CHONGQING UNIVERSITY

Non-perturbative properties of QCD basics

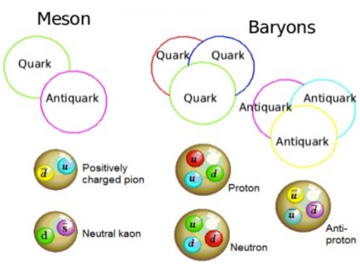
Si-Xue Qin

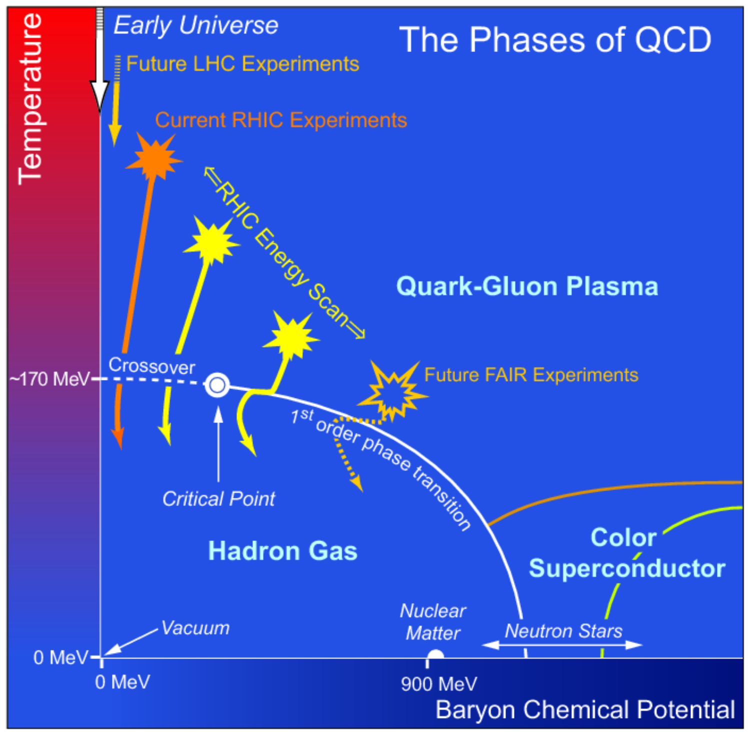
（秦思学）

Department of Physics, Chongqing University

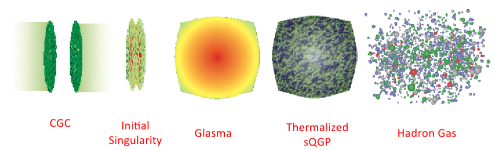
Introduction: QCD frontiers

Hadron

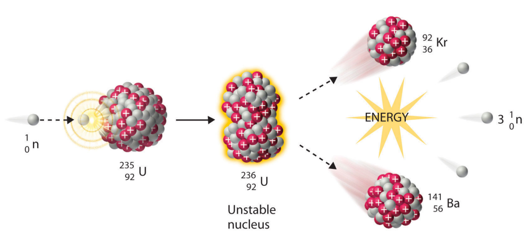




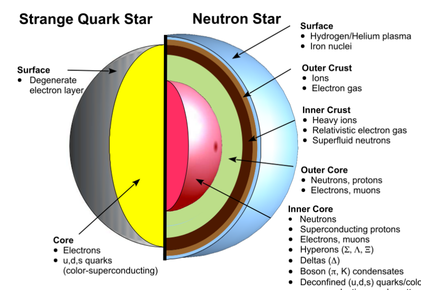
QGP



Nucleus



Compact Star



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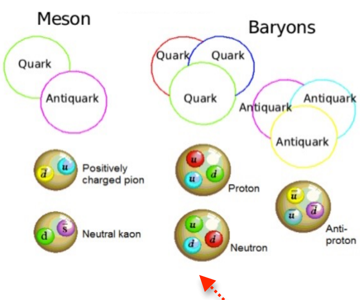

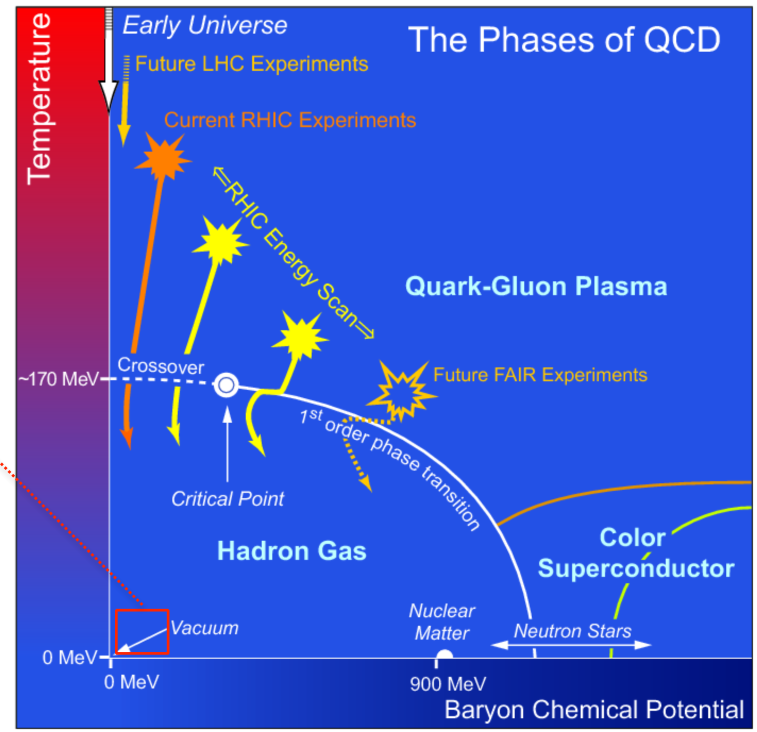

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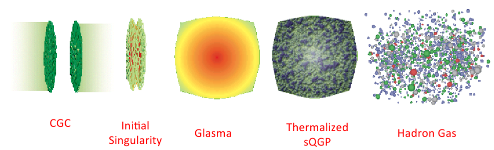

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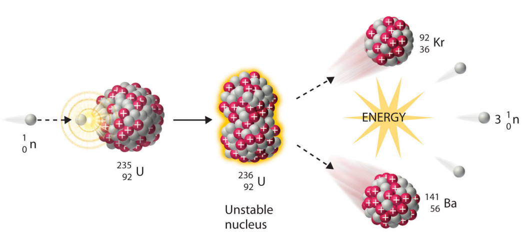

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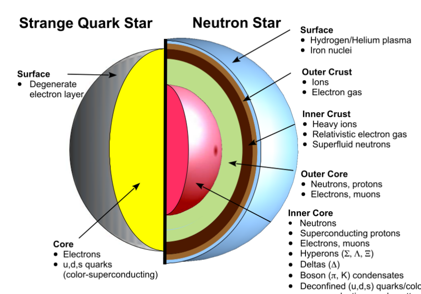

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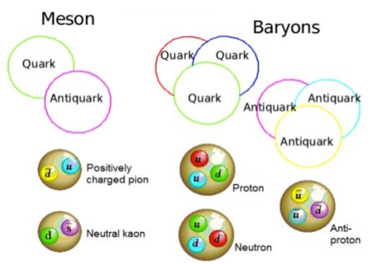

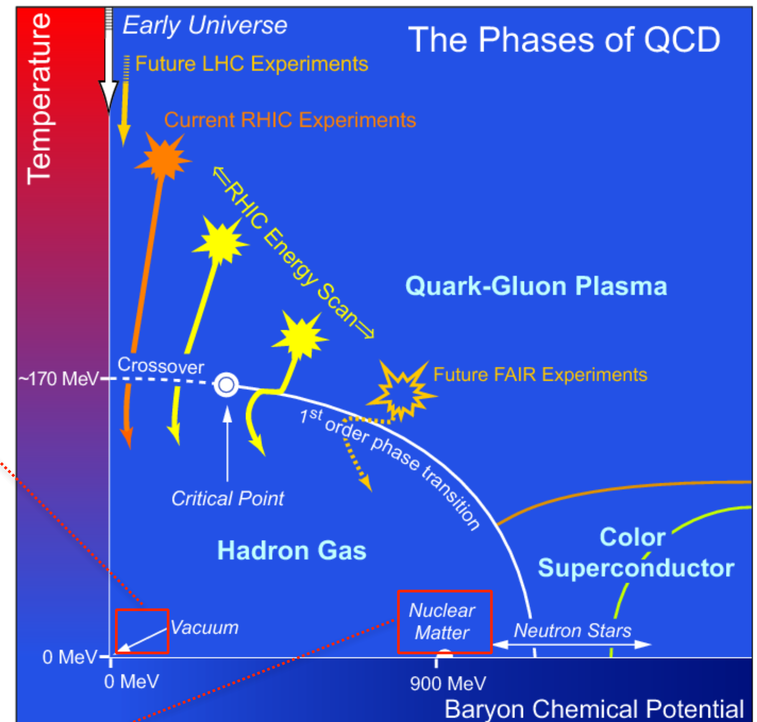

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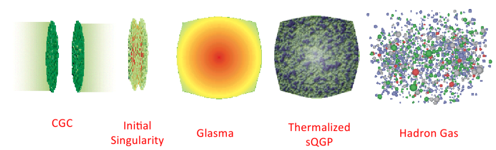

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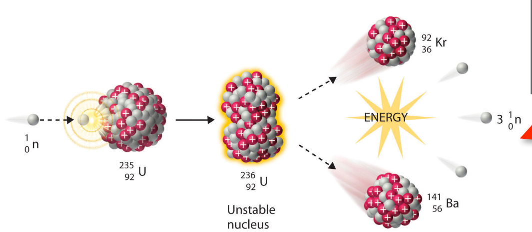

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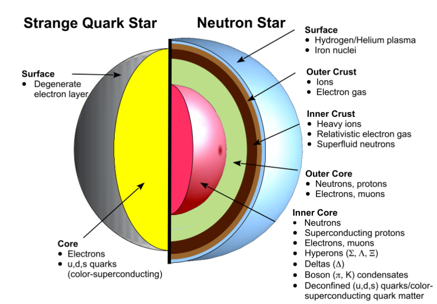

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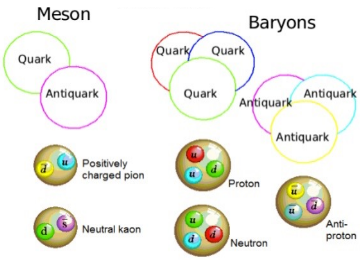

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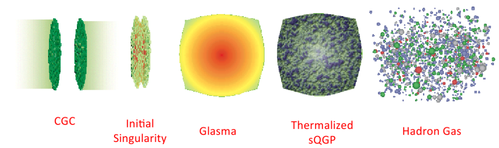

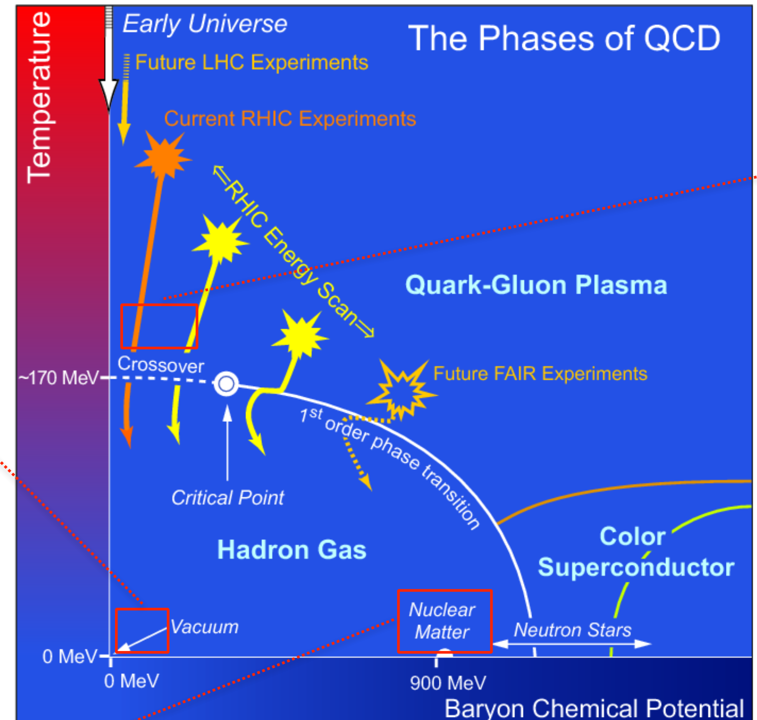

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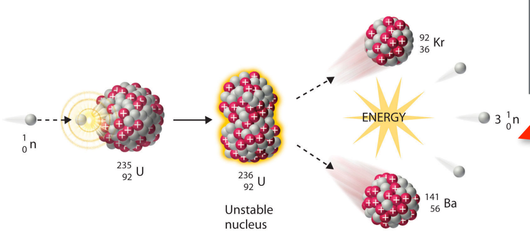

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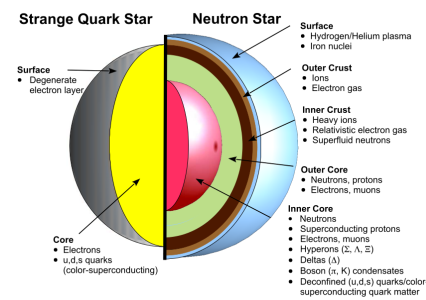

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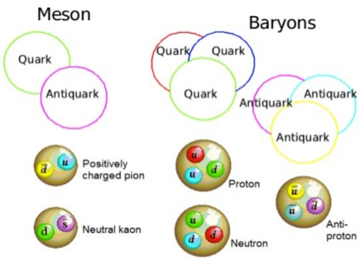

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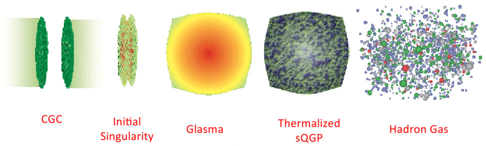

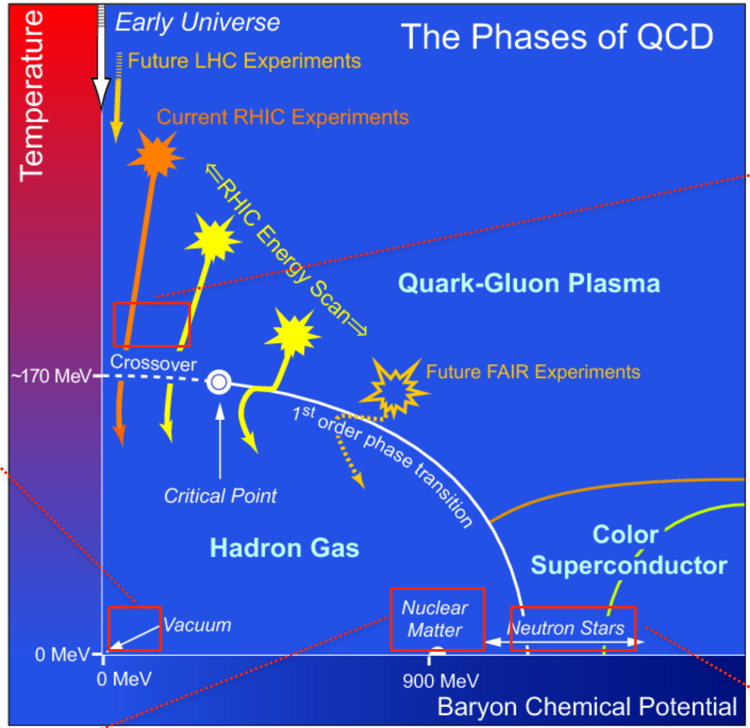

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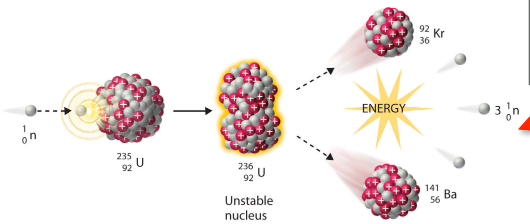

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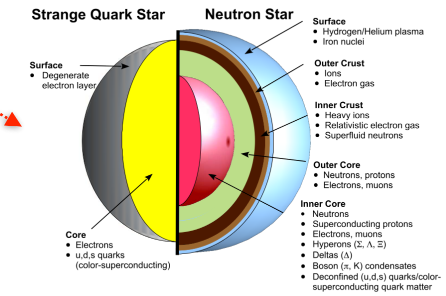

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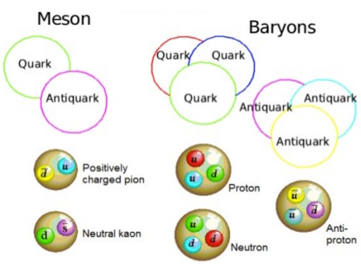

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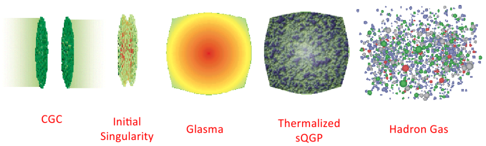

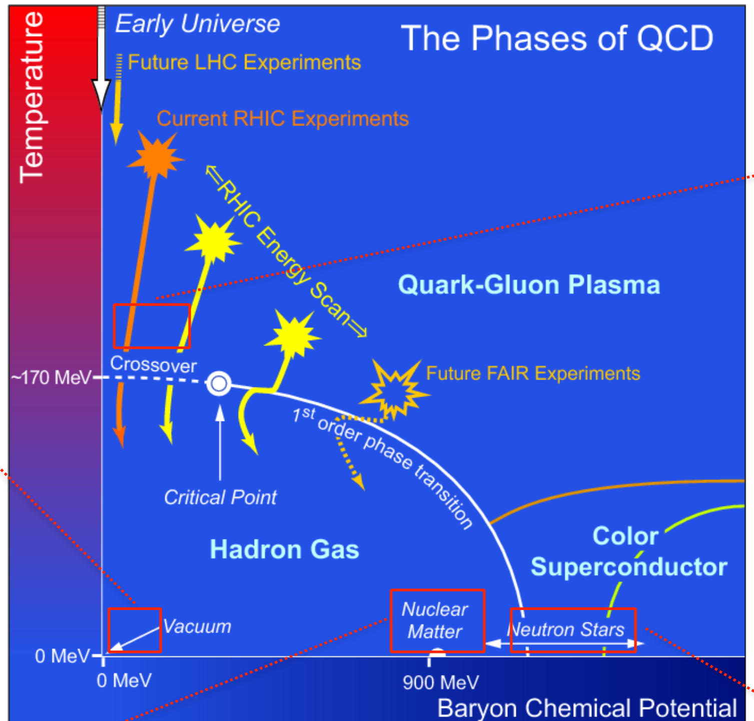

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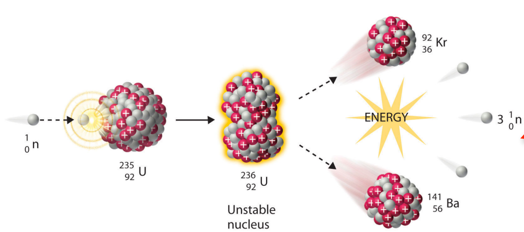

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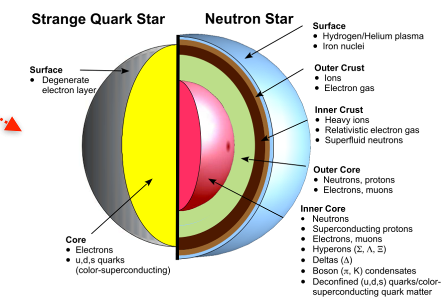
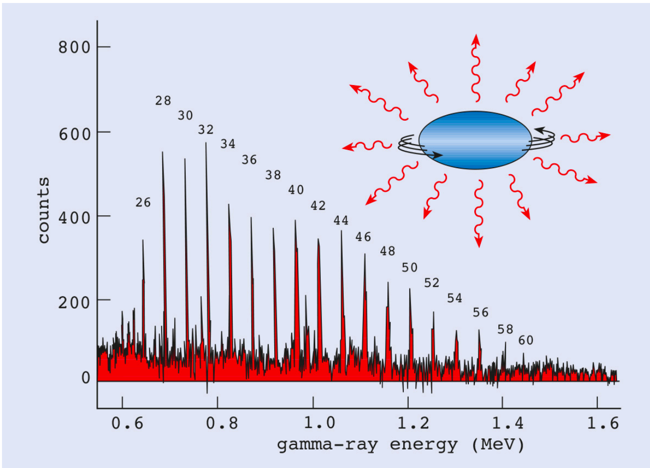


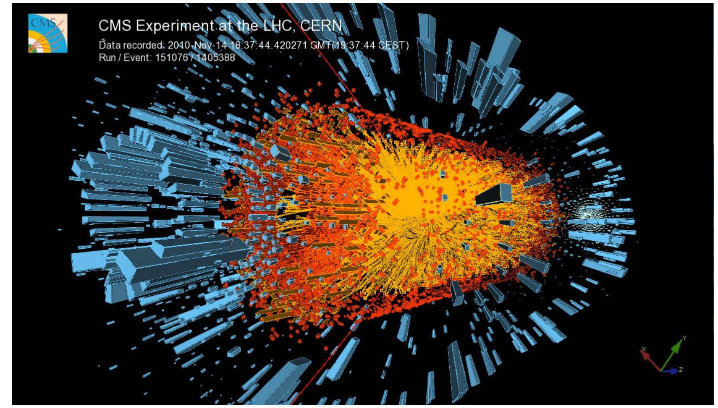
Few-body

Many-body

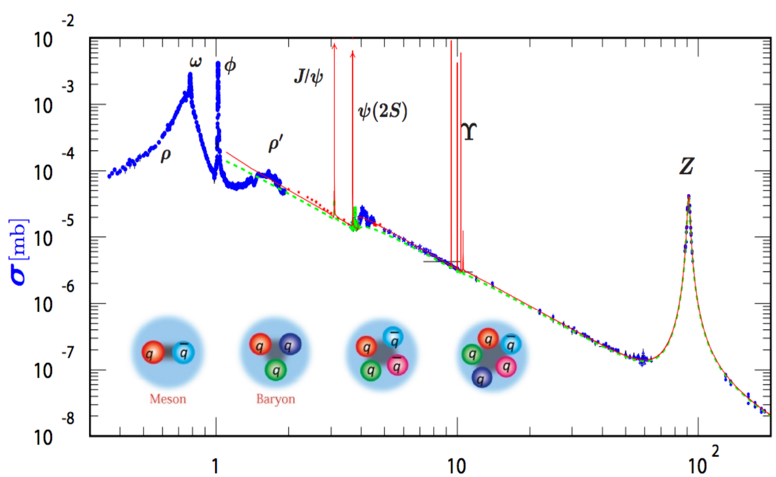
Introduction: QCD frontiers



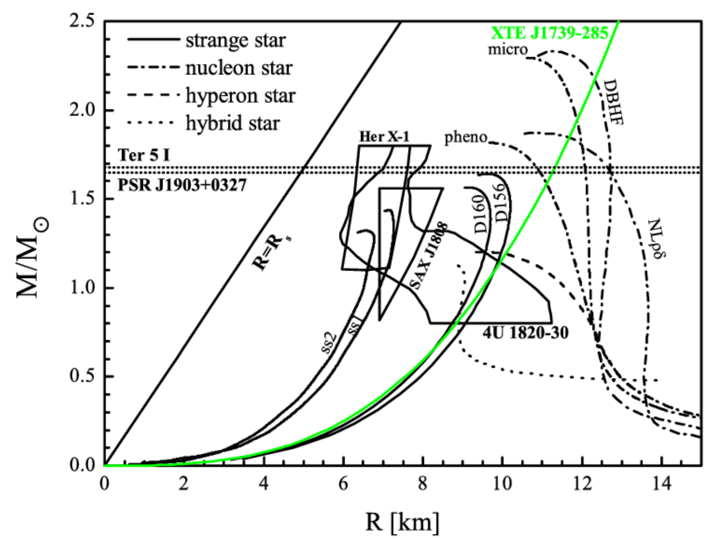
Novel states of nuclei



relativistic heavy-ion collision

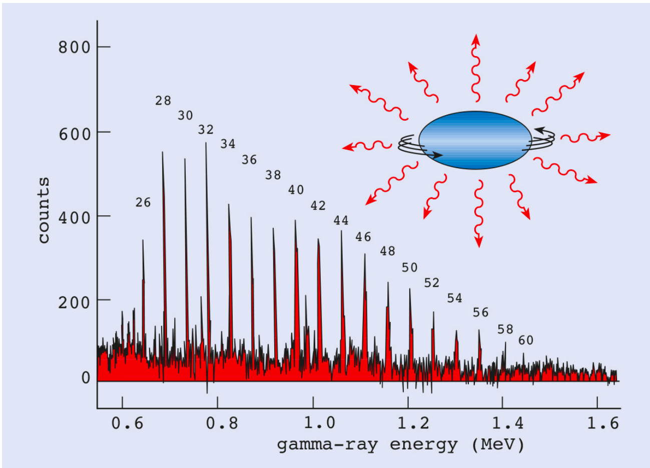


e⁺e⁻ hadronic annihilation

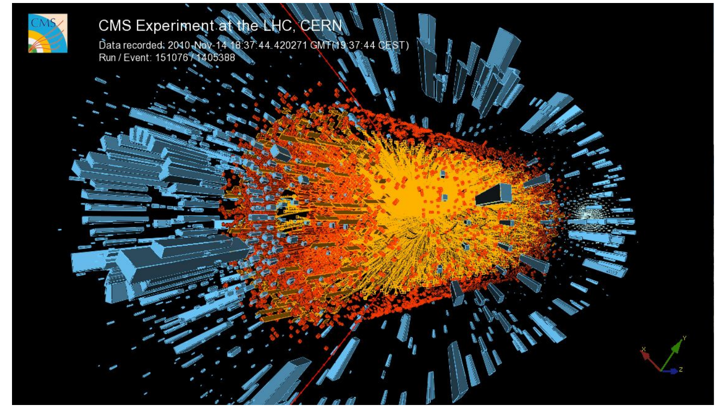


mass-radius relation of compact stars

Introduction: QCD frontiers

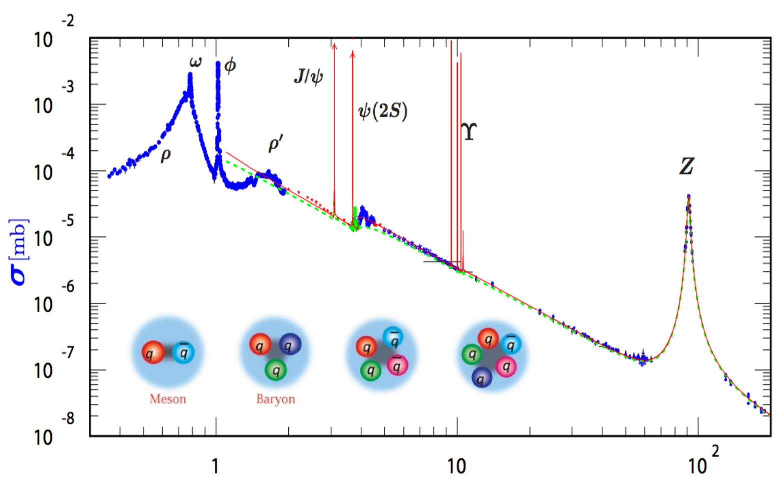


Novel states of nuclei

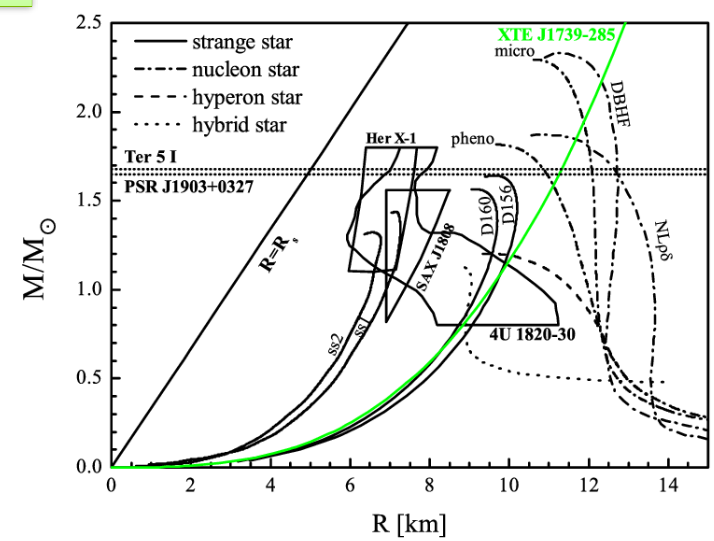


relativistic heavy-ion collision

Solve QCD



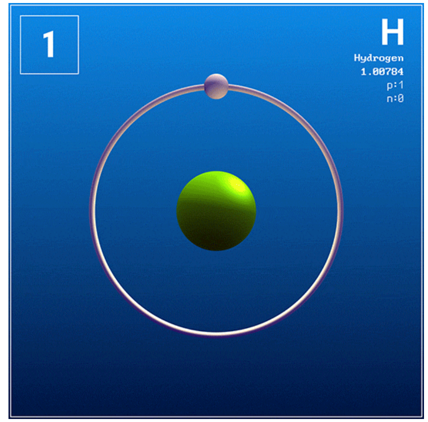
e^+e^- hadronic annihilation



mass-radius relation of compact stars

Lesson

Introduction: QED lesson



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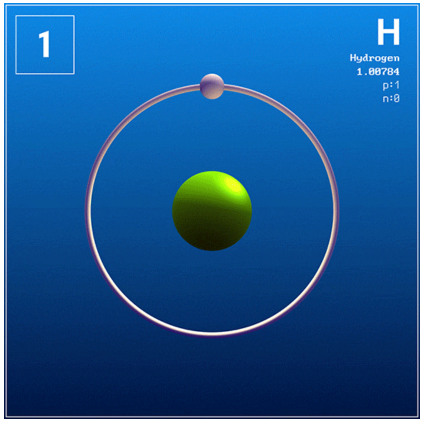
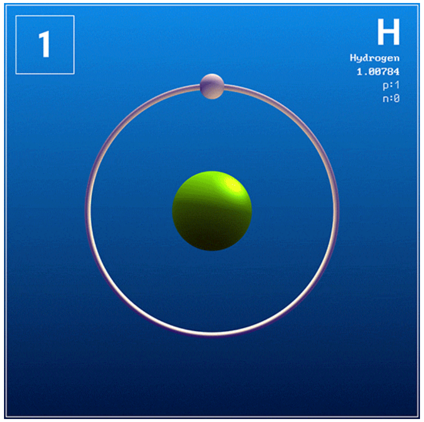


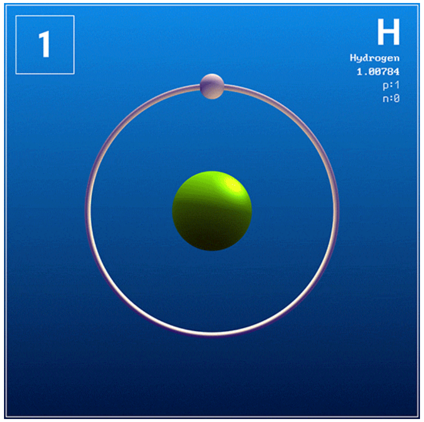
$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

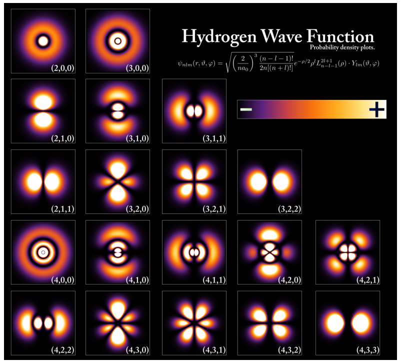


$$H\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}}$$

Introduction: QED lesson





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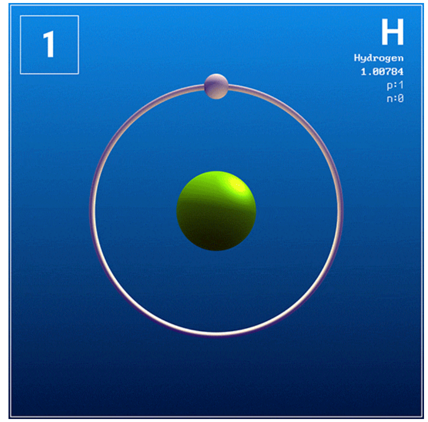
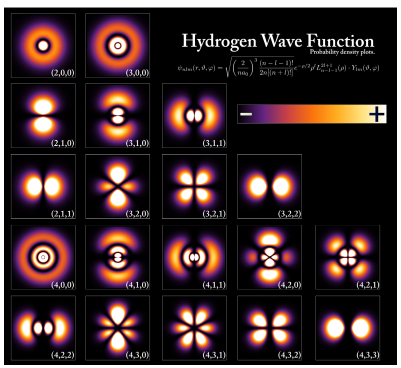
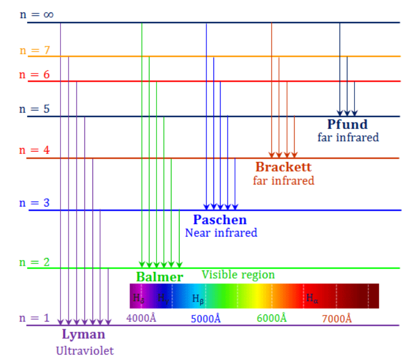
Introduction: QED lesson







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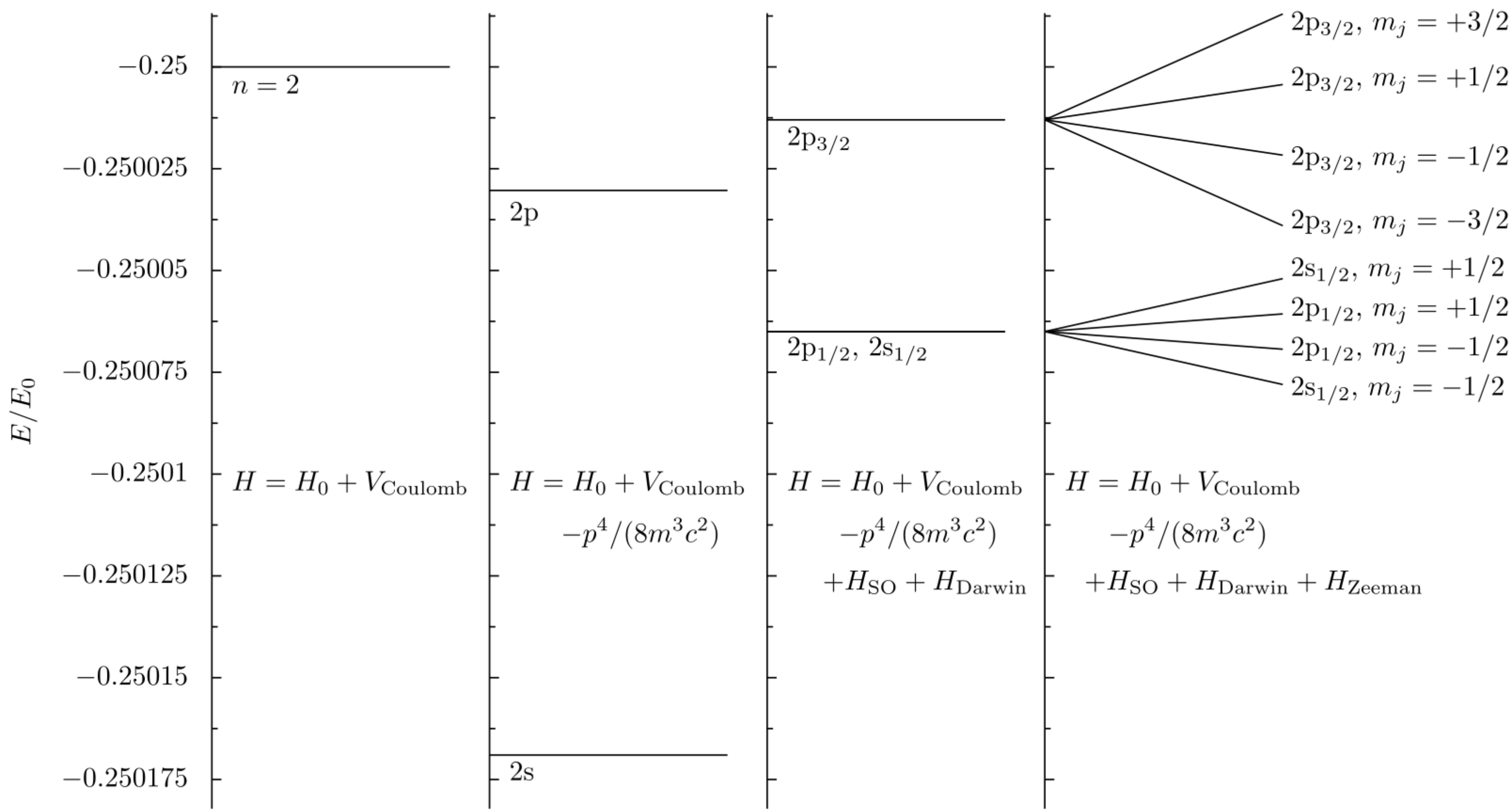

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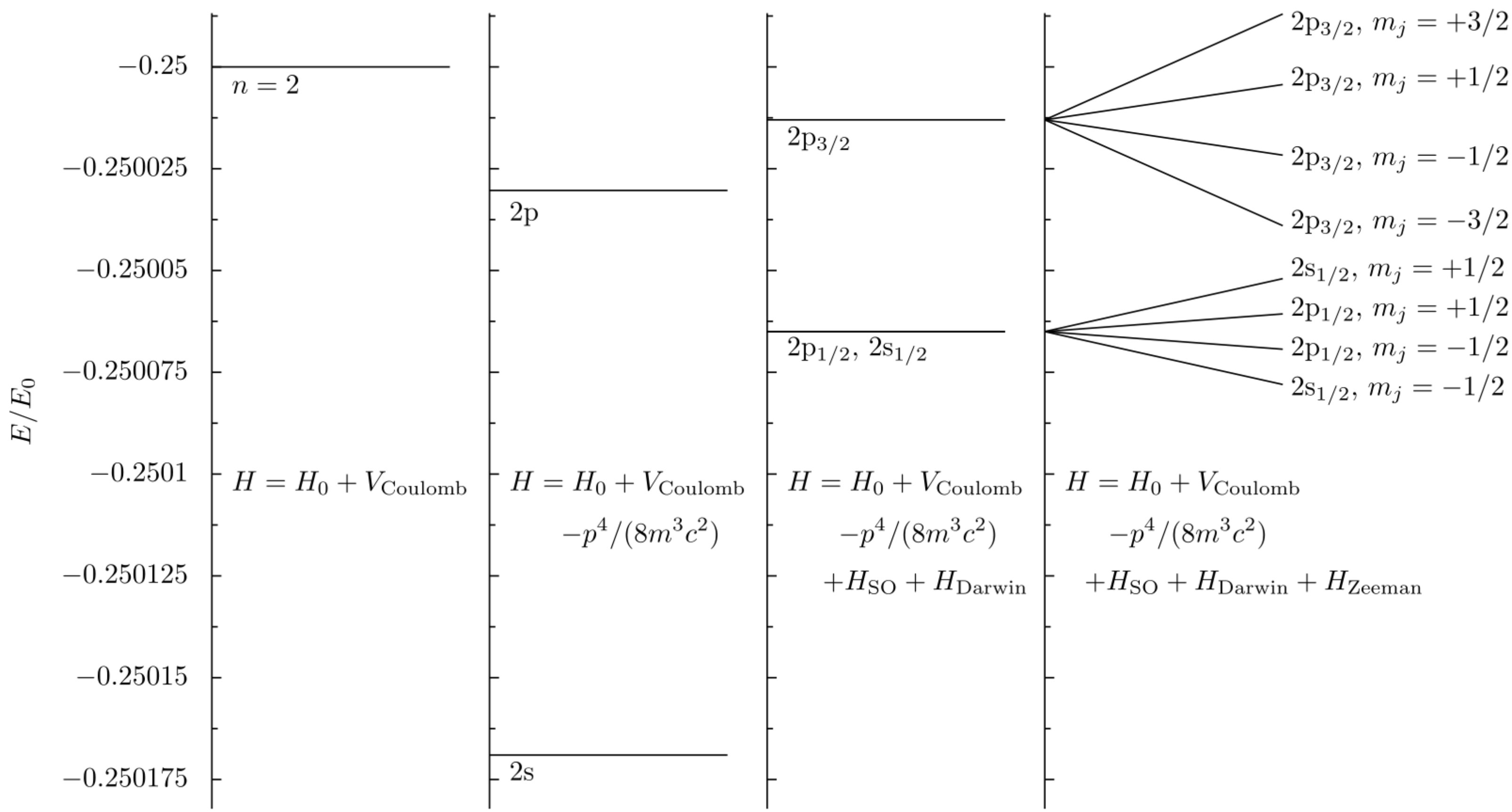
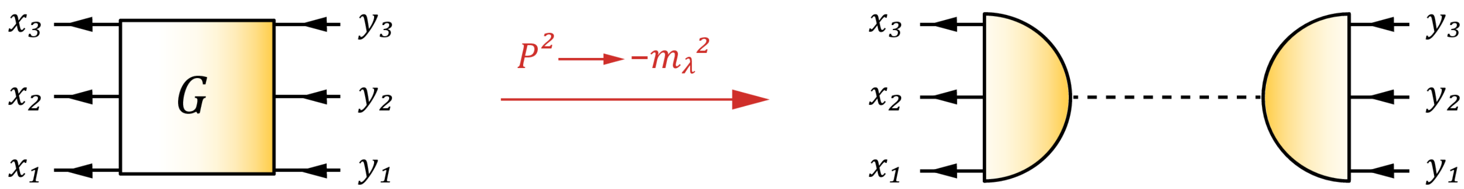


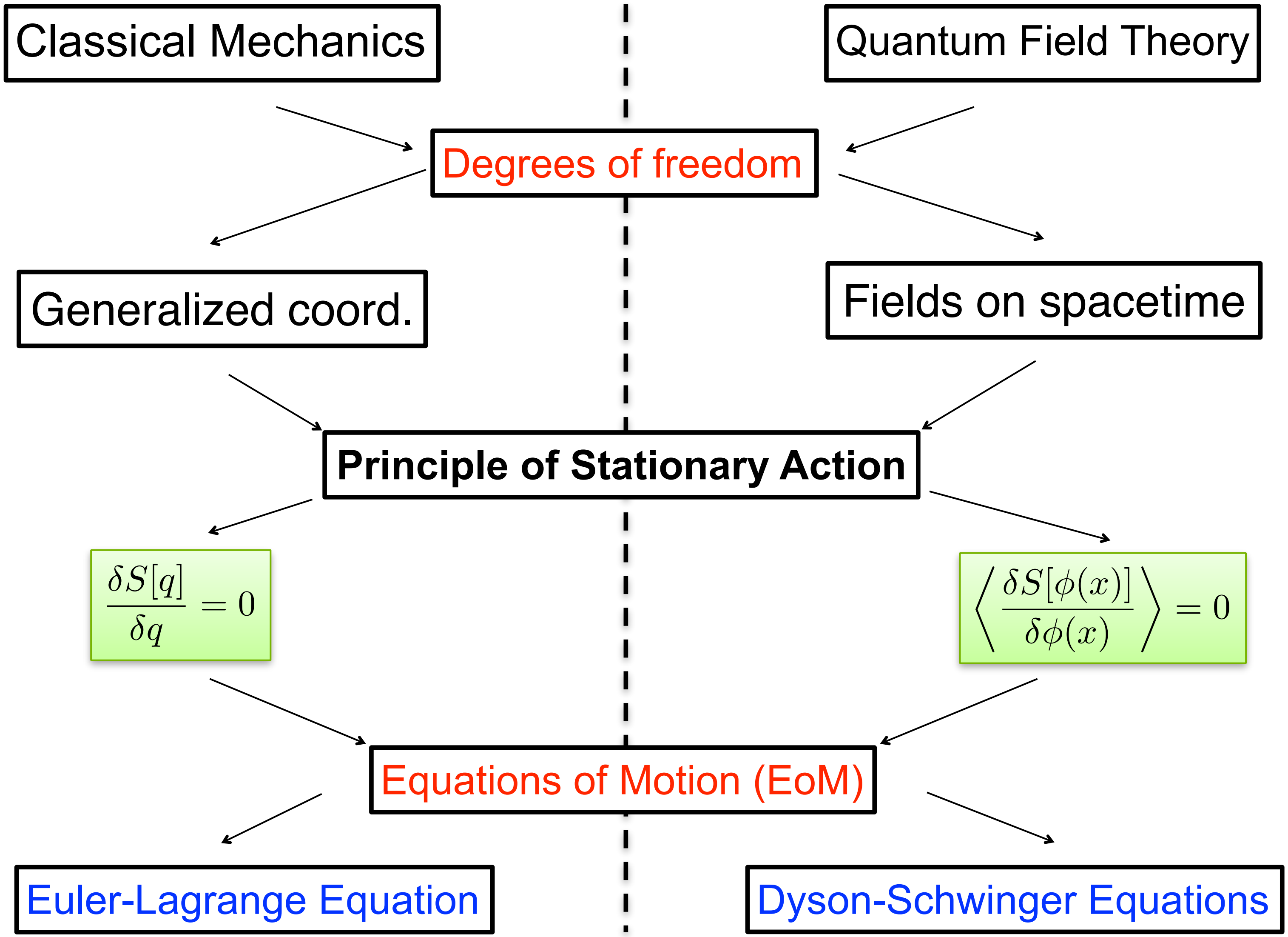
$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

Framework

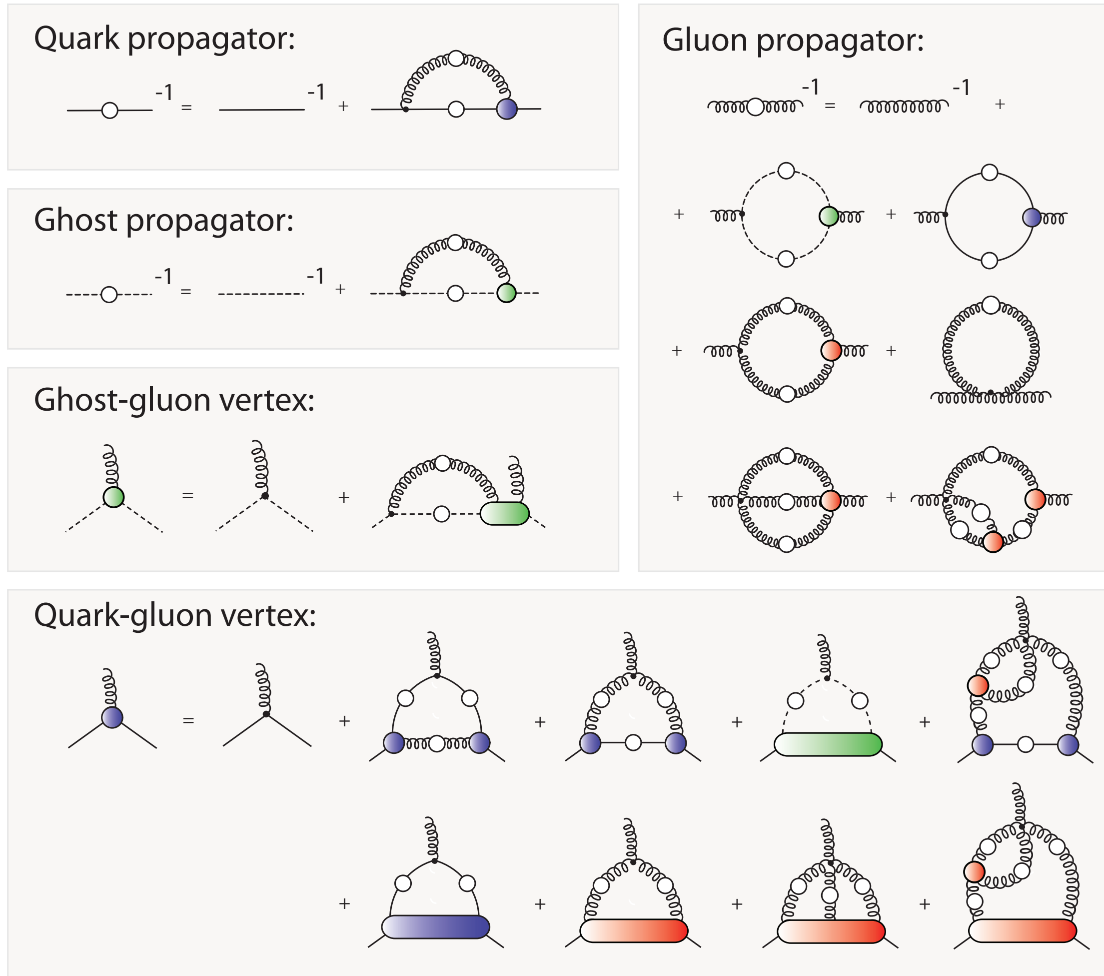
$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$



$$G^{(6)}(x_1, x_2, x_3, y_1, y_2, y_3) = \langle \Omega | q(x_1) q(x_2) q(x_3) q(y_1) q(y_2) q(y_3) | \Omega \rangle$$


Classical Mechanics
Quantum Field Theory
Degrees of freedom
Generalized coord.
Fields on spacetime
Principle of Stationary Action
$\frac{\delta S[q]}{\delta q} = 0$
$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$
Equations of Motion (EoM)
Euler-Lagrange Equation
Dyson-Schwinger Equations

Disadvantage:



Advantage:

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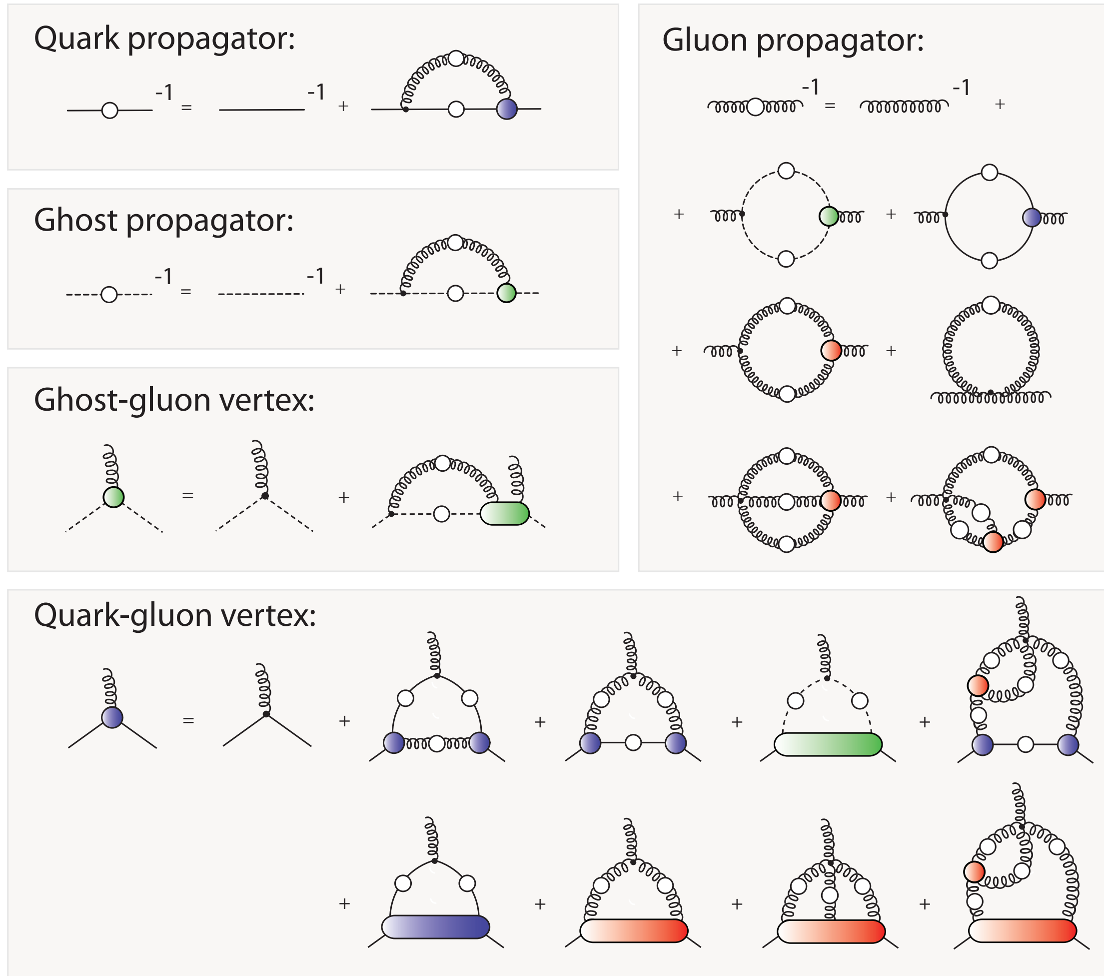
Disadvantage:

- ✦ Most equations are very complicated.

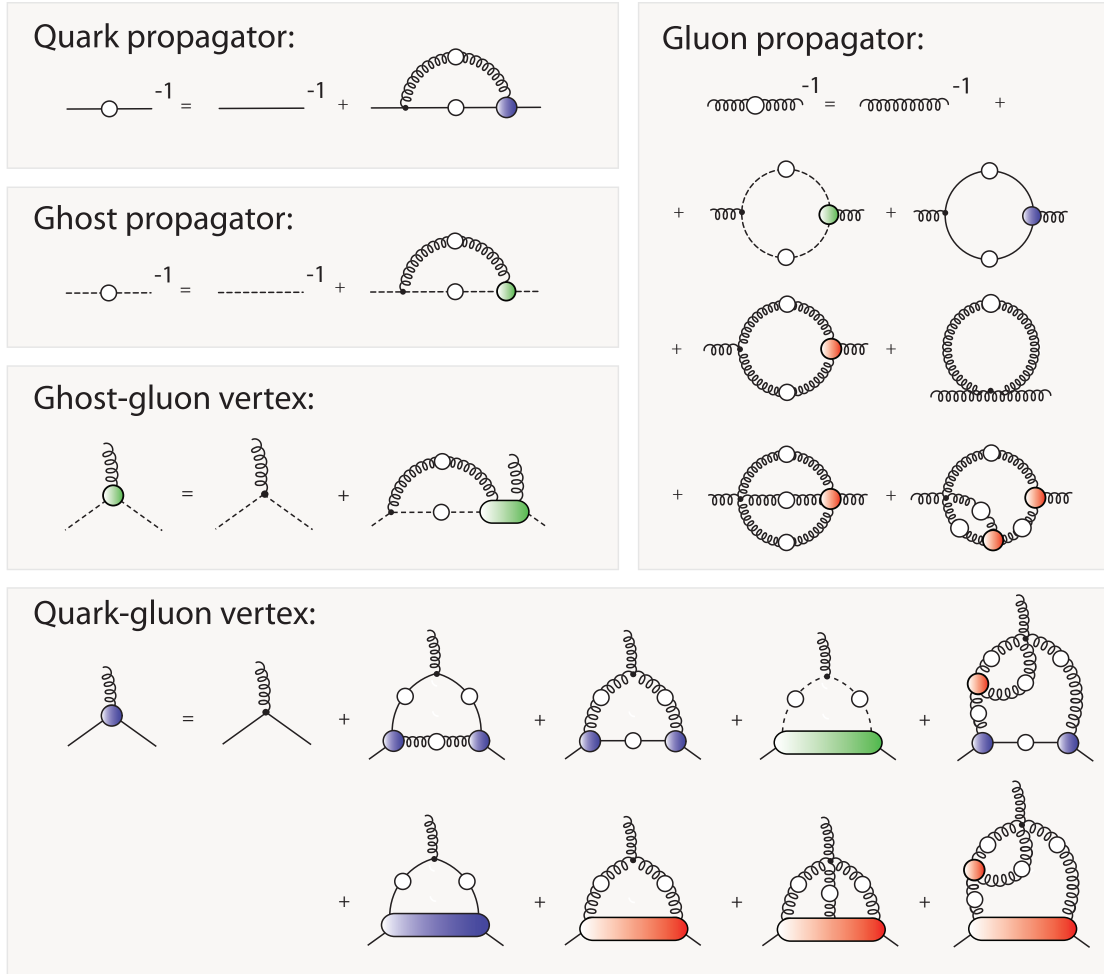


Advantage:

Introduction: Nonperturbative QCD Framework

Disadvantage:

- ✦ Most equations are very complicated.
- ✦ Multi-order Green functions couple together.



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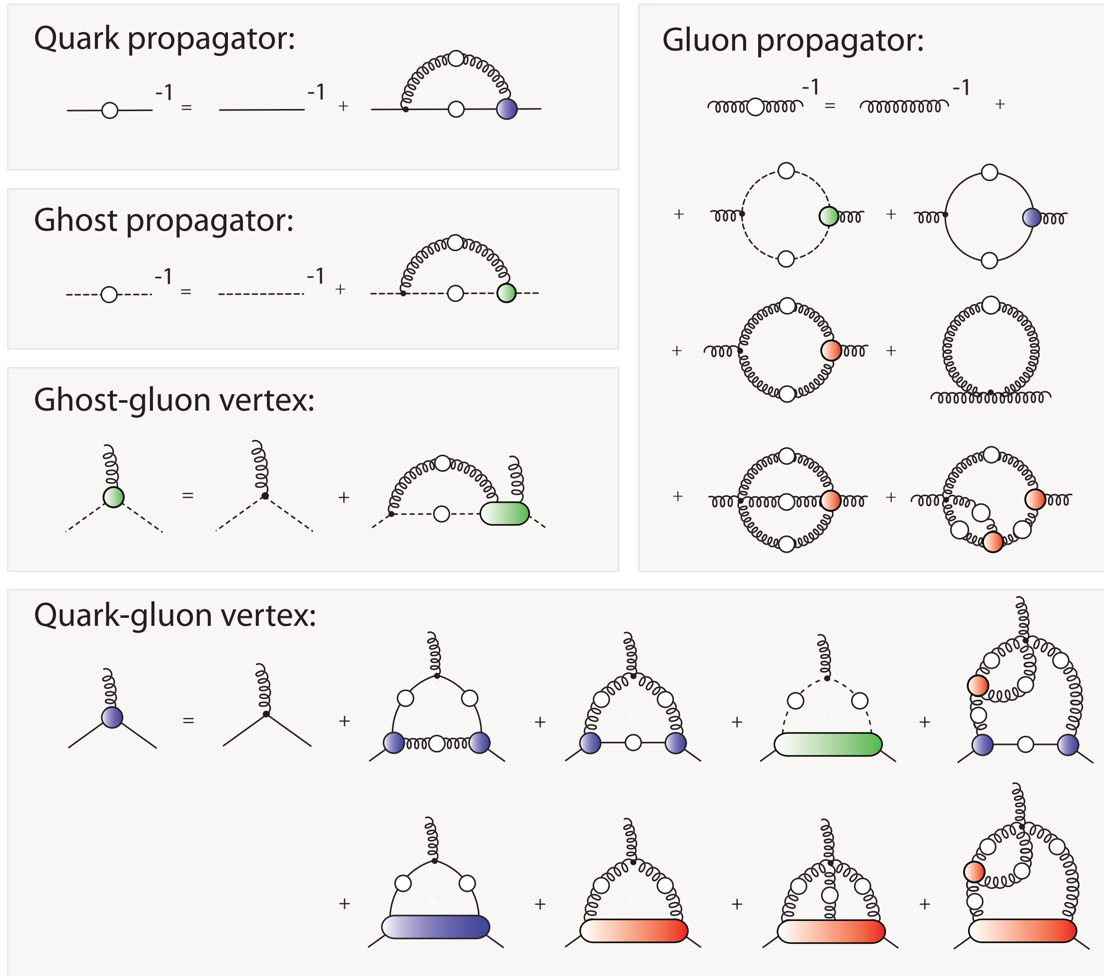
Advantage:

- ✦ Solid connection to QCD



Introduction: Nonperturbative QCD Framework

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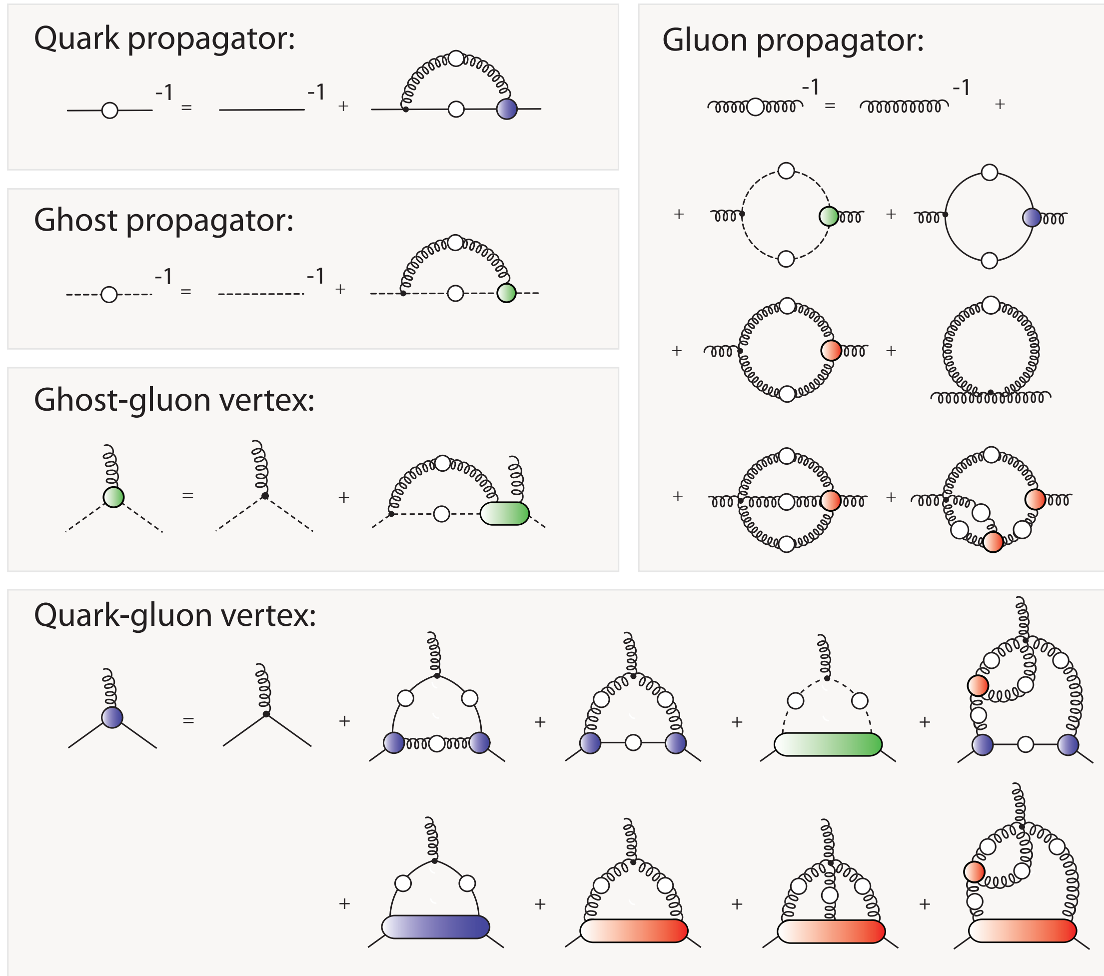

Advantage:

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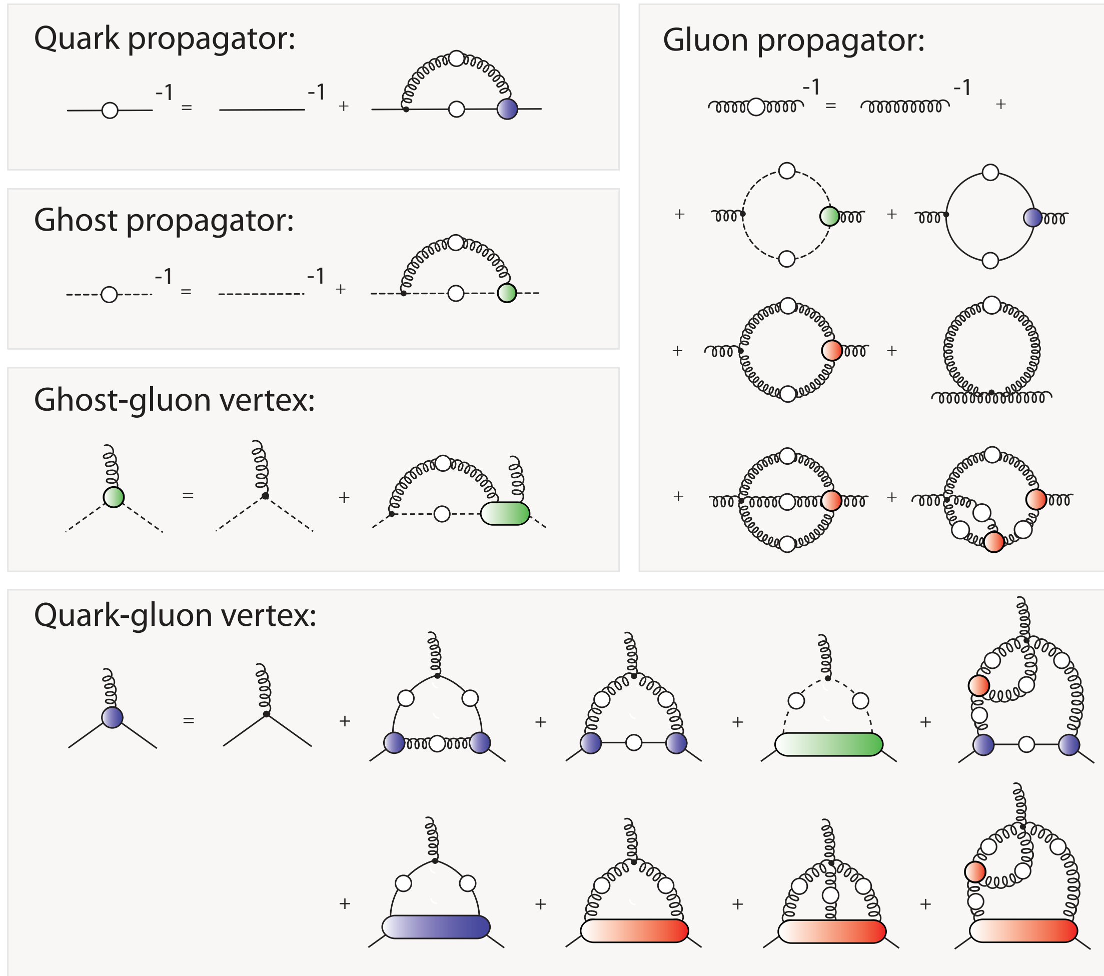

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✦ Solid connection to QCD

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❑ ***Modeling*** ❑ **Truncation**

Introduction: Nonperturbative QCD Framework

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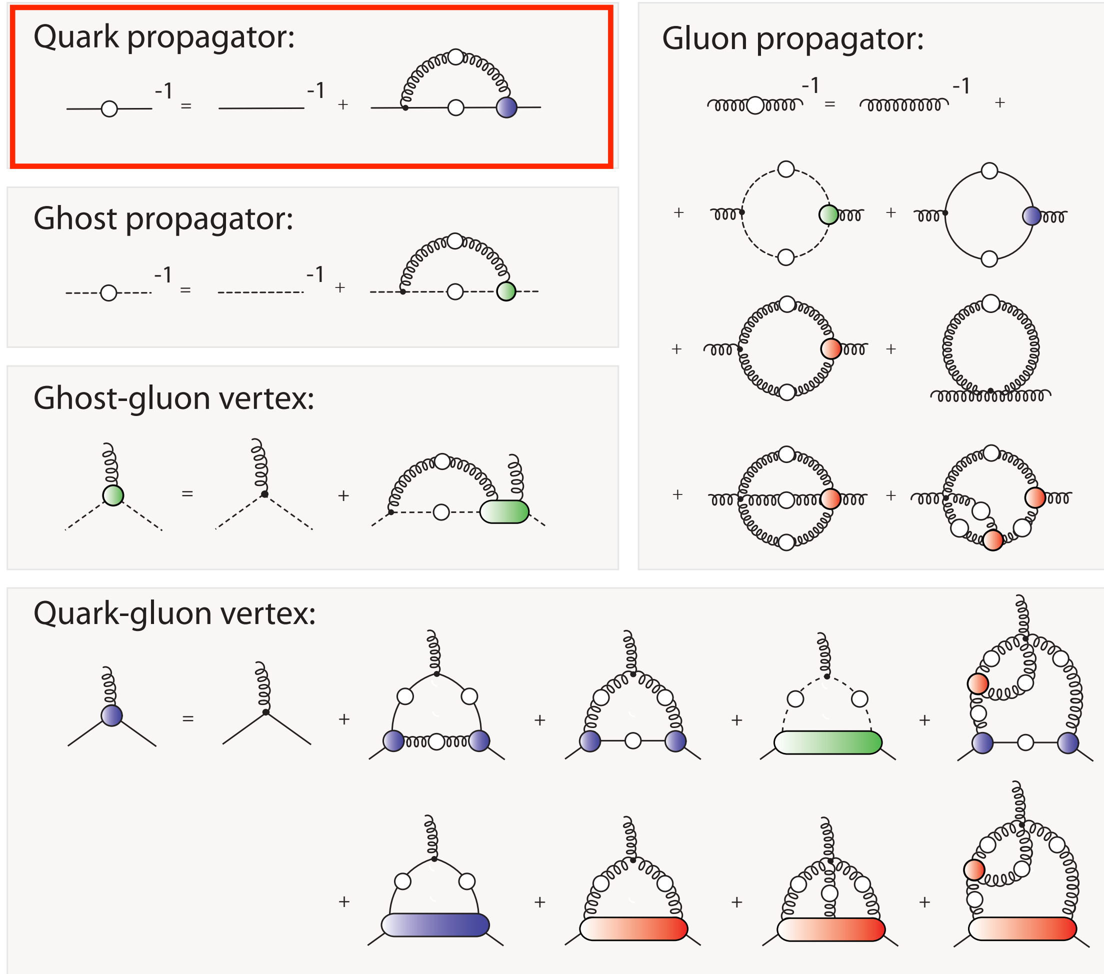

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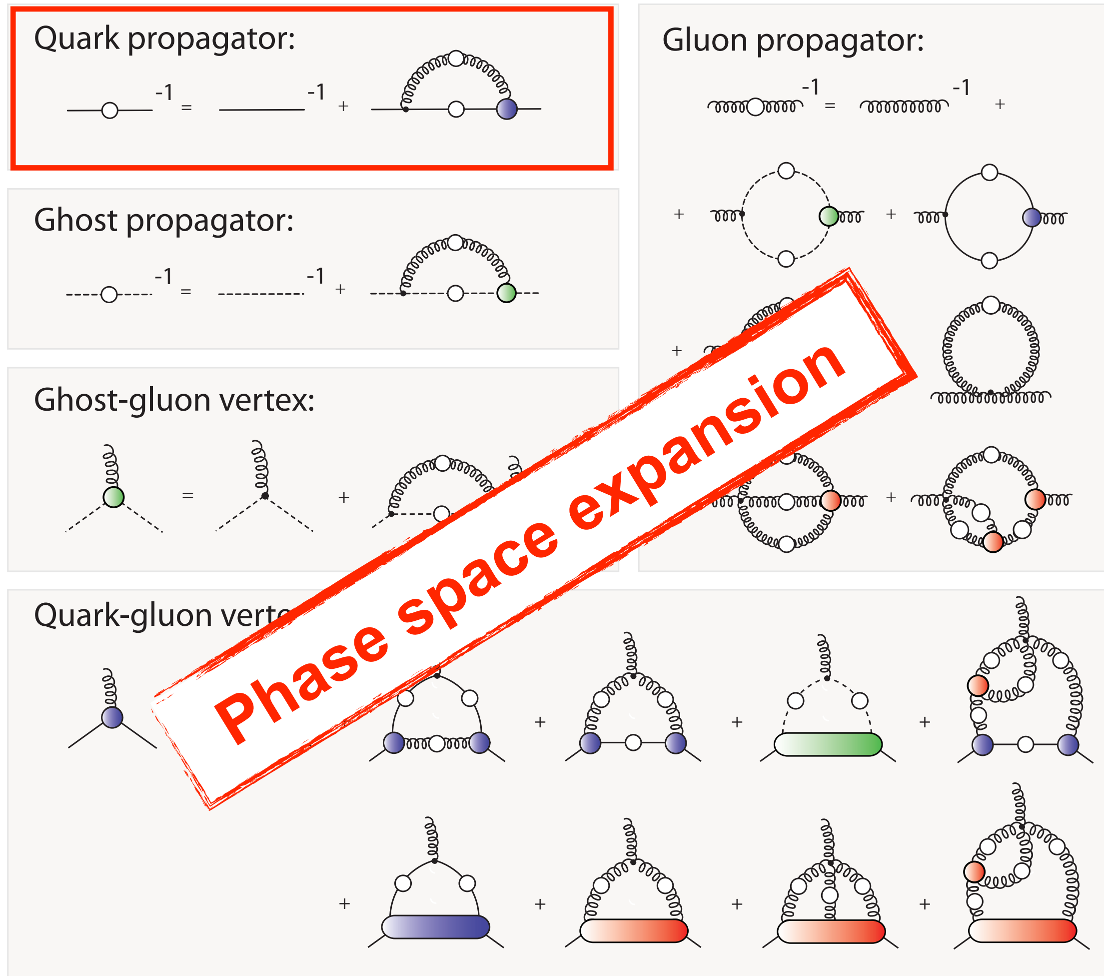

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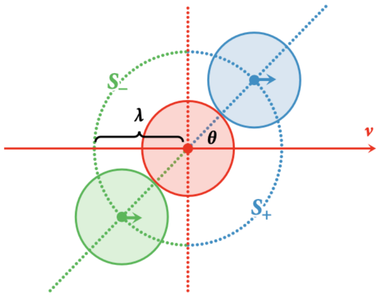
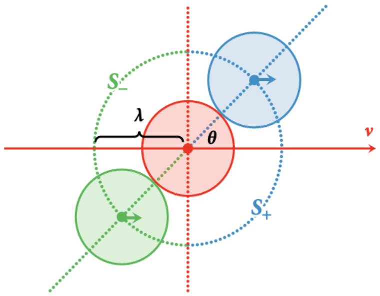


$$\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0$$

All particles:
Liouville eq.

Introduction: Nonperturbative QCD Framework



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All particles:
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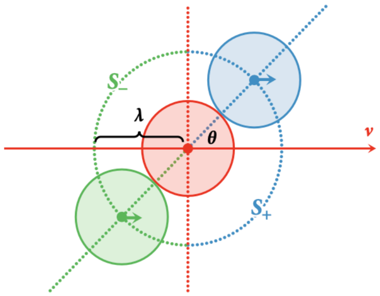
$$\frac{\partial \rho_n}{\partial t} = \{H_n, \rho_n\} + (N-n)\sum_{i=1}^{n}\int \frac{\partial \rho_{n+1}}{\partial p_i} \cdot \frac{\partial \phi_{i\,n+1}}{\partial r_i} dz_{n+1}$$

n particles:
BBGKY hierarchy

Introduction: Nonperturbative QCD Framework



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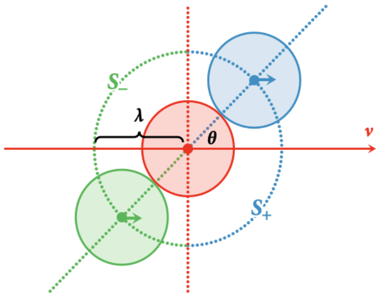
n particles: BBGKY hierarchy

$$\rho_1^{\lambda}(z_1) = \int_{D_i} \rho(Z,t)\, dz_2 \cdots dz_N \qquad D_i = \{r_i \,|\, |r_i - r_1| \geq \lambda\}$$

phase space suppression

Introduction: Nonperturbative QCD Framework



$$\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0$$

All particles: Liouville eq.

$$\frac{\partial \rho_n}{\partial t} = \{H_n, \rho_n\} + (N-n)\sum_{i=1}^{n}\int \frac{\partial \rho_{n+1}}{\partial p_i} \cdot \frac{\partial \phi_{i\,n+1}}{\partial r_i} dz_{n+1}$$

n particles: BBGKY hierarchy

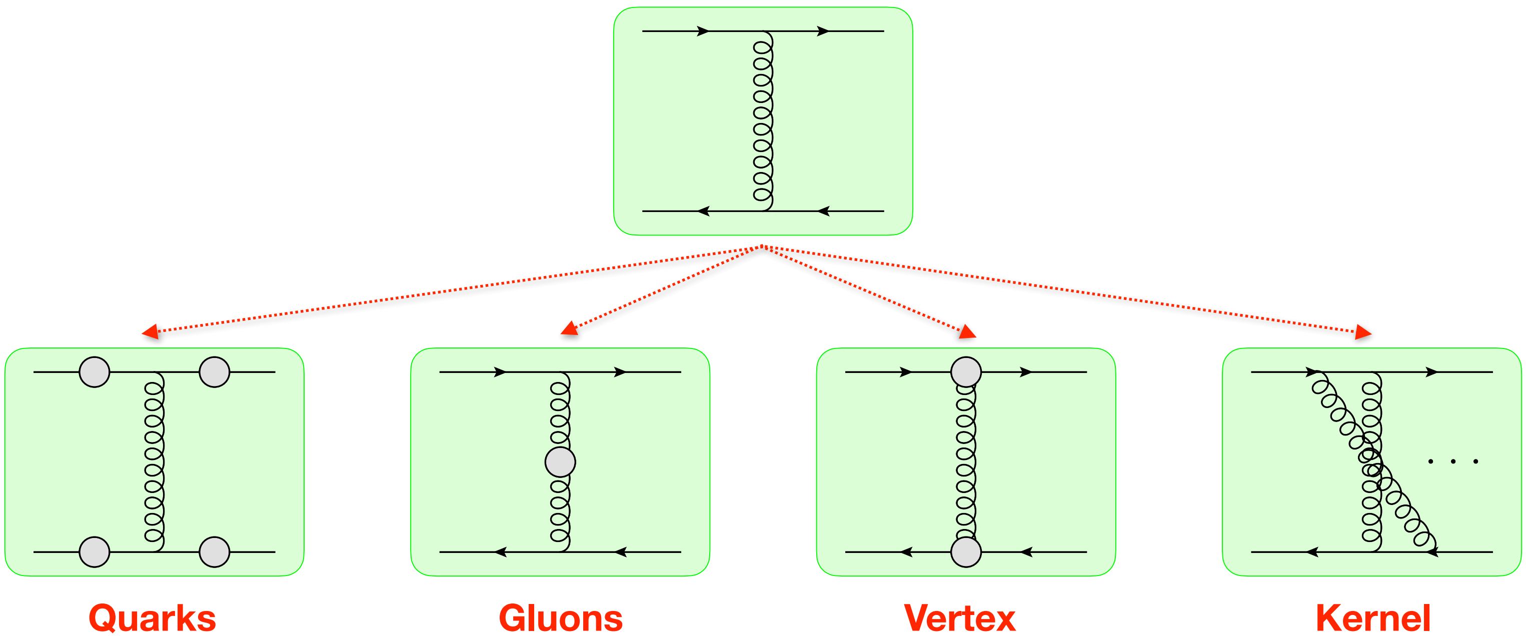
$$\rho_1^{\lambda}(z_1) = \int_{D_i} \rho(Z,t)\, dz_2 \cdots dz_N \qquad D_i = \{r_i \,|\, |r_i - r_1| \geq \lambda\}$$

phase space suppression

$$\frac{\partial \rho_1}{\partial t} = \{H_1, \rho_1\} + \int \left[\rho_1(\overline{p}_1)\rho_1(\overline{p}_2) - \rho_1(p_1)\rho_1(p_2)\right] N\nu\, d\sigma d^3 p_2$$

1 particle: Boltzmann eq.

Introduction: Nonperturbative QCD Framework



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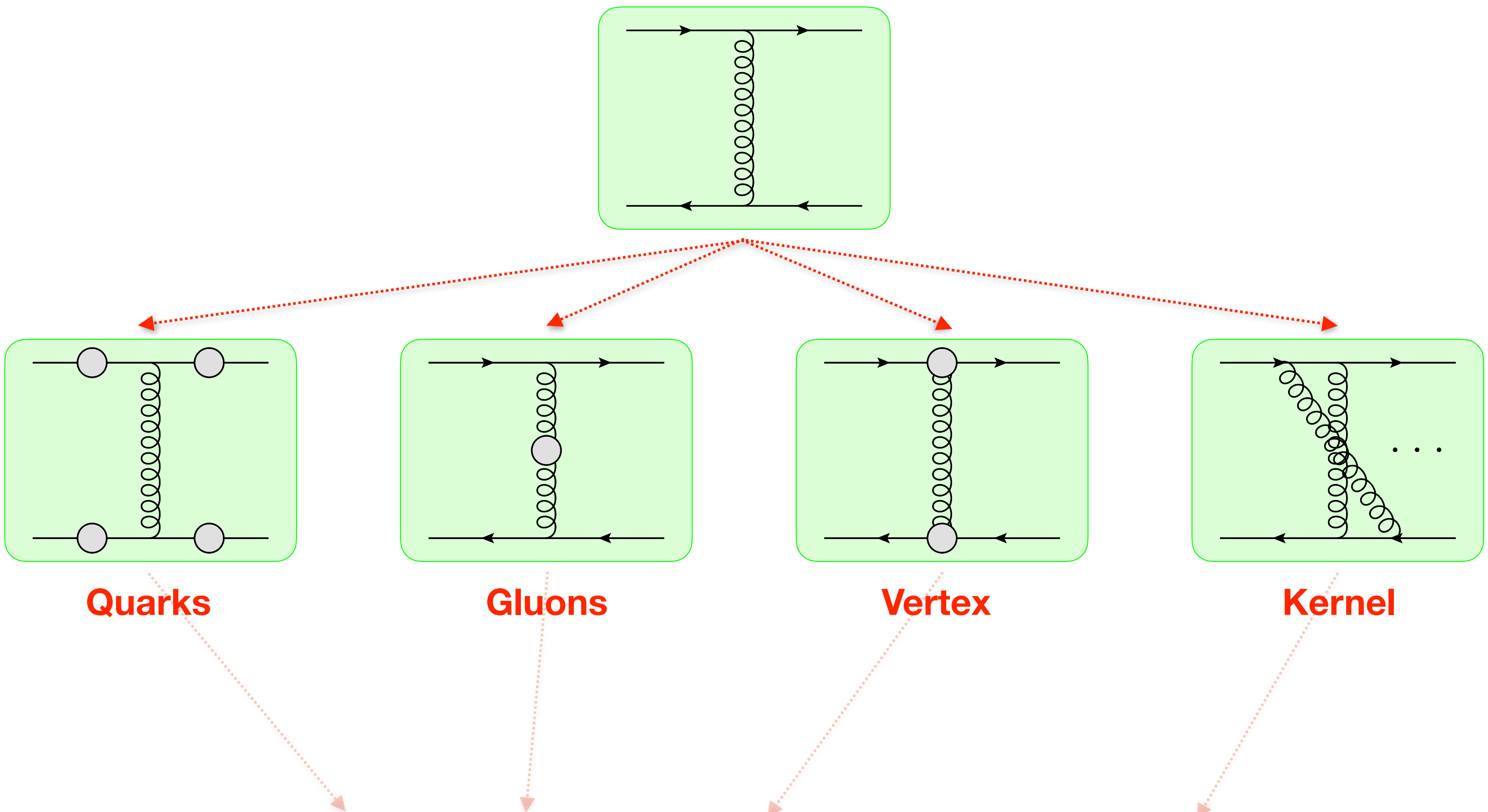

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

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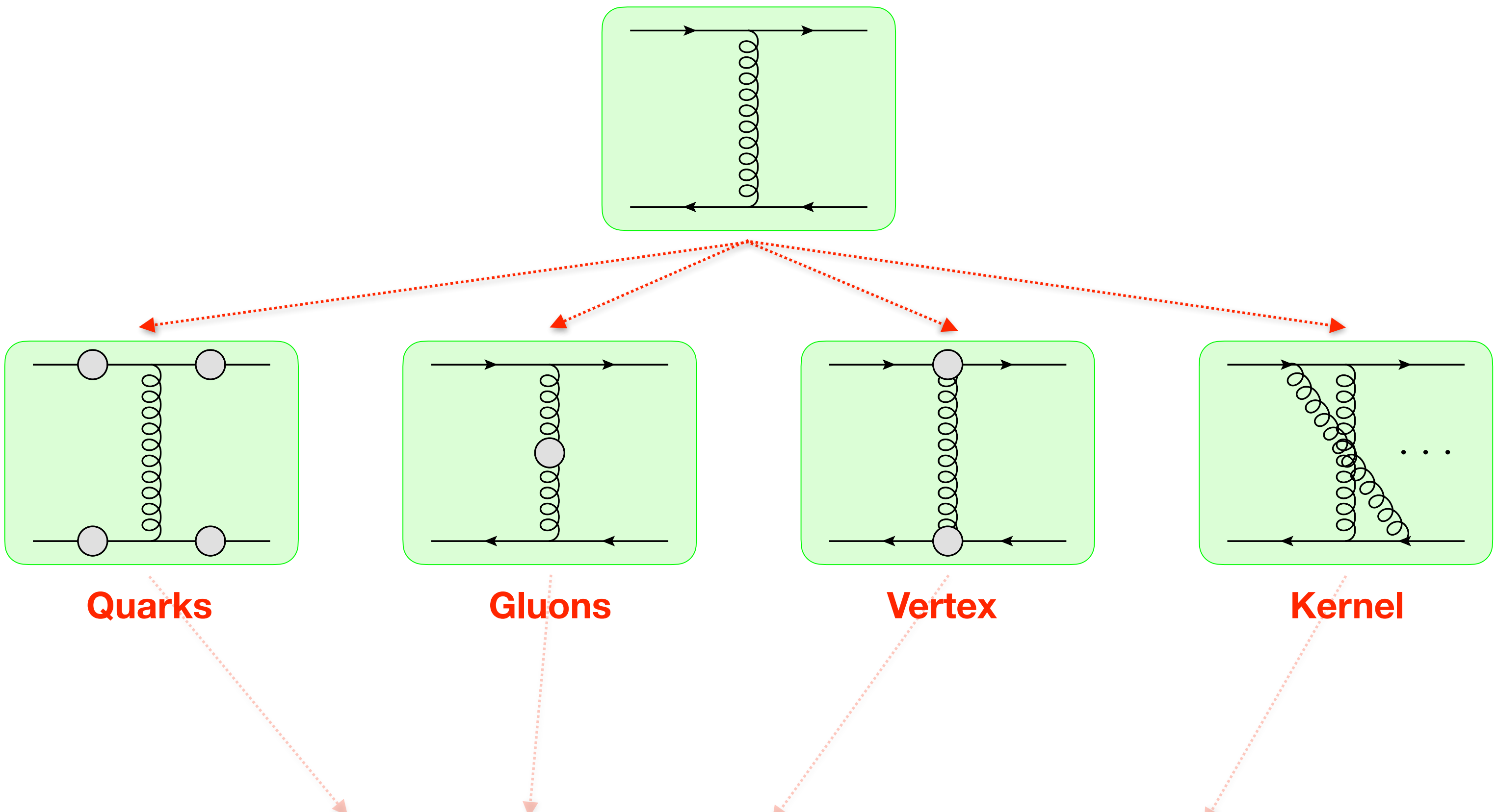

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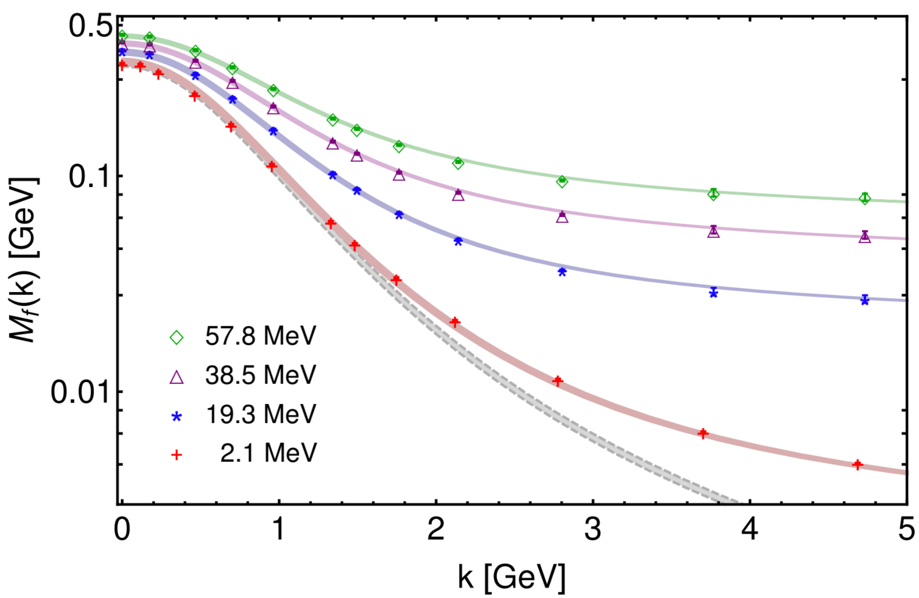
Ground states — **Excited states** → M

Basics

Basics: Quarks are dispersive quasi-particles

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



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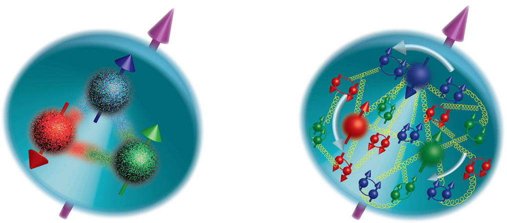

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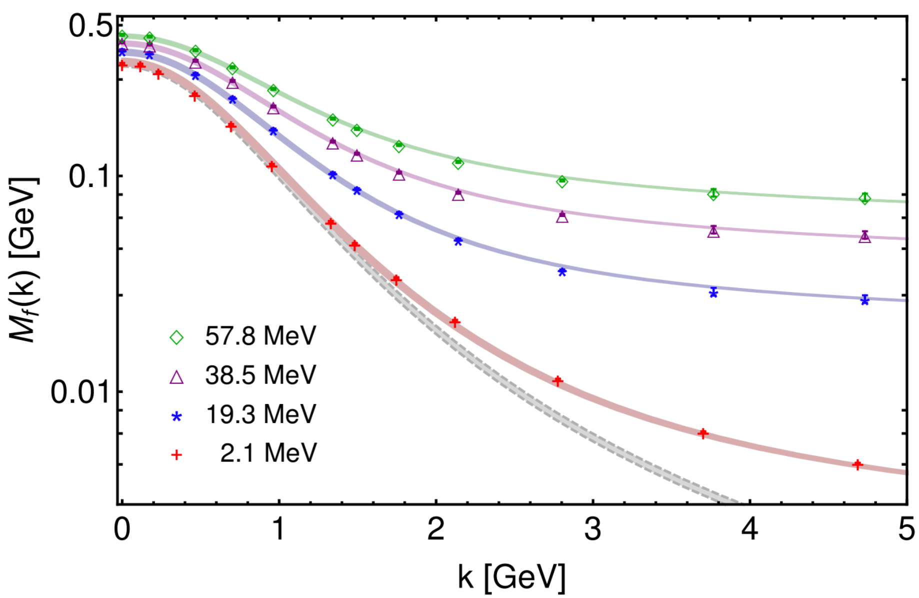

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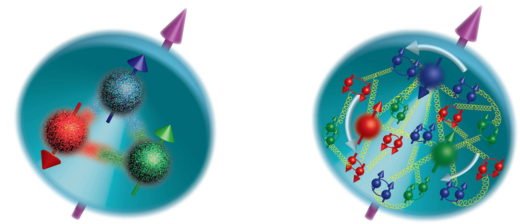

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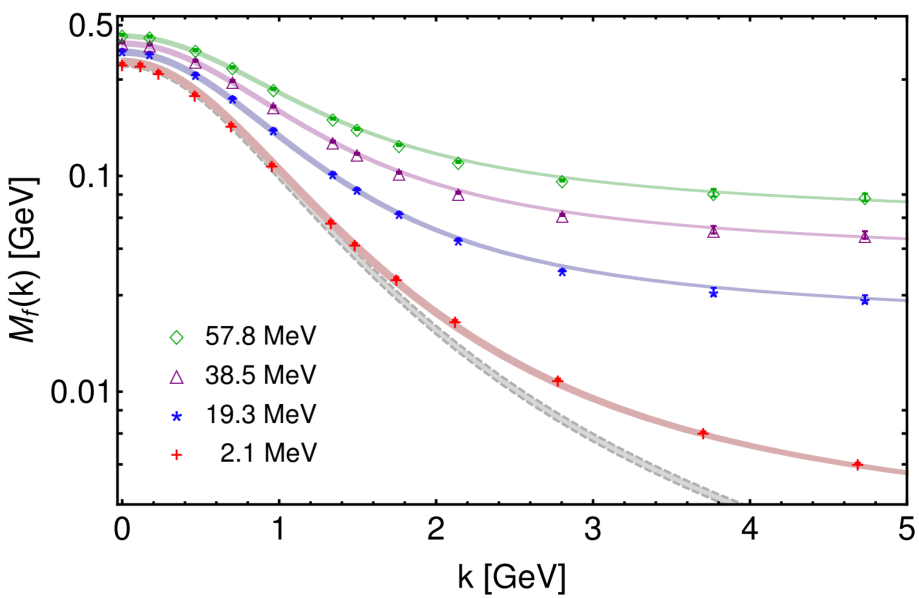

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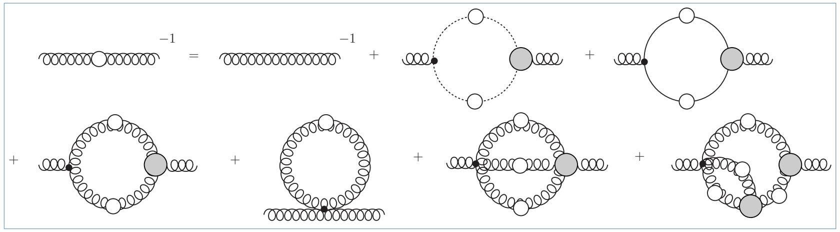
Vacuum — invisible highly dispersive medium

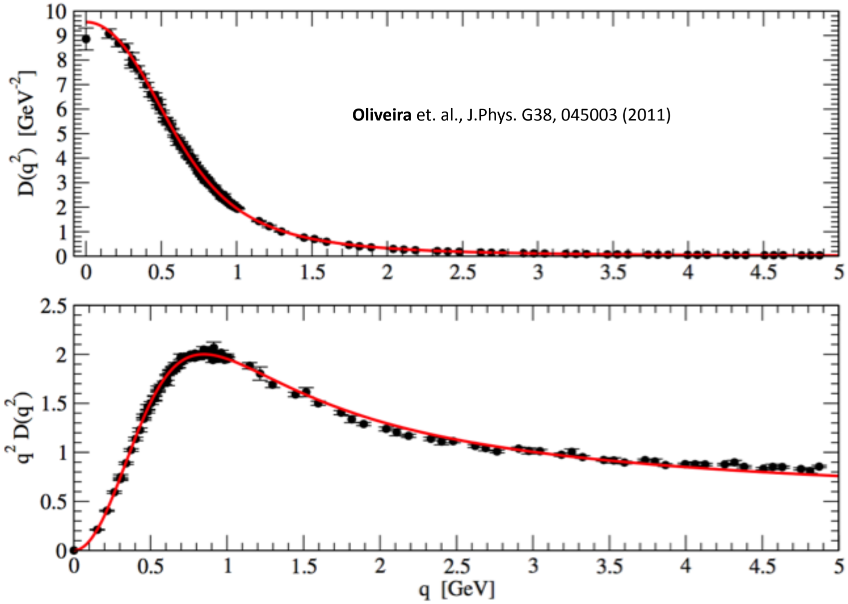
Basics: Gluons are massive quasi-particles

Gluon gap equation:

Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



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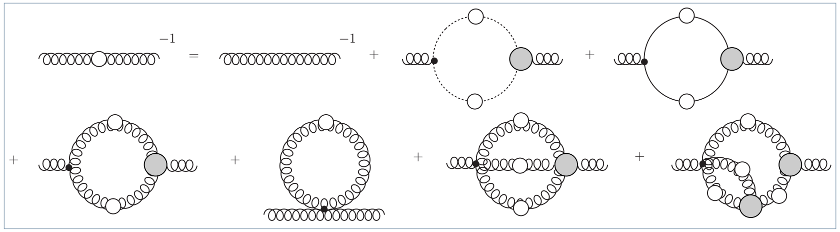

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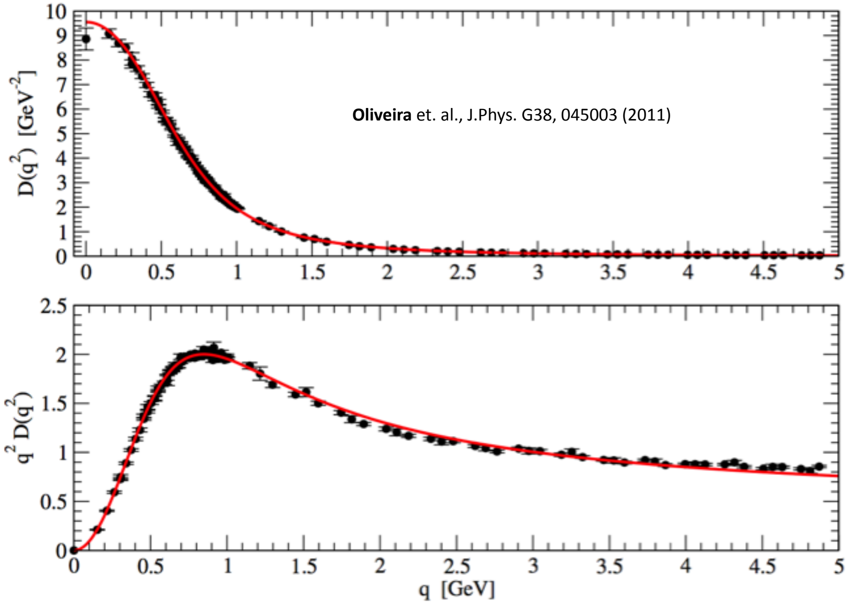

- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

- In QCD: Gluons are ***cannibals*** — a particle species whose members become **massive** by eating each other — **quasi-particles**!

Basics: Gluons are massive quasi-particles

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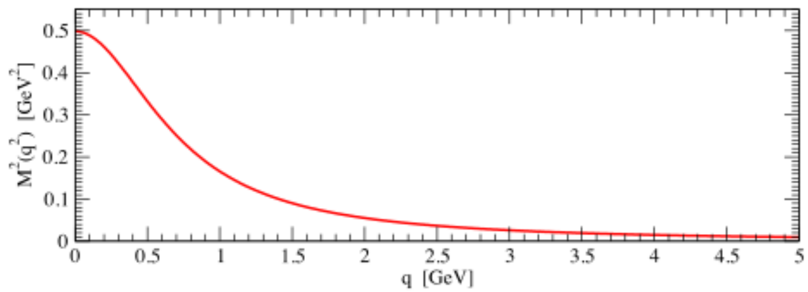
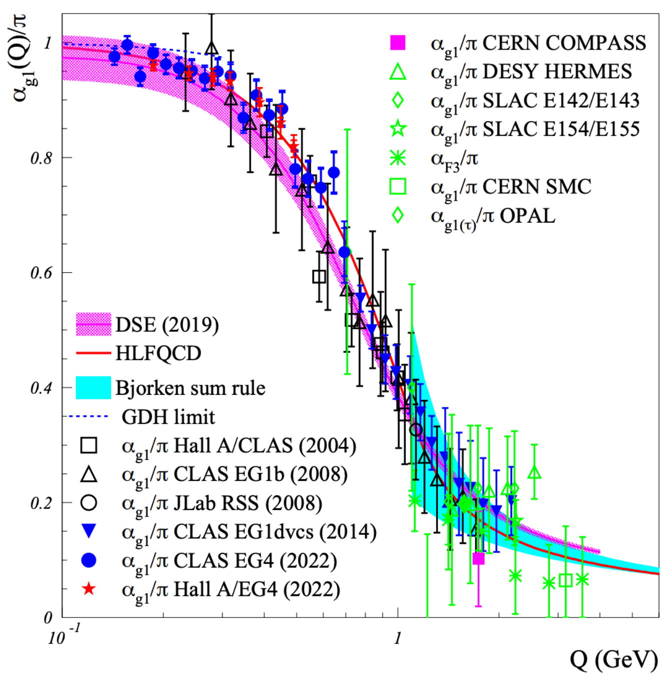
Gluon mass function: **Oliveira** et. al., J.Phys. G38, 045003 (2011)



Running coupling: **Deur, Brodsky, Roberts,** PPNP, 104081 (2024)



See, e.g., PRC 84, 042202(R) (2011)

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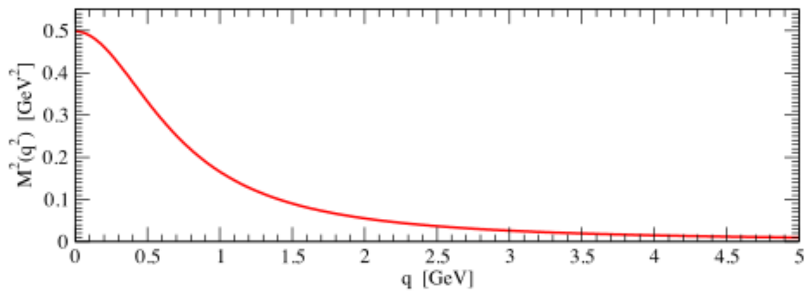

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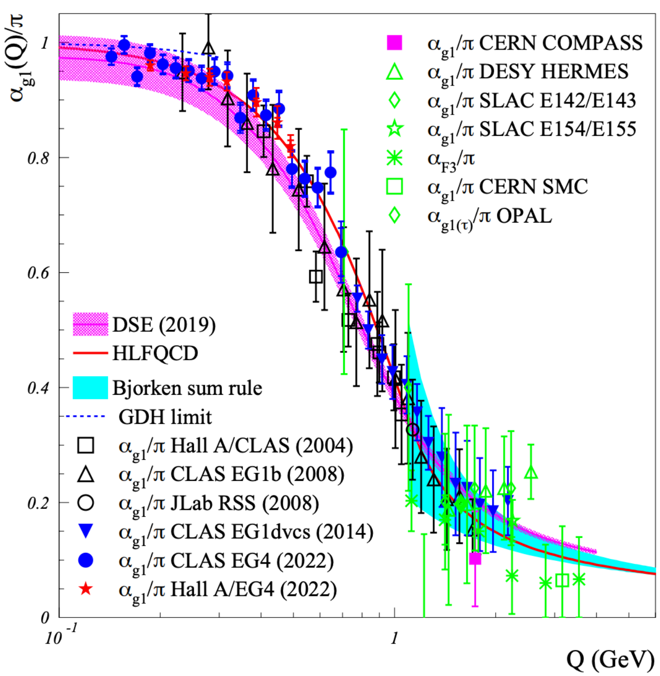


1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

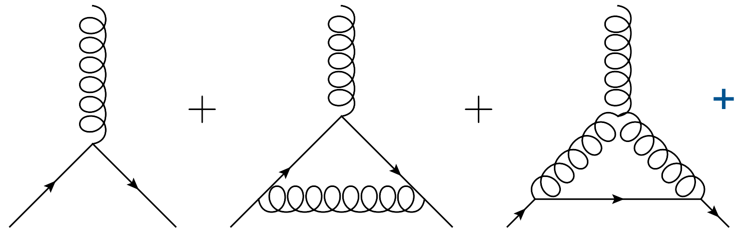
$$M_g \sim 700\ \text{MeV}$$

2. The effective running coupling **saturates** in the infrared limit.

- converge to: $\alpha_s(0) \sim \pi$
- transition at: $Q \sim 1\ \text{GeV}$

See, e.g., PRC 84, 042202(R) (2011)

Basics: Vertex has DCSB-rendered appearance

Quark-gluon vertex:  + … = ?

See, e.g., PLB722, 384 (2013)

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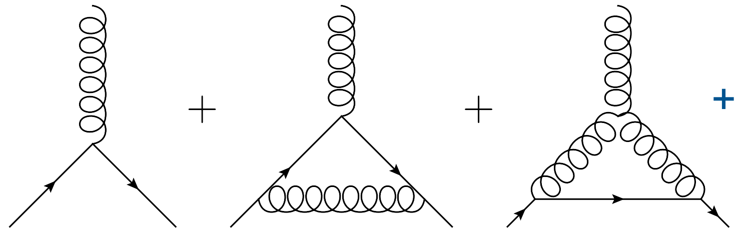
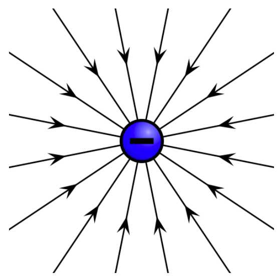
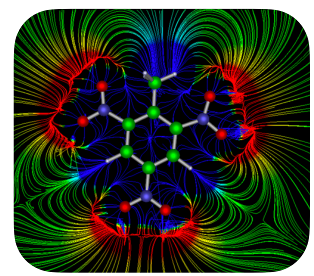


\+ … = ?

point charge





distributed charges

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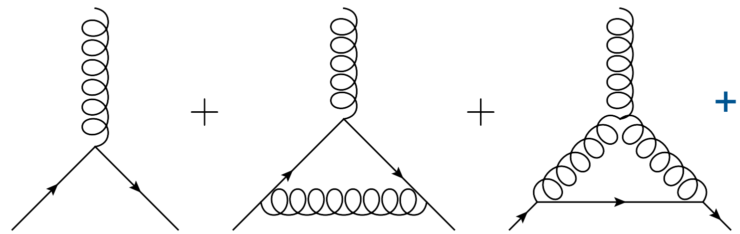

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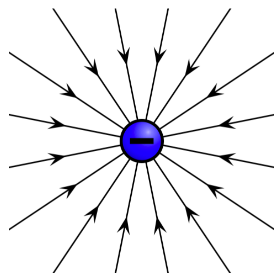

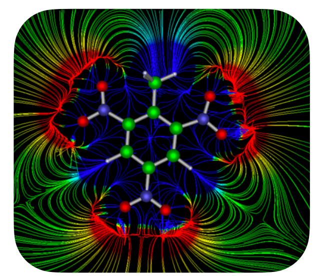

distributed charges

- The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

12 terms

- The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous moment** is proportional to the **Pauli** term.

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

Spacetime

- ❑ **Poincaré symmetry**

Fields

- ❑ **Gauge symmetry**
- ❑ **Chiral symmetry**

“Symmetry dictates interaction.” — CN Yang

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❑ **Gauge symmetry:** Longitudinal WGTI

$$iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

❑ **Lorentz symmetry + :** Transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

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Basics: Vertex has DCSB-rendered appearance

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Spacetime

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The WGTIs of the vertices can be decoupled and (partially) solved.

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma\cdot p A(p^2) + B(p^2)}$$

$$\gamma^\mu \Sigma_A + i\sigma_{\mu\nu} Q^\nu \Delta_B + \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

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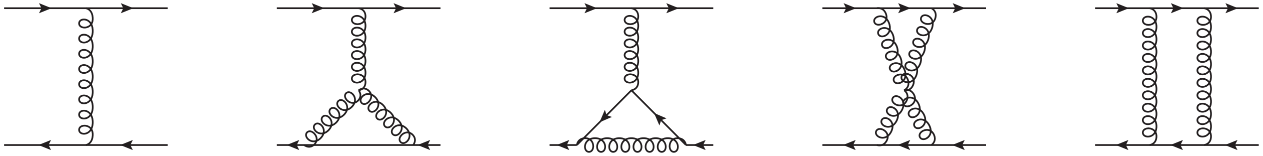
2. The **appearance** of the vertex is dramatically modified by the **dynamics**. The vertex can be phenomenologically expressed as:

$$\Gamma^\mu \sim \gamma^\mu + i\eta\sigma_{\mu\nu} Q^\nu \Delta_B$$

See, e.g., PLB722, 384 (2013)

Basics: Kernel has the Dirac and Pauli terms



+ … = ?

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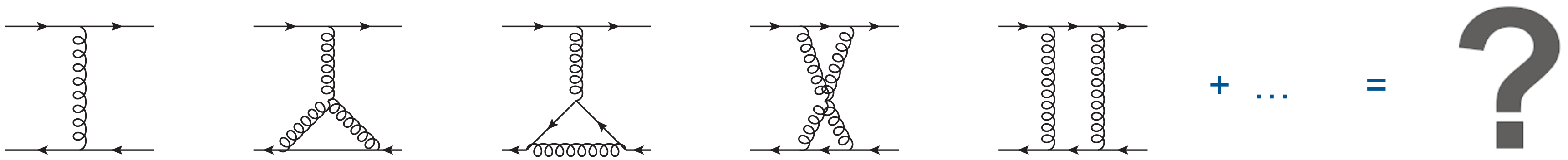

✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry	**Gauge symmetry**
C-, P-, T-symmetry	**Chiral symmetry**

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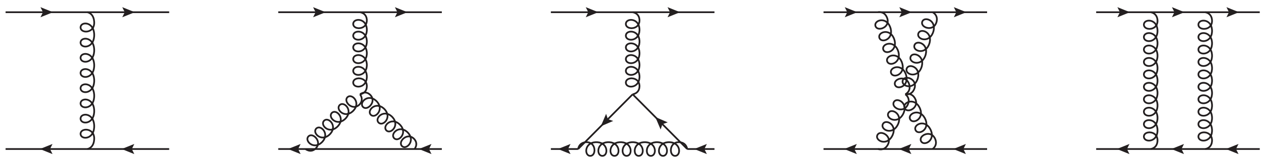

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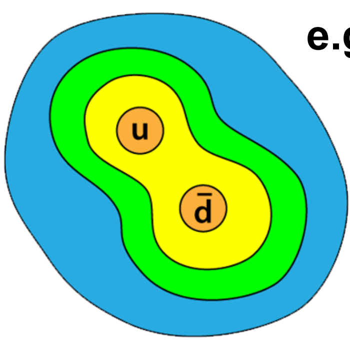

e.g., pion

1. **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

2. **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

Basics: Kernel has the Dirac and Pauli terms

✦ In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$\boxed{\partial_\mu J^\mu = 0} \qquad P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

See, e.g., PLB733, 202 (2014)

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$$\lim_{P\to 0} P_\mu \Gamma_{5\mu}(k,P) = 2i\gamma_5 B(k^2) \neq 0$$

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Pion exists if, and only if, **mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.

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Model independent

Gauge independent

Scheme independent

See, e.g., PLB733, 202 (2014)

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

✦ Deformed WTIs:

$$\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)}\right\} \\
0 &= \int_{dq} \left\{K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(+)}\right\} \\
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\end{aligned}$$

continuous

See, e.g., CPL 38 (2021) 7, 071201

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

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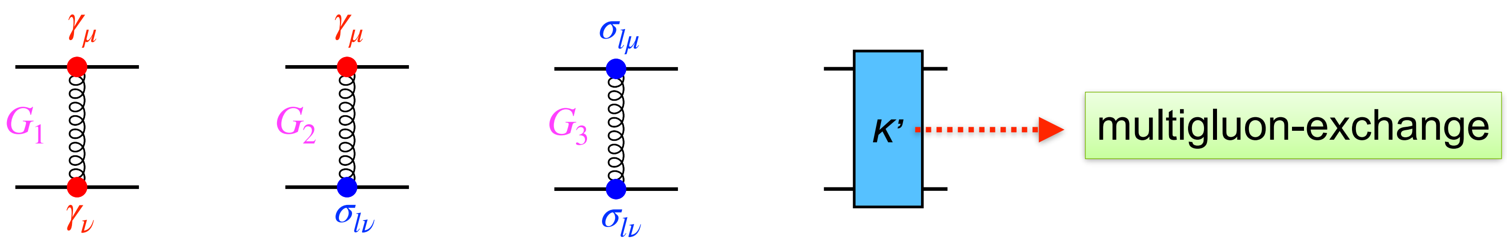
continuous

1. A realistic kernel must involves the Dirac and Pauli structures:



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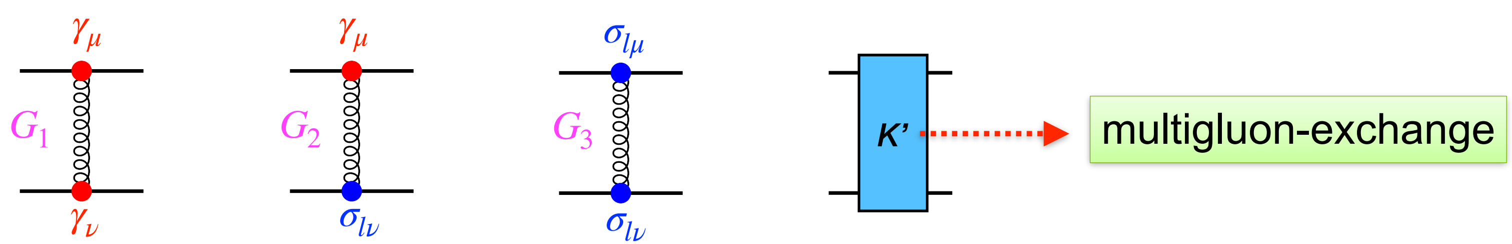


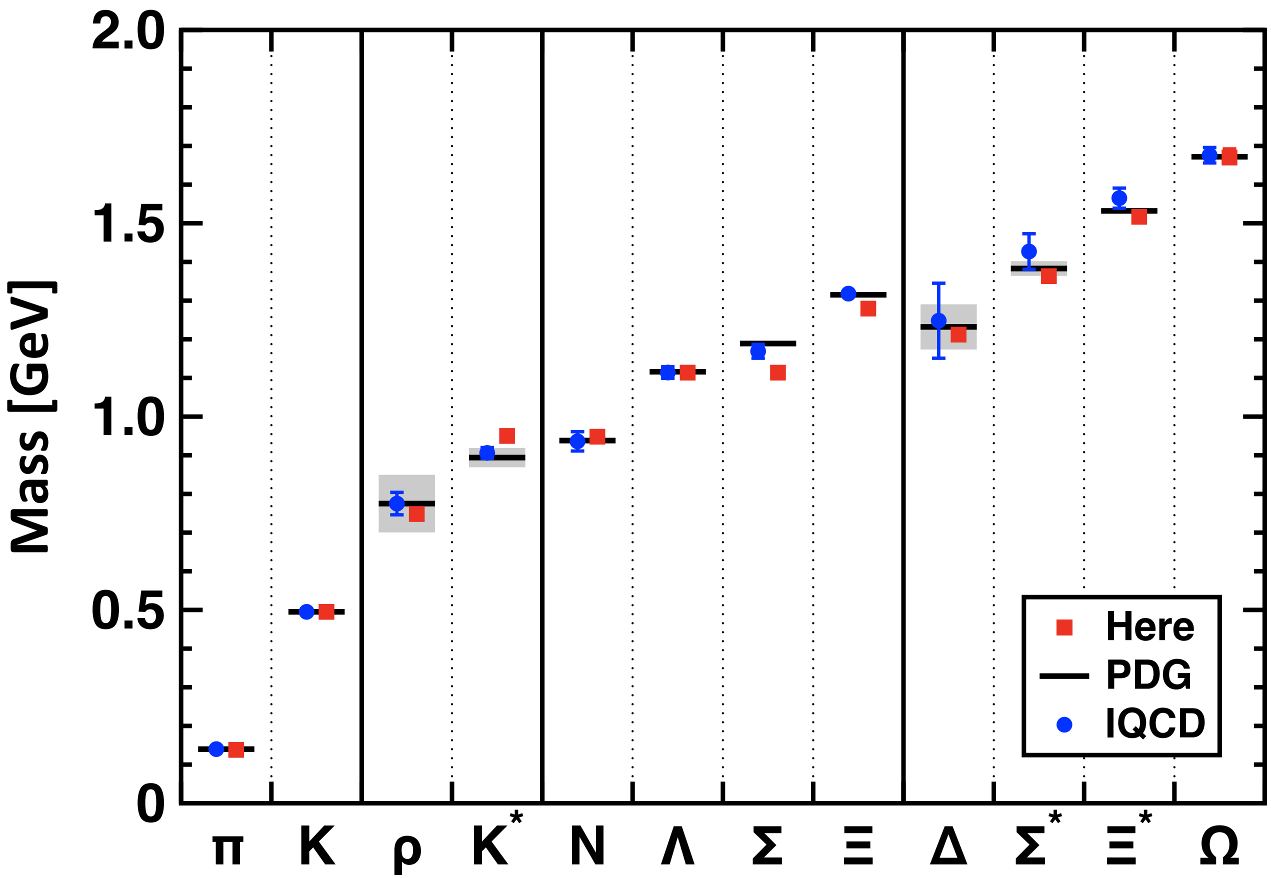
2. G_2 and G_3 are proportional to the Pauli term in the vertex, and thus to **DCSB**:

$$G_2, G_3 \sim \text{DCSB}$$

See, e.g., CPL 38 (2021) 7, 071201

Ground states

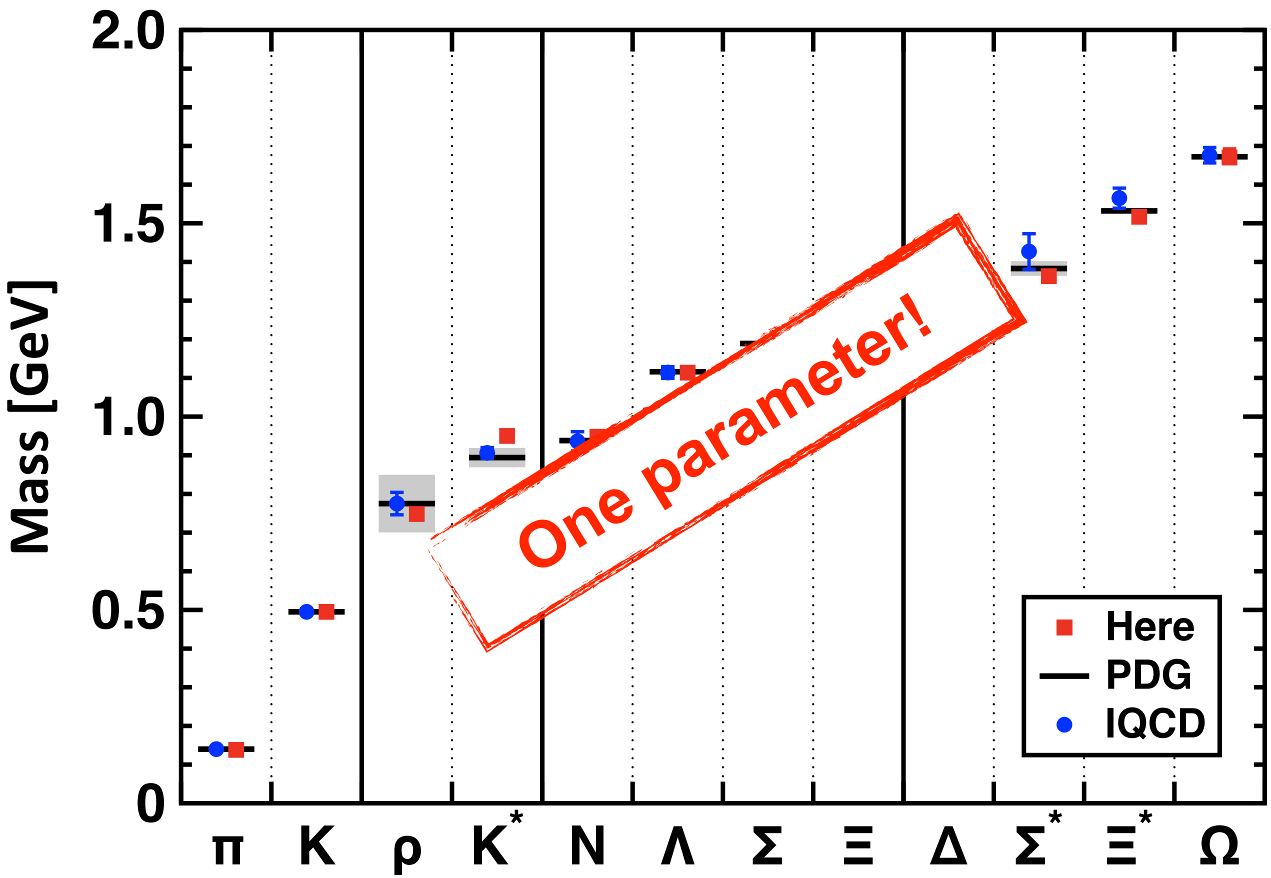
Ground states: Light & Strange flavor spectra



The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Light & Strange flavor spectra



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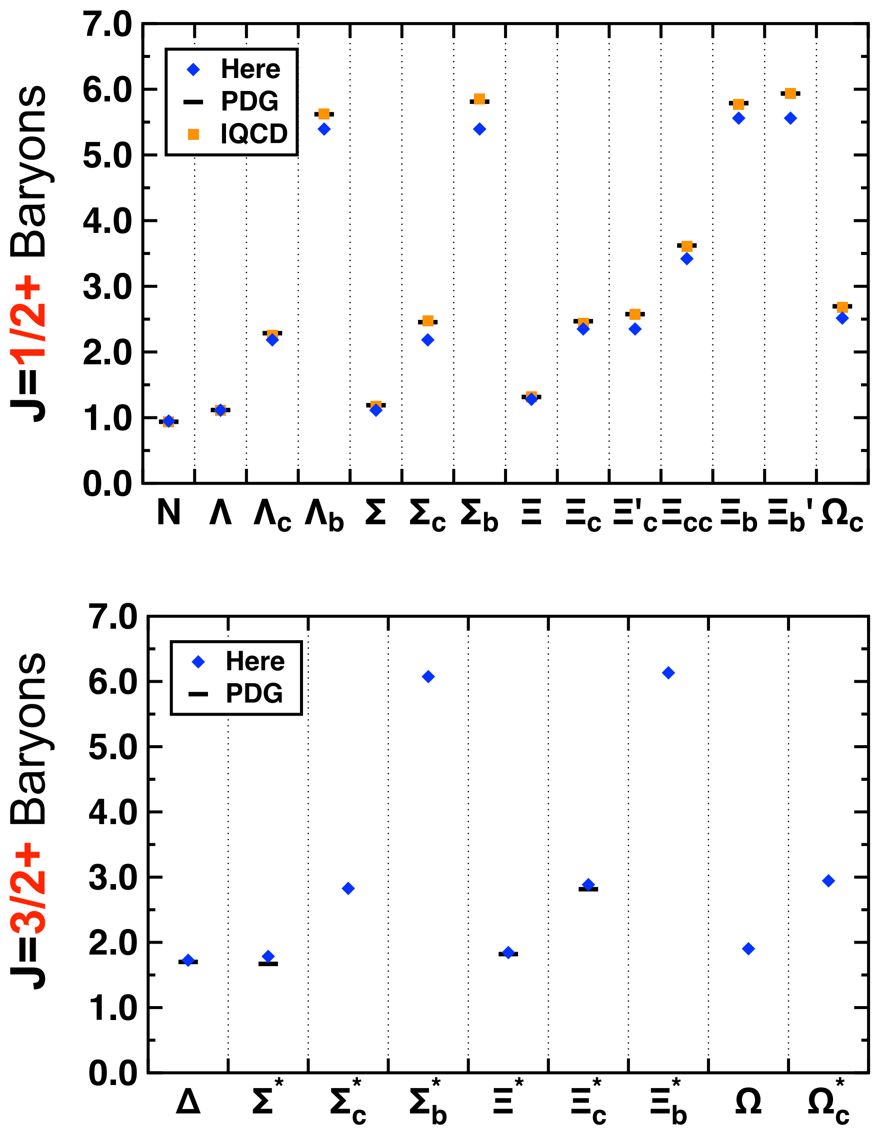
Ground states: Charm & Bottom flavor spectra



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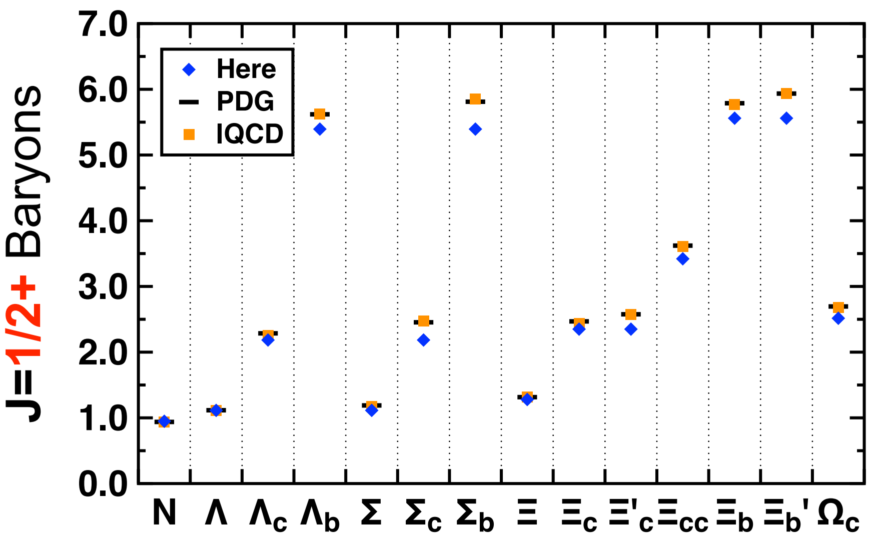
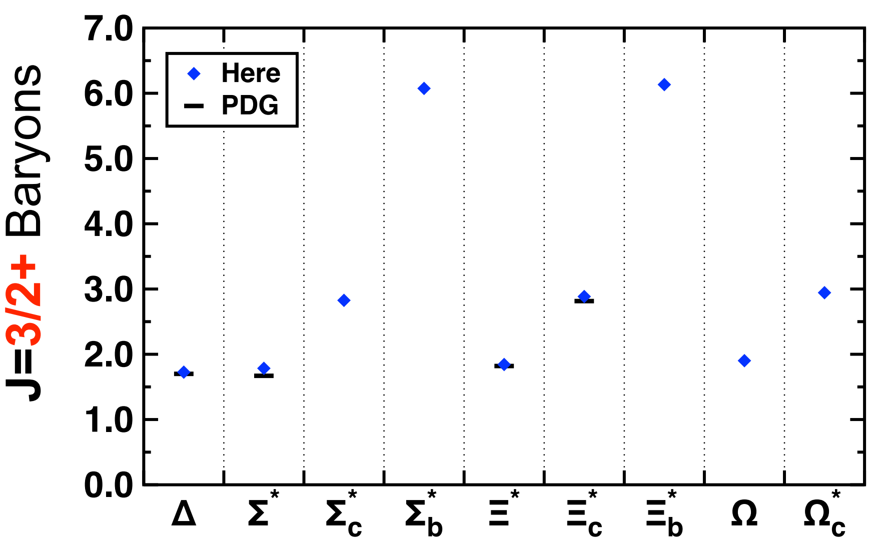
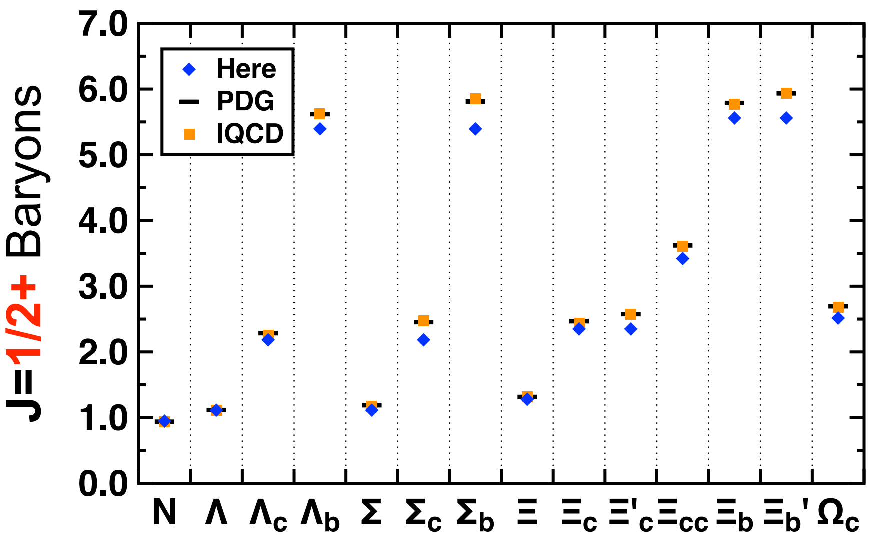


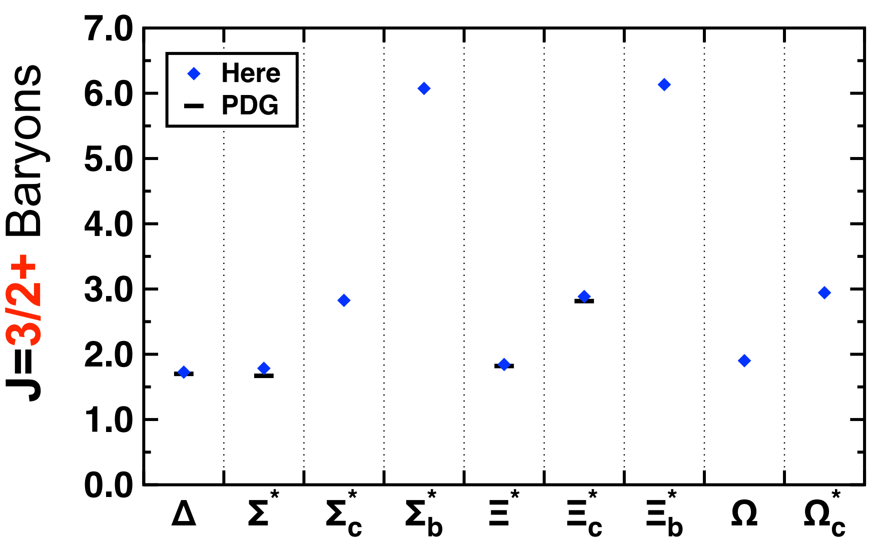


✦ The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Charm & Bottom flavor spectra



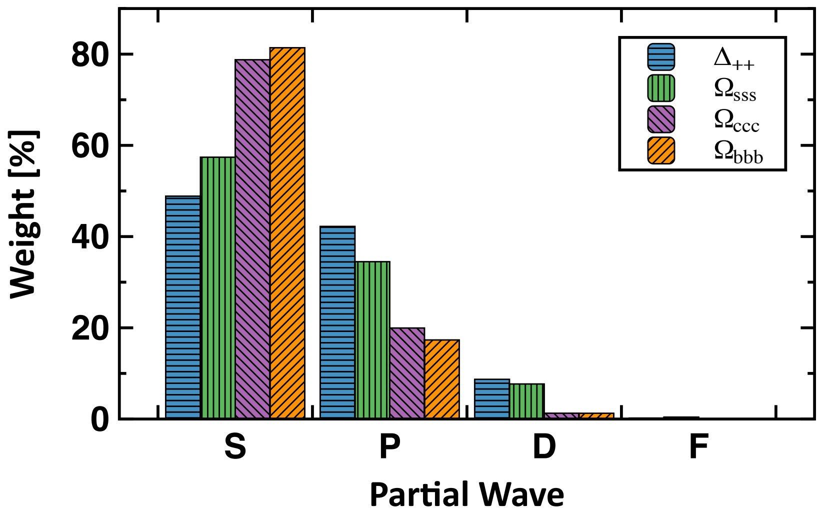


- The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.
- The ground spectra is NOT sensitive to the structures beyond the leading terms in the vertex and the kernel.

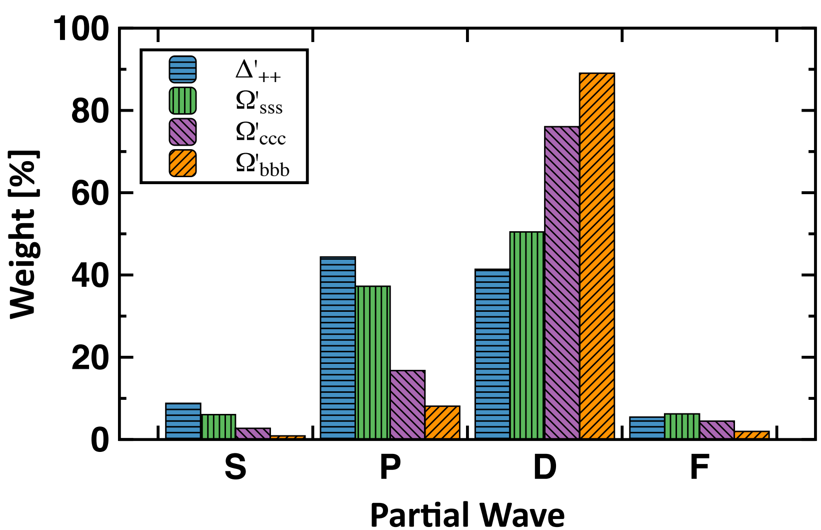
See, e.g., Few-Body Syst 60, 26 (2019)

Excited states

Excited states: Multiple partial waves



✓ S-waves dominate for ground states, but P-waves grow for light baryons.



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See, e.g., PRD 97, 114017 (2018)

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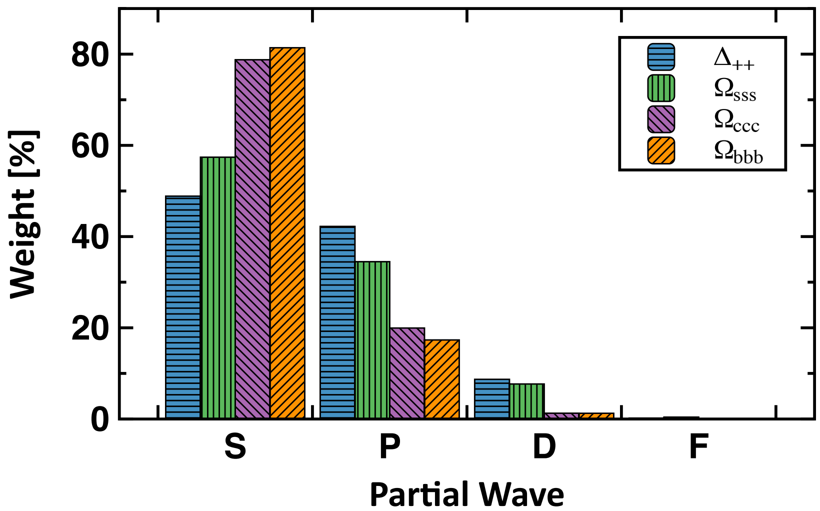

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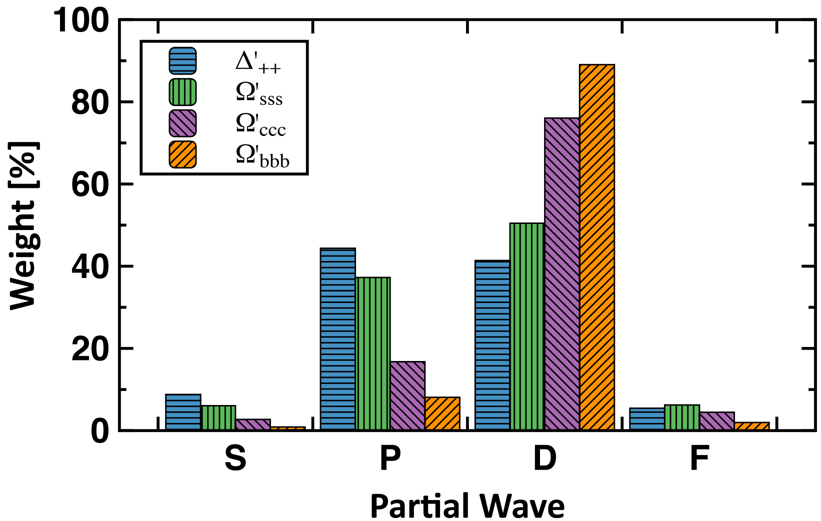
Why NR potential models work



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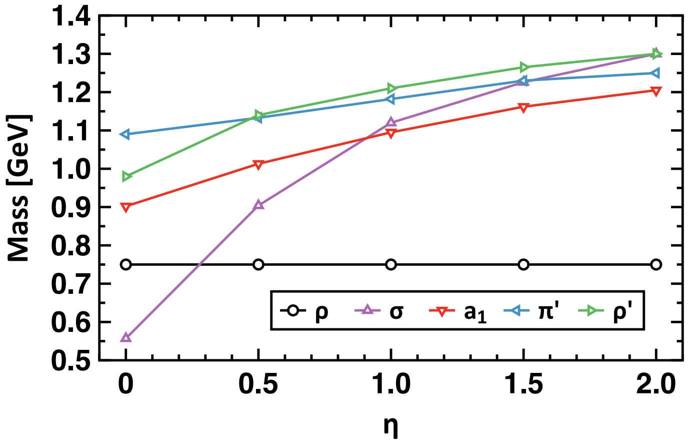
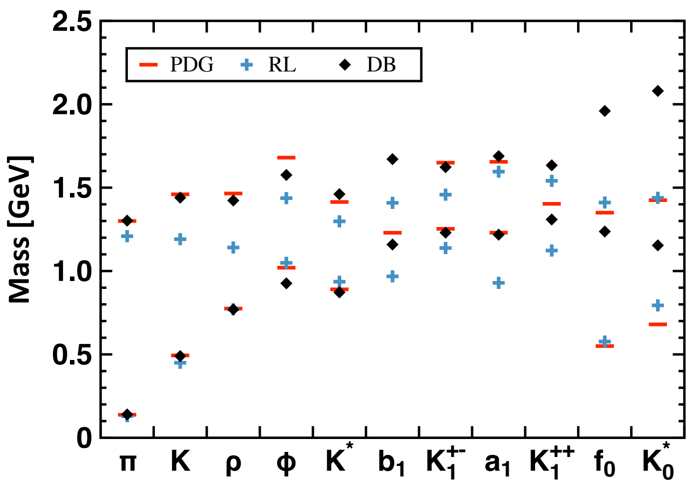
Excited states: Spin-orbit interaction

➡ Impact of the Pauli term (**a**nomalous **m**oment):



➡ Light & strange meson spectrum:



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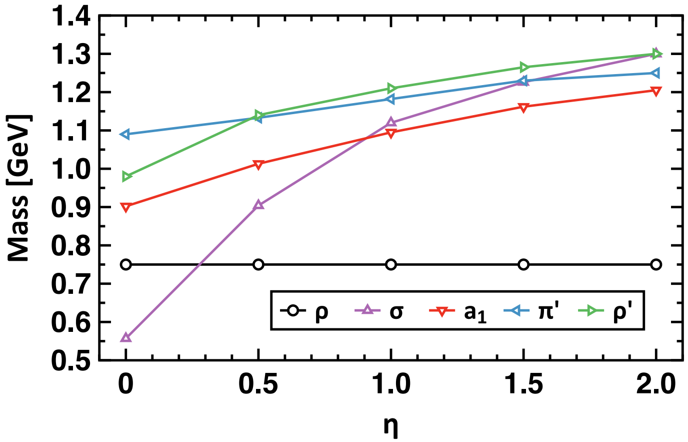

✦ With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

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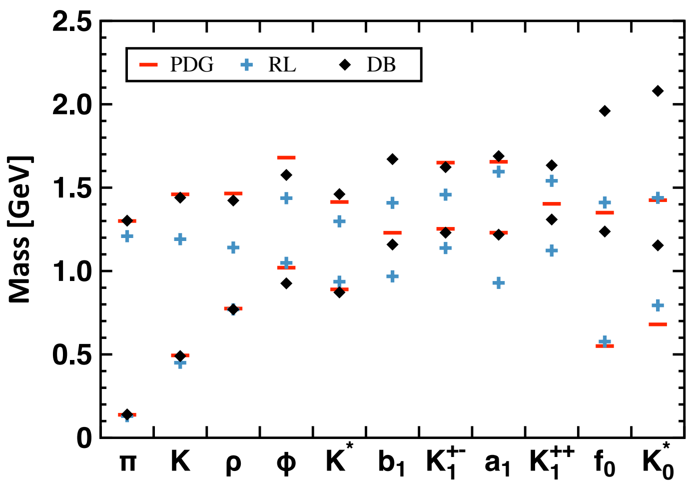

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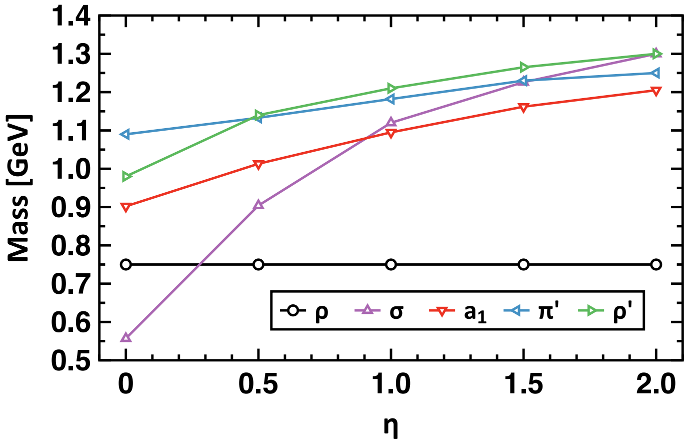

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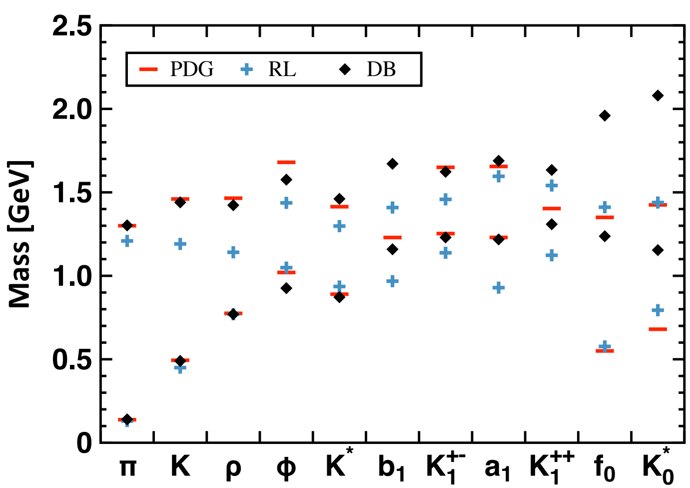
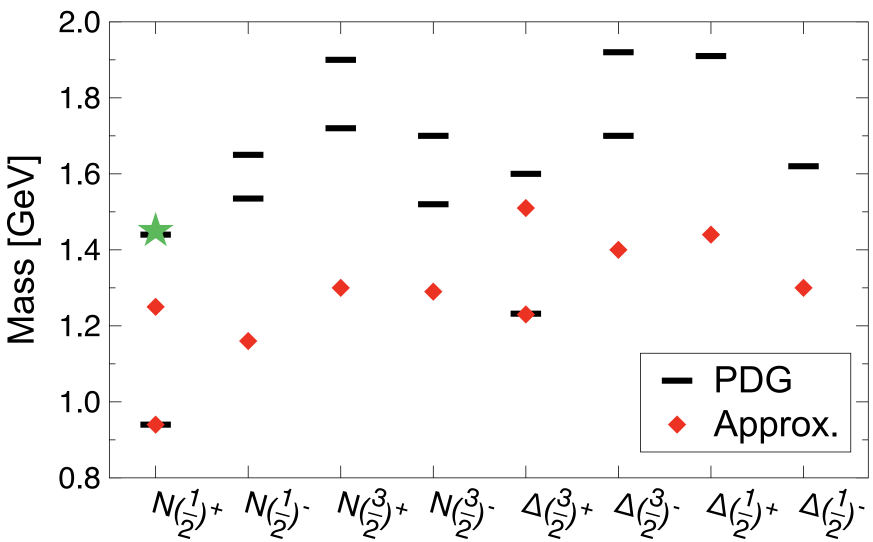
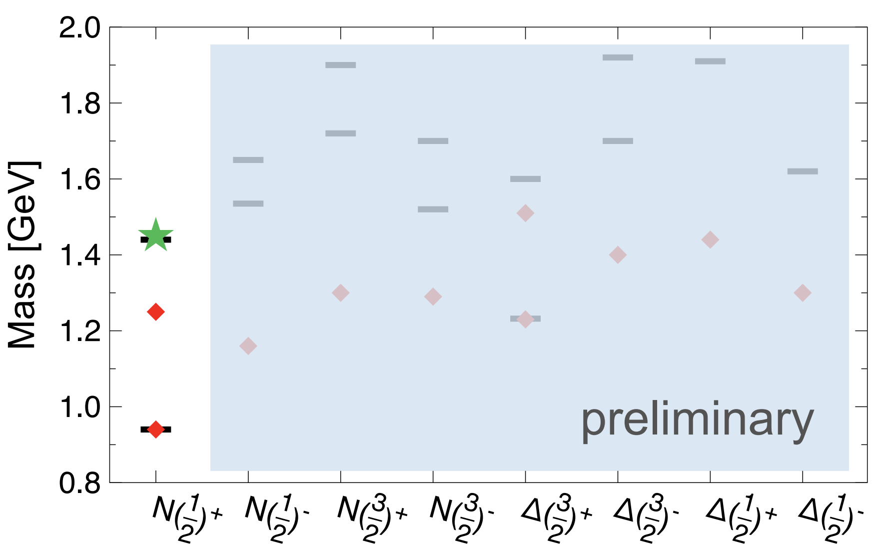


✦ The magnitude and ordering of all excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

Excited states: DCSB-rendered spectra





In progress

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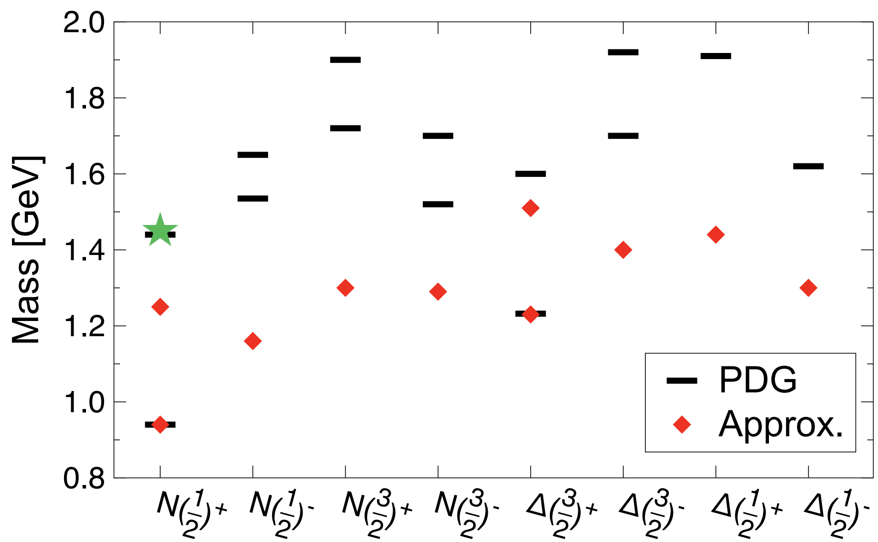
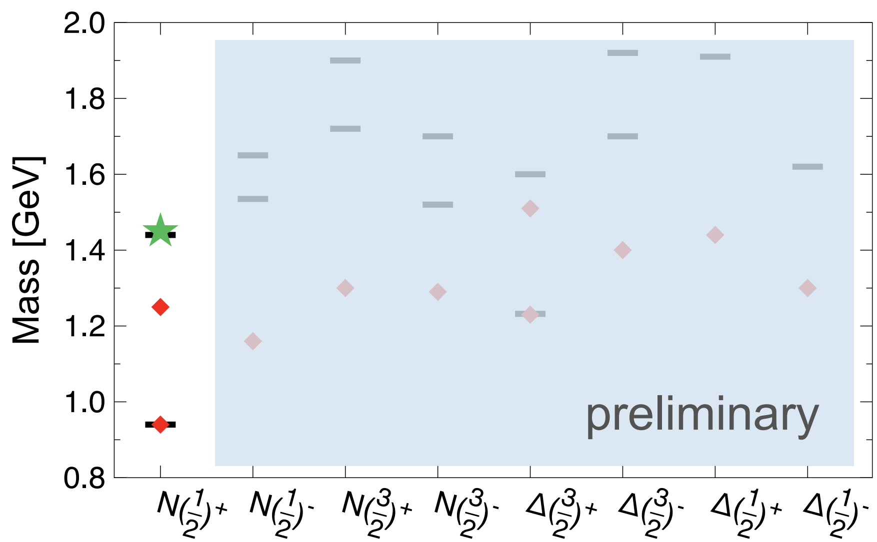
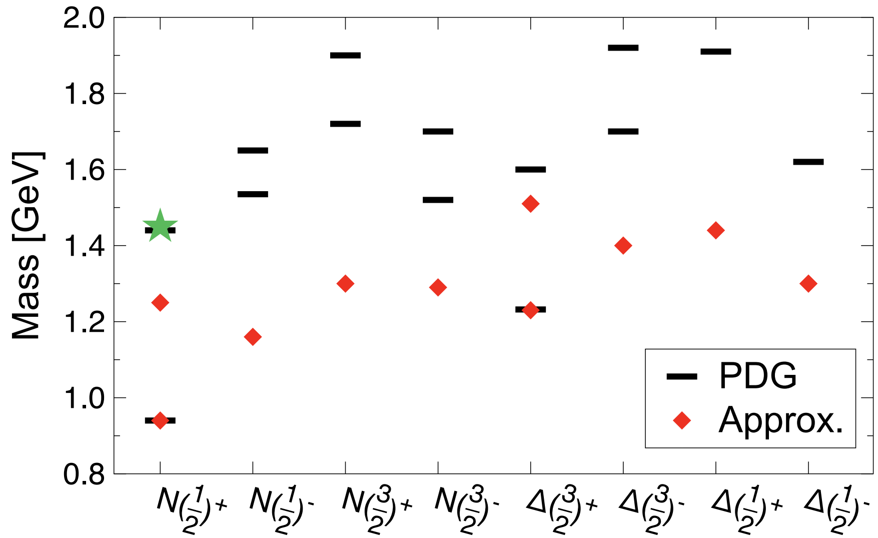




✦ The magnitude and ordering of radial or angular excitation states are WRONG in the approximation lacking of DCSB effect.

In progress

Excited states: DCSB-rendered spectra



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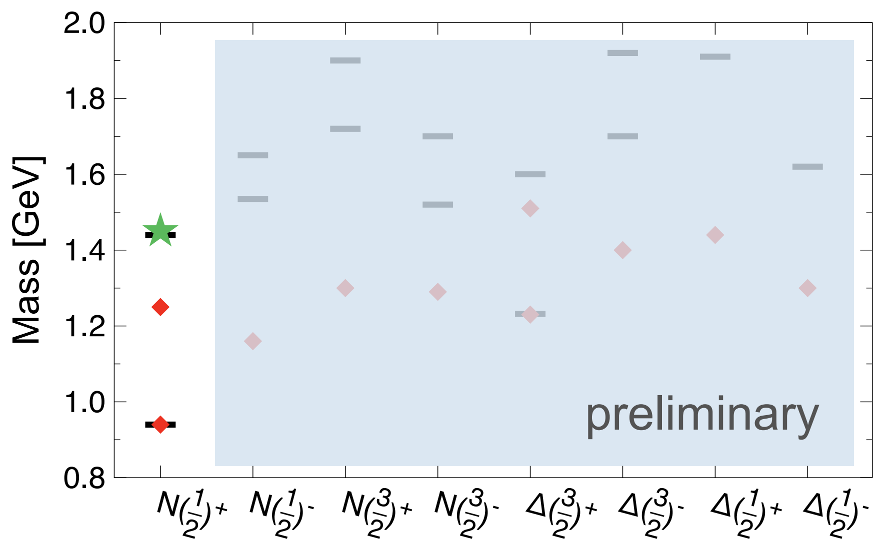

- The DCSB-enhanced kernel boost up 1st excitation nucleon, and can potentially fix the full spectra.

In progress

Summary

✦ The framework of non-perturbative Dyson-Schwinger equations, which describes hadrons in continuum QCD, and its basics (e.g., quark, gluon, vertex, kernel) are introduced.

✦ Baryon properties are studied: **a**) ground states — full mass spectrum of J=0, 1/2, 1, 3/2; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Summary

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- Baryon properties are studied: **a**) ground states — full mass spectrum of J=0, 1/2, 1, 3/2; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Outlook

- Use the three-body Faddeev equation to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.

- Hopefully, iterating with future high precision experiments on **light** and **heavy** hadrons, from spectroscopy to structures, we may provide a **faithful path** to understand **QCD**.