CHAOS AND ORDER

IN HIGH DIMENSIONAL COVARIANT DISORDERED MODELS

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JHEP 11 (2021) 211， Phys.Rev.Lett. 129 (2022) 1, 011603，*JHEP* 12 (2018) 065

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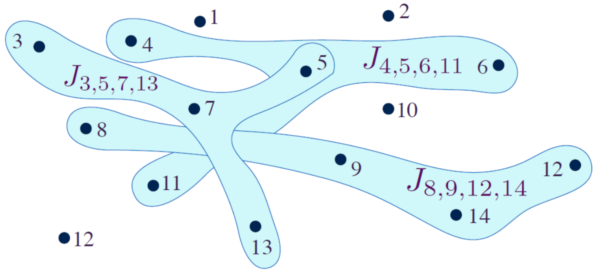
CONTENT

Recently, disordered models have attracted lots of attention in the High Energy Theory community.

THE SYK MODEL

The Sachdev-Ye-Kitaev (SYK) model:

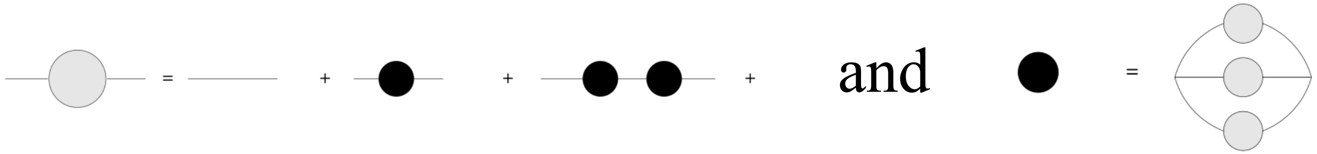
strongly coupled 0+1d quantum mechanics model



(Figure from Phys.Rev.X 5 (2015) 4, 041025)

✓ $H = \sum_{1 \le j_1 \le j_2 \cdots \le j_q \le N} J_{j_1 j_2 \cdots j_q} \psi^{j_1} \psi^{j_2} \cdots \psi^{j_q}$, $\langle J_{j_1 j_2 \cdots j_q} \rangle = 0$, $\langle J^2_{j_1 j_2 \ldots j_q} \rangle = \frac{J^2 (q-1)!}{N^{q-1}}$

✓ perturbatively solvable



✓ Correlation functions, operator spectrum, thermodynamical properties…

Sachdev, Ye,
Parcollet, Georges,
Kitaev,
Maldacena, Stanford,
…

THE SYK MODEL

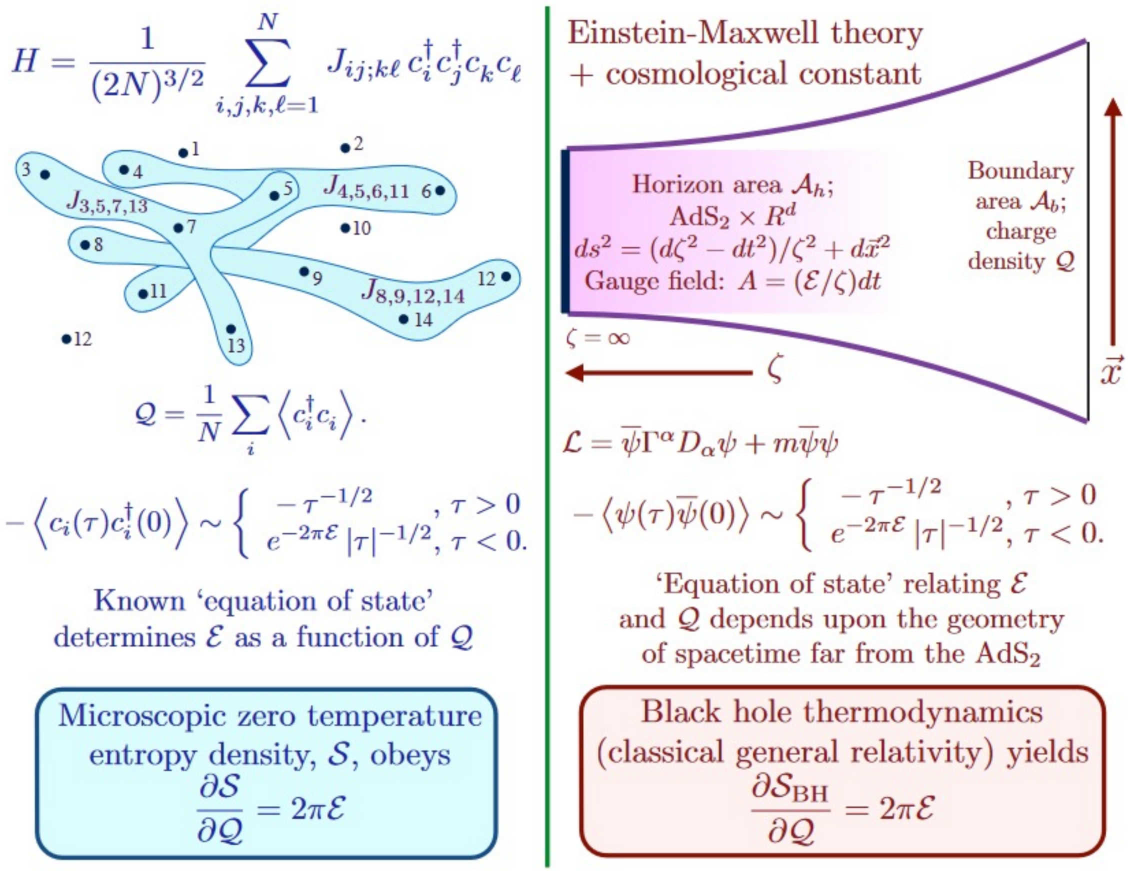
- Quantum chaos:
  Out of Time Order Correlators (OTOC)

  $$\langle \psi_1(t_1)\psi_2(0)\psi_1(t_1)\psi_2(0)\rangle \propto \frac{1}{N}\, e^{\lambda_L t}$$

  - λ_L: Lyapunov exponent
    positive λ_L indicates early time chaotic behavior of the theory
  - gravity is also chaotic

- Thermodynamics

- The SYK model leads to a simple, solvable example of the holography



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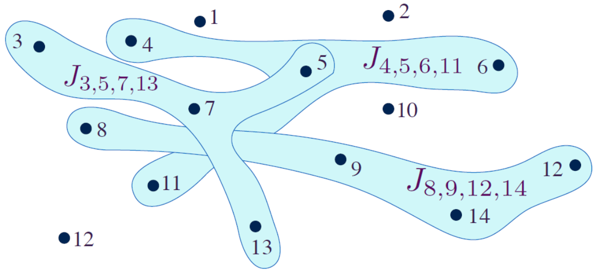

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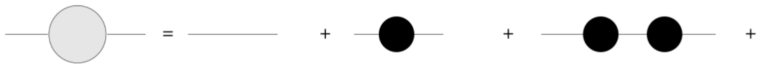

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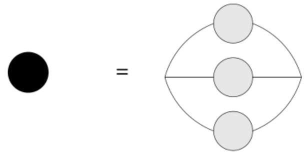

- ✓ Correlation functions, operator spectrum, thermodynamical properties…

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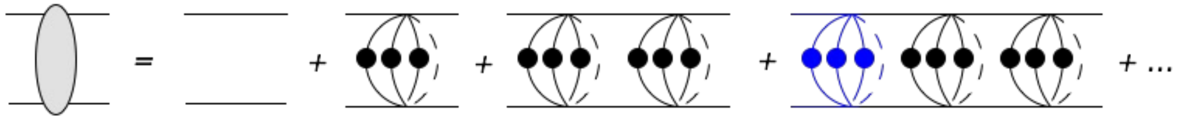
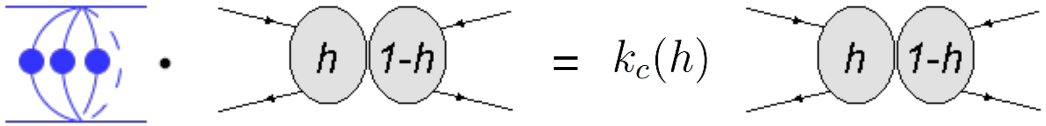
THE SYK MODEL

- Operator spectrum



$$\mathcal{F} = \frac{1}{1-K}\mathcal{F}_0$$



$$\mathcal{F}(\chi) = \frac{1}{1-K_c}\mathcal{F}_0 = \sum_h \Psi_h(\chi)\frac{1}{1-k_c(h)}\frac{\langle\Psi_h,\mathcal{F}_0\rangle}{\langle\Psi_h,\Psi_h\rangle}$$

$$k_c(h) = 1 \quad \Rightarrow \quad h_m = 2\Delta + 1 + 2m + \epsilon_m \qquad \mathcal{O}_m \sim \psi^i(\tau)\partial_\tau^{2m+1}\psi^i(\tau)$$

- An infinite tower of operators (with finite anomalous dimensions)

- A crucial difference from Random Matrix Theories

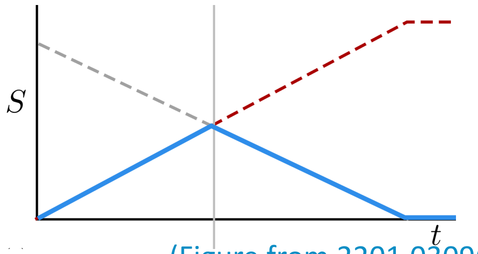
Disordered models are special cases of ensemble average theories that are often relavant in holographic dualities.

BH EVAPORATION AND ENSEMBLE AVERAGE

Penginton; Almheiri, Engelhardt, Marolf, Maxfield;Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini; Penington, Shenker, Stanford, Yang …

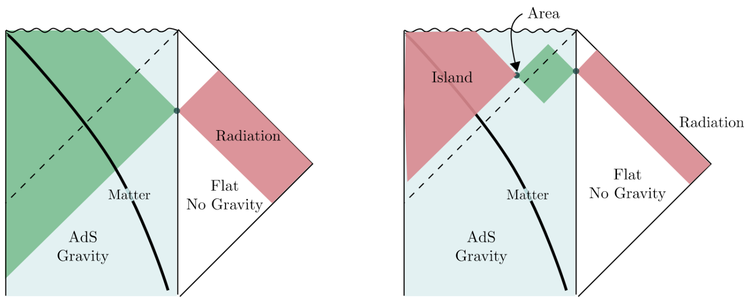
A "solvable" incarnation of the information paradox

- The information paradox:
  Are Hawking radiations from Blackholes thermal or informative?

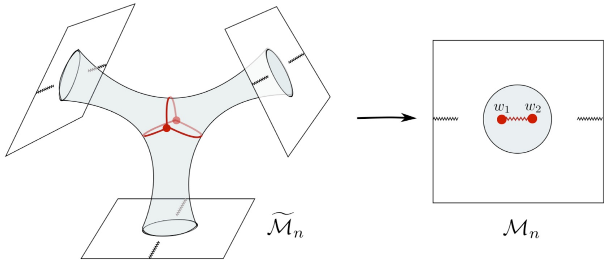


(Figure from 2201.03096)

- Recent breakthroughs in this puzzle in low-dimensional solvable toy models
  - New quantum extremal surface in an evaporating black hole
  - Alternatively, the necessity of including the spacetime wormholes in the gravitational path integral



(Figure from 1911.12333)

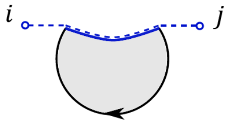


BH EVAPORATION AND ENSEMBLE AVERAGE

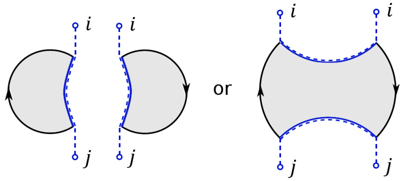
- Spacetime wormholes are tied with ensemble averages of theories

(Coleman; Giddings Strominger; Maldacena Maoz)

- Evidence including e.g.



$$\overline{\langle\psi_i|\psi_j\rangle} = \delta_{ij},$$



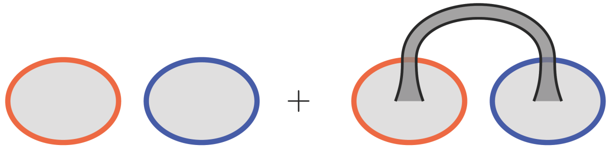
$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + \frac{Z_2}{Z_1^2}$$

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + x_{ij}, \quad \overline{x_{ij}} = 0, \quad \overline{x_{ij}^2} = \frac{Z_2}{Z_1^2}$$

(Penington, Shenker, Stanford, Yang 2020)

$$\left\langle Z[J_1]\cdots Z[J_n]\right\rangle := \int_{\Phi\sim J}\mathcal{D}\Phi\, e^{-S[\Phi]}$$

$$\left\langle Z[J_1]Z[J_2]\right\rangle =$$



$$\left\langle Z^n\right\rangle = \sum_{p\perp\{1,2,\ldots n\}} \lambda^{|p|} = B_n(\lambda) = \sum_{d=0}^{\infty} d^n p_d(\lambda) = \left\langle x^n\right\rangle_{\text{Pois}}, \qquad p_d(\lambda) = e^{-\lambda}\frac{\lambda^d}{d!}$$

(Marolf, Maxfield, 2020
CP, Tian, Yu 2021, **CP**, Tian, Yang 2022)

- Disordered models are special cases of the "ensemble average theories"

CONTENT

MOTIVATION

From a high energy physicist's point of view, the world we live in is usually described by covariant quantum field theories in high(er than 0+1) spacetime dimensions.

However, a conventional quantum field theorist would wonder if averaging over a set of different theories (or actions) is a well-defined operation.

Therefore a set of questions naturally arise

1. if there exist high dimensional covariant disordered models
2. do they fulfill the usual requirements obeyed by conventional QFTs
3. do they share similar nice features as their low dimensional counterparts
4. if there are clear connections with other well-known conventional QFTs

CONTENT

A 2D $\mathcal{N}$=(0,2) MODEL

CP, *JHEP* 12 (2018) 065

- $S = \int d^2z d\theta d\bar{\theta} \left(-\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2 z d\theta \frac{J_{i a_1 \ldots a_q}}{q!} \Lambda^i \Phi^{a_1} \ldots \Phi^{a_q}$

$$\text{Chiral:} \quad \Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z \phi^a \,, \qquad a = 1 \ldots N$$

$$\text{Fermi:} \quad \Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z \lambda^i \,, \qquad i = 1 \ldots M$$

- $N, M \gg 1$, with $\mu = \frac{M}{N}$ fixed (but tunable)

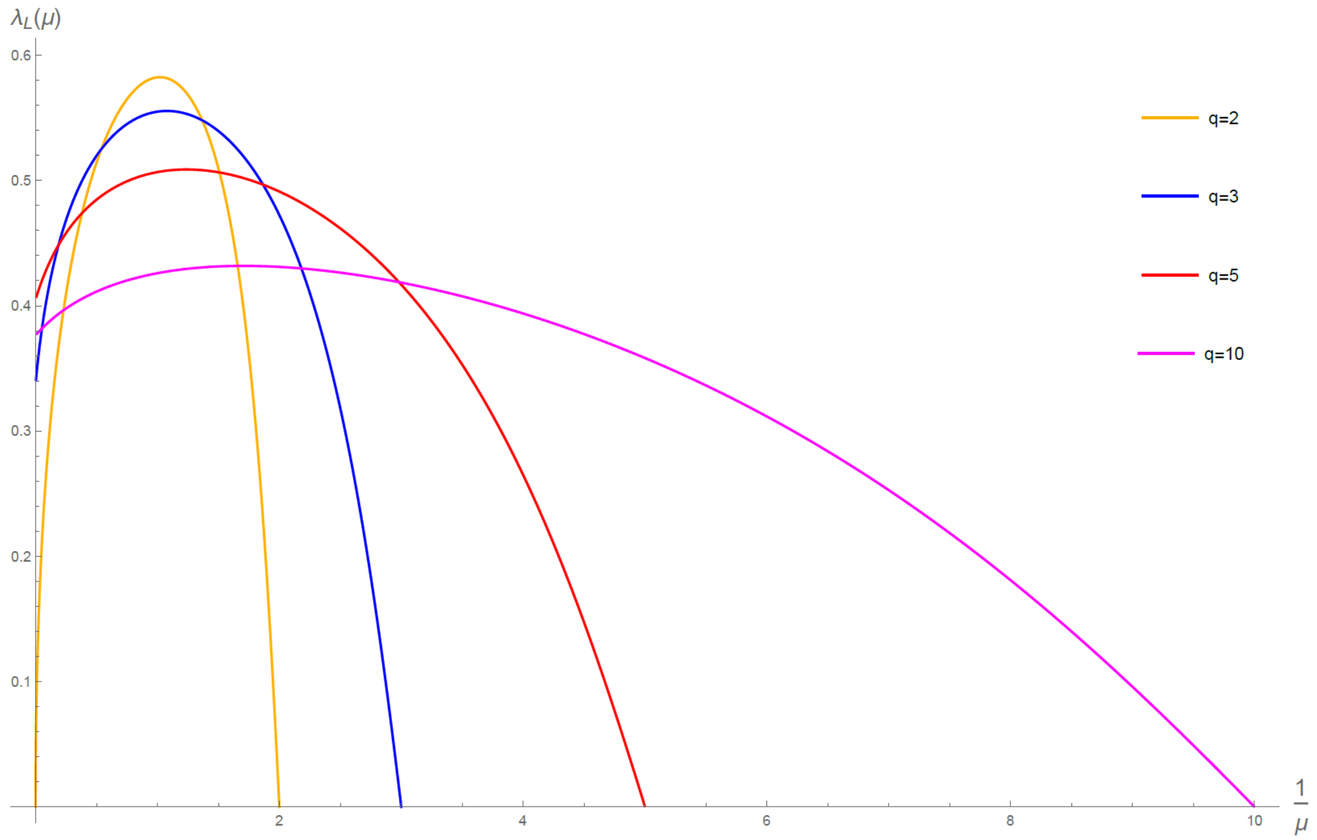
- IR solution $G_c^I(z_1, z_2) = \frac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}}$ $\qquad I = \phi, \psi, \lambda, G$

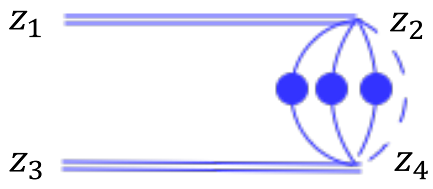
$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}, \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} .$$

CHAOS: THE LYAPUNOV EXPONENT

- Out-of-Time-Ordered Correlators

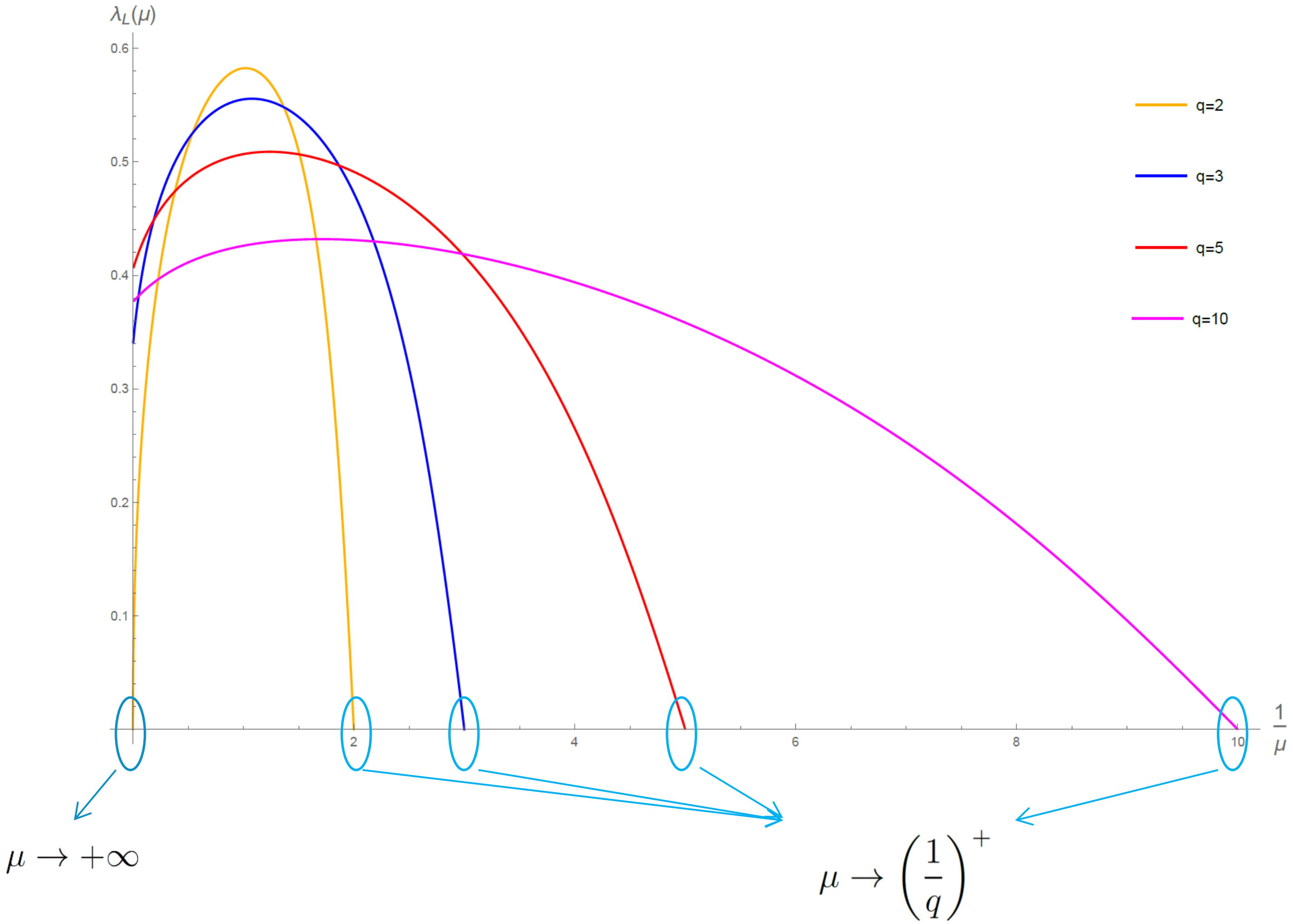




$$\langle \phi^a(t+i\tau_1, x_1)\phi^b(i\tau_2, x_2)\bar{\phi}^a(t+i\tau_3, x_3)\bar{\phi}^b(i\tau_4, x_4)\rangle$$

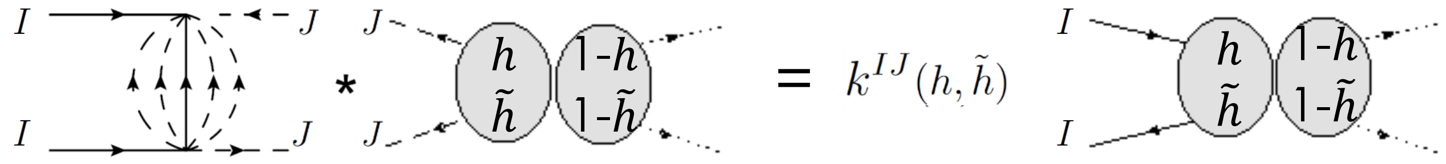
$$\propto \frac{1}{N}\, e^{\lambda_L t}$$

TWO INTERESTING LIMITS



- Lyapunov exponent drops to zero
- "Integrability" takes over ?
- Large symmetries ?

4-POINT FUNCTION

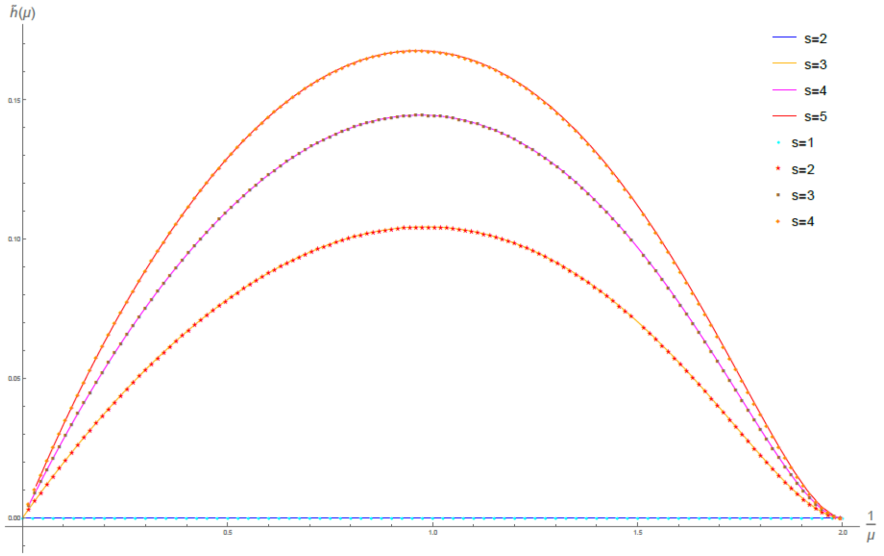
- $\langle \bar{\phi}^i \phi^i \bar{\phi}^j \phi^j \rangle \ \langle \bar{\phi}^i \phi^i \bar{\psi}^j \psi^j \rangle \ \langle \bar{\phi}^i \phi^i \bar{\lambda}^j \lambda^j \rangle$
- 

- $\begin{pmatrix} k^{\phi\phi} & k^{\phi\psi} & k^{\phi\lambda} & k^{\phi G} \\ k^{\psi\phi} & 0 & k^{\psi\lambda} & 0 \\ k^{\lambda\phi} & k^{\lambda\psi} & 0 & 0 \\ k^{G\phi} & 0 & 0 & 0 \end{pmatrix}$ whose eigenvalue x satisfies

$$E_c(x, h, \tilde{h}, \mu, q) = x^4 - k^{\phi\phi} x^3 - \left(k^{\phi G} k^{G\phi} + k^{\phi\psi} k^{\psi\phi} + k^{\phi\lambda} k^{\lambda\phi} + k^{\psi\lambda} k^{\lambda\psi}\right) x^2 + \left(k^{\phi\phi} k^{\psi\lambda} k^{\lambda\psi} - k^{\phi\psi} k^{\psi\lambda} k^{\lambda\phi} - k^{\phi\lambda} k^{\psi\phi} k^{\lambda\psi}\right) x + k^{\phi G} k^{\psi\lambda} k^{\lambda\psi} k^{G\phi} = 0$$

- Solve x=1 to get the a tower of operators $O^{\tilde{h},h}$, spin s = $|h - \tilde{h}|$.

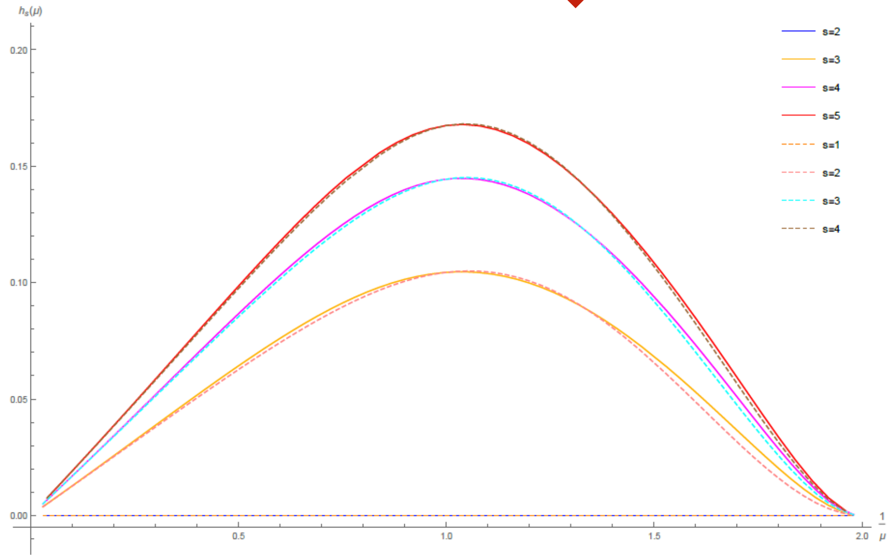
LIGHTEST OPERATORS AT EACH SPIN



$(\gamma, \gamma + s)$

Operators in the two limits have vanishing left-(right-) moving dimensions !

$(\gamma + s, \gamma)$



BACKGROUND: HIGHER-SPIN

Theories with (a large) higher-spin symmetry

- ❑ Quantum field theories
  - ❖ vector models

$$\mathrm{L} = \frac{1}{2}\left[\partial_\mu \phi_i \partial^\mu \phi_i + \frac{\lambda}{N}(\phi_i\phi_i)^2\right] \qquad J_{\mu_1\ldots\mu_s} = \phi_i \partial_{(\mu_1}\cdots\partial_{\mu_s)}\phi_i + \ldots$$

Polyakov,Klebanov,
Giombi, Yin
Aharony, Minwalla et al…

  - ❖ W_N-minimal models

$$\frac{SU(N)_k \otimes SU(N)_1}{SU(N)_{k+1}} \qquad W^{(s)} \propto \sum_{i_1,\ldots,i_s=1}^{2} \sum_{a_1,\ldots,a_s=1}^{N} d_{a_1,\ldots,a_s} J^{a_1}{}_{(i_1)} \ldots J^{a_s}{}_{(i_s)}$$

(0,s) or (s,0), if slightly broken (γ,s+γ), (s+γ, γ)

Gaberdiel,
Gopakumar…

- ❑ Higher-spin gravity

Vasiliev …

  - ❖ General relativity: graviton, spin-2
  - ❖ Higher-spin theory: graviton + higher-spin fields, spin-2,3,4,5… all fields are massless

BACKGROUND: HIGHER-SPIN

Higher-Spin theories are interesting:

- Quantum gravity contains higher-spin fields

- The most symmetric phase of quantum gravity

Gross, Mende…

$$m^2 = \frac{1}{\alpha'}(N+a)\,, \quad \alpha' \to \infty\,.$$

- A special class of solvable models of the holographic principle

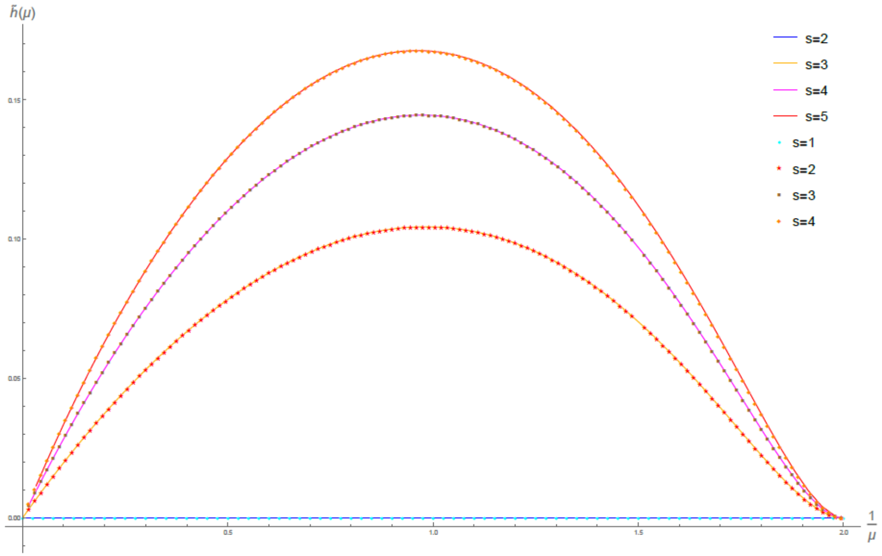
Witten, Sundborg, Gaberdiel, Gopakumar…

$$\ell_s \gg R \gg \ell_{\text{Planck}}\,, \qquad \ell_s = \sqrt{\alpha'}\,,$$

$$\Rightarrow \quad \left(\frac{R}{\ell_{\text{Planck}}}\right)^4 = N \gg 1\,, \qquad \frac{R^4}{\alpha'^2} = \lambda = g^2 N \ll 1$$

Theories with (slightly broken) higher-spin symmetry feature infinite towers of conserved operators, which is a characteristic property that could be used to detect the existence of higher-spin symmetries.

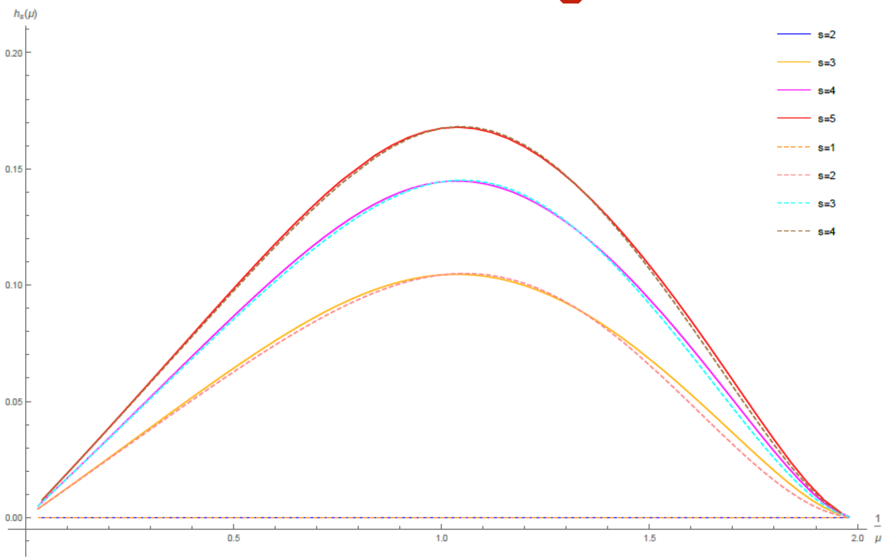
LIGHTEST OPERATORS AT EACH SPIN



⬅ $(\gamma, \gamma + s)$

$(\gamma + s, \gamma)$

⬇



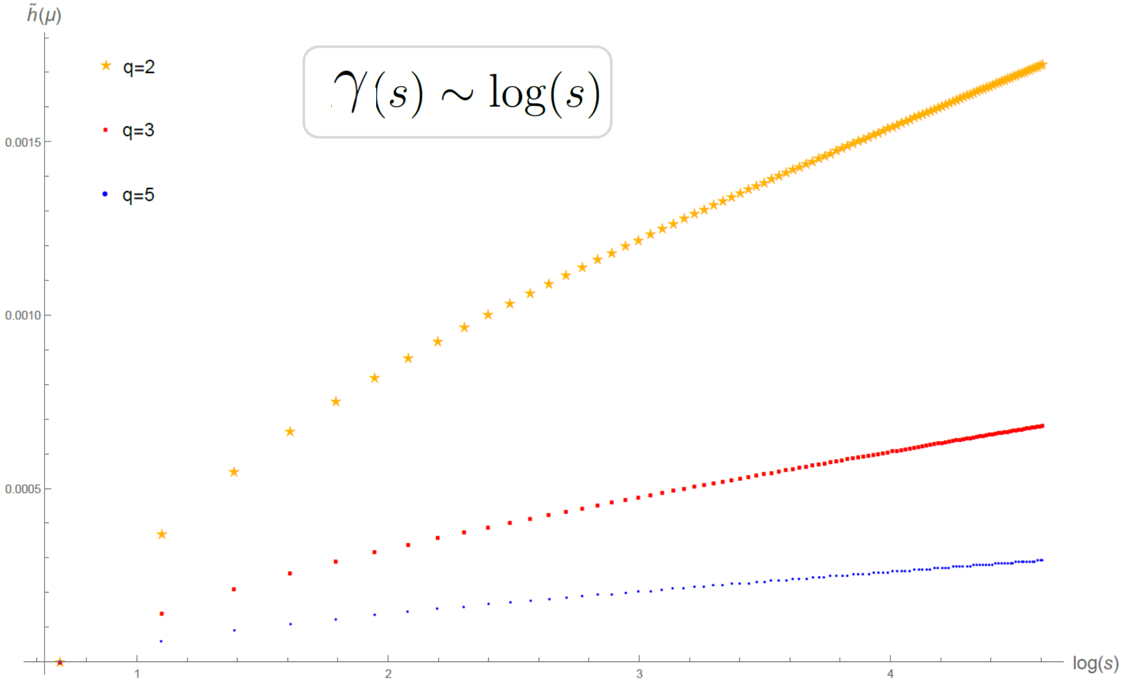
Operators in the two limits have vanishing left-(right-) moving dimensions !

Higher spin operators !

Generate large symmetry ➡ the theory becomes nonchaotic

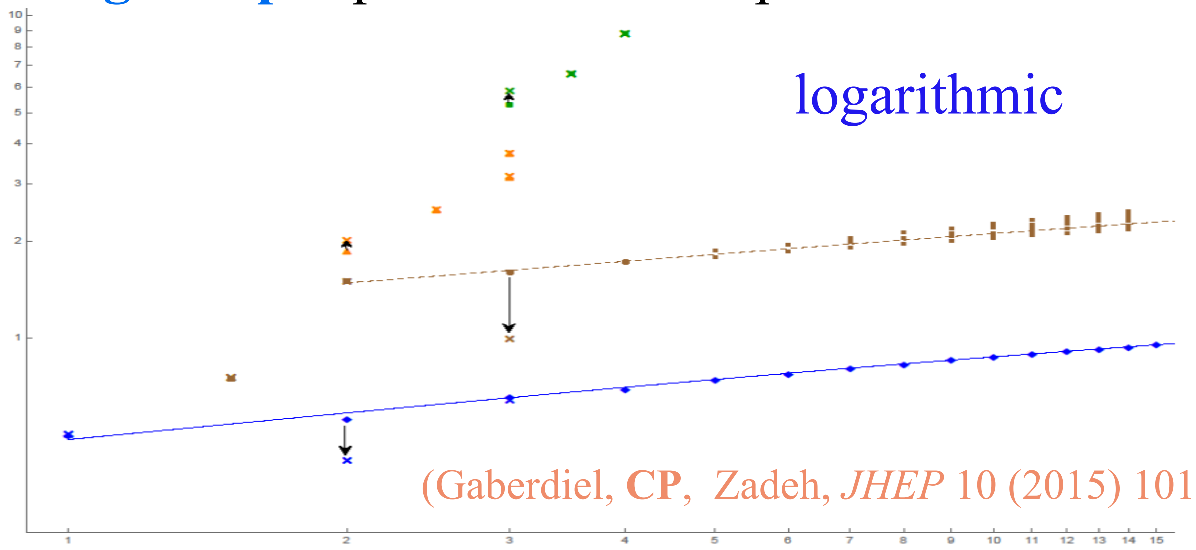
CONNECTIONS WITH CONVENTIONAL THEORIES

- Dispersion relation of this **SYK** model: the anomalous dimension γ logarithmically depends on the spin s



(**CP**, *JHEP* 12 (2018) 065)

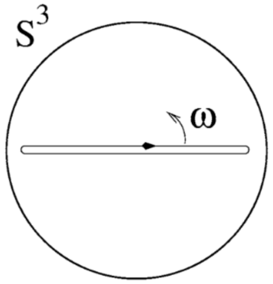
- **Higher-spin** perturbation computation



(Gaberdiel, **CP**, Zadeh, *JHEP* 10 (2015) 101)

- Rotating folded closed long **string** in AdS

$$E - S = \frac{\sqrt{\lambda}}{\pi} \ln(S/\sqrt{\lambda}) + \cdots \qquad \lambda = g_{YM}^2 N$$



logarithmic due to the AdS geometry

(Gubser, Klebanov, Polyakov, *Nucl.Phys.B* 636 (2002) 99-114)

SHORT SUMMARY

- A set of high dimensional spacetime covariant disordered models
- Tunable chaotic behavior: interpolate between chaos and integrablility
- A concrete connection to higher-spin theory and string theory.

CONTENT

- Background
- Motivation
- 1+1D disordered models
- **2+1D SYK models**
- General 2+1D disordered models

GO HIGHER…

- Many people think 1+1D is still special.

- It would be great if we can push it further to

  - Higher dimension: 2+1D

  - Get clear relations with the other known 2+1D models

A 3D SYK MODEL

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- It turns out possible to construct an N=2 Supersymmetry SYK model

$$\mathrm{L} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_i(y^\dagger)\Phi_i(y)\right) - \left[\int d^2\theta \frac{1}{3} g_{ijk}\Phi_i(y)\Phi_j(y)\Phi_k(y) + \mathrm{c.c}\right]$$

$$\mathrm{P}\left(g_{ijk}\right) \propto e^{-N^2\frac{g_{ijk}\bar{g}_{ijk}}{J}}, \qquad \left\langle g_{ijk}\right\rangle = 0, \qquad \left\langle g_{ijk}\bar{g}_{ijk}\right\rangle = \frac{J}{N^2}.$$

with N flavors of chiral multiplets

$$\Phi(X) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^2 F(y) \qquad \bar{\Phi}(X^\dagger) = \bar{\phi}(y^\dagger) + \sqrt{2}\bar{\theta}^\alpha\bar{\psi}_\alpha(y^\dagger) + \bar{\theta}^2\bar{F}(y^+)$$

- In components:

$$\mathrm{L} = -i\bar{\psi}_i\,\partial\!\!\!/\psi_i + \partial_\mu\bar{\phi}_i\partial_\mu\phi_i - \bar{F}_i F_i - g_{ijk}\left(\phi_i\phi_j F_k - \psi_i\psi_j\phi_k\right) - \bar{g}_{ijk}\left(\bar{\phi}_i\bar{\phi}_j\bar{F}_k - \bar{\psi}_i\bar{\psi}_j\bar{\phi}_k\right)$$

- The model is again solvable, and its properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

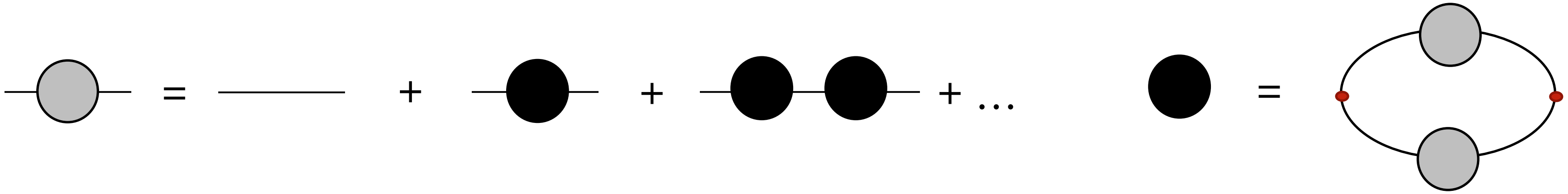
CORRELATION FUNCTIONS

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

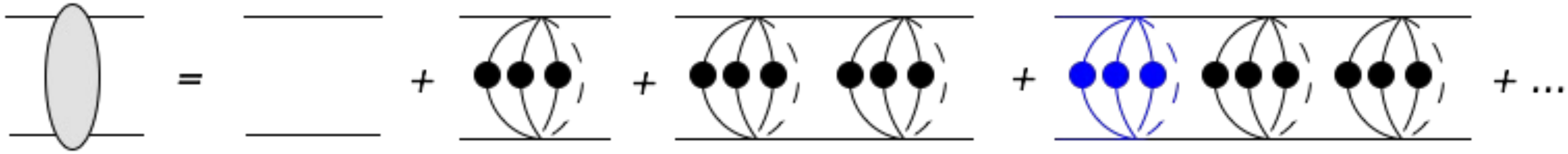
- The Green's functions

$$G_\phi(x_{12})\delta_{ij} = \langle \bar{\phi}_i(x_1)\phi_j(x_2)\rangle \qquad G_\alpha{}^\beta(x_{12})\delta_{ij} = \langle \bar{\psi}_{\alpha,i}(x_1)\psi_j^\beta(x_2)\rangle \qquad G_F(x_{12})\delta_{ij} = \langle \bar{F}_i(x_1)F_j(x_2)\rangle$$

We can efficiently sum over all loop corrections in the large-N limit



- Similarly we can compute the 4-point correlation function



SPECTRUM:

THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- From the correlation functions we get the IR spectrum, which is within the bounds obtained from numerical bootstrap

Operators	ℓ	Δ	Bootstrap bound
$(\bar{\Phi}\Phi)$	0	1.6994	<1.9098
$(\bar{\Phi}\Phi)'$	0	3.4295	<5.3
J'	1	4.2676	<5.25

Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

- Anomalous dimension $\tau = \Delta - \ell = 2\Delta_\phi + 2m + \gamma(m,\ell)$

The large-spin limit, ie fixed m, large ℓ limit

$$\gamma(m,\ell) = (-1)^{\ell+1} \frac{g_3(\Delta_\phi)}{\ell^{\Delta_\phi}} \frac{\Gamma(m-\Delta_\phi+1)}{\Gamma(m+1)}, \qquad \ell \gg 1$$

agrees with results from the light-cone analytic bootstrap

$$\gamma(m,\ell) = (-1)^{\ell} \frac{C_m}{\ell^{\tau_{\min}}} \qquad \tau_{\min} = \tau_\phi = \Delta_\phi$$

Fitzpatrick, Kaplan, Poland, Simmons-Duffin, *JHEP* 12 (2013) 004

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

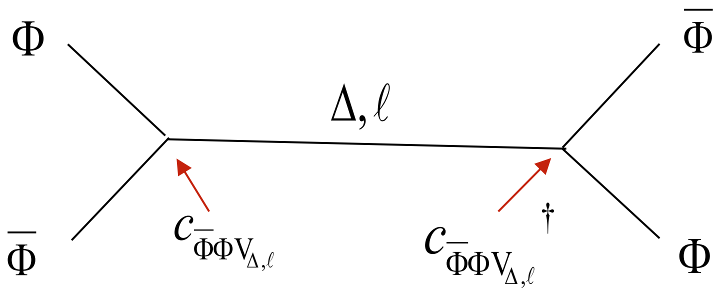
- The meaning of this check: the bootstrap bounds come from the requirement of
  - Unitarity
  - Causality
  - Locality
  - Crossing symmetry

- Consistency with the bootstrap bound means consistency with these general principles, hence no superficial contradiction to be worried about in this ensemble-average theory.

A 3D SYK MODEL v.s. LOCALIZATION

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- Can also read off the central charge



$$\frac{\langle \bar{\phi}(x_1)\phi(x_2)\phi(x_3)\bar{\phi}(x_4)\rangle}{\langle \bar{\phi}(x_1)\phi(x_2)\rangle\langle \phi(x_3)\bar{\phi}(x_4)\rangle} \supset -\frac{|C_{\bar{\phi}\phi J_R}|^2}{C_J} V_{s^2}^2\, G_{2,1}(u,v)$$

we get

$$C_J = \frac{4}{9\,|c_{\bar{\Phi}\Phi R}|^2} = N\frac{2^7}{3^4\sqrt{3}\pi}\left(\frac{2\pi}{\sqrt{3}} - \frac{9}{8}\right) \qquad C_T = 6C_J$$

- Agrees with the result from localization of the 3d N=2 Wess-Zumino model

Nishioka, Yonekura, *JHEP* 05 (2013) 165
Gang, Yamazaki, *JHEP* 02 (2020) 102

CONTENT

- Background
- Motivation
- 1+1D disordered models
- 2+1D SYK models
- **General 2+1D disordered models**

GENERAL 3D DISORDERED MODELS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- We can in fact consider more general 2+1d disordered models
  - ✓ with supersymmetry

$$\mathrm{L}_{\text{susy}} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_i(y^+)\Phi^i(y) + \bar{\Sigma}_a(y^+)\Sigma^a(y)\right) - \left[\int d^2\theta \frac{1}{2} g_{aij}\Sigma^a(y)\Phi^i(y)\Phi^j(y) + \text{c.c}\right]$$

where Φ^i and Σ^a are chiral $\mathcal{N}$=2 multiplets with $i = 1\ldots N$ and $a = 1\ldots M$

  - ✓ or without supersymmetry

$$\mathrm{L}_{\text{bos}} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i + \frac{1}{2}g_{aij}\sigma^a\phi^i\phi^j - \frac{1}{4}(\sigma^a)^2$$

where ϕ_i and σ^a are bosonic fields with $i = 1\ldots N$ and $a = 1\ldots M$

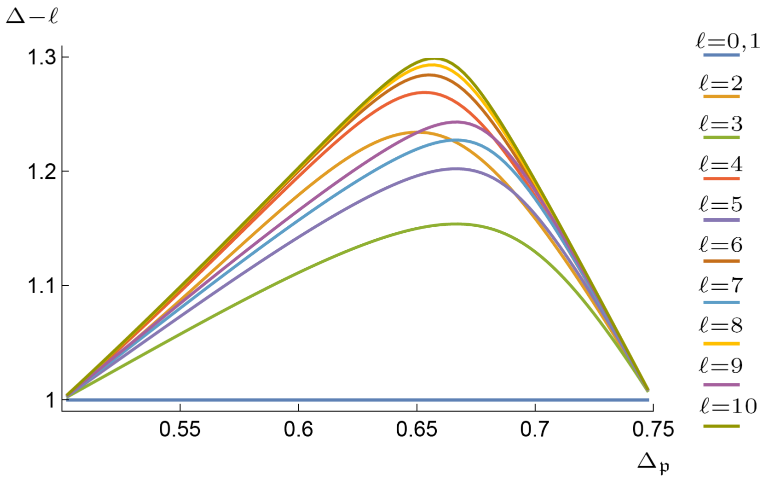
- Can solve the model in the large-N limit in the IR analytically

$$N \to \infty, \qquad \lambda \equiv M/N, \quad \text{fixed}$$

3D DISORDERED MODELS: HIGHER-SPIN LIMITS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- This model is again consistent with those physical requirements
- There is also a clear connection to higher-spin theories
- There are special limits
  - Free p — $\lambda \to 0,\ \Delta_p = \frac{1}{2},\ \Delta_s = 1$ — higher-spin
  - Free s — $\lambda \to \infty,\ \Delta_p = \frac{3}{4},\ \Delta_s = \frac{1}{2}$ — higher-spin
  - Diagonal — $\lambda \to \frac{1}{2},\ \Delta_p = \frac{2}{3},\ \Delta_s = \frac{2}{3}$ — the 2+1D SYK model



- This indicates the connection to higher-spin theories is probably universal

WHAT DO THESE NEW MODELS BUY US?

- New 2d/3d SCFT fixed points, and they look normal (not strange)
- Candidates for future bootstrap discoveries
- Proof of concept examples of disordered models in higher dimensions
- Rich relations to other known models (e.g. Higher-spin theories and String theories)
- Develop techniques for future

THANK YOU!