_{R^2} =\ & \mathbf{Y}^{ij}\mathbf{Y}_{ij} - 2\nabla^a(NL^{-1})\nabla_a(NL^{-1}) \\
& -\frac{1}{8}\varepsilon_{abcde}C^a\mathbf{G}^{bc}\mathbf{G}^{de} + NL^{-1}G^{ab}\mathbf{G}_{ab} - N^2L^{-2}G^{ab}G_{ab} \\
& +4N^2L^{-2}H^{ab}G_{ab} - \frac{1}{4}\mathbf{G}^{ab}\mathbf{G}_{ab} - 4NL^{-1}H_{ab}\mathbf{G}^{ab} \\
& +g^2\Big(\frac{1}{4}L^{ij}\nabla^a\nabla_aL_{ij} - \frac{1}{4}RL^2 - H^2L^2 + \frac{1}{8}G^2L^2 - \frac{5}{2}N^2 - \frac{1}{2}E^aE_a \\
& -\frac{1}{2}\nabla^aL\nabla_aL\Big) - 4gN^3L^{-2} + \frac{1}{16}g^4L^4\ ,
\end{aligned}
$$

where,

$$
\begin{aligned}
\mathbf{G}_{ab} =\ & 4\nabla_{[a}(L^{-1}E_{b]}) + 2L^{-1}L_{ij}(V_{ab}{}^{ij}) - 2L^{-3}L_{ij}(\nabla_{[a}L^{ik})\nabla_{b]}L_k{}^j\ , \\
\mathbf{Y}^{ij} =\ & \frac{1}{4}L^{-1}\left(4\nabla^a\nabla_aL^{ij} - 2RL^{ij} - 8H^2L^{ij} + G^2L^{ij}\right) \\
& +L^{-3}\left(-N^2L^{ij} - E^aE_aL^{ij} - 2E^aL^{k(i}\nabla_aL_k{}^{j)} - L_{kl}\nabla^aL^{k(i}\nabla_aL^{j)l}\right)\ .
\end{aligned}
$$

Supersymmetric Ricci tensor squared

Inspired by 4D, $N = 2$ result, build a composite linear multiplet from Weyl multiplet and Abelian vector multiplet

$$H^{ij} = \frac{3\mathrm{i}}{1280} \nabla^{(ij}\nabla^{kj)}\nabla_{kl} \log W$$

where W is the real primary superfield describing an Abelian vector multiplet and $\nabla^{ij} = \nabla^{\alpha(i}\nabla_{\alpha}^{j)}$. The bosonic components of H^{ij} are

$$\begin{aligned} L^{ij} &= \tfrac{8}{3} T^{\mu\nu} F_{\mu\nu}(V)^{ij} - \frac{257\mathrm{i}}{3} \lambda^{\alpha i}\lambda_{\alpha}^{j} - \rho^{-1}\Box^{C} Y^{ij} + \cdots \\ N &= \frac{128}{9} T^{\mu\nu} T^{\rho\lambda} C_{\mu\nu\rho\lambda} + \frac{8}{3} D^2 - \rho^{-1}(\Box^{C})^2 \rho + \cdots \\ E^{\mu} &= -\frac{64}{3} \nabla_{\nu} T^{\mu\nu} - \frac{25}{18} \varepsilon_{\delta}{}^{\nu\rho\sigma\lambda} T_{\nu\rho} \nabla^{\mu}\nabla^{\delta} T_{\sigma\lambda} \cdots \end{aligned}$$

The full expressions before gauge fixing can be found in [Gold, Hutomo et.al 2311.00679]

Substituting the composite the linear multiplet into the vector linear action and applying the map

$$(\rho\,,A_\mu\,,\lambda^i\,,Y^{ij}) \to (\sigma\,,C_\mu\,,\psi^i\,,\frac{1}{4}\mathrm{i}\sigma^{-1}\bar{\psi}^i\psi^j - \frac{g}{2}\sigma^{-1}L^{ij})$$

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{Ricci}^2} \;=\; & -\tfrac{1}{6}R_{ab}R^{ab} + \tfrac{1}{24}R^2 + \tfrac{1}{6}R^{ab}G^2_{ab} + \tfrac{1}{3}RH_{ab}G^{ab} - \tfrac{4}{3}R_{ab}H^{ac}G^b{}_c - \tfrac{1}{3}RH^2 - \tfrac{1}{12}\epsilon^{abcde}C_aF_{bc}{}^{ij}F_{deij} \\
& + \tfrac{1}{6}F^{abij}F_{abij} - 2(H^2)^2 + \tfrac{16}{3}H^2_{ab}H^{ac}G^b{}_c - \tfrac{4}{3}H^2H_{ab}G^{ab} + \tfrac{2}{3}H_{ab}H_{cd}\big(G^{ab}G^{cd} - 2G^{ac}G^{bd}\big) \\
& + \tfrac{2}{3}H^2G^2 - \tfrac{4}{3}H^{2ab}G^2_{ab} - \tfrac{1}{3}H_{ab}G^{ab}G^2 + G^2_{ab}H^{ac}G^b{}_c - \tfrac{1}{48}(G^2)^2 - \tfrac{1}{24}G^4 - \tfrac{1}{6}\nabla_cG^{ac}\nabla^bG_{ab} \\
& + 2\nabla_aH_{bc}\nabla^{[a}H^{bc]} + \tfrac{1}{48}\epsilon^{abcde}\nabla^fG_{ef}(4H_{ab} - G_{ab})(4H_{cd} - G_{cd}) \\
& + \tfrac{g}{6}\Big(RN - 4NH_{ab}G^{ab} - 2NG^2 + F_{ab}{}^{ij}L_{ij}(G^{ab} + 4H^{ab}) + 12NH^2 - 6\nabla^a\nabla_aN\Big) \\
& - \tfrac{g^2}{24}\Big(2RL^2 - L^2(G^2 - 4G^{ab}H_{ab} - 24H^2) + 4N^2 + 6\nabla^aL^{ij}\nabla_aL_{ij}\Big) + \tfrac{2}{3}NL^2g^3 + \tfrac{5}{24}L^4g^4\;,
\end{aligned}$$

Minimal on-shell gauged supergravity with 4-derivative corrections

Due to the off-shell nature, we can add all the supergravity actions

$$\mathscr{L}_{2\partial+4\partial} = \mathscr{L}_{\rm EH} + \lambda_1 \mathscr{L}_{\rm Weyl^2} + \lambda_2 \mathscr{L}_{\rm Log} + \lambda_3 \mathscr{L}_{R^2}$$

To go to on-tushell, one can directly substitute the leading order solutions

$$\begin{aligned} N &= -\tfrac{1}{2} gL(2+L) + \mathscr{O}(\lambda_i)\,, \quad E_\mu = \mathscr{O}(\lambda_i)\,, \quad V_\mu'^{ij} = \mathscr{O}(\lambda_i)\,, \\ V_\mu &= -\tfrac{3}{\sqrt{2}} gC_\mu + \mathscr{O}(\lambda_i)\,, \quad L = 1 + \mathscr{O}(\lambda_i)\,, \quad \widetilde{C}_\mu = C_\mu + \mathscr{O}(\lambda_i)\,. \end{aligned}$$

because the $\mathscr{O}(\lambda_i)$ terms arising from substituting the equations above to the two-derivative action either vanish or can be removed by field redefinitions.

$$(16\pi G)e^{-1}\mathscr{L}_{2\partial+4\partial} = c_0 R + 12 c_1 g^2 - \tfrac{1}{4} c_2 G_{\mu\nu} G^{\mu\nu} + \tfrac{1}{12\sqrt{3}} c_3 \varepsilon^{\mu\nu\rho\sigma\lambda} C_\mu G_{\nu\rho} G_{\sigma\lambda} + \lambda_1 \mathscr{L}_{\rm GB}|_{\rm onshell} \ ,$$

where the various coefficients are

$$\begin{aligned} c_0 &= 1 + (\tfrac{28}{3}\lambda_1 - 20\lambda_2 - 4\lambda_3) g^2 \,, \\ c_1 &= 1 + (\tfrac{50}{9}\lambda_1 - \tfrac{28}{3}\lambda_2 + \tfrac{52}{3}\lambda_3) g^2 \ , \\ c_2 &= 1 + (\tfrac{64}{9}\lambda_1 - \tfrac{92}{3}\lambda_2 - \tfrac{76}{3}\lambda_3) g^2 \ , \\ c_3 &= 1 - 12(\lambda_1 + 3\lambda_2 + 3\lambda_3) g^2 \ , \end{aligned}$$

and the on-shell Gauss-Bonnet invariant is given by

$$\mathscr{L}_{\rm GB}|_{\rm onshell} = R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 + \tfrac{1}{8} G^4 - \tfrac{1}{2} C_{\mu\nu\rho\lambda} G^{\mu\nu} G^{\rho\lambda} + \tfrac{1}{2\sqrt{3}} \varepsilon^{\mu\nu\rho\sigma\lambda} C_\mu R_{\nu\rho}{}^{ab} R_{\sigma\lambda ab} \ ,$$

- When $g = 0$, all but the supersymmetric Weyl tensor squared can be removed by field redefinition, generalizing non-renormalization theorem in $4D, N = 2$ supergravity.
- When $\lambda_2 = 0$, recover previous results by [Cassani et.al 22'].
- Keeping the dilaton field L,

$$e^{-1}\mathscr{L} = LR + R^2 \cdots \tag{1}$$

which is in string frame. By contrast, the model constructed using standard Weyl multiplet is written in Einstein-frame.

Holographic anomaly coefficients

The dual field theory should be a $D=4$, $N=1$ CFT. The a and c Weyl anomaly coefficients of the dual CFT can be computed by standard method

$$a=\frac{\pi}{8g^3G}-\frac{9\pi(\lambda_2+\lambda_3)}{2gG}\,,\quad c=\frac{\pi}{8g^3G}+\frac{\pi(2\lambda_1-9\lambda_2-9\lambda_3)}{2gG}\,,$$

As a consistency check, we compute the R-symmetry anomaly. The background field of U(1) R-current denoted by $\hat{A}_i$ is given by $\frac{\sqrt{3}g}{2}A_i$. Under $\delta\hat{A}_i=\partial_i\lambda$,

$$\delta S = \frac{1}{24\pi g^3G}\int_{\partial M}d^4x\sqrt{-g}\lambda\left(-\frac{c_3}{9}\varepsilon^{ijkl}\hat{F}_{ij}\hat{F}_{kl}-\frac{\lambda_1 g^2}{2}\varepsilon^{ijkl}R_{ijab}R_{kl}{}^{ab}\right)$$

$$\Rightarrow \mathrm{Tr}R^3=\frac{4\pi c_3}{9g^3G}=\frac{16}{9}(5a-3c),\quad \mathrm{Tr}R=-\frac{16\pi\lambda_1}{gG}=16(a-c)$$

as required by superconformal supersymmetry.

Conclusion and outlook

- We obtain all curvature squared invariants in $5D, N = 2$ minimal gauged supergravity, offering a complete basis for carrying out precision tests of holography beyond the leading order.
- It can be easily shown that the offshell supergravities based on different Weyl multiplets are on-shell equivalent.
- We can generalize the construction to include n vector multiplets.
- Supersymmetric AdS black ring solution, which does not exist in the 2-derivative theory.
- Revisit the entropy computation of BPS black holes.

Thank you!

The composite L^{ij} contains a term $\rho^{-1}\Box Y^{ij}$. Since δY^{ij} contains a term $\bar{\varepsilon}\not{D}\lambda^{i}$, and $\delta L^{ij} \propto \varepsilon^{i}\varphi^{j}$, the composite φ^{i} should contain $\Box\not{D}\lambda^{i}$. Since δl^{i} contains a term $\not{D}\rho\varepsilon^{i}$ and $\delta\varphi^{i}$ contains a term $N\varepsilon^{i}$, the composite N should contain $\Box^{C}\Box^{C}\rho$.