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实验，格点和有效模型

吴佳俊 (中国科学院大学)

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Argonne National Laboratory　The University of Adelaide　兰州大学 Lanzhou University　Bonn　上海理工大学 University of Shanghai for Science and Technology

中国科学技术大学，2021.04.08

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目录

- 物理动机
- 哈密顿有效方法介绍 (HEFT)
- 有限体积中的角动量混合
- 三体系统的有限体积能谱
- 总结和展望

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量子色动力学
QCD-Quantum chromodynamics

q表示夸克，u，d，s
a表示颜色，红，绿，蓝
μ表示四矢量指标
C表示8个SU(3)群的生成元(颜色的)

- 描述强相互作用的理论。
- 拉氏量

$$L=\sum_{q,a,\mu,C}\overline{\psi}_{q,a}\left(i\gamma^{\mu}\partial_{\mu}\delta_{ab}-g_{s}\gamma^{\mu}t^{C}_{ab\mu}A^{C}_{\mu}-m_{q}\delta_{ab}\right)\psi_{q,b}-\sum_{\mu,\nu,C}\frac{1}{4}F^{C}_{\mu\nu}F^{C\,\mu\nu}$$

ψ表示夸克场
A表示胶子场
f表示SU(3)群的结构常数

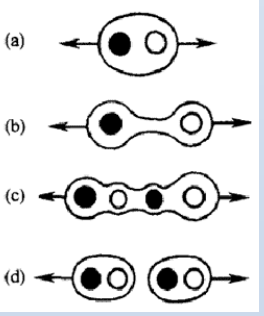
夸克协变的四动量项

夸克胶子的相互作用项

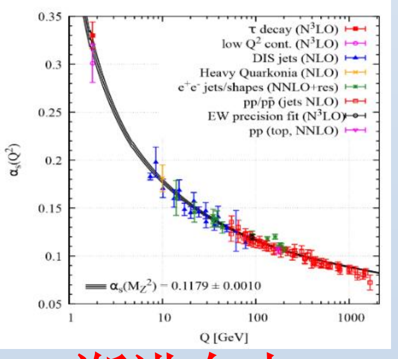
夸克质量项

胶子相互作用

$$F^{C}_{\mu\nu}=\partial_{\mu}A^{C}_{\nu}-\partial_{\nu}A^{C}_{\mu}-g_{s}f_{CAB}A^{A}_{\mu}A^{B}_{\mu}$$



色禁闭



渐进自由

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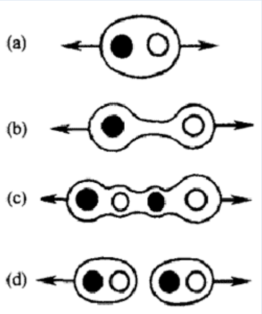
由于夸克被禁闭在强子(质子，中子，π介子等)内，所以强子是参与强相互作用中最小的可见粒子，研究强子的性质是唯一能正确理解QCD的途径，特别是谱学的研究。



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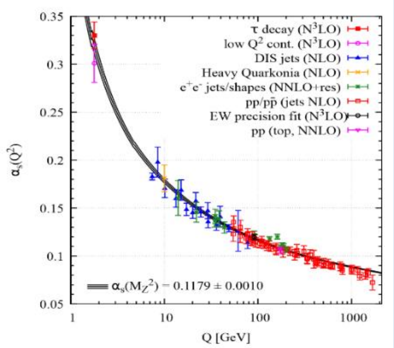

渐进自由

共振态性质：质量，宽度，极点位置，耦合常数，结构……

强子物理

实验观测量
微分截面，衰变宽度

QCD 理论

什么是Lattice QCD

- 将QCD 的基本自由度定义在离散的四维欧氏时空格子上。
- 夸克和反夸克场被定义在格点上，规范场则定义在相邻两格点间的链接上。
- 超立方格子的体积为$(N_s a)^3 \times (N_T a)$，其中格点间距a 和空间方向上长度$N_s a$ 提供了量子场论的紫外和红外截断。

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- 利用路径积分量子化进行表述，格点QCD 形式上类似于一个统计物理模型。如果将所有的场变量集中记为ϕ，相应的有效作用量记为$S[\phi]$，则我们感兴趣的任何一个物理量$O[\phi]$ 的期望值$\langle O \rangle$ 可以表达为，

$$\langle O \rangle = \int D\phi O[\phi] P[\phi] \qquad P[\phi] = \frac{1}{Z} e^{-S[\phi]} \qquad Z = \int D\phi e^{-S[\phi]}$$

- 其中$P[\phi]$ 是场应当服从的玻尔兹曼分布。这非常类似于一个统计系统中的系综平均值，Z 即为配分函数——包含了系统所有的物理。正是这种相似性使得Monte Carlo方法的使用成为可能。

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- 相应的，典型的格点QCD 数值计算也可以分为三大步骤：
- 第一，利用Monte Carlo 算法产生正确分布的规范场组态；
- 第二，从正确分布的组态中取样对感兴趣的物理量$O[\phi]$ 进行测量；
- $$\sum_{(\vec{y}-\vec{x})\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})} \left\langle T\left(\psi(t;\vec{y}), \psi^\dagger(t;\vec{y})\right)\right\rangle \sim \sum_{\Gamma,i} Z_i^\Gamma e^{-E_i^\Gamma t}$$
- 第三，把有限体积内的物理量和真实的物理量相联系。

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共振态性质：质量，宽度，极点位置，
耦合常数，结构……

T 矩阵，
相移，
非弹系数

实验观测量
微分截面，衰变宽度

格点能谱

Lattice QCD

QCD 理论

共振态性质：质量，宽度，极点位置，
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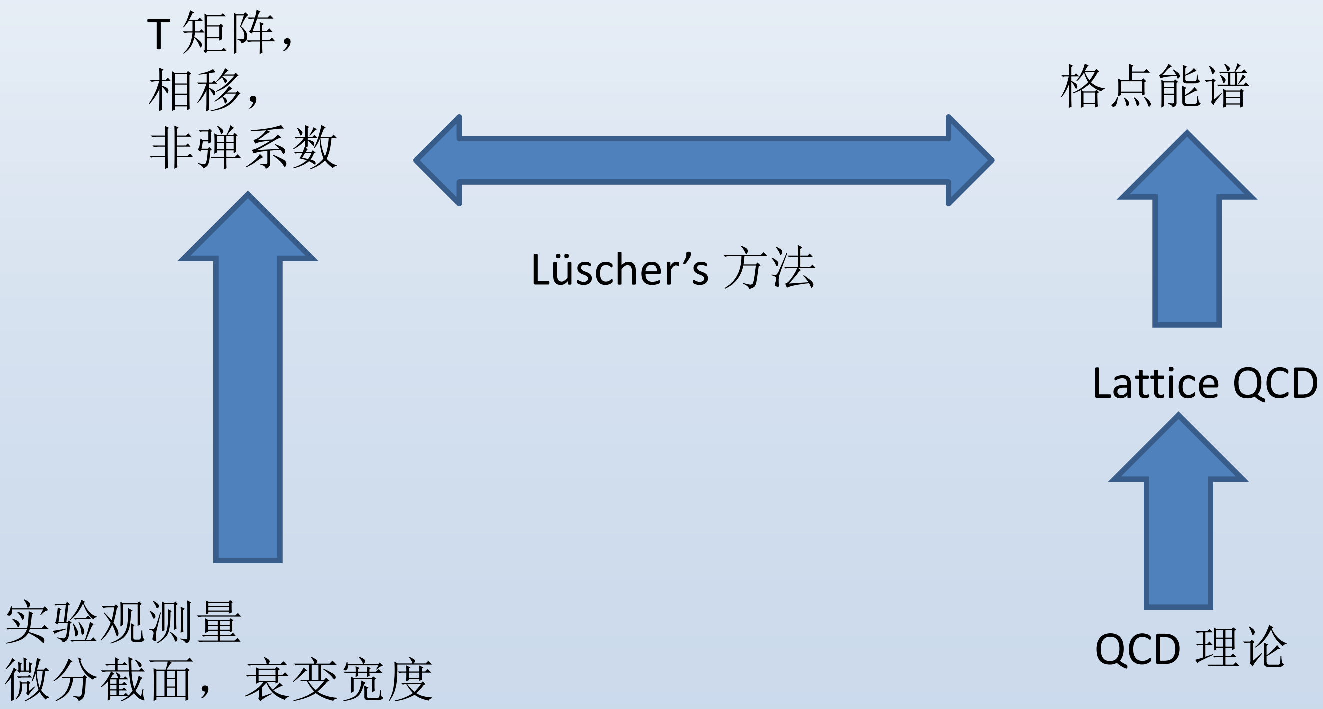
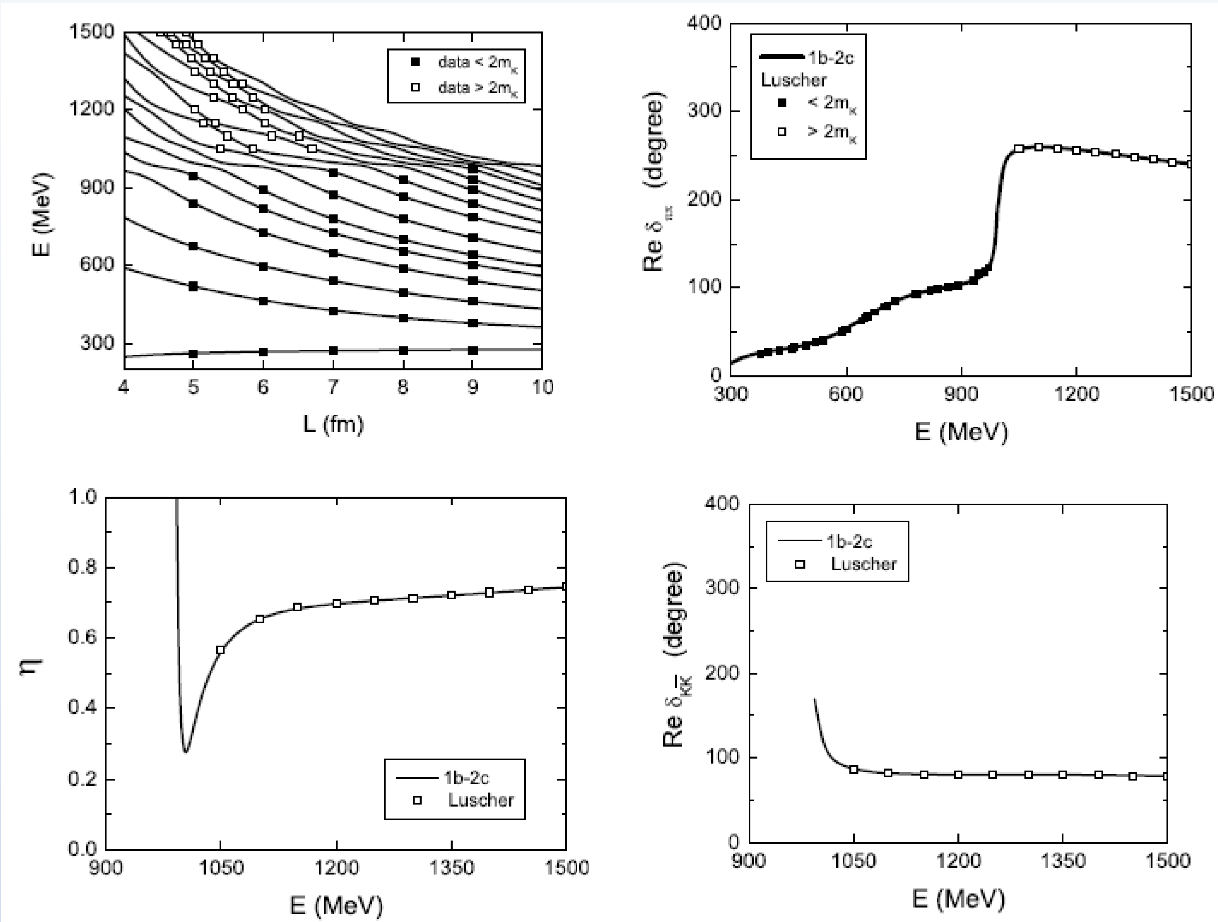


ππ –> ππ & ππ –> $\bar{K}K$ & $\bar{K}K$–> $\bar{K}K$



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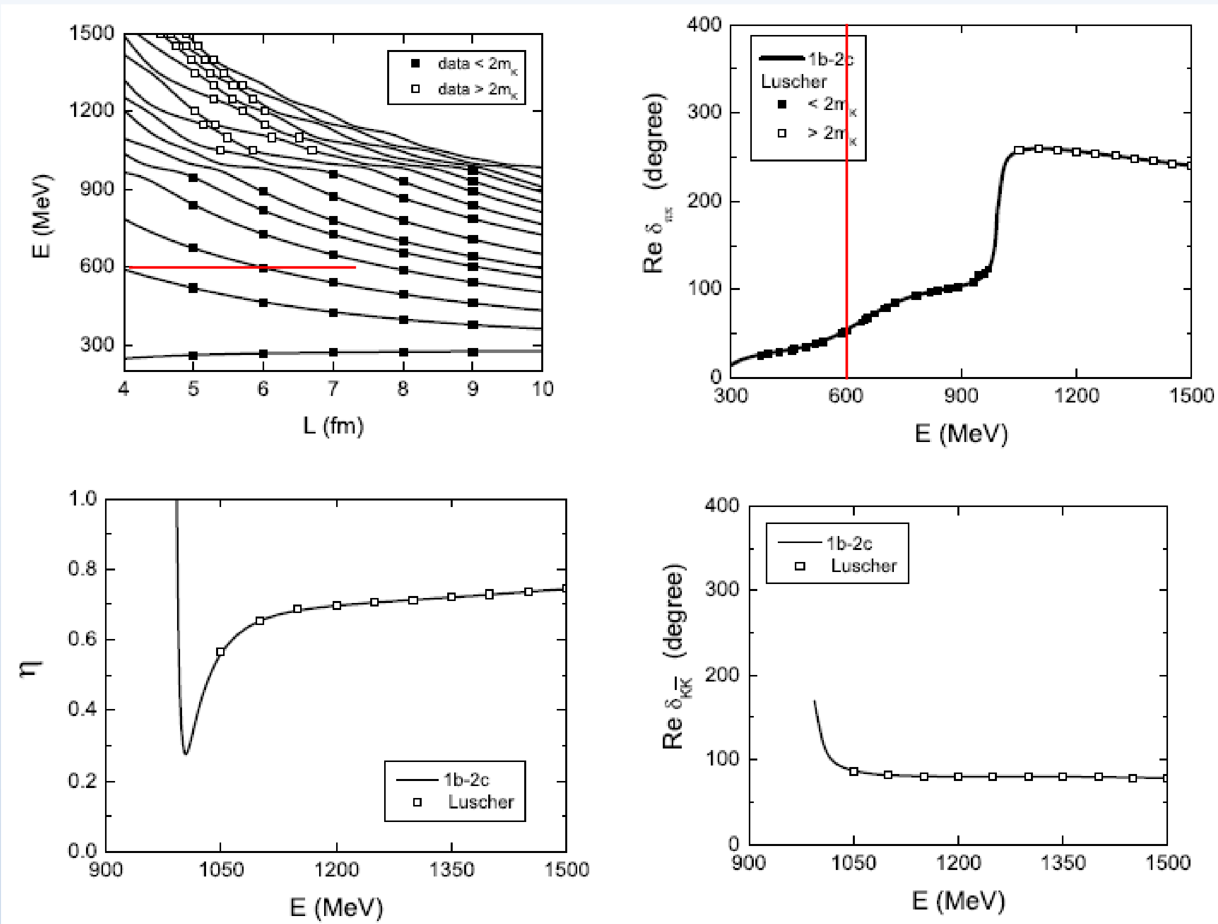
ππ -> ππ & ππ -> $\bar{K}K$ & $\bar{K}K$-> $\bar{K}K$



$\bar{K}K$ 阈下

L —— E —— $\delta_{\pi\pi}(E)$

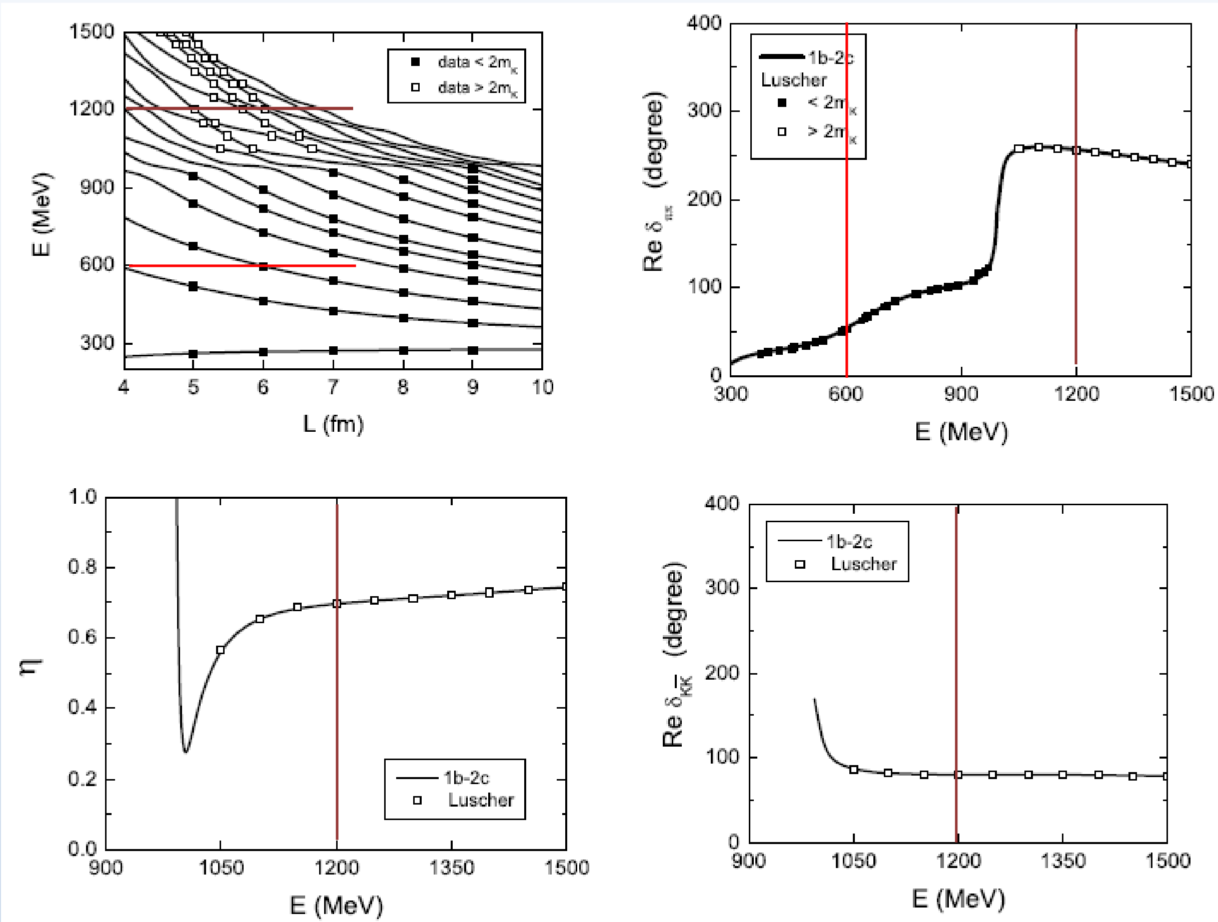
$\delta_{\pi\pi}(k)=\Delta_{\pi\pi}(L) \bmod \pi$

M. Luscher, NPB 354, 531 (1991).

$$\Delta_\alpha(L)=\tan^{-1}\left(\frac{q_\alpha \pi^{3/2}}{Z_{00}(1,q_\alpha^2)}\right)$$

$$q_\alpha=\sqrt{\frac{E(L)^2}{4}-m_\alpha^2}$$

$\pi\pi \to \pi\pi$ & $\pi\pi \to \bar{K}K$ & $\bar{K}K \to \bar{K}K$



$\bar{K}K$ 阈下

L —— E —— $\delta_{\pi\pi}(E)$

$$\delta_{\pi\pi}(k) = \Delta_{\pi\pi}(L) \bmod \pi$$

M. Luscher, NPB 354, 531 (1991).

$\bar{K}K$ 阈上

L_1, L_2, L_3 —— E

$\Updownarrow$

$\delta_{\pi\pi}(E)$, $\delta_{K\bar{K}}(E)$, $\eta(E)$

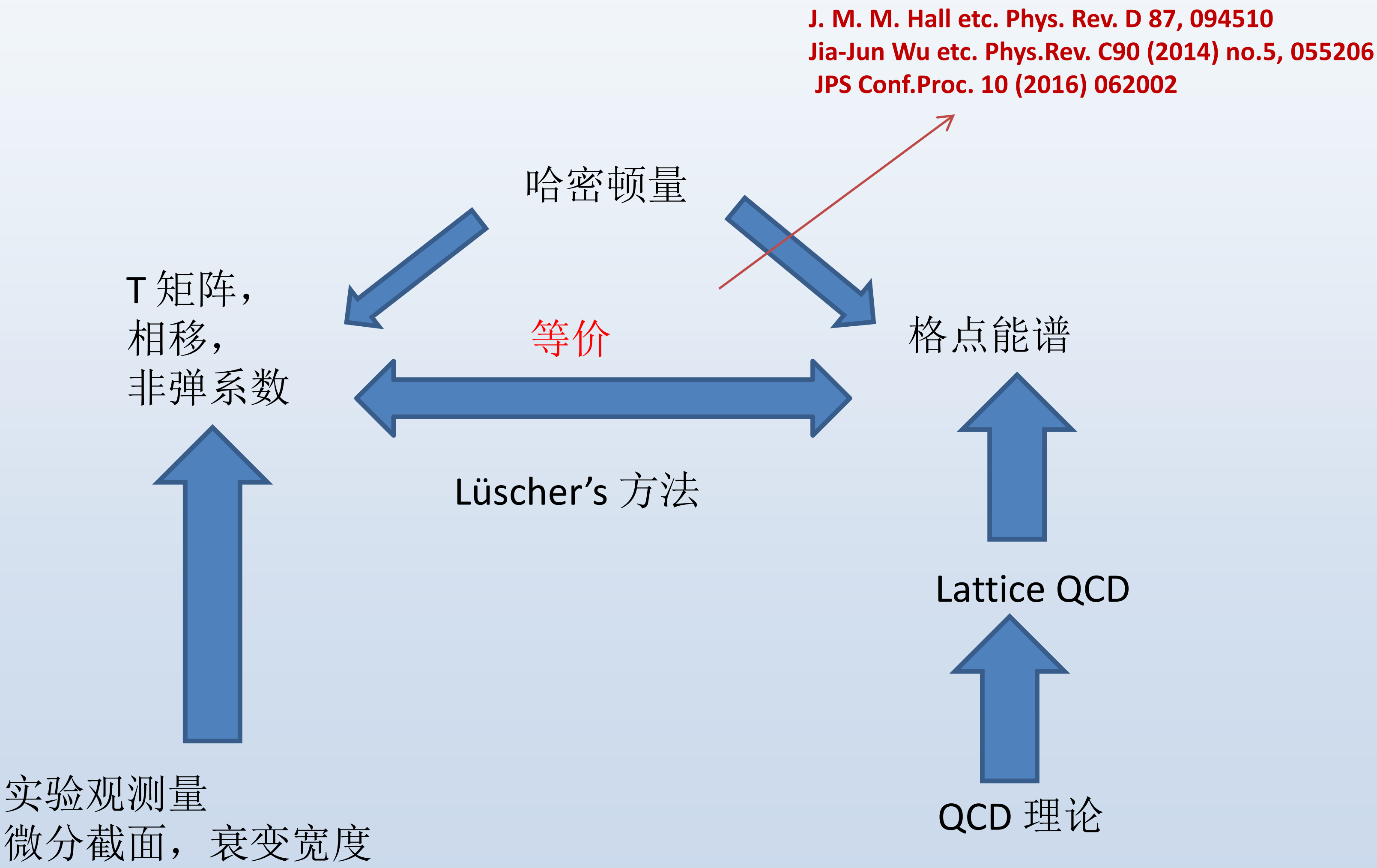
$$\Delta_\alpha(L) = \tan^{-1}\left(\frac{q_\alpha \pi^{3/2}}{Z_{00}(1, q_\alpha^2)}\right)$$

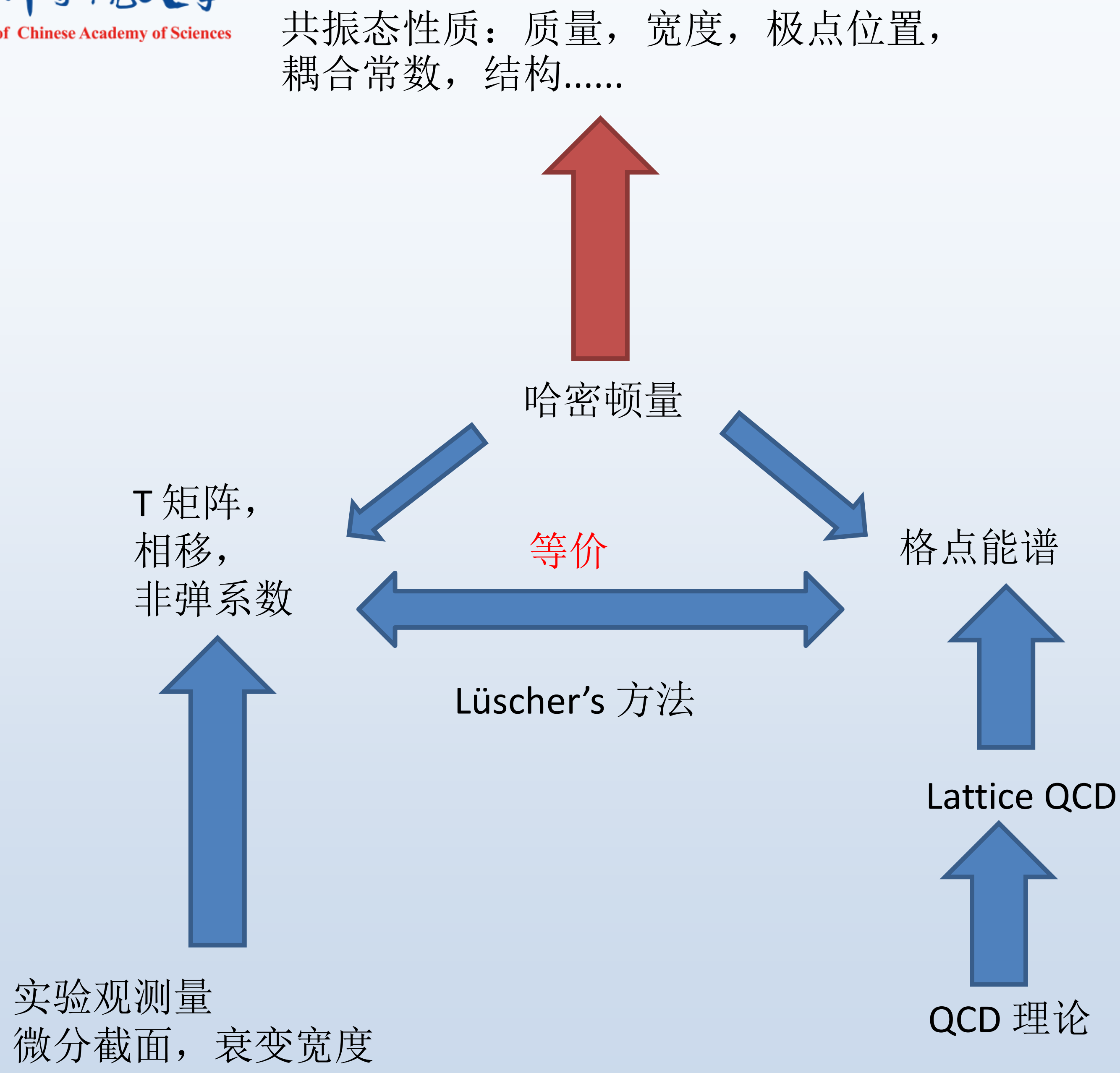
$$q_\alpha = \sqrt{\frac{E(L)^2}{4} - m_\alpha^2}$$

$$0 = \cos\left(\Delta_{\pi\pi}(L) + \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) - \delta_{K\bar{K}}(E)\right)$$
$$- \eta \cos\left(\Delta_{\pi\pi}(L) - \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) + \delta_{K\bar{K}}(E)\right)$$

S.He,X.Feng,andC.Liu, JHEP 07(2005)011

共振态性质：质量，宽度，极点位置，
耦合常数，结构……

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哈密顿量
T 矩阵，
相移，
非弹系数
等价
格点能谱
Lüscher’s 方法
Lattice QCD
实验观测量
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哈密顿量

T 矩阵，
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格点能谱

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Hamiltonian EFT

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   Phys.Rev. D87 (2013) no.9, 094510
2. **Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD**
   J-J Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young
   Phys.Rev. C90 (2014) no.5, 055206
3. **Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev.Lett. 116 (2016) no.8, 082004
4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
   J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young
   Phys.Rev.Lett. 114 (2015) no.13, 132002
5. **Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.3, 034034
6. **Structure of the Λ(1405) from Hamiltonian effective field theory**
   Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.1, 014506
7. **Nucleon resonance structure in the finite volume of lattice QCD**
   Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas
   Phys.Rev. D95 (2017) no.11, 114507
8. **Structure of the Roper Resonance from Lattice QCD Constraints**
   Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W. Thomas
   Phys.Rev. D97 (2018) no.9, 094509
9. **Partial Wave Mixing in Hamiltonian Effective Field Theory**
   Yan Li, Jia-jun Wu, C. D. Abell, Derek B. Leinweber, Anthony W. Thomas
   Phys.Rev. D101 (2020) no.11, 114501
10. **Hamiltonian effective field theory in elongated or moving finite volume**
    Yan Li, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas
    e-Print: 2103.12260 [hep-lat]

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哈密顿量

$$H = H_0 + H_I$$

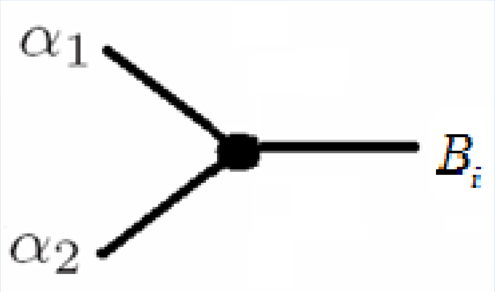
$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2} \right] \langle \alpha(k_\alpha)|$$

$|B_i>$ bare state， bare mass m_i

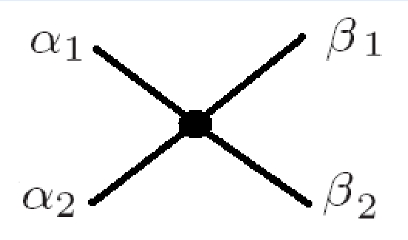
$|\alpha(k_\alpha)>$ non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$



$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



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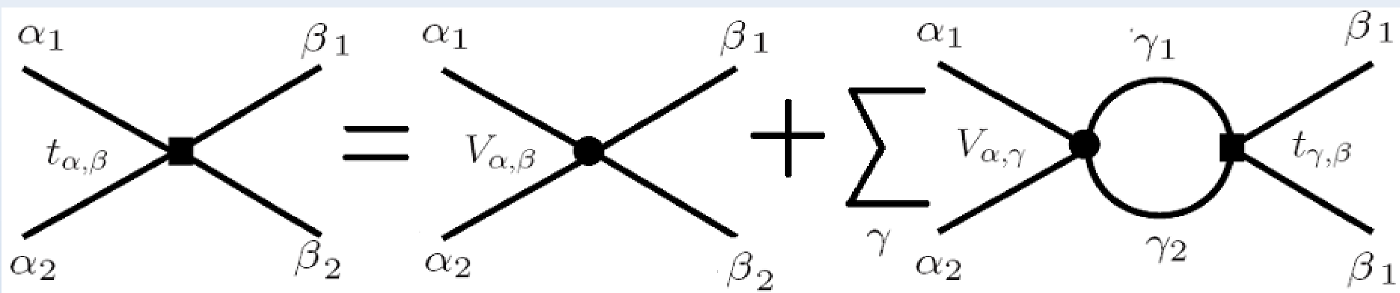
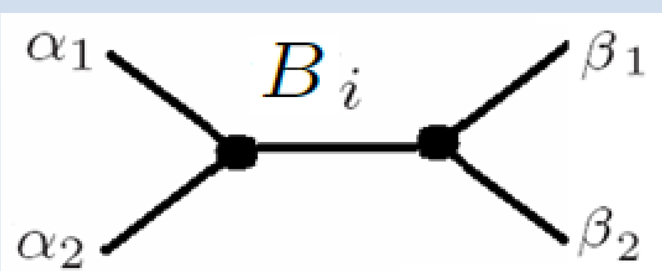
T和S矩阵

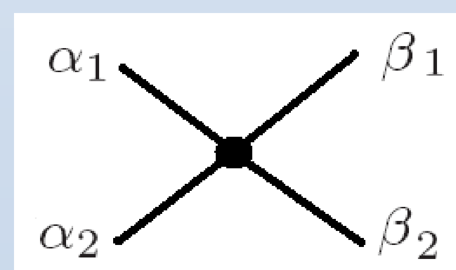
- T Matrix:

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma 1}^2+k_\gamma^2}-\sqrt{m_{\gamma 2}^2+k_\gamma^2}+i\varepsilon}$$





$$g_{i,\alpha}^{*}\frac{1}{E-m_i}g_{i,\beta}$$



$$V_{\alpha,\beta}$$

- T Matrix:

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$

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转换到有限体积

- Hamiltonian with discrete momentum

Continuous	$\int \mathrm{d}\vec{k}$	and	$\left\vert \alpha(\vec{k}_\alpha) \right\rangle$	and	$\left\langle \beta(\vec{k}_\beta) \middle\vert \alpha(\vec{k}_\alpha) \right\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓		↓		↓
Discrete	$\sum_{\mathrm{i}} \left({}^{2\pi}/_{\mathrm{L}} \right)^3$	and	$\left({}^{2\pi}/_{\mathrm{L}} \right)^{-3/2} \left\vert \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\rangle_\alpha$	and	${}_\beta\left\langle \vec{k}_{\mathrm{j}}, -\vec{k}_{\mathrm{j}} \middle\vert \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\rangle_\alpha = \delta_{\alpha\beta}\delta_{\mathrm{ij}}$

$$H_0 = \sum_{i=1,n} \left\vert B_i \right\rangle m_i \left\langle B_i \right\vert + \sum_{\alpha,\mathrm{i}} \left\vert \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2} \right]_\alpha \left\langle \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\vert$$

$$H_I = \sum_{\mathrm{j}} \left({}^{2\pi}/_{\mathrm{L}} \right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[\left\vert \vec{k}_{\mathrm{j}}, -\vec{k}_{\mathrm{j}} \right\rangle_\alpha g_{i,\alpha}^{+} \left\langle B_i \right\vert + \left\vert B_i \right\rangle g_{i,\alpha}\ {}_\alpha\left\langle \vec{k}_{\mathrm{j}}, -\vec{k}_{\mathrm{j}} \right\vert \right]$$

$$+ \sum_{\mathrm{i,j}} \left({}^{2\pi}/_{\mathrm{L}} \right)^3 \sum_{\alpha,\beta} \left\vert \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\rangle_\alpha v_{\alpha,\beta}\ {}_\beta\left\langle \vec{k}_{\mathrm{j}}, -\vec{k}_{\mathrm{j}} \right\vert$$

- Hamiltonian Matrix for discrete momentum

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0+H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Value ⟷ Lattice Spectrum

Eigen-Vector

哈密顿有效方法介绍 (HEFT)

共振态性质：质量，宽度，极点位置，
耦合常数，结构……

复连续的动量空间

哈密顿量

实连续的动量空间

实离散的动量空间

T 矩阵，
相移，
非弹系数

格点能谱

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哈密顿有效方法步骤

1. 拟合实验数据确定模型参数。
2. 模型确定后，在有限体积中写出相应的哈密顿矩阵。由于格点中π质量是非物理的，我们还需要一个和π质量相关的参数。
3. 计算本征值和本征矢量去理解格点数据，用格点数据再来筛选模型。

将来随着格点数据增多，我们也可以实现实验数据和格点数据的联合拟合！

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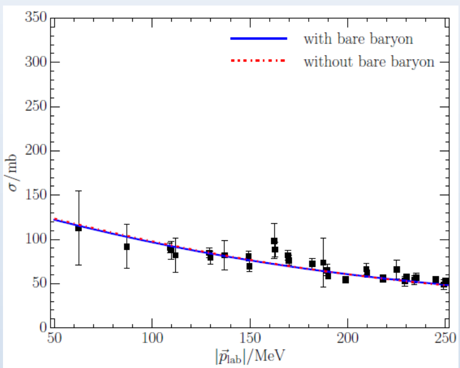
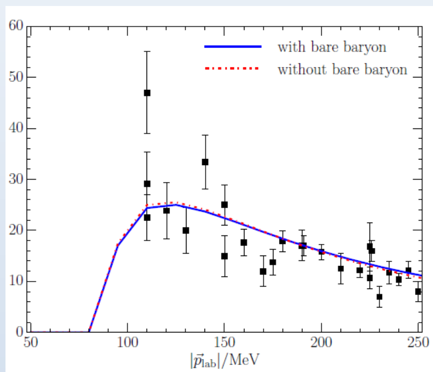
Λ*(1405)

Zhan-wei Liu etc. Phys.Rev. D95 (2017) no.1, 014506

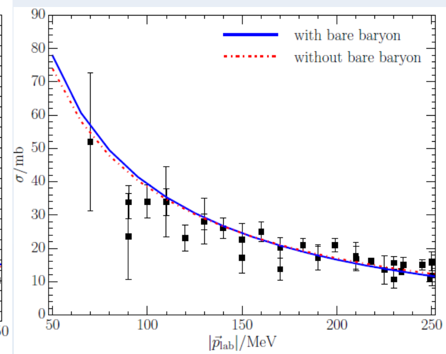
- I=0, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$ and $\bar{K}\Xi$
- I=1, $\pi\Sigma$, $\bar{K}N$, $\pi\Lambda$



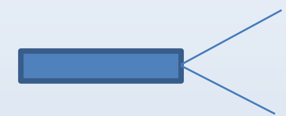
(a) $K^- p \to K^- p$



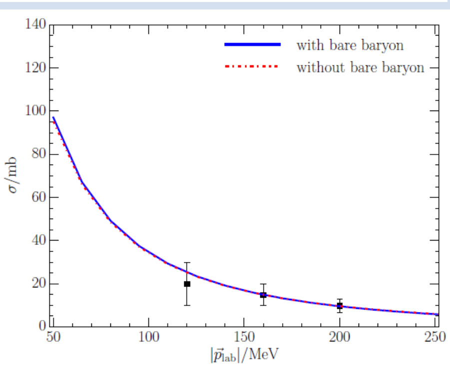
(b) $K^- p \to \bar{K}^0 n$



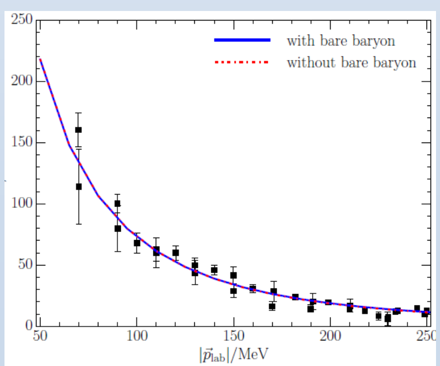
(c) $K^- p \to \pi^- \Sigma^+$



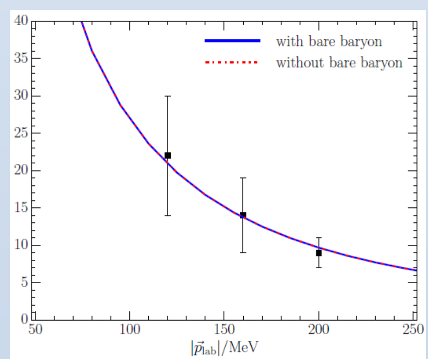
$$\frac{\sqrt{3} g^I_{\alpha,B_0}}{2\pi f}\sqrt{\omega_\pi(k)}u(k)$$



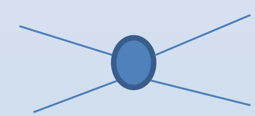
(d) $K^- p \to \pi^0 \Sigma^0$



(e) $K^- p \to \pi^+ \Sigma^-$



(f) $K^- p \to \pi^0 \Lambda$



$$g^I_{\alpha,\beta}\frac{[\omega_{\alpha_M}(k)+\omega_{\beta_M}(k')]\,u(k)u(k')}{8\pi^2 f^2\sqrt{2\omega_{\alpha_M}(k)}\sqrt{2\omega_{\beta_M}(k')}}$$

Weinberg – Tomozawa Term

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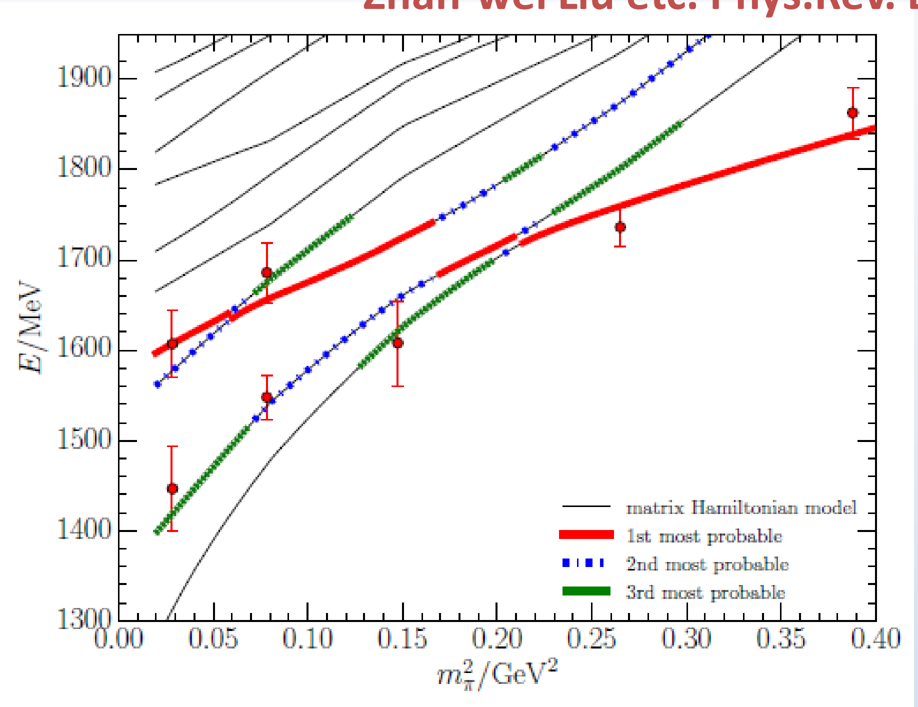
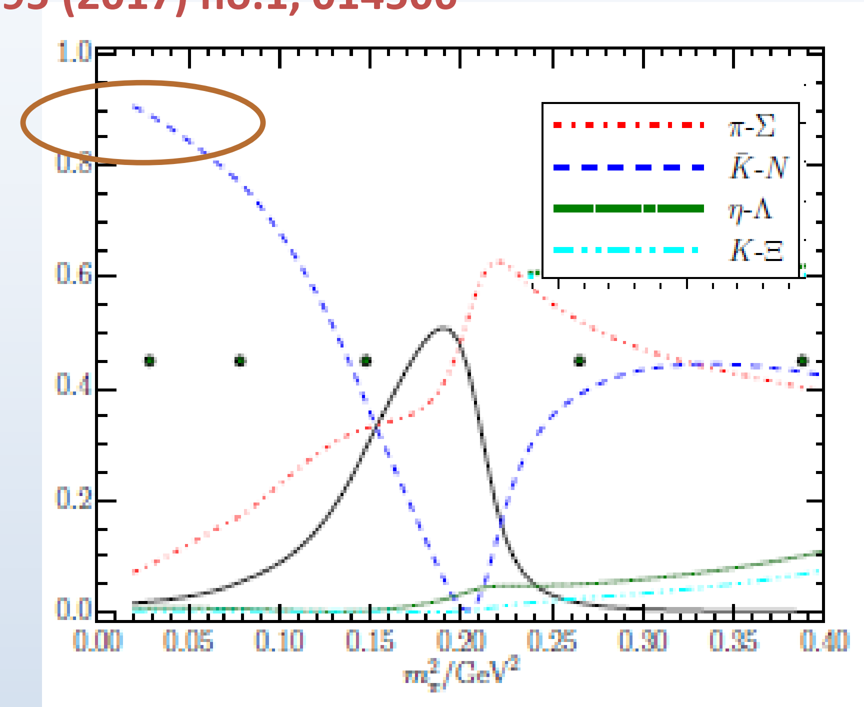
Λ*(1405)

Zhan-wei Liu etc. Phys.Rev. D95 (2017) no.1, 014506





the Λ*(1405) is predominantly a
molecular K̄ N bound State,

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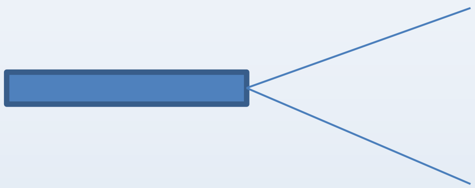
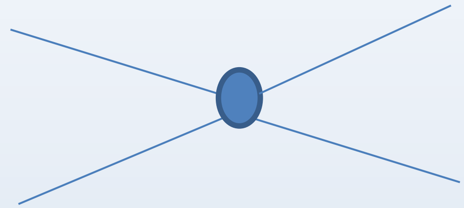
N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004

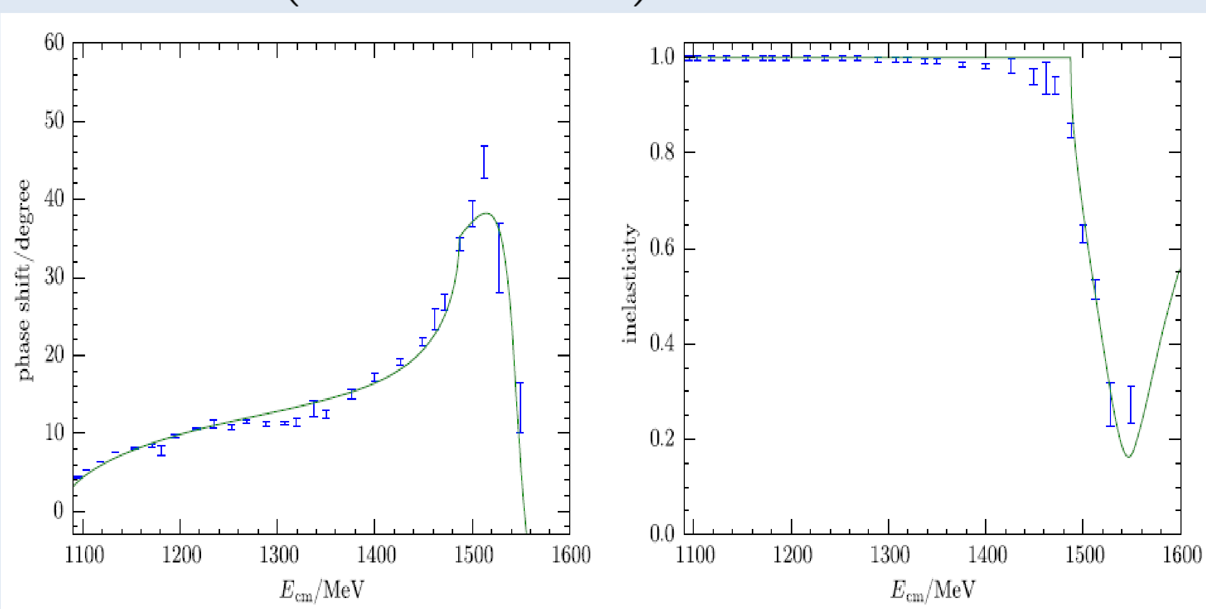
- 2 Channels: πN and ηN





$$G^2_{iN}(k) = \left(3g^2_{N_0^* iN}/4\pi^2 f^2\right)\omega_i(k)u^2(k)$$

$$\frac{3g^S_{\pi N}\tilde{u}(k)\tilde{u}(k')}{4\pi^2 f^2}$$



$$g^S_{\pi N} = -0.0608 \pm 0.0004$$
$$m_0 = 1601 \pm 14 \text{ MeV}$$
$$g_{N_0^*\pi N} = 0.186 \pm 0.006$$
$$g_{N_0^*\eta N} = 0.185 \pm 0.017.$$
$$\chi^2_{\text{DOF}} = 6.8$$
$$1531 \pm 29 - i88 \pm 2 \text{ MeV}$$

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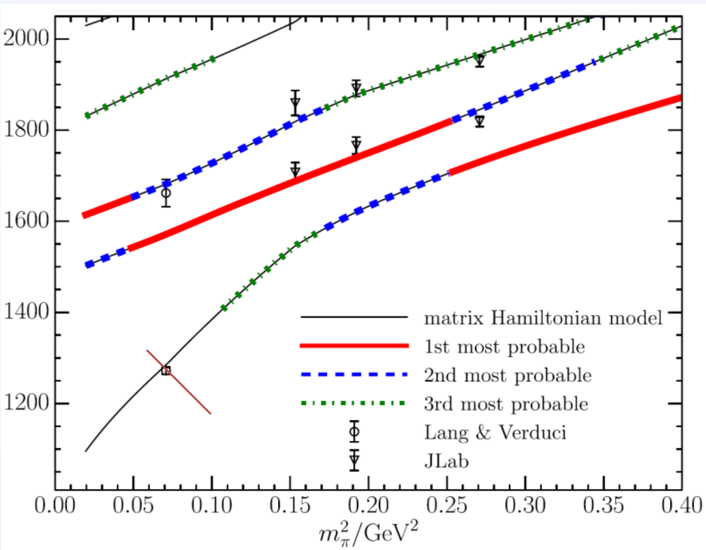
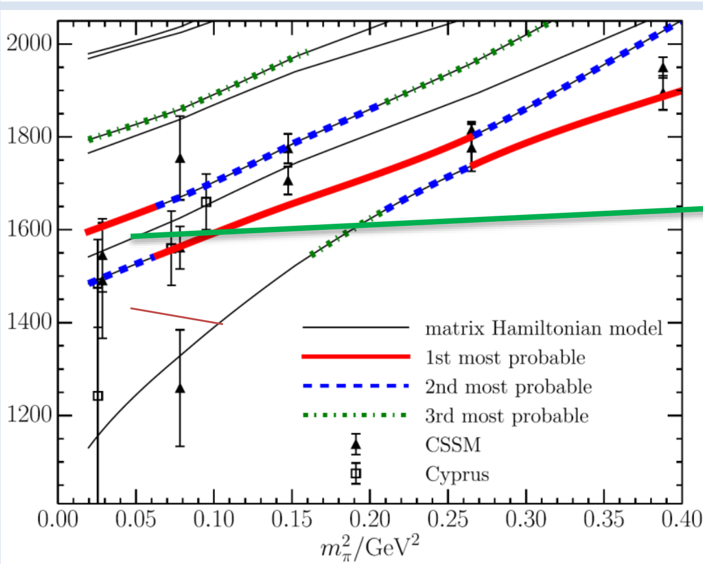
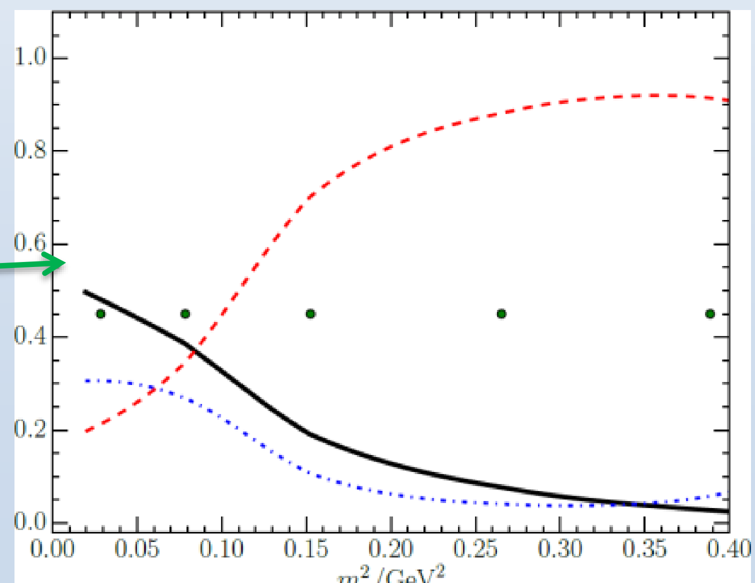
N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004

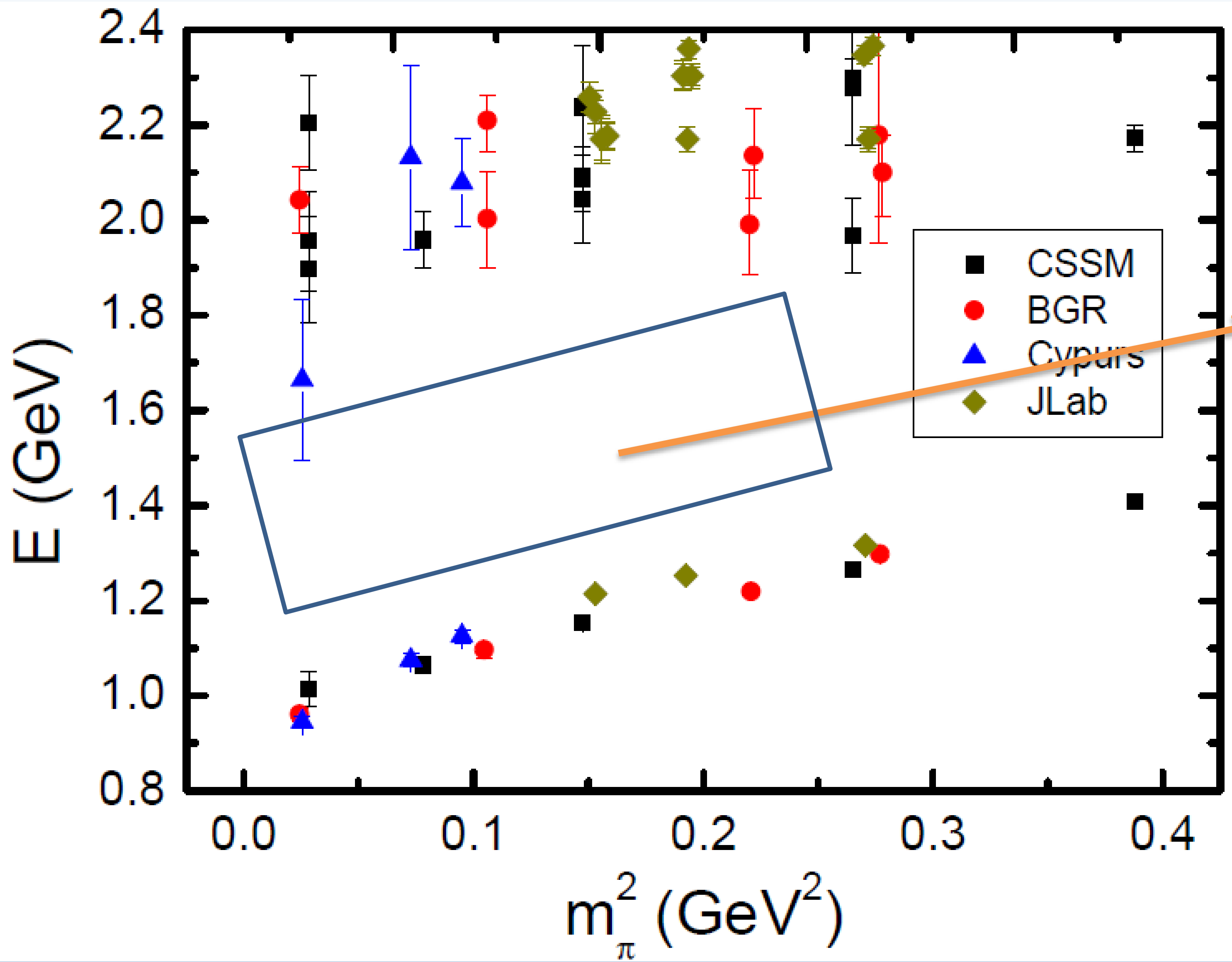


The main components (at least 50%) of N*(1535) is from the 3 quark core.





N*(1440)



No State extracted from 3 quark operator

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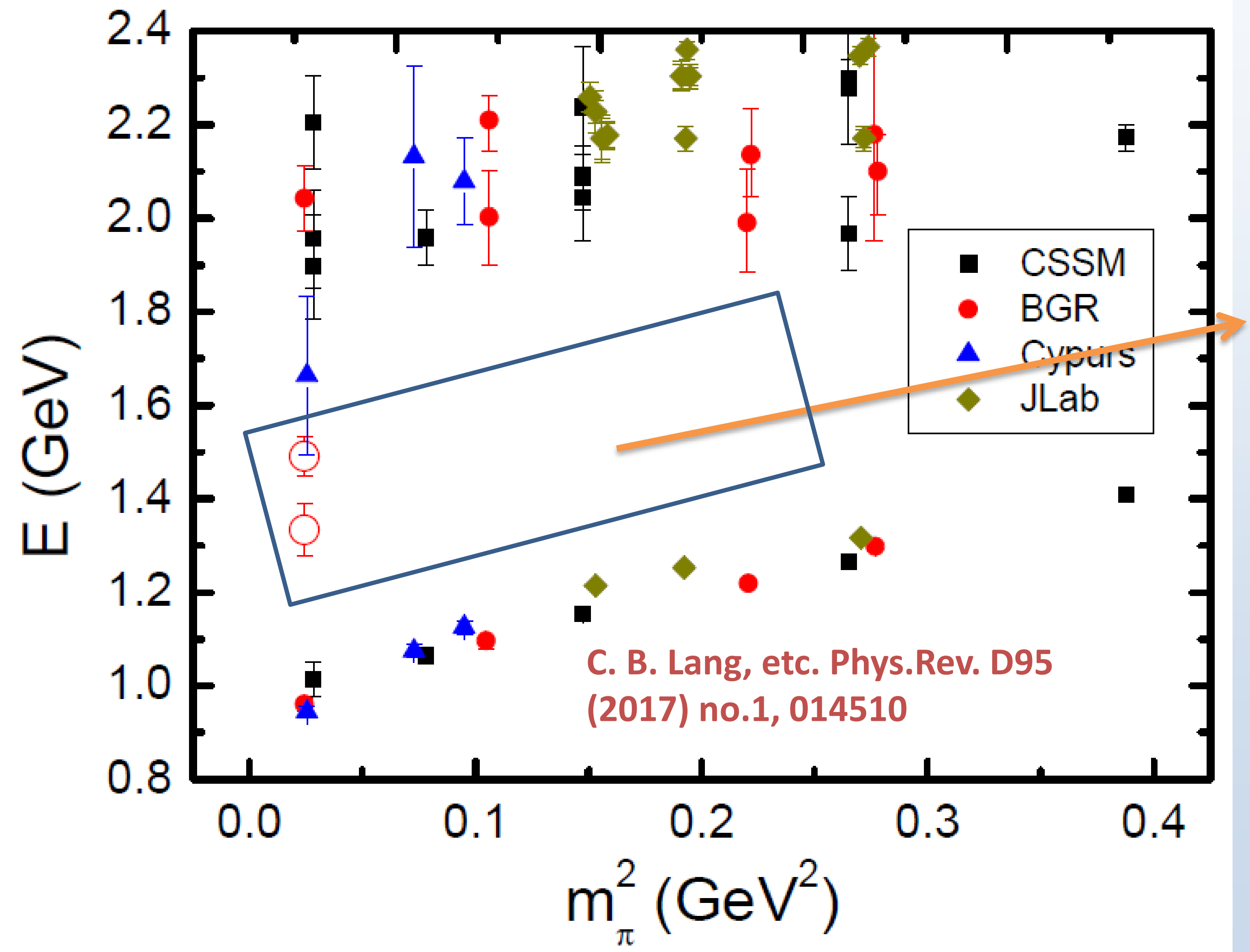
N*(1440)



No State extracted from 3 quark operator

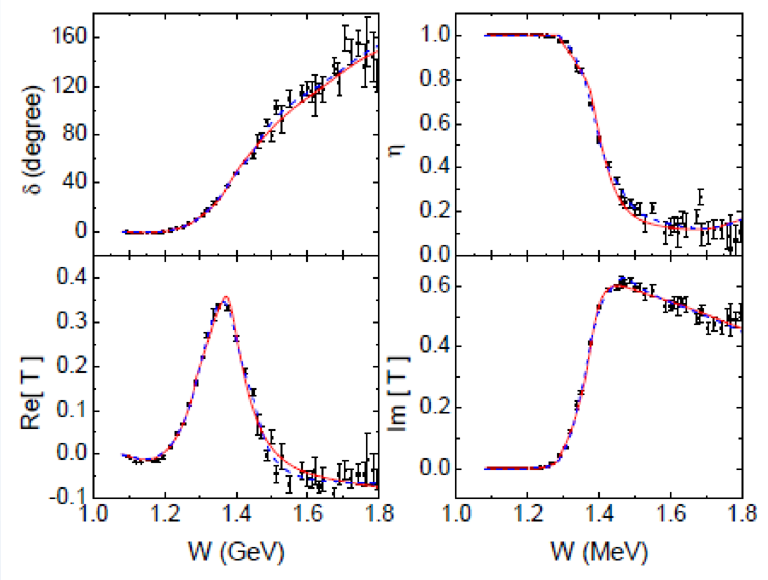
University of Chinese Academy of Sciences

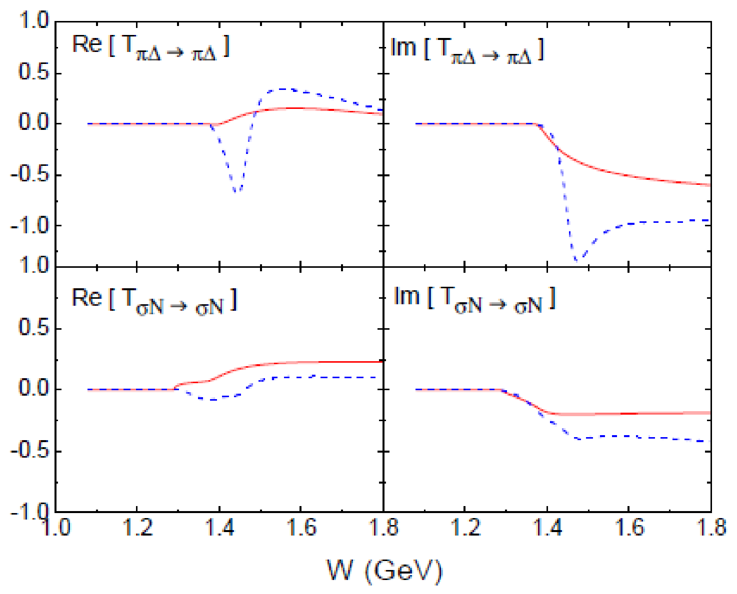
N*(1440)

Jia-jun Wu etc. arXiv: 1703.10715

Include 3 channels: πN, πΔ and σN

Parameter	I	II
$g^S_{\pi N,\pi N}$	1.156	0.634
$g^S_{\pi N,\pi\Delta}$	−0.662	−0.378
$g^S_{\pi N,\sigma N}$	−0.415	−1.738
$g^S_{\pi\Delta,\pi\Delta}$	−0.438	−0.581
$g^S_{\pi\Delta,\sigma N}$	1.332	0.964
$g^S_{\sigma N,\sigma N}$	10.000	10.000
m^0_B/GeV	2.000	1.7000
$g_{B_0\pi N}$	0.268	0.954
$g_{B_0\pi\Delta}$	1.544	−0.118
$g_{B_0\sigma N}$	–	−2.892
$\Lambda_{\pi N}$/GeV	0.5953	0.6302
$\Lambda_{\pi\Delta}$/GeV	1.5000	1.4318
$\Lambda_{\sigma N}$/GeV	1.5000	1.4533
Pole (MeV) (uuu)	$2012.28 - 42.09\,i$	$1355.57 - 70.81\,i$
Pole (MeV) (upu)	$1392.92 - 167.13\,i$	$1362.33 - 100.53\,i$





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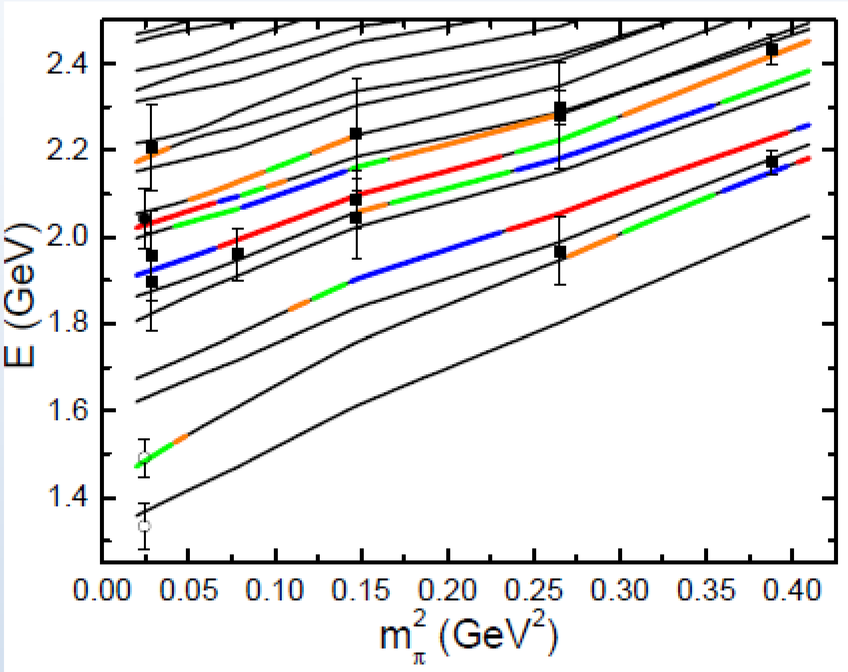
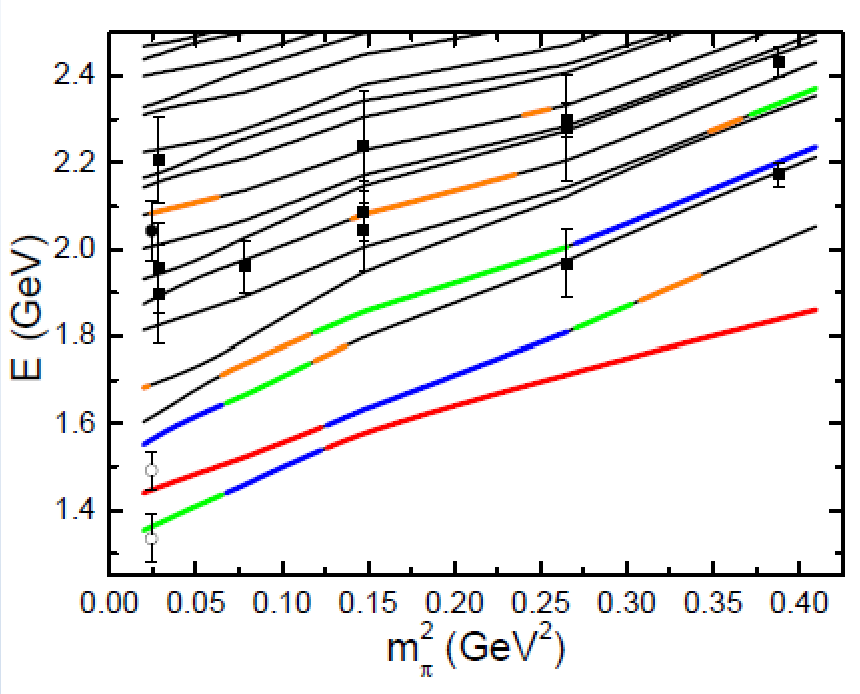
N*(1440)

Jia-jun Wu etc. arXiv: 1703.10715



The first scenario with a bare state for P11 around the pole at 2.0 GeV can fit both Lattice data and experimental data well, it indicates that N*(1440) seems a re-scattering state, and first radial excitation of nucleon should be around 2.0 GeV.



The Second scenario with a bare state for N*(1440) fit the experimental data well. But the largest possibility for bare state does not touch the lattice point. Thus, it fails to explain Lattice data.

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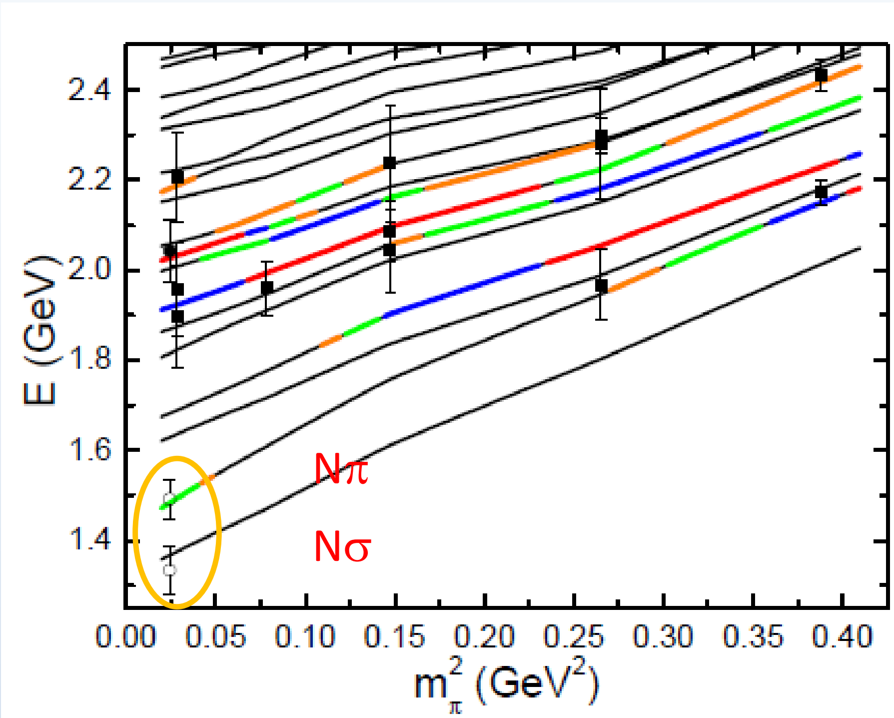
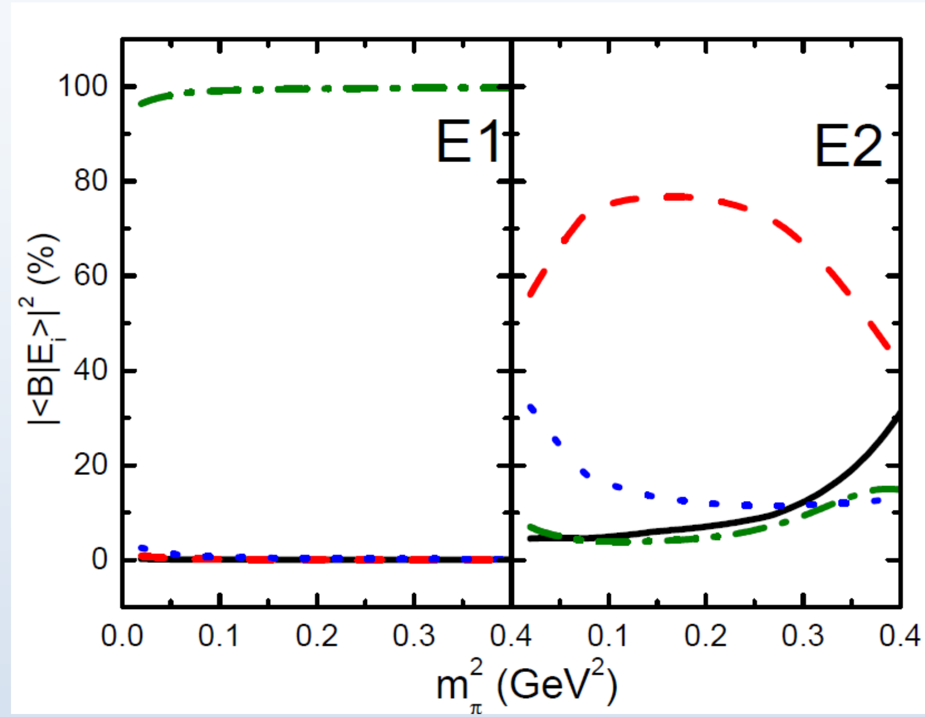
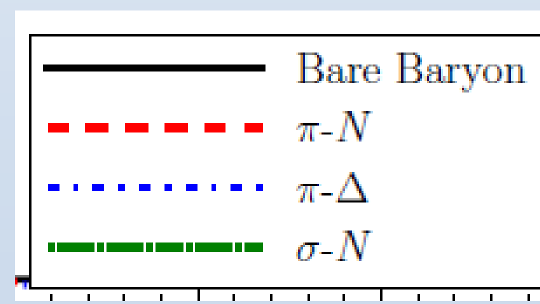
N*(1440)

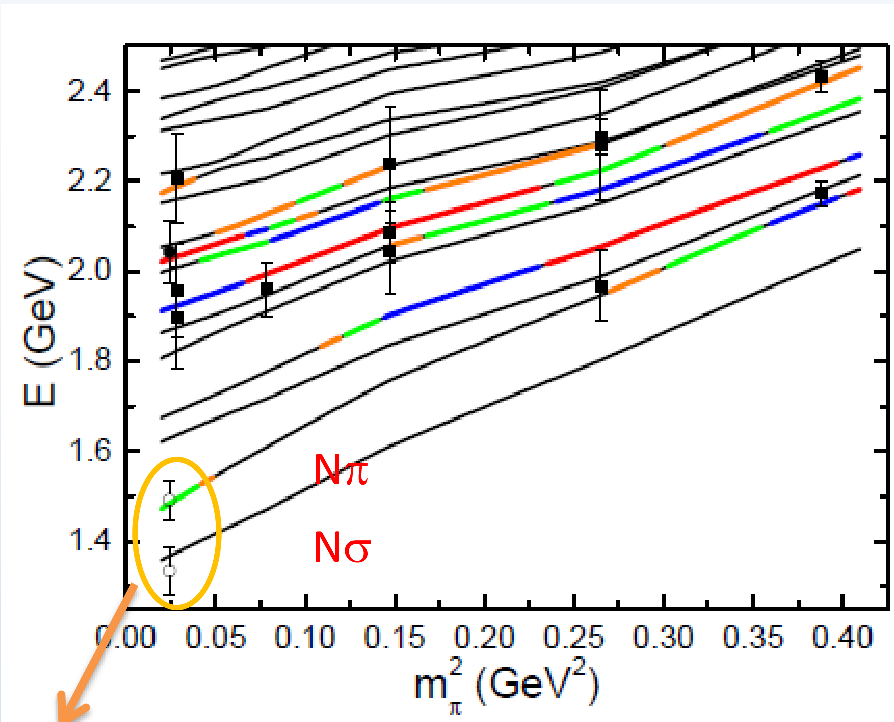


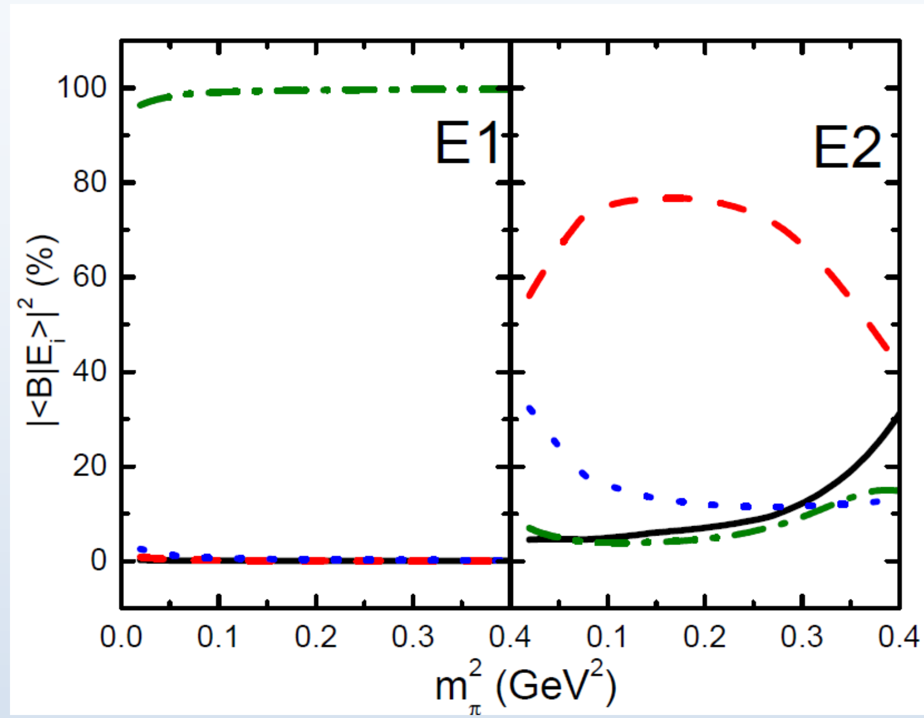


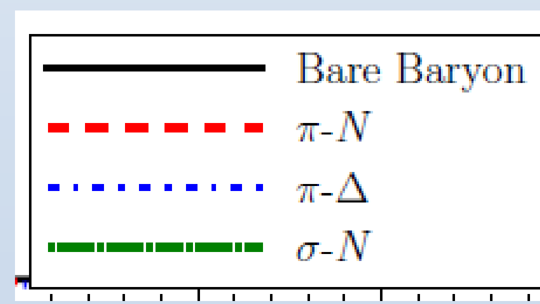


C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

N*(1440)



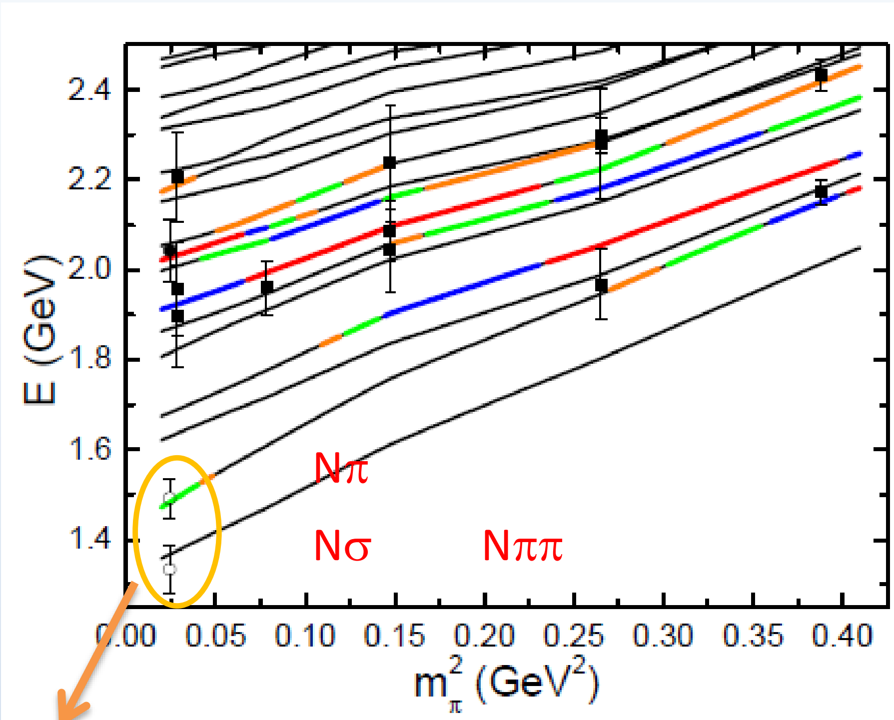


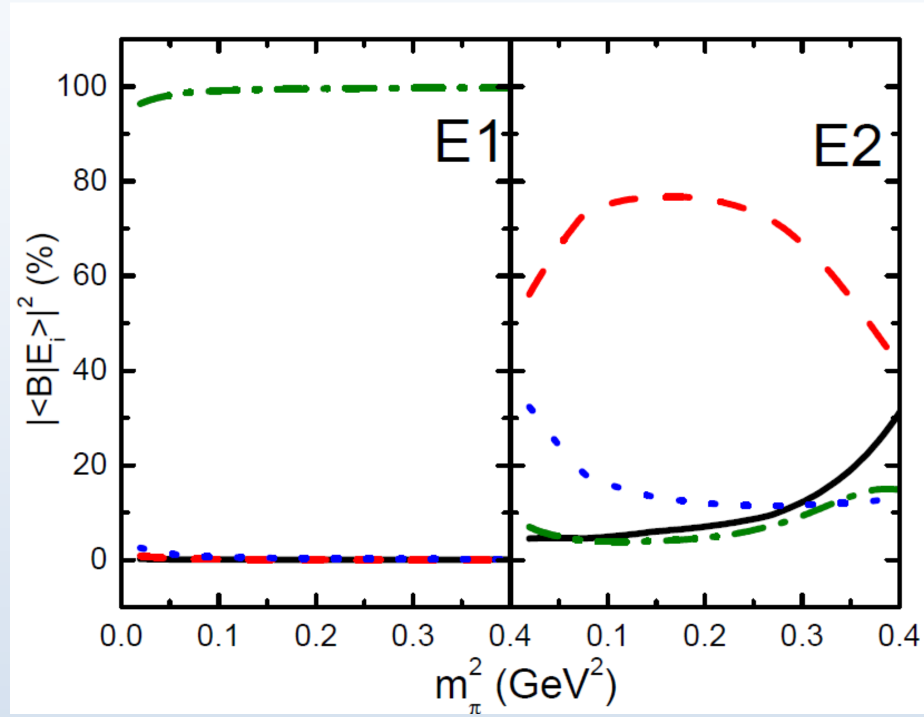


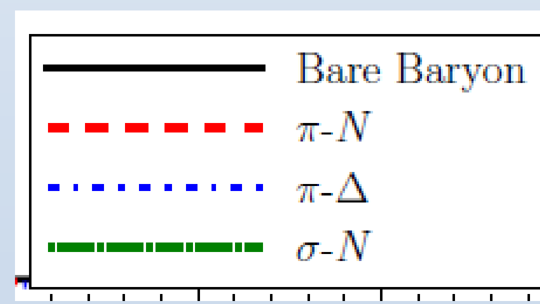
It is not a FIT !!!

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

N*(1440)







It is not a FIT !!!

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

We need more data and detailed study, for the contribution from Nππ three body.

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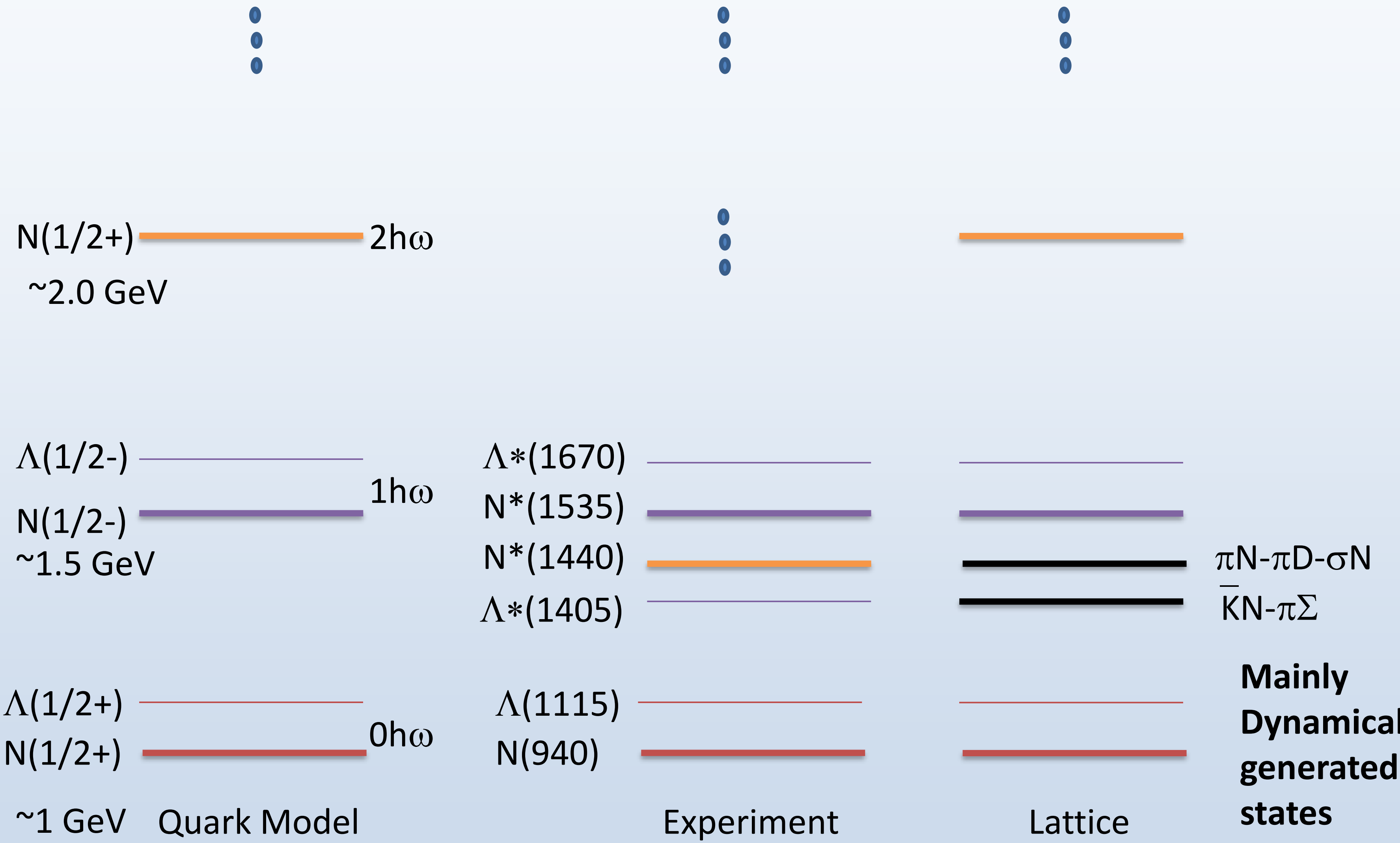
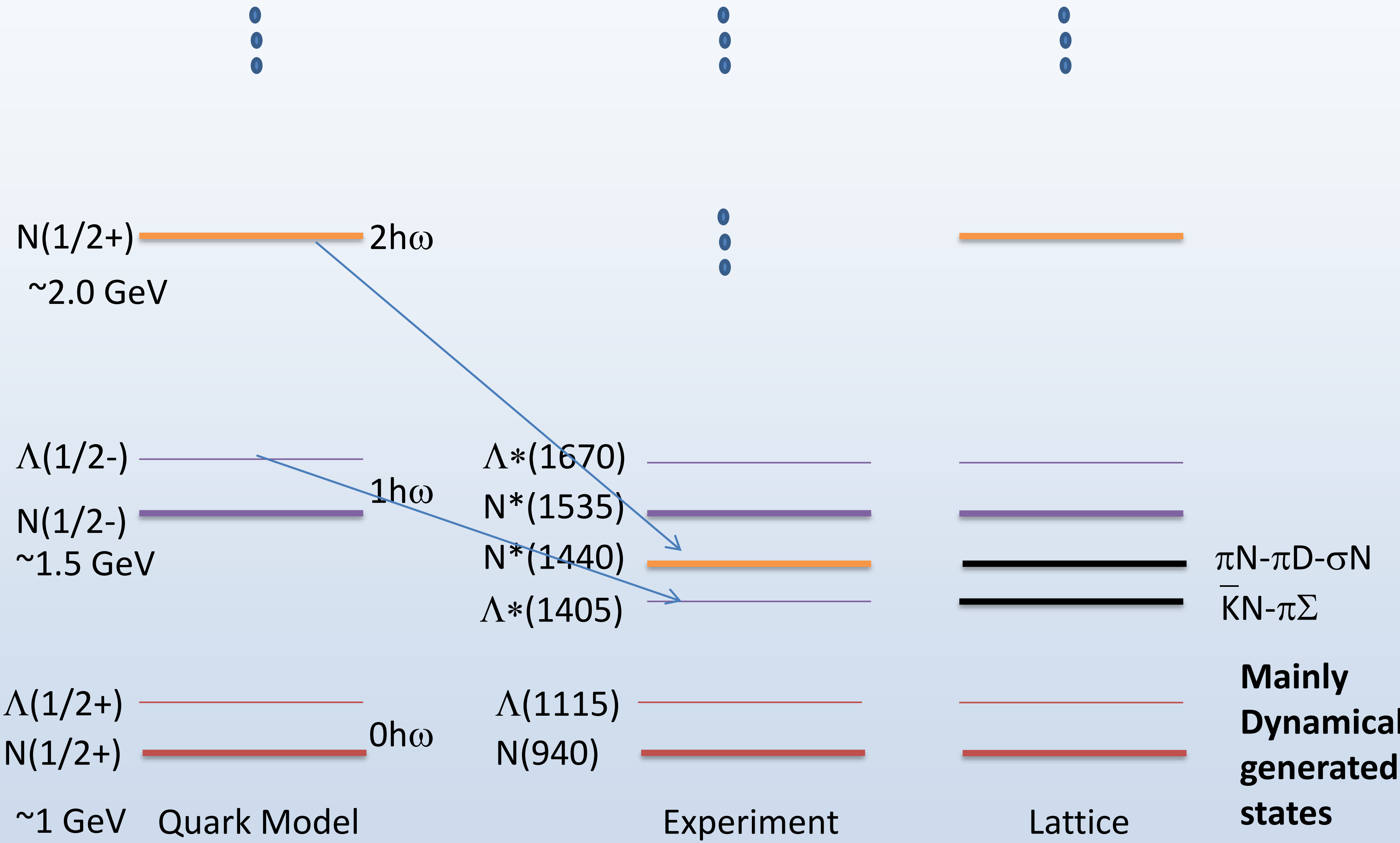


Mainly Dynamical generated states Not in the Quark Model



Mainly Dynamical generated states Not in the Quark Model

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目录

- 物理动机
- 哈密顿有效方法介绍 (HEFT)
- **有限体积中的角动量混合**
- 三体系统的有限体积能谱
- 总结和展望

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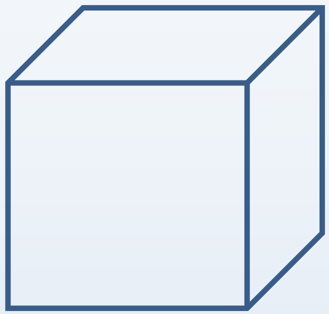
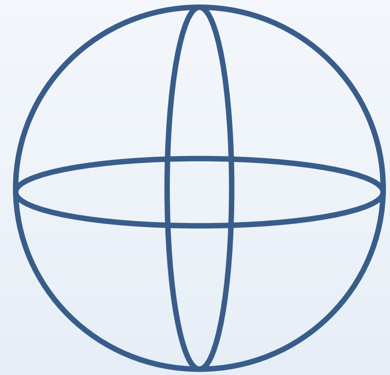
Finite volume VS Infinite volume



O_h

$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$



O(3)

$L^p = 0^+, 1^-, 2^+, \ldots\ldots$

O(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$

O_h	O(3)
A_1^+	$0^+, 4^+, \ldots.$
E^+	$2^+, 4^+, \ldots.$
T_1^+	$4^+, \ldots.$
T_2^+	$2^+, 4^+, \ldots.$
A_2^-	$3^-, 7^-, 9-, \ldots.$
T_1^-	$1^-, 3^-, \ldots.$
T_2^-	$3^-, \ldots.$

9. Partial Wave Mixing in Hamiltonian Effective Field Theory
Yan Li, Jia-jun Wu, C. D. Abell, Derek B. Leinweber, Anthony W. Thomas
Phys.Rev. D101 (2020) no.11, 114501

10. Hamiltonian effective field theory in elongated or moving finite volume
Yan Li, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas
e-Print: 2103.12260 [hep-lat]

Finite volume VS Infinite volume

$$\hat{H}_L = \hat{H}_{0L} + \sum_{\mathbf{n}',\mathbf{n}\in\mathbb{Z}^3} V_L\left(\frac{2\pi\mathbf{n}'}{L}, \frac{2\pi\mathbf{n}}{L}\right) |\mathbf{n}'\rangle \langle\mathbf{n}|$$

$$\sum_{\mathbf{n}\in\mathbb{Z}^3} \rightarrow \sum_{\mathbf{n}\in\mathbb{Z}^3}^{|\mathbf{n}|^2\leq N_{\text{cut}}}$$

For $N_{\text{cut}} = 600$, we have 61565 states.

(For $L = 3$ fm, $N_{\text{cut}} \sim 10$ GeV)

In the early works, for pure s- and p- waves,

$61565 \rightarrow$ about 600.

O(3)	O_h
0^+	A_1^+
1^-	T_1^-

- Original basis $|\mathbf{n}\rangle$: $\sum_{N=0}^{N_{\text{cut}}=600} C_3(N) \sim 60,000$
- Basis $|N;l,m\rangle$ with $l_{\text{cut}} = 4$: $\sum_{N=0}^{600} 25 \sim 600 \times 25$
- Basis $|N,l;\Gamma = \mathbf{A}_1^+, f, \alpha\rangle$: $\sum_{N=0}^{600} 2 \sim 600 \times 2$
- Orthonormalization needs the inner products——P-Matrix

$$\hat{H}_L = \hat{H}_{0L} + \sum_{N',N} \sum_{\Gamma,F',F} v_{\Gamma,F',F}(k_{N'}, k_N) \sum_{\alpha} |N';\Gamma,F',\alpha\rangle \langle N;\Gamma,F,\alpha|$$

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$$|\mathbf{n}\rangle$$

$$\sum_{\hat{\mathbf{n}}} Y_{lm}(\hat{\mathbf{n}})\,|\mathbf{n}\rangle$$

$$|N;l,m\rangle$$

$$\sum_{m}[C_l]_{\Gamma,f,\alpha;m}\,|N;l,m\rangle$$

$$|N,l;\Gamma,f,\alpha\rangle$$

Orthonormalization

$$|N;\Gamma,F,\alpha\rangle$$

$$[P_N]_{l',m';l,m} := \langle N;l',m'|N;l,m\rangle = \sum_{\hat{\mathbf{n}}} Y^*_{l'm'}(\hat{\mathbf{n}})Y_{lm}(\hat{\mathbf{n}})$$

25×25 for $l_{\text{cut}} = 4$ $\xrightarrow{[C]^\dagger[P][C]}$ block diagonal, 2×2 block $\mathbf{A}_1^+$

O(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$

$$[P_{N;\Gamma,\alpha}]_{l',f';l,f} := \langle N,l';\Gamma,f',\alpha|N,l;\Gamma,f,\alpha\rangle$$
$$= \sum_{m',m}[C_{l'}]^*_{\Gamma,f',\alpha;m'}[P_N]_{l',m';l,m}[C_l]_{\Gamma,f,\alpha;m}$$

Combination Coefficients: $[G_{l;\Gamma}]_{N',F';N,F} = \sum_f \langle N';\Gamma,F',\alpha|N';l,\Gamma,f,\alpha\rangle\,\langle N;l,\Gamma,f,\alpha|N;\Gamma,F,\alpha\rangle$

P-Matrix

s-wave	p-wave			d-wave					f-wave							g-wave								
1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.05	0	0	0	1.75	0	0	0	1.05
0	1.00	0	0	0	0	0	0	0	0	0	−0.94	0	0	0	−1.21	0	0	0	0	0	0	0	0	0
0	0	1.00	0	0	0	0	0	0	0	0	0	1.53	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1.00	0	0	0	0	0	−1.21	0	0	0	−0.94	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.25	0	0	0	1.25	0	0	0	0	0	0	0	0	0	−1.08	0	0	0	−1.08	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	2.50	0	0	0	0	0	0	0	0	0	−1.17	0	0	0	1.40	0	0	0	−1.17
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.25	0	0	0	1.25	0	0	0	0	0	0	0	0	0	−1.08	0	0	0	−1.08	0	0
0	0	0	−1.21	0	0	0	0	0	1.46	0	0	0	1.13	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	−0.94	0	0	0	0	0	0	0	0	0	0.88	0	0	0	1.13	0	0	0	0	0	0	0	0	0
0	0	1.53	0	0	0	0	0	0	0	0	0	2.33	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	−0.94	0	0	0	0	0	1.13	0	0	0	0.88	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	−1.21	0	0	0	0	0	0	0	0	0	1.13	0	0	0	1.46	0	0	0	0	0	0	0	0	0
1.05	0	0	0	0	0	−1.17	0	0	0	0	0	0	0	0	0	1.64	0	0	0	1.18	0	0	0	1.64
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	−1.08	0	0	0	−1.08	0	0	0	0	0	0	0	0	0	0.94	0	0	0	0.94	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.75	0	0	0	0	0	1.40	0	0	0	0	0	0	0	0	0	1.18	0	0	0	3.84	0	0	0	1.18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	−1.08	0	0	0	−1.08	0	0	0	0	0	0	0	0	0	0.94	0	0	0	0.94	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.05	0	0	0	0	0	−1.17	0	0	0	0	0	0	0	0	0	1.64	0	0	0	1.18	0	0	0	1.64

$$[P_{N=1}]\Big/C_3(1) \qquad C_3(1) = 6$$

$$[P_N]_{l',m';l,m} := \langle N; l', m' | N; l, m\rangle = 4\pi \sum_{|\mathbf{n}|^2=N} Y^*_{l'm'}(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}})$$

25×25 matrix ordered as $(l, m) = (0,0),\ (1,-1),\ (1,0),\ (1,1),\ \cdots,\ (4,4)$

P-Matrix

s-wave	p-wave			d-wave					f-wave							g-wave								
1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07	0	0	0	0.11	0	0	0	0.07
0	1.00	0	0	0	0	0	0	0	0	0	−0.06	0	0	0	−0.08	0	0	0	0	0	0	0	0	0
0	0	1.00	0	0	0	0	0	0	0	0	0	0.10	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1.00	0	0	0	0	0	−0.08	0	0	0	−0.06	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.02	0	0	0	0.08	0	0	0	0	0	0	0	0	0	−0.07	0	0	0	−0.09	0	0
0	0	0	0	0	0.94	0	0	0	0	0	0	0	0	0	0	0	0	0	−0.01	0	0	0	0.03	0
0	0	0	0	0	0	1.10	0	0	0	0	0	0	0	0	0	−0.09	0	0	0	0.10	0	0	0	−0.09
0	0	0	0	0	0	0	0.94	0	0	0	0	0	0	0	0	0	0.03	0	0	0	−0.01	0	0	0
0	0	0	0	0.08	0	0	0	1.02	0	0	0	0	0	0	0	0	0	−0.09	0	0	0	−0.07	0	0
0	0	0	−0.08	0	0	0	0	0	1.03	0	0	0	0.09	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.94	0	0	0	−0.03	0	0	0	0	0	0	0	0	0	0
0	−0.06	0	0	0	0	0	0	0	0	0	0.98	0	0	0	0.09	0	0	0	0	0	0	0	0	0
0	0	0.10	0	0	0	0	0	0	0	0	0	1.10	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	−0.06	0	0	0	0	0	0.09	0	0	0	0.98	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	−0.03	0	0	0	0.94	0	0	0	0	0	0	0	0	0	0
0	−0.08	0	0	0	0	0	0	0	0	0	0.09	0	0	0	1.03	0	0	0	0	0	0	0	0	0
0.07	0	0	0	0	0	−0.09	0	0	0	0	0	0	0	0	0	1.04	0	0	0	0.11	0	0	0	0.20
0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0	0	0.93	0	0	0	−0.04	0	0	0
0	0	0	0	−0.07	0	0	0	−0.09	0	0	0	0	0	0	0	0	0	1.03	0	0	0	0.08	0	0
0	0	0	0	0	−0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0.85	0	0	0	−0.04	0
0.11	0	0	0	0	0	0.10	0	0	0	0	0	0	0	0	0	0.11	0	0	0	1.30	0	0	0	0.11
0	0	0	0	0	0	0	−0.01	0	0	0	0	0	0	0	0	0	−0.04	0	0	0	0.85	0	0	0
0	0	0	0	−0.09	0	0	0	−0.07	0	0	0	0	0	0	0	0	0	0.08	0	0	0	1.03	0	0
0	0	0	0	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	−0.04	0	0	0	0.93	0
0.07	0	0	0	0	0	−0.09	0	0	0	0	0	0	0	0	0	0.20	0	0	0	0.11	0	0	0	1.04

$$[P_{N=581}]\Big/C_3(581) \qquad C_3(581) = 336$$

$$[P_N]_{l',m';l,m} := \langle N; l', m' | N; l, m\rangle = 4\pi \sum_{|\mathbf{n}|^2=N} Y^*_{l'm'}(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}})$$

25×25 matrix ordered as $(l, m) = (0,0), (1,-1), (1,0), (1,1), \cdots, (4,4)$

P-Matrix

s-wave	p-wave			d-wave					f-wave							g-wave								
1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−0.01	0	0	0	−0.02	0	0	0	−0.01
0	1.00	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0.02	0	0	0	0	0	0	0	0	0
0	0	1.00	0	0	0	0	0	0	0	0	0	−0.02	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1.00	0	0	0	0	0	0.02	0	0	0	0.01	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.00	0	0	0	−0.02	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0.02	0	0
0	0	0	0	0	1.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00	0	0	0	−0.00	0
0	0	0	0	0	0	0.98	0	0	0	0	0	0	0	0	0	0.02	0	0	0	−0.02	0	0	0	0.02
0	0	0	0	0	0	0	1.01	0	0	0	0	0	0	0	0	0	−0.00	0	0	0	0.00	0	0	0
0	0	0	0	−0.02	0	0	0	1.00	0	0	0	0	0	0	0	0	0	0.02	0	0	0	0.01	0	0
0	0	0	0.02	0	0	0	0	0	0.99	0	0	0	−0.02	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1.01	0	0	0	0.00	0	0	0	0	0	0	0	0	0	0
0	0.01	0	0	0	0	0	0	0	0	0	1.00	0	0	0	−0.02	0	0	0	0	0	0	0	0	0
0	0	−0.02	0	0	0	0	0	0	0	0	0	0.98	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0.01	0	0	0	0	0	−0.02	0	0	0	1.00	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.00	0	0	0	1.01	0	0	0	0	0	0	0	0	0	0
0	0.02	0	0	0	0	0	0	0	0	0	−0.02	0	0	0	0.99	0	0	0	0	0	0	0	0	0
−0.01	0	0	0	0	0	0.02	0	0	0	0	0	0	0	0	0	0.99	0	0	0	−0.02	0	0	0	−0.03
0	0	0	0	0	0	0	−0.00	0	0	0	0	0	0	0	0	0	1.01	0	0	0	0.00	0	0	0
0	0	0	0	0.01	0	0	0	0.02	0	0	0	0	0	0	0	0	0	1.00	0	0	0	−0.01	0	0
0	0	0	0	0	0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	1.02	0	0	0	0.00	0
−0.02	0	0	0	0	0	−0.02	0	0	0	0	0	0	0	0	0	−0.02	0	0	0	0.95	0	0	0	−0.02
0	0	0	0	0	0	0	0.00	0	0	0	0	0	0	0	0	0	0.00	0	0	0	1.02	0	0	0
0	0	0	0	0.02	0	0	0	0.01	0	0	0	0	0	0	0	0	0	−0.01	0	0	0	1.00	0	0
0	0	0	0	0	−0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00	0	0	0	1.01	0
−0.01	0	0	0	0	0	0.02	0	0	0	0	0	0	0	0	0	−0.03	0	0	0	−0.02	0	0	0	0.99

$$[P_{N=941}]\big/C_3(941) \qquad C_3(941) = 552$$

$$[P_N]_{l',m';l,m} := \langle N; l', m' | N; l, m\rangle = 4\pi \sum_{|\mathbf{n}|^2=N} Y^*_{l'm'}(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}})$$

25×25 matrix ordered as $(l, m) = (0,0),\ (1,-1),\ (1,0),\ (1,1),\ \cdots,\ (4,4)$

$$|\mathbf{n}\rangle \xrightarrow{\sum_{\hat{\mathbf{n}}} Y_{lm}(\hat{\mathbf{n}})\,|\mathbf{n}\rangle} |N;l,m\rangle \xrightarrow{\sum_m [C_l]_{\Gamma,f,\alpha;m}\,|N;l,m\rangle} |N,l;\Gamma,f,\alpha\rangle \xrightarrow{\text{Orthonormalization}} |N;\Gamma,F,\alpha\rangle$$

$$[P_N]_{l',m';l,m} := \langle N;l',m'|N;l,m\rangle = \sum_{\hat{\mathbf{n}}} Y^*_{l'm'}(\hat{\mathbf{n}})Y_{lm}(\hat{\mathbf{n}})$$

$$\begin{pmatrix} 25\times 25 \\ \text{for } l_{\text{cut}} = 4 \end{pmatrix} \xrightarrow{[C]^\dagger[P][C]} \begin{pmatrix} \boxed{A_1^+} & & \\ & \boxed{} & \\ & & \ddots \end{pmatrix}$$

2×2 → A_1^+

O(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$

$$[P_{N;\Gamma,\alpha}]_{l',f';l,f} := \langle N,l';\Gamma,f',\alpha|N,l;\Gamma,f,\alpha\rangle = \sum_{m',m} [C_{l'}]^*_{\Gamma,f',\alpha;m'}[P_N]_{l',m';l,m}[C_l]_{\Gamma,f,\alpha;m}$$

Combination Coefficients: $[G_{l;\Gamma}]_{N',F';N,F} = \sum_f \langle N';\Gamma,F',\alpha|N';l,\Gamma,f,\alpha\rangle\,\langle N;l,\Gamma,f,\alpha|N;\Gamma,F,\alpha\rangle$

Moving and Elongated system

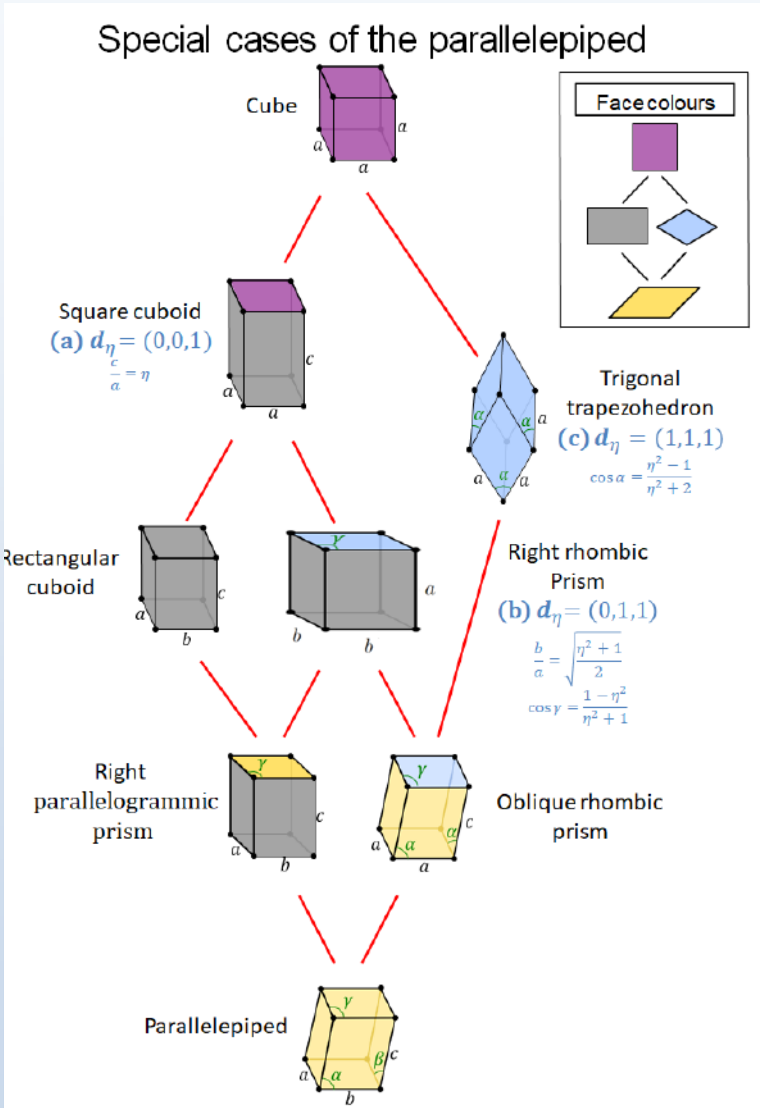
不同能量点上的数据越多，我们对整个物理系统的了解就越多，所以要尽可能的经济的产生更多的格点数据。于是非静止系，非正方体格子的能谱也对限制物理系统的参数有重要的意义。

Pure Moving system

$$H_L = \sum_{\mathbf{n}} h(k^*(\mathbf{n}))\ |\mathbf{n}\rangle\langle\mathbf{n}| + \sum_{\mathbf{n}',\mathbf{n}} L^{-3}\,\tilde{V}(\mathbf{k}(\mathbf{n}'),\mathbf{k}(\mathbf{n}))\ |\mathbf{n}'\rangle\langle\mathbf{n}|$$

$$\tilde{V}(\mathbf{k}',\mathbf{k}) = \left[\mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\,V(\mathbf{k}'^*,\mathbf{k}^*)\,\mathcal{J}^{\frac{1}{2}}(\mathbf{k})\right], \qquad \mathbf{k} = \frac{2\pi}{L}\mathbf{n}\,.$$

Moving and Elongated system



Pure Moving system

$$H_L = \sum_{\mathbf{n}} h(k^*(\mathbf{n}))\, |\mathbf{n}\rangle\langle\mathbf{n}| + \sum_{\mathbf{n}',\mathbf{n}} L^{-3}\, \tilde{V}(\mathbf{k}(\mathbf{n}'), \mathbf{k}(\mathbf{n}))\, |\mathbf{n}'\rangle\langle\mathbf{n}|$$

$$\tilde{V}(\mathbf{k}', \mathbf{k}) = \left[\mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\, V(\mathbf{k}'^*, \mathbf{k}^*)\, \mathcal{J}^{\frac{1}{2}}(\mathbf{k})\right], \qquad \mathbf{k} = \frac{2\pi}{L}\mathbf{n}.$$

Moving and Elongated system

$$H_L = \sum_{\mathbf{n}} h(k^*(\mathbf{n}))\, |\mathbf{n}\rangle\langle\mathbf{n}| + \sum_{\mathbf{n}',\mathbf{n}} \eta^{-1} L^{-3}\, \tilde{V}(\mathbf{k}(\mathbf{n}'), \mathbf{k}(\mathbf{n}))\, |\mathbf{n}'\rangle\langle\mathbf{n}|$$

$$\tilde{V}(\mathbf{k}', \mathbf{k}) = \mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\, V(\mathbf{k}'^*, \mathbf{k}^*)\, \mathcal{J}^{\frac{1}{2}}(\mathbf{k}), \qquad \mathbf{k} = \frac{2\pi}{L}\left(\mathbf{n}_\perp + \frac{1}{\eta}\mathbf{n}_\parallel\right),$$

	Moving	Elongated	mass
A:	No	No	any
B:	Yes	Yes	unequal
C1:	No	Yes	any
C2:	Yes	Yes	equal

Case	$\mathbf{d}_\eta$	η	$\mathbf{d}_\gamma$	$m_1 = m_2$?	e_n
A	any	$=1$	$\mathbf{0}$	any	$\mathbf{n}^2$
B	$\mathbf{d} \neq \mathbf{0}$	any	$\mathbf{d} \neq \mathbf{0}$	no	$(\mathbf{n}^2, (\mathbf{d}-\mathbf{n})^2)$ or $(\mathbf{n}^2, \mathbf{n}\cdot\mathbf{d})$
C1	$\mathbf{d} \neq \mathbf{0}$	$\neq 1$	$\mathbf{0}$	any	$(\mathbf{n}^2, \lvert\mathbf{n}\cdot\mathbf{d}\rvert)$
C2	$\mathbf{d} \neq \mathbf{0}$	any	$\mathbf{d} \neq \mathbf{0}$	yes	$\{\mathbf{n}^2, (\mathbf{d}-\mathbf{n})^2\}$

Moving and Elongated system

Case	A	B	C1 or C2
G_∞	O(3)	O(2)	O(2)×C_2
$(\Gamma_\infty, \alpha_\infty)$	$(l^{\mathcal{P}}, m)$	$(\lvert m\rvert, S_m)$	$(\lvert m\rvert^{\mathcal{P}}, S_m)$
$\tilde{v}_{\Gamma_\infty}$	v_l	$\mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\,\mathcal{J}^{\frac{1}{2}}(\mathbf{k})\sum_l v_l \frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}$ $\times P_{lm}(\cos\theta'^*)P_{lm}(\cos\theta^*)$	$\mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\,\mathcal{J}^{\frac{1}{2}}(\mathbf{k})\sum_{l^{\mathcal{P}}} v_l \frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}$ $\times P_{lm}(\lvert\cos\theta'^*\rvert)P_{lm}(\lvert\cos\theta^*\rvert)$
$u_{\Gamma_\infty,\alpha_\infty}$	Y_{lm}	$e^{im\phi^*}$	$S^{\mathcal{P}}(m,\theta^*)\,e^{im\phi^*}$

Moving and Elongated system

$$H_L = \sum_{\mathbf{n}} h(k^*(\mathbf{n}))\,|\mathbf{n}\rangle\langle\mathbf{n}| + \sum_{\mathbf{n}',\mathbf{n}} \eta^{-1}L^{-3}\,\tilde{V}(\mathbf{k}(\mathbf{n}'),\mathbf{k}(\mathbf{n}))\,|\mathbf{n}'\rangle\langle\mathbf{n}|$$

$$\tilde{V}(\mathbf{k}',\mathbf{k}) = \mathcal{J}^{\frac{1}{2}}(\mathbf{k}')\,V(\mathbf{k}'^*,\mathbf{k}^*)\,\mathcal{J}^{\frac{1}{2}}(\mathbf{k}), \qquad \mathbf{k} = \frac{2\pi}{L}\left(\mathbf{n}_\perp + \frac{1}{\eta}\mathbf{n}_\parallel\right),$$

$$\tilde{V}(\mathbf{k}',\mathbf{k}) = \sum_{\Gamma_\infty} \tilde{v}_{\Gamma_\infty}(e'_n, e_n) \sum_{\alpha_\infty} u_{\Gamma_\infty,\alpha_\infty}(\mathbf{n}')\,u^*_{\Gamma_\infty,\alpha_\infty}(\mathbf{n})$$

	Moving	Elongated	mass
A:	No	no	any
B:	Yes	Yes	unequal
C1:	No	Yes	any
C2:	Yes	Yes	equal

Case	$\mathbf{d}_\eta$	η	$\mathbf{d}_\gamma$	$m_1 = m_2$?	e_n
A	any	$=1$	$\mathbf{0}$	any	$\mathbf{n}^2$
B	$\mathbf{d}\neq\mathbf{0}$	any	$\mathbf{d}\neq\mathbf{0}$	no	$(\mathbf{n}^2,(\mathbf{d}-\mathbf{n})^2)$ or $(\mathbf{n}^2,\mathbf{n}\cdot\mathbf{d})$
C1	$\mathbf{d}\neq\mathbf{0}$	$\neq 1$	$\mathbf{0}$	any	$(\mathbf{n}^2,\lvert\mathbf{n}\cdot\mathbf{d}\rvert)$
C2	$\mathbf{d}\neq\mathbf{0}$	any	$\mathbf{d}\neq\mathbf{0}$	yes	$\{\mathbf{n}^2,(\mathbf{d}-\mathbf{n})^2\}$

Example of Isospin-2 Scattering

PRD 86, 034031 (2012)
Jozef J. Dudek et al.

- $l_{\text{cut}} = 4$, only s-, d- and g-waves are present

- Separable potential model:

$$v_l(p,k) = f_l(p) G_l f_l(k)$$

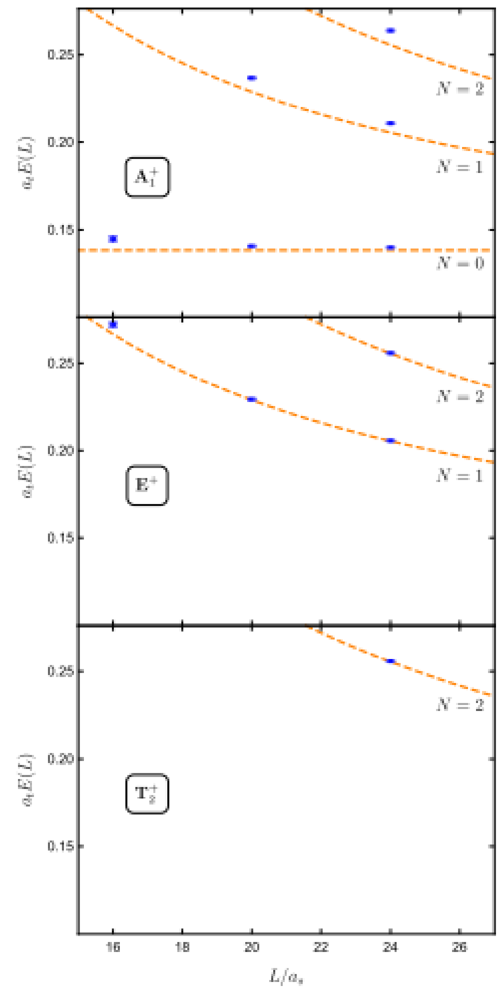
$$f_l(k) \sim \frac{(d_l \times k)^l}{(1 + (d_l \times k)^2)^{l/2+2}}$$

- 6 parameters: $G_0, G_2, G_4, d_0, d_2, d_4$

- Dimensions of Hamiltonians ($N_{\text{cut}} = 600$):

$$\mathbf{A}_1^+ : 923 \quad \mathbf{E}^+ : 965 \quad \mathbf{T}_2^+ : 963$$

- The fitted data: 11 energy levels



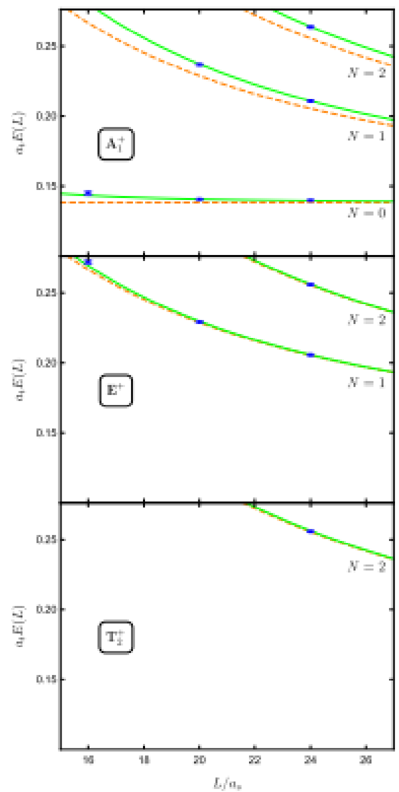
Example of Isospin-2 Scattering

Components of eigenstates

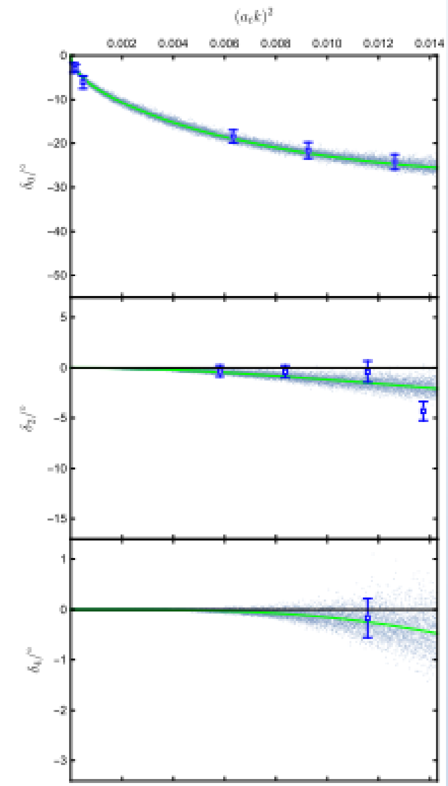
$\mathbf{A}_1^+$	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$\cdots$
1st	99.7	0.2	0.0	0.0	0.0	$\cdots$
2nd	0.1	97.4	1.9	0.2	0.0	$\cdots$
3rd	0.0	1.5	94.5	2.8	0.3	$\cdots$

- Fitting $\rightarrow$ Parameters $\rightarrow$ $\hat{H}$ and $\hat{H}_L$
- $\hat{H} \rightarrow \delta_l(E)$
- $\hat{H}_L \rightarrow E_n(\Gamma, L)$
- $\hat{H}_L \rightarrow$ Eigenstates

Volume dependent spectra



Phase shifts with errors



Example of Isospin-2 Scattering

TABLE III. The dimensions of the finite-volume Hamiltonian matrices for each of the irreducible representations Γ, for $N_{\text{cut}} = 100$ and 600.

Case: $\mathbf{d}$	Γ	$N_{\text{cut}} = 100$	$N_{\text{cut}} = 600$
A : $(0,0,0)$	$(A_1^+, A_2^+, E^+, T_1^+, T_2^+)$	$(129, 0, 145, 75, 144)$	$(923, 0, 965, 488, 963)$
C1: $(0,0,1)$	$(A_1^+, A_2^+, B_1^+, B_2^+, E^+)$	$(357, 202, 271, 249, 448)$	$(4357, 3004, 3354, 3254, 6222)$
C1: $(0,1,1)$	$(A_1^+, A_2^+, B_1^+, B_2^+)$	$(624, 467, 465, 487)$	$(8122, 6806, 6802, 6923)$
C1: $(1,1,1)$	(A_1^+, A_2^+, E^+)	$(409, 239, 652)$	$(5320, 3504, 8879)$
C2: $(0,0,1)$	$(A_1^+, A_2^+, B_1^+, B_2^+, E^+)$	$(308, 173, 234, 214, 448)$	$(4102, 2826, 3158, 3064, 6222)$
C2: $(0,1,1)$	$(A_1^+, A_2^+, B_1^+, B_2^+)$	$(558, 420, 417, 433)$	$(7772, 6516, 6518, 6625)$
C2: $(1,1,1)$	(A_1^+, A_2^+, E^+)	$(354, 215, 564)$	$(5035, 3360, 8381)$

		$\ell = 0$		$\ell = 2$		$\ell = 4$	
Data used	χ^2/N_{dof}	G_0	d_0	G_2	d_2	G_4	d_4
Rest only	10.5/(11-4)	67.8	4.57	90.6	d_B	3.40×10^2	d_B
Rest & Moving	115.9/(49-4)	67.2	4.59	68.1	d_B	2.57×10^2	d_B

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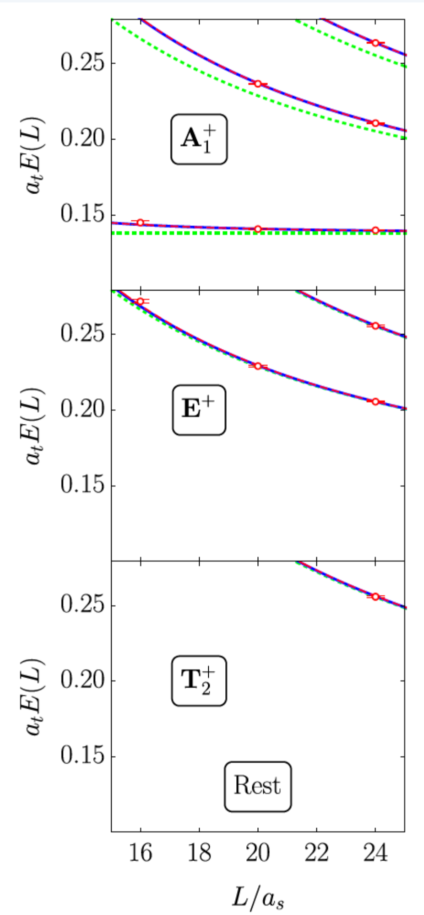
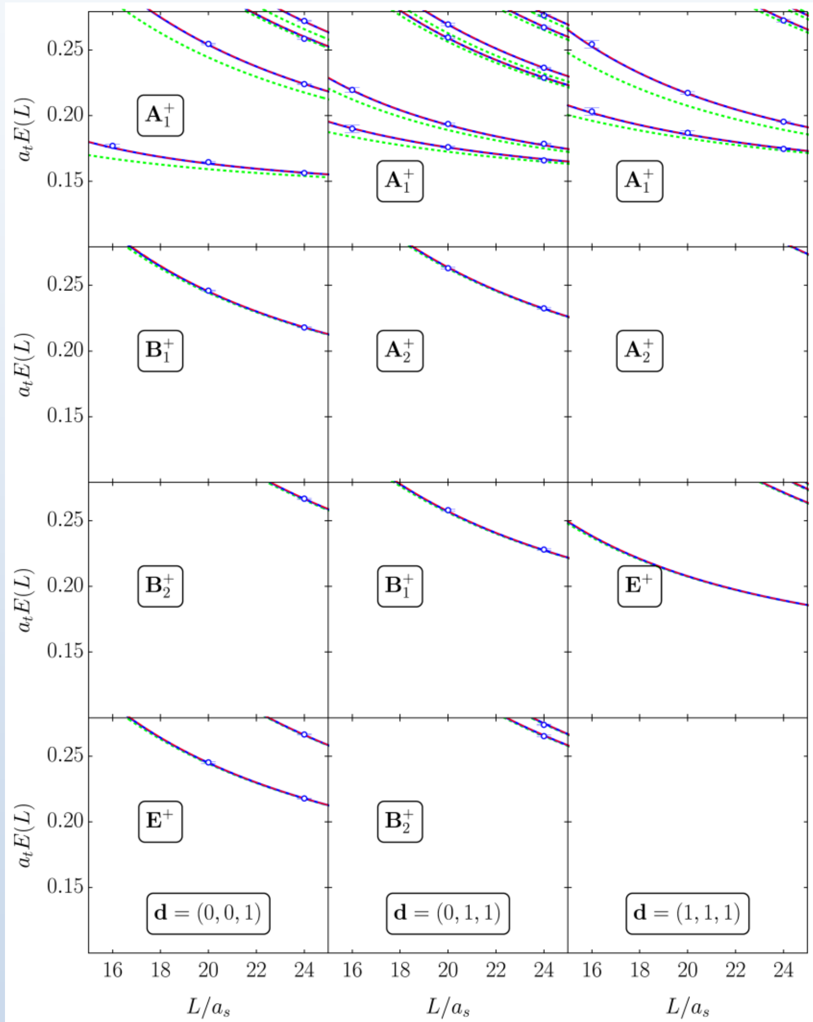
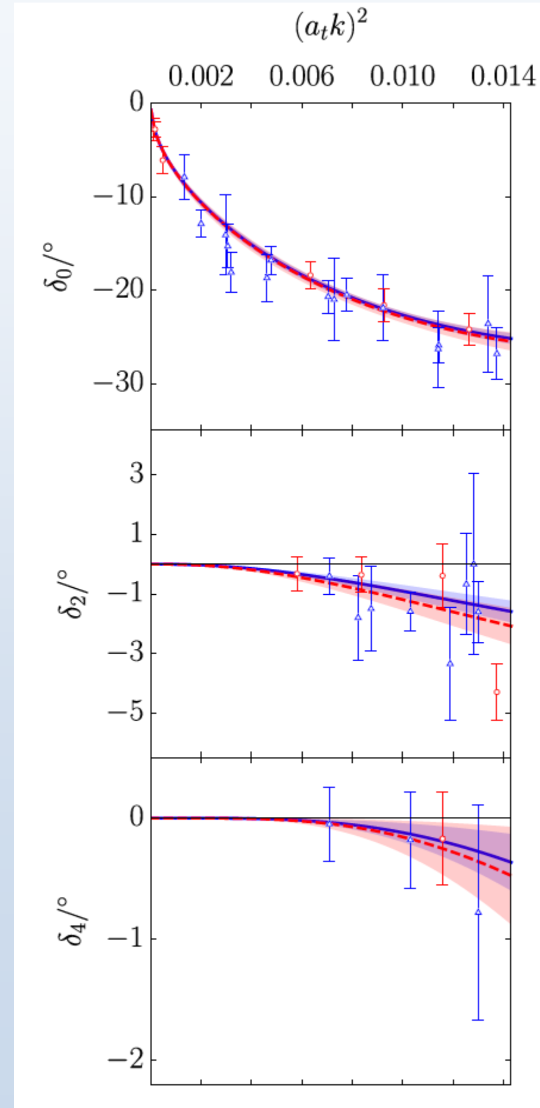
Example of Isospin-2 Scattering







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- 总结和展望

无限体积元中的三体系统

- Non-Relativistic Effective Field Theory (NREFT)

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$p\cot\delta_0 = -\frac{1}{a_0} + \frac{r_0}{2}p^2$$

$$p^5\cot\delta_2 = -\frac{1}{a_2}$$

$$\mathcal{L}_1 = \psi^\dagger\left(i\partial_0 - \frac{\nabla^2}{2m}\right)\psi$$

$$\mathcal{L}_2 = -\frac{C_0}{2}\psi^\dagger\psi^\dagger\psi\psi - \frac{C_2}{4}\left(\psi^\dagger\nabla^2\psi^\dagger\psi\psi + h.c.\right) + \cdots$$
$$-\frac{C_2'}{4}\left(\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi - 3\psi^\dagger\nabla^2\psi^\dagger\nabla^2\psi\psi + h.c.\right) + \cdots$$

$$\mathcal{L}_3 = -\frac{D_0}{6}\psi^\dagger\psi^\dagger\psi^\dagger\psi\psi\psi - \frac{D_2}{12}\left(\psi^\dagger\psi^\dagger\nabla^2\psi^\dagger\psi\psi\psi + h.c.\right) + \cdots$$
$$-\frac{D_2'}{4}\left(\psi^\dagger\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi\psi - 3\psi^\dagger\psi^\dagger\nabla^2\psi^\dagger\nabla^2\psi\psi\psi + h.c.\right) + \cdots$$

Phys.Rev. D97 (2018) no.11, 114508 Doring, Hammer, Mai, Pang, Rusetsky, Messiner, Wu
Phys.Rev. D99 (2019) no.07, 074513 Jin-Yi Pang, Jia-Jun Wu, H.-W. Hammer, A. Rusetski, U. Messiner
Phys.Rev. D102(2020)no. 11, 114515 Jin-Yi Pang, Jia-Jun Wu, Li-Sheng Geng

无限体积元中的三体系统

S-wave

$$\mathcal{L}_1 = \psi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m} \right) \psi$$

$$\mathcal{L}_2 = -\frac{C_0}{2}\psi^\dagger\psi^\dagger\psi\psi - \frac{C_2}{4}\left(\psi^\dagger\nabla^2\psi^\dagger\psi\psi + h.c.\right) + \cdots$$

$$\mathcal{L}_3 = -\frac{D_0}{6}\psi^\dagger\psi^\dagger\psi^\dagger\psi\psi\psi - \frac{D_2}{12}\left(\psi^\dagger\psi^\dagger\nabla^2\psi^\dagger\psi\psi\psi + h.c.\right) + \cdots$$

Dimer 场图像

$$(C_0,\, C_2,\, D_0,\, D_2)$$

$$\mathcal{L}_1 = \psi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m} \right) \psi$$

$$\sim (\sigma,\, f_1,\, h_0,\, h_2)$$

$$\mathcal{L}_2 = \sigma T^\dagger T + \frac{1}{2}\left[T^\dagger\left(\psi\psi + f_1\psi\nabla^2\psi + \cdots\right) + h.c.\right]$$

$$\mathcal{L}_3 = h_0 T^\dagger T\psi^\dagger\psi + h_2 T^\dagger T\left(\psi^\dagger\nabla^2\psi + h.c.\right) + \cdots$$

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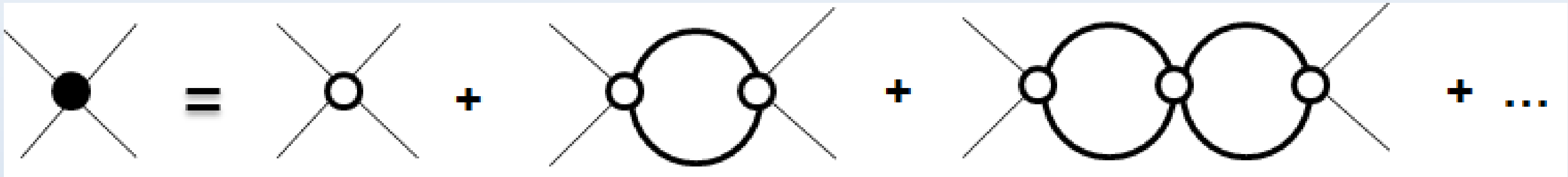
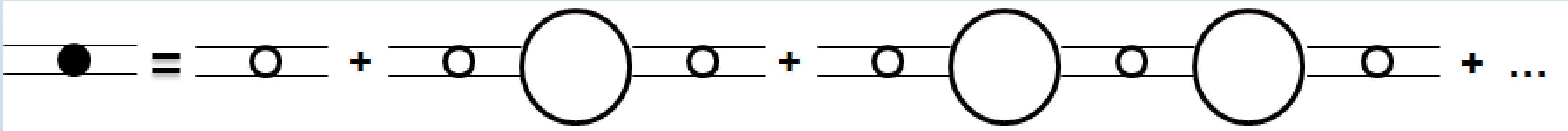
无限体积元中的三体系统

Dimer 场图像解析

$$\mathcal{L}_2 = -\frac{C_0}{2}\psi^\dagger\psi^\dagger\psi\psi - \frac{C_2}{4}\left(\psi^\dagger\nabla^2\psi^\dagger\psi\psi + h.c.\right) + \cdots$$



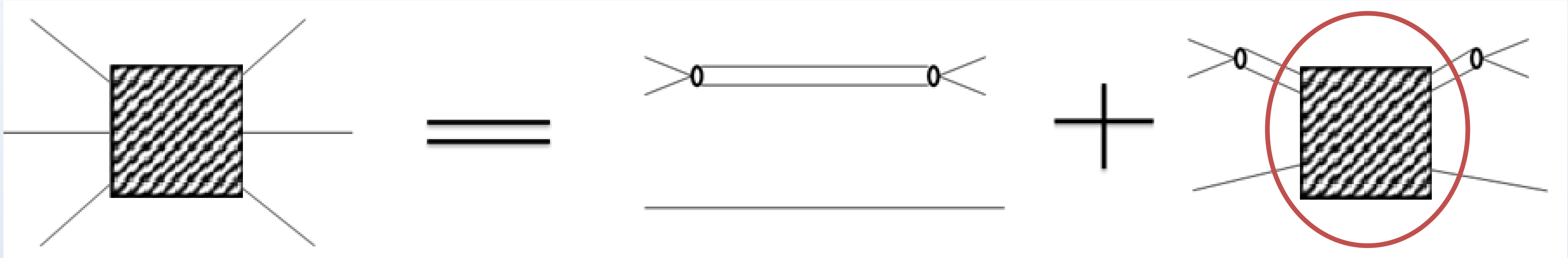
$$\mathcal{L}_2 = \sigma T^\dagger T + \left[T^\dagger\left(\psi\psi + f_1\psi\nabla\psi + \cdots\right) + h.c.\right]$$

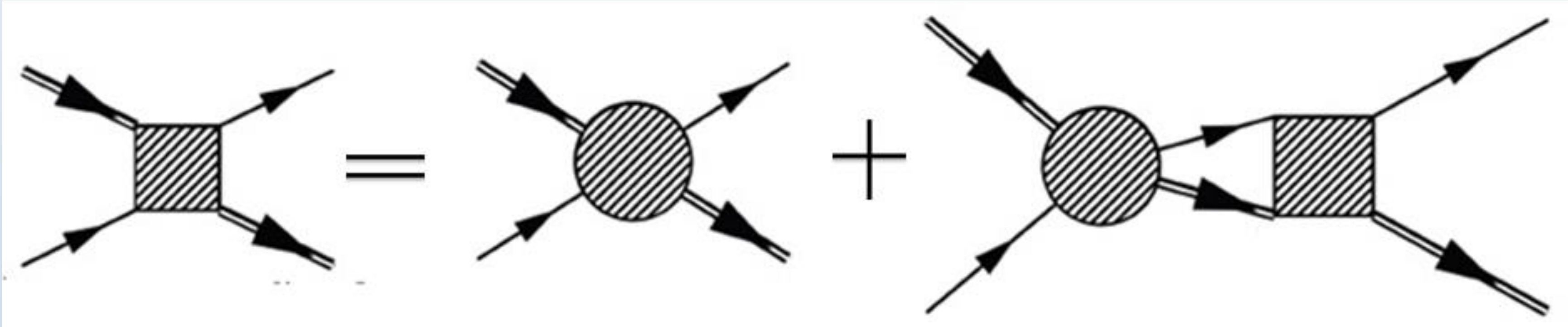


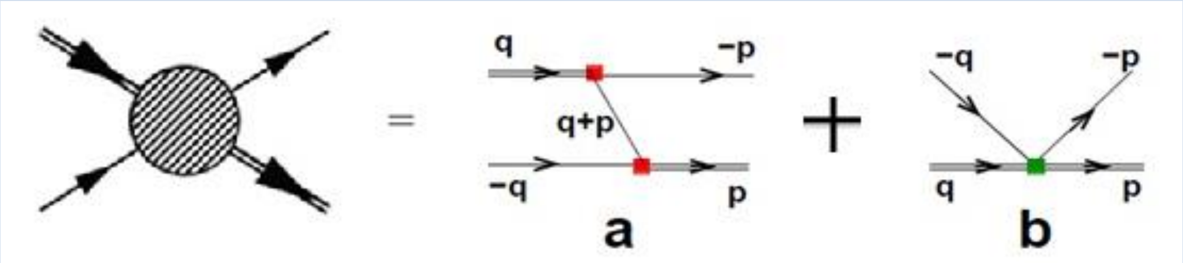
$$\tau(\vec{k}, E) = \frac{1}{k^* cot\delta_0(k^*) + ik^*}$$

无限体积元中的三体系统

Dimer Particle 散射 VS 3 Particles 散射







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无限体积元中的三体系统

简单模型

基本假设 S-wave & O(p) & Non-Relativistic

$$a_0 = m = 1$$

散射方程

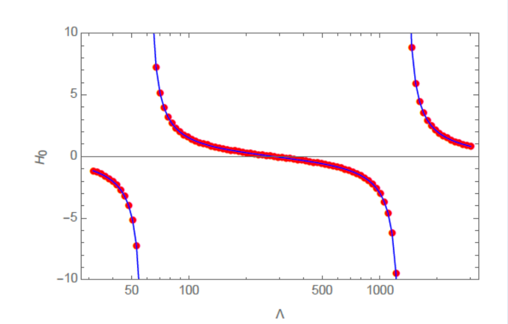
$$\mathcal{M}(\vec{p},\vec{q},E) = Z(\vec{p},\vec{q},E) + 8\pi\int_0^\Lambda \frac{d^3\vec{k}}{8\pi^3} Z(\vec{p},\vec{k},E)\tau(\vec{k},E)\mathcal{M}(\vec{k},\vec{q},E)$$

$$Z(\vec{p},\vec{q},E) = \frac{1}{\vec{p}^2+\vec{q}^2+\vec{p}\cdot\vec{q}-mE} + \frac{H_0}{\Lambda^2} \qquad \tau(\vec{k},E) = \frac{1}{-\frac{1}{a_0}+\sqrt{\frac{3}{4}k^2-mE}}$$

物理输入 $B_3(3p) = 10 \Longrightarrow H_0(\Lambda)$ vs Λ

$$\mathcal{F}(\vec{p},B_3) = \int_0^\Lambda d^3\vec{k} Z(\vec{p},\vec{k},B_3)\tau(\vec{k},B_3)\mathcal{F}(\vec{k},B_3)$$

$\mathcal{M}(\vec{p},\vec{q},E)$ is Λ independent



P.F. Bedaque, H.-W. Hammer, and U. van Kolck NPA 646 444 (1999)

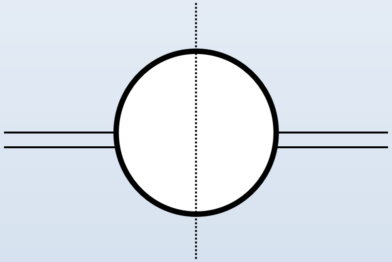
The three-boson system with short-range interactions

有限体积元三体系统的量子条件

- 无限体积 -> 有限体积， L
- 动量空间,

  连续 -> 分立　　$(2\pi/L)\,\vec{n}$ ，　$\vec{n} = (n_1, n_2, n_3)$
- Dimer场的传播子,

$$\tau(\vec{k}, E) = \frac{1}{-\frac{1}{a_0} + \sqrt{\frac{3}{4}k^2 - mE}}$$



$$\tau_L(\vec{m}, E) = \frac{1}{-\frac{1}{a_0} - \frac{4\pi}{L^3}\sum_{\vec{l}} \frac{1}{\frac{4\pi^2}{L^2}\left(m^2 + l^2 + \vec{k}\cdot\vec{l}\right) - mE}}$$

有限体积元三体系统的量子条件

- 散射方程,

$$\mathcal{M}(\vec{p},\vec{q},E) = Z(\vec{p},\vec{q},E) + 8\pi \int_0^{\Lambda} \frac{d^3\vec{k}}{8\pi^3} Z(\vec{p},\vec{k},E)\tau(\vec{k},E)\mathcal{M}(\vec{k},\vec{q},E)$$

⇒

$$\mathcal{M}(\frac{2\pi}{L}\vec{m},\frac{2\pi}{L}\vec{n},E) = Z(\frac{2\pi}{L}\vec{m},\frac{2\pi}{L}\vec{n},E) + \frac{8\pi}{L^3}\sum_{\vec{l}\in\mathcal{Z}^3}^{\frac{L\Lambda}{2\pi}} Z(\frac{2\pi}{L}\vec{m},\frac{2\pi}{L}\vec{l},E)\tau_L(\vec{l},E)\mathcal{M}(\frac{2\pi}{L}\vec{l},\frac{2\pi}{L}\vec{n},E)$$

$$\mathcal{M}_L(\vec{m},\vec{n},E) = Z_L(\vec{m},\vec{n},E) + \frac{8\pi}{L^3}\sum_{\vec{l}\in\mathcal{Z}^3}^{\frac{L\Lambda}{2\pi}} Z_L(\vec{m},\vec{l},E)\tau_L(\vec{l},E)\mathcal{M}_L(\vec{l},\vec{n},E)$$

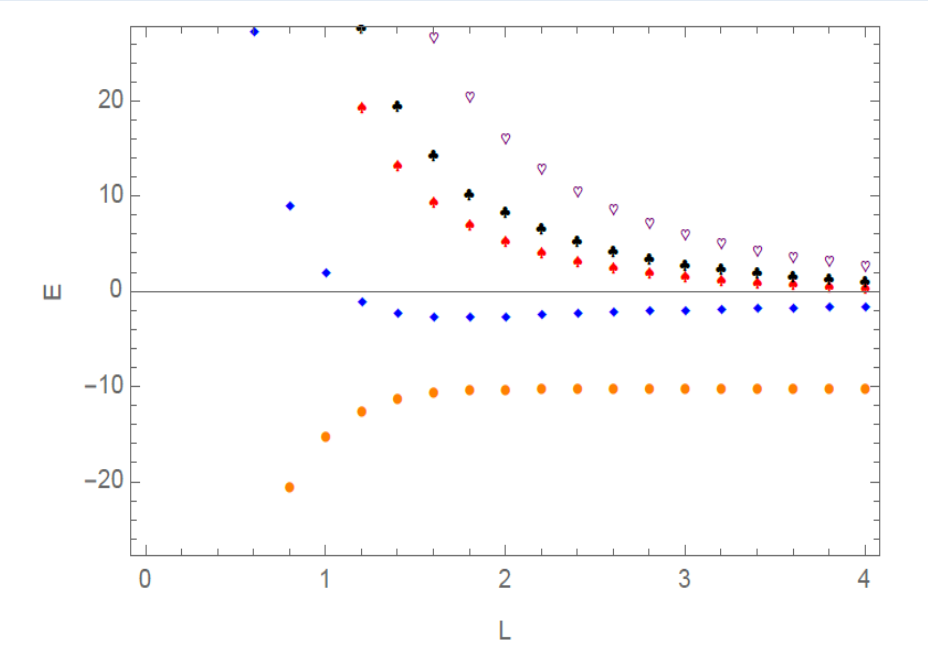
- 量子化条件:

$$\det(\tau_L^{-1} - \frac{8\pi}{L^3}Z_L) = 0$$

有限体积元三体系统的量子条件

- 简单模型的最低的五个能谱及讨论

$$\det\left(\theta^{-1}(s)\tau_L^{-1}(r,E)\delta_{rs} - \frac{8\pi}{L^3}\frac{1}{G}Z_L^{A_1^+}(r,s,E)\right) = 0$$



紫色: **Dimer-Particle 的激发态**

黑色: **3-Particle的基态**

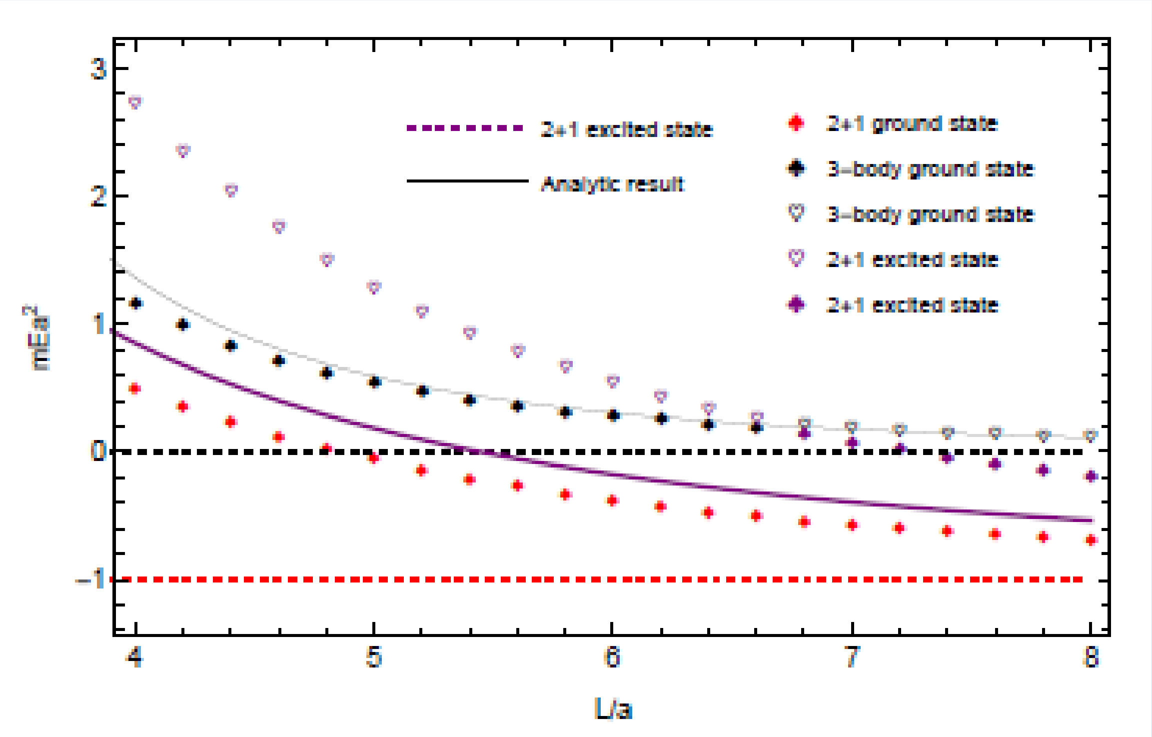
红色: **Dimer-Particle 的基态**

蓝色: **Dimer-Particle 的束缚态**

橙色: **3-Particle 的束缚态**

有限体积元三体系统的量子条件

- 这里紫色和黑色线有一个交叉。这个交叉可以帮助我们分清每条谱线的所代表物理态的意义。



紫色: **Dimer-Particle的激发态**

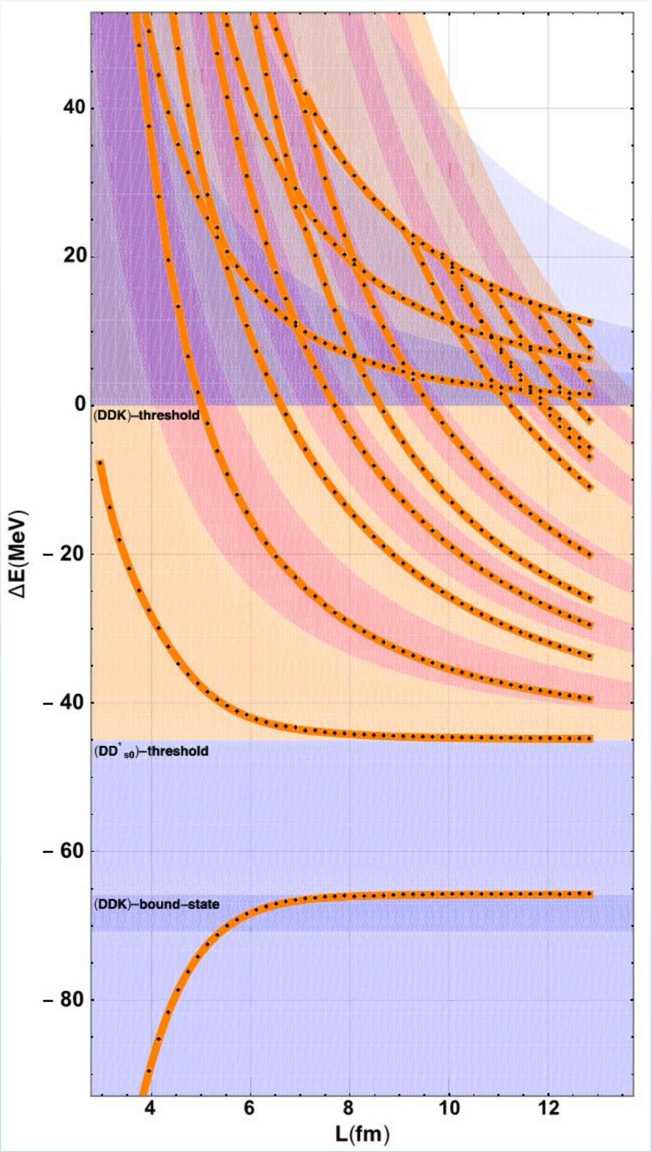
黑色: **3-Particle的基态**

红色: **Dimer-Particle的基态**

$$\Delta E = c_1 \frac{a^2}{L^3} + c_2 \frac{a^3}{L^4} + c_3 \frac{a^4}{L^5} + \mathcal{O}(L^{-6})$$

c_1, c_2, c_3 是固定的系数

DDK 系统



主要是为了能够研究D_{s0}(2317)，这个态和夸克模型预言的相差很大，而且是一个在DK阈下的束缚态，量子数0^+，我们希望通过三体系统给出其在格点上的信息。

通过具体的计算，我们得到了三个能量区域，

1. 高于DDK阈值的三体散射区
2. 低于DDK阈值高于D D_{s0}(2317)阈值的两体散射区
3. 低于D D_{s0}(2317) 阈值的三体束缚区

通过未来三体的格点数据和这个图像的比较，我们就能够检验物理模型是否正确。

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总结

- 有限哈密顿方法能够连接实验数据，格点数据以及有效模型。
- 有限哈密顿方法在不同的动量空间来研究同一个哈密顿量。 包括 复动量空间 / 实动量空间, 连续动量空间 / 离散动量空间。
- 在基本散射态作为基矢的假设下，离散态的本征矢量可以用以考察共振态的内部成分。据此我们研究了N*(1535), N*(1440), Λ*(1405)的结构，发现 N*(1440), Λ*(1405)并不能用夸克模型解释，主要是耦合道动力学产生的共振态。
- 在HEFT框架下考虑有限体积内的角动量混合问题。
- 用非相对论有效场论考虑了三体系统。

展望

- 在两体体系内搭建好了有限哈密顿方法。下一步要应用到各种具体的物理系统中。
- 非相对论有效场论考虑了三体系统，关键是要找到能够表征强子性质的特征量。

谢谢！

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Backup

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- For example： πN scattering in the Δ resonance energy region

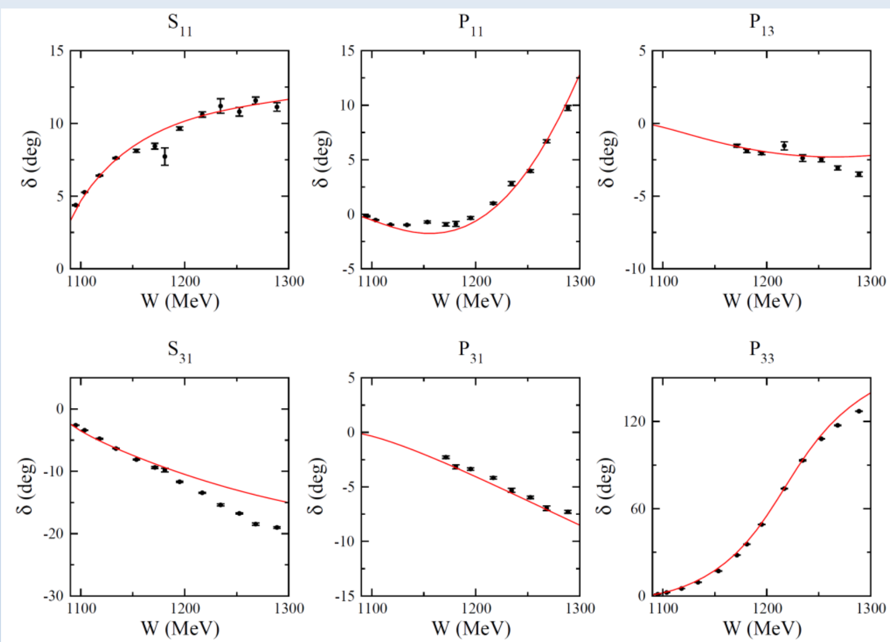
$$H_0=\begin{pmatrix} m_1 & 0 & 0 & \cdots \\ 0 & \sqrt{k_0^2+m_N^2}+\sqrt{k_0^2+m_\pi^2} & 0 & \cdots \\ 0 & 0 & \sqrt{k_1^2+m_N^2}+\sqrt{k_1^2+m_\pi^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

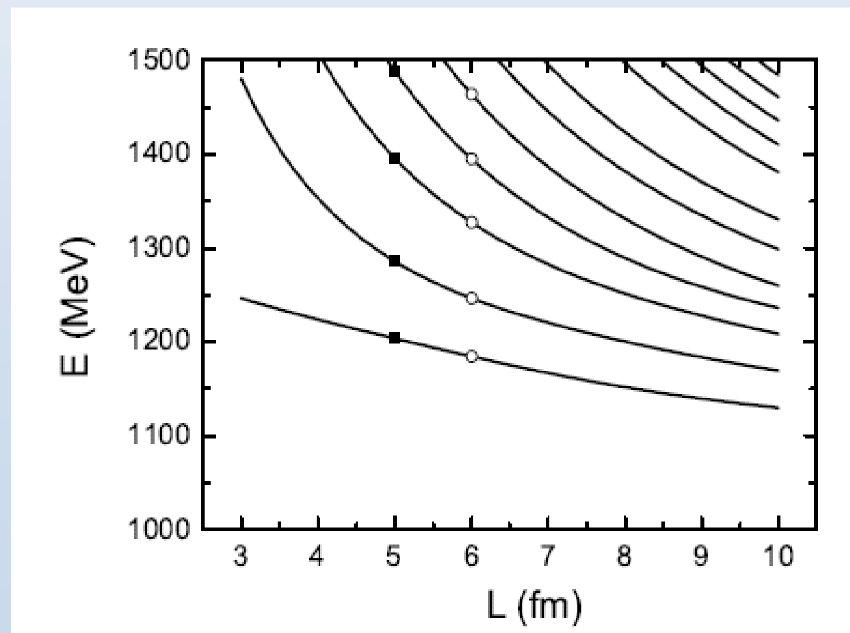
$$H_I=\begin{pmatrix} m_0 & g_{\pi N}^{fin}(k_0) & g_{\pi N}^{fin}(k_1) & \cdots \\ g_{\pi N}^{fin}(k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ g_{\pi N}^{fin}(k_1) & v_{\pi N,\pi N}^{fin}(k_1,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n)=\left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi N}(k_n) \quad v_{\pi\pi,\pi\pi}^{fin}(k_n,k_m)=\left(\frac{2\pi}{L}\right)^{3} v_{\pi N,\pi N}(k_n,k_m)$$

$$g_{\pi N}(k_n) \;\; v_{\pi N,\pi N}(k_n,k_m)$$

From Sato-Lee model





- For example： πN scattering in the Δ resonance energy region

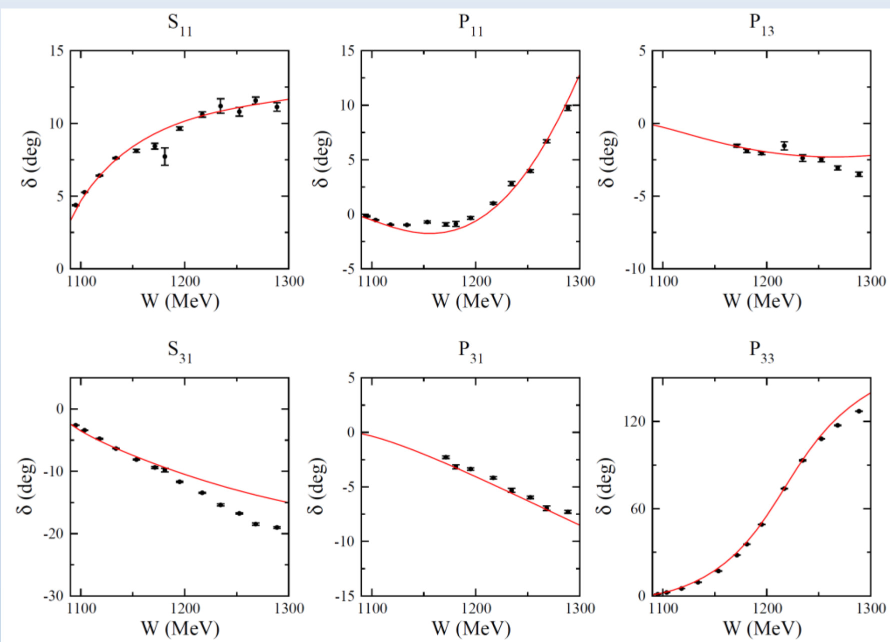
$$H_0=\begin{pmatrix} m_1 & 0 & 0 & \cdots \\ 0 & \sqrt{k_0^2+m_N^2}+\sqrt{k_0^2+m_\pi^2} & 0 & \cdots \\ 0 & 0 & \sqrt{k_1^2+m_N^2}+\sqrt{k_1^2+m_\pi^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

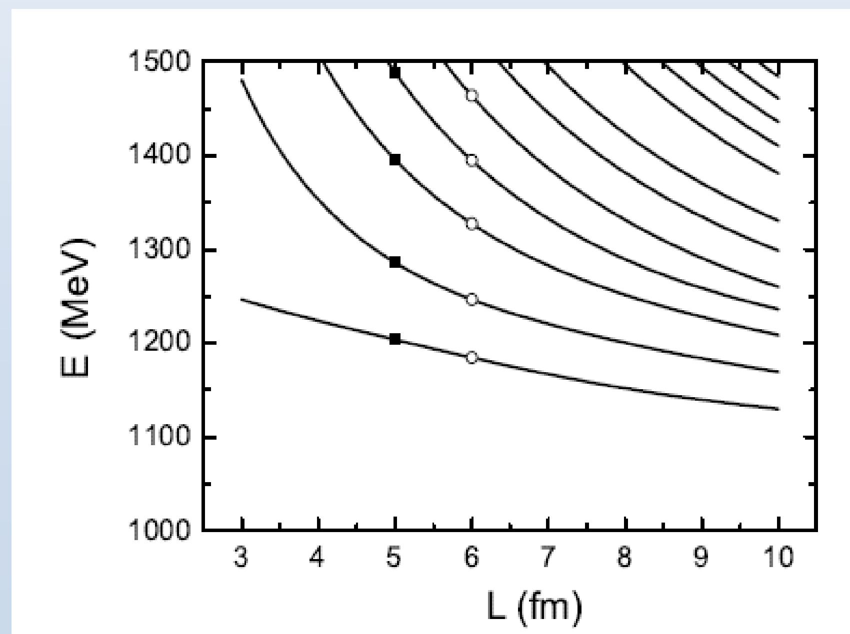
$$H_I=\begin{pmatrix} m_0 & g_{\pi N}^{fin}(k_0) & g_{\pi N}^{fin}(k_1) & \cdots \\ g_{\pi N}^{fin}(k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ g_{\pi N}^{fin}(k_1) & v_{\pi N,\pi N}^{fin}(k_1,k_0) & v_{\pi N,\pi N}^{fin}(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n)=\left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi N}(k_n) \quad v_{\pi\pi,\pi\pi}^{fin}(k_n,k_m)=\left(\frac{2\pi}{L}\right)^{3} v_{\pi N,\pi N}(k_n,k_m)$$

$g_{\pi N}(k_n)$ $v_{\pi N,\pi N}(k_n,k_m)$

From Sato-Lee model





Other Method

Resonance Properties: Mass , Width,
Pole position, Coupling, structure

Model (K matrix or others)

Lüscher Equation

T matrix
Phase Shift,
inelasticity

Lattice Spectrum

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Why equivalent with Lüscher Equation ?

$$k_{on}cot(\delta_E) = \frac{4}{\pi E}\frac{1}{g^2_{\pi N}(k_{on})}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\frac{g^2_{\pi N}(k_n)}{E_i - \sqrt{m_N^2+k_n^2} - \sqrt{m_\pi^2+k_n^2}}.$$

$$= \frac{2}{\pi}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\frac{1}{k_n^2 - k_{on}^2}$$

$\rightarrow$ $\frac{2}{\sqrt{\pi}L}Z_{00}(1;(\frac{kL}{2\pi})^2)$

Zeta Function
as for Luscher

$$+\frac{4}{\pi E}\left[\mathrm{P}\int k^2 dk - \left(\frac{2\pi}{L}\right)^3 \frac{1}{4\pi}\sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}}\right]\times$$

$$\left[\frac{g^2_{\pi N}(k_n)/g^2_{\pi N}(k_{on}) - 1}{E_i - \sqrt{m_N^2+k_n^2} - \sqrt{m_\pi^2+k_n^2}} + \ldots\right]$$

$\rightarrow$ No Singularity

Would be suppressed by e^{-Lm}

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什么是有限体积中的本征态?

$$(H_0+H_I)\,|\Psi> = E\,|\Psi>$$

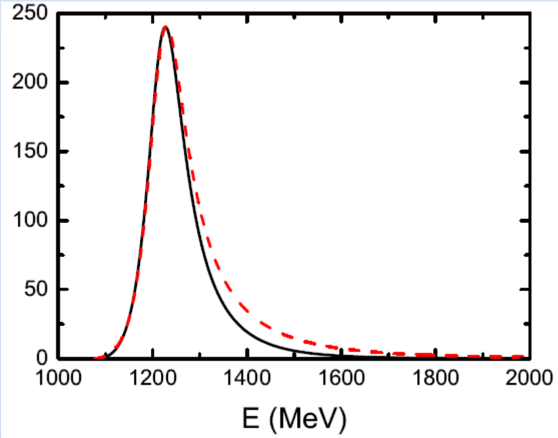
$$|\Psi_E\rangle = C_0|B> + \sum_{\vec{k}_n=\frac{2\pi}{L}\vec{n}} C_E(\vec{k}_n)|\alpha(\vec{k}_n)\rangle$$

这些本征态和共振态是什么关系?

Infinite-Volume

The eigenstate of Hamiltonian is final scattering state, which are continuum,

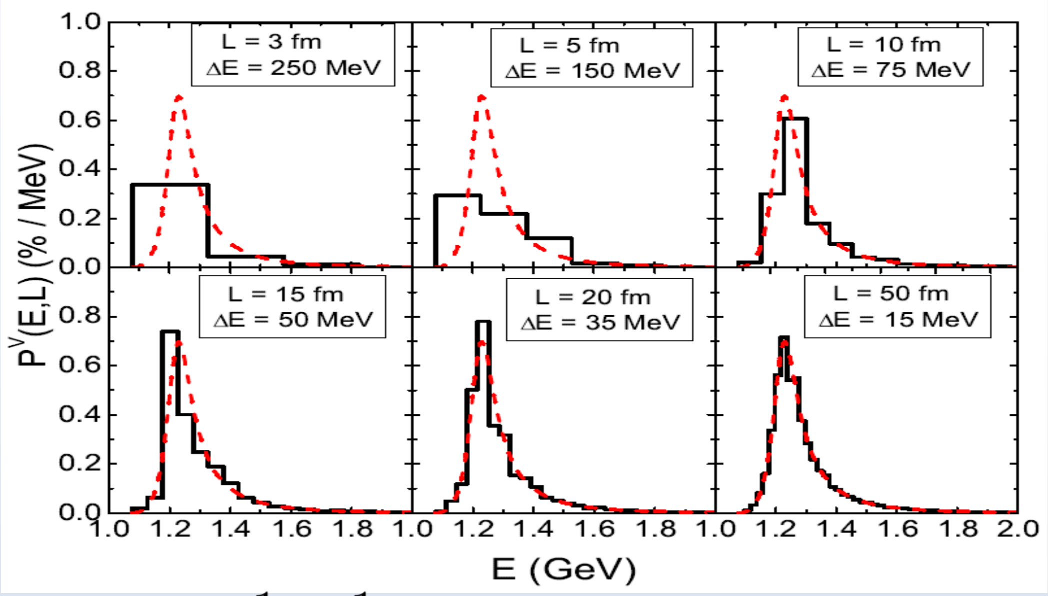
$$|\langle B|\Psi^{(+)}_{E,i}\rangle|^2 = \frac{\bar{\Gamma}_{\pi N}(k_{\pi N};E)}{E-m_0-\Sigma(E)}$$



离散本征态 VS 共振态

Sato-Lee Model for
Δ(1232) region

Jia-jun Wu etc.
Phys.Rev. D95 (2017) no.11, 114507



Finite-Volume
Black Solid line

$$P^V(E_k^{ave}, L) = \frac{1}{Z^V}\frac{1}{\Delta E}\sum_{E_k^{ave}-\frac{\Delta E}{2}\le E_\alpha \le E_k^{ave}+\frac{\Delta E}{2}} |\langle B|\Psi^V_{E_\alpha}\rangle|^2$$

Infinite-Volume
Red dashed line

$$P(E) = \frac{1}{Z}\sum_{i=1,n_c} \pi k_i E_{M_i}(k_i) E_{B_i}(k_i) |\langle B|\Psi^{(+)}_{E,i}\rangle|^2$$

LQCD -> N* resonance -> coupled-channel data

Since $P^V(E)$-> P(E) as volume size increase, P(E) which can be readily calculated using ANL-Osaka Hamiltonian can already bridge

共振态重子的结构

- 3 quark
- 5 quark
- Meson Cloud + 3 quark core
- Meson-Baryon Molecule
- Dynamical generated state

… …

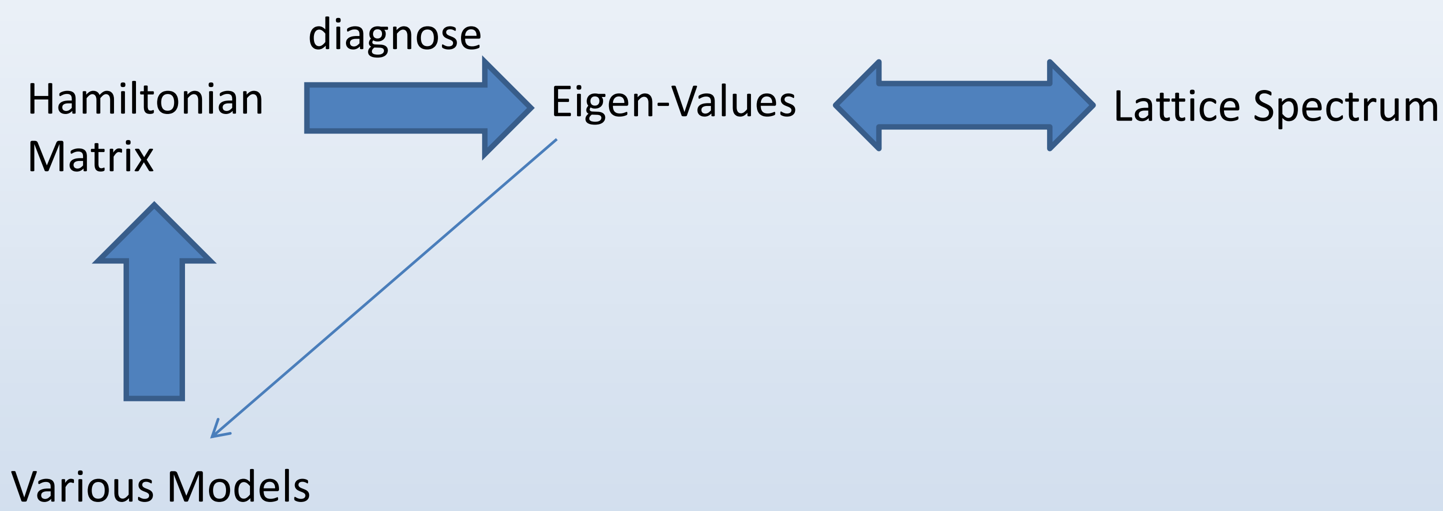
问题：如何区分???

依赖于实验和格点能谱数据的拟合。

一般而言，模型的参数很多，总有办法实现拟合 !!!

如何区分模型？？？

- 在有限哈密顿方法下:



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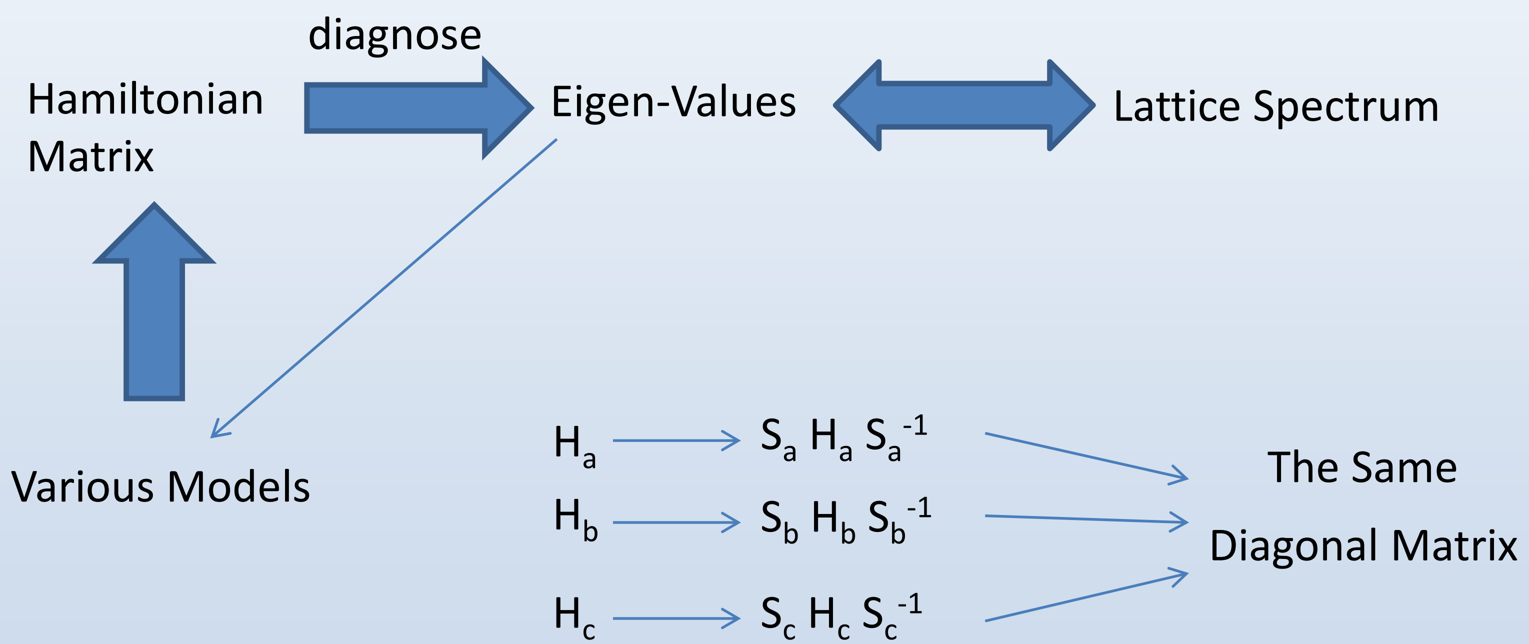

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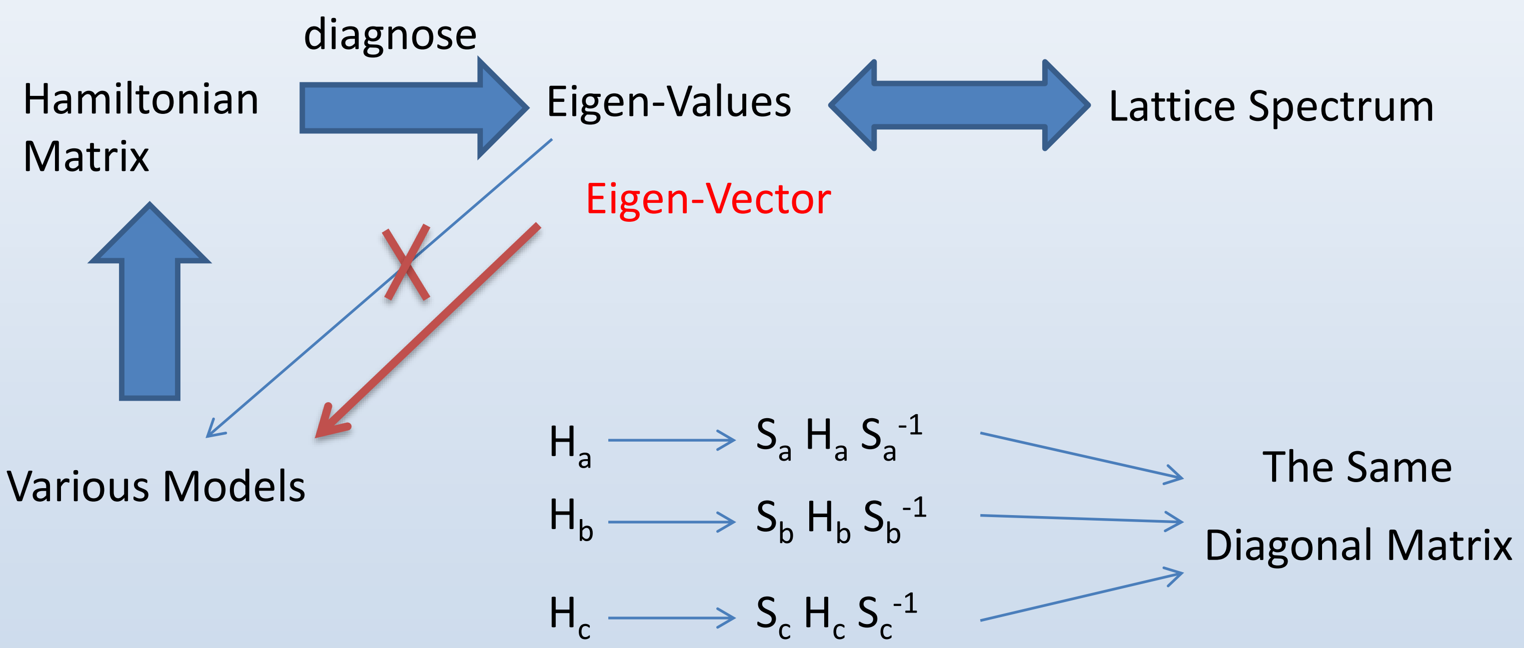

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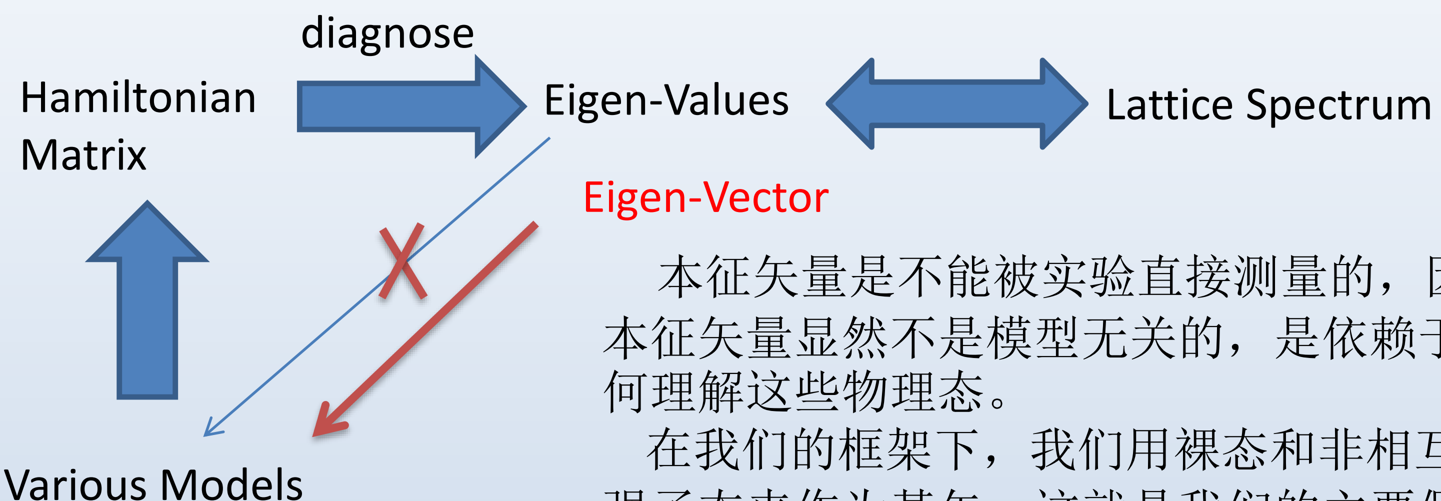


$$\left\langle \alpha \middle| \hat{O} \middle| \alpha \right\rangle \quad \text{vs} \quad \left\langle \Psi \middle| \hat{O} \middle| \Psi \right\rangle$$

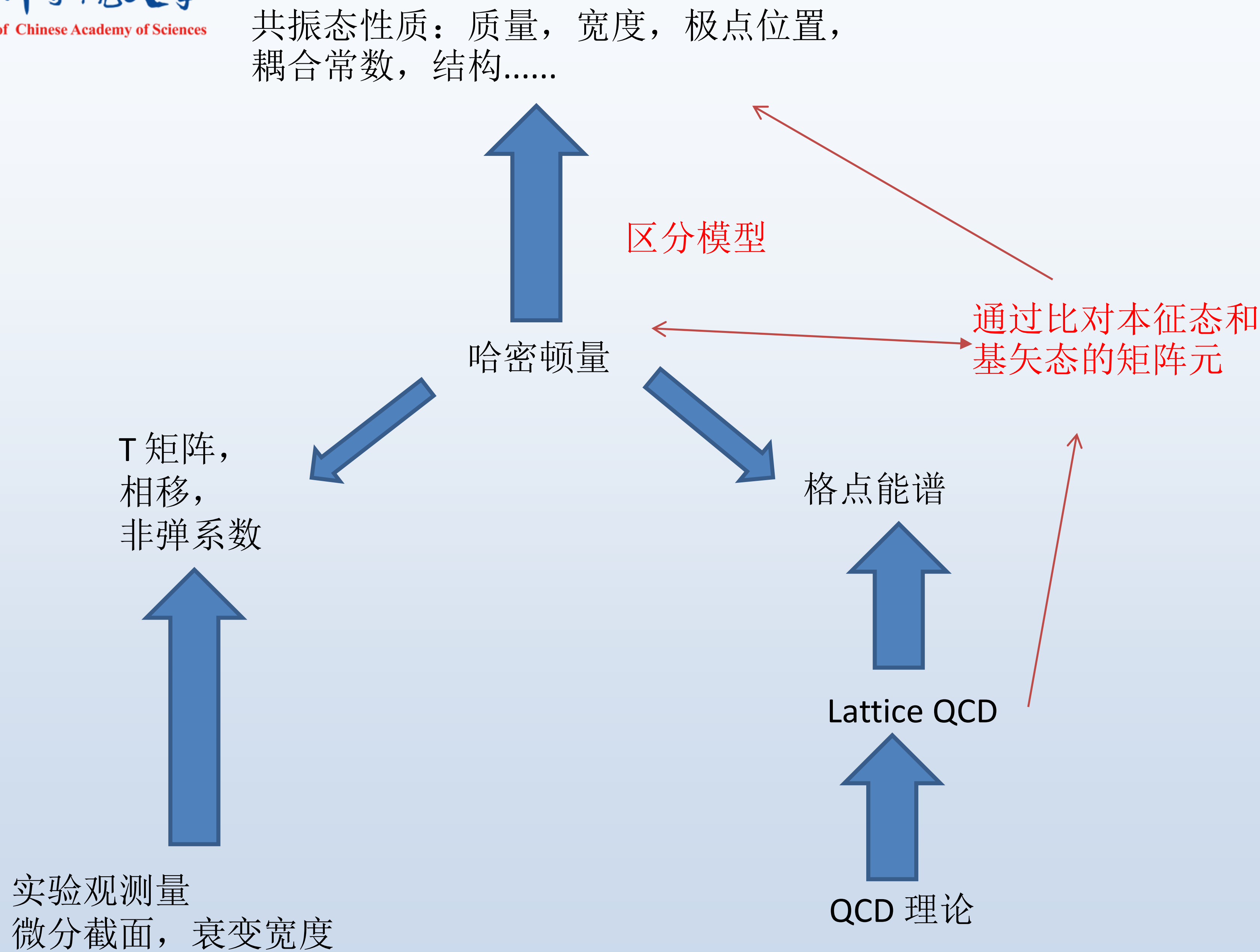
Theory or Model　　　　Lattice

本征矢量是不能被实验直接测量的，因为这个本征矢量显然不是模型无关的，是依赖于人们如何理解这些物理态。

在我们的框架下，我们用裸态和非相互作用的强子态来作为基矢，这就是我们的主要假设。在这个框架下，我们是可以确定本征矢量，虽然这些本征矢量还是会依赖于截断参数等模型参数的影响，但是定性上总可以告诉我们一些本征态的信息。而这些本征态实际上就是反映了共振态的信息，所以我们也就是间接了解我们关心的共振态的信息。

|α>: bare state or non-interaction states

|Ψ>: eigenstate of interaction Hamiltonian


共振态性质：质量，宽度，极点位置，
耦合常数，结构……
区分模型
哈密顿量
通过比对本征态和
基矢态的矩阵元
T 矩阵，
相移，
非弹系数
格点能谱
Lattice QCD
实验观测量
微分截面，衰变宽度
QCD 理论

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不幸的是，现在的格点QCD技术还不能够测量所有的本征态和相关的矩阵元，尤其是重子共振态的能区。通常，现在用的都是纯的3夸克的局域算符，介子-重子非局域的算符还用的比较少。

C. B. Lang, etc. Phys.Rev. D95 (2017) no.1, 014510

3夸克的局域算符:

- 和介子-重子散射态的耦合比较小，更多的是和3夸克态耦合，即核子激发态。
- 而裸态我们可以认为是独立于介子-重子散射态，是3夸克的激发态。
- 所以原则上，我们认为3夸克算符会激发含有裸态较多的态。
- 因此，通过3夸克算符得到的本征态应该含有较多的裸态成分。

有限体积元三体系统的能移展开

- 前人的工作:

PHYSICAL REVIEW D **96**, 054515 (2017)

Testing the threshold expansion for three-particle energies at fourth order in ϕ^4 theory

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(Received 24 July 2017; published 26 September 2017)

$$N_{\rm cut} = mL/(2\pi),\ c_L = 16\pi^3(\sqrt{3} - 4\pi/3)$$

$$\begin{aligned}\Delta E_{3,\rm thr} = \frac{12\pi a}{mL^3}\Big\{ & 1 - \left(\frac{a}{\pi L}\right)\mathcal{I} + \left(\frac{a}{\pi L}\right)^2(\mathcal{I}^2 + \mathcal{J}) \\ & + \frac{64\pi^2 a^2 \mathcal{C}_3}{mL^3} + \frac{3\pi a}{m^2 L^3} + \frac{6\pi r a^2}{L^3} \\ & + \left(\frac{a}{\pi L}\right)^3[-\mathcal{I}^3 + \mathcal{I}\mathcal{J} + 15\mathcal{K} + c_L \log(N_{\rm cut}) \\ & + \mathcal{C}_F + \mathcal{C}_4 + \mathcal{C}_5]\Big\} - \frac{\mathcal{M}_{3,\rm thr}}{48m^3L^6} + \mathcal{O}(L^{-7}), \end{aligned} \qquad (16)$$

and evaluated in Ref. [8]. The new amplitude entering at $\mathcal{O}(1/L^6)$ is the divergence-free three-to-three threshold amplitude $\mathcal{M}_{3,\rm thr}$, which begins at $\mathcal{O}(\lambda^2)$ in perturbation theory. The numerical values of $\mathcal{C}_3$, $\mathcal{C}_4$, and $\mathcal{C}_5$ depend on the choice of UV cutoff, but this dependence cancels with that of $\mathcal{M}_{3,\rm thr}$. This cancellation is necessary because $\Delta E_{3,\rm thr}$ is a physical quantity.

3-Particle的基态能移, i.e., $0 < mE < L^{-2}$

$$mE = \left(\frac{2\pi}{L}\right)^2 \left(\frac{x_1}{L/a} + \frac{x_2}{(L/a)^2} + \frac{x_3}{(L/a)^3} + \frac{x_4}{(L/a)^4}\right)$$

$$\det\left(\theta^{-1}(s)\tau_L^{-1}(r,E)\delta_{rs} - \frac{8\pi}{L^3}\frac{1}{G}Z_L^{A_1^+}(r,s,E)\right) = 0$$

3-Particle的基态能移, i.e., 0 < mE < L^{-2}

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$$\det\left(1 - \tilde{Z}_L(r,s)\tilde{\tau}(s)\right) = 0$$

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3-Particle的基态能移, i.e., 0 < mE < L^{-2}

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$$\det\left(1 - \tilde{Z}_L(r,s)\tilde{\tau}(s)\right) = 0$$

$$\begin{vmatrix} 1-\tilde{Z}(1,1)\tilde{\tau}(1) & -\tilde{Z}(1,2)\tilde{\tau}(2) & \cdots & -\tilde{Z}(1,N)\tilde{\tau}(N) \\ -\tilde{Z}(2,1)\tilde{\tau}(1) & 1-\tilde{Z}(2,2)\tilde{\tau}(2) & \cdots & -\tilde{Z}(2,N)\tilde{\tau}(N) \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{Z}(N,1)\tilde{\tau}(1) & -\tilde{Z}(N,2)\tilde{\tau}(2) & \cdots & 1-\tilde{Z}(N,N)\tilde{\tau}(N) \end{vmatrix} = 0$$

$$s = 1, 2, ..., L$$

$$N \propto \frac{L\Lambda}{2\pi}$$

$$\frac{n}{L} = \frac{0}{L}, \frac{1}{L}, \cdots \frac{\Lambda}{2\pi}$$

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$$1 - \tilde{Z}(1,1)\tilde{\tau}(1) = 1 - \frac{3a_0}{\pi x_1} + \mathcal{O}(L^{-1})$$

$$\tilde{Z}(i,j)\tilde{\tau}(j) = \mathcal{O}(L^{-1}) \text{ for i>1 or j>1}$$

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$$\begin{vmatrix} \delta & -A_{12} & -A_{13} & \cdots & -A_{1N} \\ -A_{21} & 1-A_{22} & -A_{23} & \cdots & -A_{2N} \\ -A_{31} & -A_{32} & 1-A_{33} & \cdots & -A_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_{N1} & -A_{N2} & -A_{N3} & \cdots & 1-A_{NN} \end{vmatrix} = 0$$

$$A_{ij} \propto \frac{1}{L}$$

$$\delta \propto \frac{1}{L}$$

$$\delta = \sum_{j_1 \neq 1} A_{1j_1} A_{j_1 1} + \sum_{j_1, j_2 \neq 1} A_{1j_1} A_{j_1 j_2} A_{j_2 1} + \sum_{j_1, j_2, j_3 \neq 1} A_{1j_1} A_{j_1 j_2} A_{j_2 j_3} A_{j_3 1} + \cdots$$

$$\begin{vmatrix} \delta & -A_{12} & -A_{13} & \cdots & -A_{1N} \\ -A_{21} & 1-A_{22} & -A_{23} & \cdots & -A_{2N} \\ -A_{31} & -A_{32} & 1-A_{33} & \cdots & -A_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_{N1} & -A_{N2} & -A_{N3} & \cdots & 1-A_{NN} \end{vmatrix} = 0$$

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$$1 - \tilde{Z}(1,1)\tilde{\tau}(1) = 1 - \frac{3a_0}{\pi x_1} + \mathcal{O}(L^{-1})$$

$$\Longrightarrow \quad x_1 = \frac{3a_0}{\pi}$$

$$\tilde{\tau}^{-1}(1) - \tilde{Z}(1,1) = \sum_{j_1 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,1) + \sum_{j_1, j_2 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,1)$$
$$+ \sum_{j_1, j_2, j_3 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,j_3)\tilde{\tau}(j_3)\tilde{Z}(j_3,1) + \cdots$$

$$x_1 = \frac{3a_0}{\pi} \quad \Longrightarrow \quad mE = \frac{12\pi a}{L^3}\left(1 + \frac{x_1}{L/a} + \frac{x_2}{(L/a)^2} + \frac{x_3' Log[L/a]}{(L/a)^3} + \frac{x_3}{(L/a)^3}\right)$$

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$$\tilde{\tau}^{-1}(1) - \tilde{Z}(1,1) = \frac{1}{L/a}\left(-x_1 - \frac{\mathbb{I}(\vec{0})}{\pi}\right) + \frac{1}{(L/a)^2}\left(-x_2 + x_1^2 - \frac{3\mathbb{J}(\vec{0})}{\pi^2}\right) + \frac{-x_3' \log[L/a]}{(L/a)^3}$$
$$+ \frac{1}{(L/a)^3}\left(-x_3 + 2x_1x_2 - x_1^3 + \frac{6\pi r}{a} - \frac{1}{\pi^3}\left(3x_1\pi\mathbb{J}(\vec{0}) + 9\mathbb{K}(\vec{0})\right)\right) - \frac{1}{(L/a)^3}\left(\frac{8\pi H_0(\Lambda)}{a^2\Lambda^2}\right) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,1) = -\frac{4\mathbb{J}(\vec{0})}{(\pi L/a)^2} - \frac{16\sqrt{3}\log(L/a)}{(L/a)^3} + \frac{1}{(\pi L/a)^3}(X_0(\Lambda) + C_2 - 24\mathbb{K}(0)) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1,j_2 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,1) = \frac{64\pi\log(L/a)}{3(L/a)^3} + \frac{1}{(\pi L/a)^3}X_1(\Lambda) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1,j_2,j_3 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,j_3)\tilde{\tau}(j_3)\tilde{Z}(j_3,1) + \cdots = X_2(\Lambda)\frac{1}{(\pi L/a)^3} + \mathbb{O}(L^{-3-\epsilon})$$

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$$x_1 = \frac{3a_0}{\pi} \quad \Longrightarrow \quad mE = \frac{12\pi a}{L^3}\left(1 + \frac{x_1}{L/a} + \frac{x_2}{(L/a)^2} + \frac{x_3' Log[L/a]}{(L/a)^3} + \frac{x_3}{(L/a)^3}\right)$$

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$$+ \frac{1}{(L/a)^3}\left(-x_3 + 2x_1x_2 - x_1^3 + \frac{6\pi r}{a} - \frac{1}{\pi^3}\left(3x_1\pi\mathbb{J}(\vec{0}) + 9\mathbb{K}(\vec{0})\right)\right) - \frac{1}{(L/a)^3}\left(\frac{8\pi H_0(\Lambda)}{a^2\Lambda^2}\right) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,1) = -\frac{4\mathbb{J}(\vec{0})}{(\pi L/a)^2} - \frac{16\sqrt{3}\log(L/a)}{(L/a)^3} + \frac{1}{(\pi L/a)^3}(X_0(\Lambda) + C_2 - 24\mathbb{K}(0)) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1,j_2 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,1) = \frac{64\pi\log(L/a)}{3(L/a)^3} + \frac{1}{(\pi L/a)^3}X_1(\Lambda) + \mathbb{O}(L^{-3-\epsilon})$$

$$\sum_{j_1,j_2,j_3 \neq 1} \tilde{Z}(1,j_1)\tilde{\tau}(j_1)\tilde{Z}(j_1,j_2)\tilde{\tau}(j_2)\tilde{Z}(j_2,j_3)\tilde{\tau}(j_3)\tilde{Z}(j_3,1) + \cdots = X_2(\Lambda)\frac{1}{(\pi L/a)^3} + \mathbb{O}(L^{-3-\epsilon})$$

非微扰效应

有限体积元三体系统的能移展开

$$mE = \frac{12\pi a}{L^3}\left(1 + \frac{x_1}{L/a} + \frac{x_2}{(L/a)^2} + \frac{x_3' \log[L/a]}{(L/a)^3} + \frac{x_3}{(L/a)^3} + \cdots\right)$$

$$x_1 = -\frac{\mathbb{I}(0)}{\pi} \quad x_2 = \frac{\mathbb{I}^2(0) + \mathbb{J}(0)}{\pi^2} \quad x_3' = 16\sqrt{3} - 64\pi/3$$

$$x_3 = +\frac{6\pi r}{a} + \frac{1}{\pi^3}\left(-\mathbb{I}^3(0) + \mathbb{I}(0)\mathbb{J}(\vec{0}) + 15\mathbb{K}(\vec{0}) + C_2\right) - \frac{1}{\pi^3}(X_0(\Lambda) + X_1(\Lambda) + X_2(\Lambda)) - \frac{8\pi H_0(\Lambda)}{a^2\Lambda^2}$$

有限体积元三体系统的能移展开

$$mE = \frac{12\pi a}{L^3}\left(1 + \frac{x_1}{L/a} + \frac{x_2}{(L/a)^2} + \frac{x_3' \log[L/a]}{(L/a)^3} + \frac{x_3}{(L/a)^3} + \cdots\right)$$

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$$x_3 = +\frac{6\pi r}{a} + \frac{1}{\pi^3}\left(-\mathbb{I}^3(0) + \mathbb{I}(0)\mathbb{J}(\vec{0}) + 15\mathbb{K}(\vec{0}) + C_2\right)$$

$$-\frac{1}{\pi^3}(X_0(\Lambda) + X_1(\Lambda) + X_2(\Lambda)) - \frac{8\pi H_0(\Lambda)}{a^2\Lambda^2}$$

$$-(L/a)^3\left(X(\Lambda)\frac{8\pi a}{L^3} + \frac{4\mathbb{J}(\vec{0})}{(\pi L/a)^2} + \frac{(16\sqrt{3} - 64\pi/3)\log(L/a)}{(L/a)^3}\right)$$

$$X(\Lambda) = \frac{H_0(\Lambda)}{\Lambda^2} + \sum_{\vec{i}\in\mathbb{Z}^3}^{[1,N]} \frac{8\pi}{L^3} z(\vec{0},\vec{i},0)\tau(\vec{i},0)M^L(\vec{i},\vec{0}),$$

$$M^L(\vec{j},\vec{0}) = z(\vec{j},\vec{0},0) + \sum_{\vec{i}\in\mathbb{Z}^3}^{[1,N]} \frac{8\pi}{L^3} z(\vec{j},\vec{i},0)\tau(\vec{i},0)M^L(\vec{i},\vec{0}).$$

截断无关

X_3：Λ 无关

$$X(\Lambda) = \frac{H_0(\Lambda)}{\Lambda^2} + \sum_{\vec{i}\in\mathbb{Z}^3}^{[1,N]} \frac{8\pi}{L^3} z(\vec{0},\vec{i},0)\tau(\vec{i},0)M^L(\vec{i},\vec{0}),$$

$$M^L(\vec{j},\vec{0}) = z(\vec{j},\vec{0},0) + \sum_{\vec{i}\in\mathbb{Z}^3}^{[1,N]} \frac{8\pi}{L^3} z(\vec{j},\vec{i},0)\tau(\vec{i},0)M^L(\vec{i},\vec{0}),$$

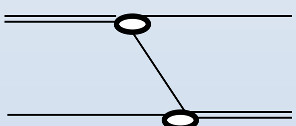
$$z(\vec{i},\vec{j},n) = \frac{L^2}{4\pi^2}\frac{1}{\vec{i}^2+\vec{j}^2+\vec{i}\cdot\vec{j}-n} + \frac{H_0}{\Lambda^2},$$

$$\tau(\vec{i},n) = -\frac{1}{a} + \frac{\pi}{L}\sqrt{3\vec{i}^2-4n}$$

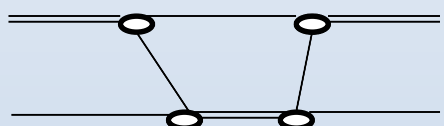
$$\sim M(\vec{0},\vec{0},0)$$

发散来源

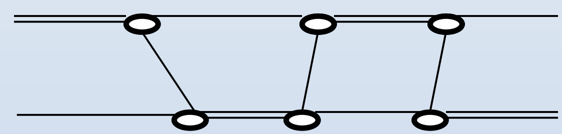
树图，单圈图，两圈图。



$\frac{1}{\epsilon^2}$



$\frac{1}{\epsilon}$ & $\log[\epsilon]$



$\log[\epsilon]$

$$\mathcal{A}_b = \lim_{\epsilon\to 0^+}\left(M_\epsilon(\vec{0},\vec{0},0) + \frac{4a}{\pi\epsilon} + \left(\frac{8}{3} - \frac{2\sqrt{3}}{\pi}\right) a^2 \log\left(\frac{a\epsilon}{2\pi}\right)\right) - a^2\frac{112\zeta(3)}{9}$$

$$M_\epsilon(\vec{0},\vec{0},0) \equiv \frac{H_0(\Lambda)}{\Lambda^2} + 8\pi\int_\epsilon^\Lambda \frac{d^3\vec{k}}{(2\pi)^3} Z(\vec{0},\vec{k},0)\tau(\vec{k},0)M(\vec{k},\vec{0},0)$$

X_3：Λ 无关

$$X(\Lambda) + La\frac{\mathbb{J}(\vec{0})}{2\pi^3} + \frac{a^2(16\sqrt{3} - 64\pi/3)\log(L/a)}{8\pi} = \mathcal{A} + a^2\left(X_1 - \frac{\sqrt{3}\mathbb{I}_3}{2\pi^2}\right).$$

$$x_3 = +\frac{1}{\pi^3}\left(-\mathbb{I}^3(0) + \mathbb{I}(0)\mathbb{J}(\vec{0}) + 15\mathbb{K}(\vec{0}) + C_2\right) - \frac{8\pi}{a^2}\mathcal{A} - 8\pi\left(X_1 - \frac{\sqrt{3}\mathbb{I}_3}{2\pi^2}\right)$$

$$X_1 = \frac{1}{\pi^4}\left(\sum_{\vec{n},\vec{m}\in\mathbb{Z}^3}^{[1,\infty)} - \int_1^\infty d\vec{n}\int_0^\infty d\vec{m}\right)\frac{1}{\vec{n}^2}\frac{1}{\vec{n}^2+\vec{m}^2+\vec{n}\cdot\vec{m}}\frac{1}{\vec{m}^2}$$

amplitude $\mathcal{M}_{3,\text{thr}}$, which begins at $\mathcal{O}(\lambda^2)$ in perturbation theory. The numerical values of $\mathcal{C}_3$, $\mathcal{C}_4$, and $\mathcal{C}_5$ depend on the choice of UV cutoff, but this dependence cancels with that of $\mathcal{M}_{3,\text{thr}}$. This cancellation is necessary because $\Delta E_{3,\text{thr}}$ is a physical quantity.