Simplicity of AdS Super Yang-Mills

Xinan Zhou

Kavli Institute for Theoretical Sciences

University of Chinese Academy of Sciences

East Asia Joint Workshop on Fields and Strings (Xian, Nov. 2023)

Based on

- JHEP 07 (2018) 147
- **JHEP 06 (2021) 020 (with L. F. Alday, C. Behan, P. Ferrero)**
- Phys.Rev.Lett. 127 (2021) 14, 141601
- **JHEP 02 (2022) 105 (with L. F. Alday, A. Bissi)**
- Phys.Rev.Lett. 128 (2022) 16, 161601 (with L. F. Alday, V. Goncalves)
- JHEP 07 (2023) 053 (with Z. Huang, B. Wang, Y. Yuan)
- 2307.06884 (with L. F. Alday, V. Goncalves, M. Nocchi)
- **2309.14413 (with Z. Huang, B. Wang, Y. Yuan)**

Motivations

- **Gauge theory in AdS**
  - The curvature of AdS offers a gauge invariant regulator for infrared divergences.
  - Exponential growth of area w.r.t. distance — massive photons. Also particles diffuse very rapidly, effectively infinitely dimensional spacetime.
  - The radius gives a new scale in addition to Λ_{QCD} and we can probe different phases of the theory by dialing the radius.

INFRARED BEHAVIOR AT NEGATIVE CURVATURE

Curtis G. CALLAN *

Physics Department, Princeton University, Princeton, NJ 08544, USA

Frank WILCZEK * *

School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA

[Callan, Wilczek ’90]

Motivations

• A subsector of holography

- Adding probe branes naturally leads to subsectors of AdS gauge theory which decouple from the rest in the large N limit.
- For example, consider N D3 with $N_F \ll N$ D7. We get $\mathcal{N} = 1$ SYM on $AdS_5 \times S^3$ coupled to IIB SUGRA in $AdS_5 \times S^5$. The couplings scale as $\langle AAA \rangle \sim 1/\sqrt{N}$, $\langle AAh \rangle \sim 1/N$. So gravity decouples at large N.
- This is quite common and can be done in other ways and in other dimensions. Such duality (albeit in an approximate sense) without gravity is also known in the literature as *rigid holography*.

Motivations

- **Scattering amplitude in AdS**
  - Holographic correlators are on-shell scattering amplitudes in AdS.
  - In flat space, amplitudes satisfy various remarkable relations which give deep insight into the nature of gauge theory and gravity. For example, the double copy relation tells us that gravity is the square of YM.
  - How robust are such lessons? Do they hold in curved spacetimes? The maximally symmetric AdS space is an ideal starting point for exploring such questions.

$\mathcal{N}=1$ SYM on $AdS_5 \times S^3$

- Symmetries

$$PSU(2,2|2) \times SU(2)_L \times G_F$$

$PSU(2,2|2) \to SO(4,2)$, $SU(2)_R$; $SU(2)_R \times SU(2)_L \to SO(4)$; $G_F \to SU(N_F)$

$SO(4,2) \times SO(4)$ isometry of $AdS_5 \times S^3$

- AdS spectrum (from KK reducing an 8d vector multiplet)

	super gluon		spinning gluon			
quantum numbers	s_k	$\xrightarrow{Q^2}$	V_k^μ	$\xrightarrow{Q^2}$	r_k	+ fermions
Δ	k		$k+1$		$k+2$	
$SU(2)_R \times SU(2)_L$	$(\frac{k}{2}, \frac{k}{2}-1)$		$(\frac{k}{2}-1, \frac{k}{2}-1)$		$(\frac{k}{2}-2, \frac{k}{2}-1)$	

adjoint under gauge group G_F

$\mathcal{N}=1$ SYM on $AdS_5\times S^3$

- The AdS super gluons S_k are dual to scalar operators O_k. Our main focus is the holographic correlators $G_{\{k_i\}}=\langle O_{k_1}O_{k_2}O_{k_3}O_{k_4}\rangle$.

- It's also interesting to think about the dual interpretation. The theory is dual to 4d $\mathcal{N}=4$ SYM coupled to N_F $\mathcal{N}=2$ fundamental hypermultiplets. Then the super gluons in AdS are mesons in the 4d SCFT and gravitons are glue states

$$O_2\sim q\bar{q}\equiv m \qquad\qquad \mathrm{tr}(\phi^I)^2\equiv g$$

Then in large N QCD, we have

$$\langle mmm\rangle\sim 1/\sqrt{N} \qquad\qquad \langle mmg\rangle\sim 1/N$$

- For AdS tree level, we are computing the meson 4pt correlators at leading order in $1/N$ and infinite 't Hooft coupling.

As correlators…

- Supersymmetry dictates

2-component spinors for $SU(2)_{L,R}$ polarizations.

$$G_{\{k_i\}}(x_i; v_i, \bar{v}_i) = G^{\text{protected}}_{\{k_i\}}(x_i; v_i, \bar{v}_i) + R\, H_{\{k_i\}}(x_i; v_i, \bar{v}_i)$$

R is determined by susy to be

$$R = (v_1 \cdot v_2)^2 (v_3 \cdot v_4)^2 x_{13}^2 x_{24}^2 (1 - z\alpha)(1 - \bar{z}\alpha)$$

where

$$U = z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad \alpha = \frac{(v_1 \cdot v_3)(v_2 \cdot v_4)}{(v_1 \cdot v_2)(v_3 \cdot v_4)}$$

$$V = (1 - z)(1 - \bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

- Dynamical information is in the reduced correlator H: $\Delta_i = k_i + 1,\ j_L = j_R = \frac{k_i}{2} - 1.$

- Equivalent to the superconformal Ward id

$$(z\partial_z - \alpha\partial_\alpha)\mathcal{G}\,|_{\alpha=z} = 0$$

As amplitudes…

- We can define AdS amplitudes via Mellin representation [Mack '09, Penedones '09]

$$G_{\{k_i\}}(x_i) = \int [d\delta](\prod_{i<j} x_{ij}^{-2\delta_{ij}} \Gamma[\delta_{ij}]) \mathcal{M}(\delta_{ij}; v_i, \bar{v}_i)$$

- Conformal symmetry requires

$$\delta_{ij} = \delta_{ji} \qquad \delta_{ii} = -\Delta_i \qquad \sum_j \delta_{ij} = 0$$

which can be understood as Mandelstam variables upon introducing fictitious momenta

$$\delta_{ij} = \vec{p}_i \cdot \vec{p}_j \qquad \text{where} \qquad \vec{p}_i^{\,2} = -\Delta_i \qquad \sum_i \vec{p}_i = 0$$

- Mellin amplitude shares a lot of similarities with flat-space tree-level amplitudes
  - Contact diagrams are just constants
  - Exchange diagrams have poles
- Great simplification in terms of analytic structure and we can use flat-space intuition.

As amplitudes…

- We can also similarly define a reduced Mellin amplitude from the reduced correlator

$$H \to \widetilde{\mathcal{M}}$$

- The protected part does not contribute to the Mellin amplitude. Then we have

$$\mathcal{M} = \widehat{R} \circ \widetilde{\mathcal{M}}$$

- Then this relation completely takes care of the effect of superconformal symmetry. This similar to the flat-space super amplitude

$$\mathcal{A} = \delta^8(Q)\,\widehat{\mathcal{A}}$$

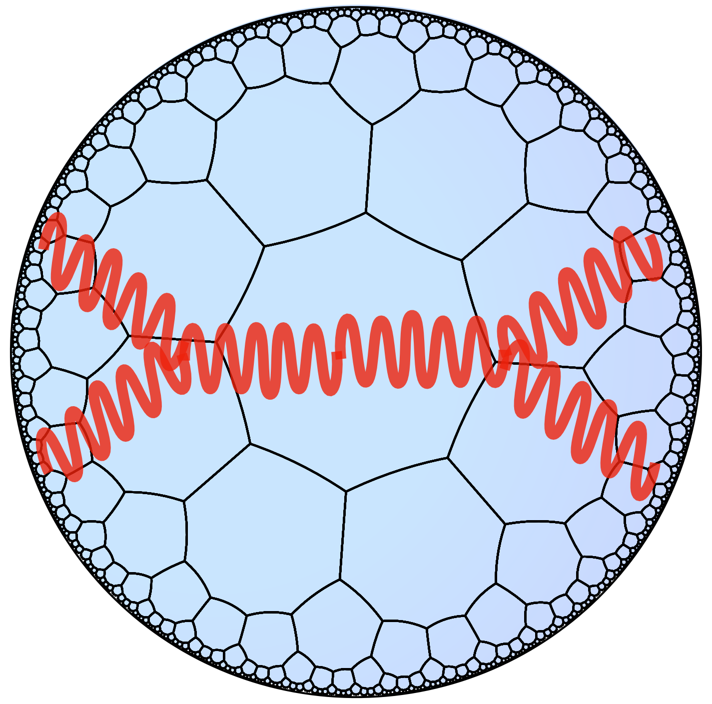
Tree level

- Holographic correlators are difficult to compute even at tree level. The effective Lagrangian is very complicated which makes the traditional diagrammatic expansion very cumbersome.
- A better method: bootstrap by using superconformal symmetry [Rastelli, XZ '16].

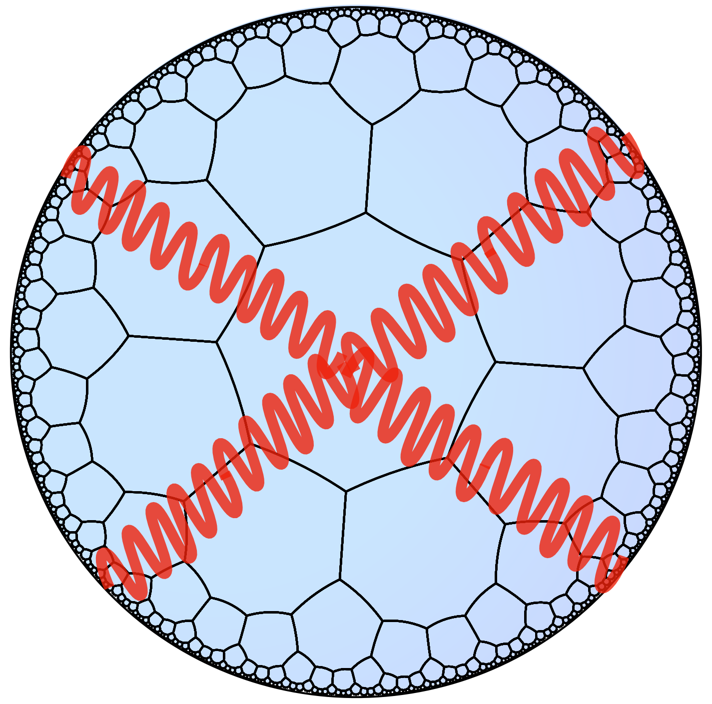
Schematically,

- We write down an ansatz as the sum of all possible diagrams with unfixed coefficients

$$\mathscr{A} = \sum_X \lambda_X$$



$$+ \ (t) \ + \ (u) \ + \sum_i c_i$$



- We can solve all coefficients by imposing the superconformal Ward identities

Tree level

- This can be efficiently implemented in Mellin space [XZ '18]. For example, for $\langle 22kk \rangle$

$$\mathcal{M} = \mathtt{c}_s M_s + \mathtt{c}_t M_t + \mathtt{c}_u M_u$$

$$M_s = \frac{-2\alpha(k+2)+k+\alpha t+\alpha u-u+2}{s-2} \qquad M_t = \frac{(1-\alpha)(-2\alpha k-k+\alpha s+u-2)}{t-k} \qquad M_u = \frac{-\alpha(2\alpha k-3k-\alpha s+s+t-2)}{u-k}$$

$$s+t+u=2k+4 \qquad \mathtt{c}_s = f^{I_1I_2J}f^{JI_3I_4} \qquad \mathtt{c}_t = f^{I_1I_4J}f^{JI_2I_3} \qquad \mathtt{c}_u = f^{I_1I_3J}f^{JI_4I_2}$$

- Using "MRV" techniques developed in [Alday, XZ PRL '20, Alday, XZ PRX '21], tree-level amplitudes for arbitrary k_i can also be obtained in a closed form. The reduced Mellin amplitudes read [Alday, Behan, Ferrero, XZ '21]

$$\widetilde{\mathcal{M}} \sim \sum_{\substack{i+j+k=\mathcal{E}-2 \\ 0\leq i,j,k\leq \mathcal{E}-2}} \frac{\sigma^i\tau^j}{i!j!k!(i+\frac{\kappa_u}{2})!(j+\frac{\kappa_t}{2})!(k+\frac{\kappa_s}{2})!}\left(\frac{\mathtt{c}_s}{(s-s_M+2k)(\tilde{u}-u_M+2i)} - \frac{\mathtt{c}_t}{(t-t_M+2k)(\tilde{u}-u_M+2i)}\right)$$

Tree level

- From the result, we find a remarkable hidden structure

Generating function $\quad \mathbf{H} = H_{2222}(x_{ij}^2 - t_i \cdot t_j)$ $\qquad t_{\alpha\beta} = v_\alpha \bar{v}_\beta$ polarization vector of $SO(4)$

Taylor expand in $t_i \cdot t_j$ $\downarrow$

$$H_{k_1k_2k_3k_4}(x_{ij}^2)$$

E.g.,

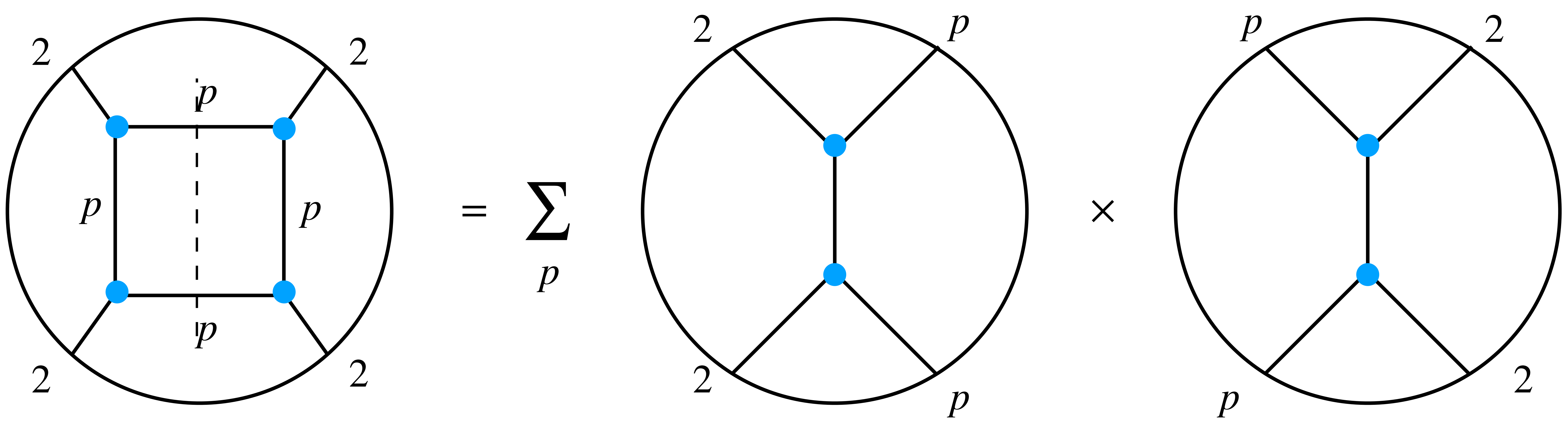
$$H_{3333} = \left(\frac{\partial}{\partial x_{12}^2}\frac{\partial}{\partial x_{34}^2} + \frac{\partial}{\partial x_{13}^2}\frac{\partial}{\partial x_{24}^2} + \frac{\partial}{\partial x_{14}^2}\frac{\partial}{\partial x_{23}^2} \right) H_{2222}$$

- The new variable $x_{ij}^2 - t_i \cdot t_j$ is the distance in $AdS_5 \times S^3$ (or $\mathbb{R}^8$ after Weyl trans.), therefore the structure is usually called hidden conformal symmetry.

- The same structure was first observed for IIB sugra in $AdS_5 \times S^5$ [Caron-Huot, Trihn '18], then soon in $AdS_3 \times S^3 \times K3$ [Rastelli, Roumpedakis, XZ '19]. But in this case, there is no gravity!

One-loop level

- Going forward, let's consider correlators at one loop. They are interesting because they capture quantum fluctuations.
- Diagrammatic methods are impossible because we don't know how to compute diagrams.
- But we can use an AdS version of the unitarity method [Aharony, Alday, Bissi, Perlmutter]



One-loop level

- In practice, we use the lower order data as follows

$$\mathcal{G}^{\text{disc}} = \sum_{n,\ell} a^{(0)}_{n,\ell}\, g_{n,\ell}(U,V)$$

$$\mathcal{G}^{\text{tree}}\big|_{\log U} = \sum_{n,\ell} \frac{1}{2} a^{(0)}_{n,\ell}\, \gamma^{(1)}_{n,\ell}\, g_{n,\ell}(U,V)$$

$$\Rightarrow \quad \mathcal{G}^{1-\text{loop}}\big|_{\log^2 U} = \sum_{n,\ell} \frac{1}{4} a^{(0)}_{n,\ell} \left(\gamma^{(1)}_{n,\ell}\right)^2 g_{n,\ell}(U,V)$$

Gives the leading log singularity as $U \to 0$, contains $\log^2 V$ as $V \to 0$ after resummation

Mellin space ($k_i = 2$):

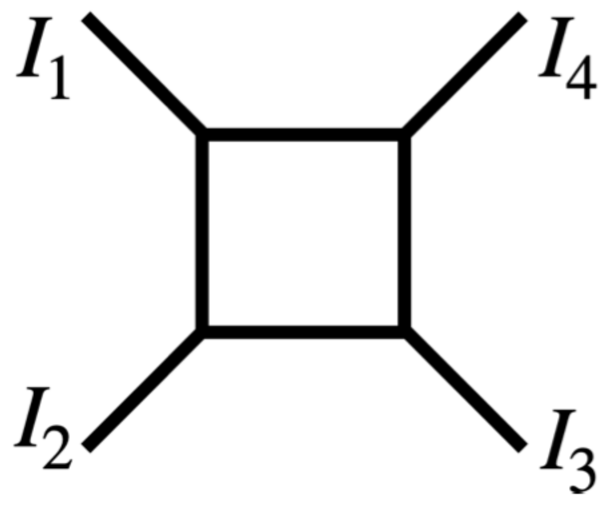
$$H \sim \int \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma^2\left(\frac{4-s}{2}\right) \Gamma^2\left(\frac{4-t}{2}\right) \Gamma^2\left(\frac{4-\tilde{u}}{2}\right) \widetilde{\mathcal{M}}(s,t)$$

Tells us the reduced Mellin amplitude must have simultaneous poles at $s = 2m$ and $t = 2n$ for $m, n = 2,3,\ldots$

One-loop level

- We therefore make the ansatz [Alday, Bissi, XZ '22]

$$\widetilde{\mathcal{M}}(s,t) = \sum_{m,n=2}^{\infty} \mathtt{d}_{st} \frac{c_{mn}}{(s-2m)(t-2n)} + (\text{crossing})$$

$\mathtt{d}_{st} =$ 

- We can use the leading logs to fix the coefficients

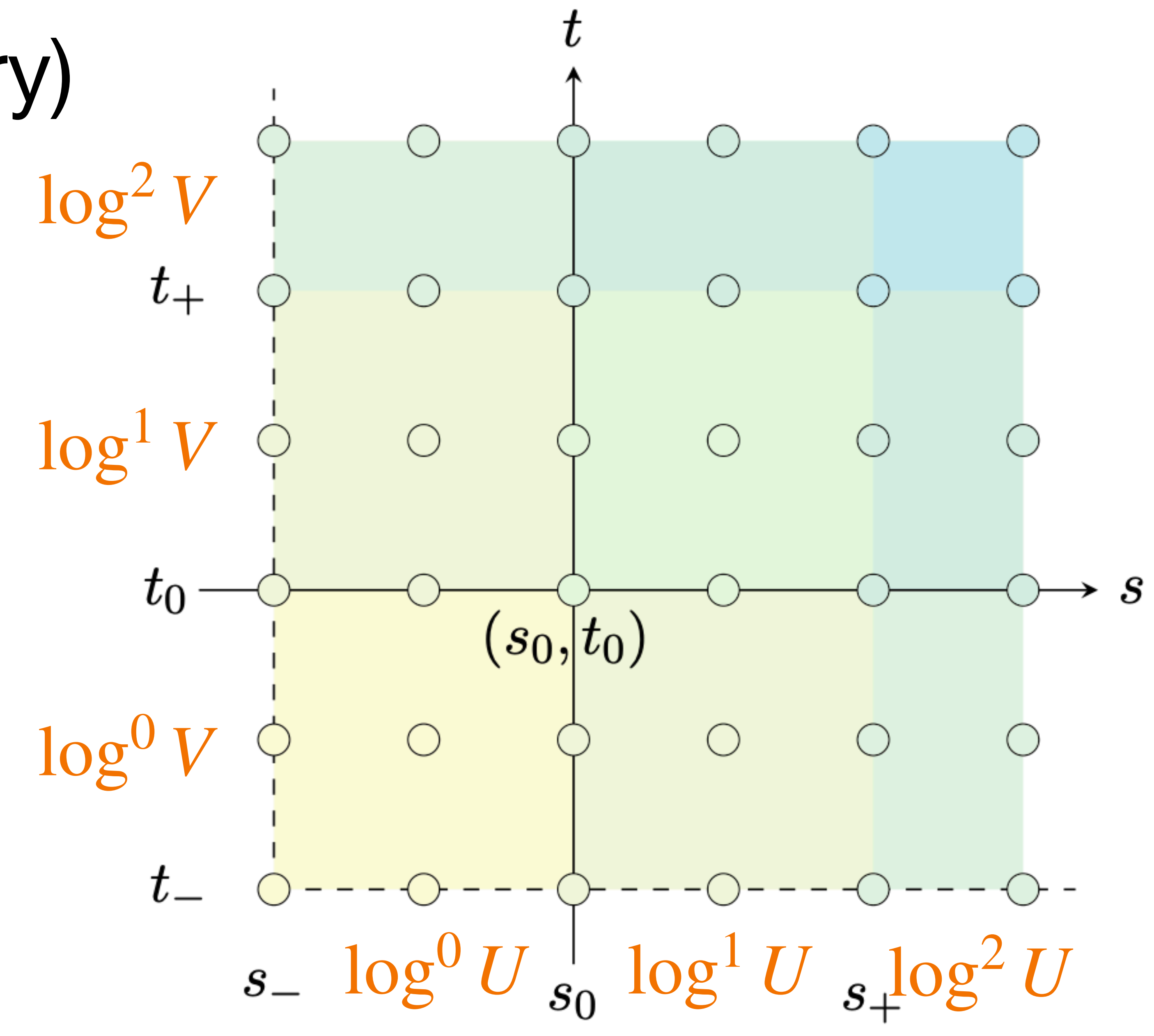
$$c_{mn} \sim \frac{3m^2n - 4m^2 + 3mn^2 - 16mn + 15m - 4n^2 + 15n - 12}{(m+n-4)(m+n-3)(m+n-2)}$$

- Then we can check there cannot be other terms and therefore the ansatz is complete. The Mellin amplitude also correct reduces to 8D box diagrams in the flat-space limit.

- This is already quite remarkable. The lowest weight super gluon 4pt amplitude at one loop in Mellin space has such a simple form!

One-loop level

- Now what about correlators with arbitrary KK levels? What happens to the hidden conformal symmetry at tree level?
- We find yet another surprise [Huang, Wang, Yuan, XZ '23]: hidden conformal symmetry extends way beyond its expected regime of validity and fixes almost entirely the one-loop amplitude!
- We still have simultaneous poles with constant coefficients. But not all poles are determined by the leading logs (determined by hidden conformal symmetry)



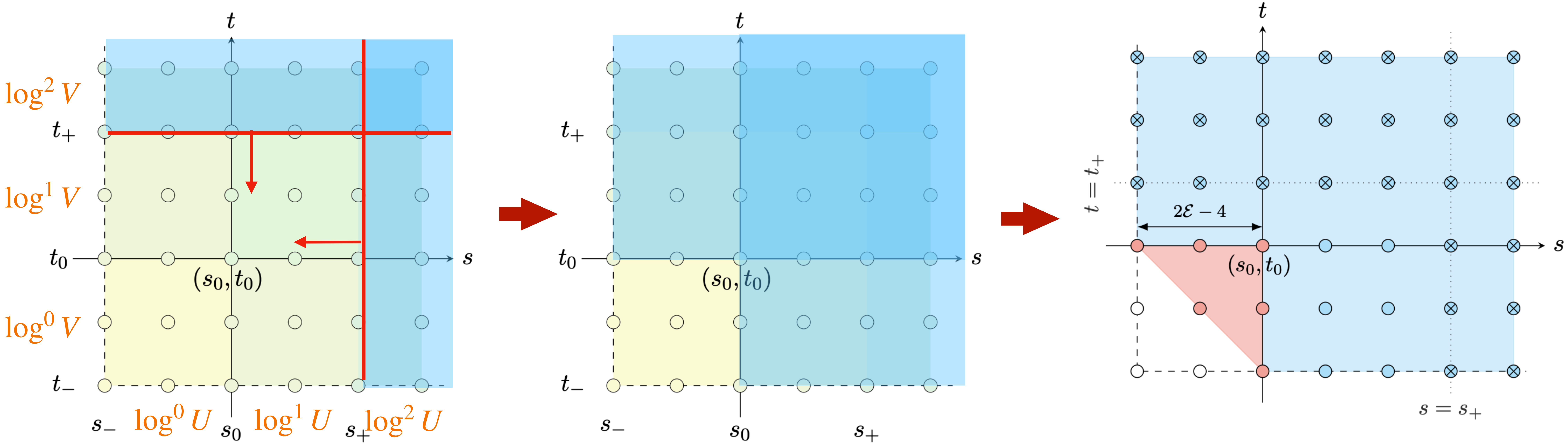
$$\widetilde{\mathcal{M}}(s,t) = \sum_{m,n} \mathrm{d}_{st} \frac{c_{mn}}{(s-s_0-2m)(t-t_0-2n)}$$

$$+(\text{crossed channels})$$

Gamma factors: $\Gamma\left(\frac{k_1+k_2-s}{2}\right)\Gamma\left(\frac{k_3+k_4-s}{2}\right)\ldots$

One-loop level

- But we can still compute the residues by using the tree-level data and then check that the ansatz is complete.
- Surprisingly, hidden conformal symmetry determines almost all infinite simultaneous poles up to just finitely many!



Discussion and outlook

- We have showed that SYM in AdS has remarkable hidden simplicity, which makes it a good testing ground for new ideas. There are many other developments which were not discussed in this talk and lead to research avenues.
- Double copy in AdS
  - Double copy was found for three-level 4pt amplitudes, relating SYM in $AdS_5 \times S^3$ and IIB SUGRA in $AdS_5 \times S^5$ (also bi-adjoint scalar in $AdS_5 \times S^1$) [XZ PRL '21].
  - Double copy structure was also found in position space for spinning 4pt correlators [Bissi, Fardelli, Manenti, XZ '22].
- Super gluon 4pt amplitudes are now known up to 2 loops [Huang, Wang, Yuan, Zhou '23] and also for super gravitons [Huang, Yuan, '21]. Can we find generalization at loop level?

Discussion and outlook

- Efficient methods for computing higher-point amplitudes
  - Super gluon 5pt amplitude has been bootstrapped using superconformal symmetry [Alday, Goncalves, XZ PRL '22].
  - A more systematic and powerful method for higher-point amplitude was also found which uses instead flat-space limit and factorization [Alday, Goncalves, Nocchi, XZ '23], and computes the example of 6pt.
  - Can we find a recursive method similar to BCFW?
- Stringy corrections
  - Recently, there has been a lot of progress in studying AdS Virasoro-Shapiro amplitudes [Alday, Hansen, Silva].
  - Can we do something similar for super gluons which gives rise to open string amplitudes in AdS?

Thank you!