D-branes and Orbit Average

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Based on papers with Peihe Yang, Yunfeng Jiang, Shota Komatsu, 2103.16580[hep-th]

ICTS-PCFT, USTC

August 27, 2021

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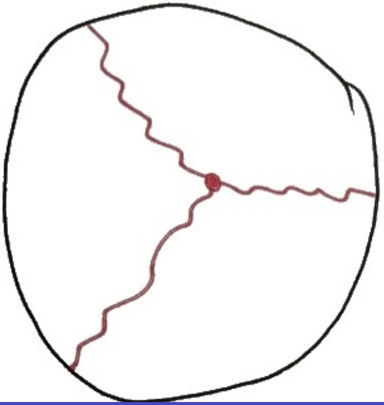
- Some non-BPS local operators with large conformal weights are dual to semi-classical string solutions. *[GKP, 02, for $\mathcal{N}=4$ SYM][Bin Chen, JW 08, for ABJM]*

- The three point function of single trace light operators (dual to supergravitons) are computed holographically using Witten diagrams. *[GKP, 98][Witten, 98]*.



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- In ABJM theory, the 3pt functions of $1/3$-BPS operators do not enjoy such a non-renormalization theorem *[Hirano, Kristjansen, Young, 12]*.
- Computation of such functions for most general case is still great challenge for supersymmetric localization and integrability method.

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- In planar limit, this problem is essentially solved by integrability. (Review: *[Beisert etal, 12]*)
- The holographic computation of the conformal weight is just compute the energy of the dual string solutions.

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- Contributions from open string attached on such D-branes was also computed.

- Later on, more examples of 3pt HHL correlators for D-branes were computed, both for $\mathcal{N} = 4$ SYM *[Bissi, Kristjansen, Young, Zoubos, 11]* and ABJM theories *[Hirano, Kristjansen, Young, 12]*.

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- Contributions from the wave functions of the heavy states.
- This two effects were studied in *[Bajnok, Janik, Wereszczynski, 14]* for semiclassical string cases. But their treatment seems incomplete.

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- The results at weaking coupling and strong coupling are different, as expected.
- It is interesting to get wrapping corrects at strong coupling from the holographic result.

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- Here by light, we mean the quantum numbers of $\mathcal{O}$ are small.

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and the path integral

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- Here we have already assume that $\langle \theta | \mathcal{O} | \theta' \rangle = \mathcal{O}[\theta]\delta(\theta - \theta')$.

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- Now, suppose we found one solution satisfying the equation (6), $\theta_0^*(t)$. Then, it immediately follows from the $U(1)$ invariance (1) that there should be a family of solutions, or equivalently a moduli of solutions, given by

$$\theta_c^*(t) \equiv \theta_0^*(t) + c\,, \qquad c \in [0, 2\pi]\,. \tag{7}$$

- Therefore, the correct saddle-point formula is given by

$$\langle J|\mathcal{O}(t=0)|J\rangle \overset{\text{WKB}}{=} \int_0^{2\pi} \frac{\mathrm{d}c}{2\pi}\, e^{-iJ\theta_c^*(+\epsilon)}\mathcal{O}[\theta_c^*(0)]e^{iJ\theta_c^*(-\epsilon)}e^{\frac{i}{\hbar}S[\theta_c^*]}\,. \tag{8}$$

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- As we can see, the final result is given by an average over the parameter c and this is precisely the orbit average discussed in [Bajnok, Janik, Wereszczynski, 14].

Boundary term

- Let us now generalize the computation slightly and consider the situation in which the bra and ket states are not identical: $\langle J+q|\mathcal{O}|J\rangle$. We assume J is again large $(J \sim 1/\hbar \gg 1)$ while q is taken to be $O(1)$.

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- Especially, for $\mathcal{O}$ being $\mathcal{O}_p \equiv e^{ip\theta}$, an operator with $U(1)$ charge p, we have

$$\langle J+q|\mathcal{O}_p(t=0)|\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]}\delta_{p,q}, \quad (12)$$

where $\delta_{p,q}$ is manifestation of the $U(1)$ charge conservation.

Two lessons on boundary term

- First, when the bra and ket states are different, there is a nontrivial (boundary-term) contribution from the wave functions.
- Second, such contributions, together with the orbit average, are essential for reproducing a correct charge conservation $\delta_{p,q}$.

HHL $3-$point functions

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- The sub-determinant operator with charge M will be denoted by $\mathcal{D}_M$ and the single trace operator with charge L will be denoted by $\mathcal{O}_L$.
- Structure constant:

$$\langle\hat{\mathcal{D}}_{M+k}|\hat{\mathcal{O}}_L(t=0)|\hat{\mathcal{D}}_M\rangle = \int DX\,\Psi^*_{M+k}[X]\hat{\mathcal{O}}_L[X(t=0)]$$
$$\Psi_M[X]e^{-S_{\text{DBI+WZ}}[X]}\,.$$

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- Shift in wave functions $\Psi \sim \exp(-i\Delta t + iJ\phi)$,
$$\Psi \mapsto e^{-\Delta\tau_0+iJ\phi_0}\Psi. \tag{14}$$

Master equation

$$\langle\hat{\mathcal{D}}_{M+k}|\hat{\mathcal{O}}_L(t=0)|\hat{\mathcal{D}}_M\rangle = \underbrace{\int \mathrm{d}\tau_0 \int \frac{\mathrm{d}\phi_0}{2\pi}}_{\text{orbit average}} \hat{\mathcal{O}}_L[X^*_{\tau_0,\phi_0}(t=0)]$$
$$\underbrace{e^{(\Delta_{M+k}-\Delta_M)\tau_0} e^{-i(J_{M+k}-J_M)\phi_0}}_{\text{wave function}} . \tag{15}$$

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  as in the old approach *[Bak, Chen, JW, 11][Bissi, Kristjansen, Young, Zoubos, 11]*.
- *Remark:* The last step is similar to the holographic computations of correlators of BPS Wilson loops (surfaces) and local BPS operators *[Berenstein, Corrado, Fischler, Maldacena, 98][Giombi, Ricci, Trancanelli, 06][Chen, Liu, JW, 07]*

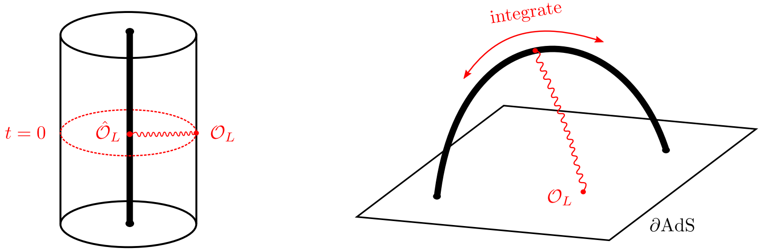

Figure: Comparison of new and old approaches.

- We consider the following sub-determinant operator in $\mathcal{N}=4$ super Yang-Mills

$$\mathcal{D}_M = \chi_M(Z) \equiv \frac{1}{M!}\delta^{[b_1 b_2 \cdots b_M]}_{[a_1 a_2 \cdots a_M]} Z^{a_1}_{b_1} \cdots Z^{a_M}_{b_M} , \tag{17}$$

$$\delta^{[b_1 \cdots b_M]}_{[a_1 \cdots a_M]} \equiv \sum_{\sigma \in S_M} (-1)^{|\sigma|} \delta^{b_1}_{a_{\sigma_1}} \cdots \delta^{b_M}_{b_{\sigma_M}} . \tag{18}$$

and the following single trace operator

$$\mathcal{O}_L \equiv \mathrm{tr}\tilde{Z}^L , \qquad \tilde{Z} = \frac{Z + \bar{Z} + Y - \bar{Y}}{2} , \tag{19}$$

The metric of $AdS_5 \times S^5$ with unit radius and in terms of the global coordinates,

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\text{AdS}} + \mathrm{d}s^2_{S^5} \,, \tag{20}$$

where

$$\begin{aligned} \mathrm{d}s^2_{\text{AdS}} &= -\cosh^2\rho\, \mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\rho\, \mathrm{d}\widetilde{\Omega}_3^2 \,, \\ \mathrm{d}s^2_{S^5} &= \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 + \cos^2\theta\, \mathrm{d}\Omega_3^2 \,. \end{aligned} \tag{21}$$

where $\mathrm{d}\widetilde{\Omega}_3^2$ and $\mathrm{d}\Omega_3^2$ are the metric on S^3 which we parametrize as

$$\begin{aligned} \mathrm{d}\tilde{\Omega}_3^2 &= \mathrm{d}\tilde{\chi}_1^2 + \sin^2\tilde{\chi}_1\, \mathrm{d}\tilde{\chi}_2^2 + \cos^2\tilde{\chi}_1 \mathrm{d}\tilde{\chi}_3^2 \,, \\ \mathrm{d}\Omega_3^2 &= \mathrm{d}\chi_1^2 + \sin^2\chi_1\, \mathrm{d}\chi_2^2 + \cos^2\chi_1 \mathrm{d}\chi_3^2 \,. \end{aligned} \tag{22}$$

- The D-brane dual to $\mathcal{D}_M$ is localized at $\theta = \theta_0$ and extended along $\chi_{1,2,3}$ directions. It is rotating along the ϕ direction at the speed of light. The worldvolume coordinates of the D3 brane σ^μ $(\mu = 0, 1, 2, 3)$ are identified with the target space coordinates as follows:

$$\rho = 0, \qquad \sigma^0 = t, \quad \phi = t, \qquad \sigma^i = \chi_i, \quad i = 1, 2, 3. \tag{23}$$

- The value of θ_0 is related to the charge of the giant graviton as;

$$\cos^2 \theta_0 = \frac{M}{N}\,, \tag{24}$$

- Note that the classical D3-brane equations of motion lead to $\phi = t$.
- The holographic dual of $\mathcal{O}_L$ is the fluctuation of the background fields (super-graviton). I omit the details here.

Results for $\mathcal{N} = 4$ SYM

- Diagonal structure constant

$$C_{\mathcal{D}_M \mathcal{D}_M \mathcal{O}_L} = -\frac{i^L + (-i)^L}{2\sqrt{L}} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0) \right) . \quad (25)$$

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- The old computations without orbit average failed to reproduce the field theory results here.
- The holographic off-diagonal structure constant, with orbit average and contributions of wave functions included, matches the field theory results for non-extremal cases as well.

Results for $\mathcal{N} = 4$ SYM

- Off-diagonal structure constant,

$$C_{\mathcal{D}_{M+k}\mathcal{D}_M\mathcal{O}_L} = -\frac{1}{2}\sqrt{L}\left(i^{L-k}+(-i)^{L-k}\right)\frac{\Gamma(\frac{L+k}{2})\cos^2\theta_0\sin^k\theta_0}{\Gamma(1+k)\Gamma(1+\frac{L-k}{2})}\,{}_2F_1\left(1+\tfrac{k-L}{2},1+\tfrac{k+L}{2},1+k;\sin^2\theta_0\right)\,.$$

for $L > k$.

Results for $\mathcal{N} = 4$ SYM

- Off-diagonal structure constant,

$$C_{\mathcal{D}_{M+k}\mathcal{D}_M\mathcal{O}_L} = -\frac{1}{2}\sqrt{L}\left(i^{L-k} + (-i)^{L-k}\right)\frac{\Gamma(\frac{L+k}{2})\cos^2\theta_0\sin^k\theta_0}{\Gamma(1+k)\Gamma(1+\frac{L-k}{2})}\, {}_2F_1\left(1+\frac{k-L}{2}, 1+\frac{k+L}{2}, 1+k; \sin^2\theta_0\right) .$$

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- This matches the field theory results for non-extremal cases as well.

Application to ABJM theory

- Diagonal structure constant,

$$
\begin{aligned}
C_{\mathcal{D}_M \mathcal{D}_M \mathcal{O}_L} &= \left(\frac{\lambda}{2\pi^2}\right)^{1/4} \frac{\sqrt{2L+1}}{L}(1+(-1)^L) \\
& \frac{(-1)^{\frac{L}{2}+1} 2^L \sqrt{\pi}\,\Gamma(\frac{L}{2}+1)}{\Gamma(\frac{L+3}{2})}(1-4\alpha^4)^{\frac{1}{2}(L-1)} \\
\times & \left[(1-4\alpha^4)\,{}_2F_1\left(-\tfrac{1}{2}(L+1), -\tfrac{L}{2}; 1; \tfrac{4\alpha^4}{4\alpha^4-1}\right)\right. \\
& \left. +2\alpha^4(L+1)\,{}_2F_1\left(-\tfrac{1}{2}(L-1), -\tfrac{L}{2}+1; 2; \tfrac{4\alpha^4}{4\alpha^4-1}\right)\right] .
\end{aligned} \tag{26}
$$

with the relation among M, N and α is

$$
\frac{M}{N} = \sqrt{1-4\alpha^4} - 4\alpha^4 \log\left(\frac{1+\sqrt{1-4\alpha^4}}{2\alpha^2}\right) . \tag{27}
$$

Application to ABJM theory

- The strong coupling results are different from the weakly coupling ones.
- This is as expected, since there are no non-renormalization theorems for BPS $3-$pt functions in ABJM theory.
- The result is to be tested against integrability.

Conclusion

- We compute HHL correlators from branes dual to sub-determinant operators, including orbit average and wave function contributions.

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- For ABJM theory, where there are no such non-renormalization theorems, the holographic computations provide a non-trival prediction for field theory computations at strong coupling.
- For off-diagonal case $\langle \mathcal{D}_{M+k} | \mathcal{O}_J | \mathcal{D}_M \rangle$, the holographic result is sensitive to k, though $k \leq M, N$.

Outlook

- Compute the HHL correlators at arbitrary coupling in planar limit using integrability.

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- Compute the HHL correlators at arbitrary coupling in planar limit using integrability.
- Revisit the holographic computations of HHL correlators for GKP strings.

Thanks for Your Attention !