Gravitational waves from phase transitions

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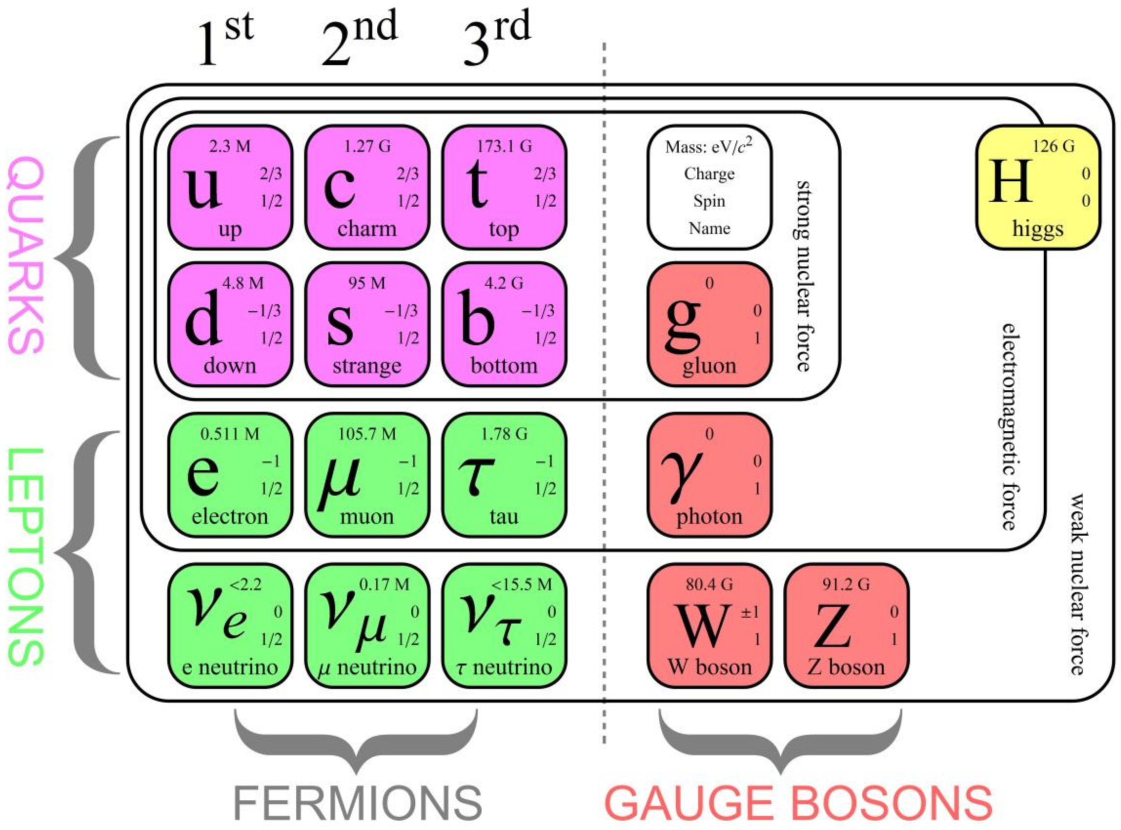
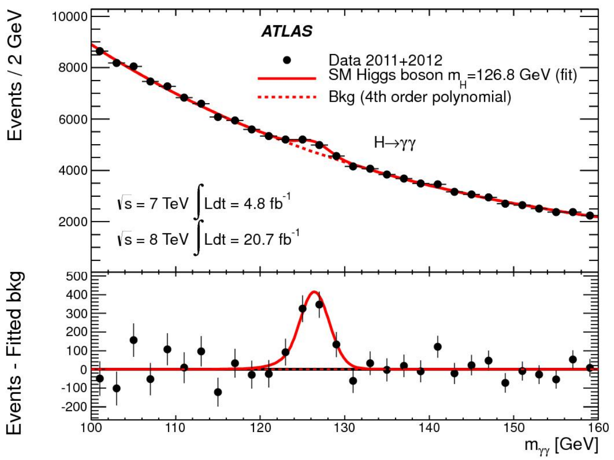
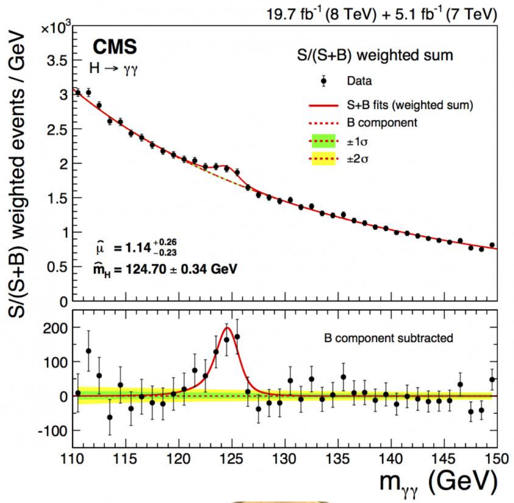
2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou
2208.14857 w/ Xi Tong and Siyi Zhou
2304.02361 w/ Chen Yang
2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang
2409.05833 w/ Qi Chen, Yuan Yin
2410.XXXXX w/ Qi Chen, Yuhang Li, Yuan Yin

The two Standard Models







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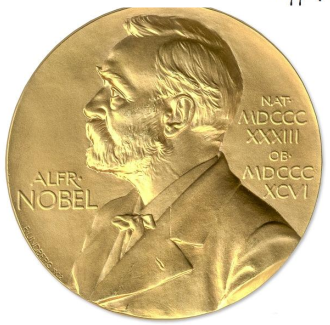
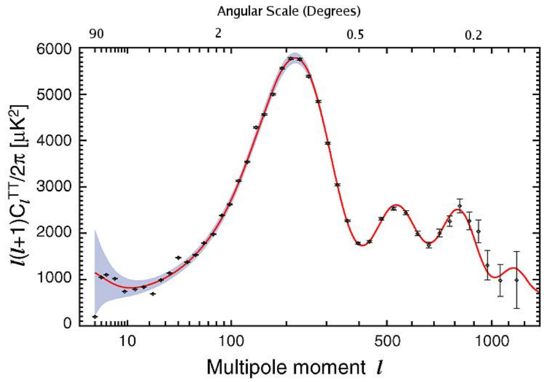
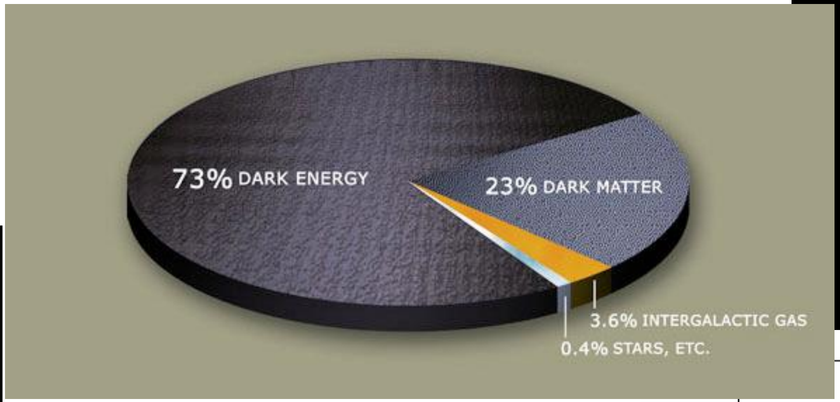
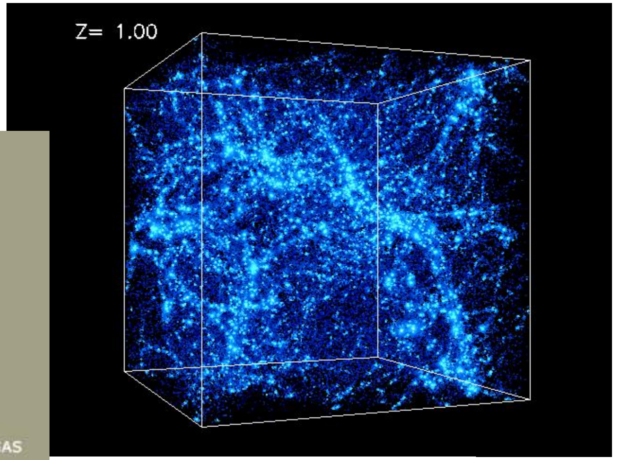
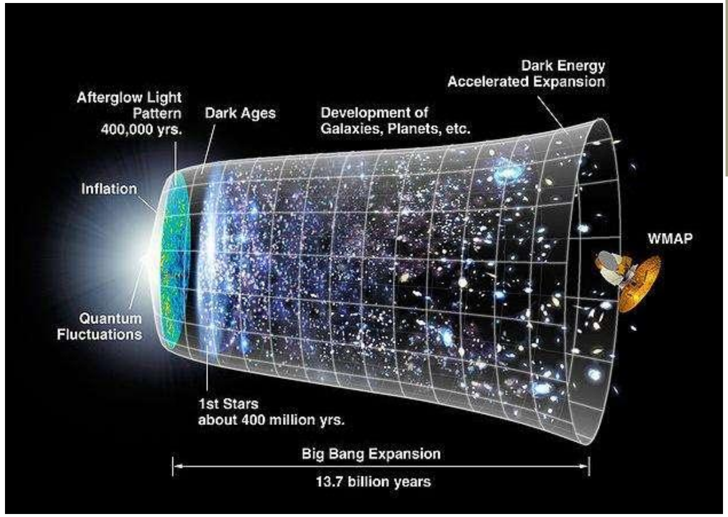


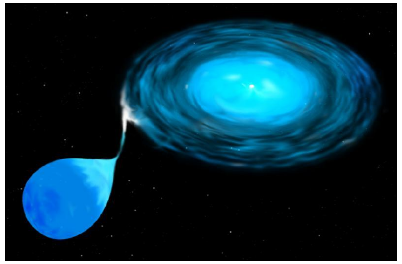
The two Standard Models

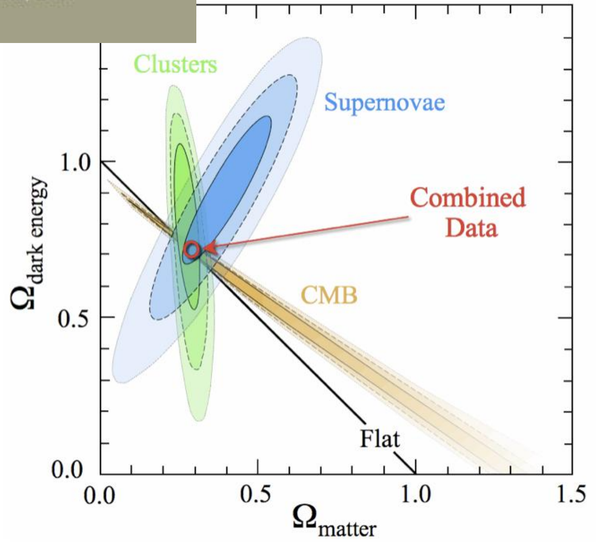






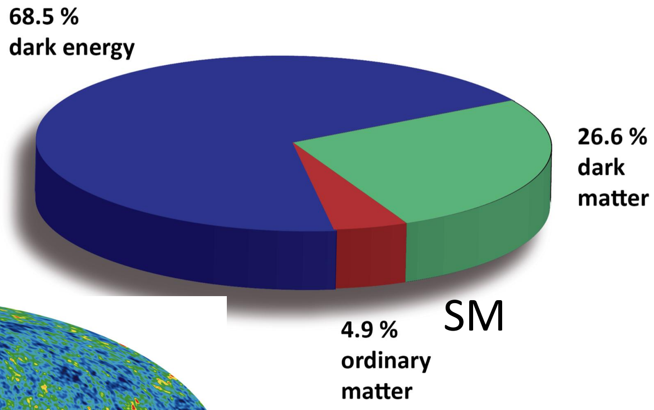


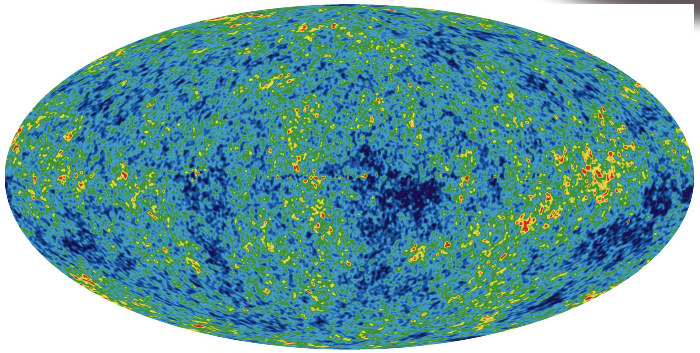


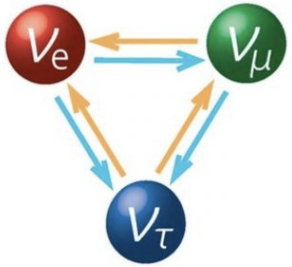


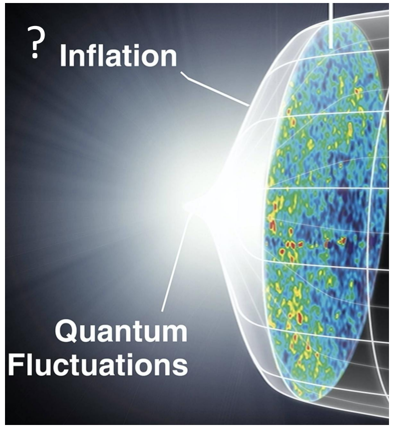
The big problems in particle phenomenology

- What is the particle nature of dark matter?
- What is dark energy?
- What is the mechanism for baryogenesis?
- What is the theory for neutrino masses?
- What is the origin of the large scale structure?
- Had the Universe experienced inflation? Any alternative scenarios?

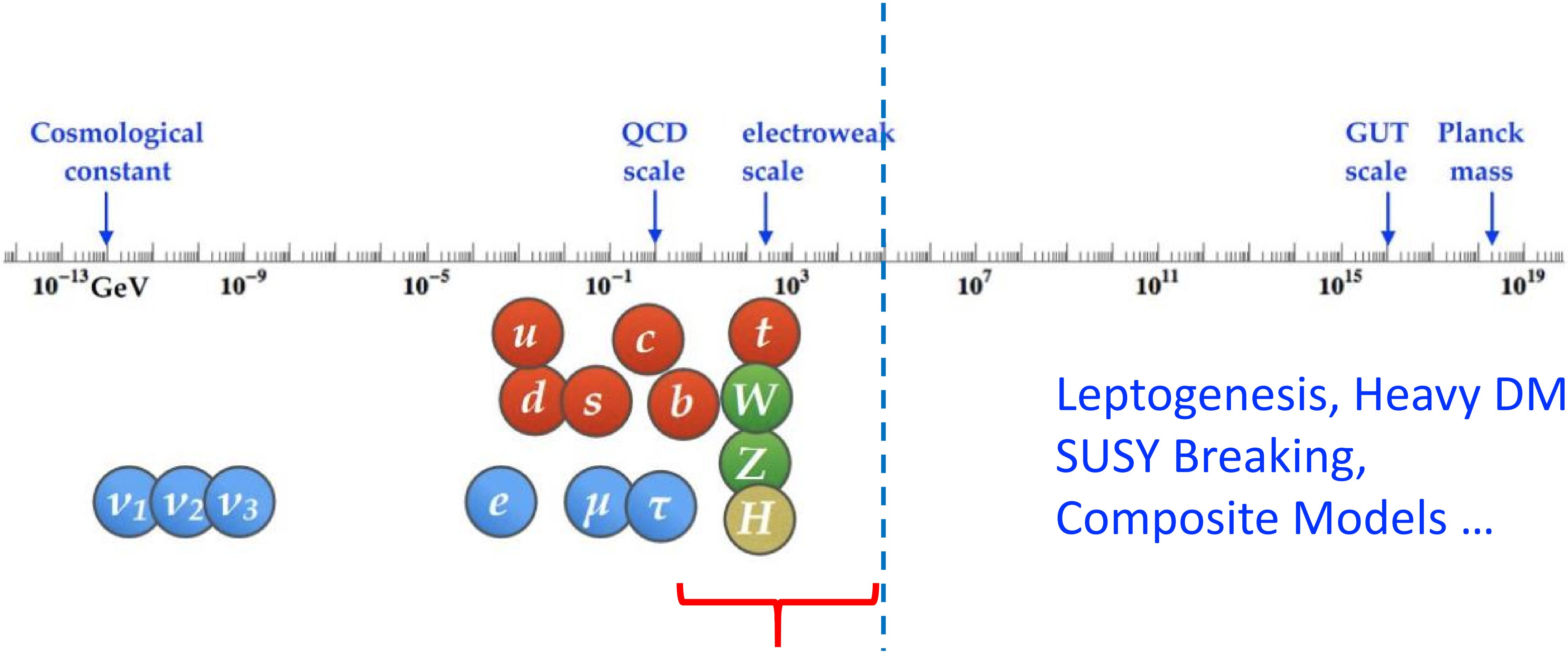








The scales of particle physics

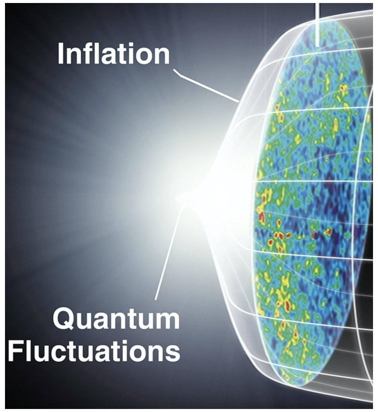


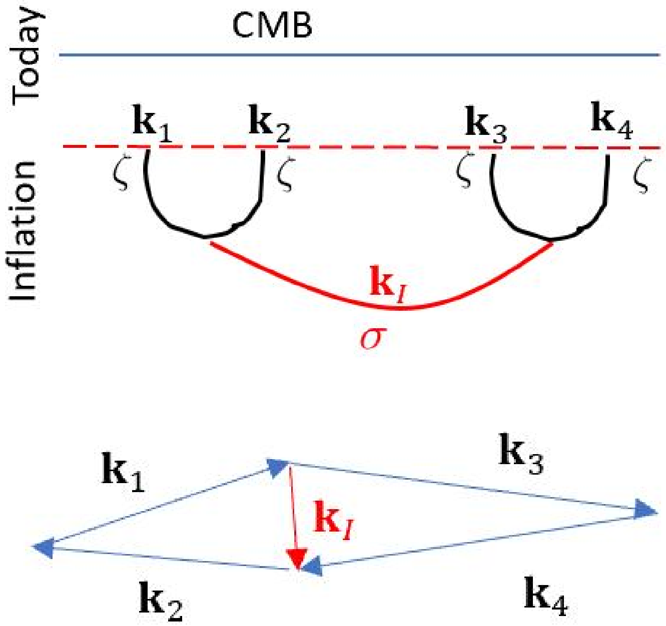
Leptogenesis, Heavy DM,
SUSY Breaking,
Composite Models …

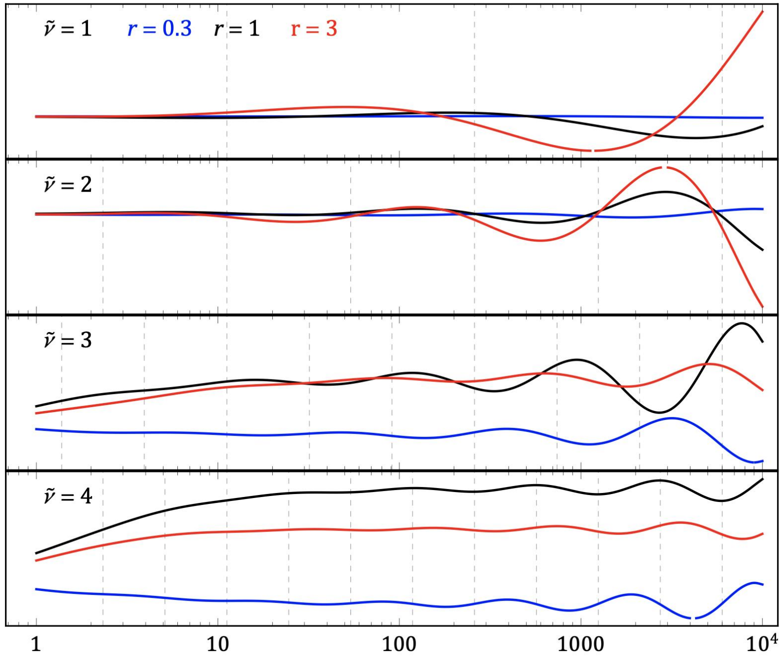
LEP, TEVATRON, LHC
and Future Colliders

Early Universe as ultra high energy particle physics labs

- The large scale structure of the Universe (Cosmological Collider Physics) Chen, Wang, … Arkani-Hamed, Maldacena, … Xianyu, …

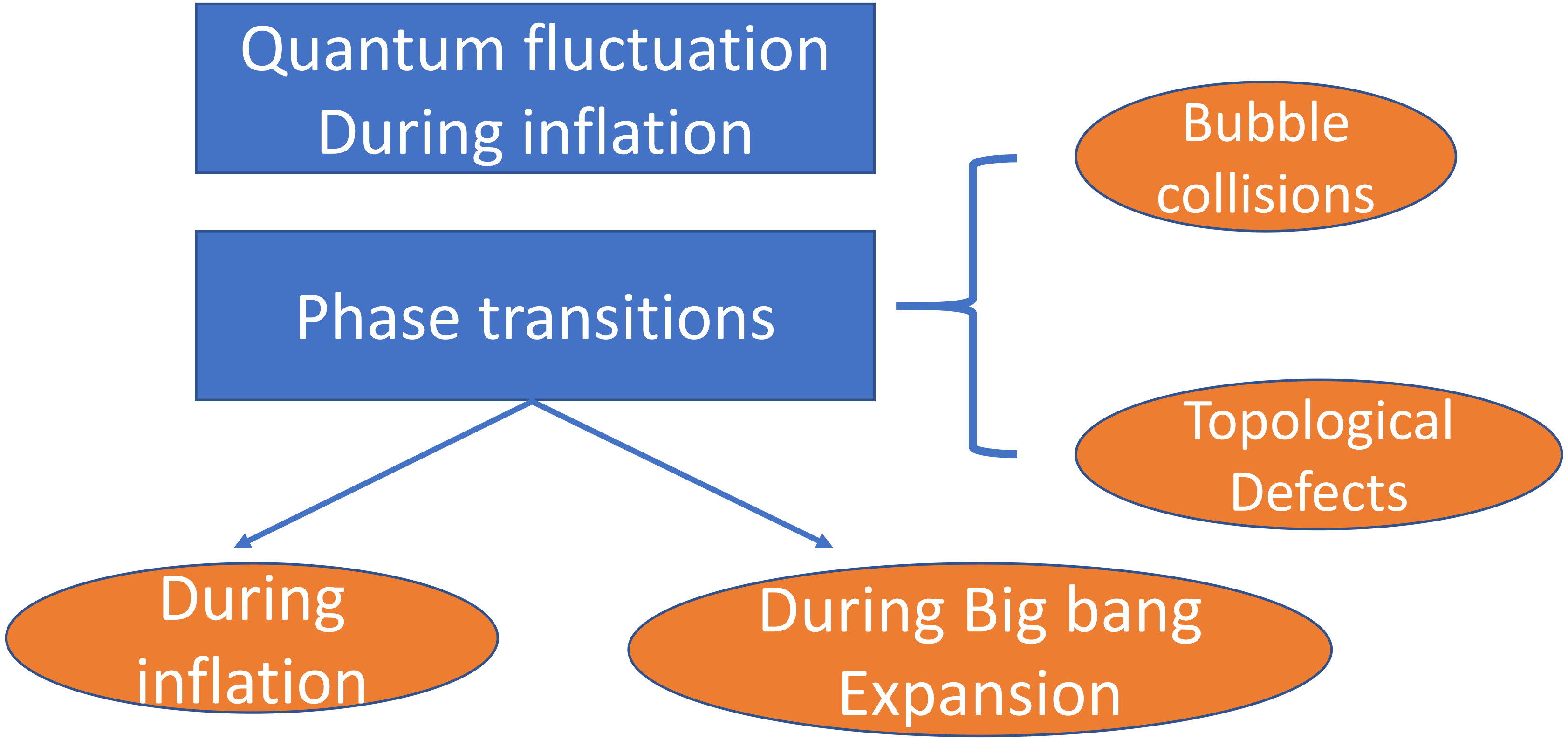


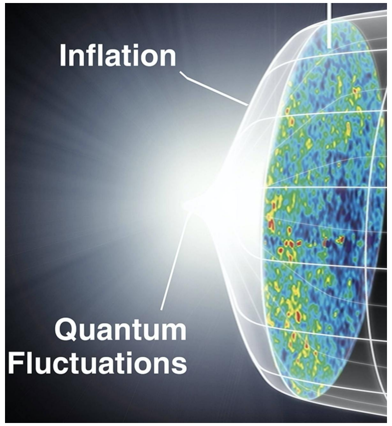


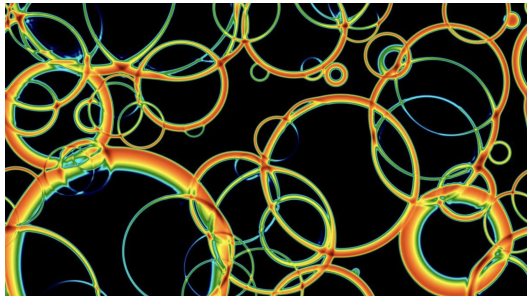


Early Universe as ultra high energy particle physics labs

- Stochastic primordial gravitational waves







Early Universe as ultra high energy particle physics labs

- Stochastic primordial gravitational waves

Quantum fluctuation
During inflation

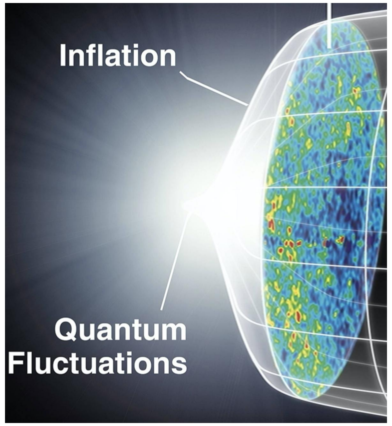
Phase transitions

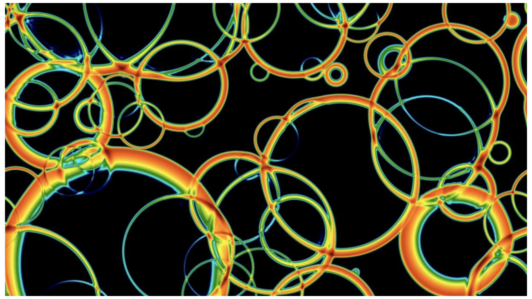
Bubble
collisions

Topological
Defects

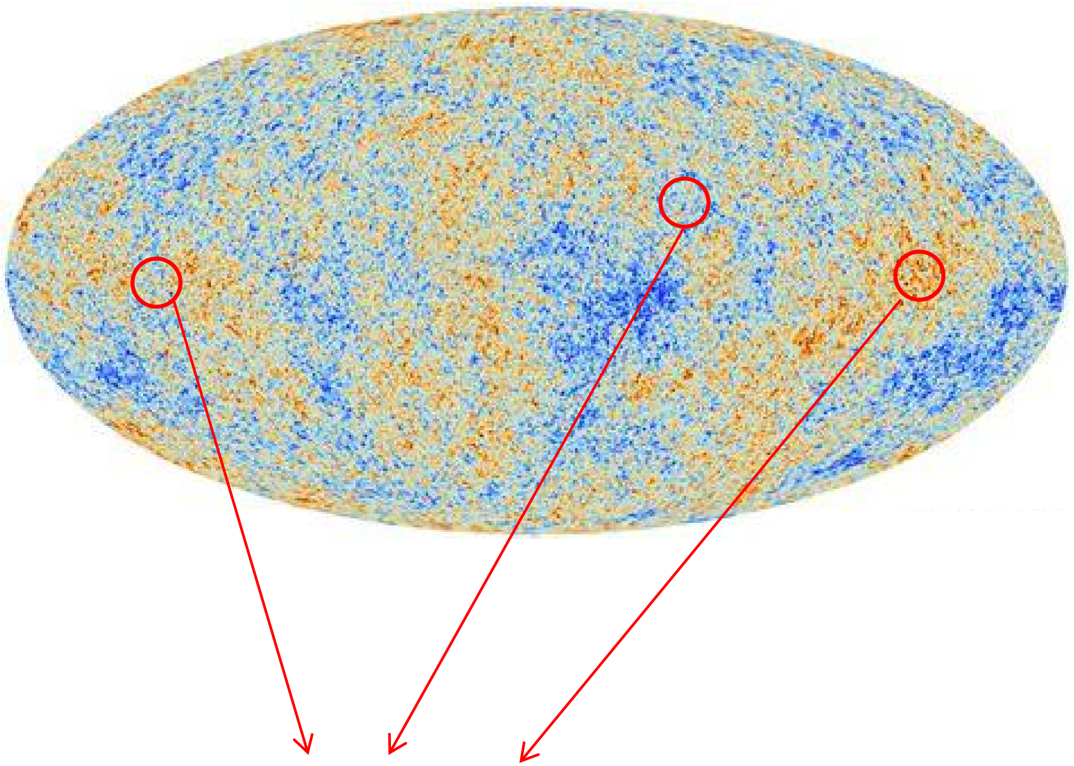
During
inflation

During Big bang
Expansion

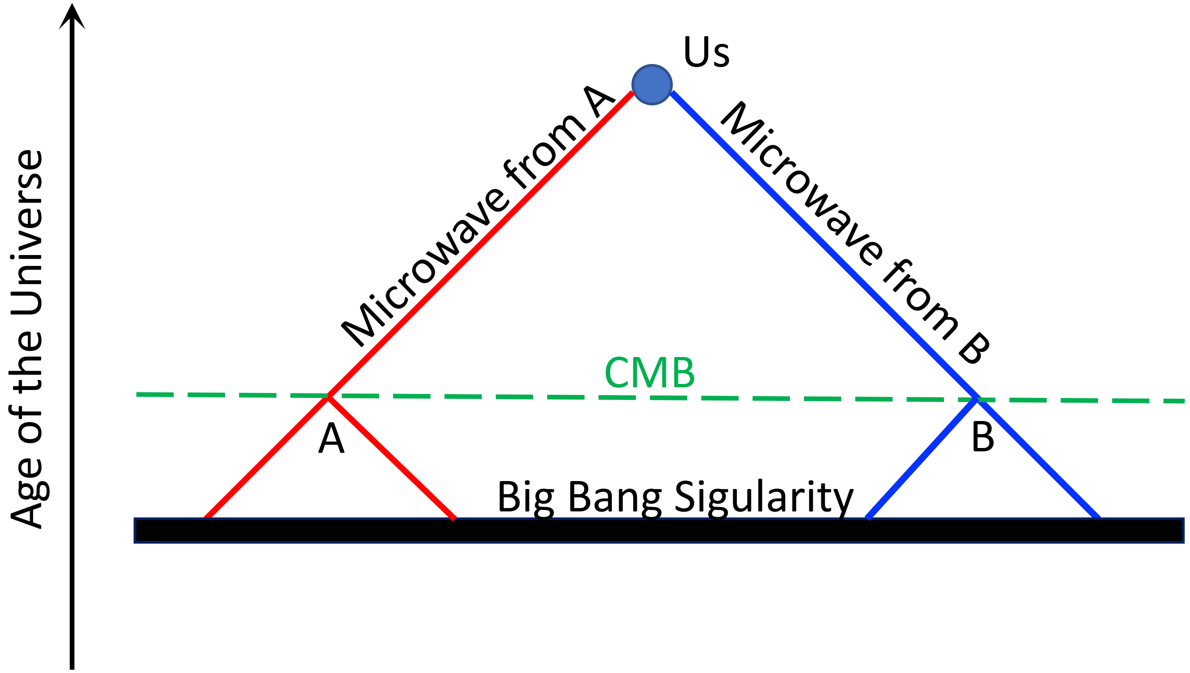




The causality problem

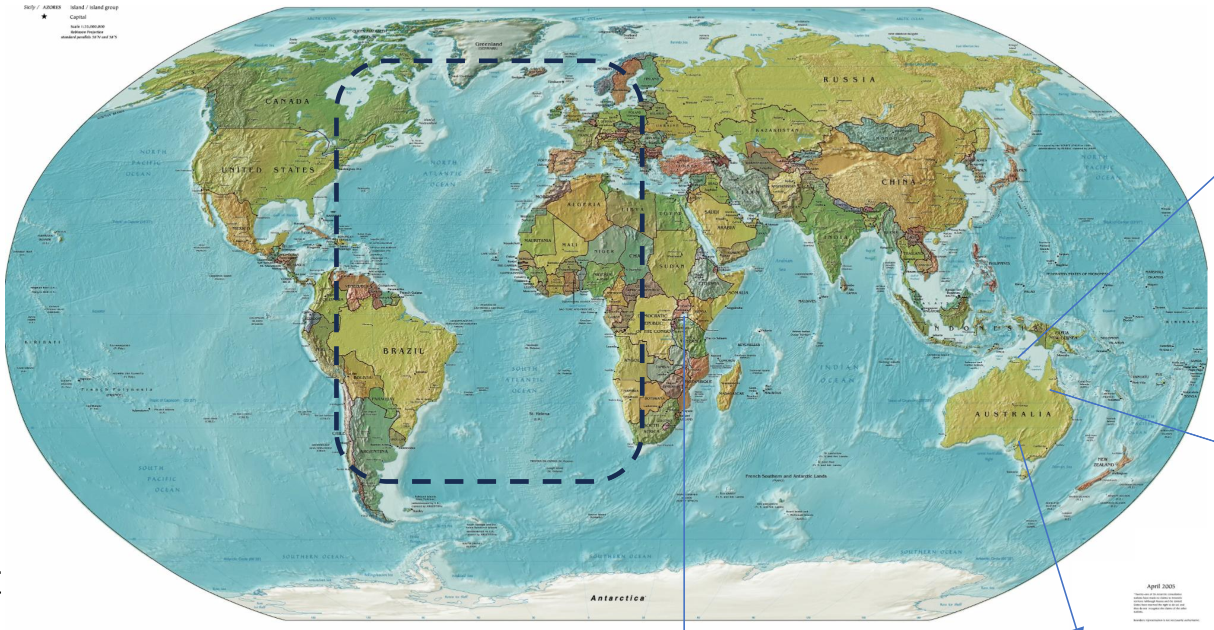


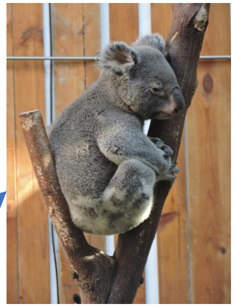
Same temperature and similar fluctuations.



Causality problems usually indicate big discoveries!

Alfred Wegener: Continental drift hypothesis





Animals with brood pouch

育儿袋

ostrich

emu

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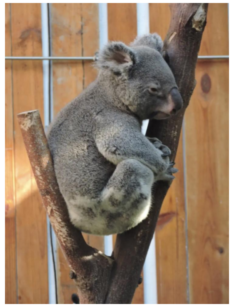

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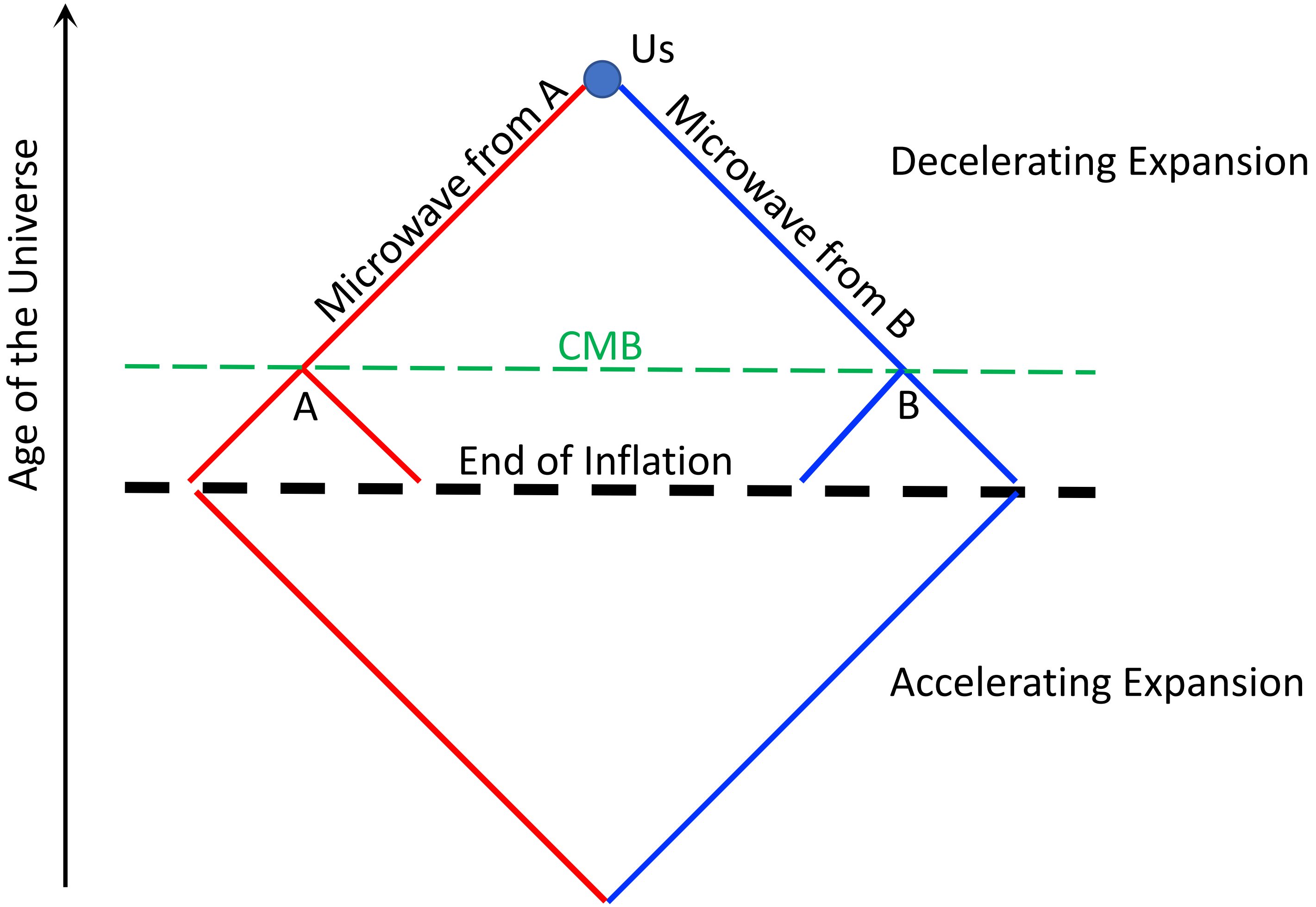
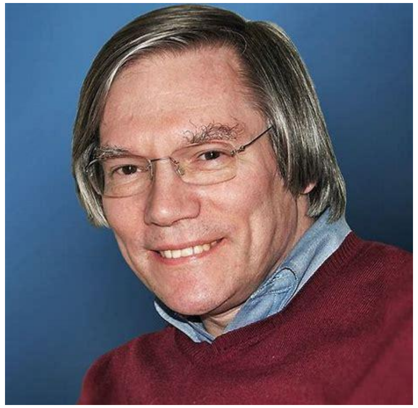
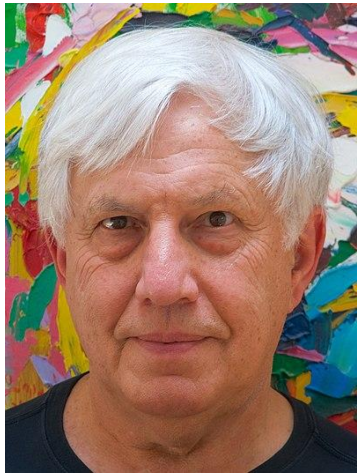
Inflation theory

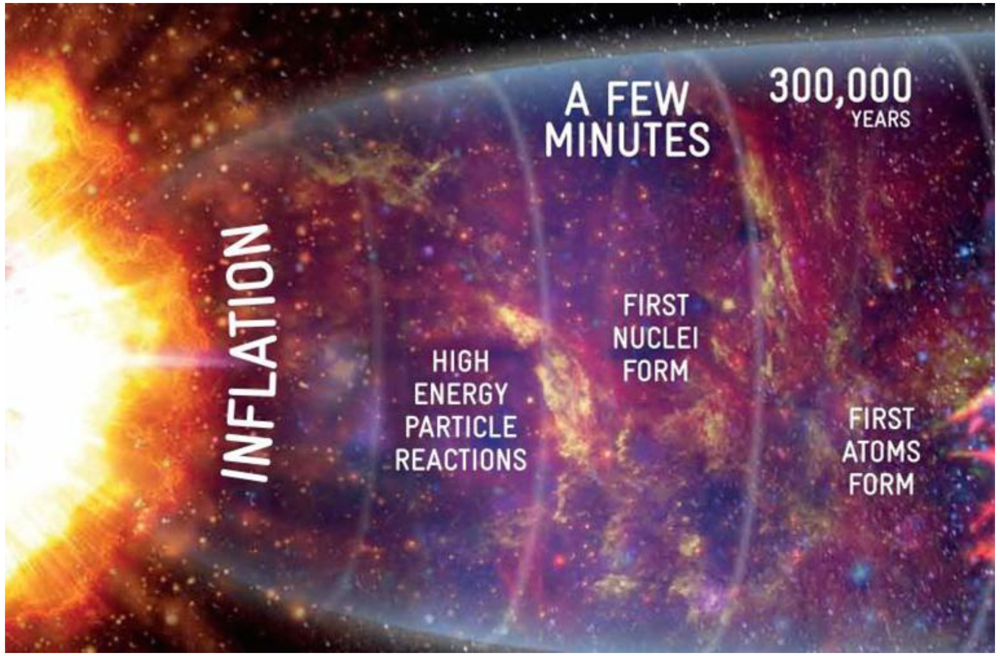




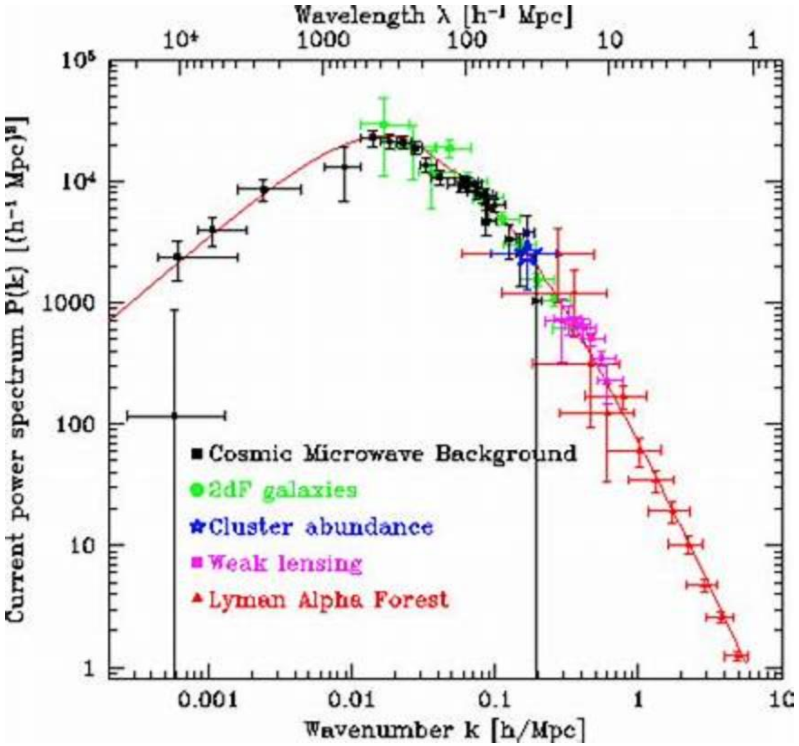


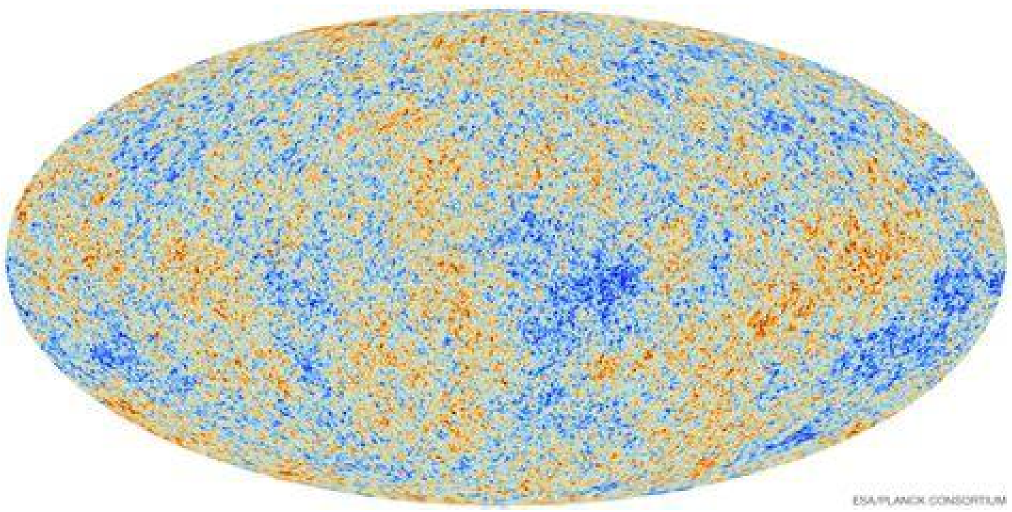
Motivations for inflation

1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure



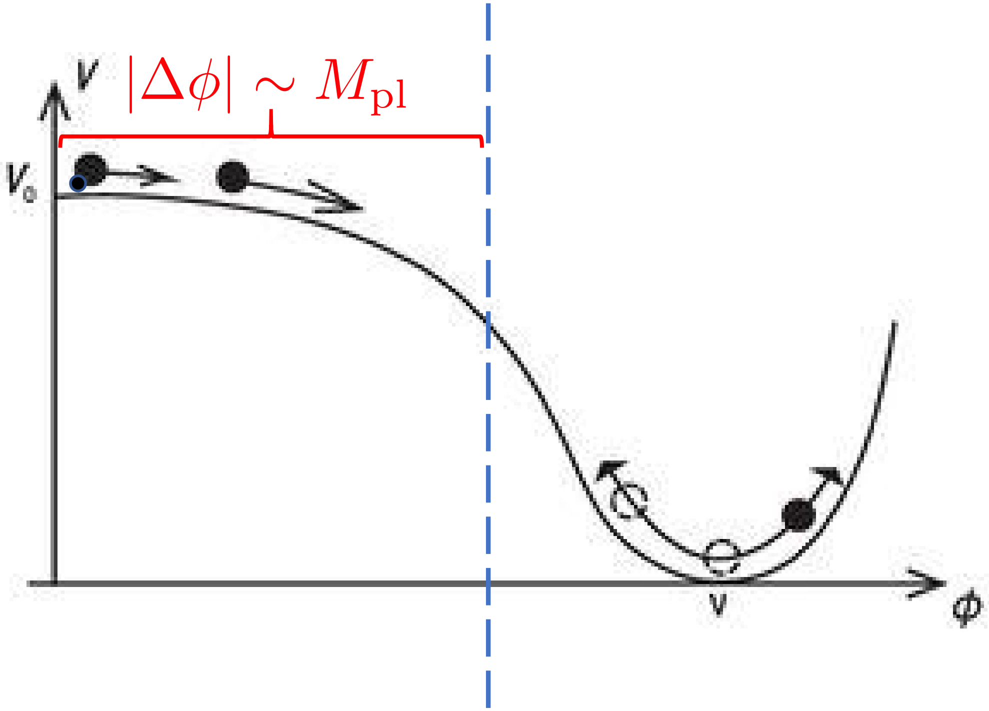
Current status of measurement

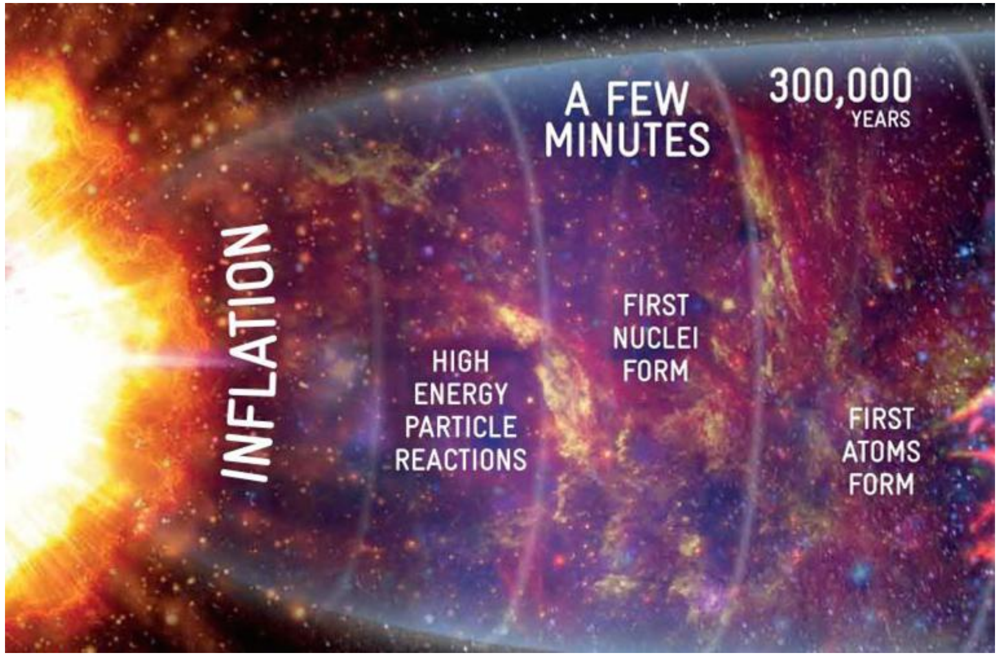




- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll inflation





To generate enough e-folds, the excursion of the inflaton field must be very large, **comparable or even larger than M_{pl}.**

Evolutions in the early universe

- Inflation: ϕ coupled to spectator sectors $f(\phi)g(\sigma)$

$f(\phi)$

Grand Unification | Baryogenesis | SUSY Breaking | Dark sector Phase transitions | …

- Thermal expansion: temperature coupled to SM sector $T^2|H^2|$

T

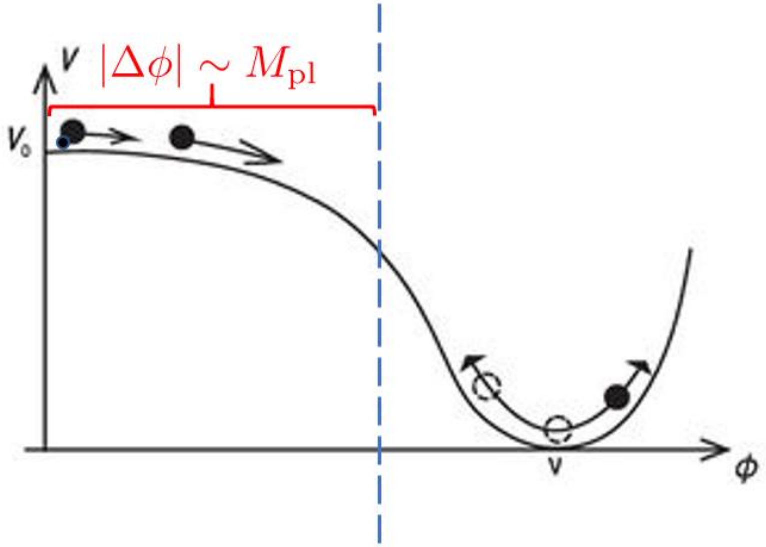
T_{RH} | T_{EW} | T_{QCD}

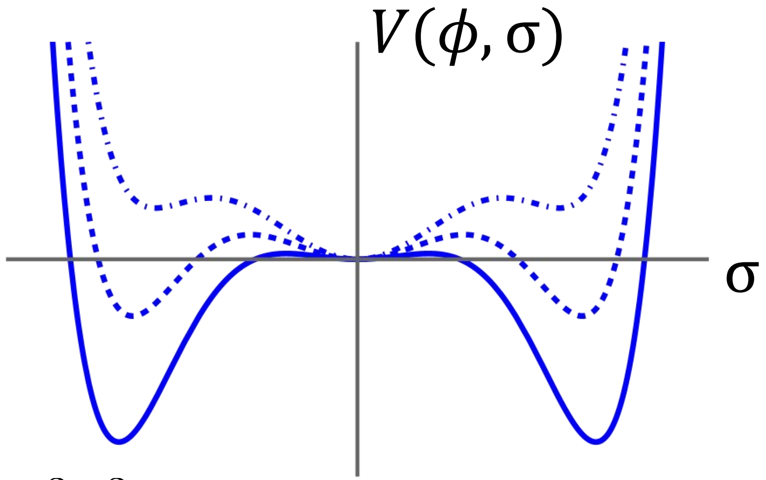
Phase transitions in spectator sector triggered by the evolution of the inflaton field

- ϕ: inflaton field $\qquad$ σ: spectator field

Example 1:

$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

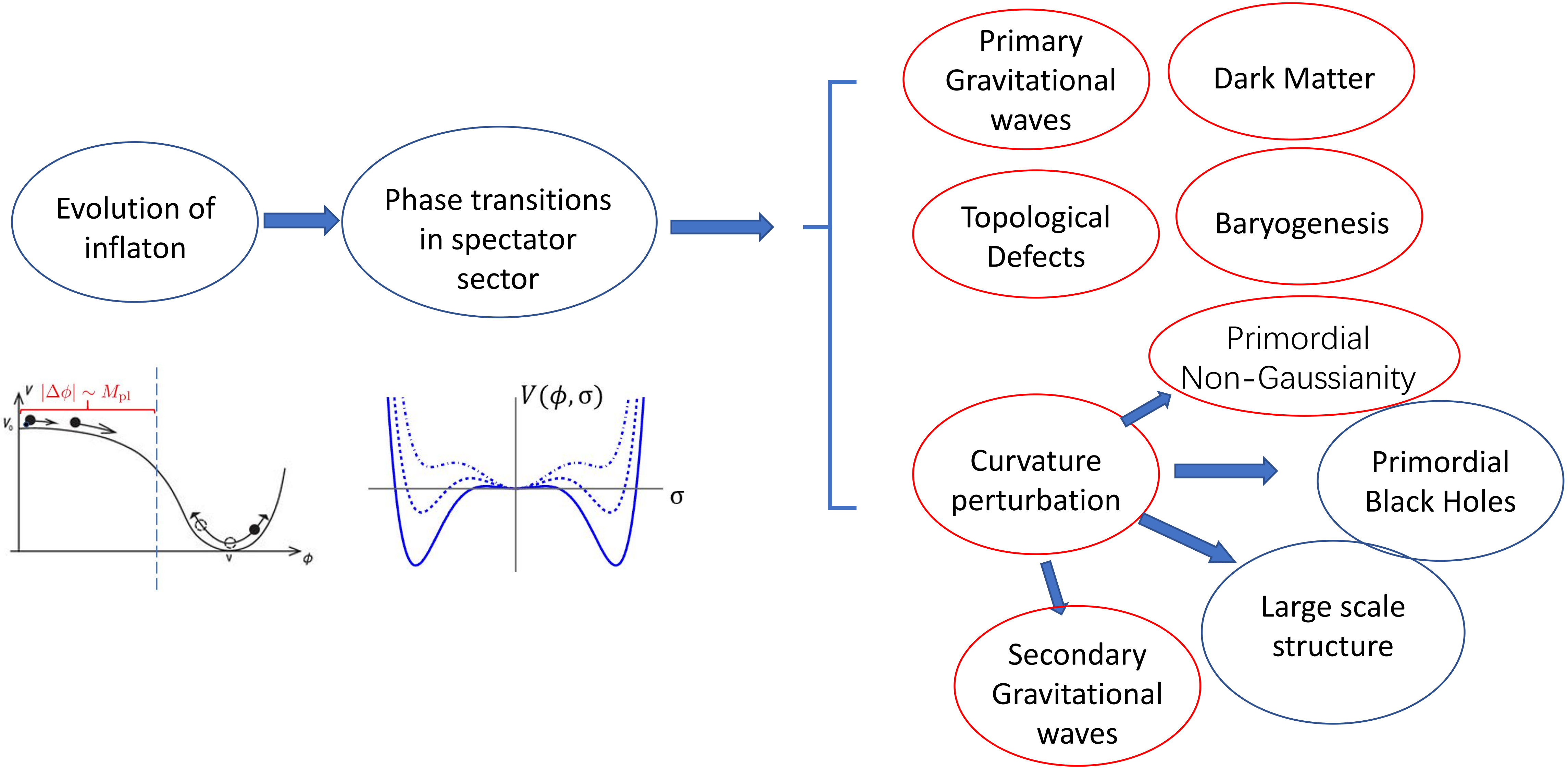




Example 2:

$$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

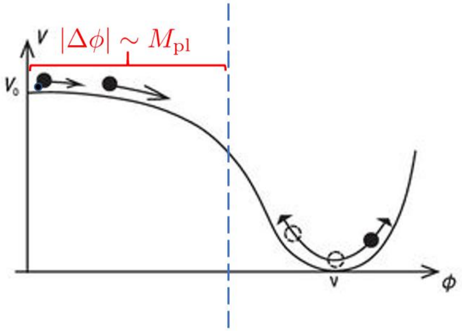
Consequences of the phase transitions

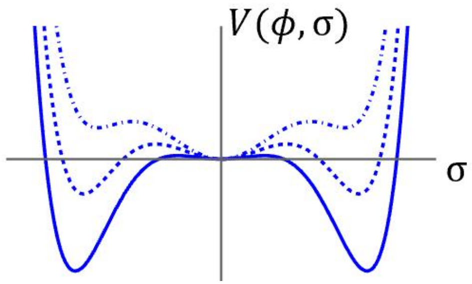


Consequences of the phase transitions

Evolution of inflaton → Phase transitions in spectator sector →

- Primary Gravitational waves
- Dark Matter
- Topological Defects
- Baryogenesis
- Curvature perturbation
  - → Primordial Non-Gaussianity
  - → Primordial Black Holes
  - → Large scale structure
  - → Secondary Gravitational waves





GWs from first-order phase transitions during inflation

- Cosmology 101

Robertson-Walker Metric with conformal time and flat space

$$ds^2 =- a^2(d\tau^2 - dx^2 - dy^2 - dz^2)$$

Perturbation of the metric

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

GWs are in the traceless and transverse part of h_{ij}.

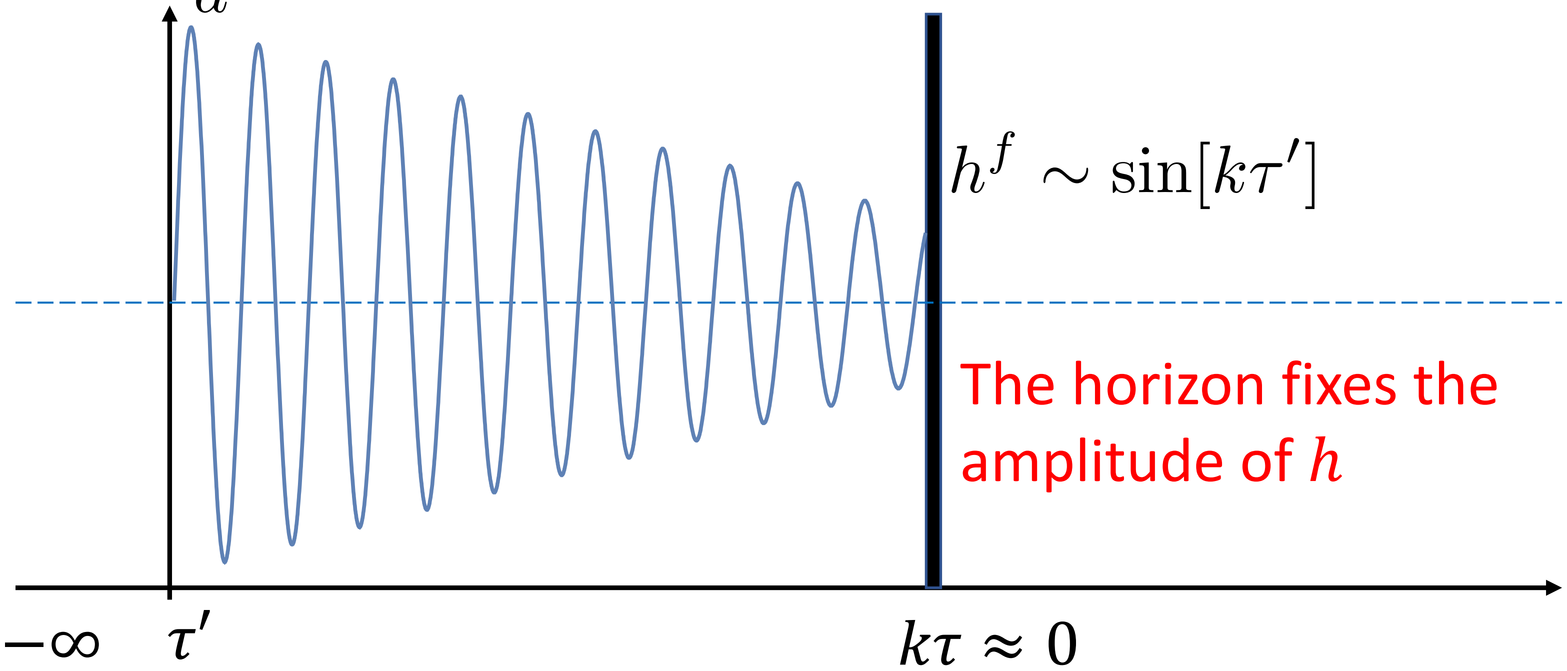
GWs from first-order phase transitions during inflation

- How to calculate GWs?
- In E&M: $\partial_\mu F^{\mu\nu} = J^\nu$
  - We solve the Green's function first.
  - We convolute the Green's function with the source.
- In GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  - We linearize the Einstein equation: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$. GW is h^{TT}_{ij}.
  - We solve the Green's function first. (instantaneous and local source)
  - We convolute the Green's function with the source.

GWs from first-order phase transitions during inflation

$$ds^2 = -a^2(d\tau^2 - dx^2 - dy^2 - dz^2)$$

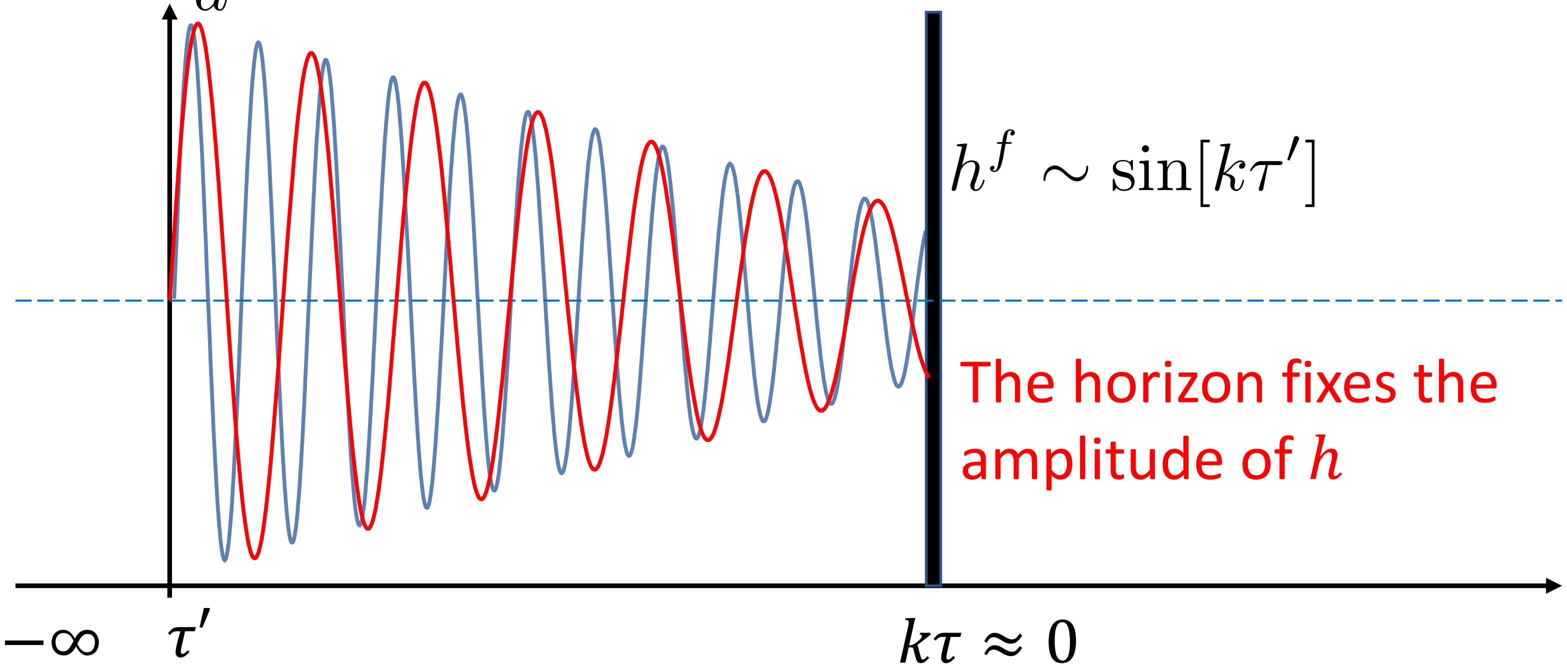
- $h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$



$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$
GW is h^{TT}_{ij}.

GWs from first-order phase transitions during inflation $ds^2 = -a^2(d\tau^2 - dx^2 - dy^2 - dz^2)$

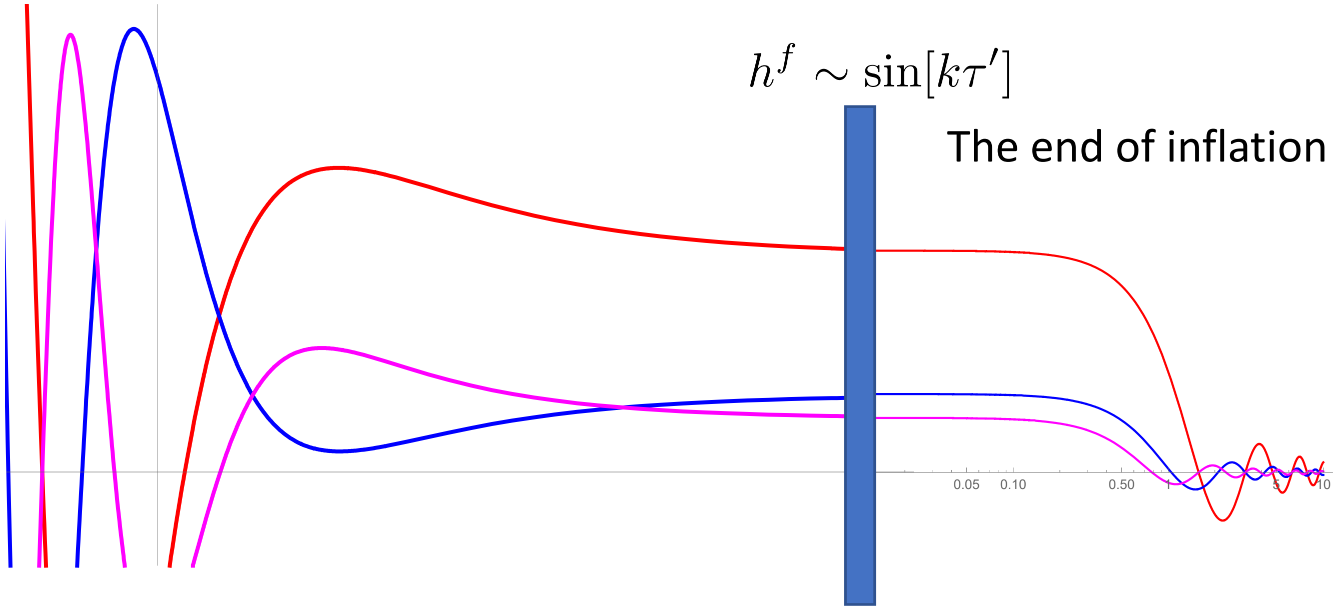
- $h''(\tau, \mathbf{k}) + \dfrac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$



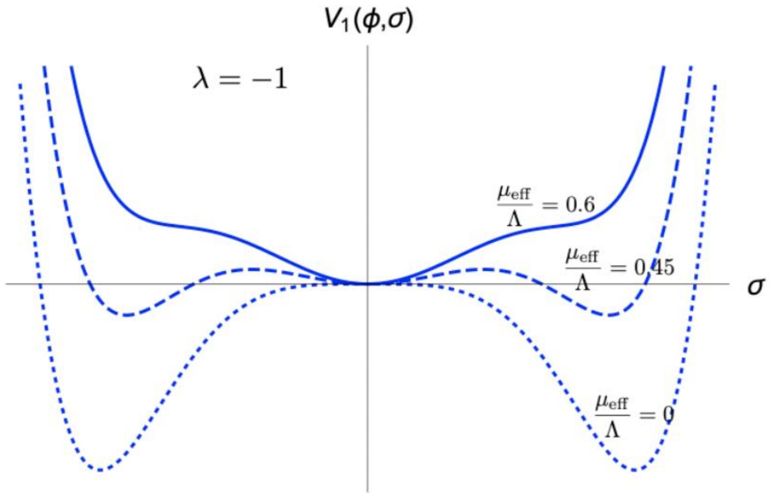
$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$
GW is h_{ij}^{TT}.

After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau/k\tau$.



First-order phase transition during inflation



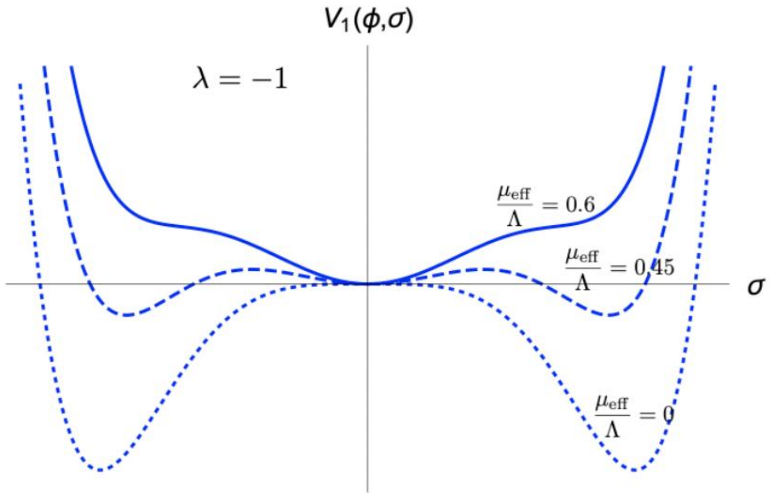
S_4 becomes smaller during

$$S_4 \approx S_4^* - \beta(t - t_*)$$

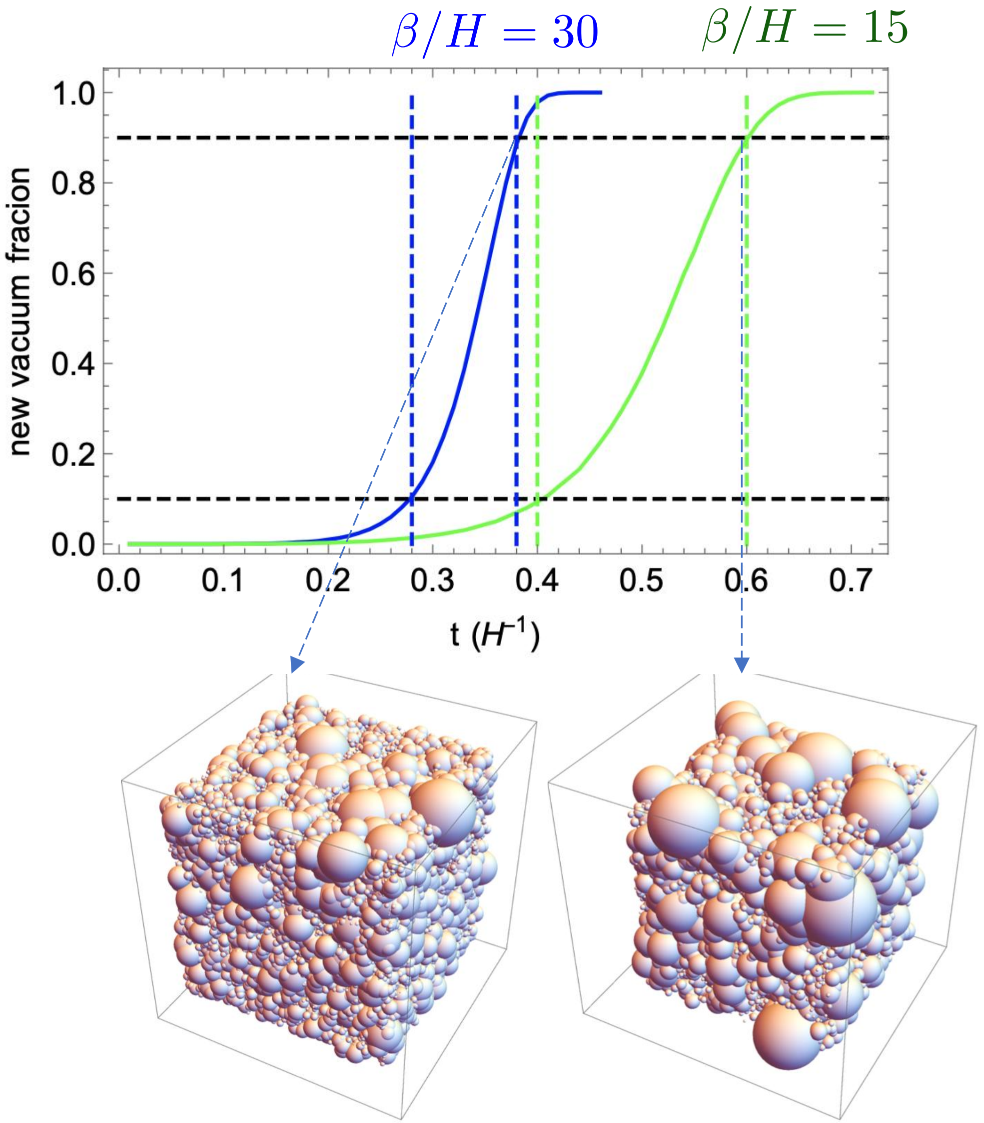
$$\beta = -\frac{dS_4}{dt}$$

- $\frac{\Gamma}{V} \sim m_\sigma^4 e^{-S_4}$, bubble nucleation rate per unit volume.
- Phase transition completes: $\frac{\Gamma}{V} \times H^{-4} \sim O(1)$

First-order phase transition during inflation

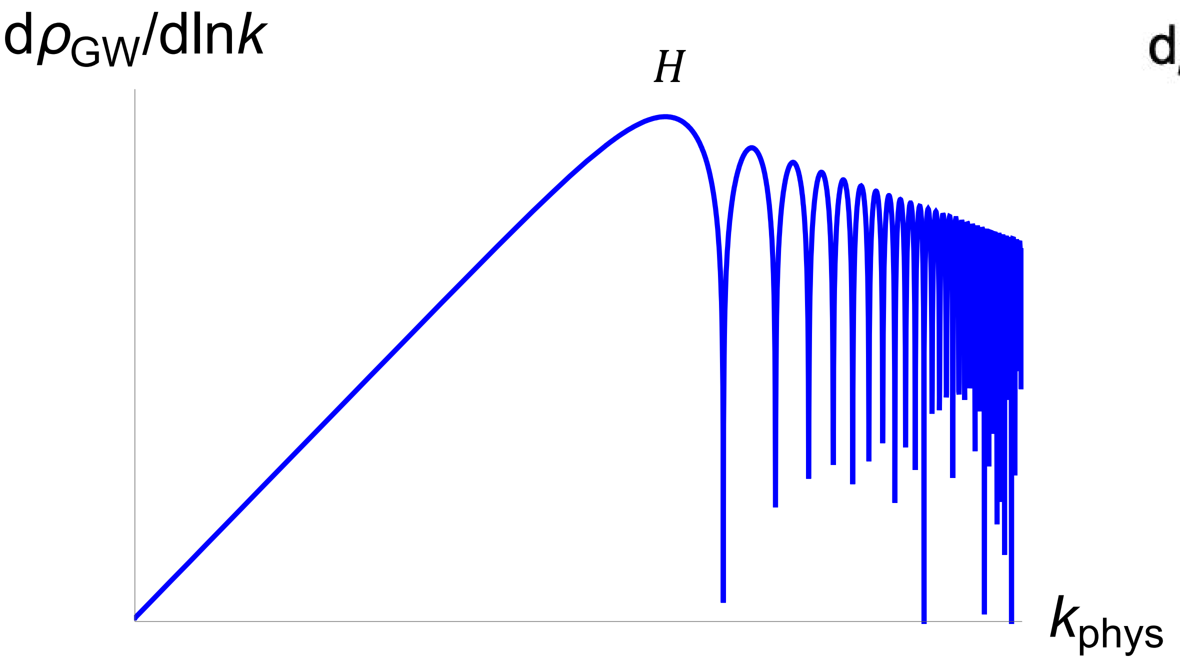


S_4 becomes smaller during

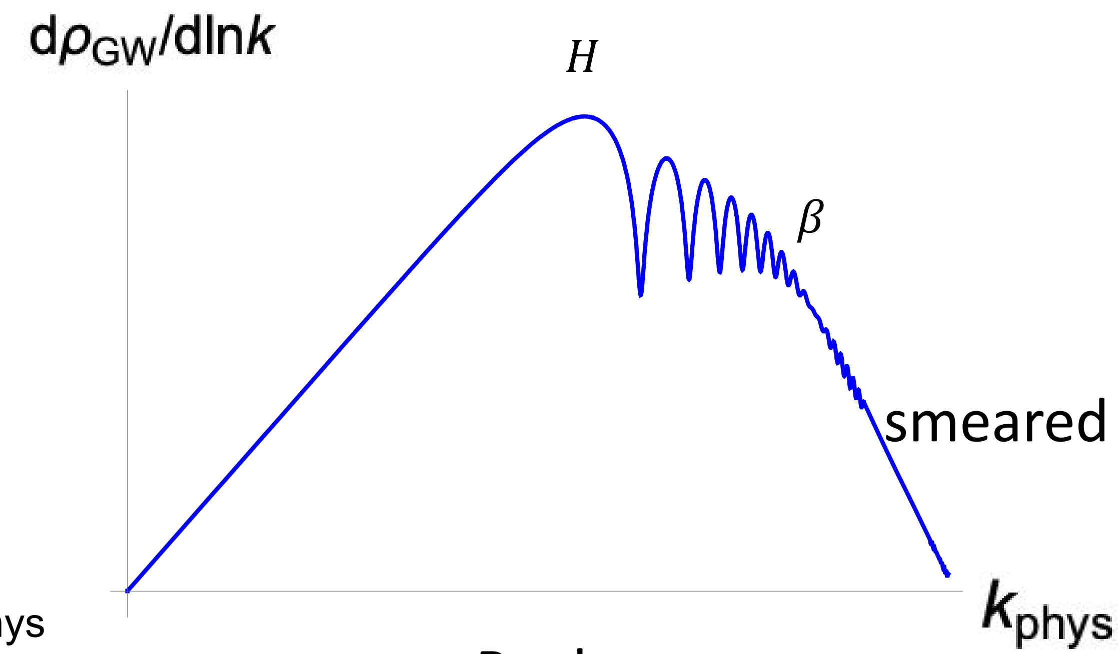


- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

Spectrum of GW from a real source



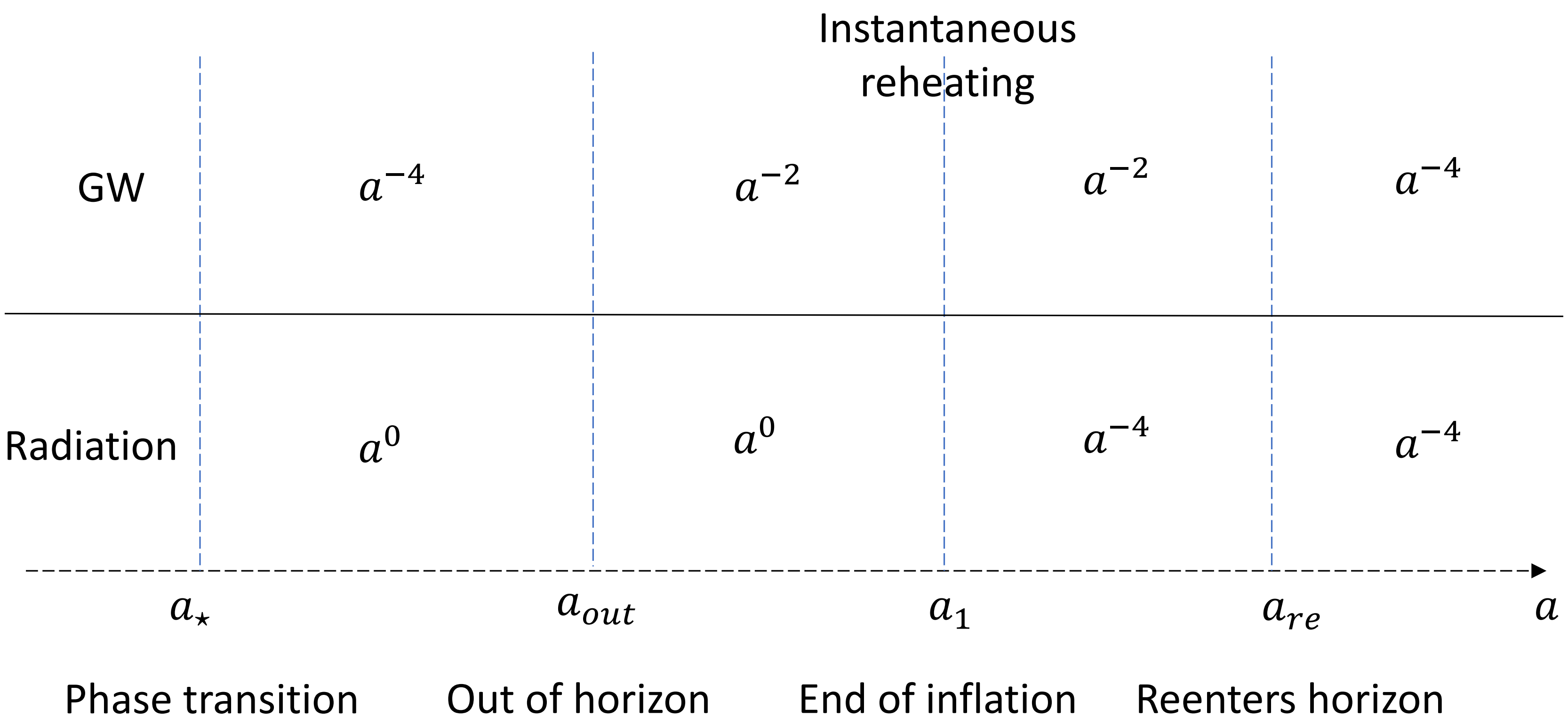
Instantaneous source



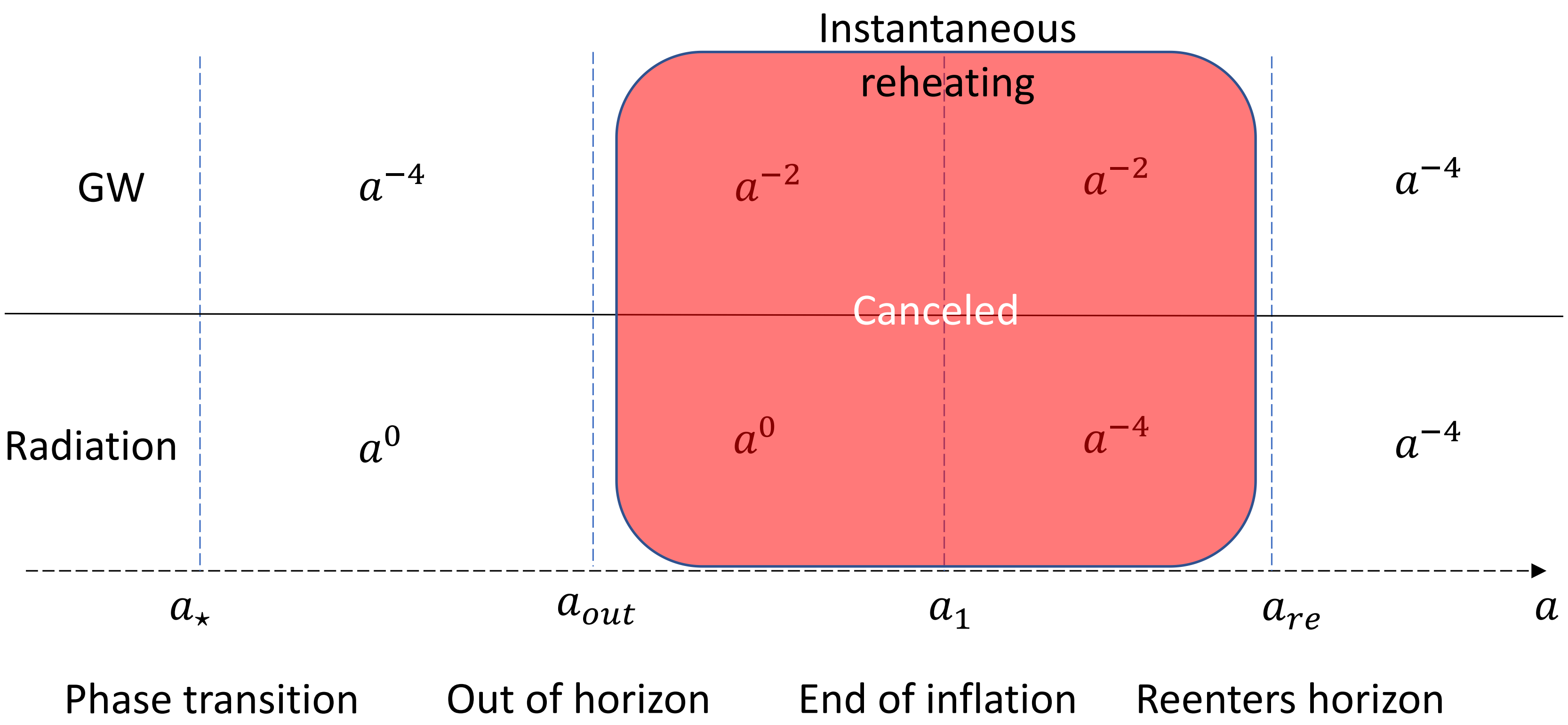
Real source

$$\beta = -\frac{dS_b}{dt} \gg H$$

Redshifts of the GW signal

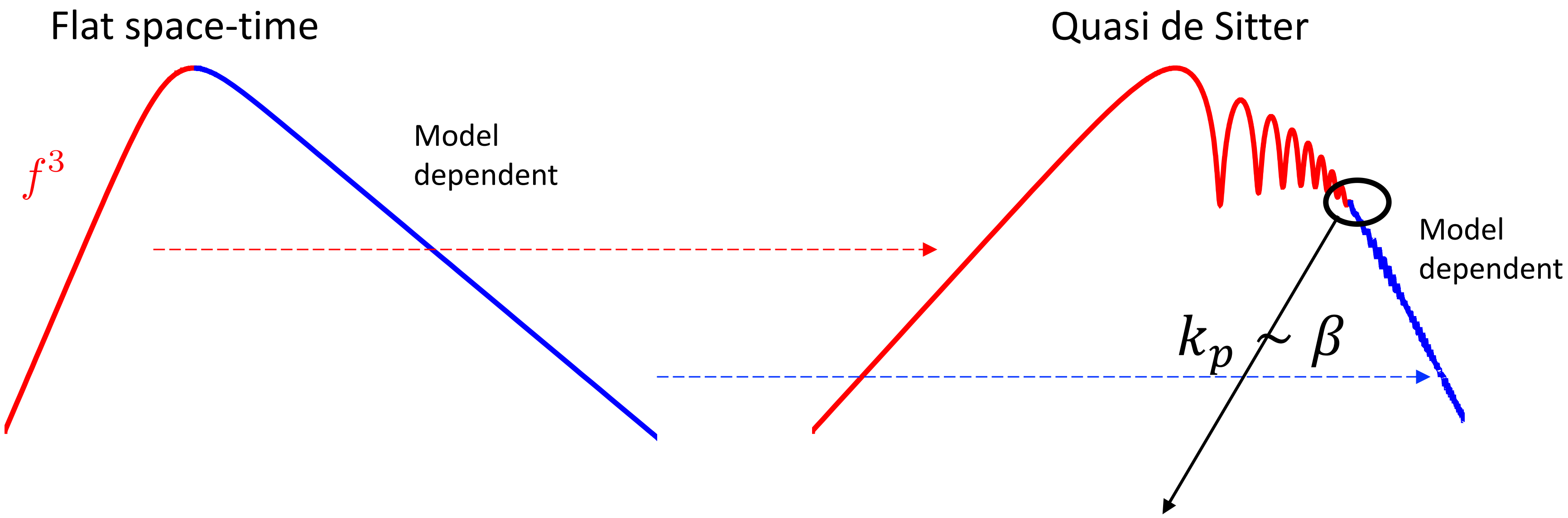


Redshifts of the GW signal



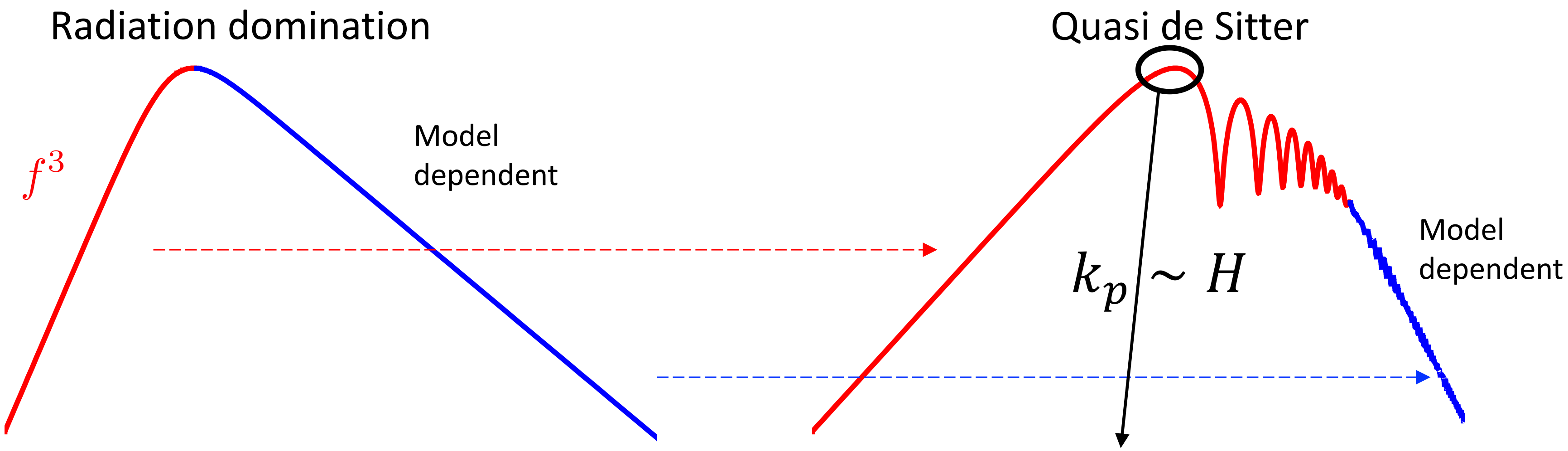
$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

Spectrum distortion by inflation



$$\Omega_{\mathrm{GW}} \approx \Omega_R \left(\frac{H_{\mathrm{inf}}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\mathrm{vac}}}{\rho_{\mathrm{inf}}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\mathrm{inf}}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\mathrm{vac}}}{0.1\rho_{\mathrm{inf}}}\right)^2$$

$$\approx 10^{-17} \times \left(\frac{H_{\mathrm{inf}}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\mathrm{vac}}}{0.1\rho_{\mathrm{inf}}}\right)^2$$

First-order phase transition during inflation

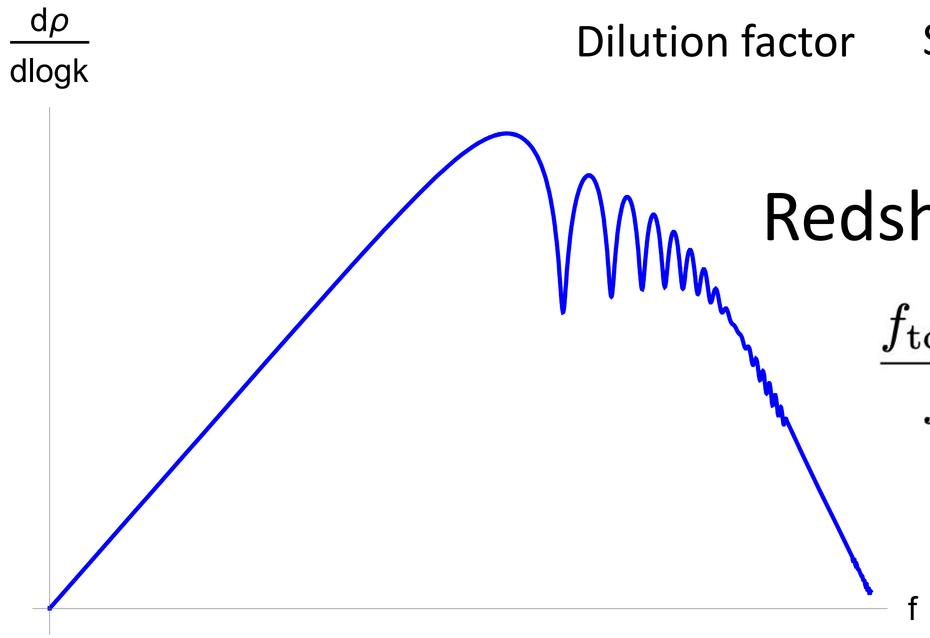
- Assume quasi-dS inflation, RD re-entering, and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \underbrace{\frac{H_{\rm inf}^4}{k_p^4}}_{\text{Dilution factor}} \Big[\frac{1}{2} + \underbrace{\mathcal{S}(k_p\beta^{-1})}_{\text{Smearing}} \cos\Big(\frac{2k_p}{H_{\rm inf}}\Big)\Big] \underbrace{\Big(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\Big)^2}_{\text{Suppressed by the energy faction}} \frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac} d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction



Redshift

$$\frac{f_{\rm today}}{f_\star} = \underbrace{\frac{a(\tau_\star)}{a_1}}_{e^{-N_e}} \left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

e^{-N_e}

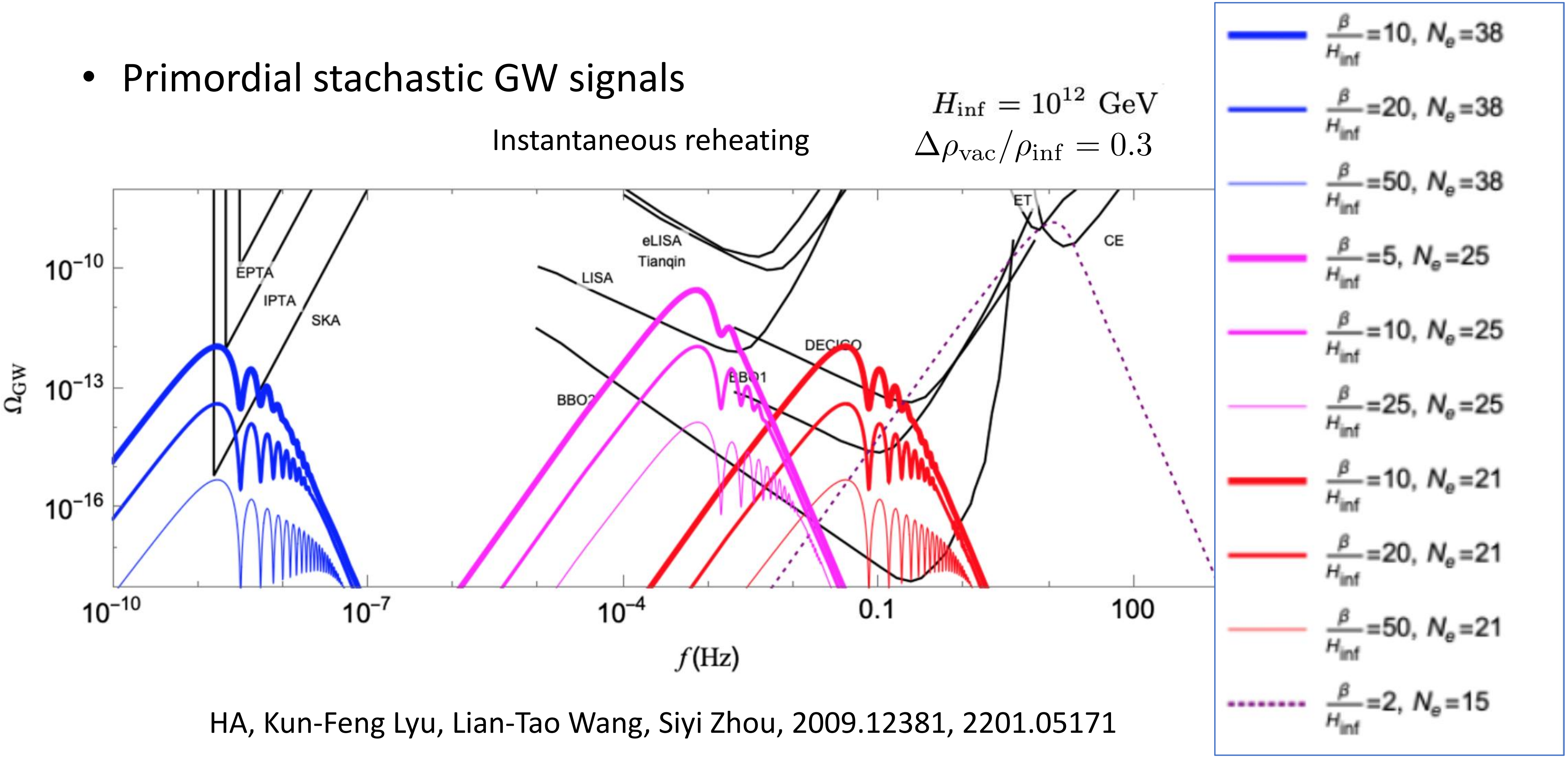
N_e: e-folds before the end of inflation

First-order phase transition during inflation

- Primordial stachastic GW signals

Instantaneous reheating

$$H_{\rm inf} = 10^{12}\ \text{GeV}$$
$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.3$$

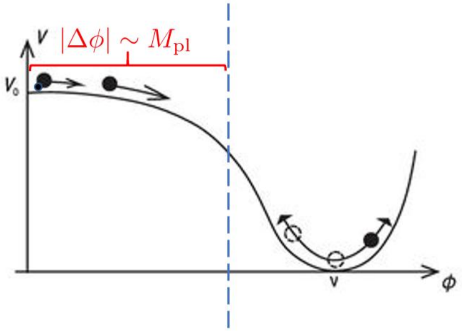


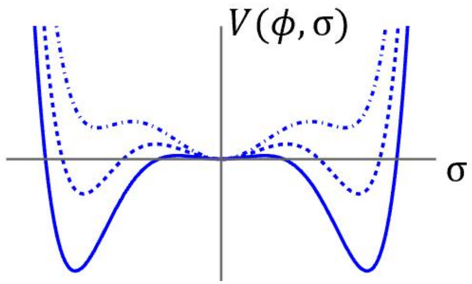
HA, Kun-Feng Lyu, Lian-Tao Wang, Siyi Zhou, 2009.12381, 2201.05171

Consequences of the phase transitions

Evolution of inflaton → Phase transitions in spectator sector →

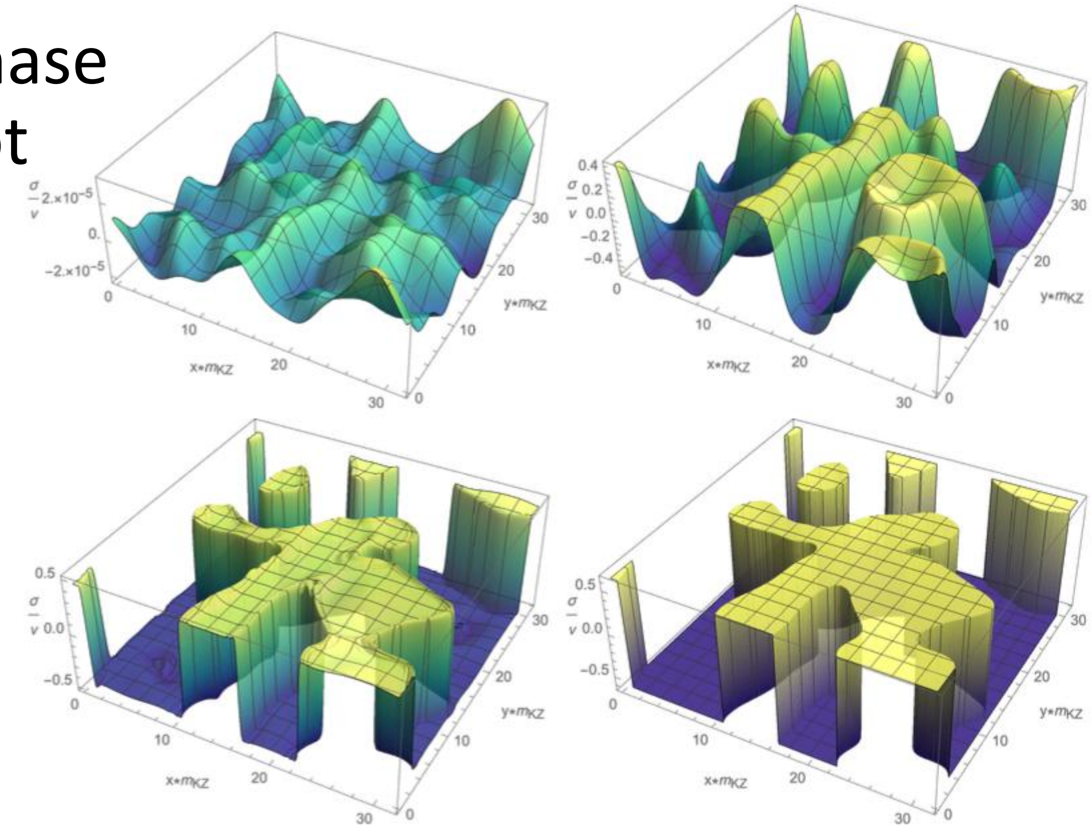
- Primary Gravitational waves
- Dark Matter
- Topological Defects
- Baryogenesis
- Primordial Non-Gaussianity
- Curvature perturbation
- Primordial Black Holes
- Large scale structure
- Secondary Gravitational waves





GWs from topological defects

- GWs directly from second-order phase transitions are small, usually cannot be detected.
- Phase transitions can produce topological defects:
  - Domain walls
  - Cosmic strings
  - Monopoles



Formation of domain walls

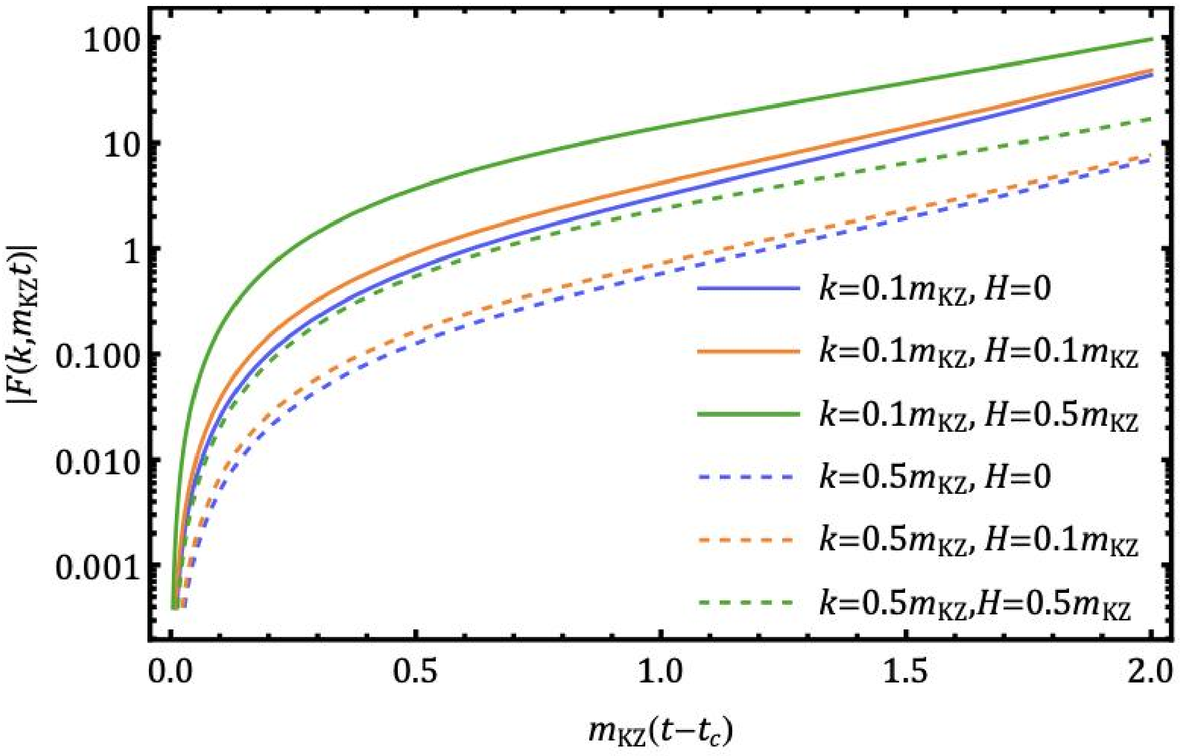
- Tachyonic growth

$$V_{\mathrm{KZ}} = -\frac{1}{2} m_{\mathrm{KZ}}^3 a_c^{-1} (\tau - \tau_c) \sigma^2 + \frac{\lambda}{4} \sigma^4$$

$$\sigma_{\mathbf{k}}'' + \frac{2a'}{a} \sigma_{\mathbf{k}}' + \omega_{\mathbf{k}}^2(\tau) \sigma_{\mathbf{k}} = 0$$

$$k^2 - a_c^2 m_{\mathrm{KZ}}^3 (\tau - \tau_c) + \frac{\lambda}{2} \langle \sigma^2(\tau, \mathbf{x}) \rangle$$

$\omega_k^2 < 0$ for small k around τ_c.



$F(k, m_{KZ}t)$ can be seen as the occupation number in the k mode.

Formation of domain walls

- Matching to classical nonlinear evolution

$$\tilde{\pi}(\mathbf{k},\tau) = a_{\mathbf{k}} a(\tau)^2 f'(k,\tau) + a^{\dagger}_{-\mathbf{k}} a(\tau)^2 f'^{*}(k,\tau),$$
$$\tilde{\sigma}(\mathbf{k},\tau) = a_{\mathbf{k}} f(k,\tau) + a^{\dagger}_{-\mathbf{k}} f^{*}(k,\tau).$$

Quantum essemble → Classical essemble

$$F(k,\tau) = a(\tau)^2 \mathrm{Re}\left[f'(k,\tau) f^{*}(k,\tau)\right]$$

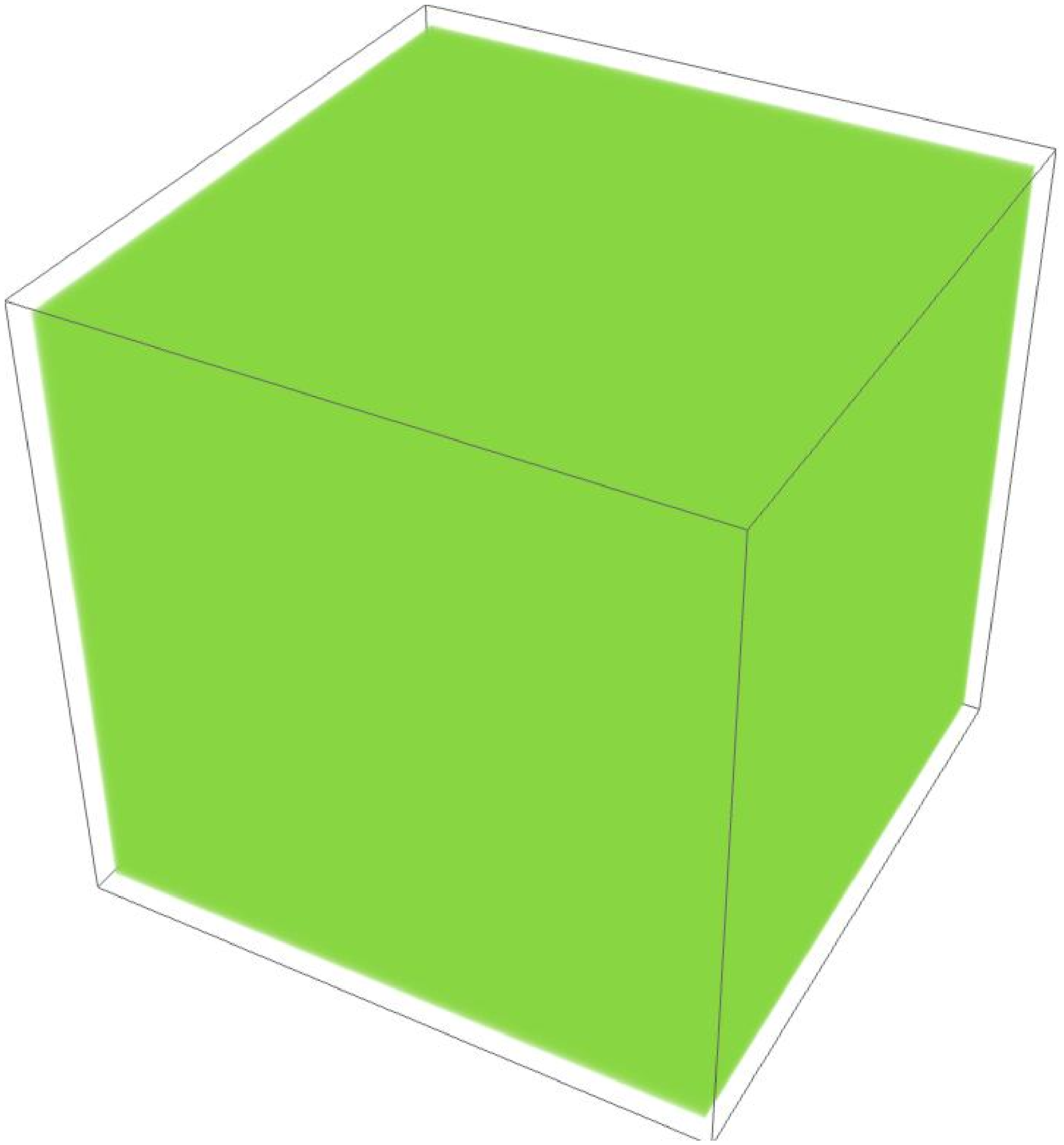
$$W(\sigma_{\mathbf{k}},\pi_{\mathbf{k}}) = \frac{1}{\pi^2}\exp\left[-\frac{|\sigma_{\mathbf{k}}|^2}{|f(\mathbf{k},\tau)|^2} - 4|f(\mathbf{k},\tau)|^2\left|\pi_{\mathbf{k}} - \frac{F(\mathbf{k},\tau)}{|f(\mathbf{k},\tau)|^2}\sigma_{\mathbf{k}}\right|^2\right]$$

We randomly generate the σ_k and π_k according to W as the initial condition for classical lattice simulation.

Polarski and Starobinsky 1996,
Lesgourgues, Polarski and Starobinsky, gr-qc/9611019
Kiefer, Polarski and Starobinsky, gr-qc/9802003
…

Formation of domain walls

- Symmetry breaking via a second order phase transition.
- We numerically solve the nonlinear evolution of σ field with $1000 \times 1000 \times 1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Formation of domain walls

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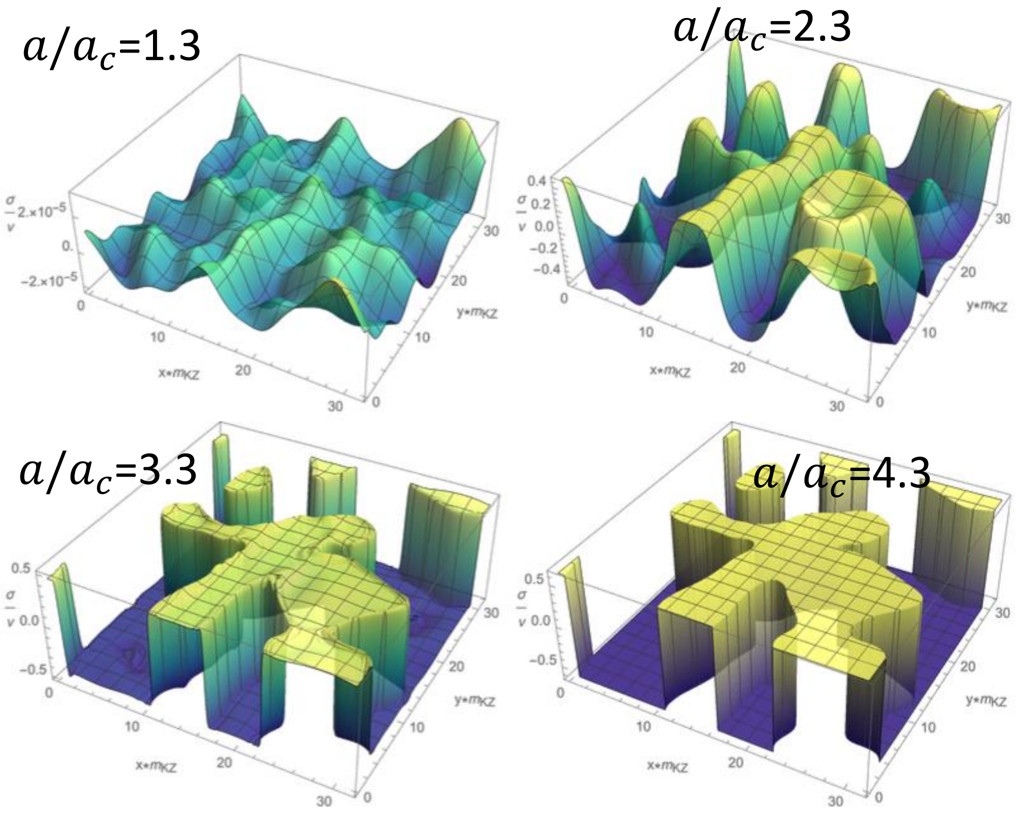
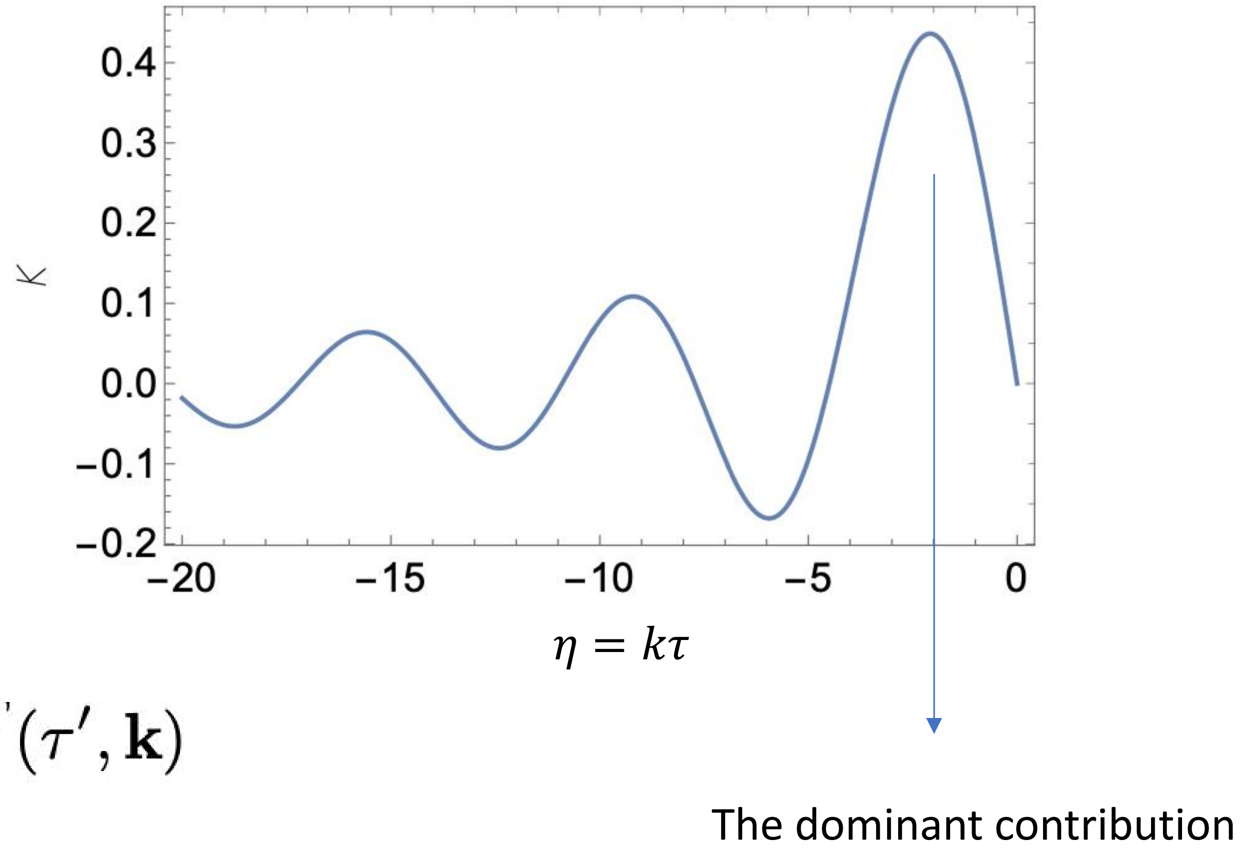


Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.



$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k}\int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau')\tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$

Numerical results for GWs

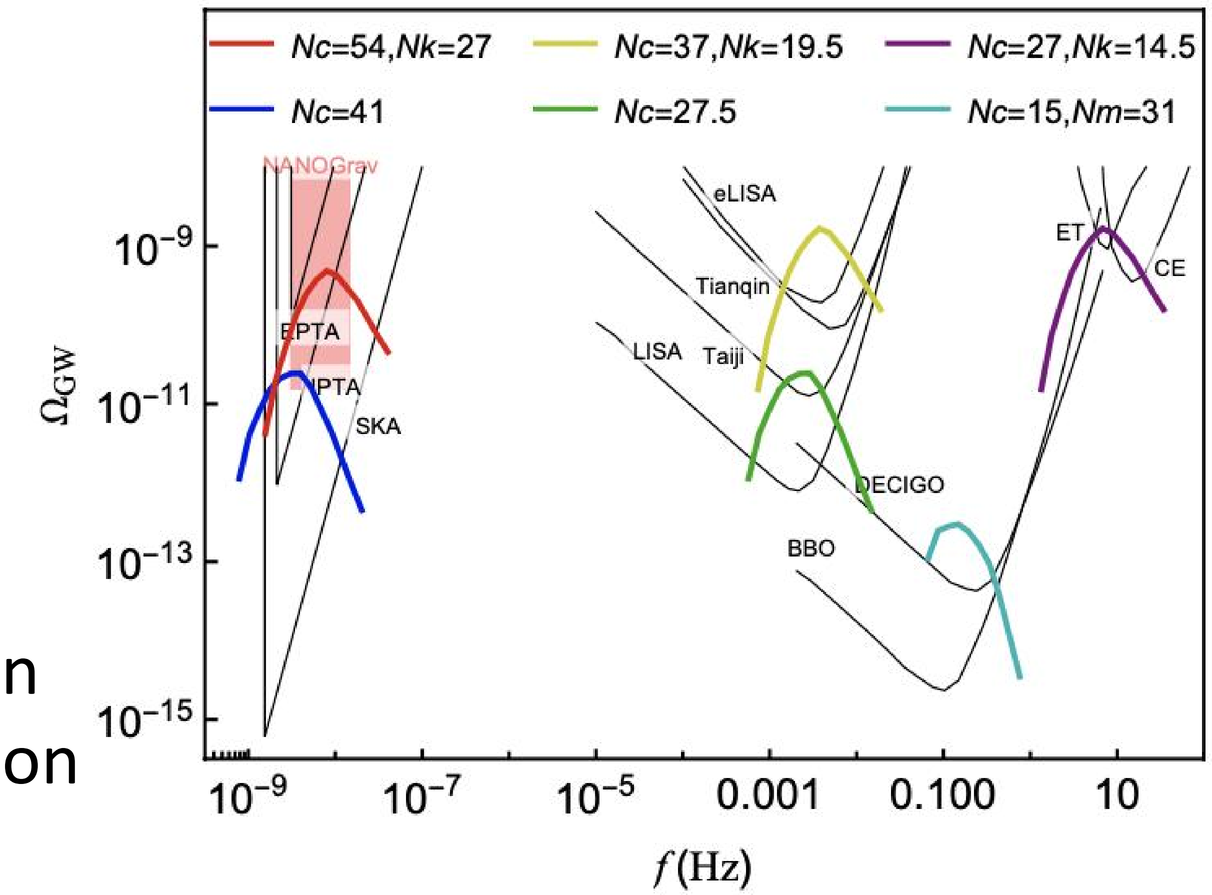
HA, Chen Yang, 2304.02361

$$\Omega_{\rm GW}(f) = \Omega_R \times \rho_R^{-1} \left.\frac{d\rho_{\rm GW}}{d\ln f}\right|_{\rm today}$$

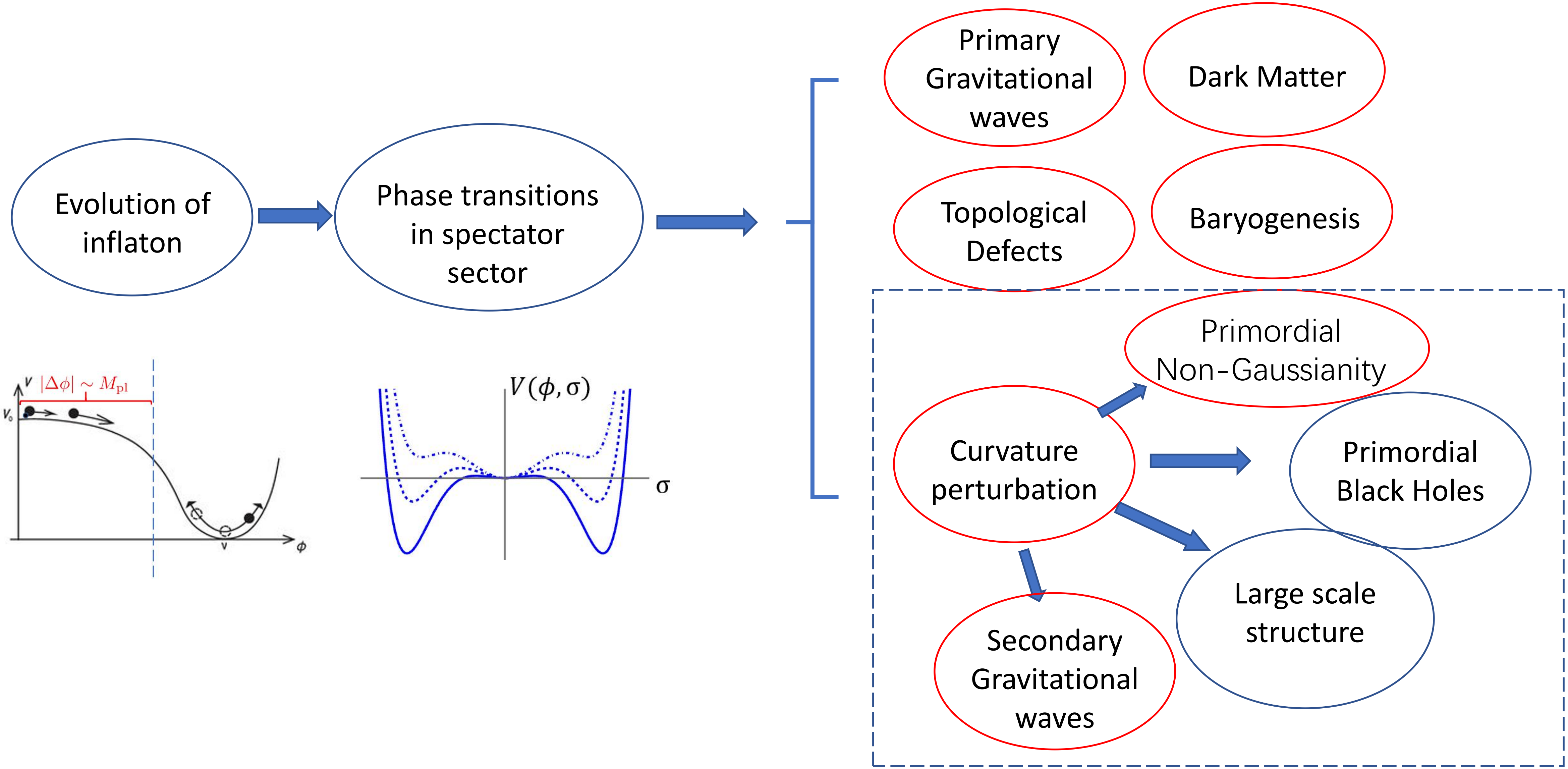
$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

Intermediate stages matter:

- Instantaneous reheating
- Intermediate matter domination
- Intermediate kination domination



Consequences of the phase transitions

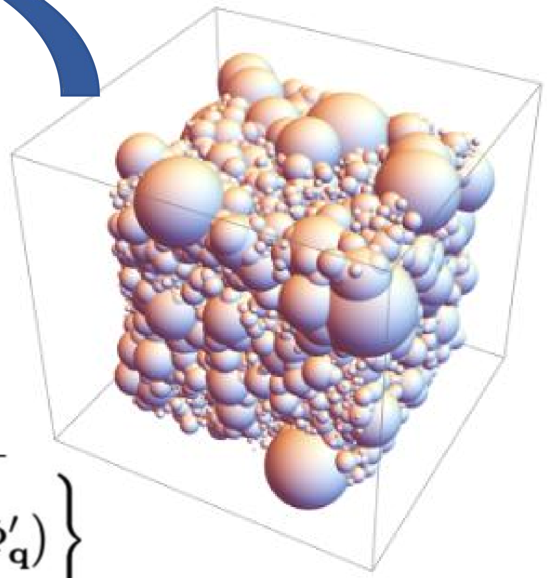


Induced classical scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial\phi_0}\delta\phi$$



$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}}\ ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2}\left[\frac{\partial V_1}{\partial\phi}\right]_{\mathbf{q}} \underbrace{-\left\{\frac{2\Phi_{\mathbf{q}}}{H^2\tau^2}\left(\frac{\partial V_0}{\partial\phi_0} + \left[\frac{\partial V_1}{\partial\phi}\right]_0\right) + \frac{\dot{\phi}_0}{H\tau}\left(3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}\right)\right\}}_{\text{Pure gravitational, subdominant}}$$

Induced classical scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial \phi_0}\delta\phi$$

Source term for $\delta\phi$

- The source is different from T_{ij}^{TT}
- No one has done the simulation before

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}}\ ,$$

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Induced curvature perturbation ζ

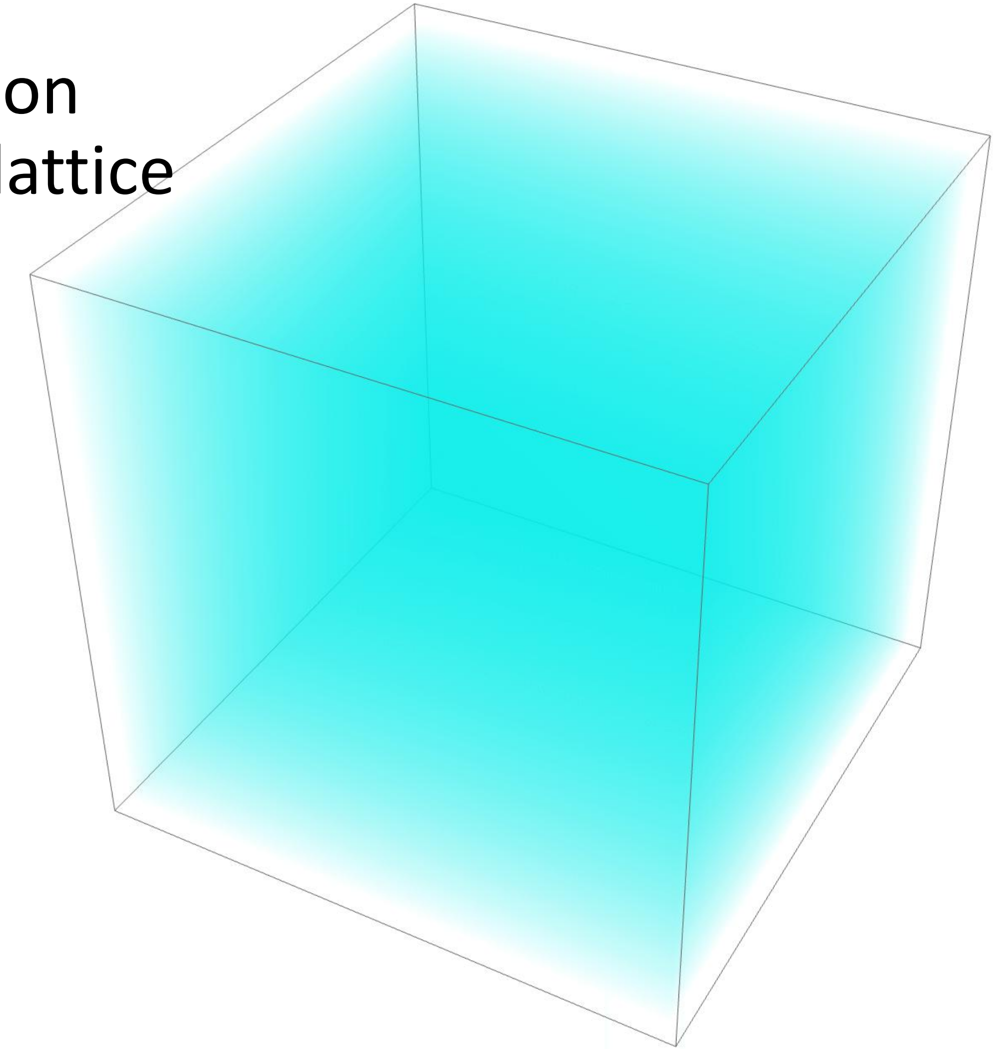
- We solve the following equations of motion numerically with a $1000 \times 1000 \times 1000$ lattice

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} \ ,$$

$$\tilde{\Psi}'_{\mathbf{q}} - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N \left(\frac{\dot{\phi}_0\delta\tilde{\phi}_{\mathbf{q}}}{H_{\rm inf}\tau} + \left[\frac{\partial_i}{\partial^2}(\sigma'\partial_i\sigma)\right]_{\mathbf{q}}\right)$$

$$\tilde{\pi}^S_{\mathbf{q}} = -\frac{3}{2}H_{\rm inf}^2\tau^2 q_i q_j q^{-4}\left[(\partial_i\sigma\partial_j\sigma)^{\rm TL}\right]_{\mathbf{q}}$$

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\rm inf}\delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

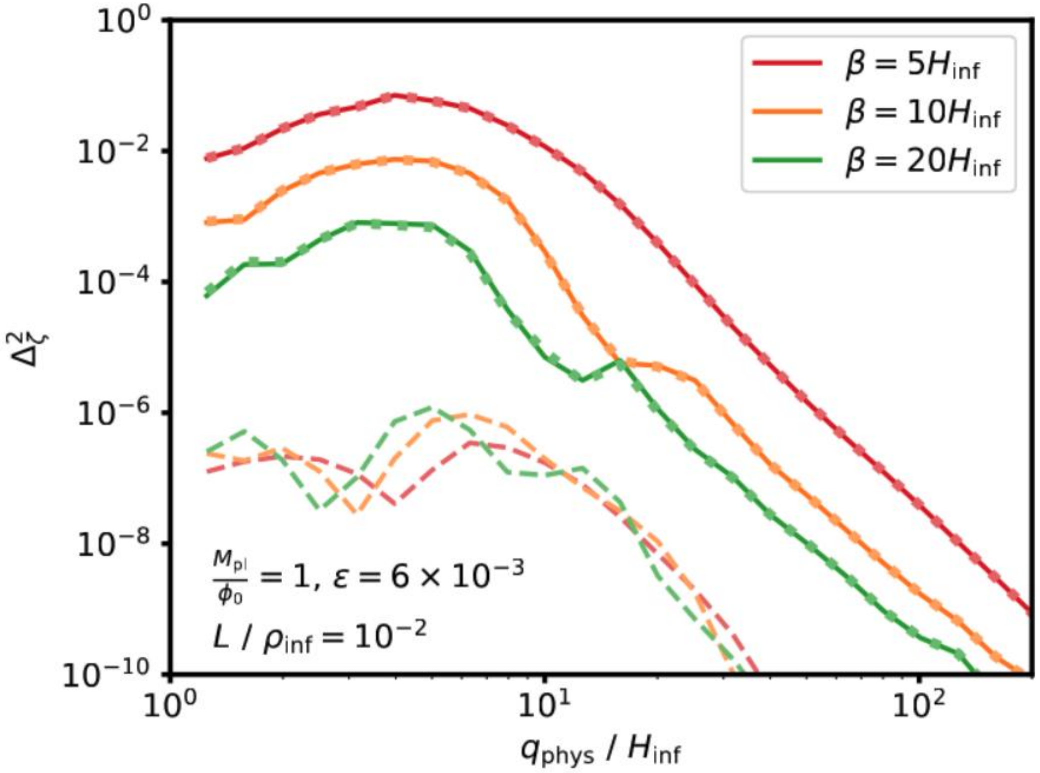
- After the collision of the bubbles, σ field oscillates and keeps producing ζ.
- The production of ζ lasts about H^{-1}, longer than β^{-1}.

$$\Delta^{2\,(\mathrm{emp})}_{\zeta}(q) = A_{\mathrm{ref}}\mathcal{F}\left(\frac{q_{\mathrm{phys}}}{H_{\mathrm{inf}}}\right)$$

$$\mathcal{F}(x) = \frac{x^3}{1+(\alpha_1 x)^4+(\alpha_2 x)^9}$$

$$A_{\mathrm{ref}} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\mathrm{pl}}}{\phi_0}\right)^2\left(\frac{H_{\mathrm{inf}}}{\beta}\right)^3\left(\frac{\Delta\rho}{\rho_{\mathrm{inf}}}\right)^2$$

$\mathcal{A} \approx 24$ $\qquad$ $\alpha_1 \approx 0.31,\ \alpha_2 \approx 0.17$



Secondary GWs

- After inflation $\zeta \to \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}^{\;lm}\mathcal{S}_{lm}\,,$$

$$\begin{aligned}\mathcal{S}_{ij} \equiv\; & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i\left(\Psi' + \mathcal{H}\Phi\right)\partial_j\left(\Psi' + \mathcal{H}\Phi\right) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)\,.\end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278
Mollerach, Harari, and Matarrese, astro-hp/0310711
Ananda, Clarkson, and Wands, gr-qc/0612013
Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290
…

Secondary GWs

$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

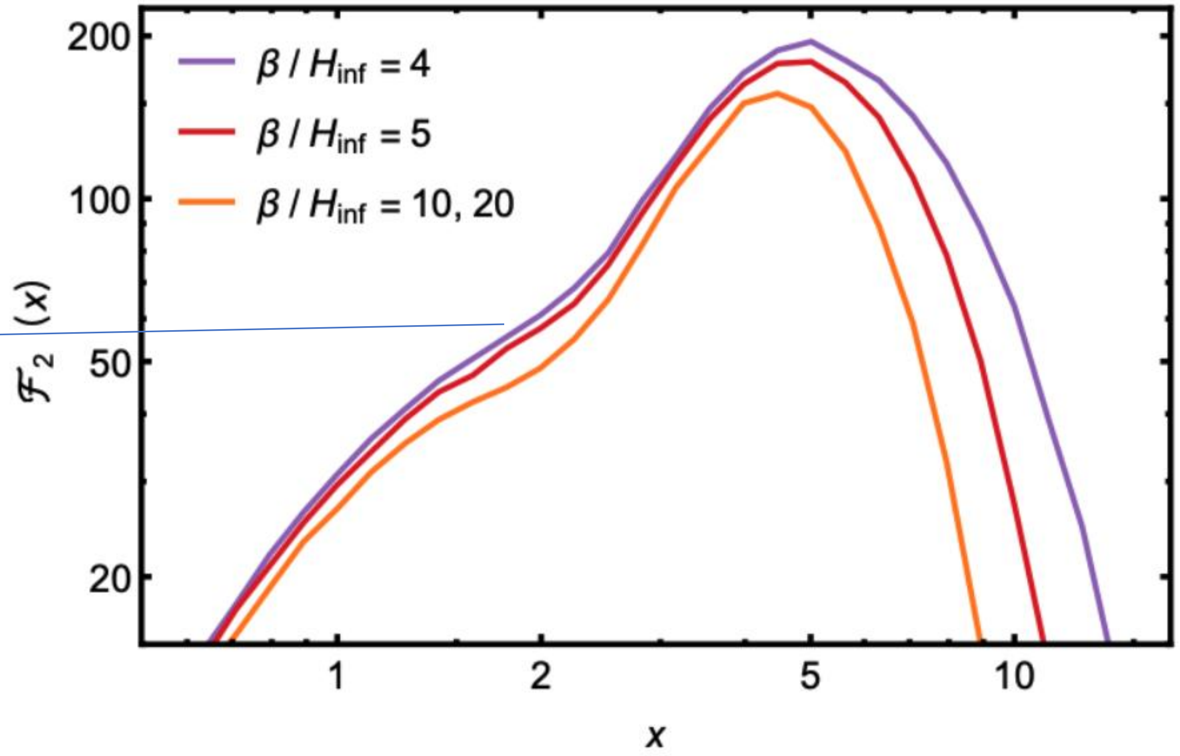
$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

$\mathcal{F}_2$ collects information of the transfer functions.

$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^2$$

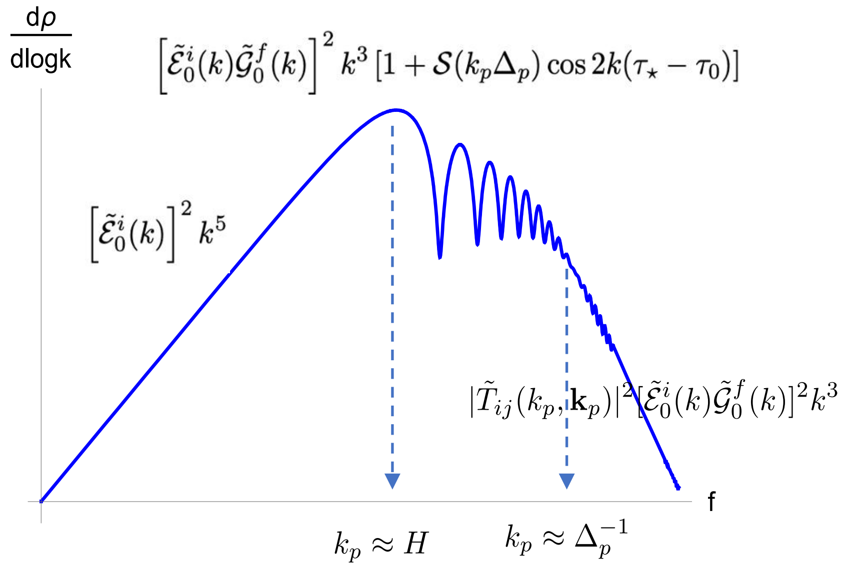
$$\mathcal{F}_2^{\rm max} \approx 200$$



Comparison between primary GW and secondary GW

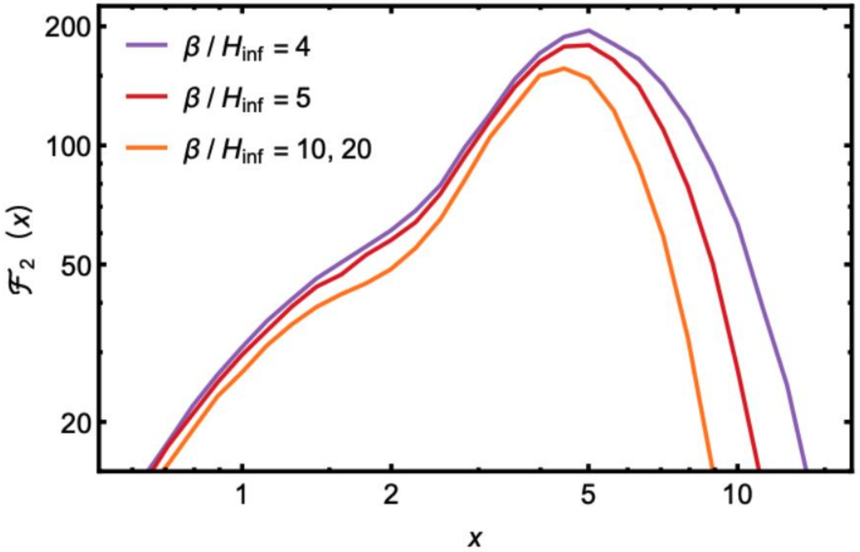
- Primary

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

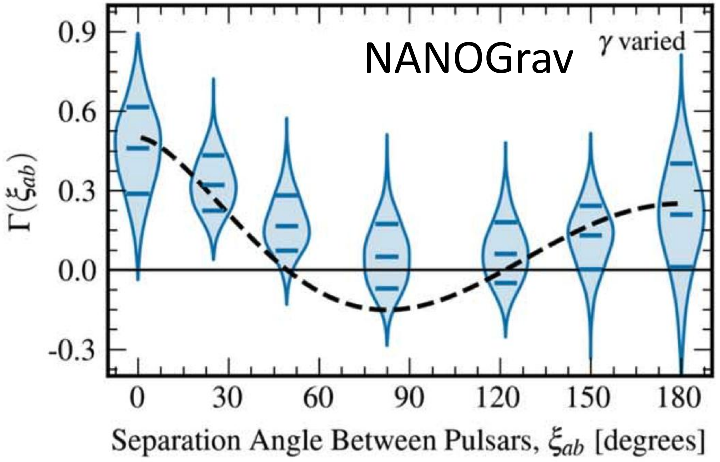


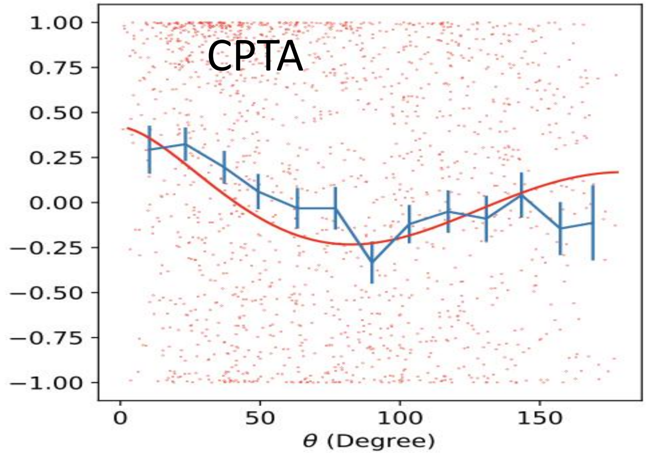
- Secondary

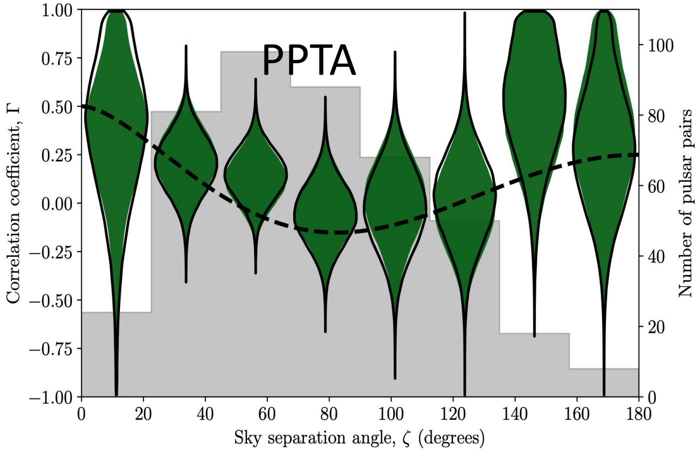
$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^4$$

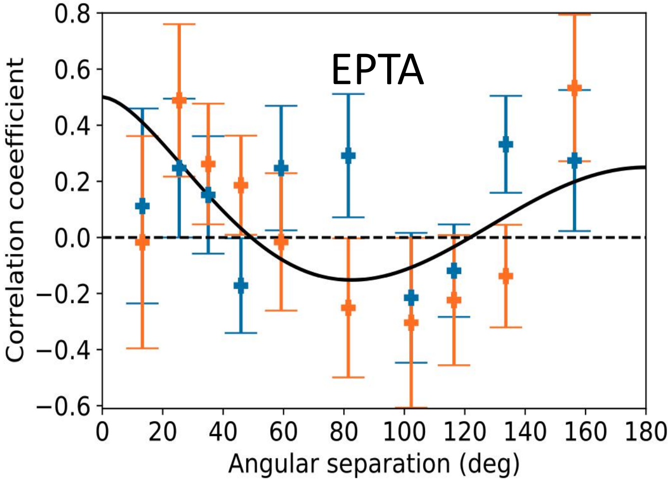


Observation from PTAs







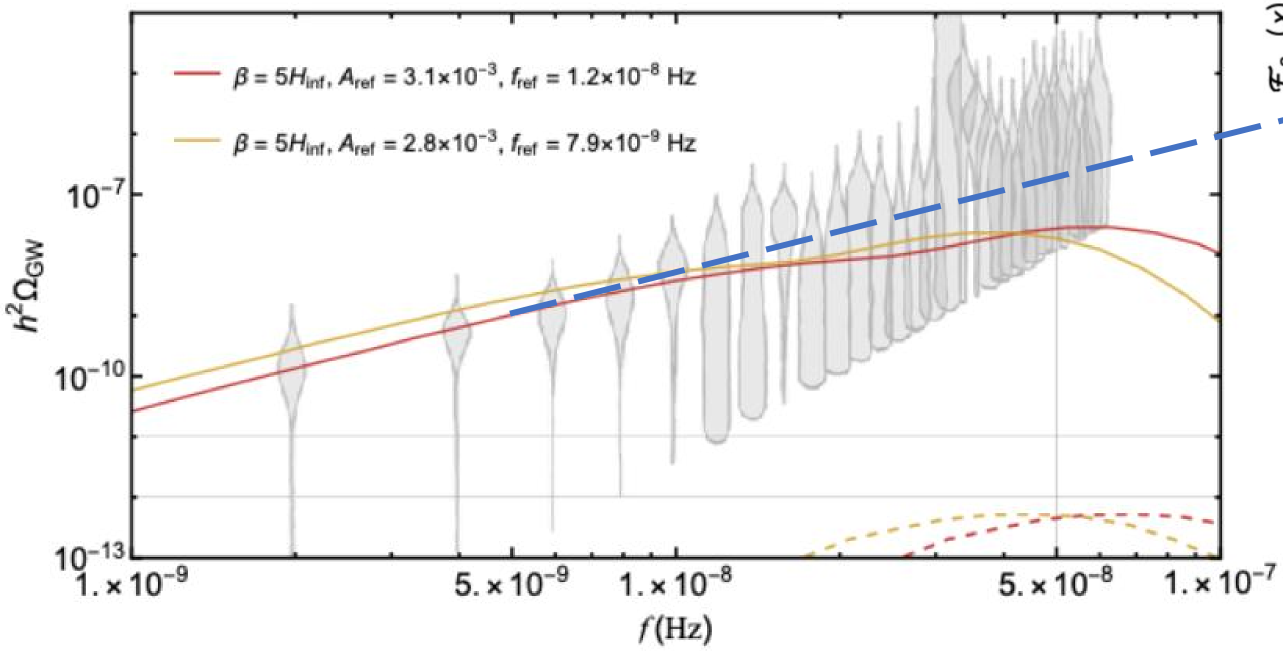


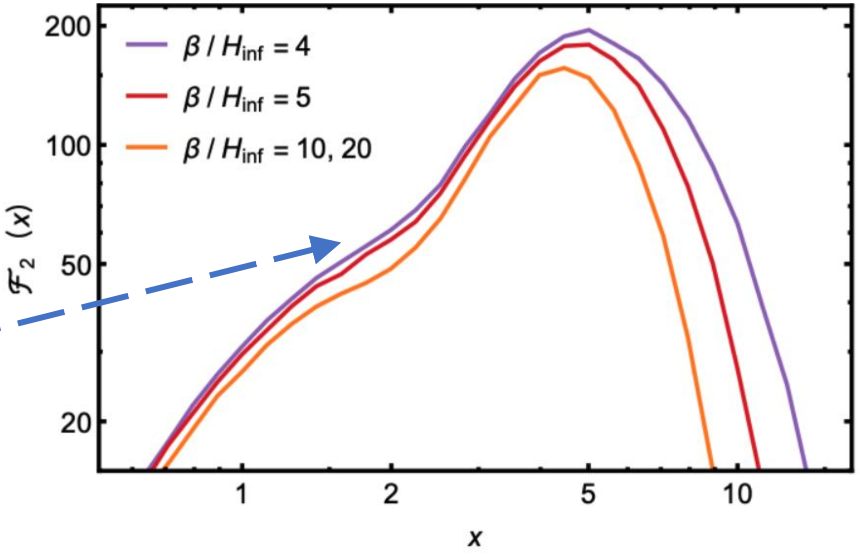
Observation from PTAs

HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

- The slope is around 2 in the IR region

$$\mathcal{F}_2^{\mathrm{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$





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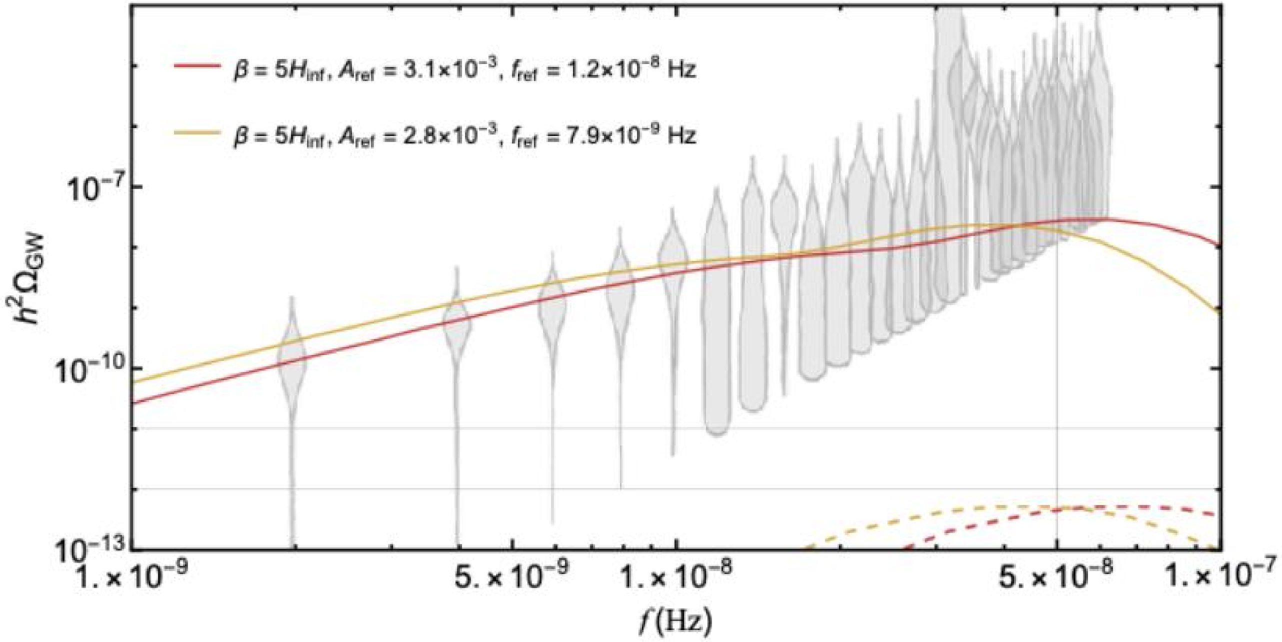
$$f = \frac{q}{2\pi a_0} = f_{\mathrm{ref}} \times \frac{q_{\mathrm{phys}}}{H_{\mathrm{inf}}}$$

$$f_{\mathrm{ref}} = 10^{-9}\ \mathrm{Hz} \times e^{40-N_e} \left(\frac{H_{\mathrm{inf}}}{10^{14}\ \mathrm{GeV}} \right)^{1/2}$$

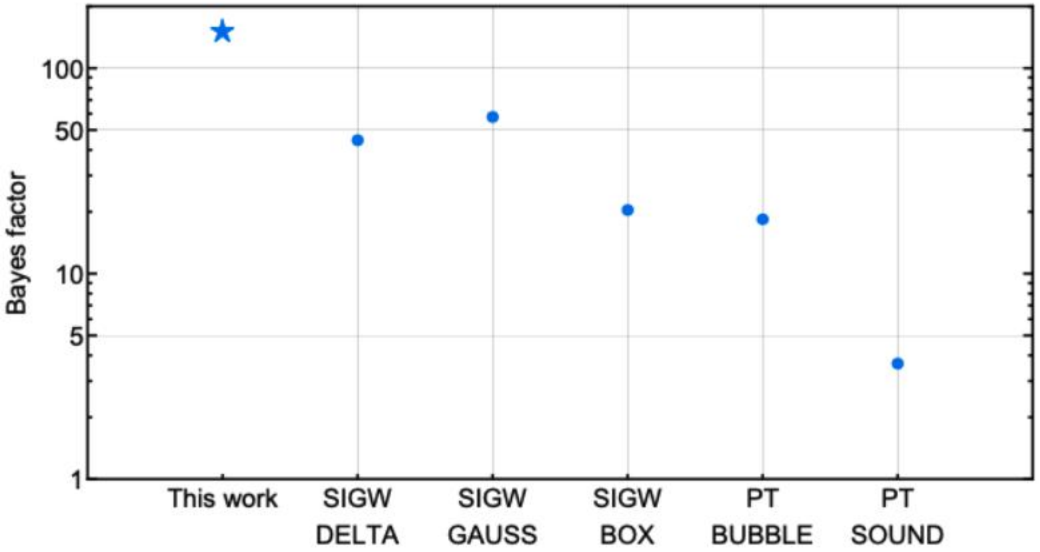
Observation from PTAs

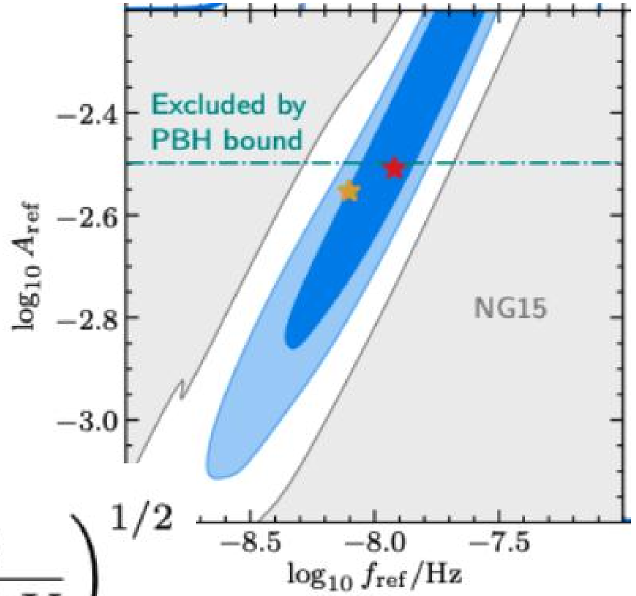
HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

- Bayes factor against SMBHB



Our model





$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R \underline{A_{\rm ref}^2} \mathcal{F}_2\left(\frac{q_{\rm phys}}{\underline{H_{\rm inf}}}\right)$$

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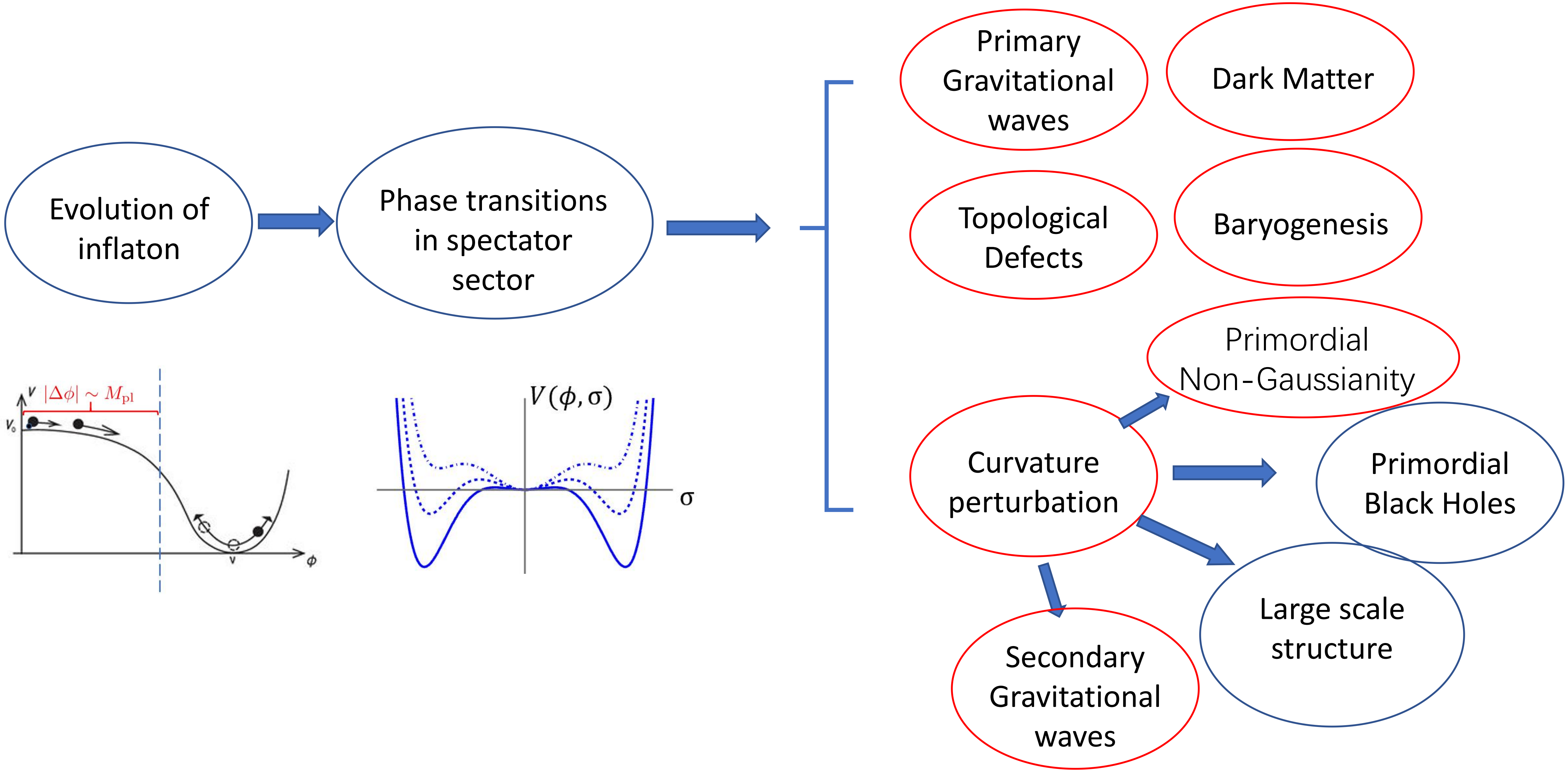
$$f_{\rm ref} = 10^{-9}\ \mathrm{Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ \mathrm{GeV}}\right)^{1/2}$$

Summary


Evolution of inflaton
Phase transitions in spectator sector
$|\Delta\phi| \sim M_{\rm pl}$
V
V_0
ϕ
$V(\phi, \sigma)$
σ
Primary Gravitational waves
Dark Matter
Topological Defects
Baryogenesis
Primordial Non-Gaussianity
Curvature perturbation
Primordial Black Holes
Large scale structure
Secondary Gravitational waves

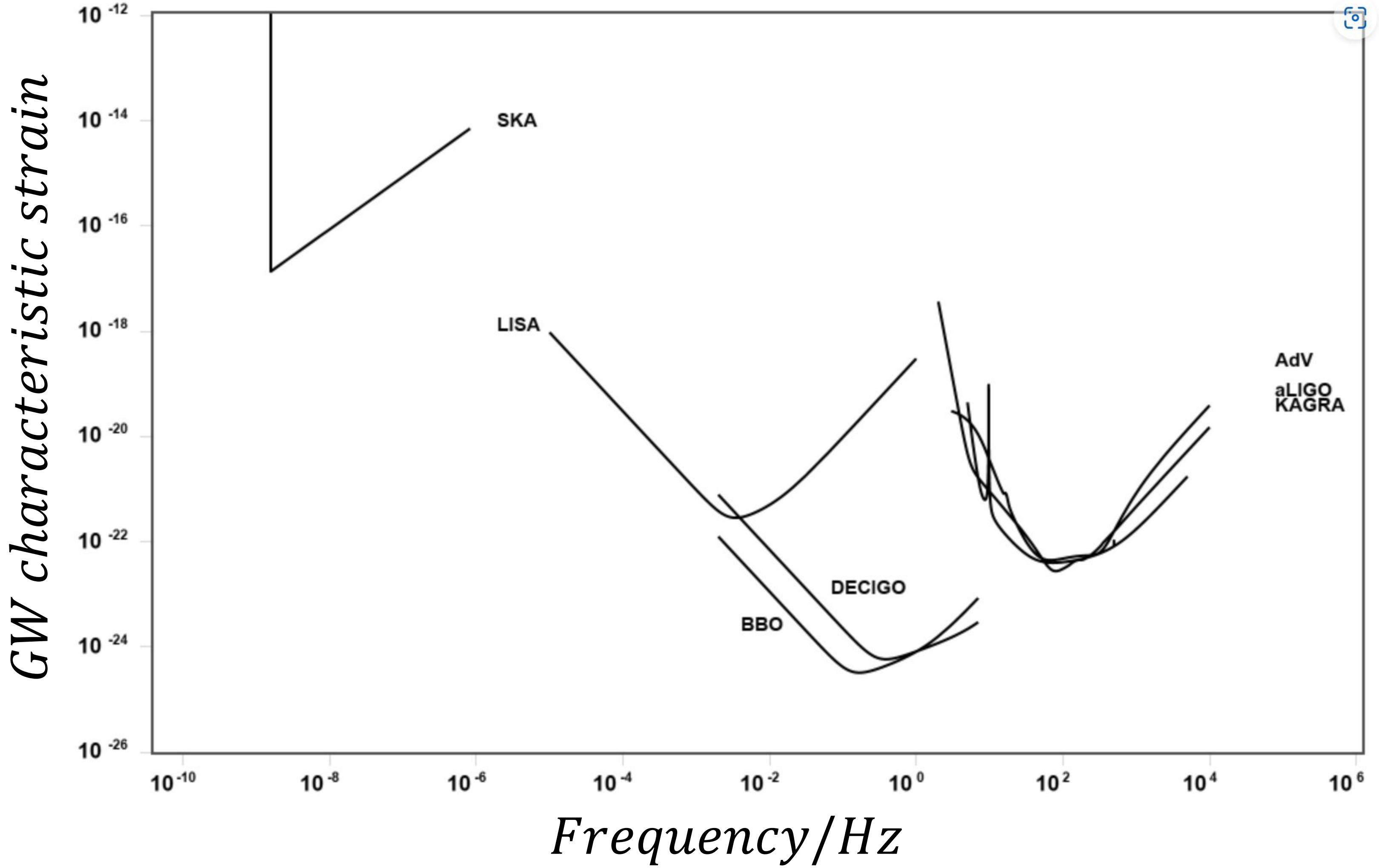
Computational challenges

- 1 double : 8bit (c++)
- 1000^3 -> 8GB   1 field
- Complex field: (minimal)
- 2(phi)+2(pi)+9(Tij)+9(hij,sin)+9(hij,cos)+2(Fourier, auxi)+1(fft) ~ 250 GB

- Two scales are important: the wall thickness and the size of the universe
- We need larger box to get more accurate GW spectrum.

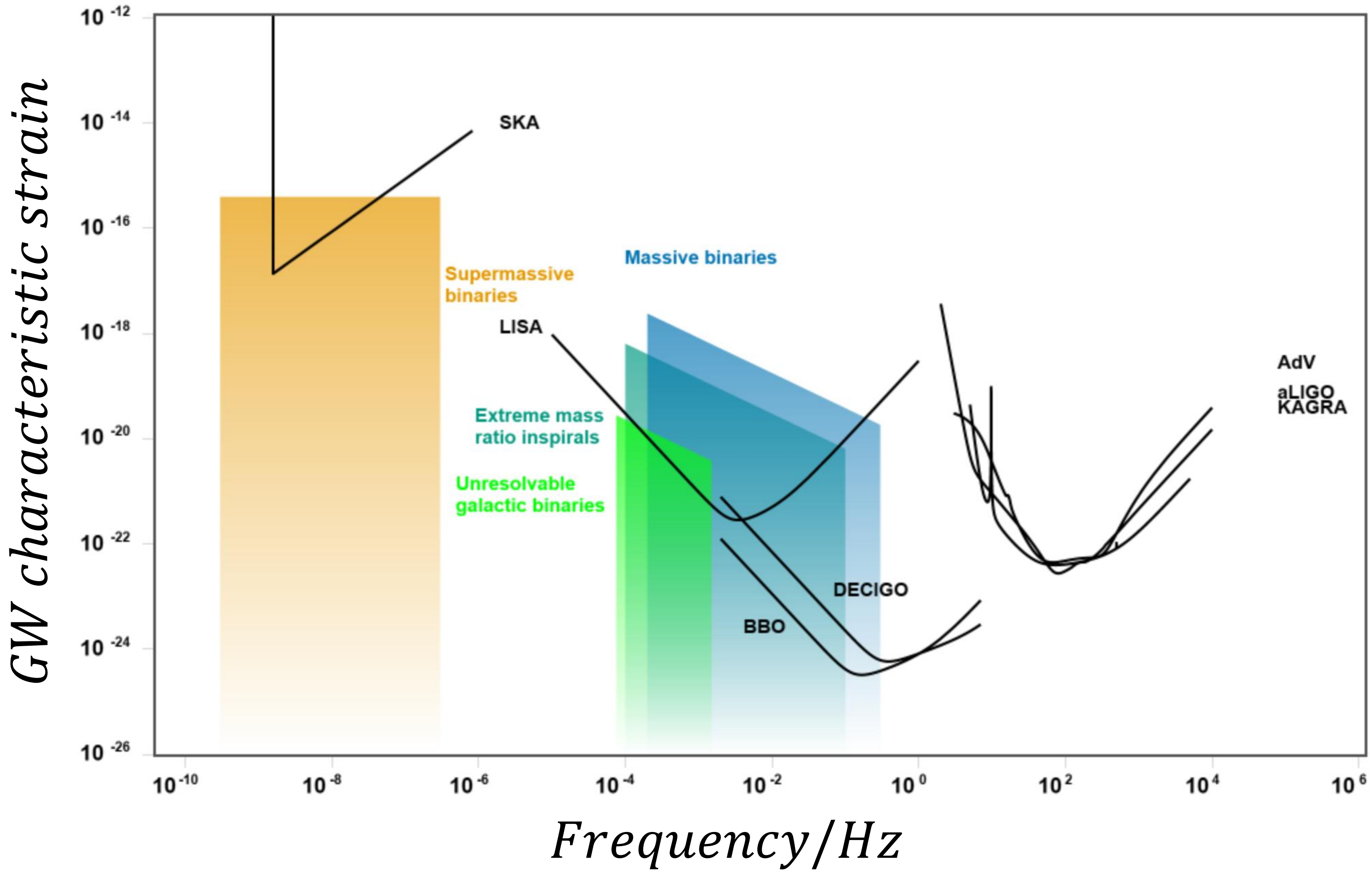
Observational challenges: astrophysics foreground

[Gravitational Wave Sensitivity Curve Plotter (gla.ac.uk)]



Observational challenges: astrophysics foreground

[Gravitational Wave Sensitivity Curve Plotter (gla.ac.uk)](#)



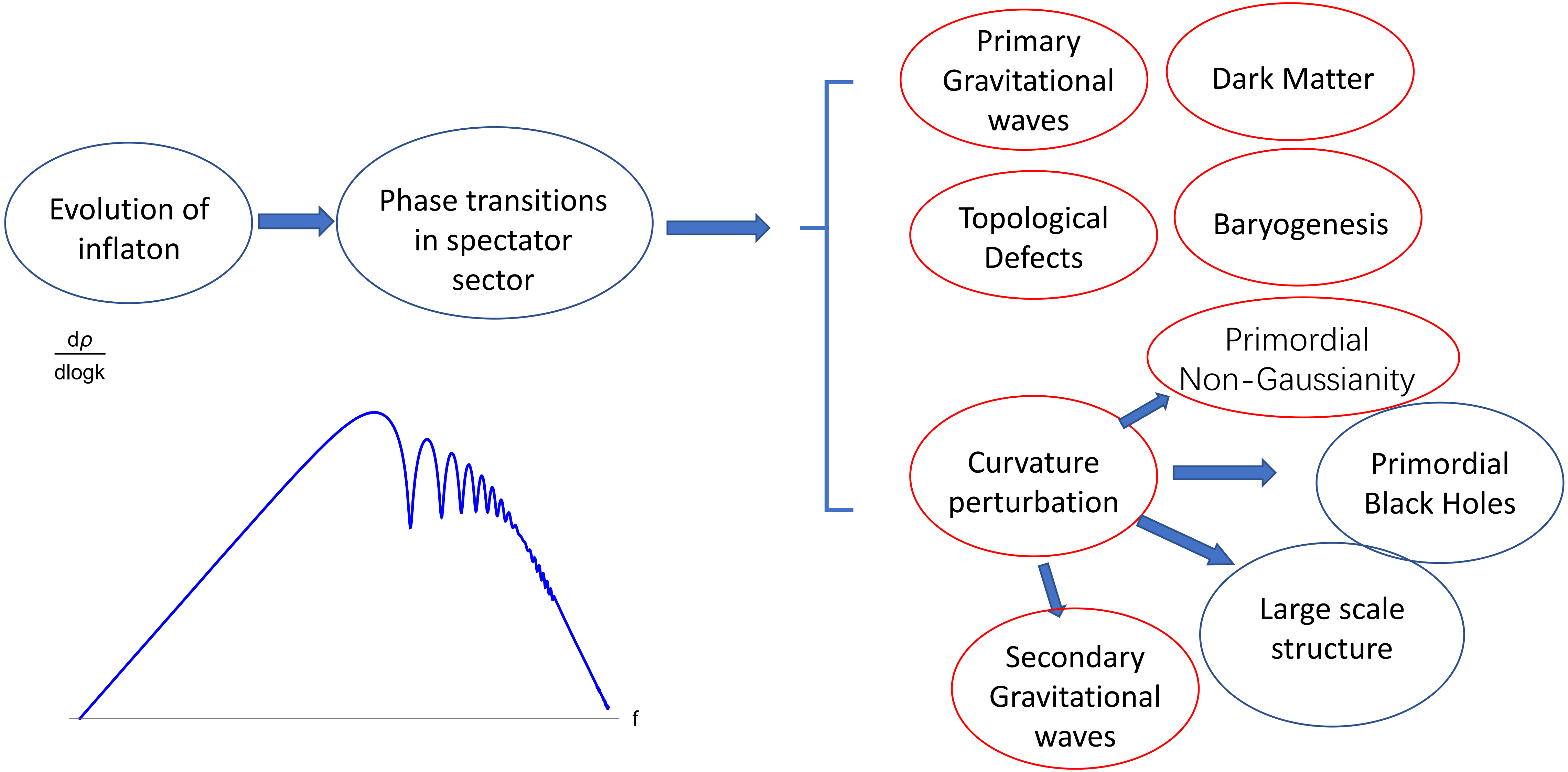
Observational challenges: astrophysics foreground

- Its better that the primordial GW spectrum has features.
- We need to accurately simulate the primordial GW spectrum and the foreground.

Computational challenges

- Two scales are important: the wall thickness and the size of the universe
- The bubble walls are usually boosted.
- We need larger box to get more accurate GW spectrum.
- It cost a lot of memory to do the simulation:
  - 1 double : 8bit
  - 1000^3 -> 8GB   1 field
  - Complex field
    - 2(phi)+2(pi)+9(Tij)+9(hij,sin)+9(hij,cos)+2(Fourier, auxi)+1(fft) ~ 250 GB

Summary



Evolution of inflaton
Phase transitions in spectator sector
dρ
dlogk
f
Primary Gravitational waves
Dark Matter
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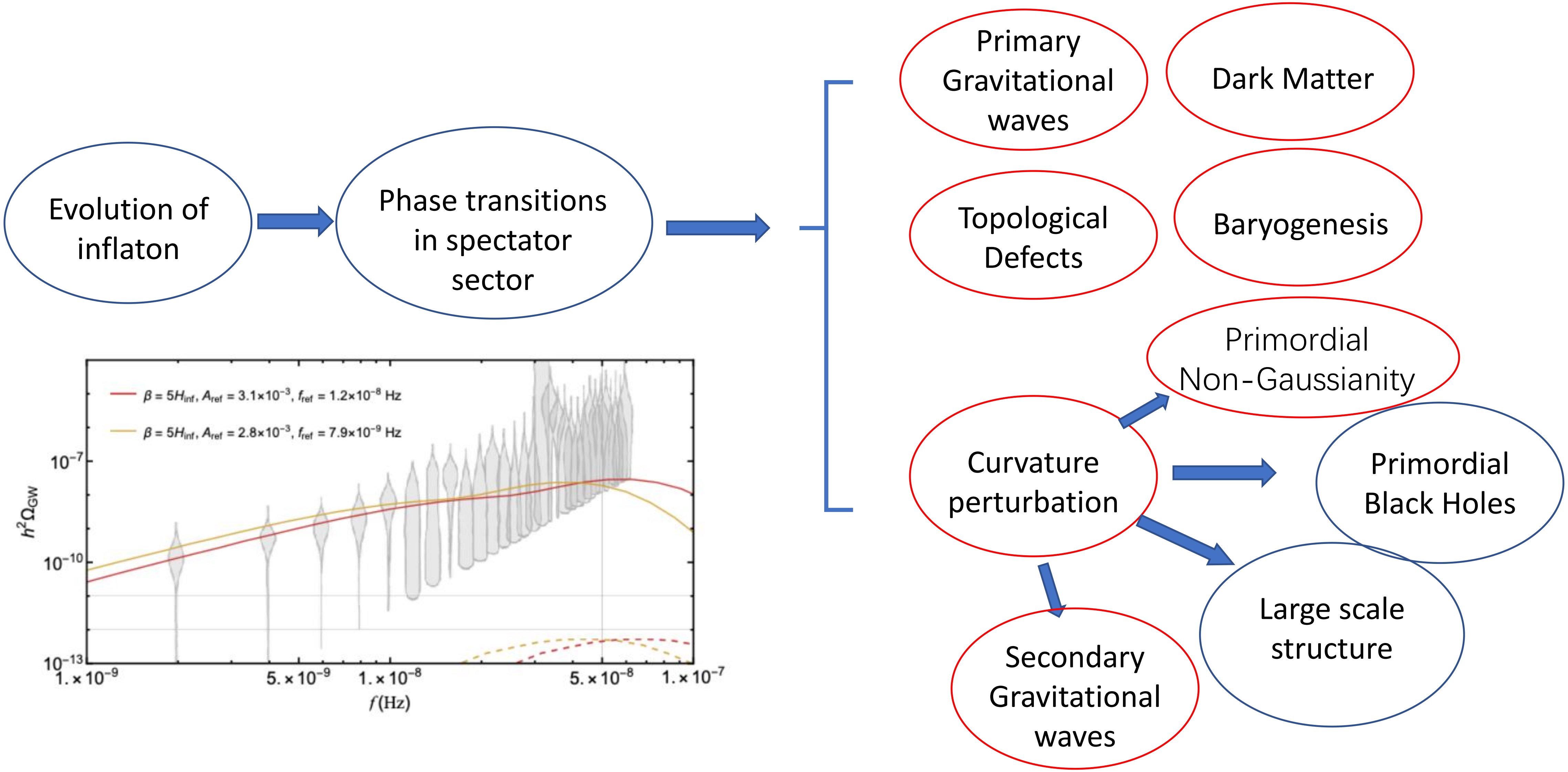
Primordial Non-Gaussianity
Curvature perturbation
Primordial Black Holes
Large scale structure
Secondary Gravitational waves

Summary



Evolution of inflaton
Phase transitions in spectator sector
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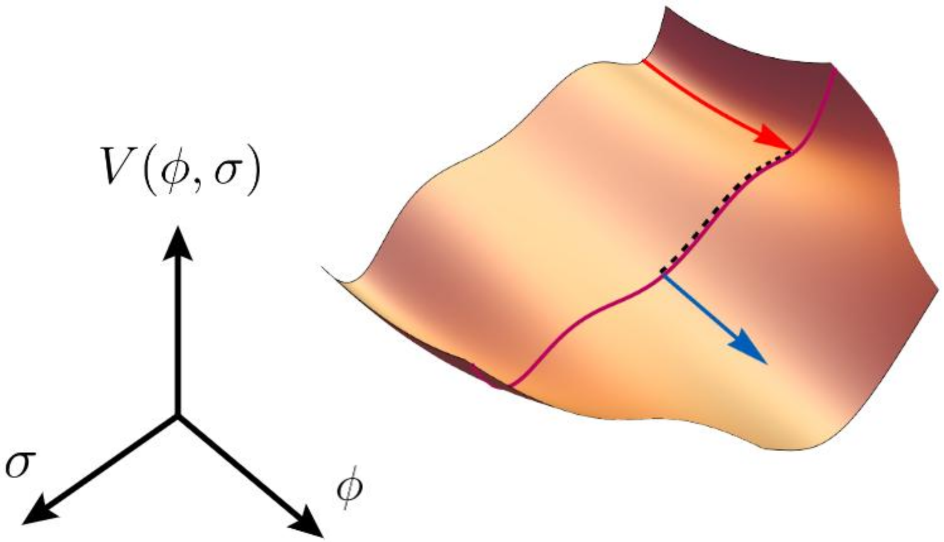
Large scale structure
Secondary Gravitational waves
$\beta = 5H_{\rm inf}$, $A_{\rm ref} = 3.1\times10^{-3}$, $f_{\rm ref} = 1.2\times10^{-8}$ Hz
$\beta = 5H_{\rm inf}$, $A_{\rm ref} = 2.8\times10^{-3}$, $f_{\rm ref} = 7.9\times10^{-9}$ Hz
$h^2\Omega_{\rm GW}$
10^{-7}
10^{-10}
10^{-13}
$1.\times10^{-9}$
$5.\times10^{-9}$
$1.\times10^{-8}$
$5.\times10^{-8}$
$1.\times10^{-7}$
f(Hz)

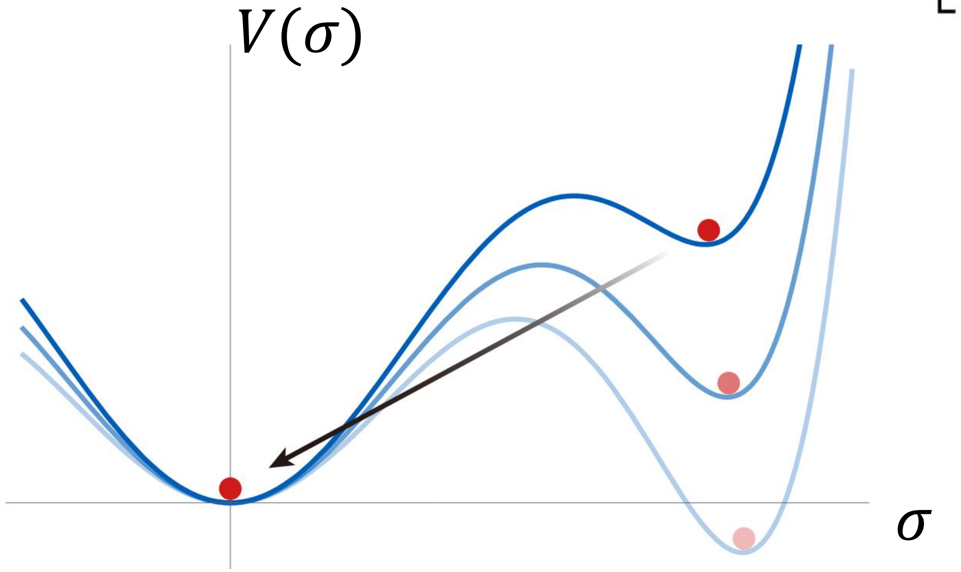
Summary

Primordial non-Gaussianity (quantum fluctuation)

- The evolution of ϕ_0 induces the evolution of σ_0.

$$\zeta = -H\left[\frac{\dot{\phi}_0\delta\phi + \dot{\sigma}_0\delta_\sigma}{\dot{\phi}_0^2 + \dot{\sigma}_0^2}\right]$$

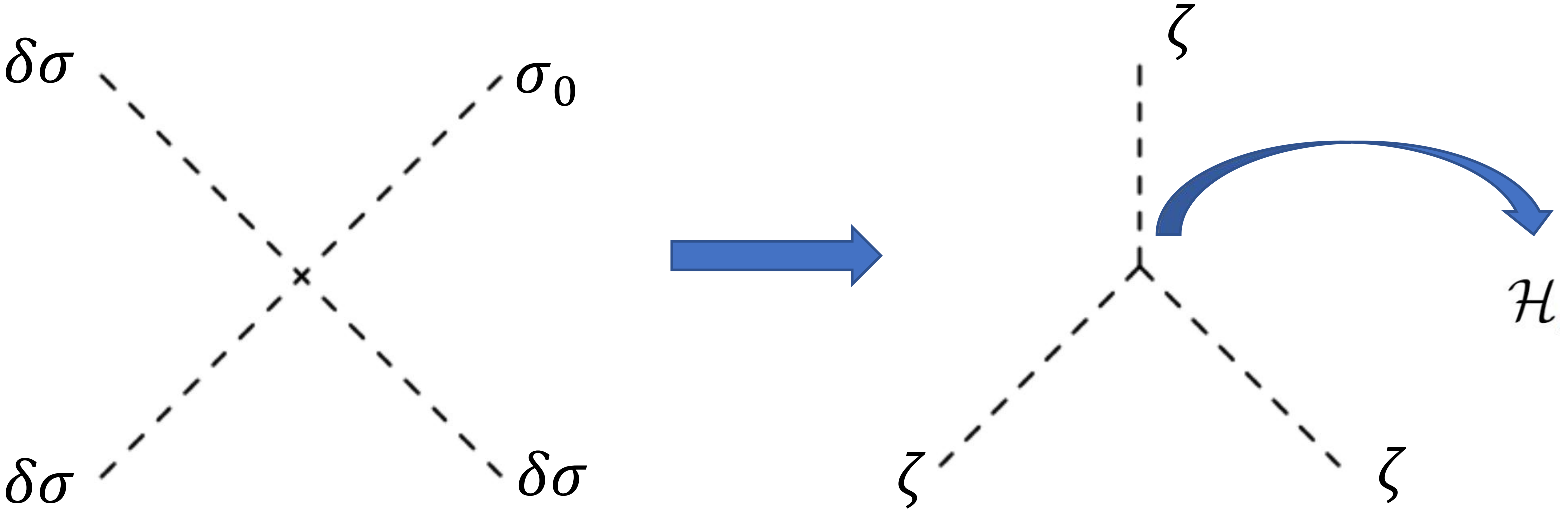




- $\delta\sigma$ also contributes to the curvature perturbation, and the interaction in the σ sector is strong.

Primordial non-Gaussianity (quantum fluctuation)

- 3pt function in the symmetry breaking phase



$$\mathcal{H}_I \sim \left[\frac{1}{6}\frac{\partial^3 V_\sigma}{\partial\sigma^3}\dot{\sigma}_0^3\right]\zeta^3$$

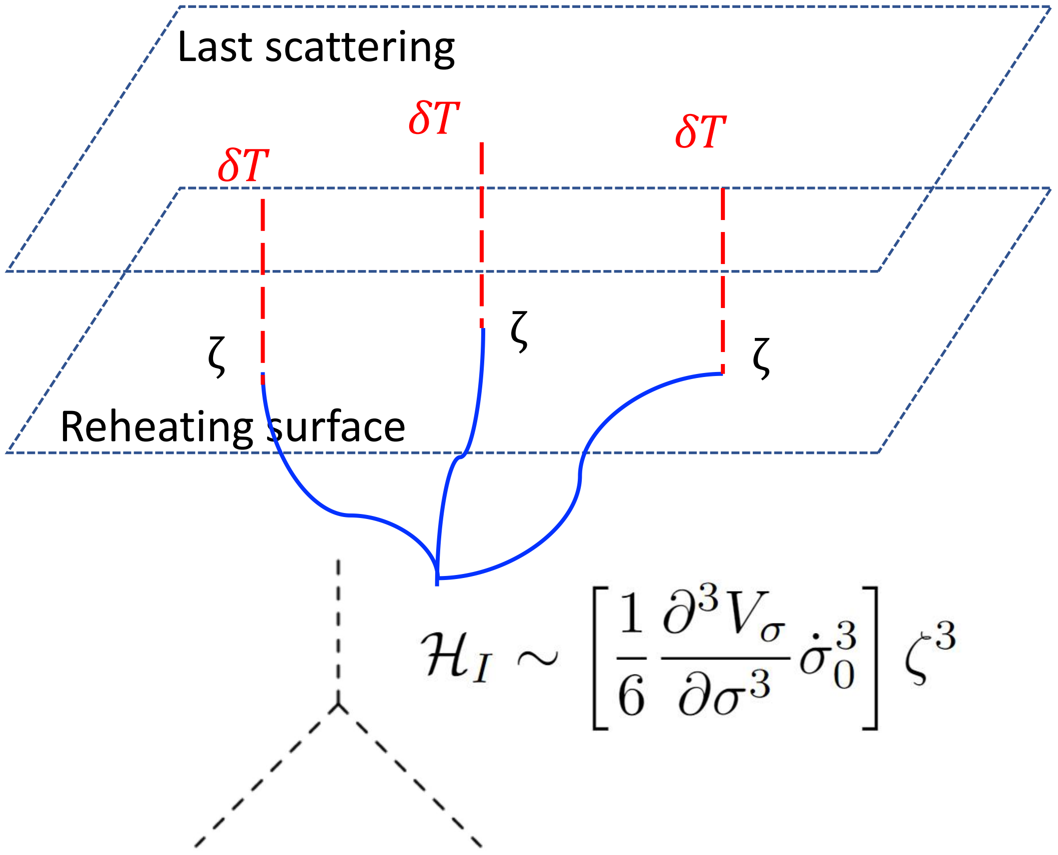
- Relevant operator, important in the IR.

Primordial non-Gaussianity

- Calculate the three-point function using the in-in formalism.

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t)\right] \cdots\right]\right]\right\rangle$$

S. Weinberg, hep-th/0506236



Primordial non-Gaussianity

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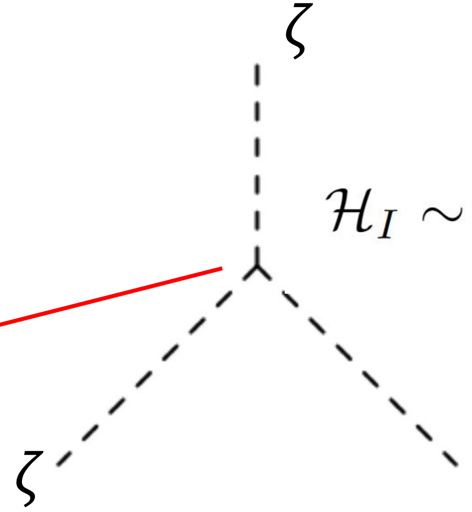
S. Weinberg, hep-th/0506236



$$\mathcal{H}_I \sim \left[\frac{1}{6}\frac{\partial^3 V_\sigma}{\partial\sigma^3}\dot{\sigma}_0^3\right]\zeta^3$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle' = \frac{3}{4}\int_{-\infty}^{0} \frac{d\tau}{\tau}\frac{H^8}{\dot{\phi}_0^6}\frac{\lambda(\tau)}{k_1^2 k_2^3 k_3^2} f(k_1, k_2, k_3)$$

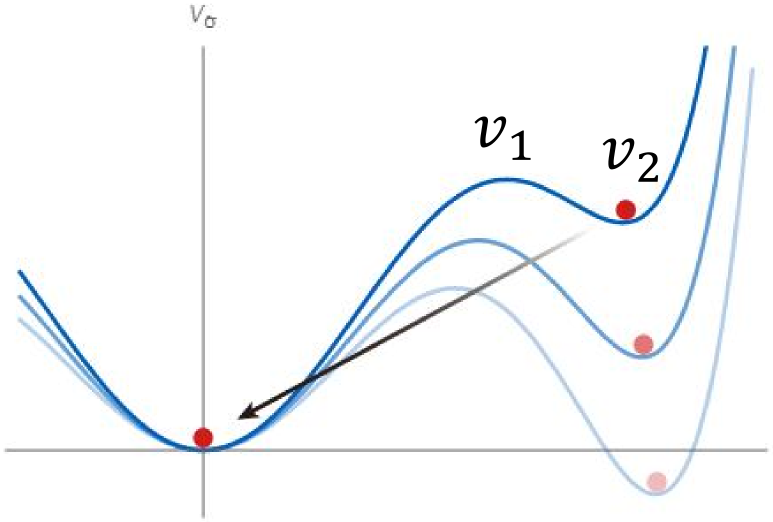
Dominated in the region
$|k_1\tau| \ll 1, |k_2\tau| \ll 1, |k_3\tau| \ll 1$

$$\mathrm{Re}\left[\left(1+\frac{i}{k_1\tau}\right)\left(1+\frac{i}{k_2\tau}\right)\left(1+\frac{i}{k_3\tau}\right)e^{i(k_1+k_2+k_3)\tau}\right]$$

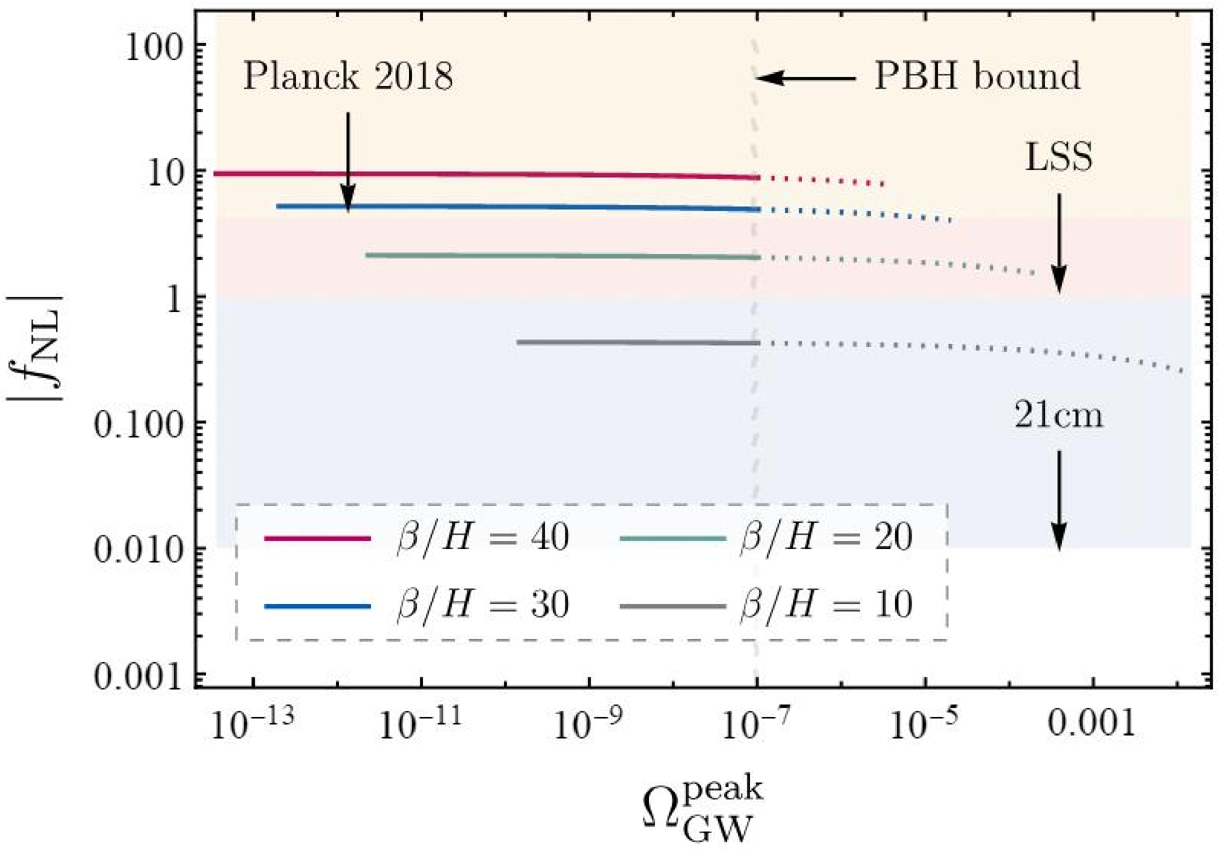
Primordial non-Gaussianity

$$f_{\rm NL} = \left(\frac{\beta_*}{H}\right)^3 \left(\frac{L}{\rho_{\rm inf}}\right) \left(\frac{\Delta_\star}{\mathcal{S}_{E\star}}\right)^3 \mathcal{F}(\Delta_\star)$$

$$\Omega_{\rm GW} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^4$$

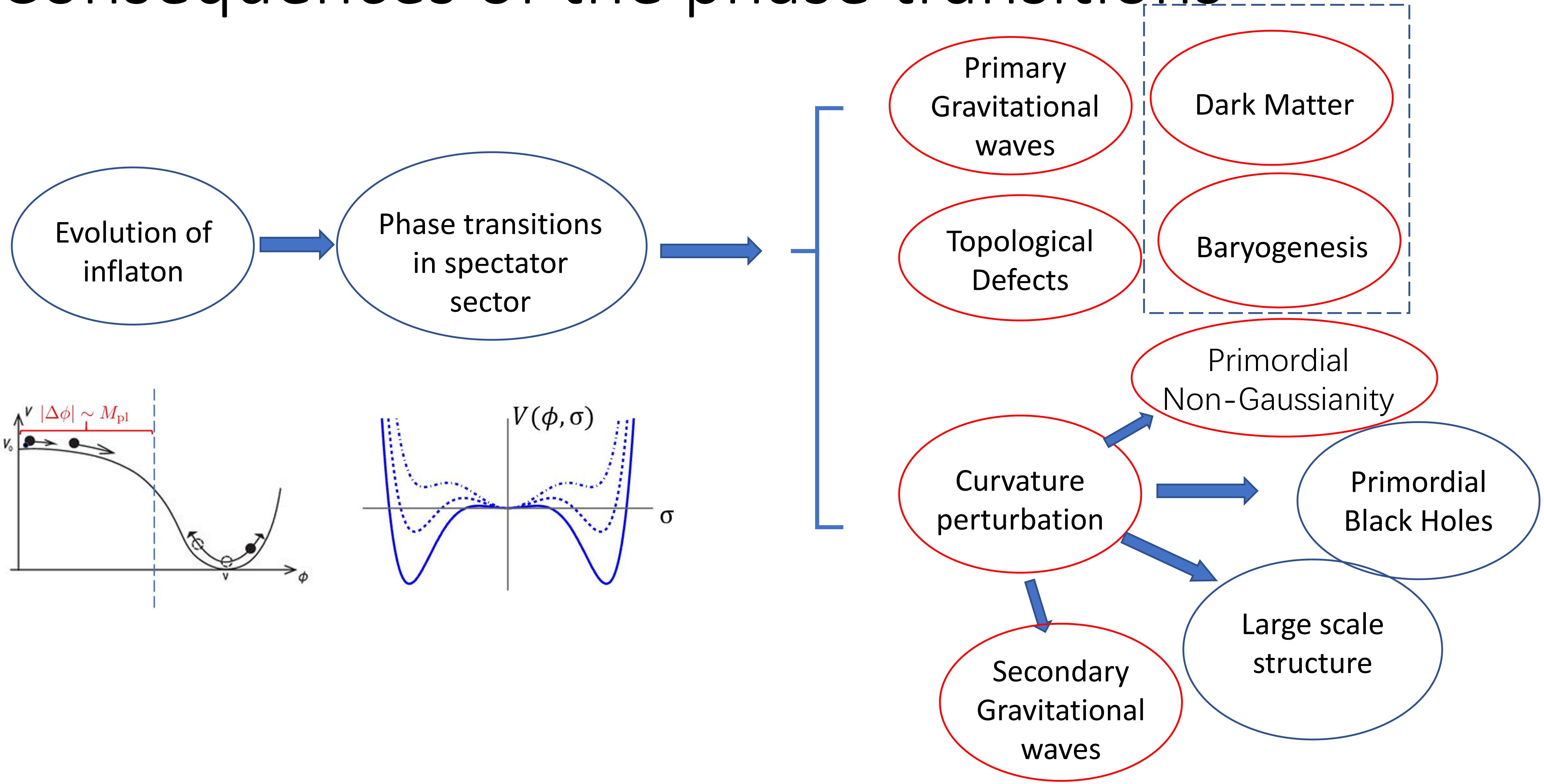


$$\frac{v_2}{v_1} = 1 + \Delta$$



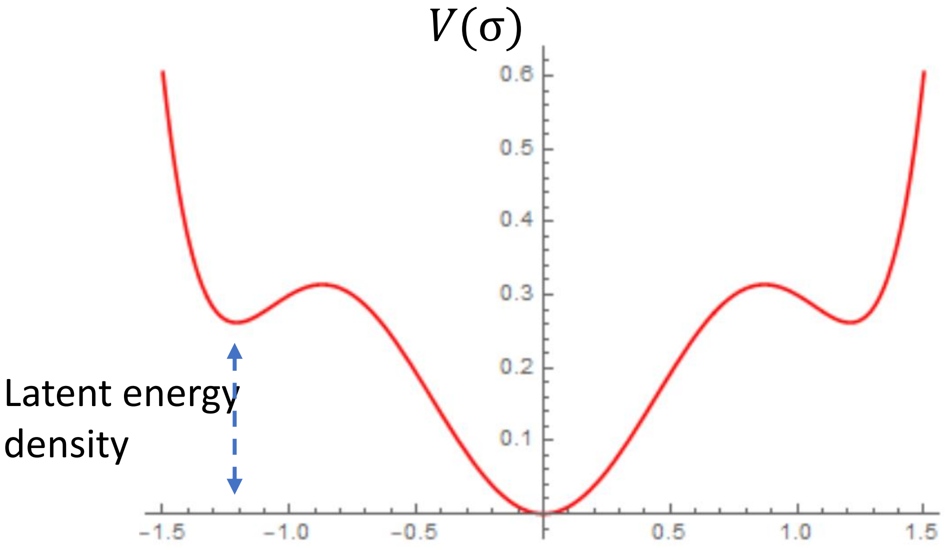
HA, Qi Chen, Yuhang Li, Yuan Yin, in progress

Consequences of the phase transitions



Producing superheavy DM

- Where does the latent energy go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.



Producing superheavy DM

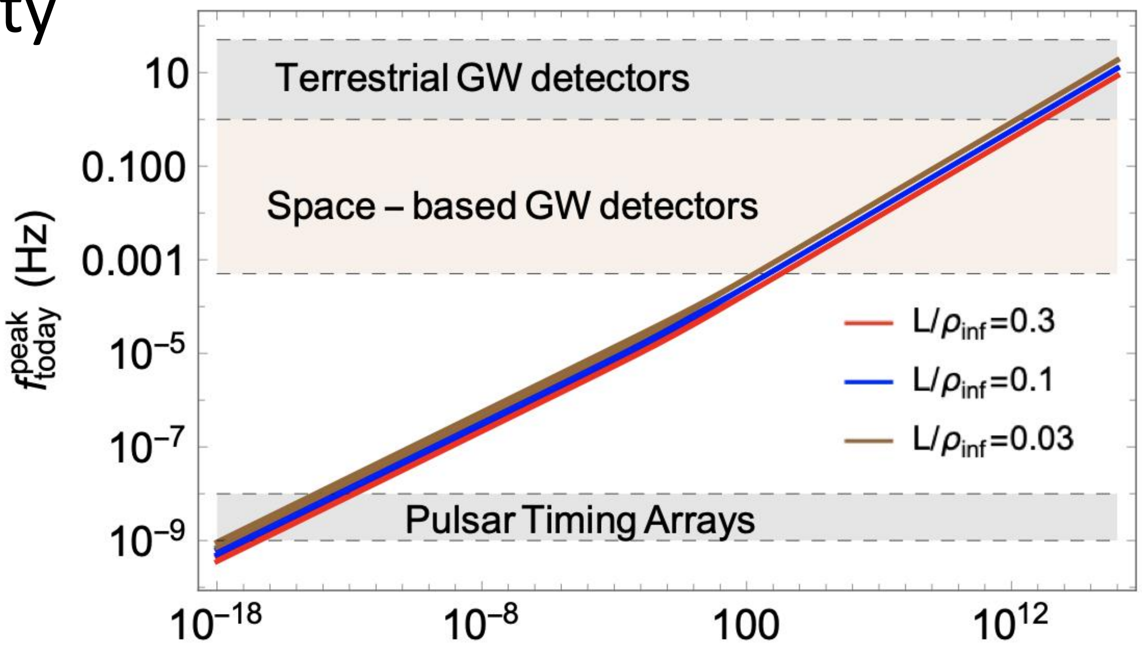
- Today's dark matter energy density

$$\rho_{DM}^{(0)} \approx \Delta\rho_{\text{vac}} e^{-3(N_\star - N_{\text{aftar}})}$$

$$\Omega_{\text{DM}} \sim \frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \times \eta^{-1} \times e^{-3N_\star}$$

$$\boxed{\eta = \frac{n_B^{(0)}}{n_\gamma} \approx 10^{-9}}$$

$$f_{\text{today}}^{\text{peak}} \sim \frac{1}{2\pi} \left(\frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}}\right)^{-1/3} (H_{\text{inf}} H_0^2)^{1/3}$$



m_{DM}~1 GeV m_{DM}~10^{15} GeV

HA, Xi Tong, Siyi Zhou, 2208.14857

Baryogenesis during inflation

- Conserved numbers must be diluted as a^{-3}, even for spontaneously broken symmetries.
- For a U(1) number to survive inflation, it must be broken explicitly.
- In our model:

$$\mathcal{L}_{\text{dim-5}} = -\frac{i}{\Lambda}\partial_\mu\phi(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi)$$

$$-i\frac{\dot{\phi}_0}{\Lambda}(\chi\dot{\chi}^* - \chi^*\dot{\chi})$$

	ϕ	χ	σ
$U(1)$	0	1	0
$\mathcal{Z}_2$	1	1	-1

- Explicit U(1) breaking term: $A\sigma^2\chi + h.c.$

Trivial if no explicit broken.

Baryogenesis during inflation

- We use the phase transition of σ as a switch:
- In the Z_2breaking phase: $A\sigma^2\chi \to A\sigma_0^2\chi$, a tadpole for χ.

$$\sqrt{-g}\mathcal{L} = a^3|\dot{\chi}|^2 - a|\partial_i\chi|^2 + ia^3\mu(\chi\dot{\chi}^* - \chi^*\dot{\chi}) - a^3\left(m_\chi^2|\chi|^2 - Av_\sigma^2(\chi+\chi^*)\right)$$

Chemical potential

Initial U(1) number density:

$$n_\chi^{(\mathrm{ini})} = -2\mu v_\chi^2 = -\frac{2\mu A^2 v_\sigma^4}{m_\chi^4 + 9H^2\mu^2}$$

does not dilute with inflation!

Baryogenesis during inflation

- We use the phase transition of σ as a switch:
- In the Z_2restored phase:
  - No tadpole for χ, the U(1) breaking interactions become perturbative.
  - We need to consider the washout effects from the explicit breaking term.
  - We need to further engineer the model to transfer this U(1) number to the baryon number.
- The phase transition happened in a very short period ($\beta \gg H$), the change of U(1) number during the phase transition can be neglected.

Baryogenesis during inflation

- Today's baryon number

$$n_B^{(0)} = \frac{2\mu A^2 v_\sigma^4}{m_\chi^4 + 9H^2\mu^2} z_{\rm ph}^{-3}$$

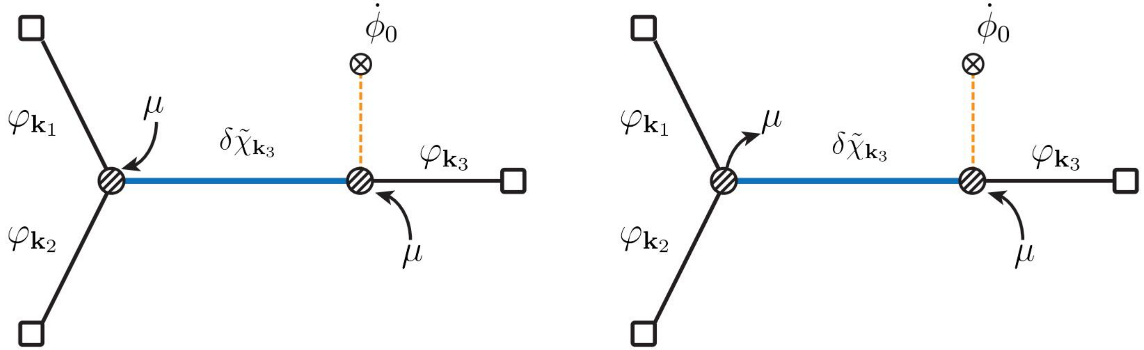
$$\eta = \frac{n_B^{(0)}}{n_\gamma} \approx 10^{-9} \times \left(\frac{H}{10^{14}\ \mathrm{GeV}}\right)^{-1/2} \times \frac{c_A^2 c_\mu \theta}{9c_\mu^2 + c_{m_\chi}^4} \times e^{-(3N_e - 29)}$$

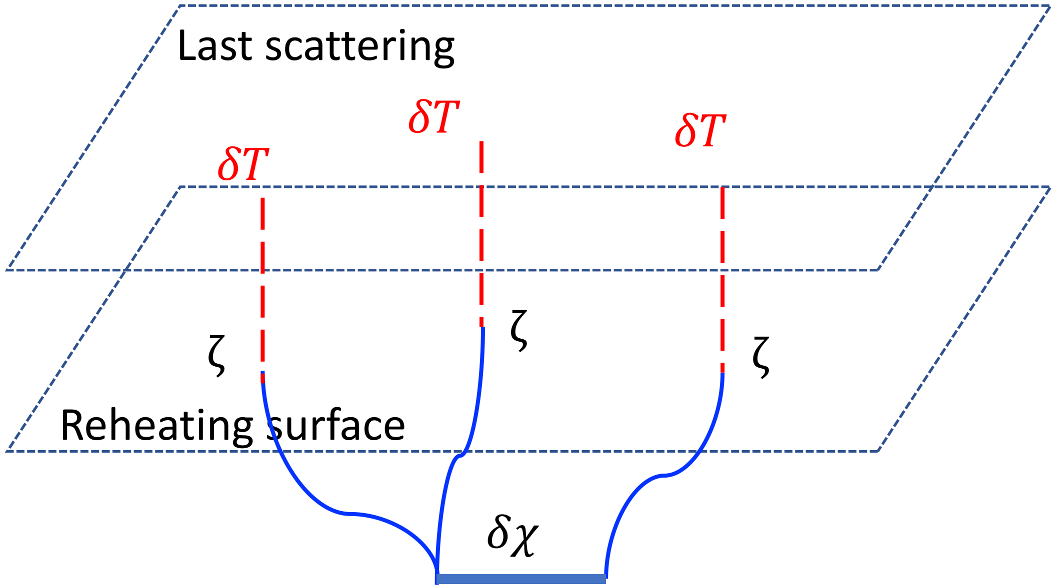
$$c_A = \frac{A}{H}\ ,\ \ c_\mu = \frac{\mu}{H}\ ,\ \ c_{m_\chi} = \frac{m_\chi}{H}\ ,\ \ \theta = \frac{v_\sigma^4}{\rho_{\rm inf}}$$

HA, Qi Chen, Yuan Yin, 2409.05833

Baryogenesis during inflation

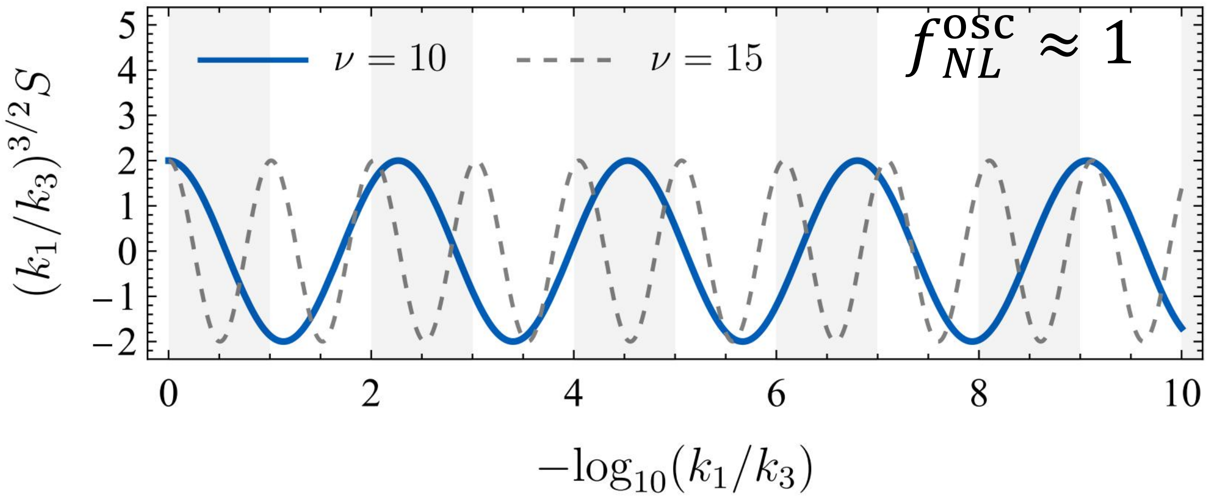
- Cosmological collider signal
  Bodas, Kumar, Sundrum, 2010.04727



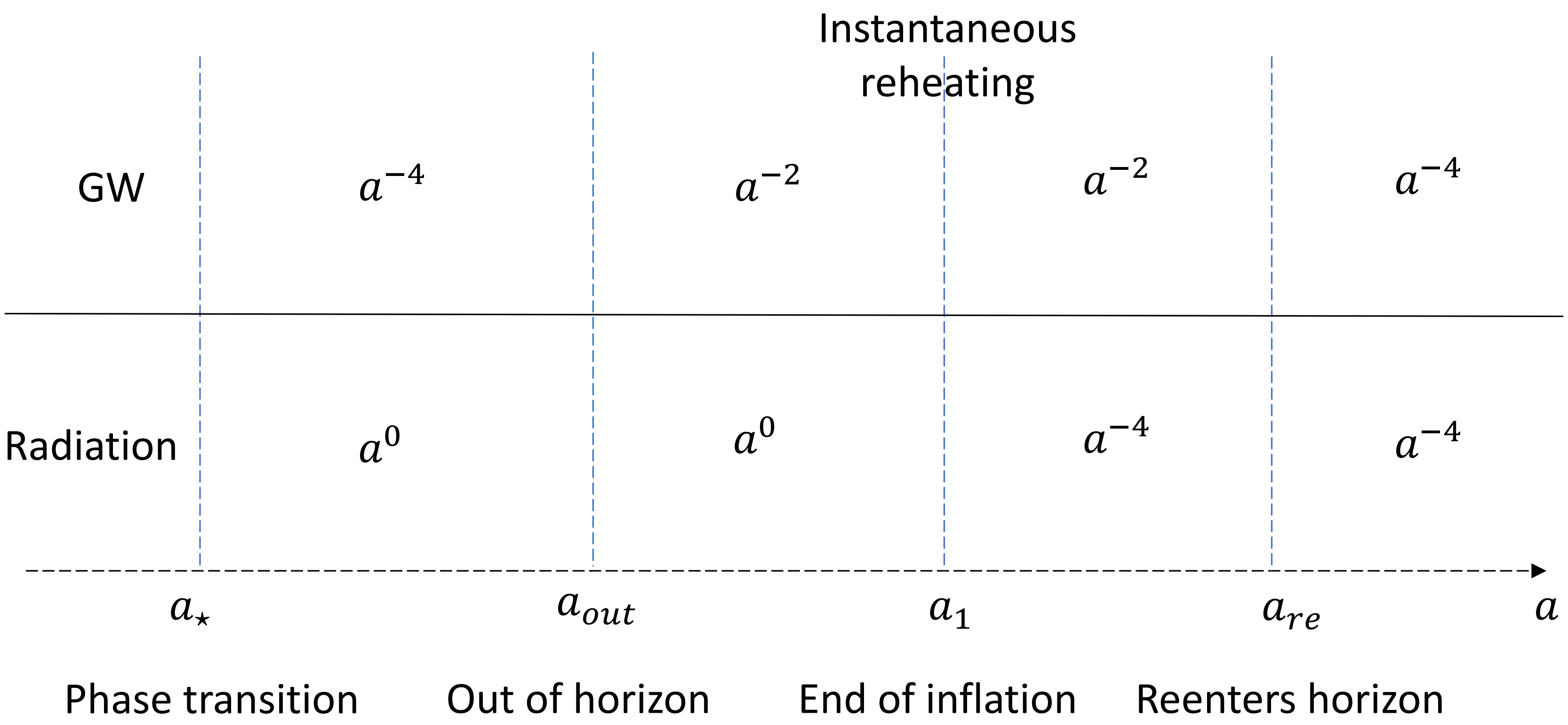


$$\nu \equiv \sqrt{m_{\text{eff}}^2/H^2 - 9/4}$$

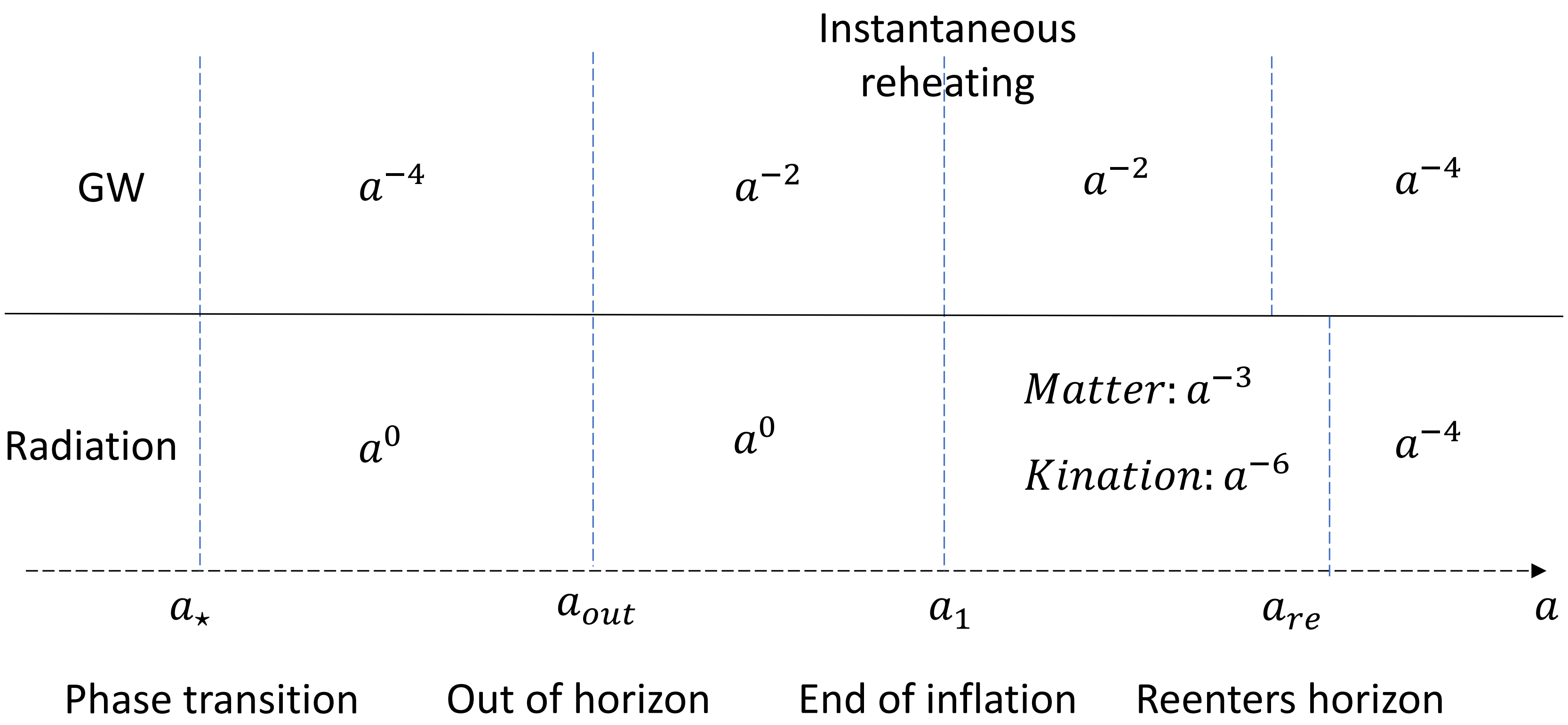
$$m_{\text{eff}}^2 \equiv m_\chi^2 + \mu^2$$



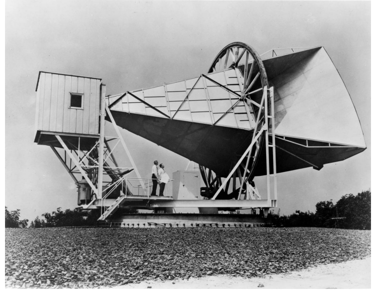
Redshifts of the GW signal

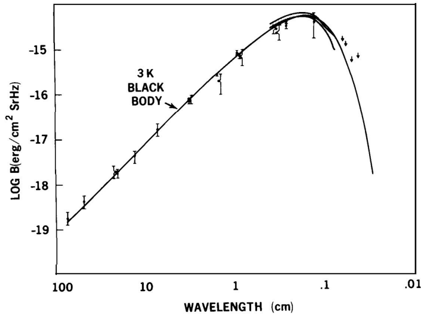


Intermediate stages between inflation and reheating

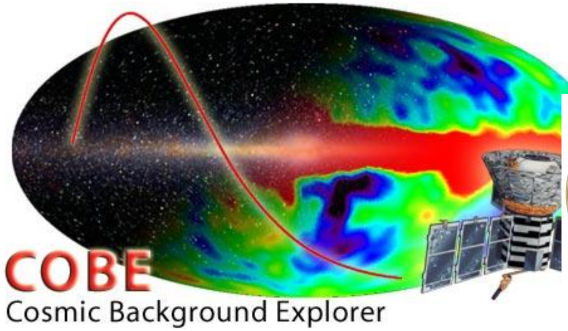


Why do we need inflation?

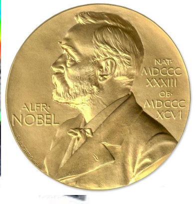




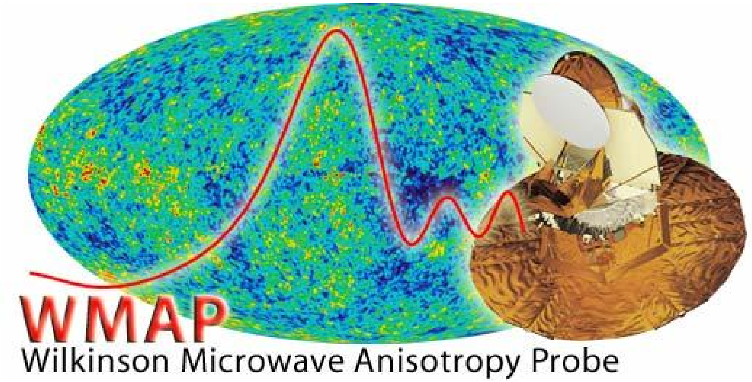
Bell Laboratory Penzias and Wilson 1964



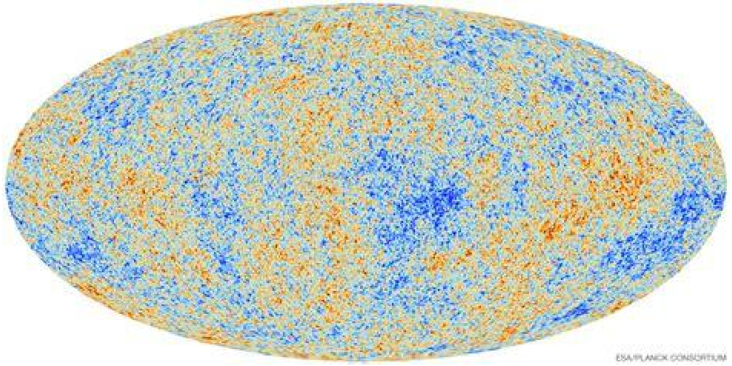
NASA 1989-1996



2006

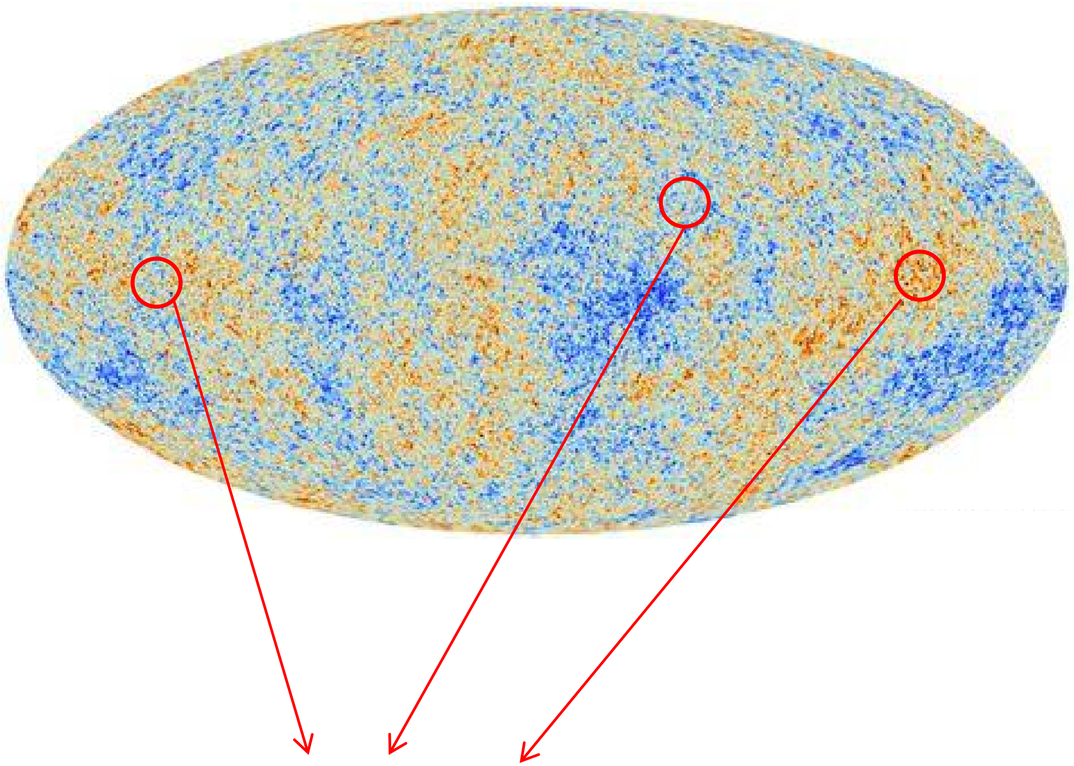


NASA 2001-2010

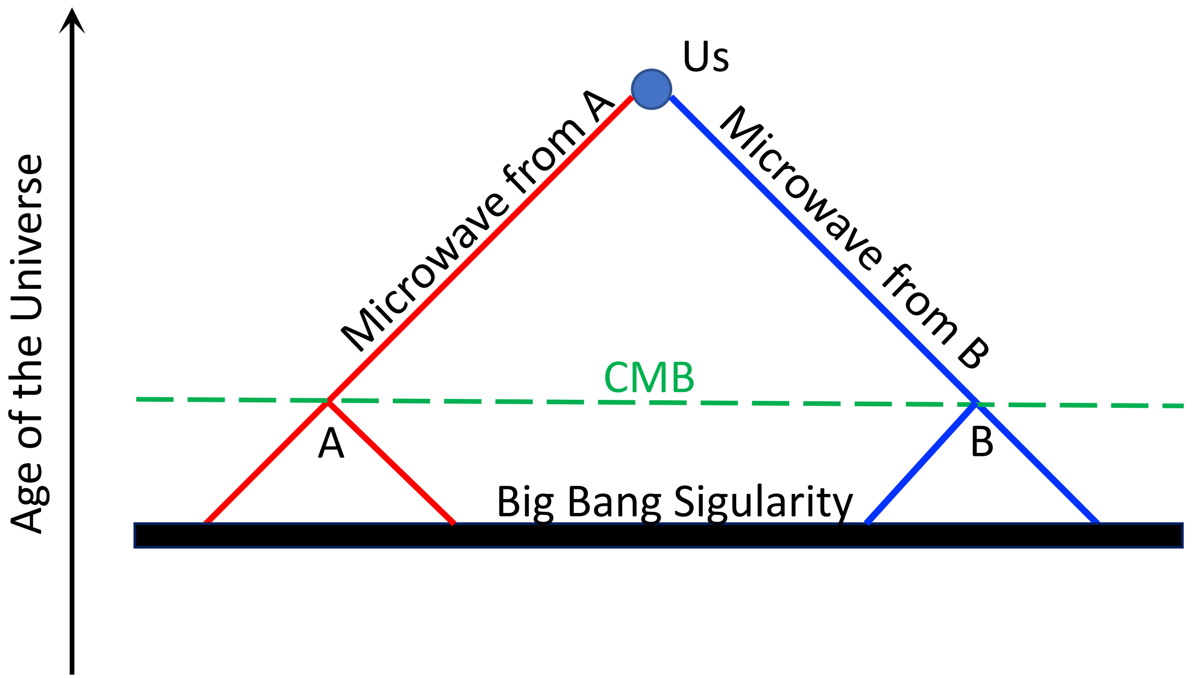


European space agency
Planck spacecraft

The causality problem

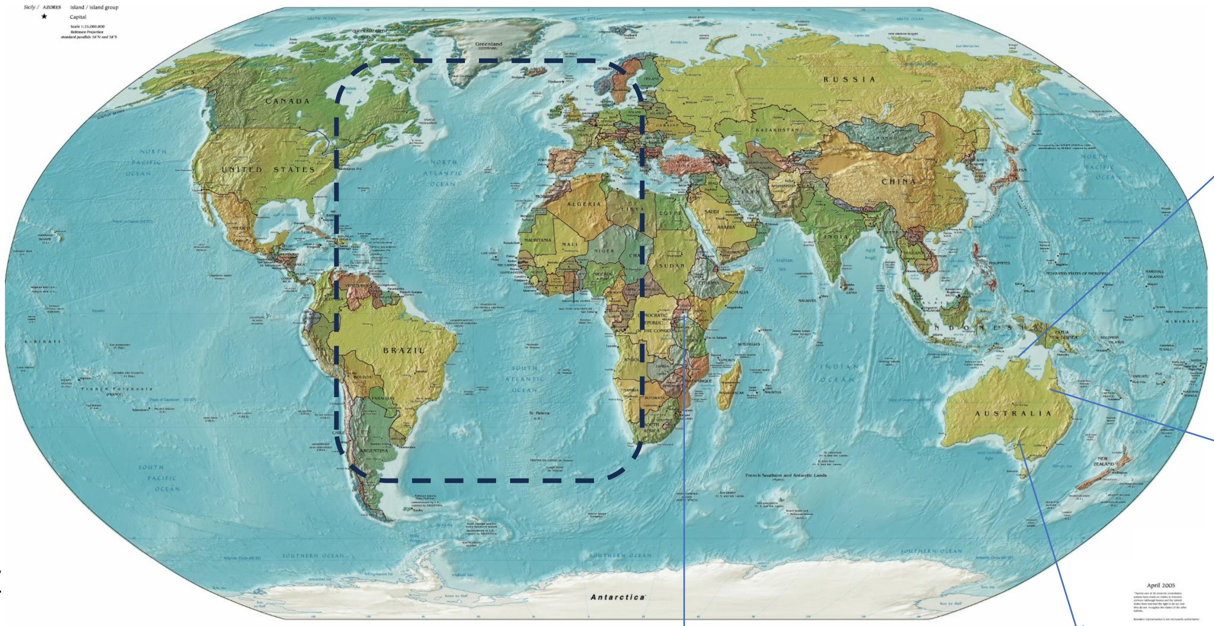


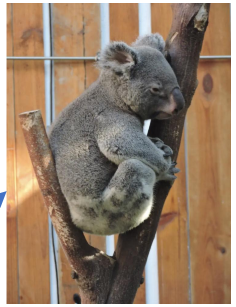
Same temperature and similar fluctuations.



Causality problems usually indicate big discoveries!

Alfred Wegener: Continental drift hypothesis





Animals with brood pouch

育儿袋

ostrich

emu

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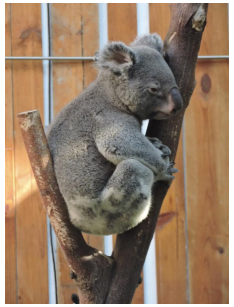

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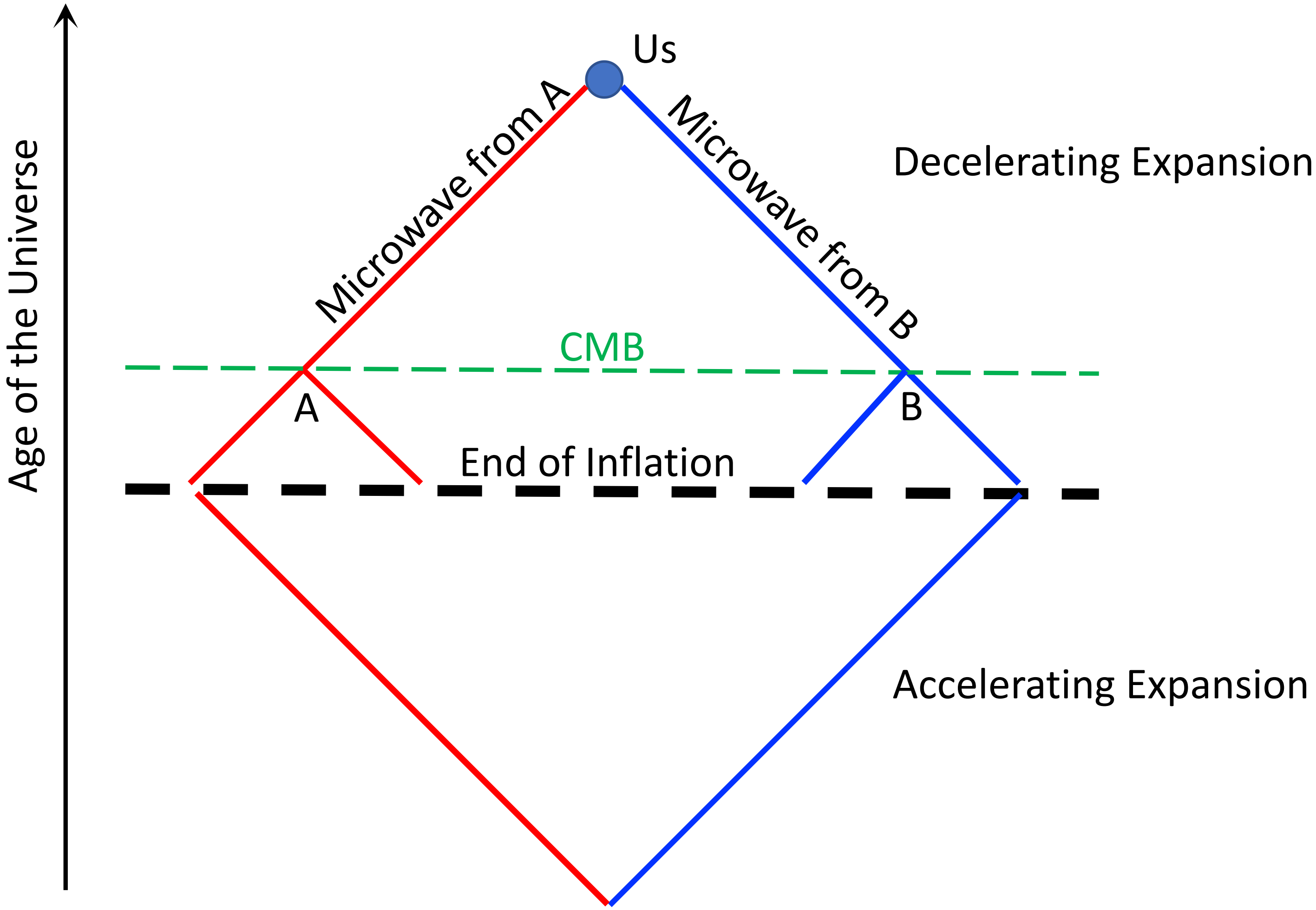
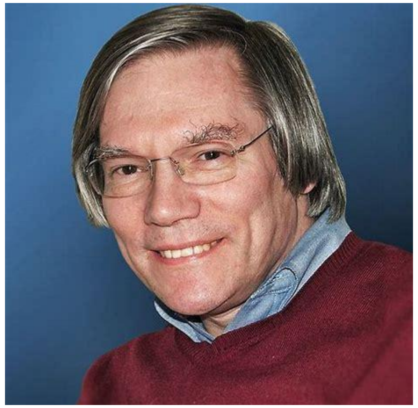
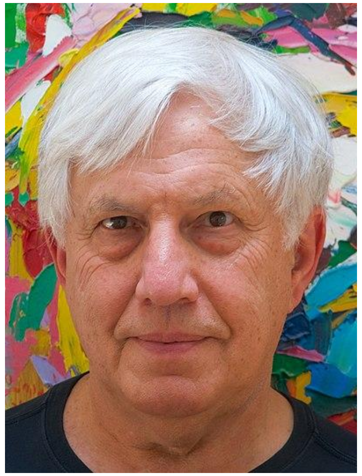
Inflation theory

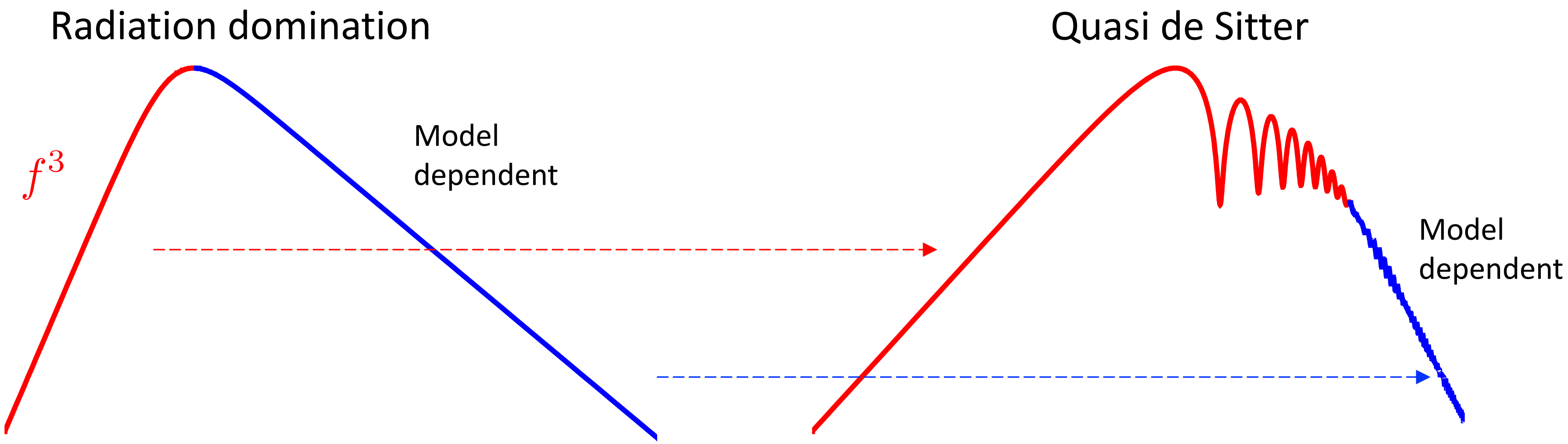






Backups

Spectrum distortion by inflation



GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Traceless and transverse

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

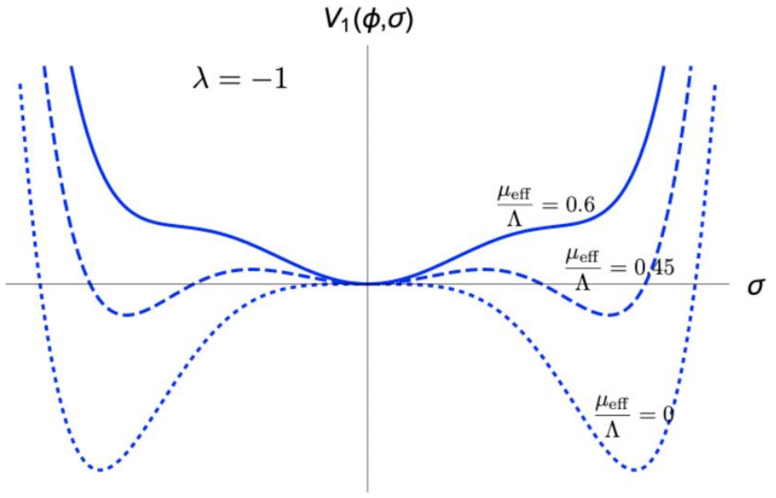
- For an instantaneous and local source,

$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

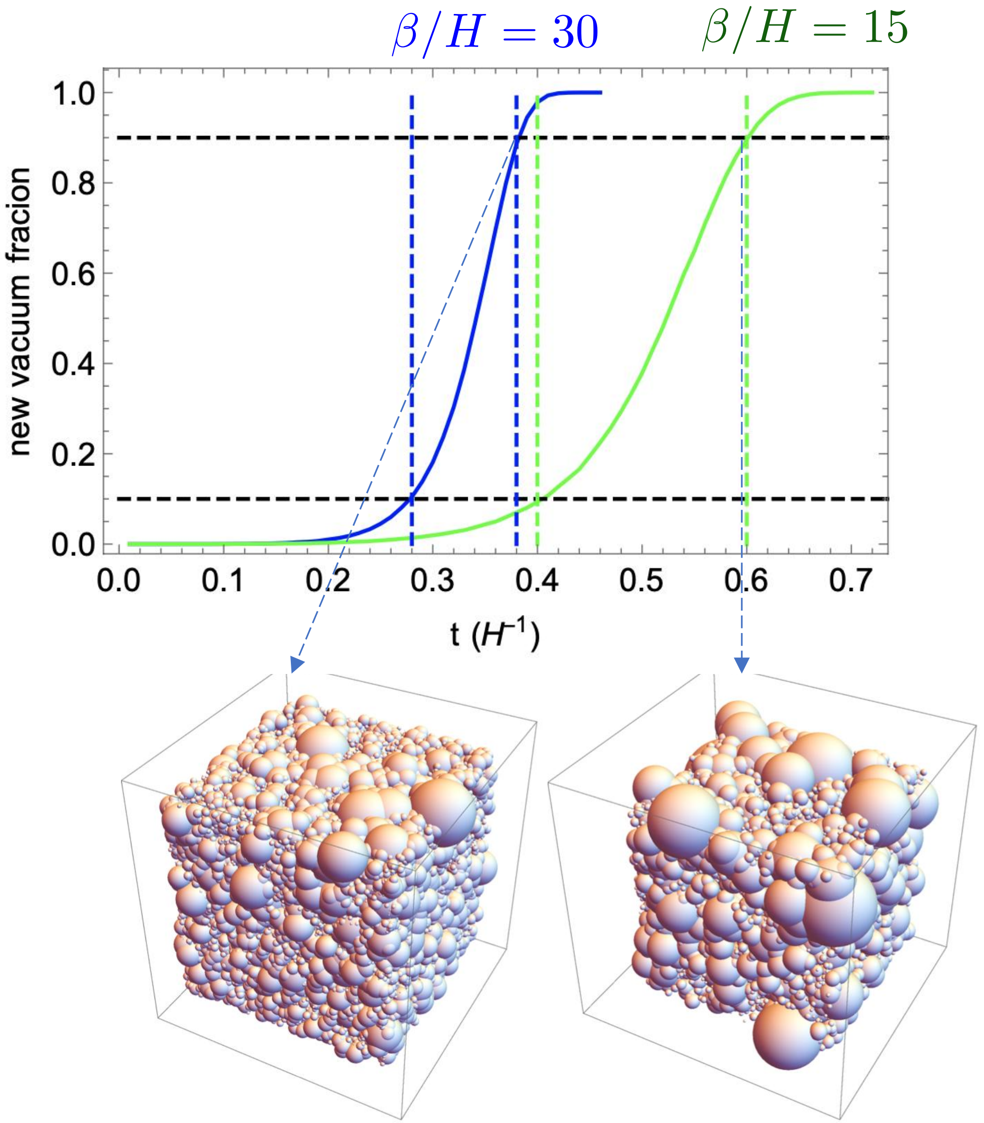
- E.O.M. in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

First-order phase transition during inflation

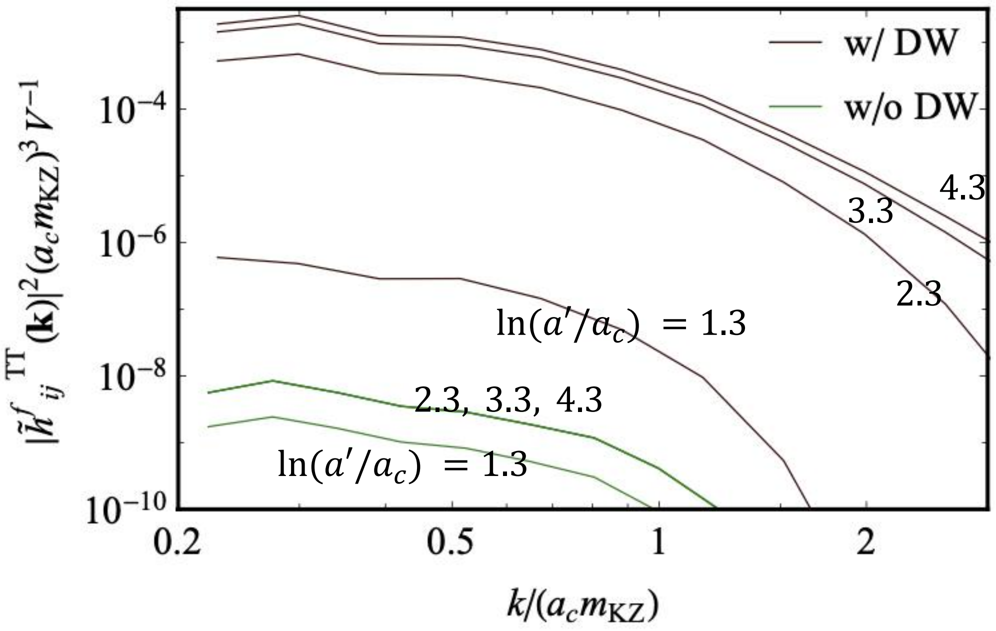


S_4 becomes smaller during



- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

Calculation of GWs



$$\tilde{h}^{f}_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k}\int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau')\tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$

With domains, the dominant contribution to $\tilde{h}^f$ happens around $\ln(a'/a_c) \sim 2$ to 3.

Without domains ($\delta\sigma \to |\delta\sigma|$), the dominant contribution to $\tilde{h}^f$ stops around $\ln(a'/a_c) \sim 2$, and the magnitude is much smaller.

The dominant contribution to GWs is from domain walls.

Formation of domain walls

- Landau-Ginzburg type

$$V = -\frac{1}{2}m_{\rm eff}^2\sigma^2 + \frac{\lambda}{4}\sigma^4$$

$$m_{\rm eff}^2 = y\phi^2 - m^2$$

Inflaton field

$$m^3_{\rm KZ(B)} = -y a_c \frac{d\phi_0^2}{d\tau} = \frac{2^{3/2}\varepsilon^{1/2}m^2 H M_{\rm pl}}{\phi_0(\tau_c)}$$

Murayama & Shu, 0905.1720

$$H^2 \ll m^2_{\rm KZ} \ll m^2$$

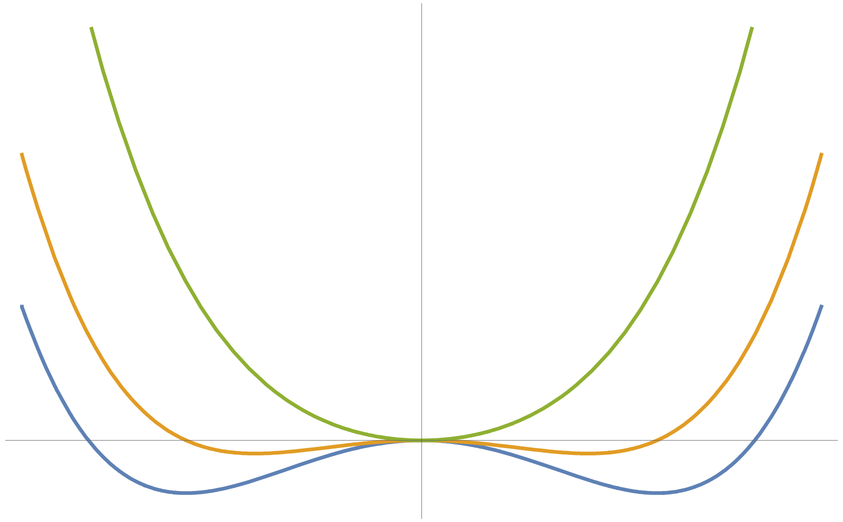
- Kibble-Zurek mechanism

c for critial

$$V_{\rm KZ} = -\frac{1}{2}m^3_{\rm KZ}a_c^{-1}(\tau-\tau_c)\sigma^2 + \frac{\lambda}{4}\sigma^4$$

- $m_{\rm KZ}$ determines the average distances between the domain walls.

Kibble 1976, Zurek 1985



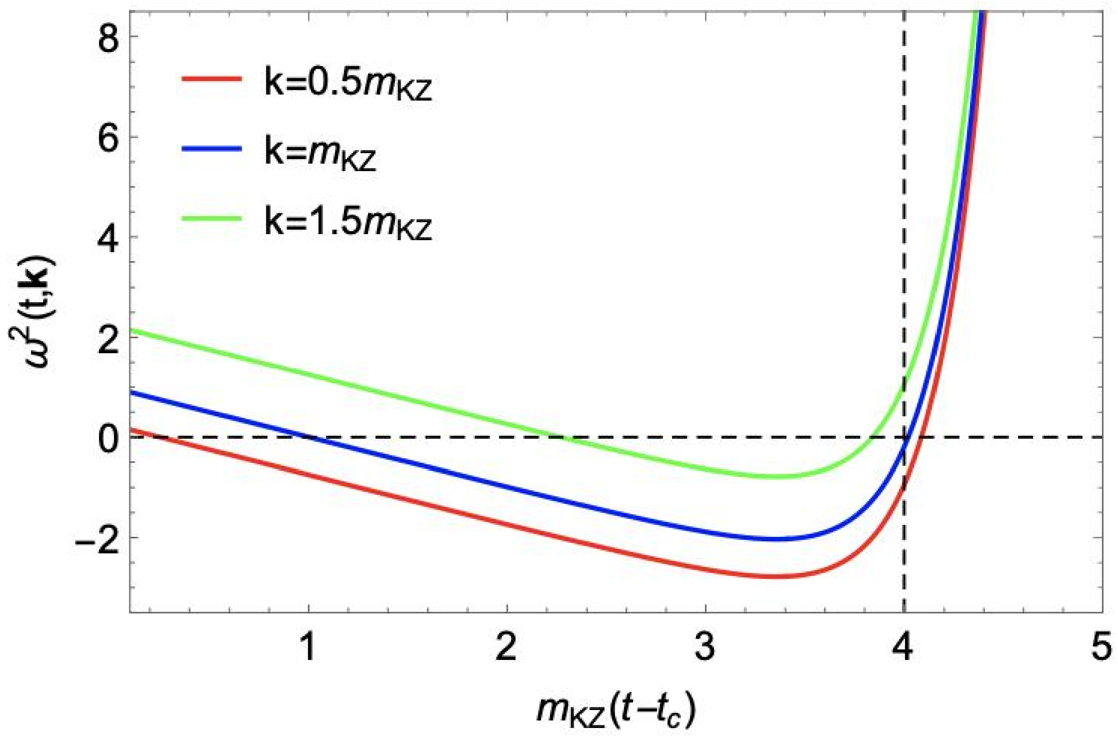
Formation of domain walls

- Stop of the tachyonic growth

$$k^2 - a_c^2 m_{\mathrm{KZ}}^3(\tau - \tau_c) + \frac{\lambda}{2}\langle \sigma^2(\tau, \mathbf{x})\rangle$$

Growth exponentially

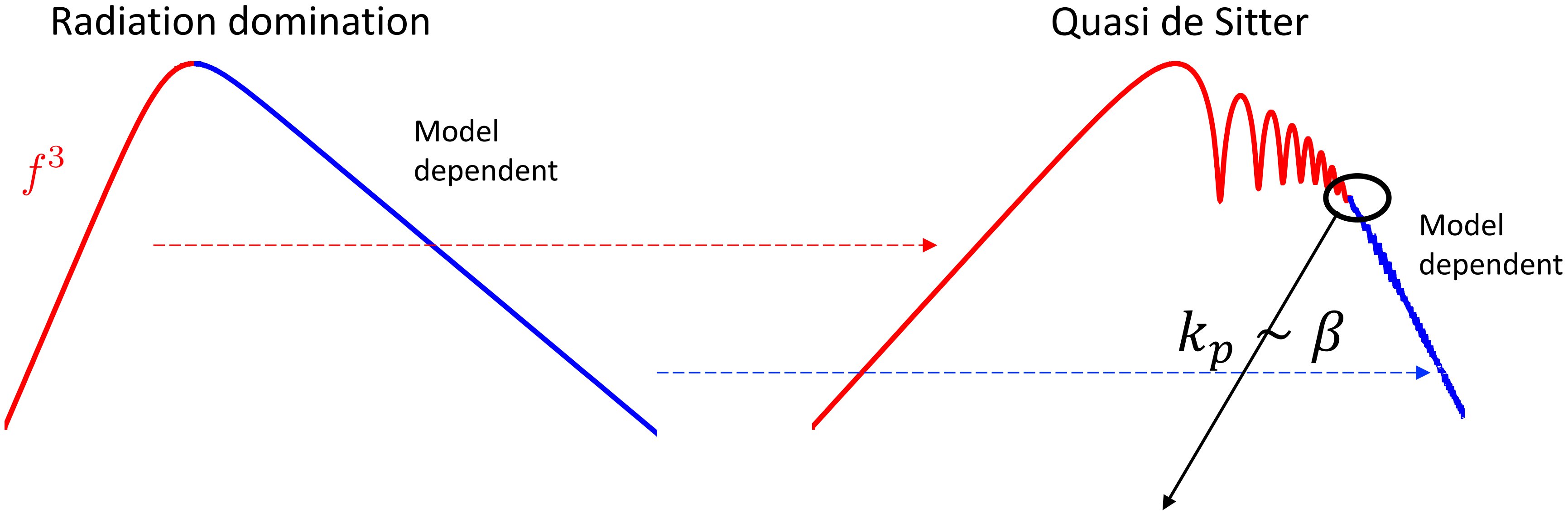
- Only modes with k smaller than about m_{KZ} can have a chance to grow exponentially.



Outlook

- The fate of the domain walls.
- Other topologcial defects.
- Application to high scale particle physics models.
- Baryogenesis (work in progress)

Spectrum distortion by inflation

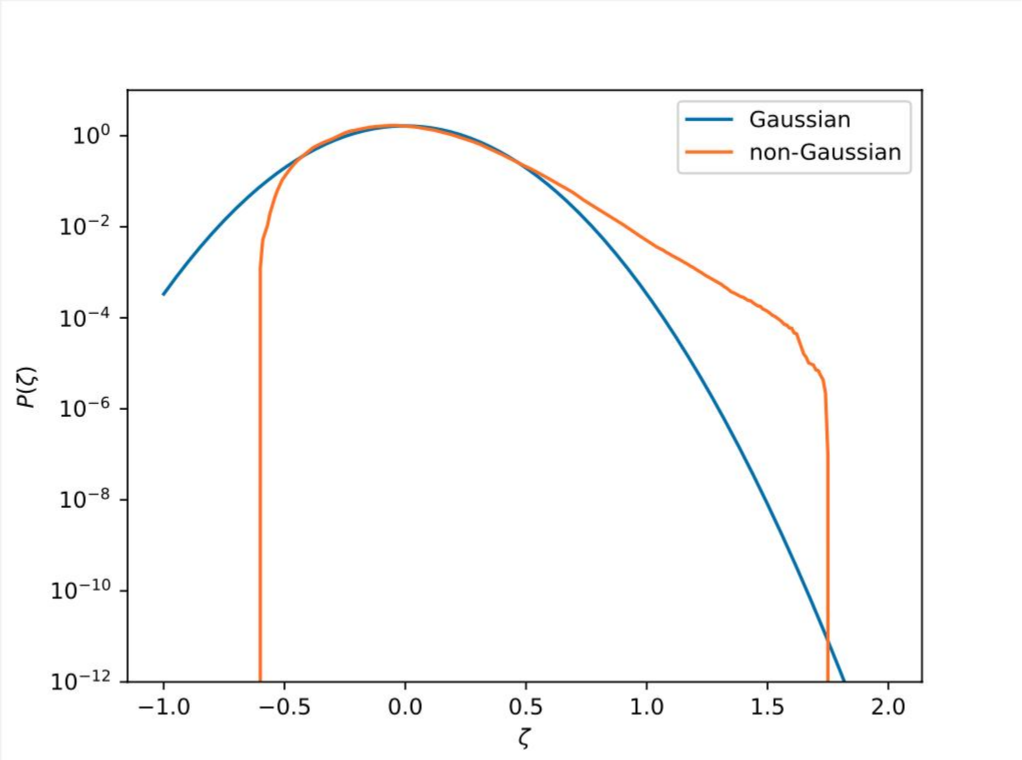


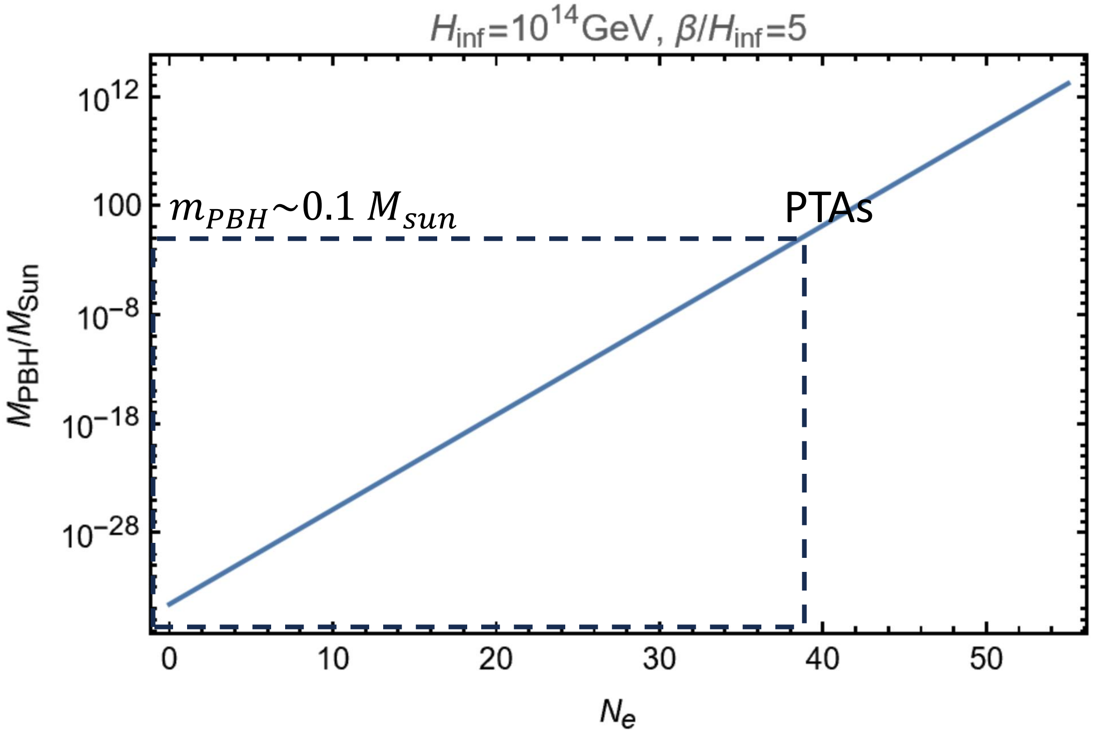
$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

Primordial Black Holes

HA, Boye Su, Lian-Tao Wang, Chen Yang, work in progress

- PBHs will form if $\Delta_\zeta^2 \sim 0.01$
- The power spectrum is highly non-Gaussian



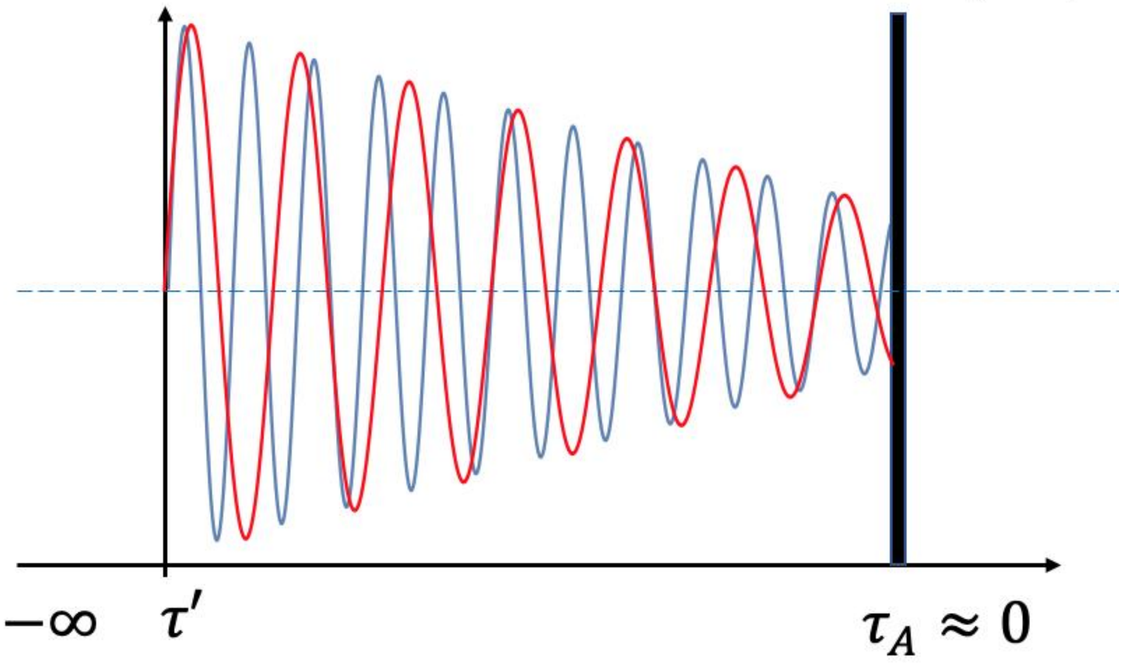


GW from instantaneous and local sources (qualitative study)

- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon oscillates with k.
- h^f is the initial condition for

  later evolution.

$h^f \sim \sin[k\tau']$



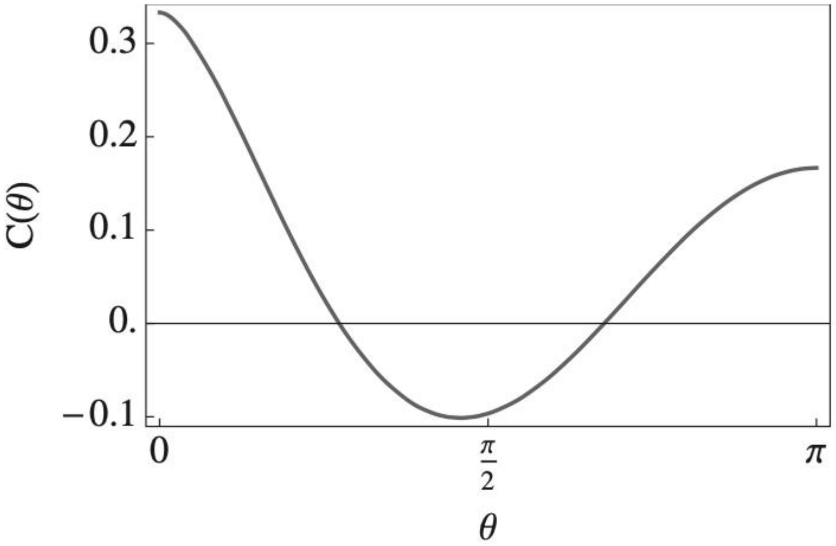
$k\tau_A \approx 0$

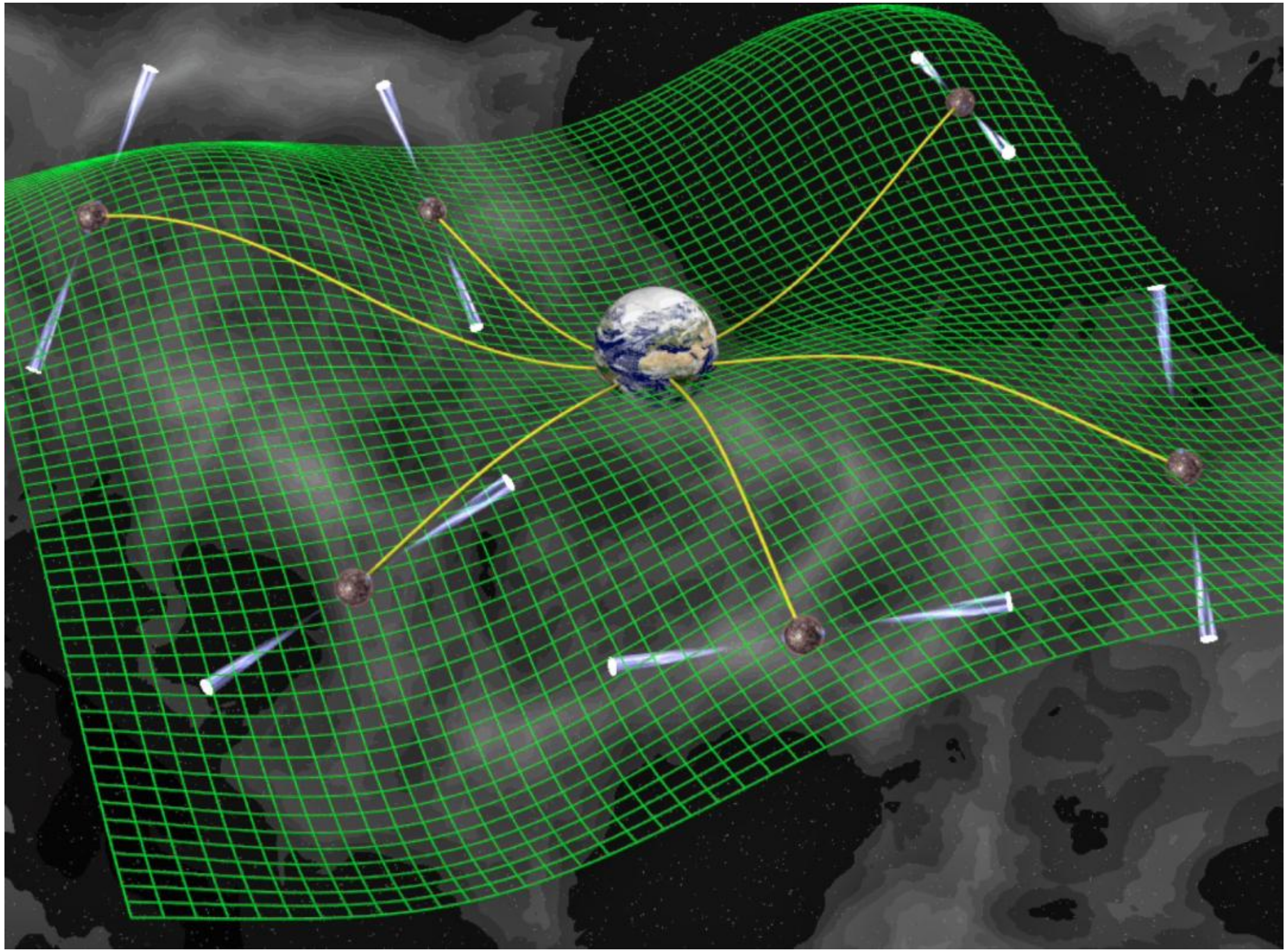
Observation from PTAs

- Hellings-Downs curve

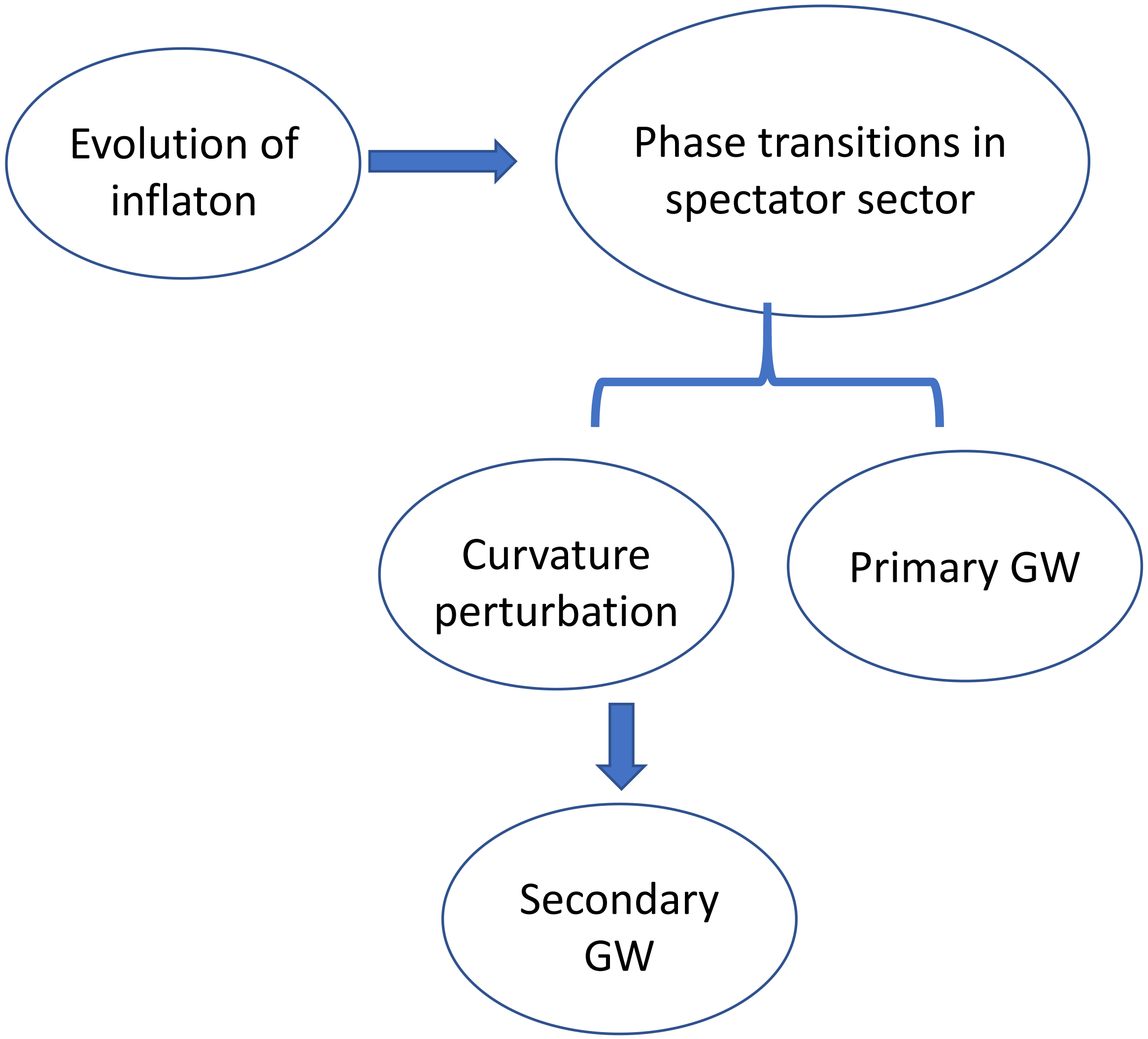
$$\langle z_a(t) z_b(t) \rangle = \underbrace{C(\theta_{ab})}_{\text{Angular correlation}} \int_0^\infty df\, S_h(f)$$

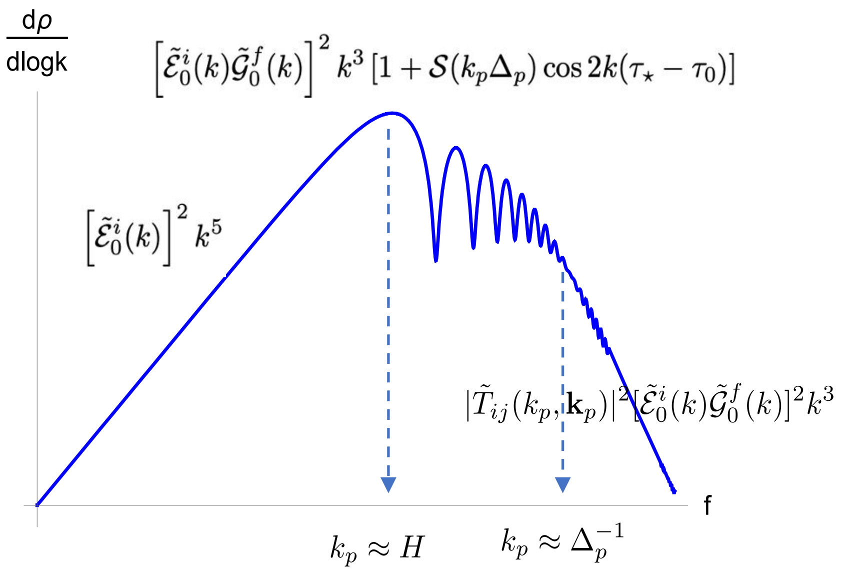
$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a / T_a$$

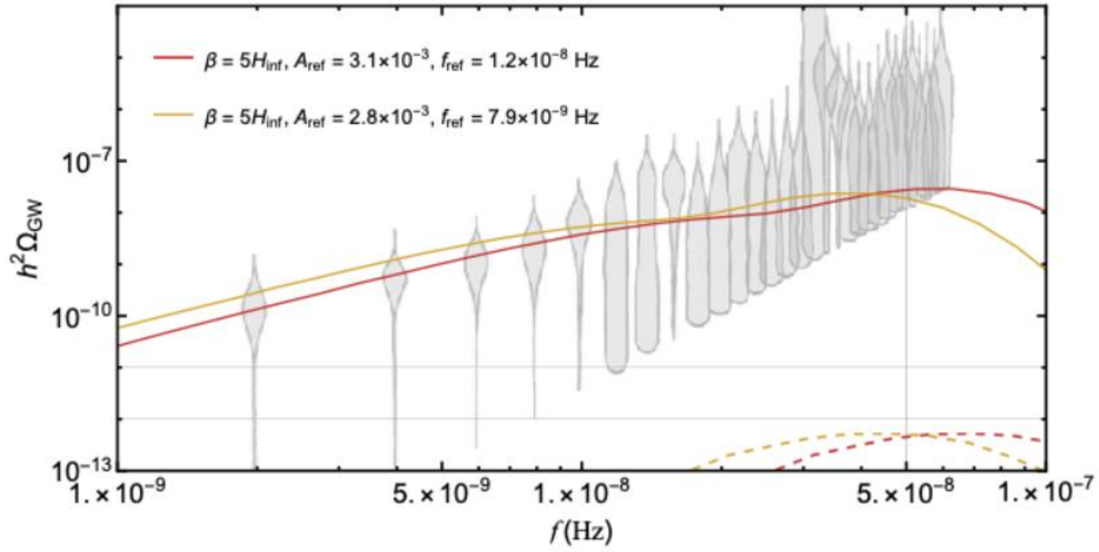




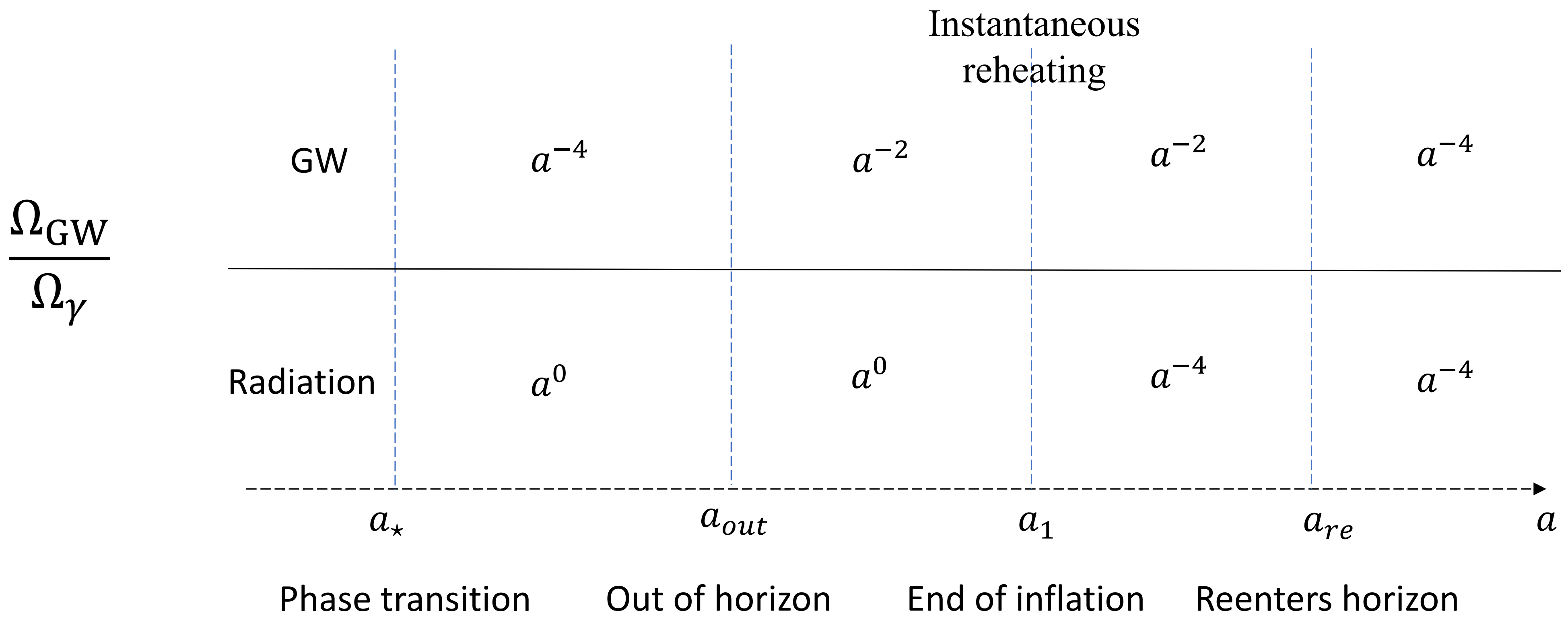
Summary for FOPT



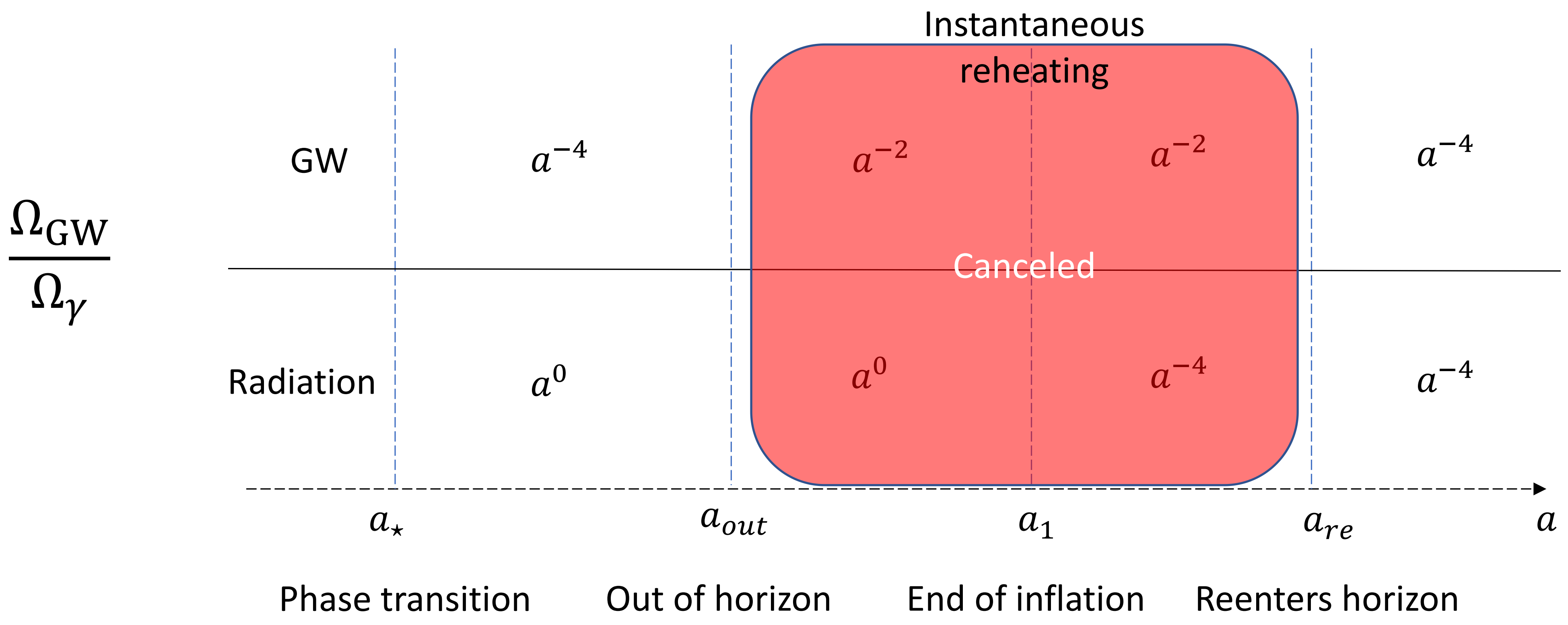




Redshifts of the GW signal

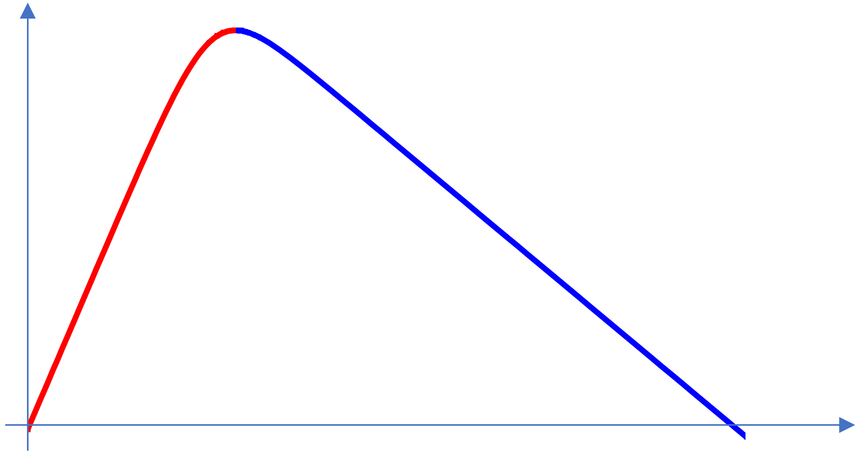


Redshifts of the GW signal



$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

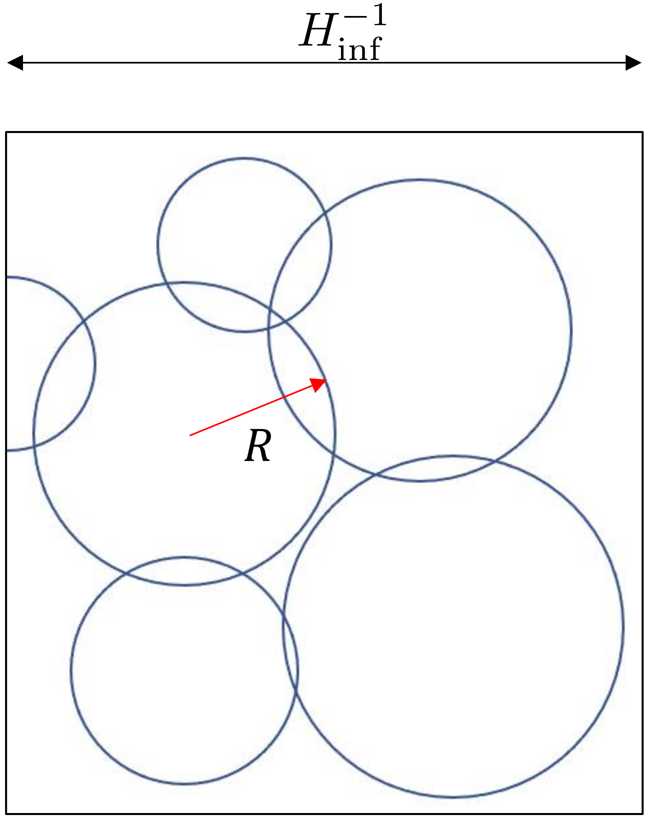
GWs produced in flat space-time



$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$

Huber and Konstandin, 0806.1828

$$\Omega_{\rm GW}^{(0)} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^2 \frac{\beta k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$



First order phase transition during inflation

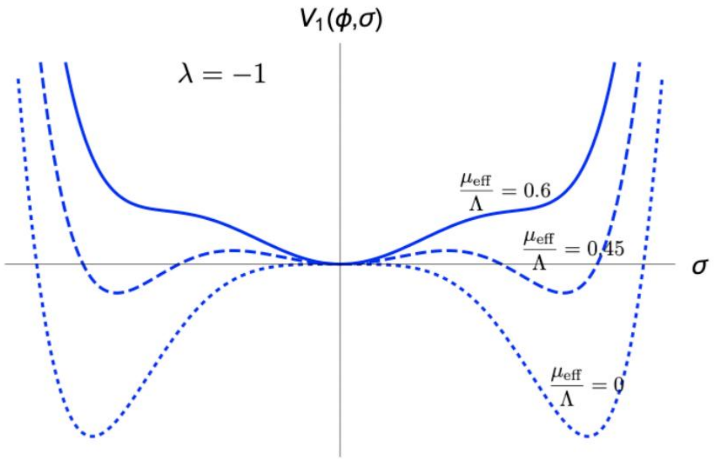
- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$$

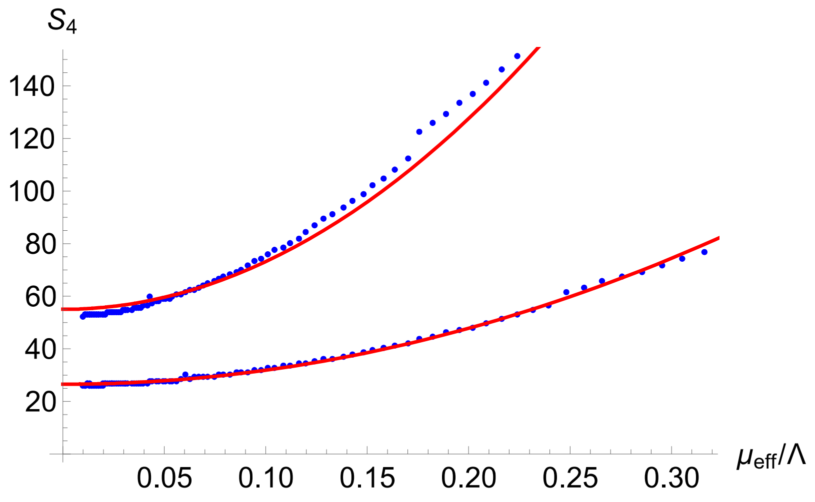
$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\underbrace{\left(1-\frac{\mu^2}{c^2\phi^2}\right)}_{\sim\,\mu_{\rm eff}^2/\Lambda^2}\right|}$$

$$\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

It is natural to have $\beta/H \sim O(10)$.





First order phase transition during inflation

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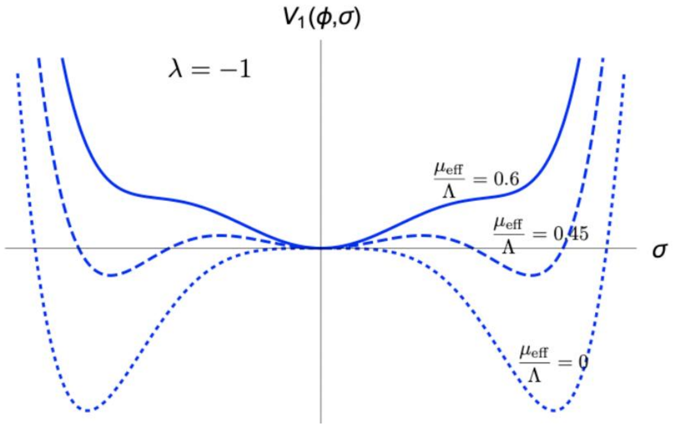
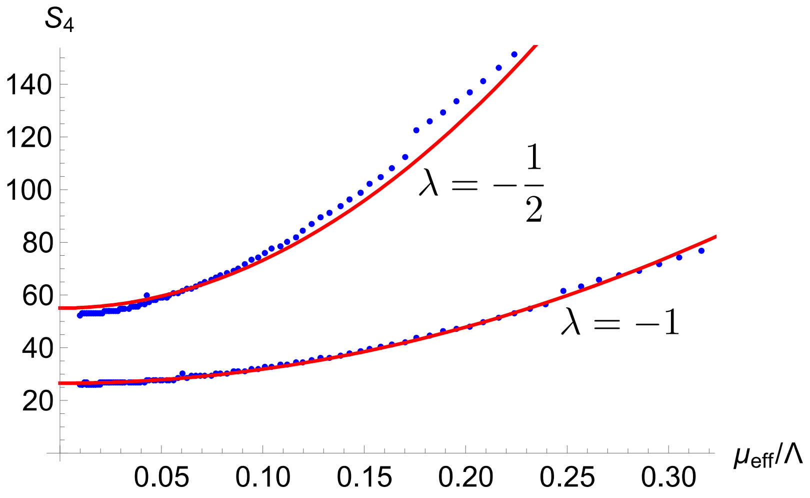
$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$



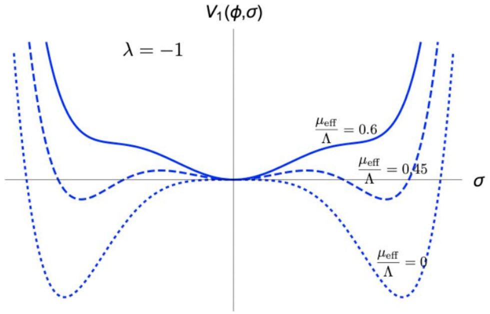
$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



CosmoTransitions

First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$$



$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

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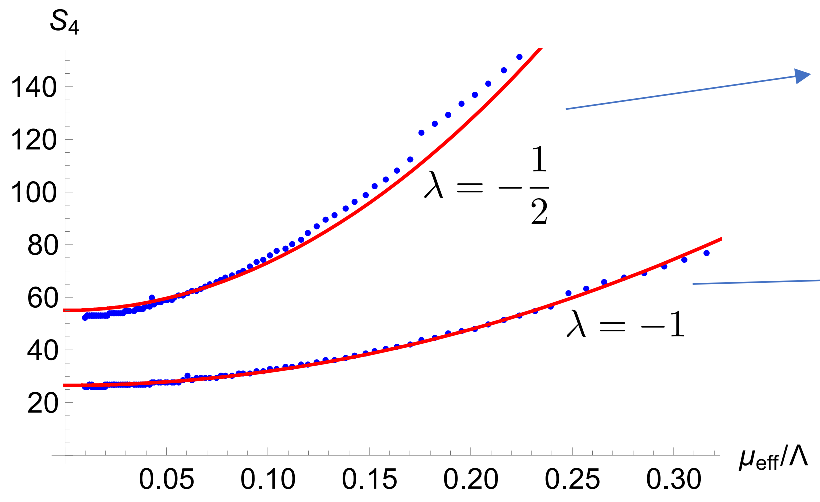
$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\rm eff}^2} \right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

N_e: e-folds before the end of inflation

$$V_1(\phi, \sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

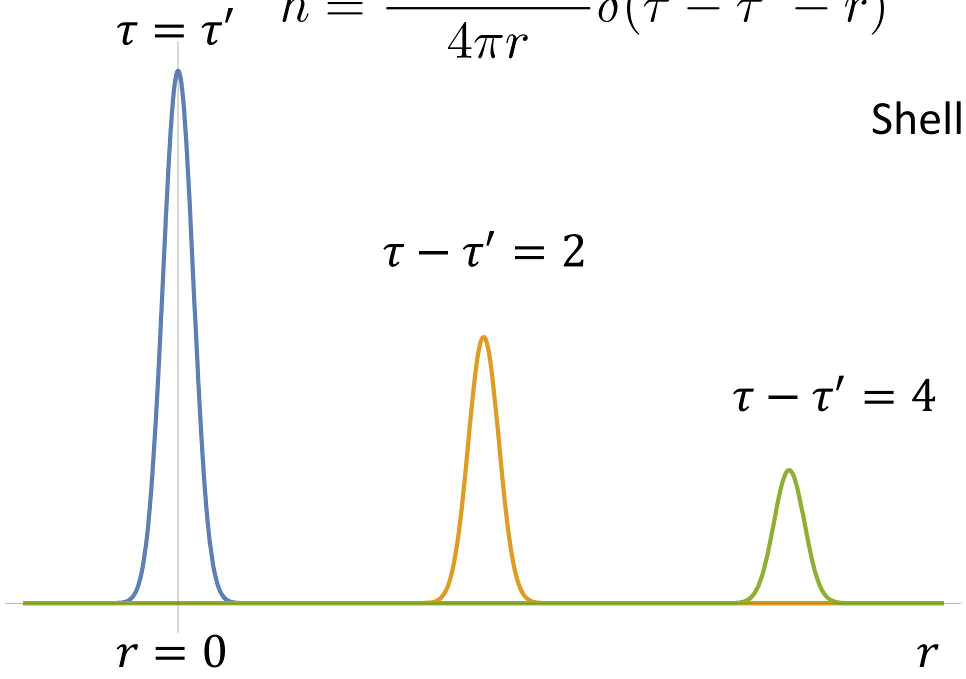
$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In Minkovski space

$$h = \frac{16\pi G_N T}{4\pi r}\delta(\tau - \tau' - r)$$

Shell with radius $|\tau - \tau'|$



de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h_{ij}(\tau,\mathbf{k}) = -16\pi G_N H T_{ij}\tau\Theta(\tau-\tau')\left[\frac{\sin k(\tau-\tau')}{k} + \left(\frac{1}{k^2\tau}-\frac{1}{k^2\tau'}\right)\cos k(\tau-\tau') + \frac{1}{k^3\tau\tau'}\sin k(\tau-\tau')\right]$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$\frac{\tau}{4\pi x}\delta(\tau - \tau' - |\mathbf{x}|)$$

$$\begin{aligned} h_{ij}(\tau,\mathbf{k}) = -16\pi G_N H T_{ij}\tau\Theta(\tau-\tau')\Bigg[&\frac{\sin k(\tau-\tau')}{k} \\ &\underbrace{+\left(\frac{1}{k^2\tau}-\frac{1}{k^2\tau'}\right)\cos k(\tau-\tau') + \frac{1}{k^3\tau\tau'}\sin k(\tau-\tau')\Bigg]}_{\frac{1}{4\pi}\Theta(\tau-\tau'-|\mathbf{x}|)} \end{aligned}$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h(\tau, \mathbf{x}) \sim \underbrace{\frac{\tau}{4\pi x}\delta(\tau - \tau' - x)}_{\text{Similar to Minkovski}} + \underbrace{\frac{1}{4\pi}\Theta(\tau - \tau' - x)}_{\text{Intrinsic in de Sitter}}$$

Decreases with both x and τ

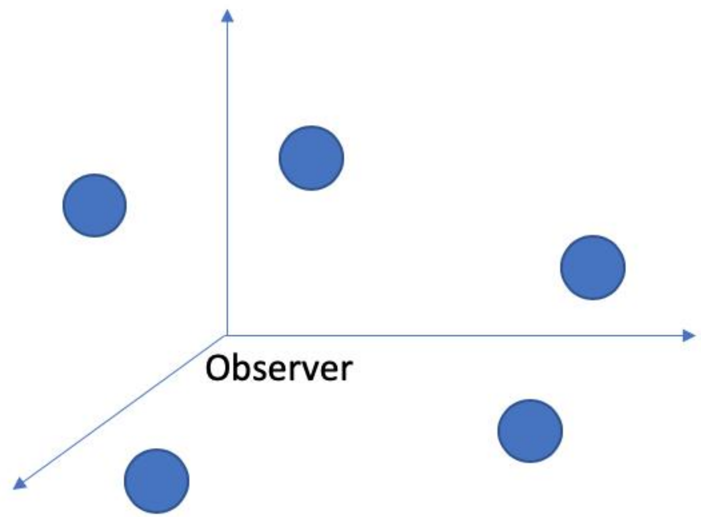
constant

Vanishes out of horizon

de Sitter inflation as an example

- At $\tau \to 0$ $\quad h(\tau, \mathbf{x}) \sim \frac{1}{4\pi}\Theta(|\tau'| - x)$

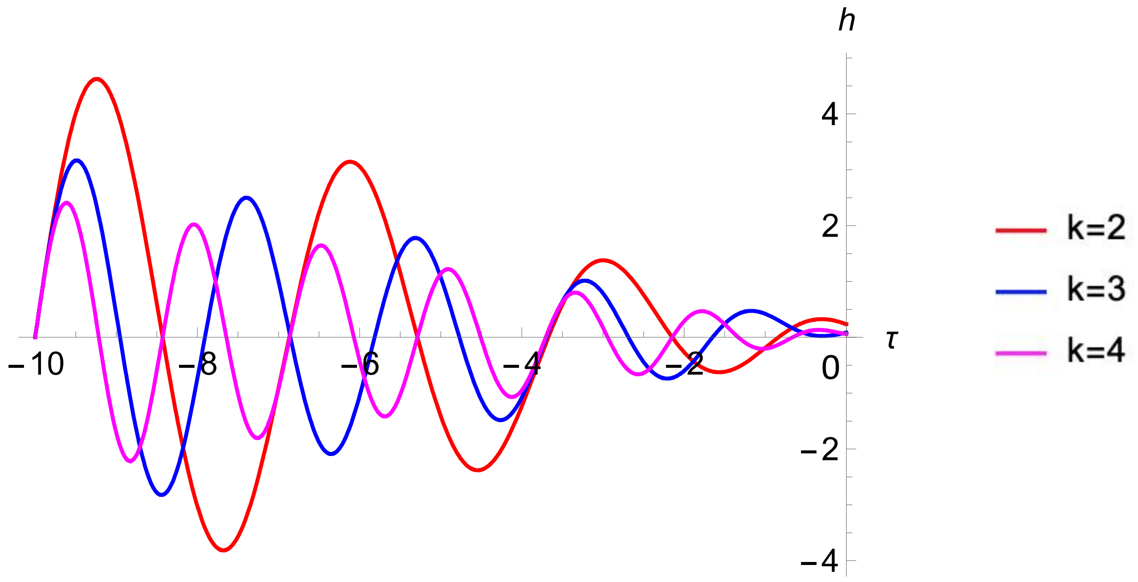
- A ball of GW, with radius $|\tau'|$
- h uniformally distributed inside the GW balls.
- All the balls have the same radius.



Quasi-de Sitter inflation as an example

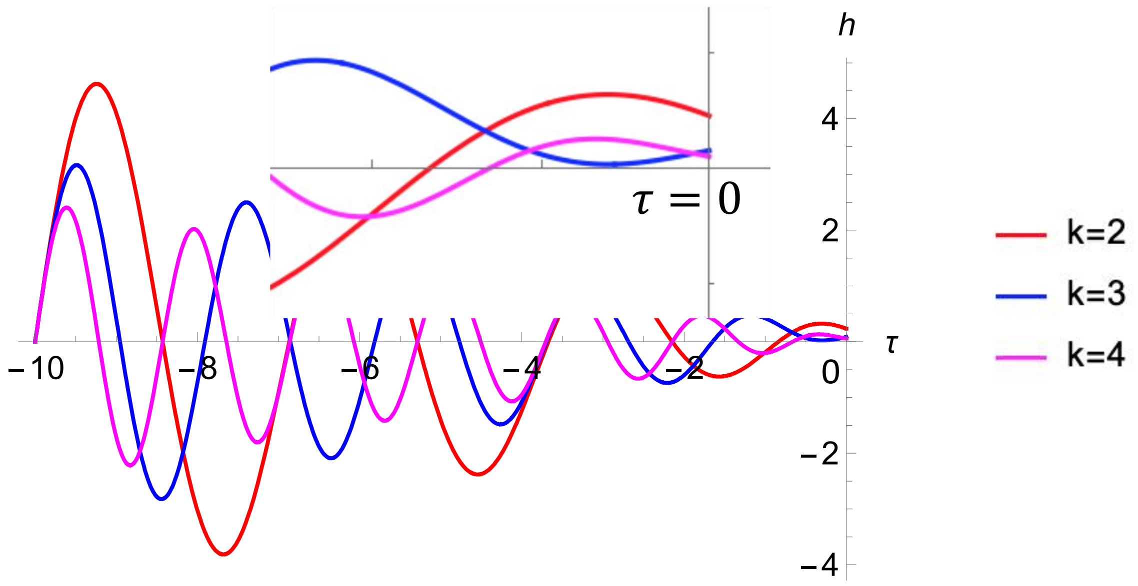
- $$a = -\frac{1}{H\tau}$$

- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



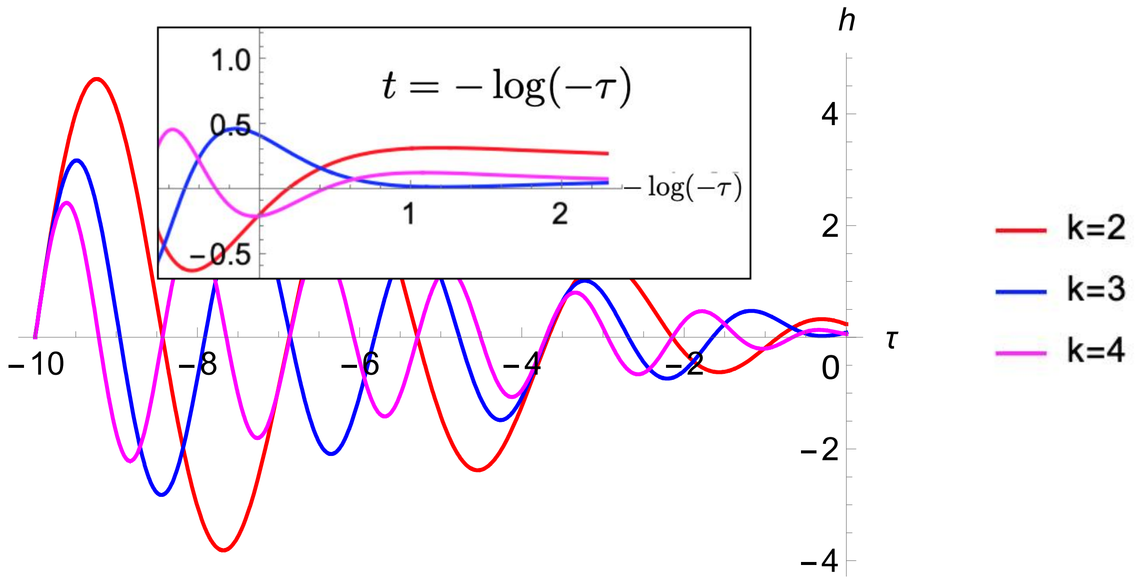
De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



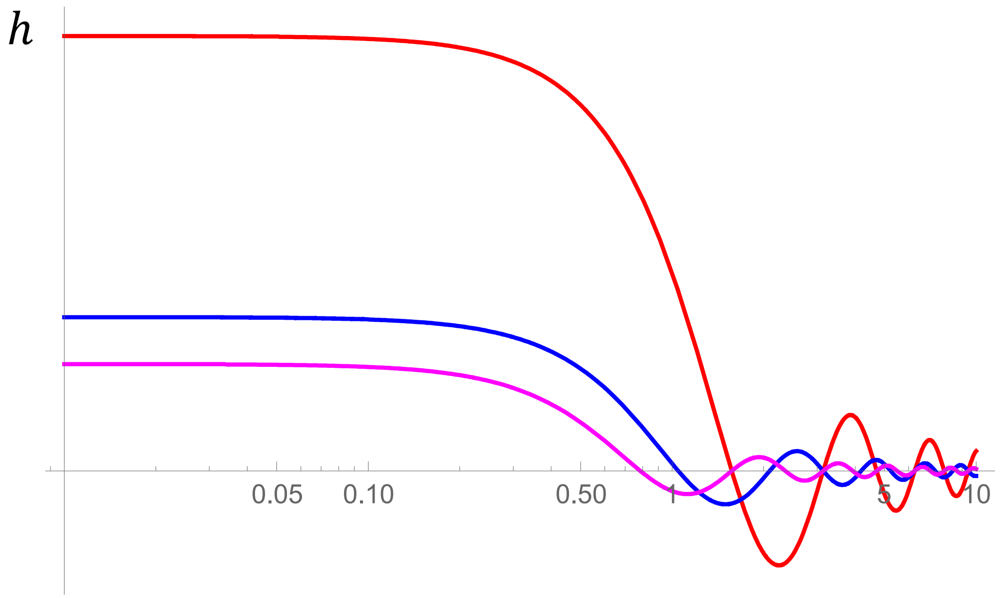
De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



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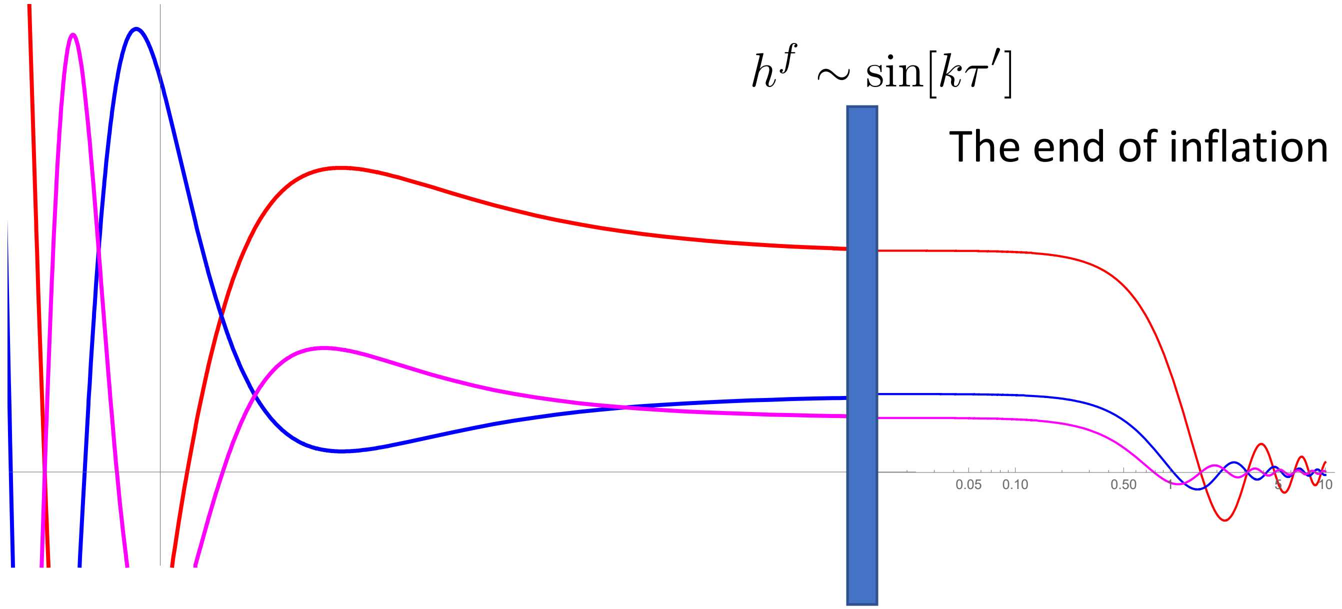
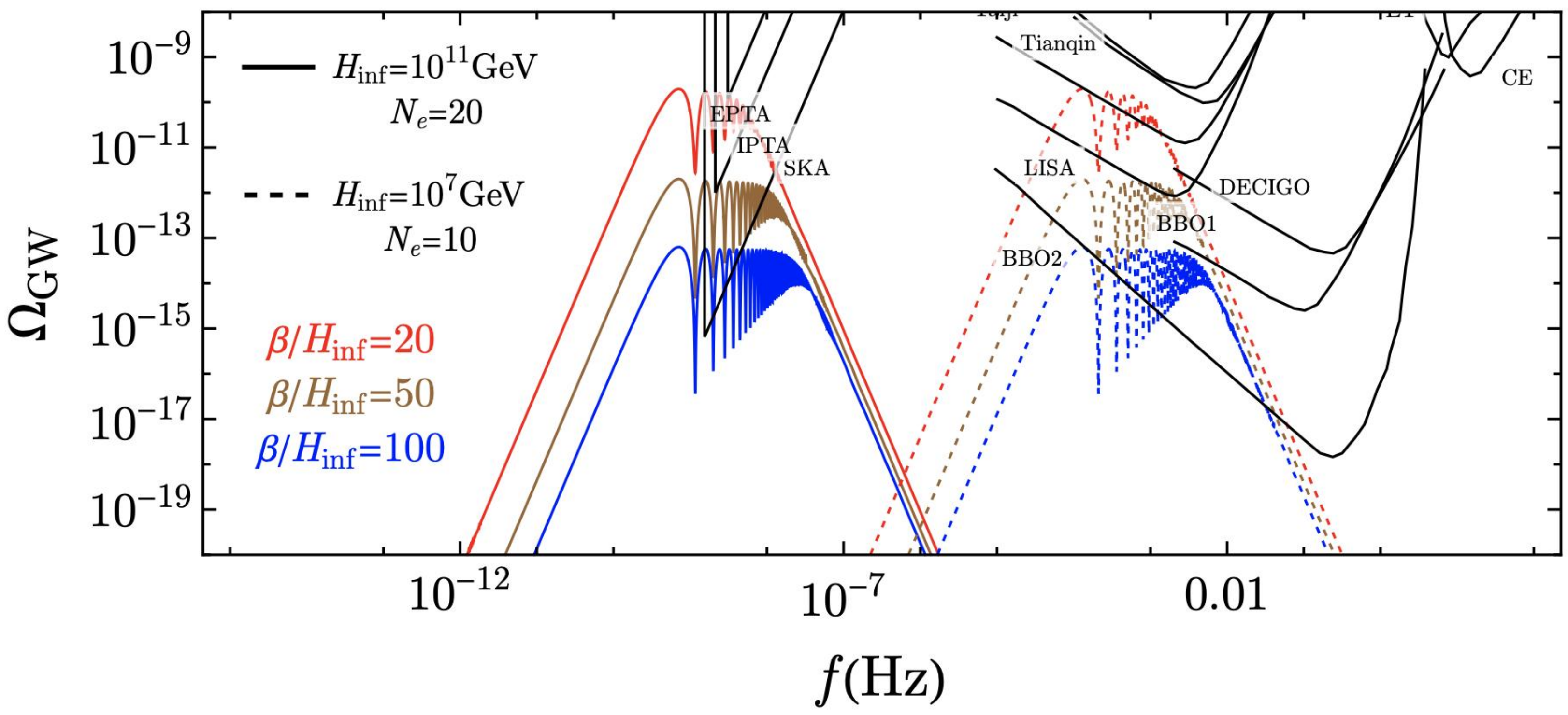


First order phase transition during inflation

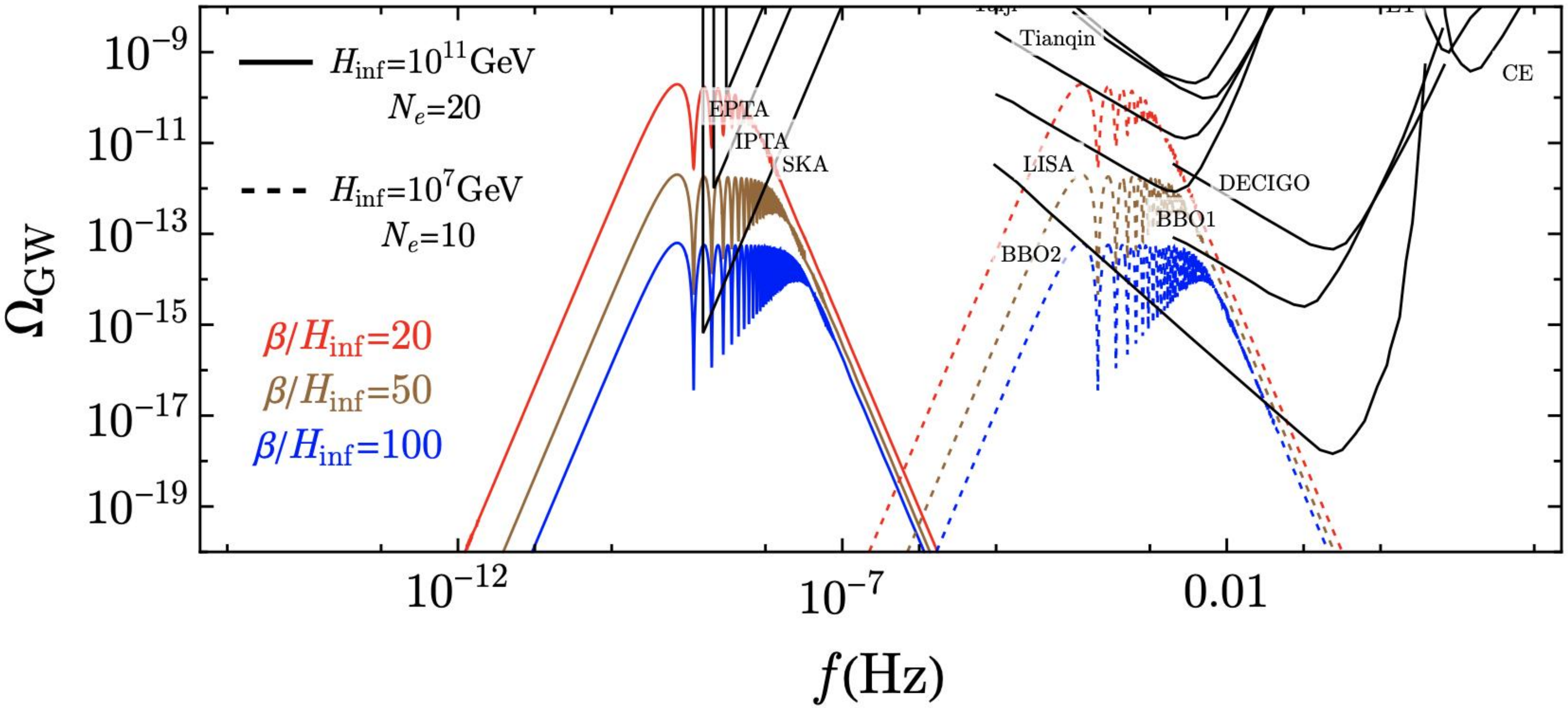
- Signal strength is also sensitive to intermediate stages

Kination domination



First order phase transition during inflation

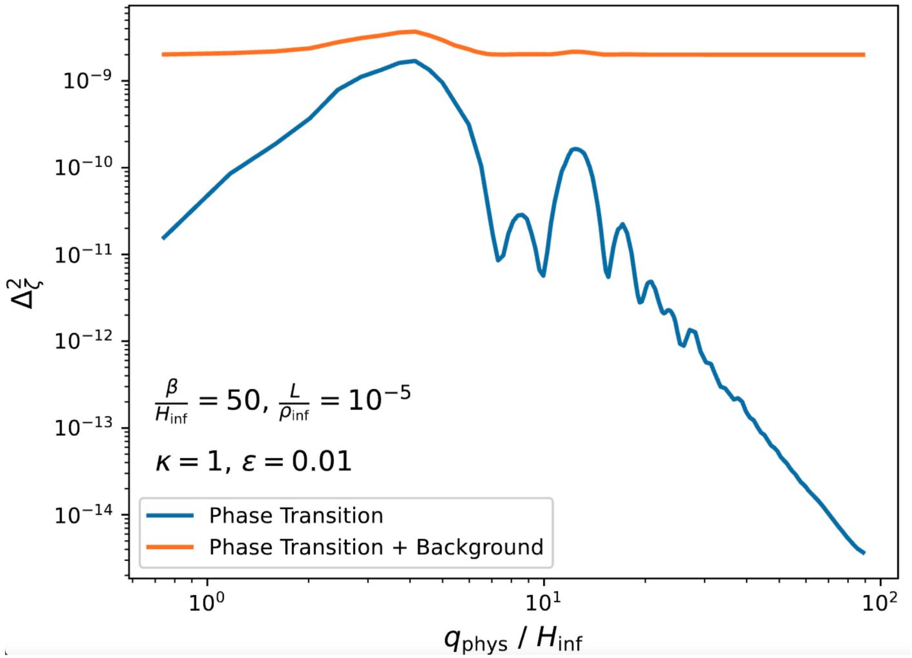
With kination domination intermediate stage



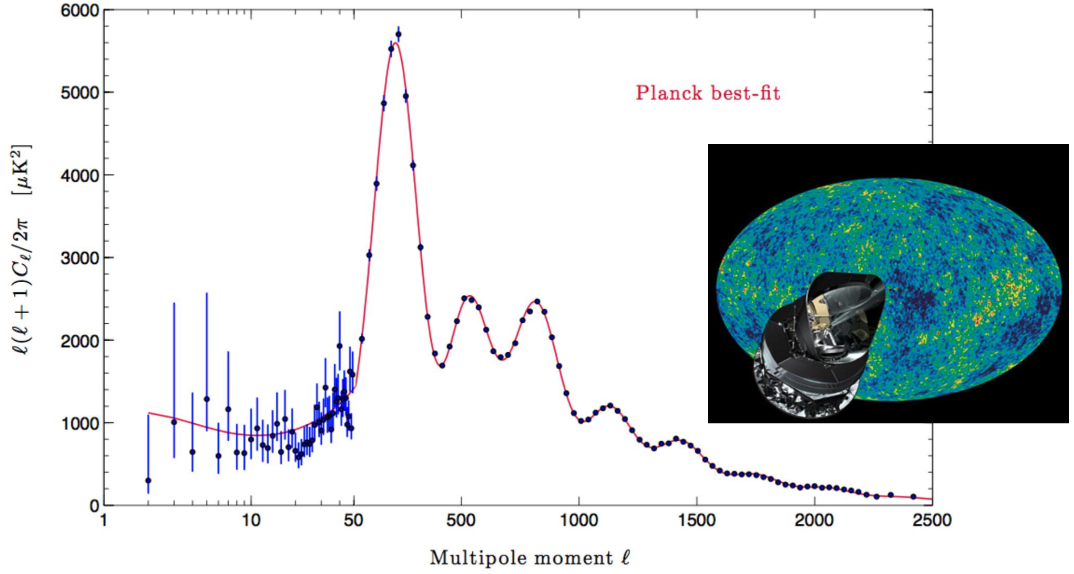
Power spectrum of ζ

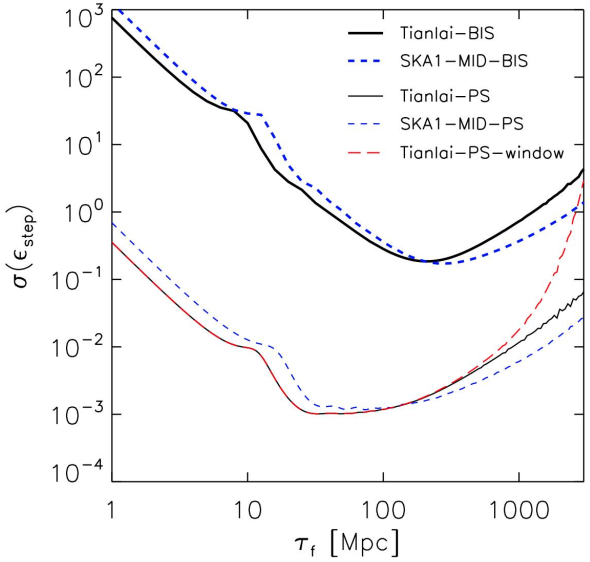
$$k_{\text{today}} = (2000\ \text{Mpc})^{-1} \times e^{60-N_e} \times \left(\frac{H_{\text{inf}}}{10^{14}\ \text{GeV}}\right)$$

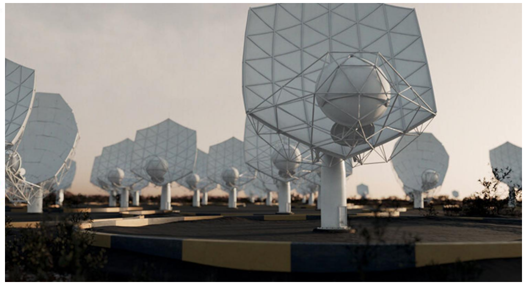
$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\text{pl}}}{\phi_0}\right)^2 \left(\frac{H_{\text{inf}}}{\beta}\right)^3 \left(\frac{L}{\rho_{\text{inf}}}\right)^2$$



HA, Boye Su, Yidong Xu, Chen Yang, work in progress.



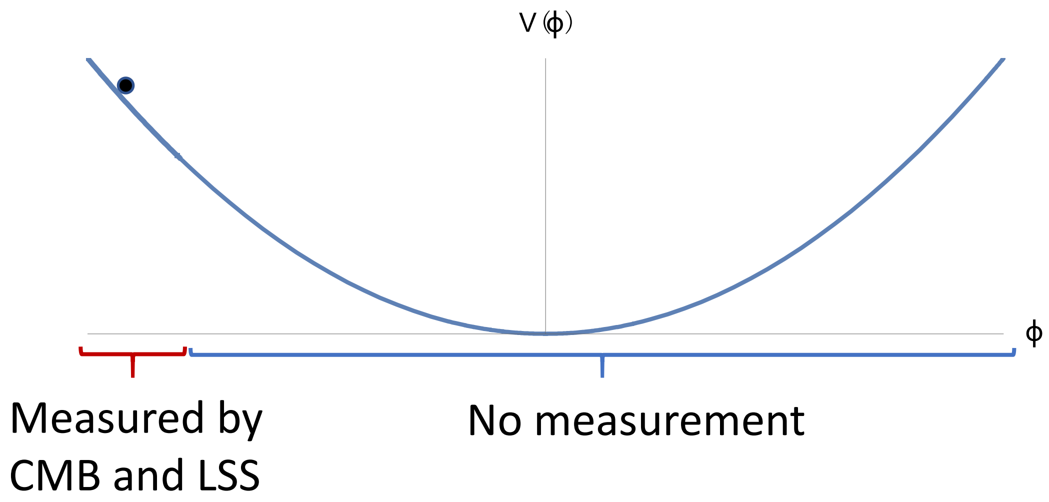




Xu, Hamann, Chen, 1607.00817

Slow roll models

- We usually assume a potential.
- Use it to calculate n_s, r ...



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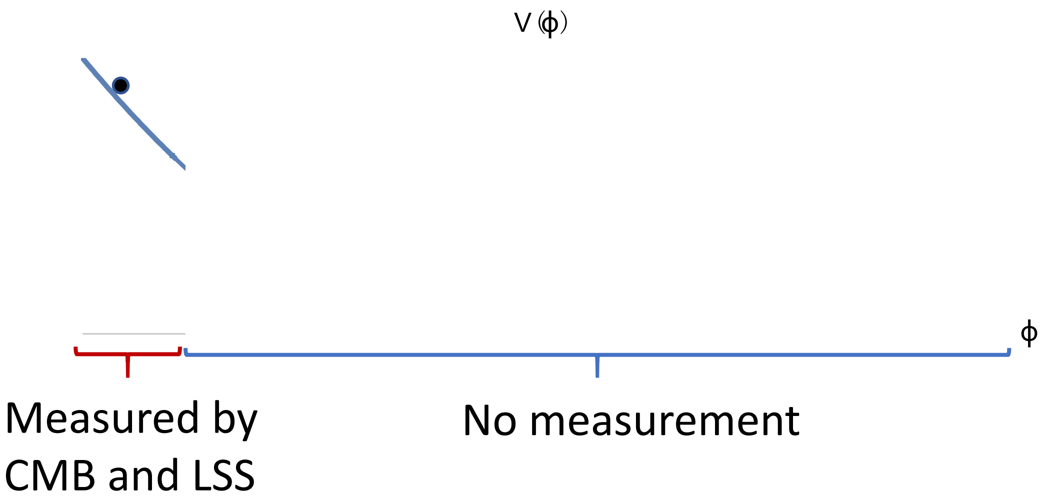

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