Hyperon-Nucleon Interaction from Lattice QCD:

some preliminary analyses

Shanghai Normal University　Hang Liu

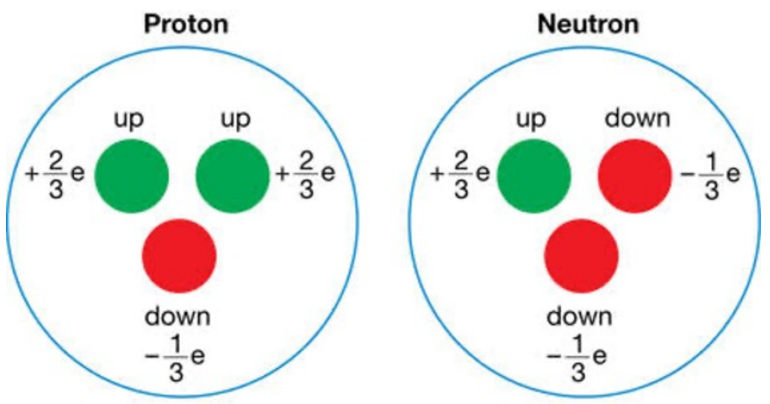
Collaborators：　Liuming Liu, Wei Wang、Jin-Xin Tan、Qian-Teng Zhu

Outline

- Hyperon-nucleon interactions
- $p-\Lambda$ interaction from the HALQCD approach
- $p-\Lambda$ scattering from the Lüscher′s finite volume method
- Summary and Prospect

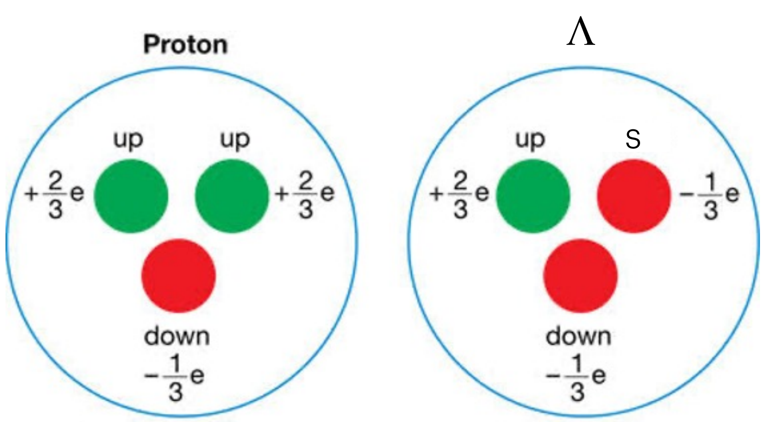
Motivation

- Nucleon-nucleon interactions



✓ hadronic molecule?
✓ multiquark state?
✓ the effective potential ?
✓ the binding energy?

- Hyperon-Nucleon interactions

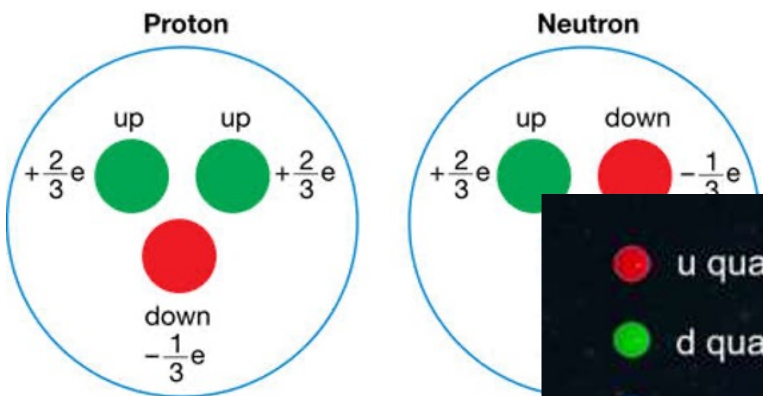


Also concerned

✓ hadronic molecule?
✓ multiquark state?
✓ the effective potential ?
✓ the binding energy?
✓ "hyperon puzzle" in neutron stars

Motivation

- Nucleon-nucleon interactions



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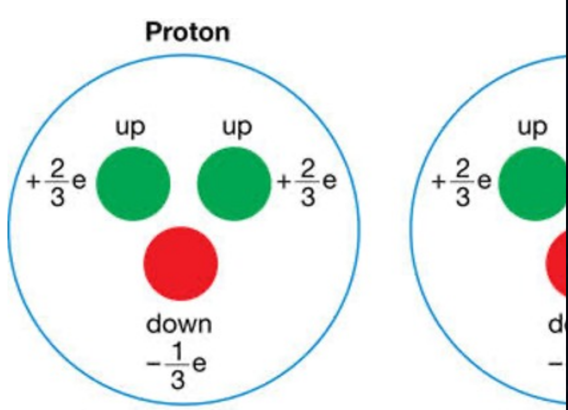
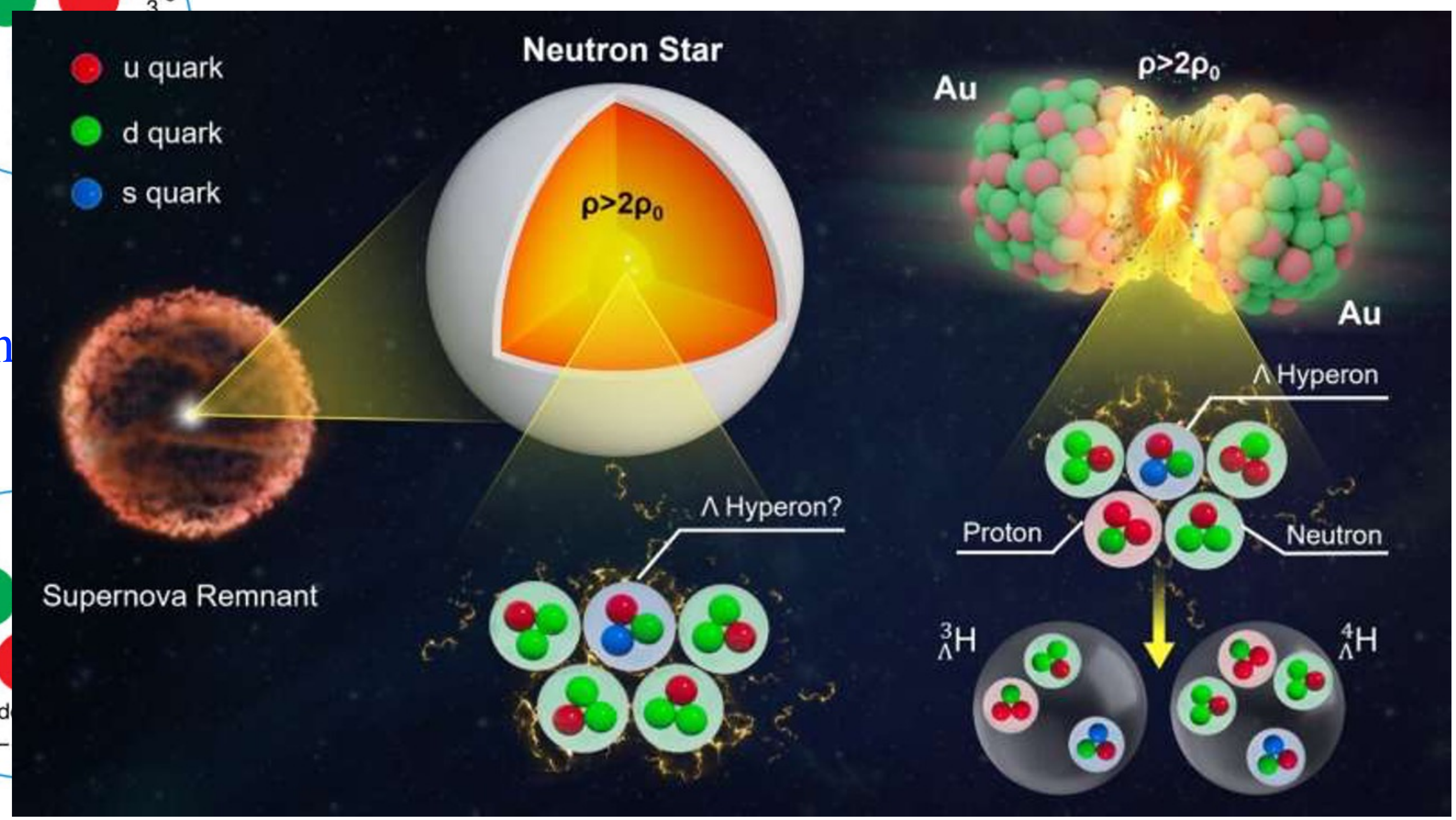
- Hyperon-Nucleon in





From Karen McNulty Walsh, Brookhaven National Laboratory

Motivation

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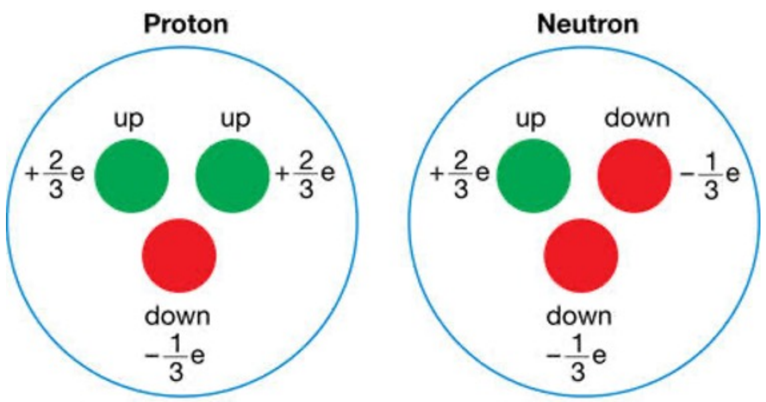

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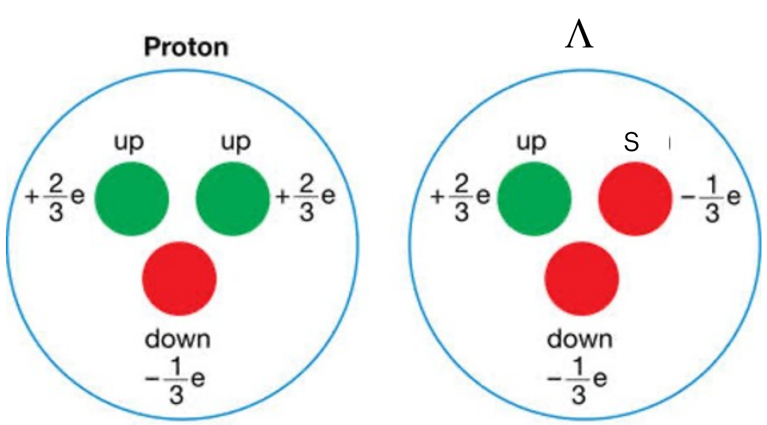

Hep-ex:

✓ YN correlation functions in heavy-ion collisions:

J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006)
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[J-PARC E07 Collaboration], Phys. Rev. Lett. 126, 062501 (2021)

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BESIII Collaboration,PhysRevLett.132.231902(2024)

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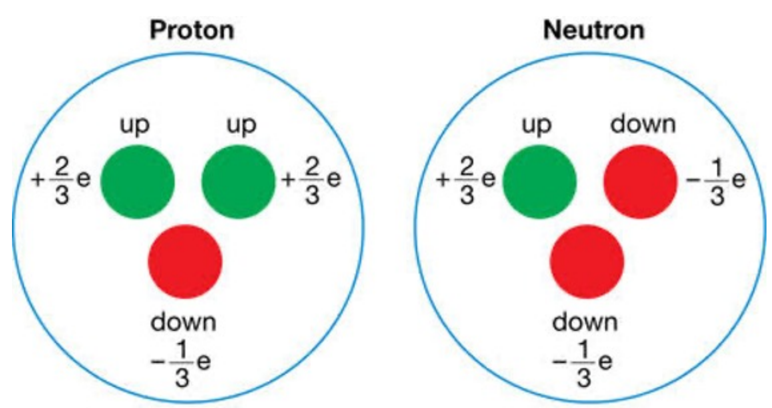

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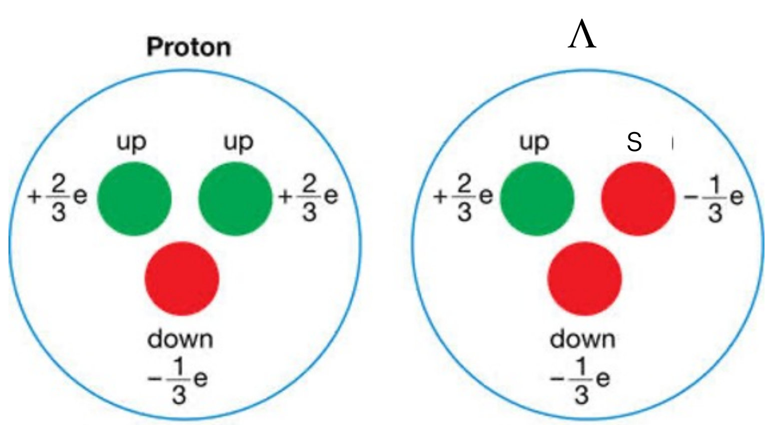

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$$C(\boldsymbol{k}^*) \approx 1 + \frac{|f(k)|^2}{2{R_G}^2} F(d_0) + \frac{2\mathrm{Re}f(k)}{\sqrt{\pi}R_G} F_1(2kR) - \frac{\mathrm{Im}f(k)}{R_G} F_2(2kR_G)$$

$$\frac{1}{f(k)} \approx \frac{1}{f_0} + \frac{d_0 k^2}{2} - ik$$

Different f_0 and d_0 for different spin states

(2019)

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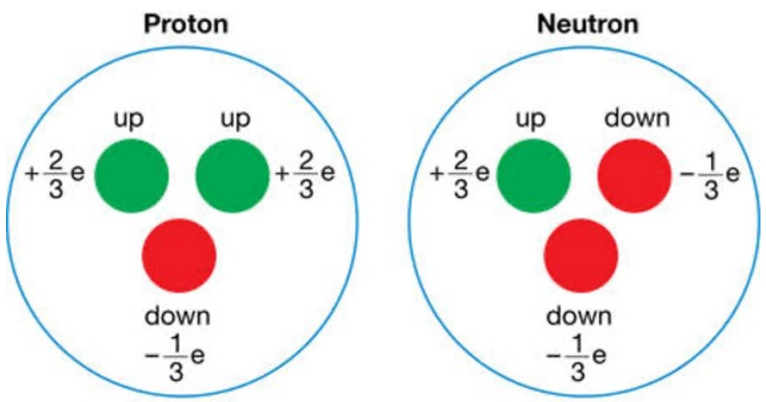

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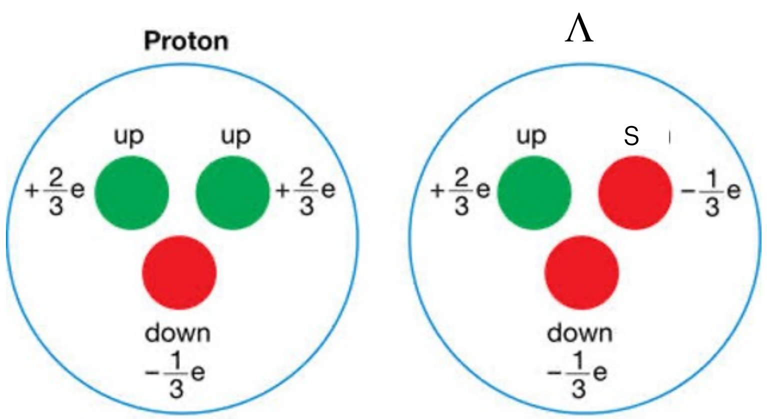

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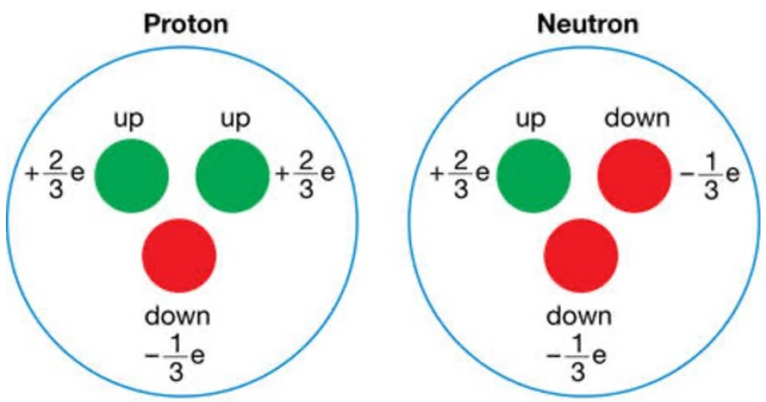

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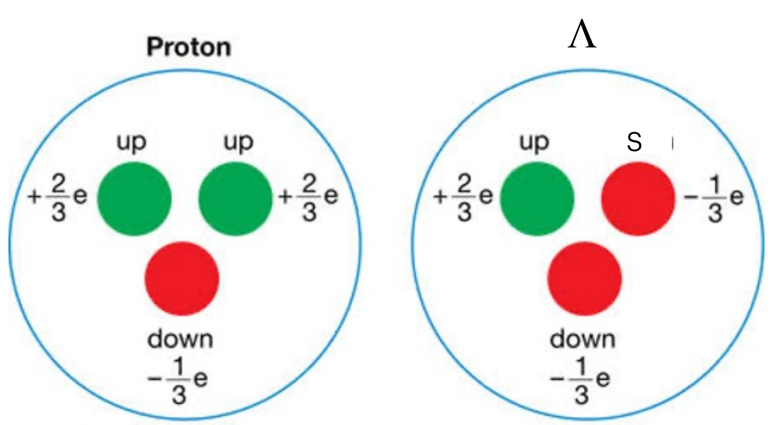

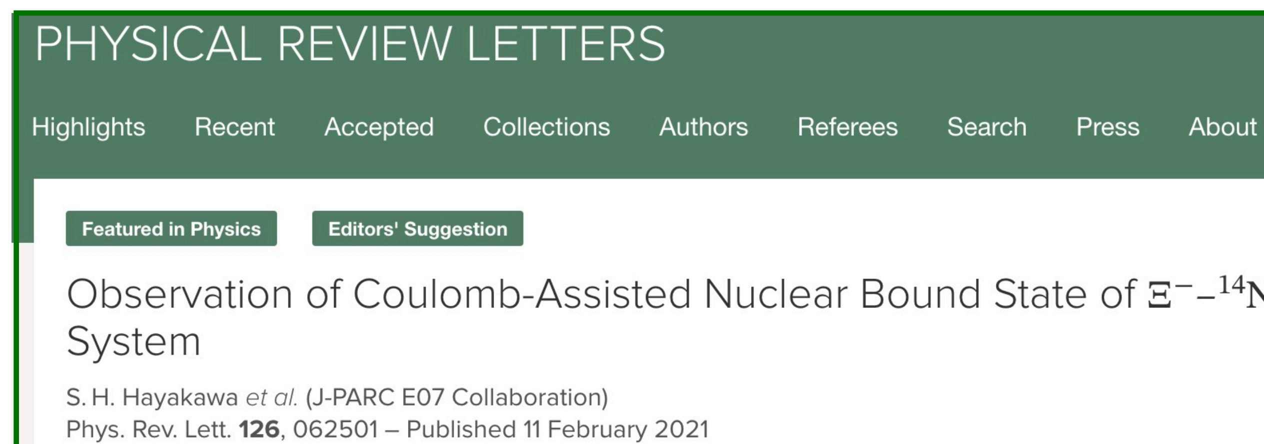
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Observation of Coulomb-Assisted Nuclear Bound State of Ξ^-–^{14}N System

S. H. Hayakawa *et al.* (J-PARC E07 Collaboration)
Phys. Rev. Lett. **126**, 062501 – Published 11 February 2021

2006)
9)
2002 (2019)

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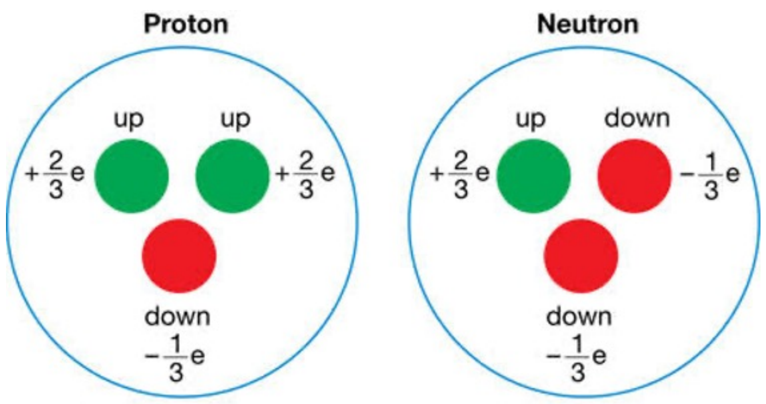

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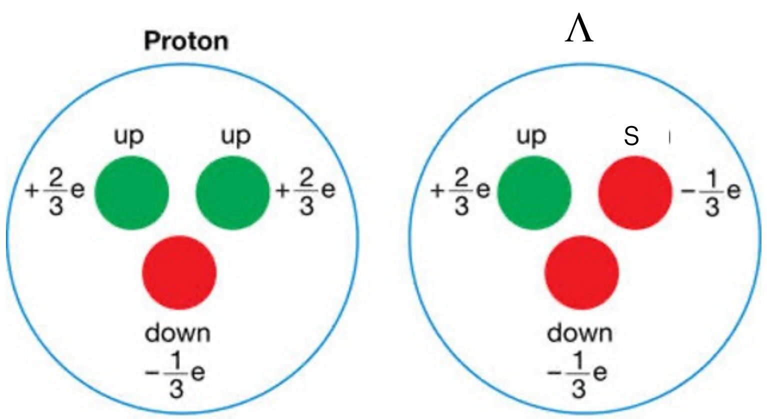

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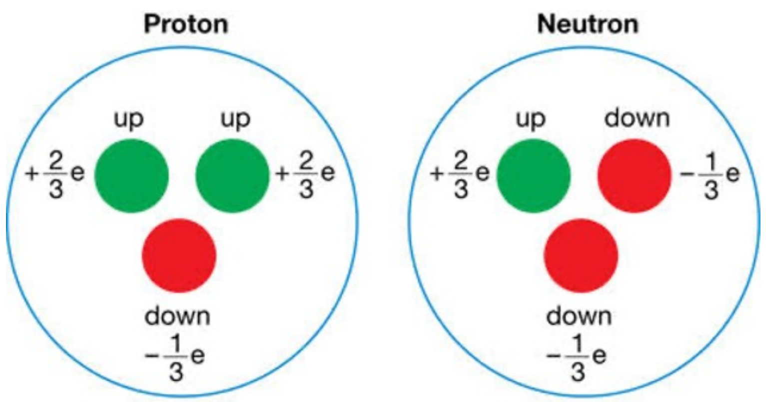

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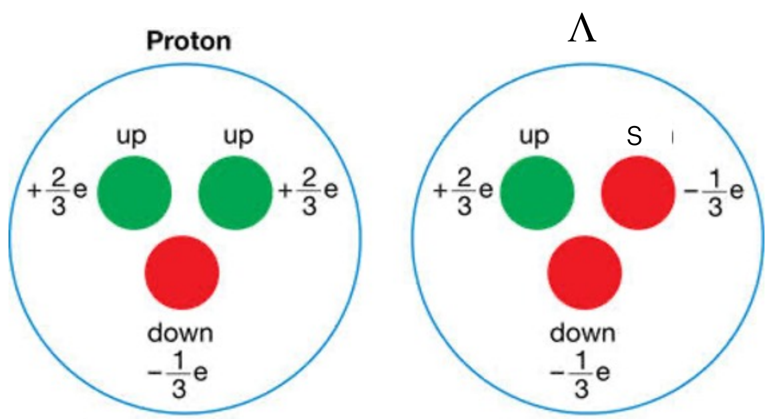

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large uncertainty due to lack of relevant measurements
the availability and short-lifetime of hyperon beams

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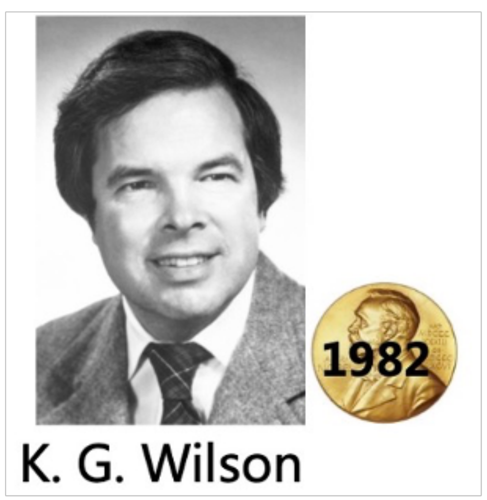
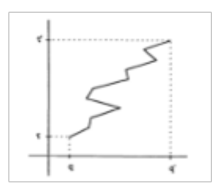
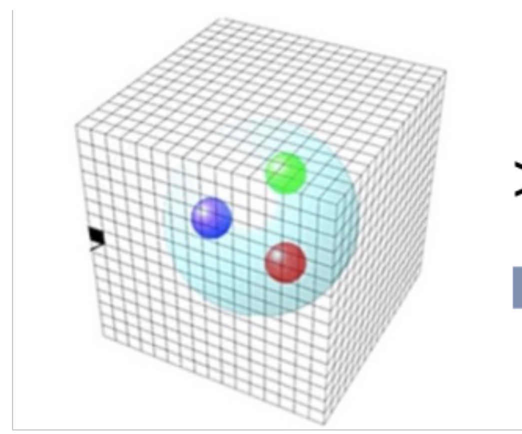
Lattice QCD

LatticeQCD(Wilson,1974): the ab-initio non-perturbative method





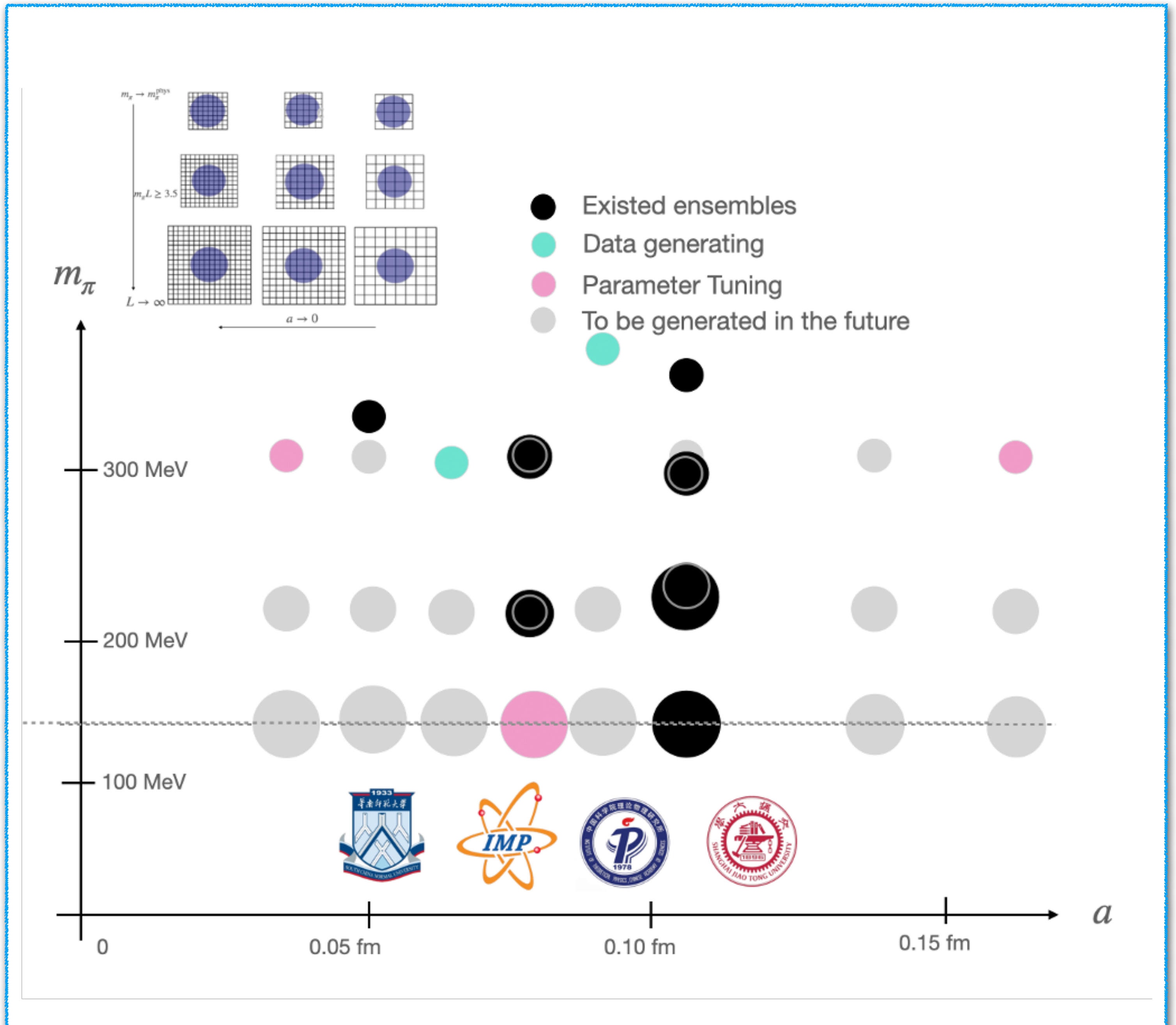


Lattice QCD

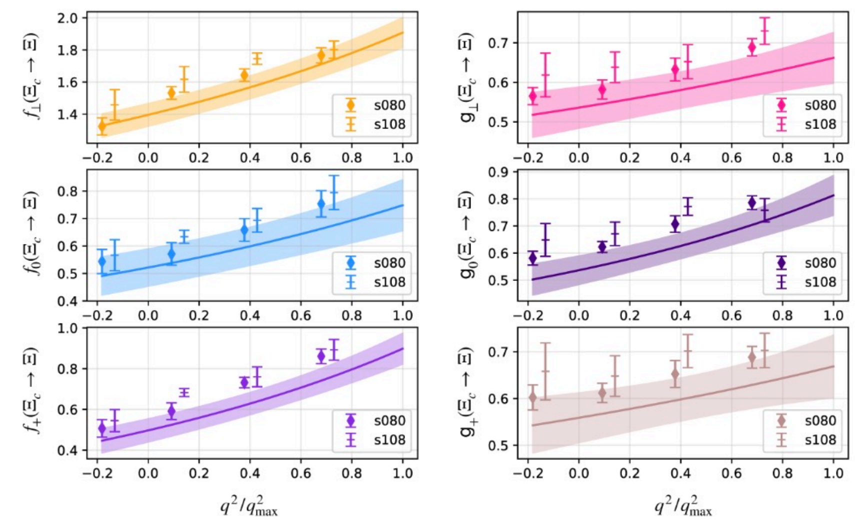
$> 2^{1000000}$

Supercomputer

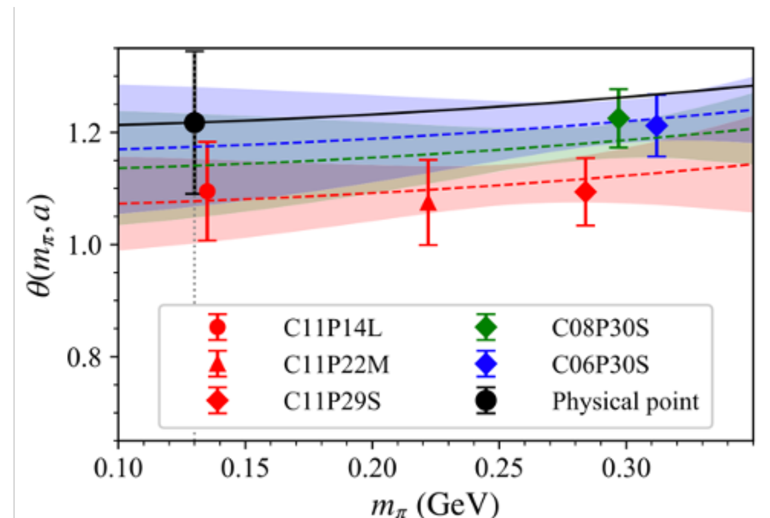
New Lattice QCD configurations



Hu, et.al., PRD 109, 054507 (2024)



Chin.Phys.C 46 (2022) 1, 011002



Phys.Lett.B 841 (2023) 137941

Outline

- Hyperon-nucleon interactions
- $p - \Lambda$ interaction from the HALQCD approach
- $p - \Lambda$ scattering from the Lüscher′s finite volume method
- Summary and Prospect

The HALQCD method

The key quantity in HALQCD method is Nambu-Bethe-Salpeter wave function:

$$\Psi^{W}(\vec{r}) = \sum_{\vec{x}} \langle 0 | T\{p(\vec{x},0)\Lambda(\vec{x}+\vec{r},0)\} | p\Lambda, W\rangle$$

In lattice simulations，NBS wave function is obtained from three-point correlator:

$$C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0\rangle$$

By inserting the complete set of energy eigenstates:

$$\sum_{\vec{x}} \langle 0 | p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t)\bar{J}_{p\Lambda}(0) | 0\rangle = \sum_{n} A_n \Psi^{W_n}(\vec{r}) e^{-W_n t}$$

The HALQCD method

Defining a nonlocal potential $U(\vec{r}, \vec{r'})$ so as to satisfy

$$(E_k - H_0)\Psi^W(\vec{r}) = \int d^3\vec{r'}\, U(\vec{r}, \vec{r'})\Psi^W(\vec{r'})$$

assume the nonlocal potential is energy independent

$$R_{p\Lambda}(\vec{r}, t) = \frac{C_{p\Lambda}(\vec{r}, t)}{C_p(t)C_\Lambda(t)} = \sum_n A'_n \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

$$\left[-H_0 - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right] R(\vec{r}, t) = \int d^3\vec{r'}\, U(\vec{r}, \vec{r'}) R(\vec{r'}, t)$$

Then the leading order analysis neglecting higher orders leads to

$$U(\vec{r}, \vec{r'}) = V_0^{LO}(\vec{r})\delta(\vec{r} - \vec{r'})$$

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{(\partial/\partial t)R(\vec{r}, t)}{R(\vec{r}, t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r}, t)}{R(\vec{r}, t)}$$

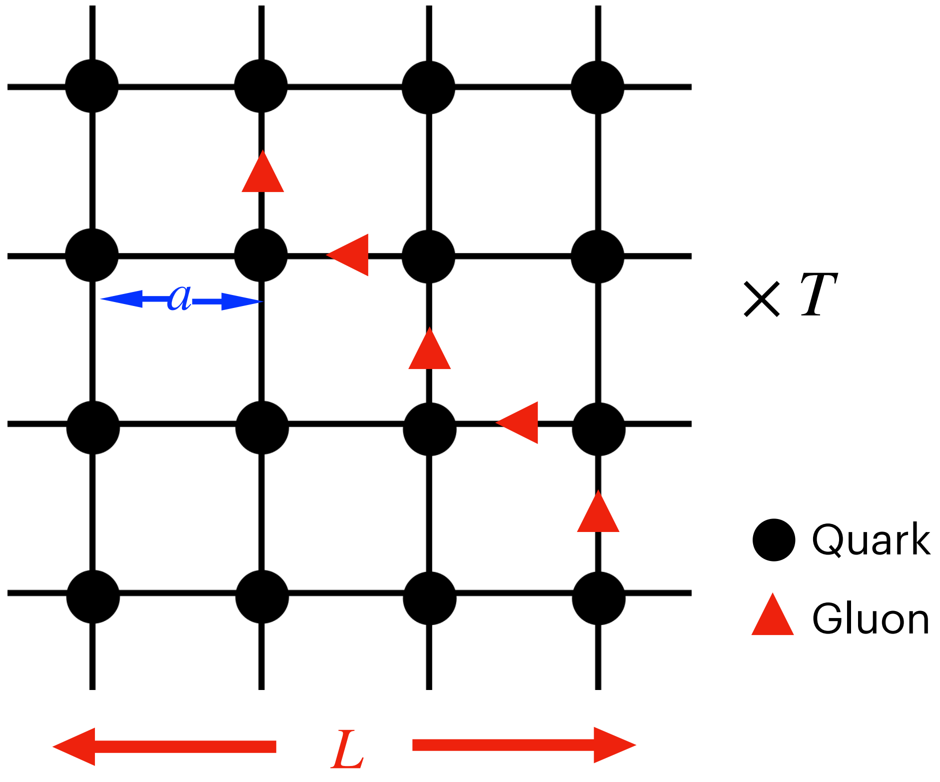
Lattice setup for $p - \Lambda$

- Two ensembles:

  C24P29, $n_s^3 \times n_t = 24^3 \times 72$;

  C32P29, $n_s^3 \times n_t = 32^3 \times 64$
- $a = 0.10530(18)\mathrm{fm}$;
- $m_\pi \simeq 293\mathrm{MeV}, m_K \simeq 509\mathrm{MeV}$
- Coulomb gauge fixed-wall source

  for HALQCD method



Two-point Correlation Function

We define the map from color-spin index to weight index:

$$
\begin{aligned}
p_\sigma &= \epsilon^{abc}\frac{1}{\sqrt{2}}[u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi} d^b_\xi(x) - d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi} u^b_\xi(x)] \\
&\times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho} u^c_\rho(x)
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_\sigma &= \epsilon^{abc}\frac{1}{\sqrt{2}}[d^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi} u^b_\xi(x) - u^a_\zeta(x)(C\gamma_5 P_+)_{\zeta\xi} d^b_\xi(x)] \\
&\times [P_+(1-(-1)^\sigma i\gamma_1\gamma_2)]_{\sigma\rho} s^c_\rho(x)
\end{aligned}
$$

$$p_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} u^{i(\alpha)}(x) d^{j(\alpha)}(x) u^{k(\alpha)}(x)$$

$$\Lambda_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} d^{i(\alpha)}(x) u^{j(\alpha)}(x) s^{k(\alpha)}(x)$$

$$(C\gamma_5 P_+) - (C\gamma_5 P_+)^T = \begin{pmatrix} 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_+\left(1-(-1)^0 i\gamma_1\gamma_2\right) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_+\left(1-(-1)^1 i\gamma_1\gamma_2\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

α	$i(\alpha)$	$j(\alpha)$	$k(\alpha)$	$w_\alpha^{[N]0}$
1	0, 0	1, 1	0, 2	$-2\sqrt{2}$
2	0, 0	1, 2	0, 1	$2\sqrt{2}$
3	0, 1	1, 0	0, 2	$2\sqrt{2}$
4	0, 1	1, 2	0, 0	$-2\sqrt{2}$
5	0, 2	1, 0	0, 1	$-2\sqrt{2}$
6	0, 2	1, 1	0, 0	$2\sqrt{2}$
7	1, 0	0, 1	0, 2	$2\sqrt{2}$
8	1, 0	0, 2	0, 1	$-2\sqrt{2}$
9	1, 1	0, 0	0, 2	$-2\sqrt{2}$
10	1, 1	0, 2	0, 0	$2\sqrt{2}$
11	1, 2	0, 0	0, 1	$2\sqrt{2}$
12	1, 2	0, 1	0, 0	$-2\sqrt{2}$

α	$i(\alpha)$	$j(\alpha)$	$k(\alpha)$	$w_\alpha^{[N]1}$
1	0, 0	1, 1	1, 2	$-2\sqrt{2}$
2	0, 0	1, 2	1, 1	$2\sqrt{2}$
3	0, 1	1, 0	1, 2	$2\sqrt{2}$
4	0, 1	1, 2	1, 0	$-2\sqrt{2}$
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$$
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\end{aligned}
$$

→

$$
p_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} u^{i(\alpha)}(x) d^{j(\alpha)}(x) u^{k(\alpha)}(x)
$$

$$
\Lambda_\sigma(x) = \sum_\alpha w_\alpha^{[N]\sigma} d^{i(\alpha)}(x) u^{j(\alpha)}(x) s^{k(\alpha)}(x)
$$

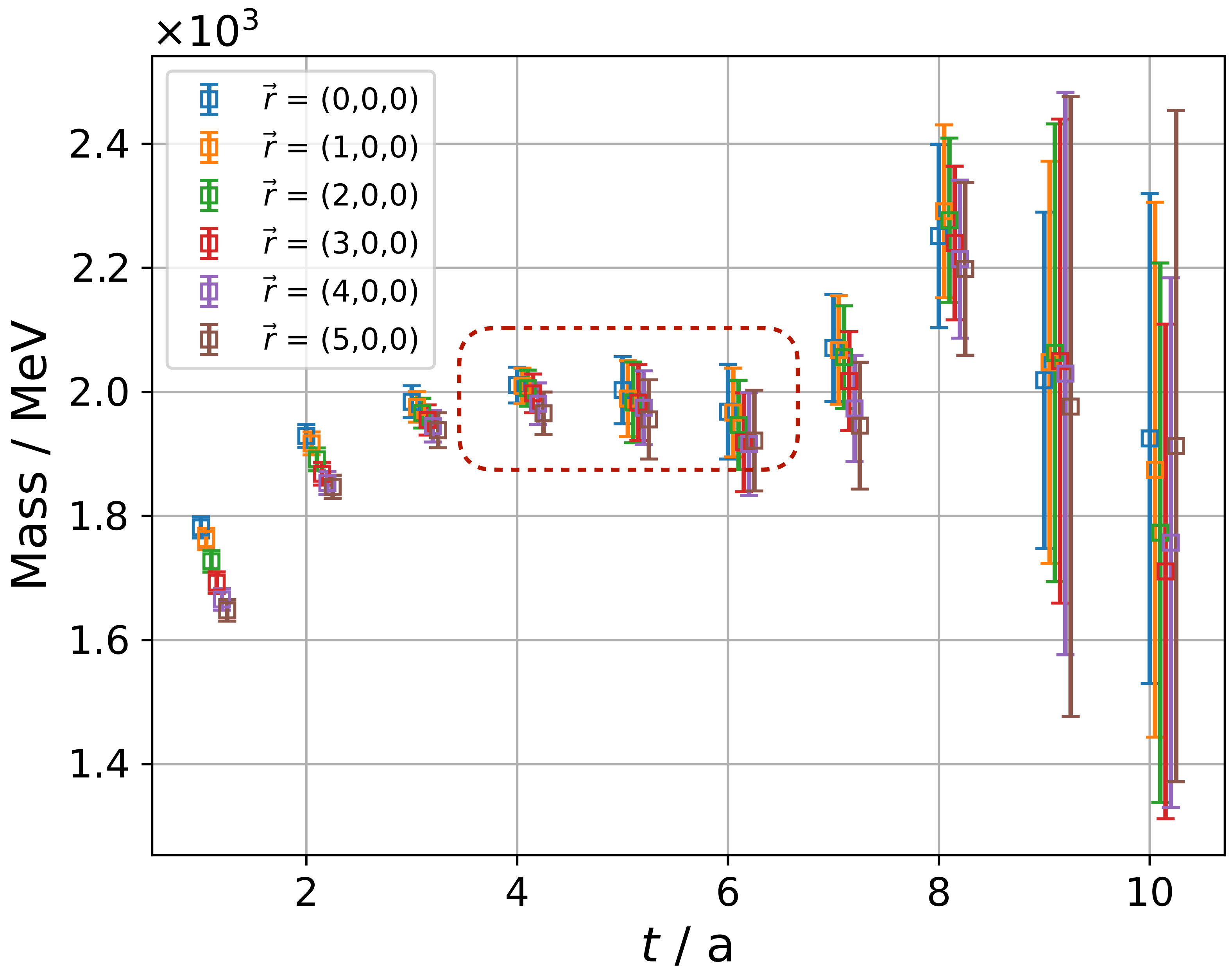
For the p-Λ system, after test different types of operators, we choose construct the correlation functions with operator Dibaryon(sink) Hexaquark(source)

$$
C_{p\Lambda}(\vec{r},t) = \sum_{\vec{x}} \langle 0 | (p(\vec{x},t)\Lambda(\vec{x}+\vec{r},t))^D \bar{J}^H_{p\Lambda}(0) | 0 \rangle
$$

We also test different types of the source.

Effective mass

We try to find the appropriate time slice for the ground state saturation of the system from the effective mass of the dibaryon system.

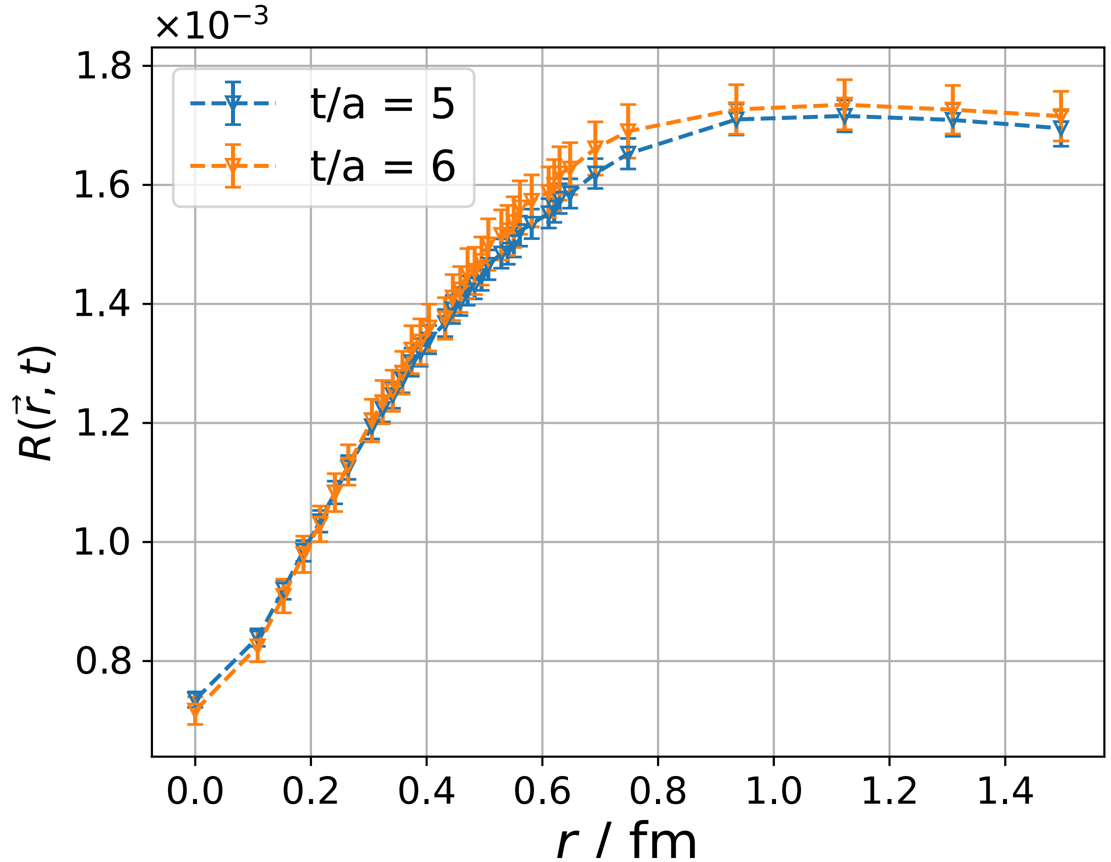


The HALQCD method

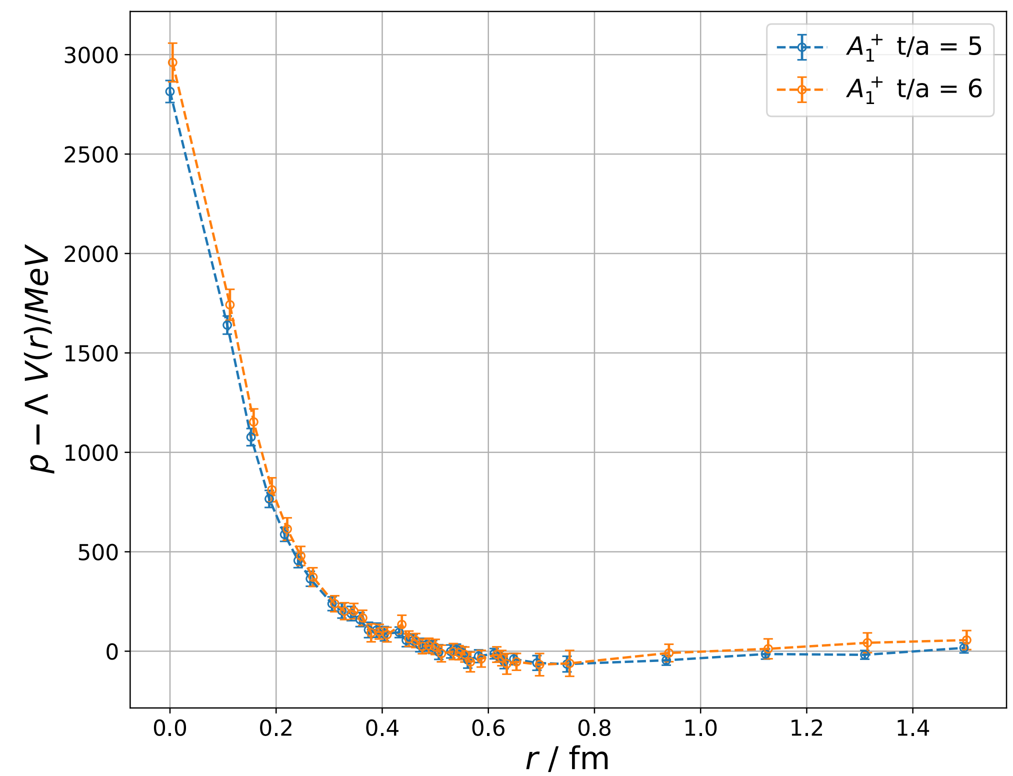
We extract the NBS wave function and the effective potential for the $^1S_0$ channel on the timeslice $t/a = 5{,}6$.

$$R_{p\Lambda}(\vec{r},t) = \frac{C_{p\Lambda}(\vec{r},t)}{C_p(t)C_\Lambda(t)} = \sum_n A_n' \Psi^{W_n}(\vec{r}) e^{-\Delta W_n t}$$

$$V_0^{LO}(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} + \frac{1}{8\mu}\frac{(\partial^2/\partial t^2)R(\vec{r},t)}{R(\vec{r},t)}$$



NBS wave function



effective potential

Phase shift from the HALQCD method

We parameterize the effective potential in this form

$$V(r) = v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$

Phase shift from the HALQCD method

We parameterize the effective potential in this form

Argonne-type form factor

$$V(r) = v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$

Gaussian form

two-pion exchange

Phase shift from the HALQCD method

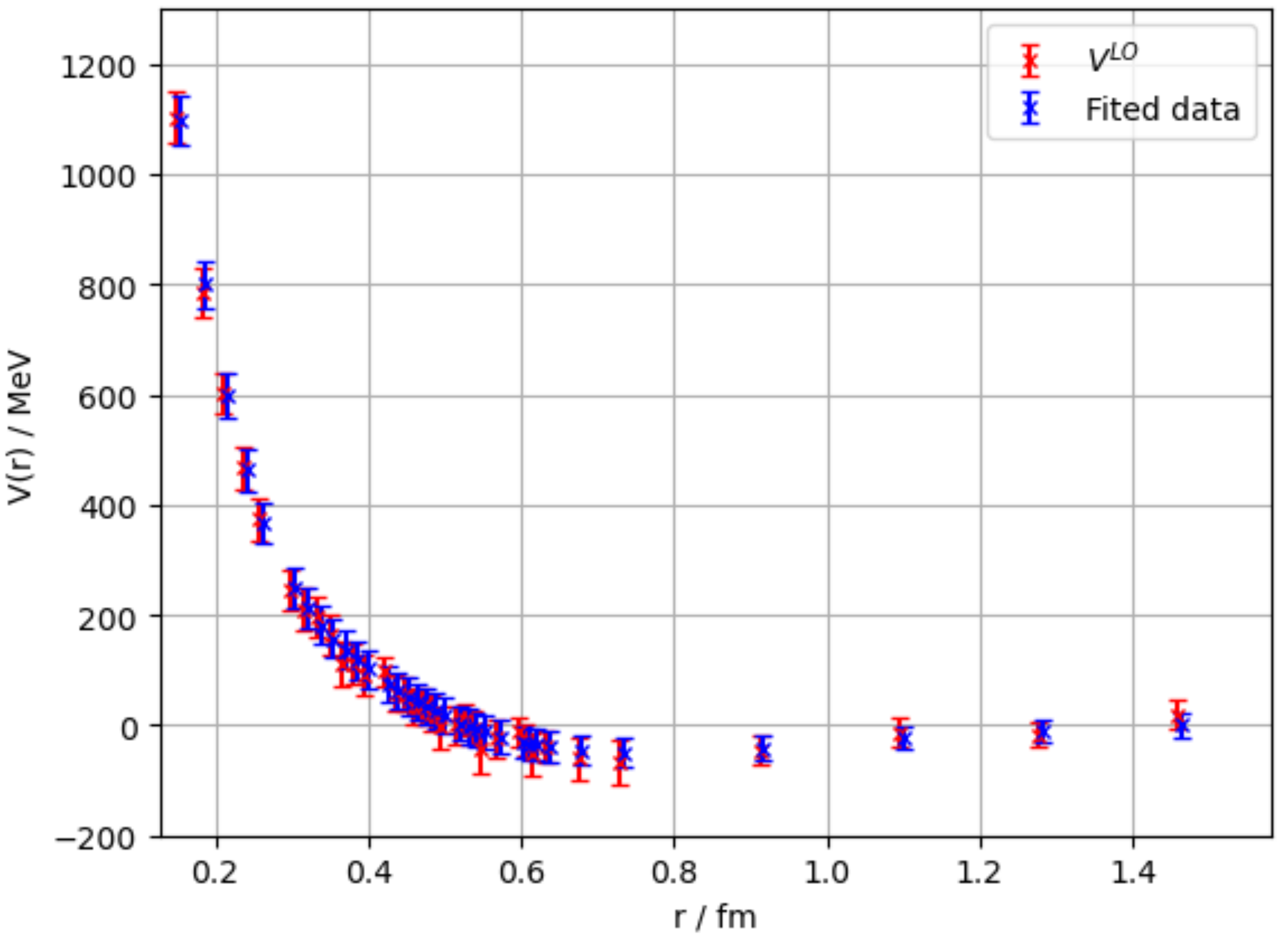
We parameterize the effective potential in this form

Argonne-type form factor

$$V(r) = v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$

Gaussian form

two-pion exchange



Phase shift from the HALQCD method

We parameterize the effective potential in this form

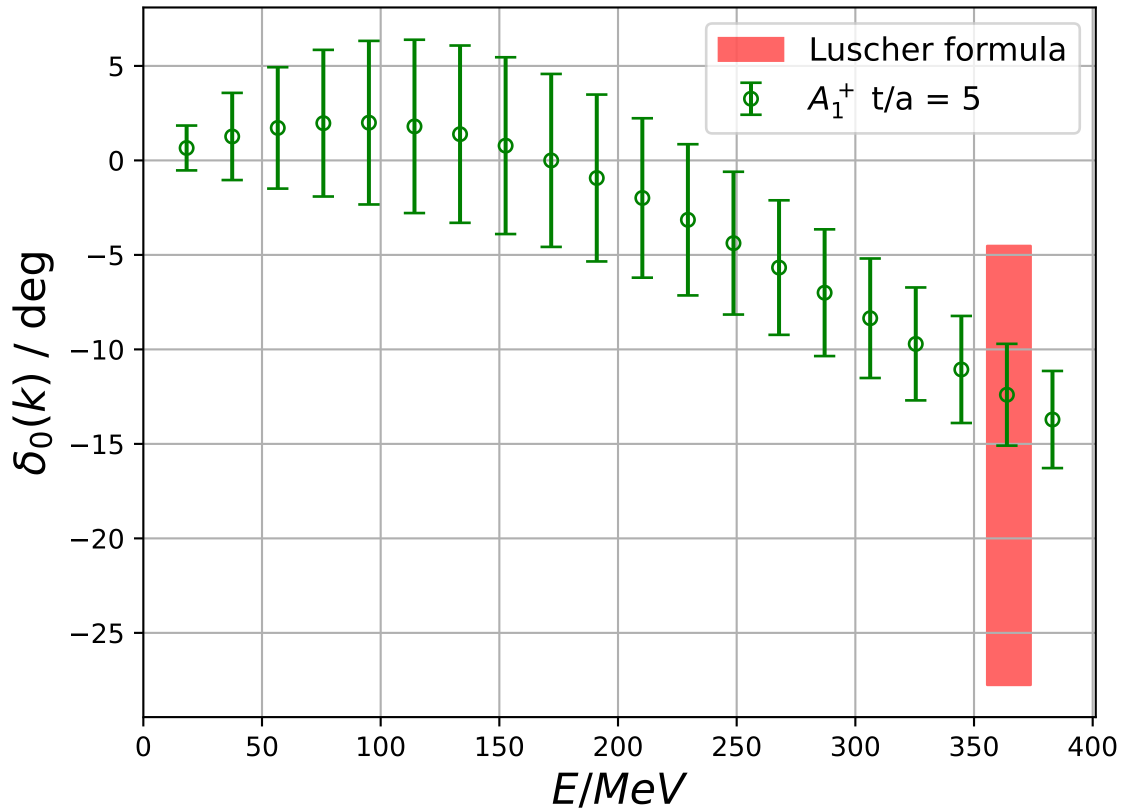
Argonne-type form factor

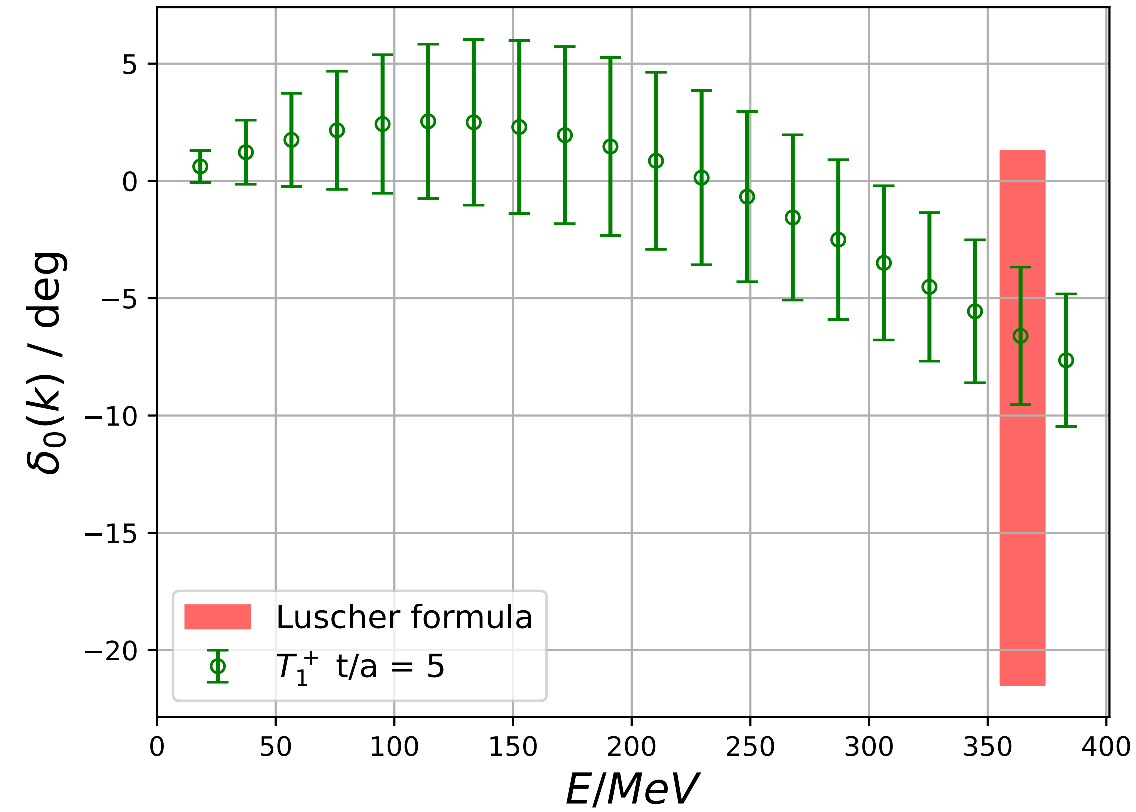
$$V(r) = \underbrace{v_{C1}\mathrm{e}^{-\kappa_{C1}r^2} + v_{C2}\mathrm{e}^{-\kappa_{C2}r^2}}_{\text{Gaussian form}} + v_{C3}\left(1-\mathrm{e}^{-\alpha_C r^2}\right)^2\left(\frac{\mathrm{e}^{-\beta_C r}}{r}\right)^2$$

Gaussian form

two-pion exchange

scattering phase shift can be obtained by solving the Schrodinger equation





Outline

- Hyperon-nucleon interactions
- $p-\Lambda$ interactions from the HALQCD approach
- $p-\Lambda$ scattering from the Lüscher′s finite volume method
- Summary and Prospect

Lüscher′s **finite volume formula**

The direct method for scattering on the lattice: Lüscher′s **finite volume method**

M. Lüscher, Nucl. Phys. B354, 531(1991)

Finite volume energies ⟷ (Lüscher formula) ⟷ Infinite volume scattering

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

$$E_{1\cdot 2}(k) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2}$$

$$k = 2n\pi/L$$

$$\delta_l \,.\, a_l \,.\, r_l$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length effective range

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$\bar{k}^2$ differs from its free counter-part k^2

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$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

$$E_{1,2}(k) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2}$$

$$q^2 = \frac{\bar{k}^2 L^2}{(2\pi)^2}$$

$$\delta_l \,.\, a_l \,.\, r_l$$

Due to the interaction,
$\bar{k}^2$ differs from its free counter-part k^2

$$\begin{cases} q^2 = n^2 \\ q^2 < n^2 \\ q^2 > n^2 \end{cases}$$

ERE(effective range expansion)

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \ldots$$

scattering length　　effective range

Lüscher′s **finite volume formula**

We construct the two-particle operators on the ${}^3S_1(T_1^+)$ and ${}^1S_0(A_1^+)$ channels

https://github.com/wittscien/OpTion

By haobo Yan

A_1^+ :

$$\mathcal{O}_{A_1^+} = p_{\frac{1}{2}}(0)\Lambda_{-\frac{1}{2}}(0) - p_{-\frac{1}{2}}(0)\Lambda_{\frac{1}{2}}(0)$$

$$\begin{aligned}\mathcal{O}'_{A_1^+} = & \ p_{\frac{1}{2}}(e_x)\Lambda_{-\frac{1}{2}}(-e_x) - p_{-\frac{1}{2}}(e_x)\Lambda_{\frac{1}{2}}(-e_x) + p_{\frac{1}{2}}(-e_x)\Lambda_{-\frac{1}{2}}(e_x) - p_{-\frac{1}{2}}(-e_x)\Lambda_{\frac{1}{2}}(e_x) \\ & + p_{\frac{1}{2}}(e_y)\Lambda_{-\frac{1}{2}}(-e_y) - p_{-\frac{1}{2}}(e_y)\Lambda_{\frac{1}{2}}(-e_y) + p_{\frac{1}{2}}(-e_y)\Lambda_{-\frac{1}{2}}(e_y) - p_{-\frac{1}{2}}(-e_y)\Lambda_{\frac{1}{2}}(e_y) \\ & + p_{\frac{1}{2}}(e_z)\Lambda_{-\frac{1}{2}}(-e_z) - p_{-\frac{1}{2}}(e_z)\Lambda_{\frac{1}{2}}(-e_z) + p_{\frac{1}{2}}(-e_z)\Lambda_{-\frac{1}{2}}(e_z) - p_{-\frac{1}{2}}(-e_z)\Lambda_{\frac{1}{2}}(e_z)\end{aligned}$$

We evaluate a correlation matrix of the form

$$C_i^{\alpha\beta}(t) = \langle 0 | \mathcal{O}_i^{\alpha}(t)\mathcal{O}_i^{\beta\dagger}(0) | 0 \rangle$$

Solving the so-called Generalized Eigenvalue Problem

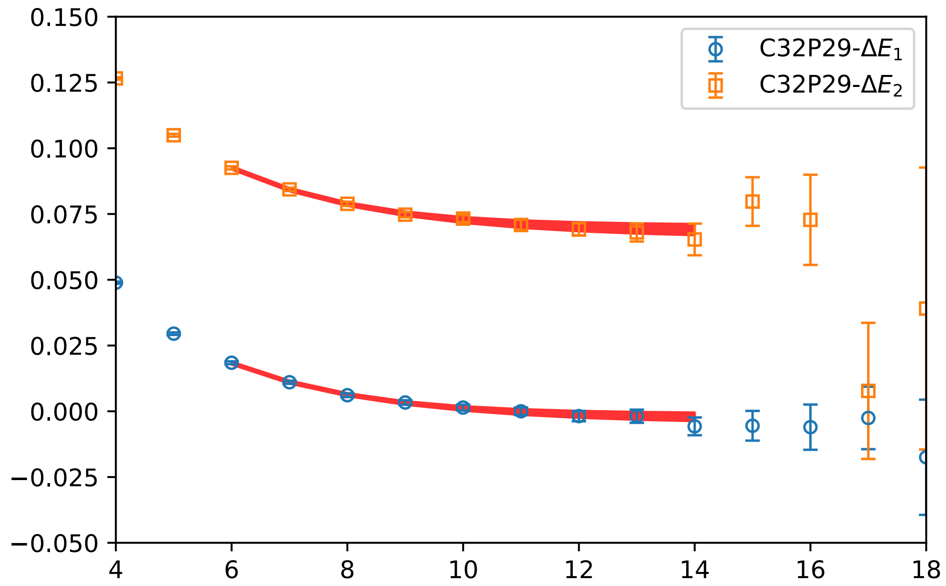
$$C(t)v_n(t) = \lambda_n(t)C(t_r)v_n(t)$$

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

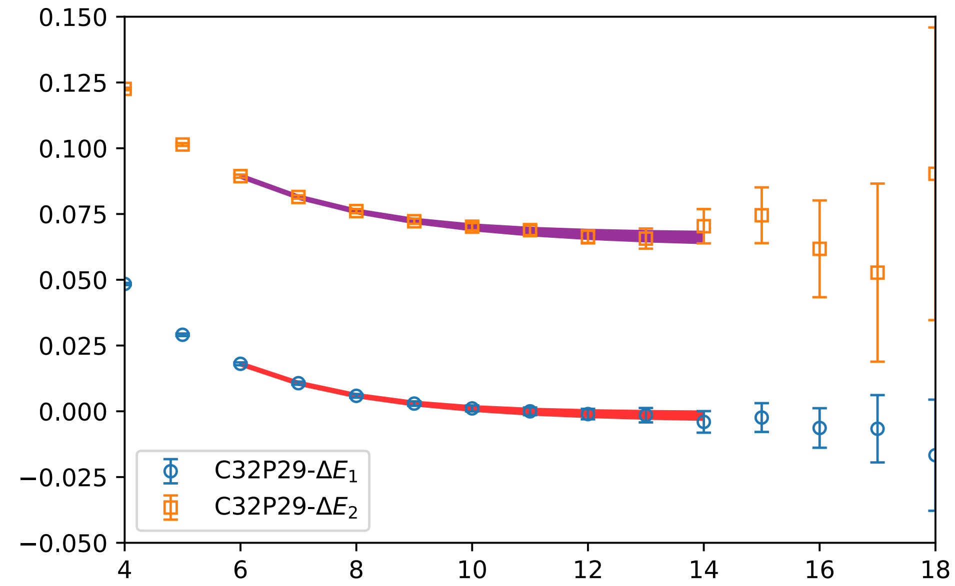
Effective mass of the YN system

To enhance the signal, the following ratio was attempted:

$$R_\alpha(t, t_0) = \frac{\lambda_\alpha(t, t_0)}{C^p(t - t_0, \mathbf{0})C^\Lambda(t - t_0, \mathbf{0})} \propto e^{-\Delta E_\alpha (t - t_0)}$$



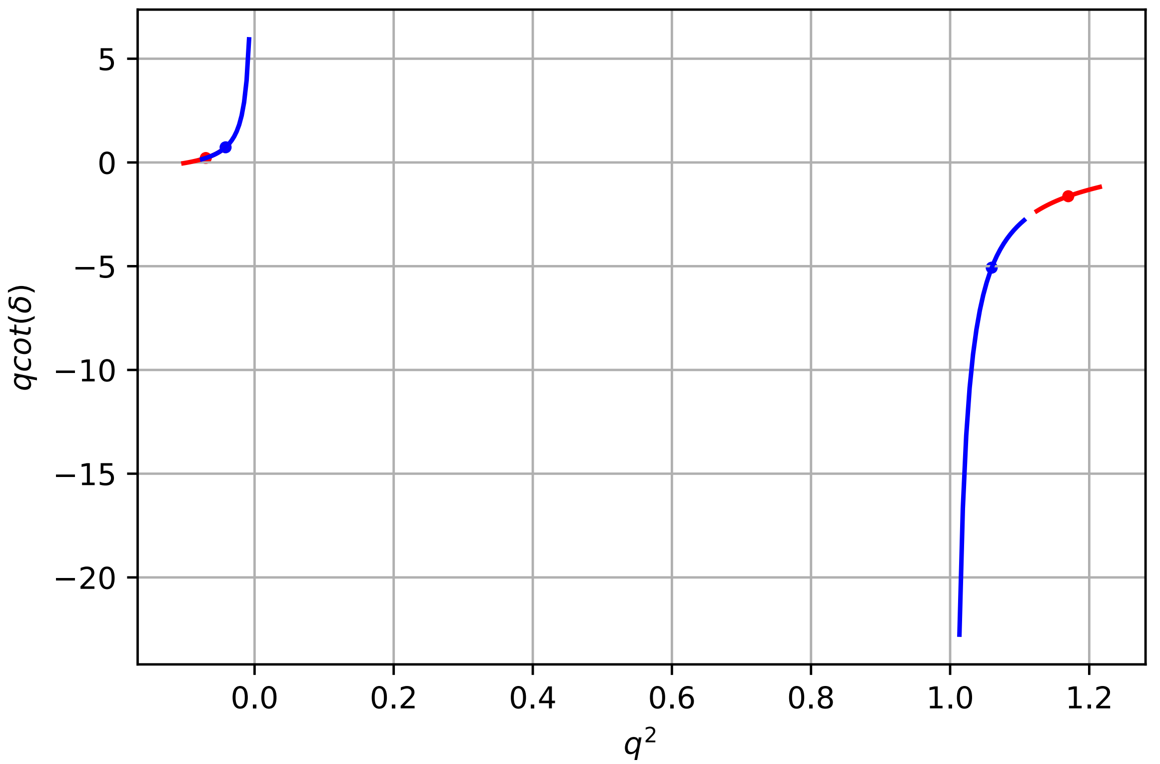
A_1^+



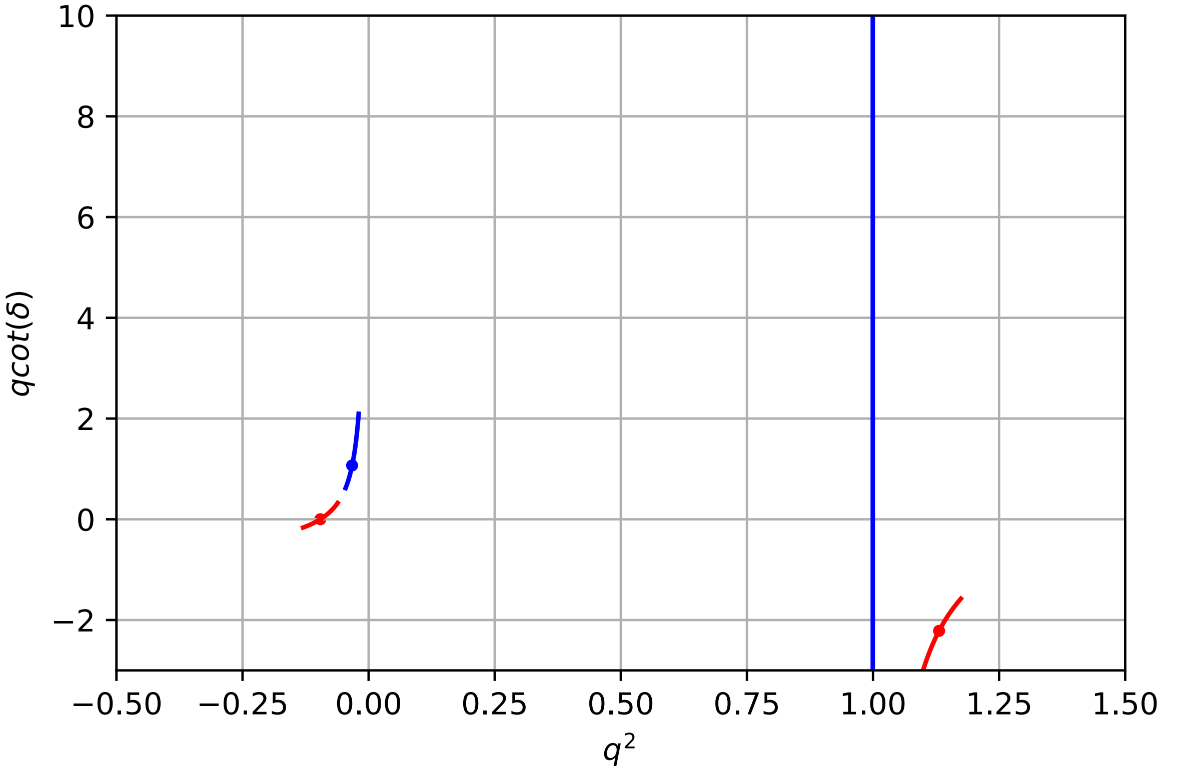
T_1^+

	C24P29		C32P29	
Irrep	ΔE_0(MeV)	ΔE_1(MeV)	ΔE_0(MeV)	ΔE_1(MeV)
A_1^+	−14.4(6.7)	238.9(8.6)	−5.1(3.7)	128.0(4.9)
T_1^+	−19.5(7.7)	230.5(9.0)	−3.9(3.4)	122.2(4.9)

$qcot\delta_0$ vs q^2



A_1^+



T_1^+

ERE(effective range expansion):

$$k^{2l+1}cot\delta_l(k) = a_l^{-1} + \frac{1}{2}r_l k^2 + \dots$$

preliminary results

Irrep	a_0	r_0
A_1^+	0.41(0.69)	6.48(21.05)
T_1^+	0.42(0.23)	7.41(4.51)

Summary and Prospect

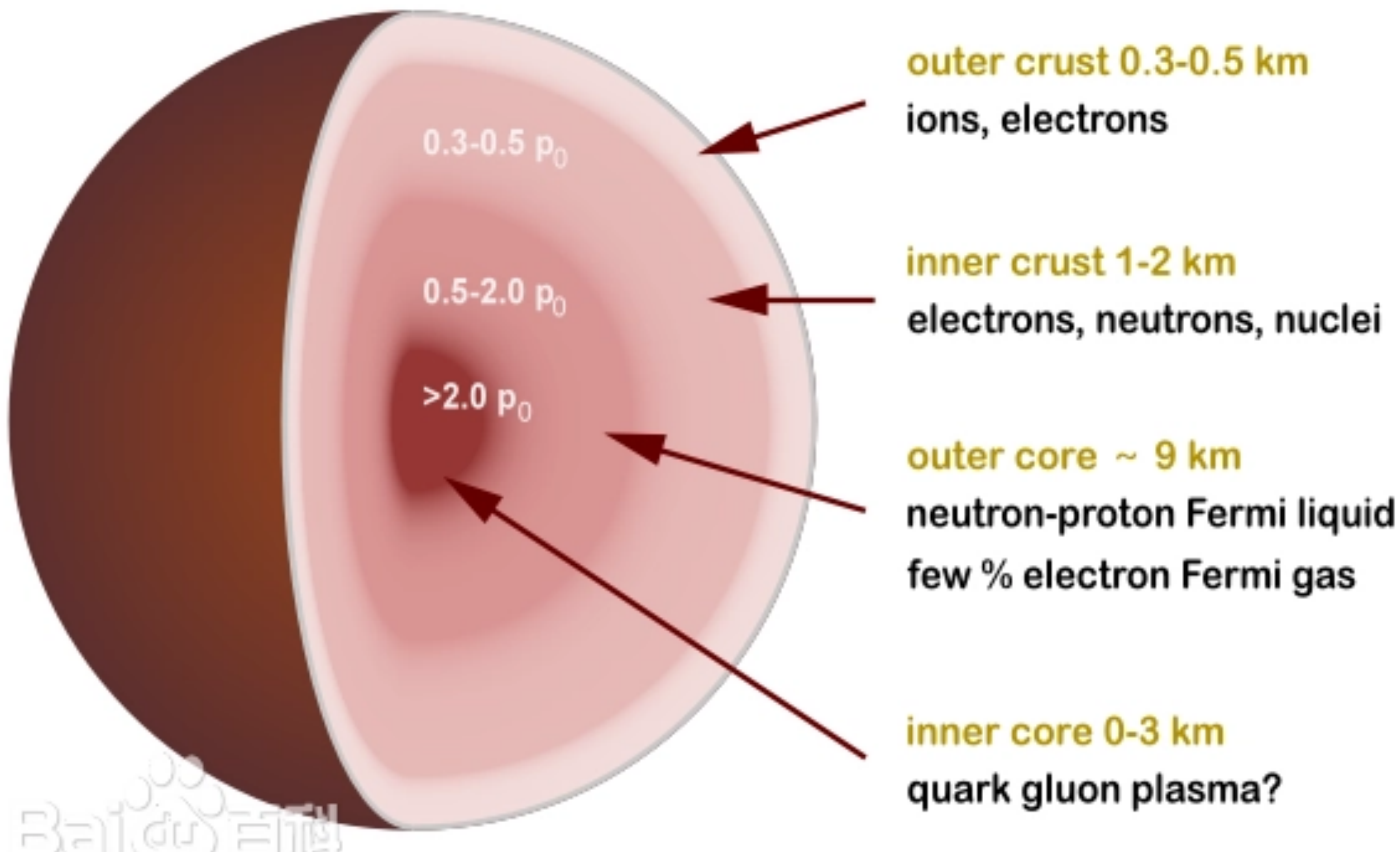
- ✓ HALQCD method: preliminary results for p-Λ NBS wave function, the interaction potential, and phase shift;
- ✓ Lüscher′s finite volume method: preliminary results for finite volume energies and phase shift ;

  Effective range expansion: scattering length and effective range;

- ➡ HALQCD method： Schrodinger equation+ the interaction potential
- ➡ more ensembles, discretization error, pion mass, finite volume effect
- ➡ p-Λ: p-Σ coupled channel
- ➡ ${}^{3}_{\Lambda}H$: three-body problem

Thank you!

Backup


0.3-0.5 p_0
0.5-2.0 p_0
>2.0 p_0
outer crust 0.3-0.5 km
ions, electrons
inner crust 1-2 km
electrons, neutrons, nuclei
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas
inner core 0-3 km
quark gluon plasma?

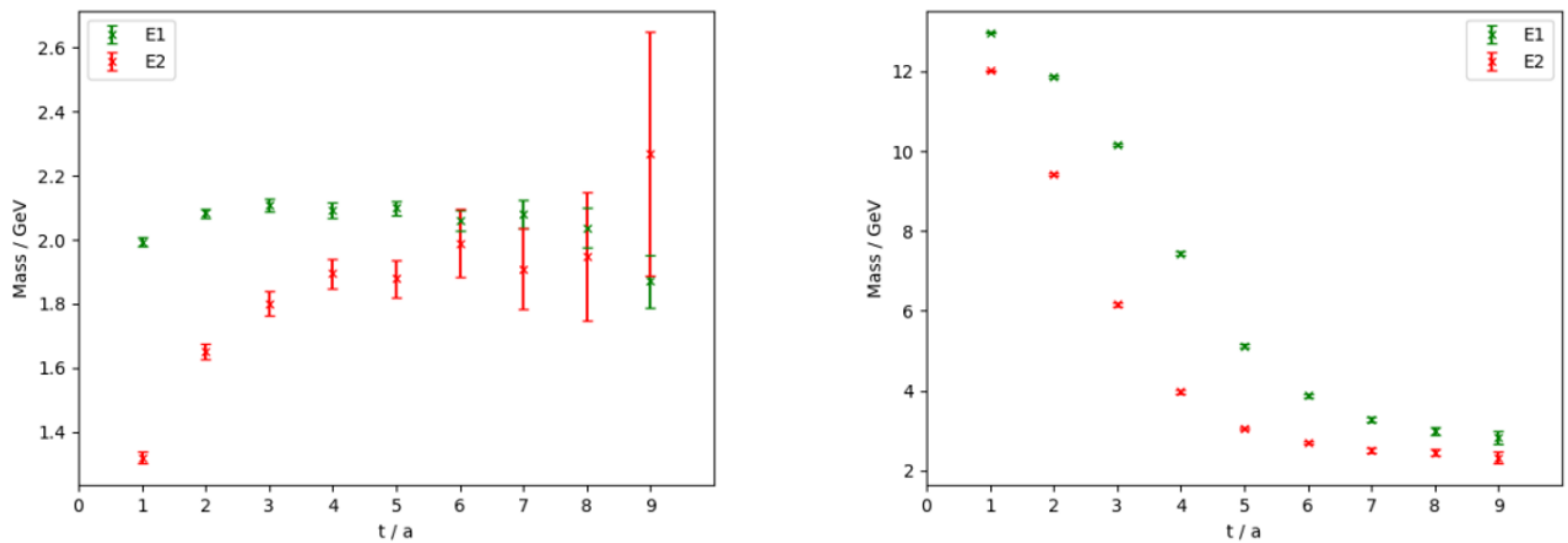
$$p\Lambda_{\rho\mathfrak{m}g}(t) \;= \sum_{\vec{x}_1,\vec{x}_2\in\Lambda_S} \psi^{[D]}_{\mathfrak{m}}(\vec{x}_1,\vec{x}_2) \sum_{\sigma,\sigma'} v^{\rho}_{\sigma\sigma'} \tfrac{1}{\sqrt{2}} [p_{\sigma g}(\vec{x}_1,t) n_{\sigma' g}(\vec{x}_2,t) + (-1)^{1-\delta_{\rho 0}} n_{\sigma g}(\vec{x}_1,t) p_{\sigma' g}(\vec{x}_2,t)],$$

$$\begin{aligned}
v^0_{\sigma\sigma'} &= \frac{1}{\sqrt{2}}(\delta_{\sigma 0}\delta_{\sigma' 1} - \delta_{\sigma 1}\delta_{\sigma' 0}),\\
v^1_{\sigma\sigma'} &= \delta_{\sigma 0}\delta_{\sigma' 0},\\
v^2_{\sigma\sigma'} &= \frac{1}{\sqrt{2}}(\delta_{\sigma 0}\delta_{\sigma' 1} + \delta_{\sigma 1}\delta_{\sigma' 0}),\\
v^3_{\sigma\sigma'} &= \delta_{\sigma 1}\delta_{\sigma' 1}.
\end{aligned}$$

Variational Analysis

We constructed the correlation matrix and calculate the eigenvalues

$$C = \begin{pmatrix} C_{HH} & C_{DH} \\ C_{DH} & C_{DD} \end{pmatrix}$$



Wallsource

Pointsource