Stringy Effect on Hawking Radiation

Pei-Ming Ho
National Taiwan University

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[Chau-PMH-Kawai-Shao-Wang 23]
[PMH-Imamura-Kawai-Shao 23]

Hawking radiation has been assumed to be an adiabatic process.

→ Black holes evaporate until they are Planckian.

→ information loss paradox (after the Page time). $t_{Page} \sim a^3/\ell_p^2$.

This is a prediction of low-energy effective theories.

Effective theories break down around the scrambling time. $t_{scr} \sim 2a \log(a^2/\ell_p^2)$.

[PMH-Kawai-Yokokura 21] [PMH-Kawai 22]

→ No reliable evidence of information loss.

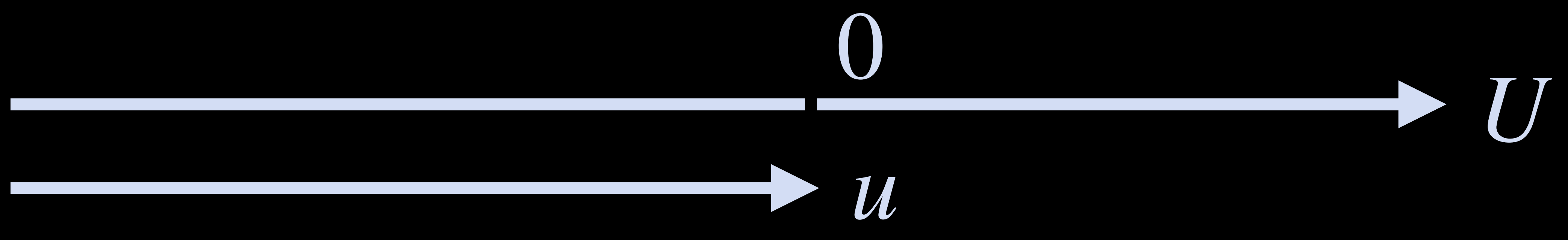
What about string theory? Other UV theories?

UV theories with stringy effects turn off Hawking radiation around t_{scr}.

[PMH-Kawai-Yokokura 21] [PMH-Kawai 22]

→ Black holes are essentially classical. → No paradox.

Hawking radiation [Hawking 74]



$$U \simeq -2ae^{-u/2a}$$

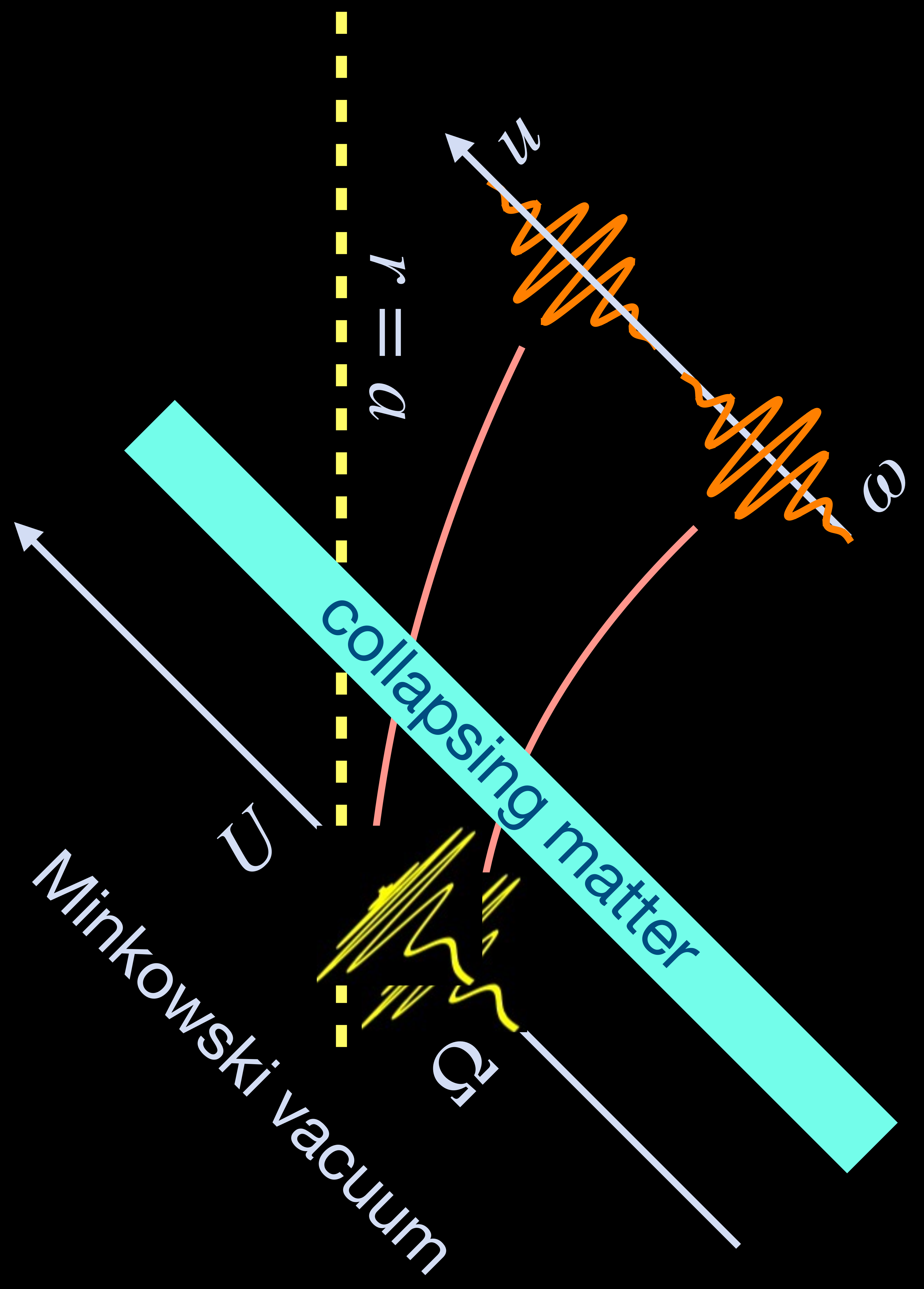
$$\langle \Omega \rangle \sim \langle \omega \rangle \frac{du}{dU} \simeq \langle \omega \rangle \, e^{u/2a} \lesssim \Delta\Omega$$

Minkowski vacuum $|\,0\rangle$ of the infinite past

$\simeq$ Hawking radiation

$\Rightarrow$ No information in Hawking radiation.

What's new? Time dependence.



$\langle \Omega \rangle$ is higher at later times (larger u):

$$\langle \Omega \rangle \sim \langle \omega \rangle \exp(u/2a)$$

For $\langle \omega \rangle \sim \mathcal{O}(1/a)$ in Hawking radiation.

$$\text{At } u \sim 2a \log(a/\ell_p), \quad \langle \Omega \rangle \sim \ell_p^{-1}.$$

$$4a \log(a/\ell_p), \quad \langle \Omega \rangle \sim (a/\ell_p)\, \ell_p^{-1}.$$

$$6a \log(a/\ell_p), \quad \langle \Omega \rangle \sim (a/\ell_p)^2\, \ell_p^{-1}.$$

$$\vdots$$

For an in-going mode of momentum $P_V \sim \mathcal{O}(1/a)$,

Lorentz-invariant CoM energy $\sqrt{\langle \Omega \rangle P_V} \sim \mathcal{O}\left((a/\ell_p)^{(n-1)/2} \ell_p^{-1} \right)$.

→ characteristic time scale: **scrambling time** $\boxed{\mathcal{O}\left(2a \log(a/\ell_p)\right)}$

Robustness of Hawking Radiation?

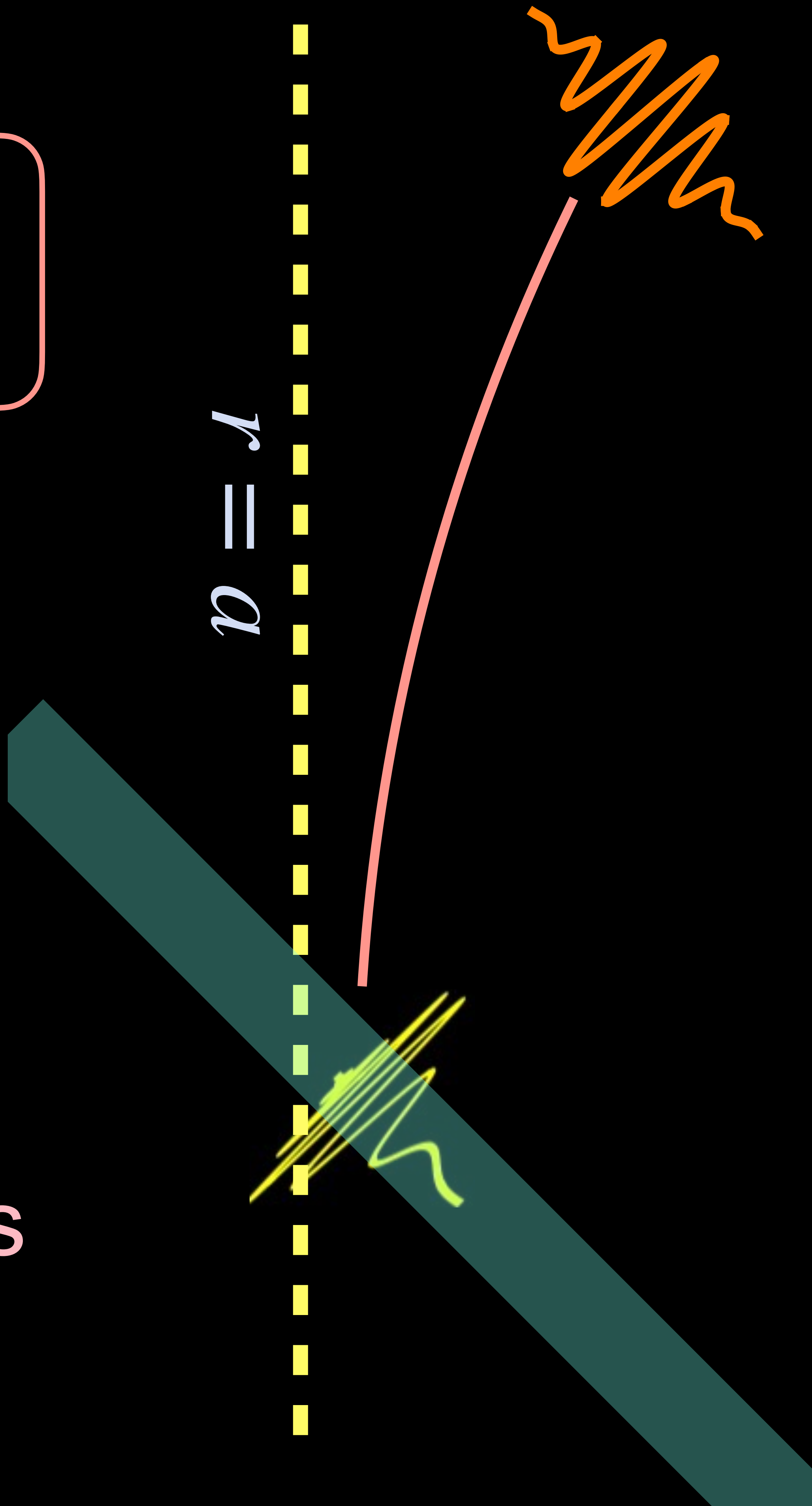
Previously, [PMH-Kawai-Yokokura 21] [PMH-Kawai 22]

Effective theory breaks down around scrambling time:

$$u \sim 2na \log(a/\ell_p) \quad \rightarrow \quad \sqrt{\langle \Omega \rangle P_V} \sim \mathcal{O}\left((a/\ell_p)^{(n-1)/2} \ell_p^{-1} \right)$$

$$S_{EFT} = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3 + \cdots \right.$$
$$\left. + \frac{\lambda}{2}\partial^m\psi\partial^n\psi\partial^k\phi\partial^q\phi + \frac{h}{2}\partial^m\phi\partial^n\phi\partial^k\mathcal{R} + \cdots \right]$$

Non-renormalizable higher-derivative (Lorentz-inv) interactions starts to dominate around scrambling time.



People forgot

1. non-renormalizable higher-derivative interactions
   radiation field and collapsing matter/spacetime curvature.
2. to compute time dependence of HR amplitude.

What about (hypothetical) UV theories?

quadratic action with higher derivatives (non-perturbative)

Stringy Effects

New results:

H.R. in examples of UV models (motivated by string theory)

(1) Generalized Uncertainty Principle (GUP) [Chau-PMH-Kawai-Shao-Wang 23]

$$\Delta x \Delta p \gtrsim 1 + \ell_p^2 \Delta p^2$$

(2) String Field Theory [PMH-Imamura-Kawai-Shao 23]

$$S_{SFT} = \int d^D x \left[\frac{1}{2} \phi_\alpha (\partial^2 - m_\alpha^2) \phi_\alpha + g_{\alpha\beta\gamma} \tilde{\phi}_\alpha \tilde{\phi}_\beta \tilde{\phi}_\gamma + \cdots \right]$$

$$\tilde{\phi}_\alpha \equiv e^{\frac{1}{2}\ell^2 \partial_\mu \partial^\mu} \phi_\alpha$$

Hawking radiation stops at scrambling time $u \sim 2a \log(a/\ell_p)$.

Generalized Uncertainty Principle (GUP)

[Chau-PMH-Kawai-Shao-Wang 23]

GUP:

$$\Delta x \Delta p \gtrsim 1 + \ell_p^2 \Delta p^2$$

[Amati-Ciafaloni-Veneziano 87, 89] [Gross-Mende 87, 88] [Konishi-Paffuti-Provero 90]

$$\Leftarrow \quad [x, p] = i\left(1 + \ell_p^2 p^2\right)$$

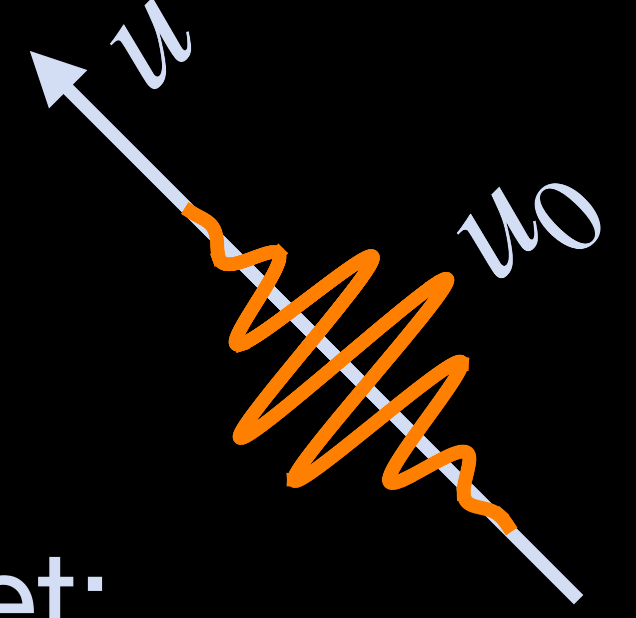
[Kempf-Mangano-Mann 95]

Schwarzschild metric in a freely falling frame

[Brout-Gabriel-Lubo-Spindel 98]

We examine the **time dependence** of the **amplitude** of Hawking radiation.

Consider a wave packet $\Psi_{(\omega_0,u_0)}$ at $r \to \infty$ centered around u_0 :



Hawking radiation = the VEV of the number operator for this wave packet:

$$\langle 0 \mid b_\Psi^\dagger b_\Psi \mid 0 \rangle \simeq \frac{1}{2}\frac{\omega_0}{e^{4\pi a \omega_0} - 1} \int_{-\infty}^{2a\log(a/\ell_p)} du \left| \Psi_{(\omega_0,u_0)}(u) \right|^2$$

Same Hawking temperature.

It goes to 0 when $u_0 \gg 2a\log(a/\ell_p)$ (scrambling time).

At late times, the wave packet has a large Δp, GUP implies a large Δx.

When Δx is much larger than a, it does not contribute to Hawking radiation.

(Same result in a non-perturbative calculation.)

The scrambling time $t_{scr} \sim 4a \log(a/\ell_p)$

is much smaller than the Page time $t_{Page} \sim \dfrac{a^3}{\ell_p^2}$.

Hawking radiation takes away a negligible portion of the black hole mass

$$\frac{\Delta M}{M} \sim \frac{\ell_p^2}{a^2} \log\left(\frac{a}{\ell_p}\right)$$

The black hole is essentially *classical*.

→ No information loss paradox.

Hawking temperature is not modified.

→ Bekenstein-Hawking entropy applies.

String Field Theory

[PMH-Imamura-Kawai-Shao 23]

$$S_{SFT} = \int d^D x \left[\frac{1}{2} \phi_\alpha (\partial^2 - m_\alpha^2) \phi_\alpha + g_{\alpha\beta\gamma} \tilde{\phi}_\alpha \tilde{\phi}_\beta \tilde{\phi}_\gamma + \cdots \right],$$

$$\tilde{\phi}_\alpha \equiv e^{\frac{1}{2}\ell^2 \partial_\mu \partial^\mu} \phi_\alpha \qquad \eta = (-++\cdots+)$$

→ exponential suppression of UV interactions in string theory.

Change of variables $\phi_\alpha \to \tilde{\phi}_\alpha$

$$S_{SFT} = \int d^D x \left[\tilde{\phi}_\alpha (\partial^2 - m_\alpha^2) e^{-\ell^2 \partial^2} \tilde{\phi}_\alpha + g_{\alpha\beta\gamma} \tilde{\phi}_\alpha \tilde{\phi}_\beta \tilde{\phi}_\gamma + \cdots \right]$$

→ coupling to large-k background suppressed for large-k modes.

cf: Energy-dependent Newton constant $G_N(k^2)$,

asymptotically safe gravity, rainbow gravity, …

Problem:

Feynman propagator in spacetime coordinates

$$\frac{1}{p^2 + m_\alpha^2 - i\epsilon} e^{-\ell^2 p^2} \rightarrow \frac{1 - e^{t^2/\ell^2} e^{-x^2/\ell^2}}{t^2 - x^2}$$

$\rightarrow$ unphysical in time-like directions

Problem does not show up in S-matrix by Euclideanizing momenta.

Problem arises in time-dependent calculations.

We solve the problem by [PMH-Imamura-Kawai-Shao 23]

1. complexify modular parameter: $\ell^2 = i\ell_E^2$ [Witten 13]

2. light-cone frame: $V =$ time. [Erler-Gross 04]

$$\langle 0 \mid \hat{\tilde{\phi}}(U,V)\hat{\tilde{\phi}}(U',V') \mid 0\rangle = \int_0^{V-V'\ /4\ell_E^2} \frac{d\Omega}{4\pi\Omega} e^{-i\Omega(U-U')}$$

$\rightarrow$

$$\hat{\tilde{\phi}}(U,V) = \int_0^{\infty} \frac{d\Omega}{\sqrt{4\pi\Omega}} \left[a_\Omega(V) e^{-i\Omega U} + a_\Omega^\dagger(V) e^{i\Omega U} \right]$$

$$[a_\Omega(V), a_{\Omega'}^\dagger(V')] = \Theta\left(\mid V - V' \mid - 4\ell_E^2 \Omega \right) \ \delta(\Omega - \Omega')$$

$$[a_\Omega(V), a_{\Omega'}(V')] = 0 \qquad [a_\Omega^\dagger(V), a_{\Omega'}^\dagger(V')] = 0$$

$$a_\Omega(V) \mid 0\rangle = 0$$

$$\Delta V \equiv \ \ V - V' \ \ \geq 4\ell_E^2 \Omega$$

Spacetime Uncertainty Principle From String Field Theory

[PMH-Imamura-Kawai-Shao 23]

Lorentz invariant constraint:

$$\Delta V \geq 4\ell_E^2 \Omega \quad \Rightarrow \quad \Delta U \Delta V \gtrsim \ell_E^2$$

Recall the spacetime uncertainty principle [Yoneya 87]

$$\Delta X \Delta T \gtrsim \ell_s^2$$

Hawking radiation:

$$\langle 0 \mid \hat{n}_\Psi(\Delta V) \mid 0 \rangle \simeq \frac{2\omega_0}{e^{4\pi a \omega_0} - 1} \int_{-\infty}^{u_{cut}} du \left| \Psi(u) \right|^2$$

$$u_{cut} \equiv 2a \log\left(\frac{a\Delta V}{4\ell^2}\right)$$

Recall that, at $u \sim 2(n+1)a \log(a/\ell_p)$, $\langle \Omega \rangle \sim (a/\ell_p)^n \, \ell_p^{-1}$,

$u_{cut} > u \quad \Rightarrow \quad \Delta V \gtrsim 4\ell_p^2 \langle \Omega \rangle \sim (a/\ell_p)^{n-1} a$ ~ universe for some n

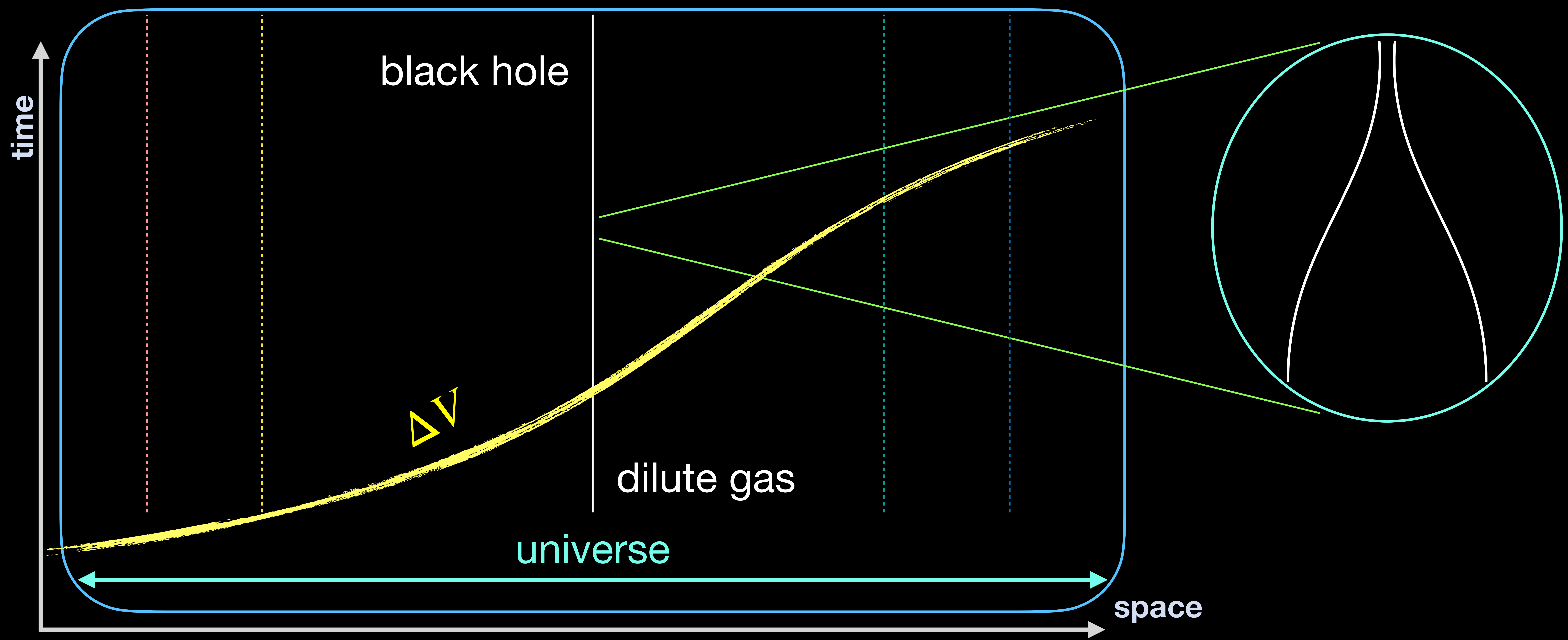
(assuming $\ell \sim \ell_p$)

$\rightarrow$ Same conclusion as GUP.

Ex: $n = 1.26$ for a solar mass black hole ($a \sim 3\,km$) for $\Delta V \sim$ universe.

$\rightarrow$ H.R. stops around $u \sim 5a \log(a/\ell_p) \sim 5\,ms$. (vs.$10^{64}$ yrs.)

$\rightarrow$ Black holes are essentially classical.



Conclusion and Outlook

Explicit calculations of Hawking radiation in stringy UV models.

→ Non-trivial time dependence (previously unnoticed).

Hawking radiation is turned off at scrambling time due to stringy effects.

→ Information loss paradox resolved.

Trans-Planckian modes have large spatial extensions ~ universe.

They cannot be localized in the near-horizon region.

Hawking temperature persists → same entropy

What does it imply on the entanglement entropy?

Is it related to the “islands”?

Other trans-Planckian problems? [Blamart-Laliberte-Brandenbergere 23]

Thank you!