

# *The dynamical holographic QCD method for QCD matter*

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Based on arXiv: 2405.06386, 2408.17080

Oct. 26, 2024

# Outline

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**I. Introduction**

**II. QCD phase transition under rotation**

**III. Pion Condensation**

**IV. Conclusion and discussion**

# I. Introduction

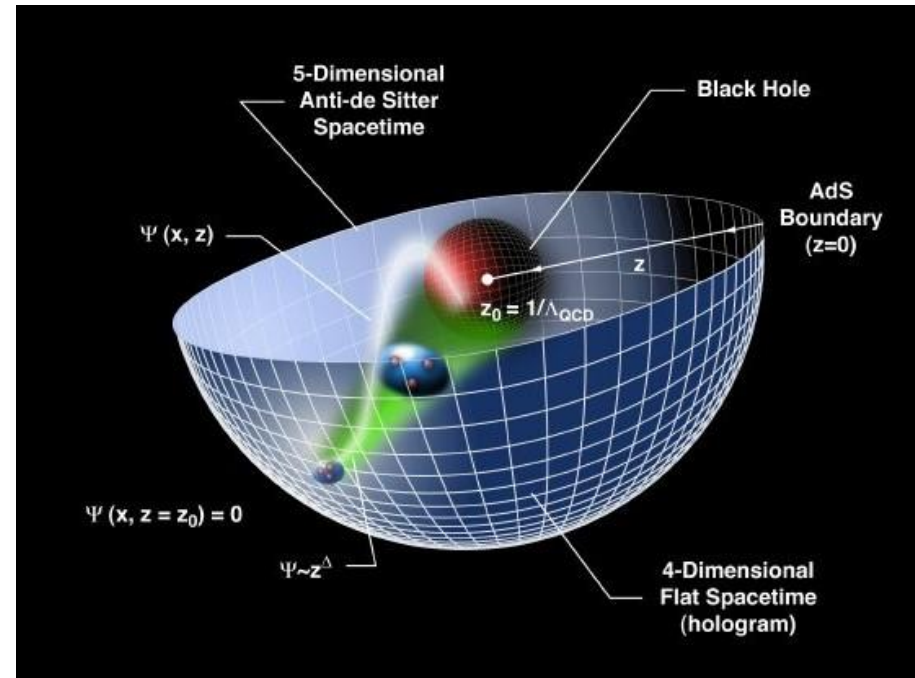
# Gauge/Gravity duality

## AdS/CFT correspondence

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

relates gravity theories on anti-de Sitter spacetimes (AdS) to conformal field theories

## General Gauge/Gravity duality

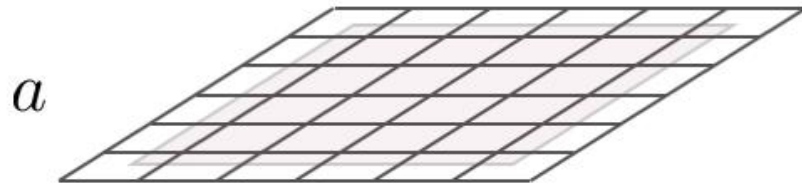


# The Renormalization Group Flow

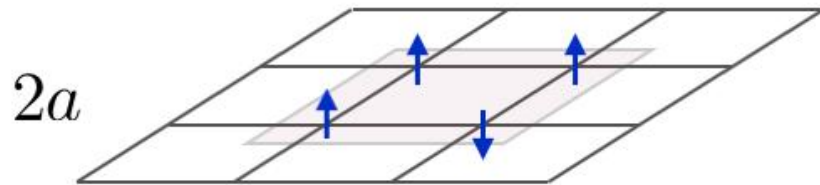
Adams, Allan et al.  
New J.Phys.(2012),  
arXiv:1205.5180

$$H = \sum_{x,i} J_i(x) \mathcal{O}^i(x)$$

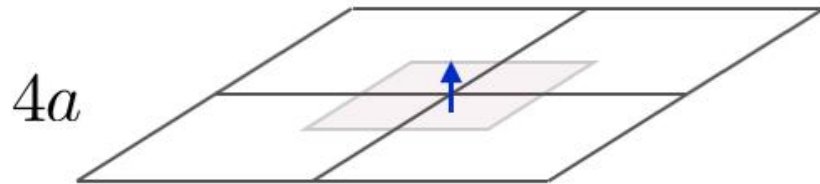
$J(x)$ : coupling constant or source for the operator



$$H = \sum_i J_i(x, a) \mathcal{O}^i(x)$$



$$H = \sum_i J_i(x, 2a) \mathcal{O}^i(x)$$



$$H = \sum_i J_i(x, 4a) \mathcal{O}^i(x)$$

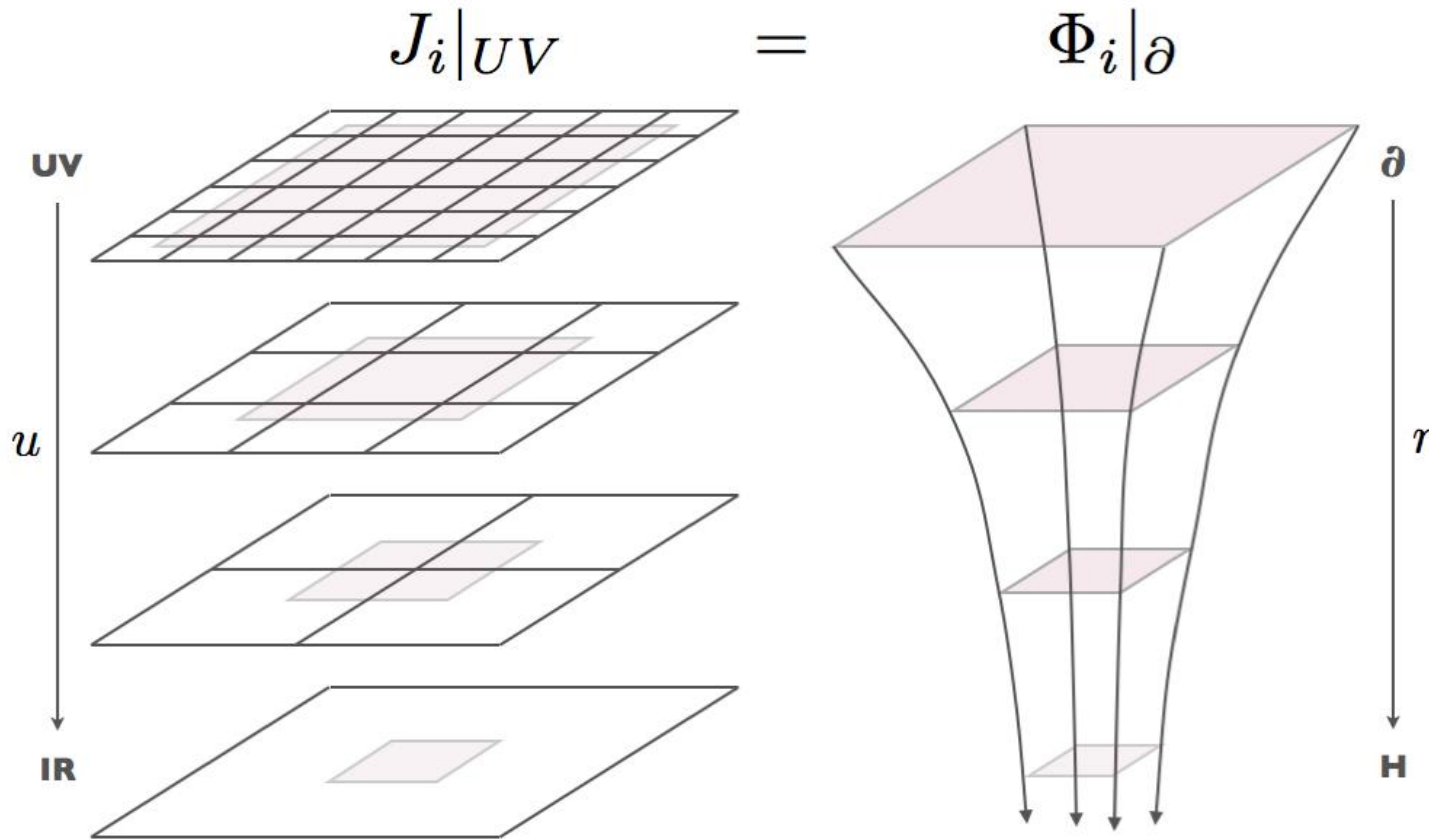
$$u \frac{\partial}{\partial u} J_i(x, u) = \beta_i(J_j(x, u), u)$$

# Holographic Duality & RG flow

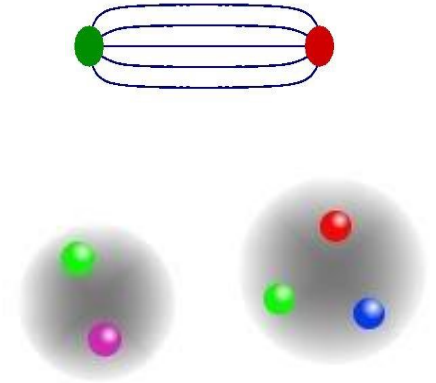
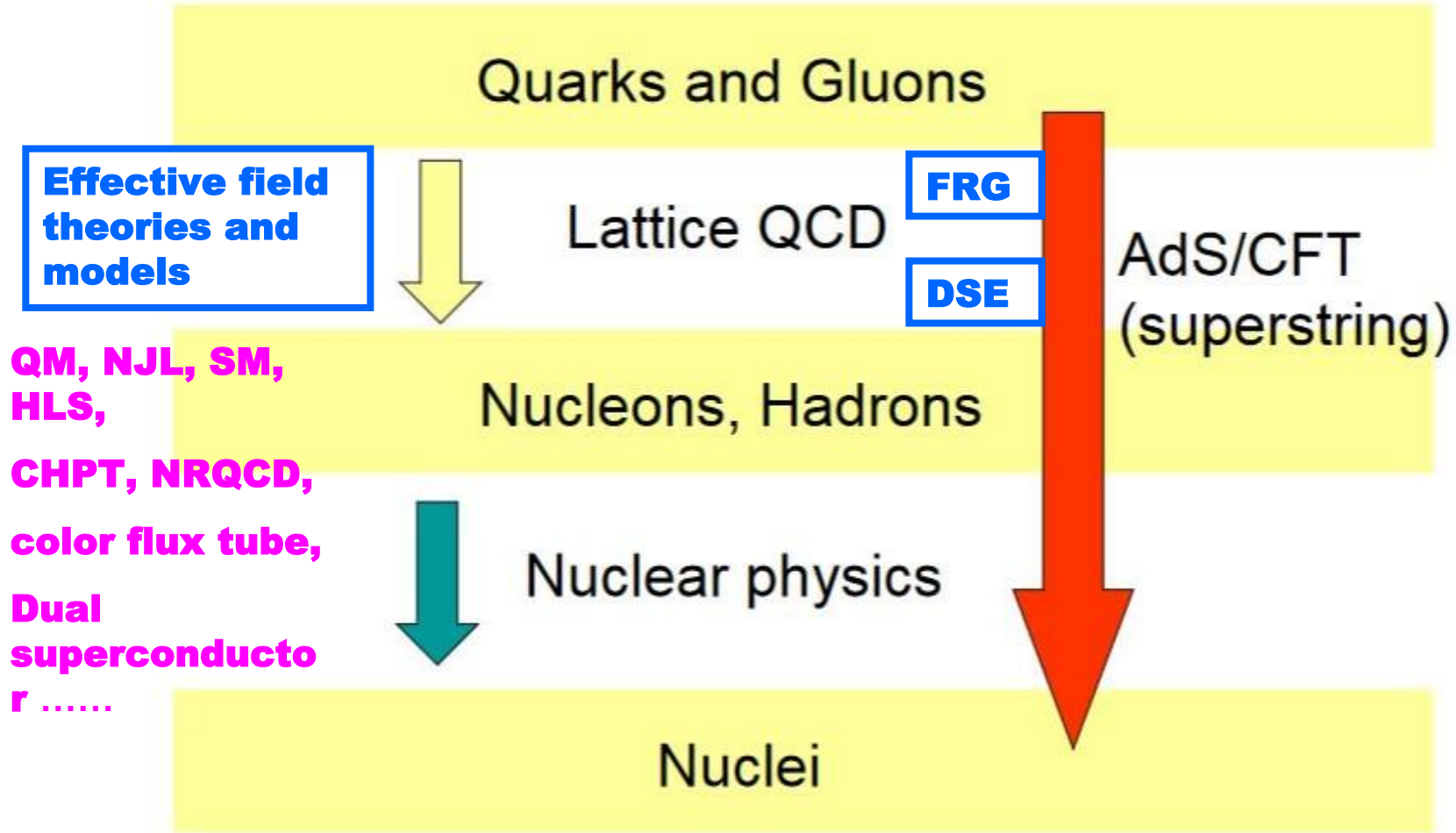
Adams, Allan et al.  
New J.Phys.(2012),  
arXiv:1205.5180

QFT on lattice  
RG scale  
Coupling constant

Gravitational problem  
Extra spatial dimension  
Dynamical field

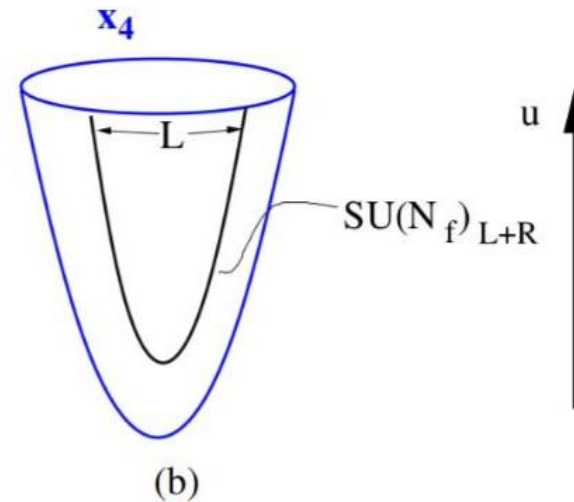
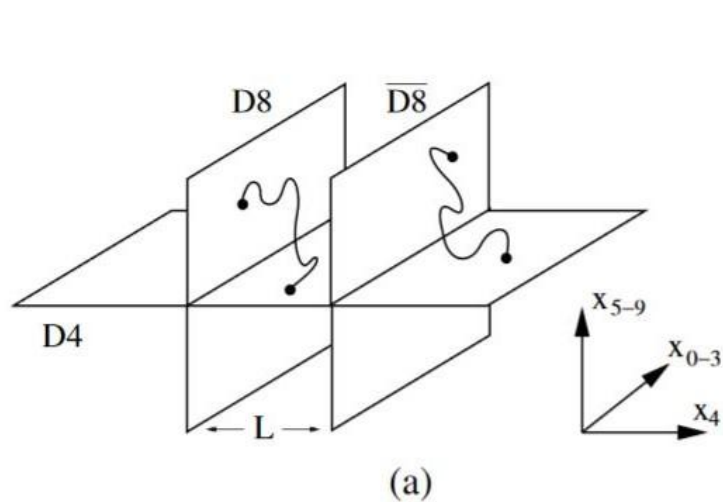


# QCD



# Top-down vs Bottom-up

Top-down: based on string theory and D-brane



D3-D7,  
D4-D6,  
D4-D8,  
STU model  
...

Bottom-up: from symmetry of QCD

- 1) 5D effective action
- 2) IR cutoff or dilaton to realize confinement

hard-wall, soft-wall, Gubser model,  
improved holographic QCD,  
dynamical holographic QCD ...

Hadron spectra, chiral symmetry breaking & linear confinement, phase transitions, equation of state, transport properties



# Holographic dictionary

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## Boundary QFT

**Local operator**  $\mathcal{O}_i(x)$

## Gravity

**Field**  $\Phi_i(x, r)$

conformal dimension  $\Delta$ , p-form field  $p$   $(\Delta - p)(\Delta + p - d) = m_5^2 L^2$

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**Strongly coupled**

**Weakly coupled semi-classical**

$$Z_{\text{QFT}}[J_i] = Z_{\text{QG}}[\Phi[J_i]]$$

$$Z_{\text{QFT}}[J] \simeq e^{-I_{\text{GR}}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \frac{\delta^n I_{\text{GR}}[\Phi[J_i]]}{\delta J_1(x_1) \dots \delta J_n(x_n)} \Big|_{J_i=0}$$

5D action

$$S_{DHQCD} = S_{GDM} + \lambda S_{KKSS}$$

Dilaton profile

the Graviton–Dilaton–Maxwell action in the string frame

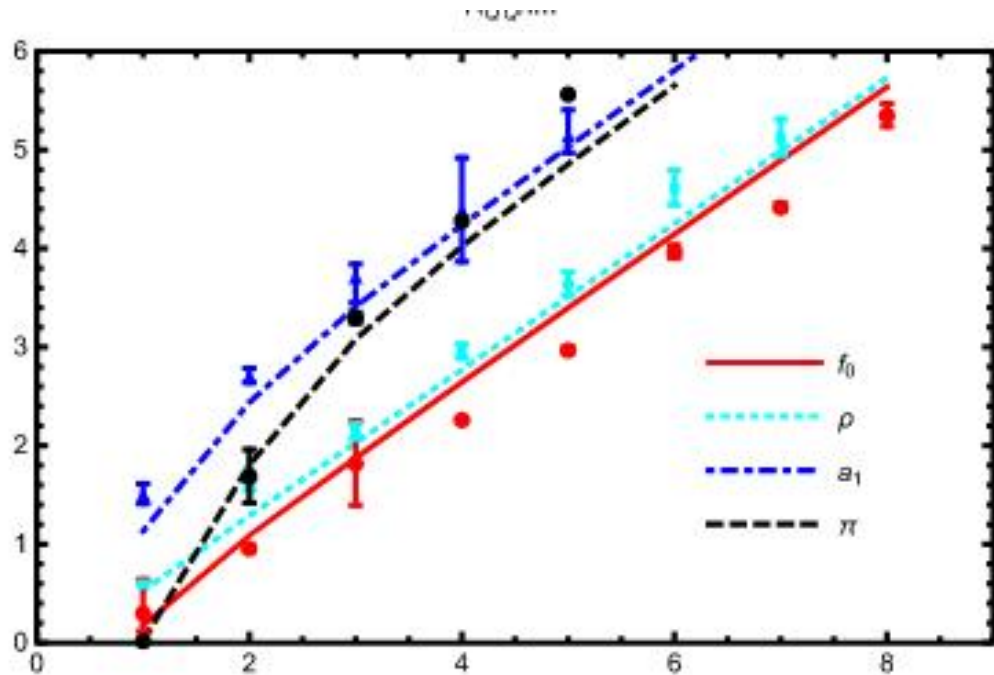
$$\Phi(z) = \mu_G^2 z^2$$

$$S_{GDM} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} e^{-2\Phi} \left[ R + 4(\partial\Phi)^2 - V(\Phi) - \frac{h(\Phi)}{4} F^2 \right]$$

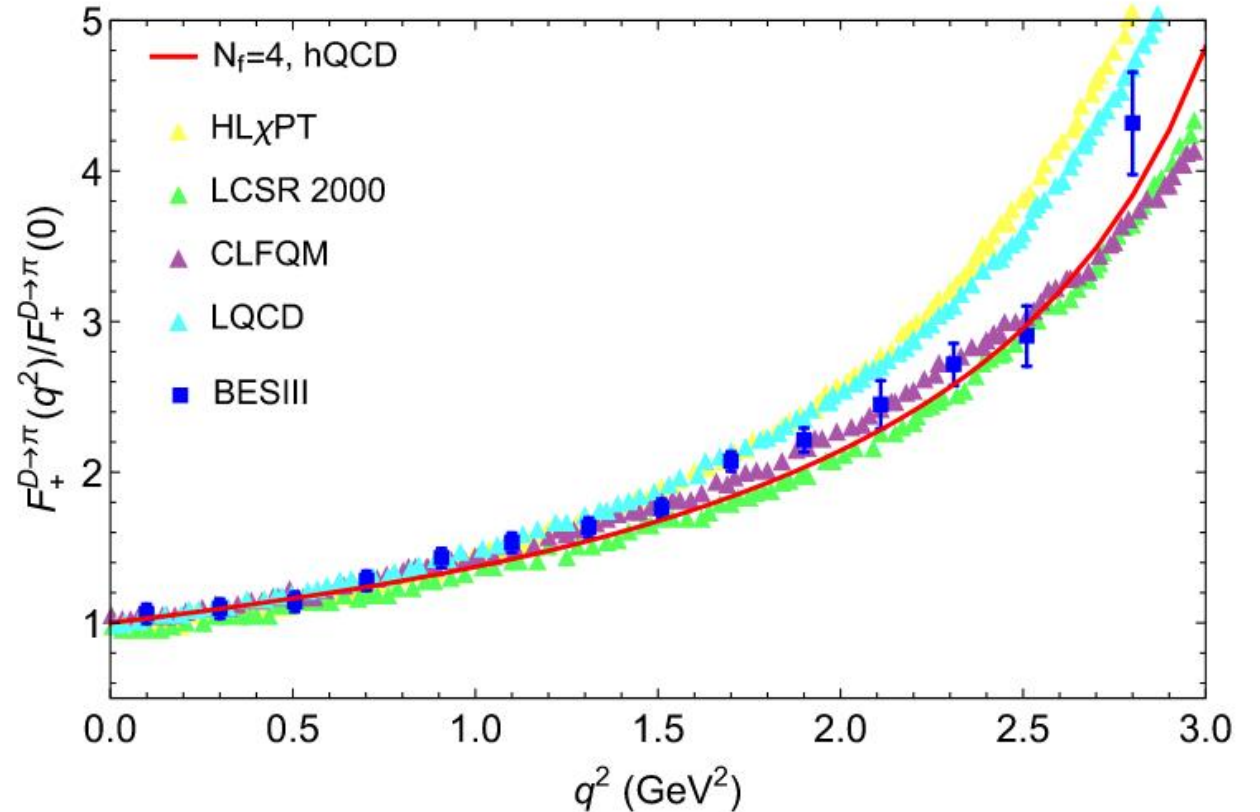
The action of KKSS model takes the following form

$$S_{KKSS} = - \int d^5x \sqrt{-g} e^{-\Phi} \text{Tr} \left[ |DX|^2 + V_X + \frac{1}{4} (F_L^2 + F_R^2) \right]$$

# DHQCD for Hadron Physics



**Li et al., JHEP(2013)**



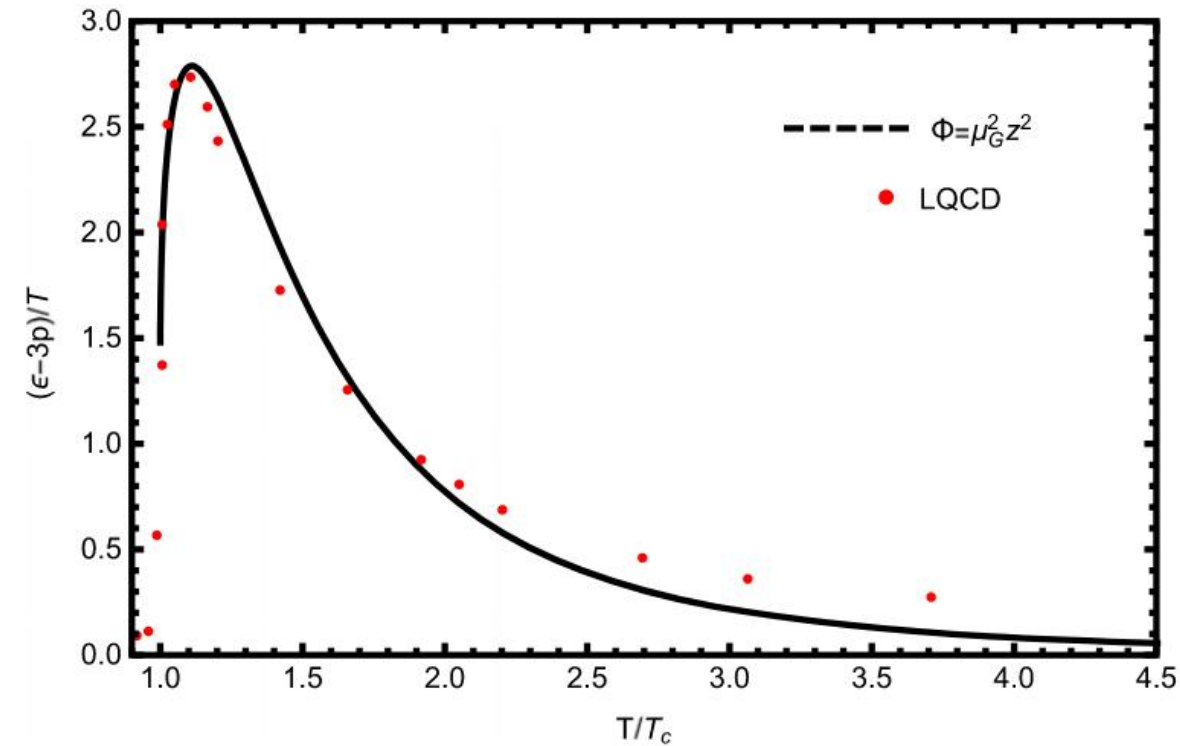
**Chen et al., PRD(2022)**

**Ahmed et al., PRD108.086034 (2023)**

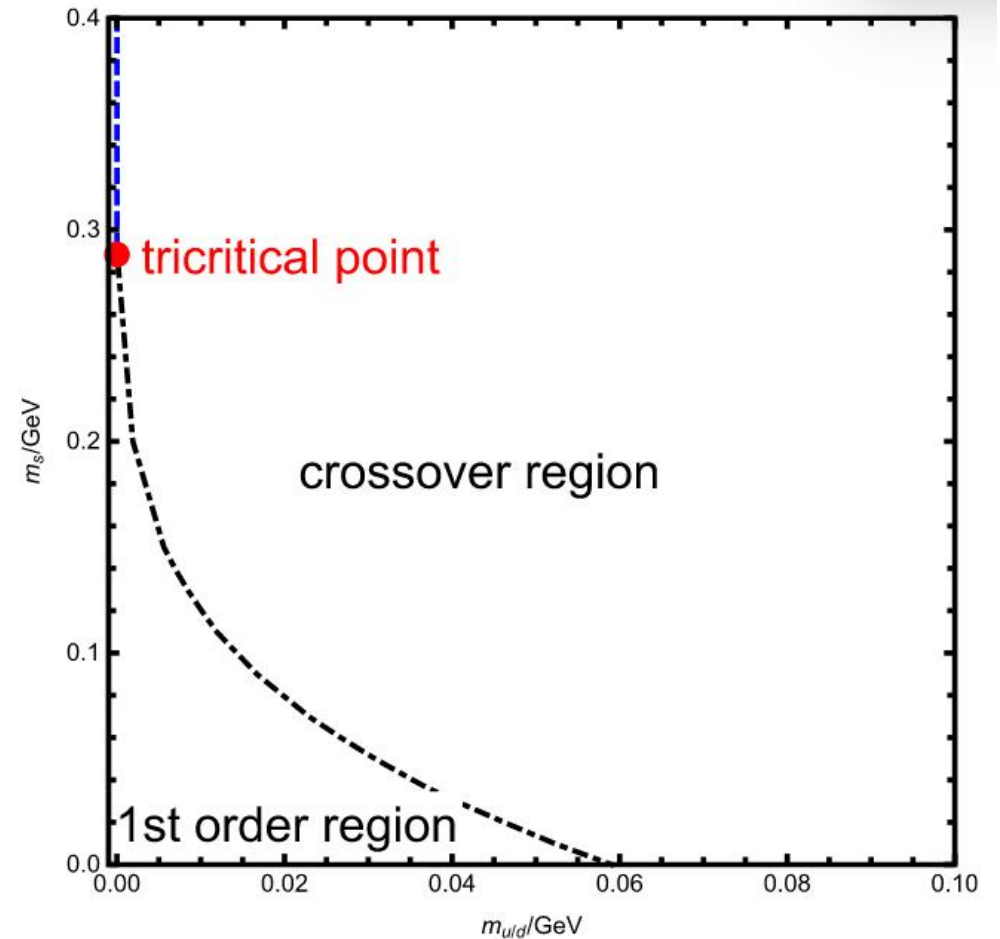
**Ahmed et al., PRD109.026008 (2023)**

Results of the light meson spectra and semileptonic form factor from the DHQCD model

# DHQCD for QCD Phase Transition

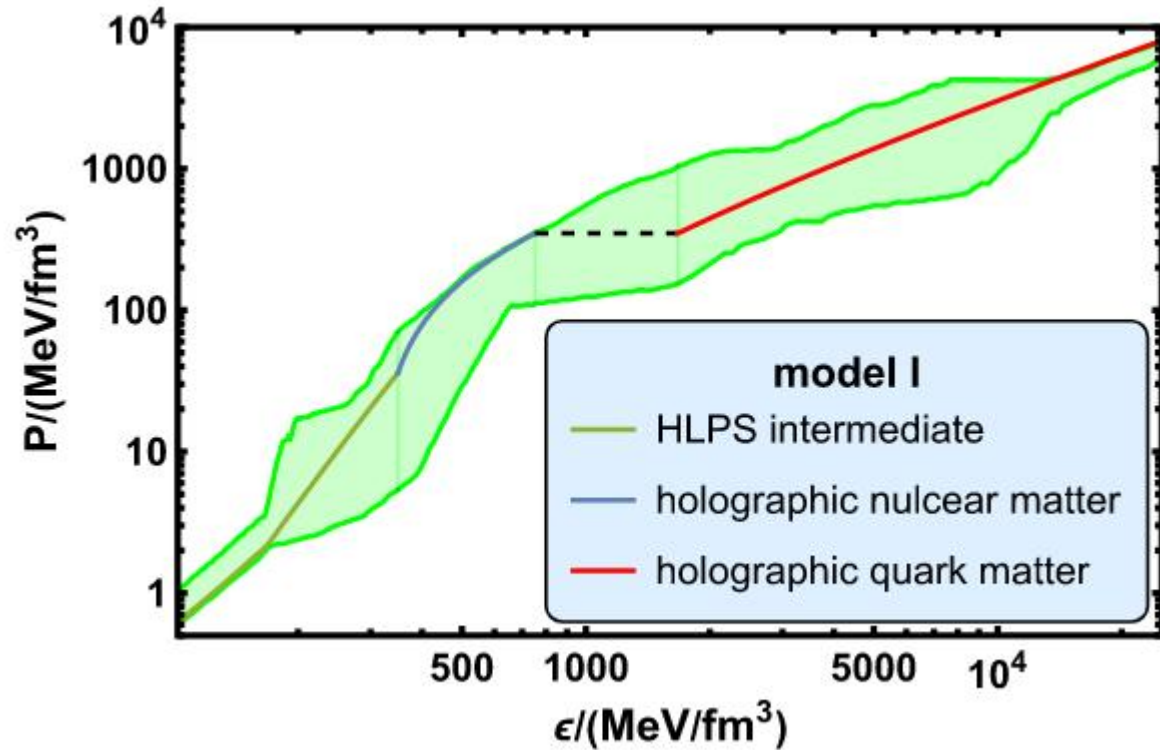


**Li et al., JHEP(2011)**

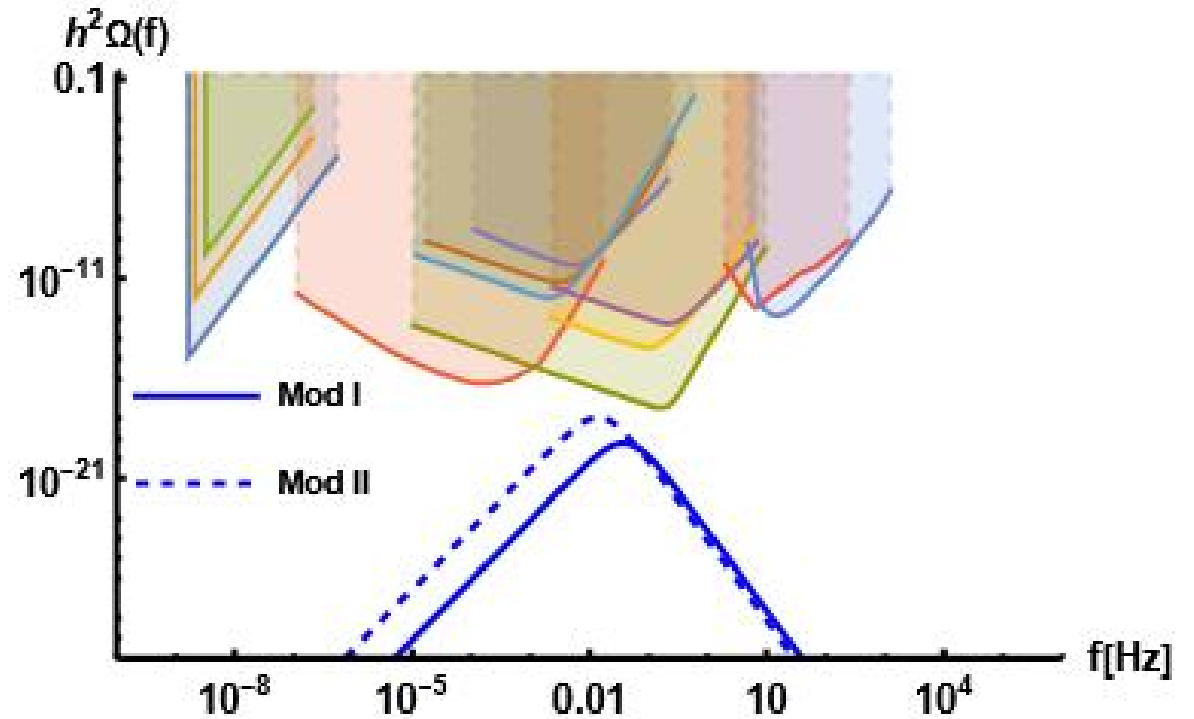


**Chelabi et al., PRD(2016)**  
**Chelabi et al., JHEP(2016)**

# DHQCD for QCD Phase Transition



Zhang et al., PRD(2022)

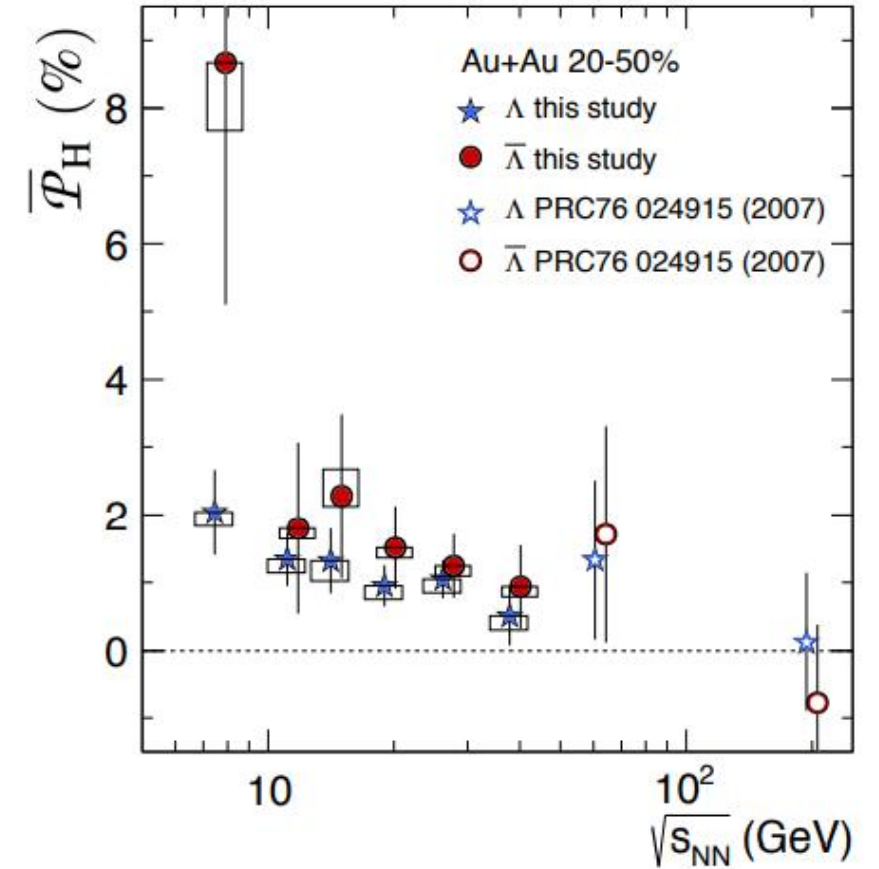
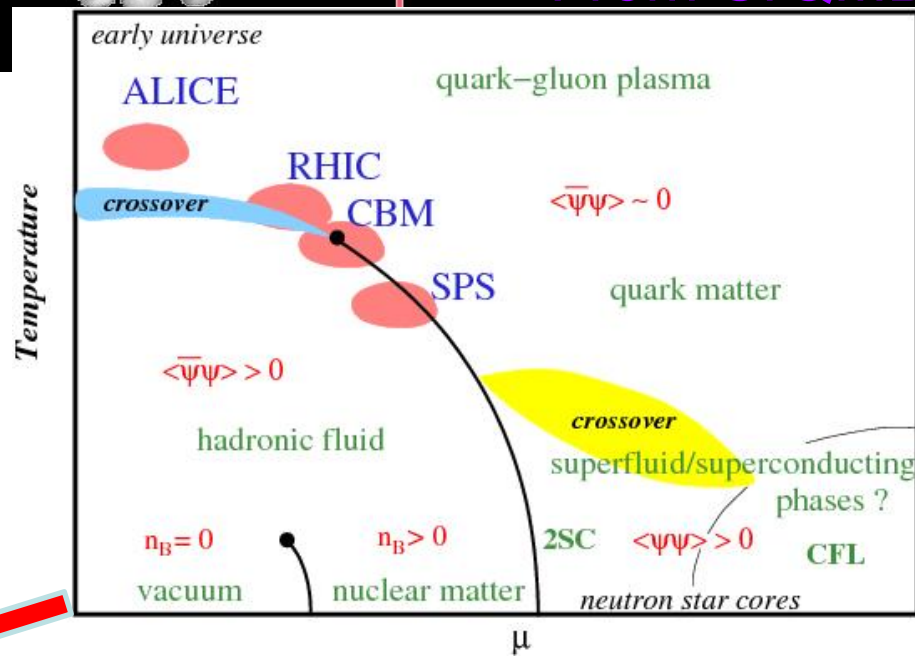
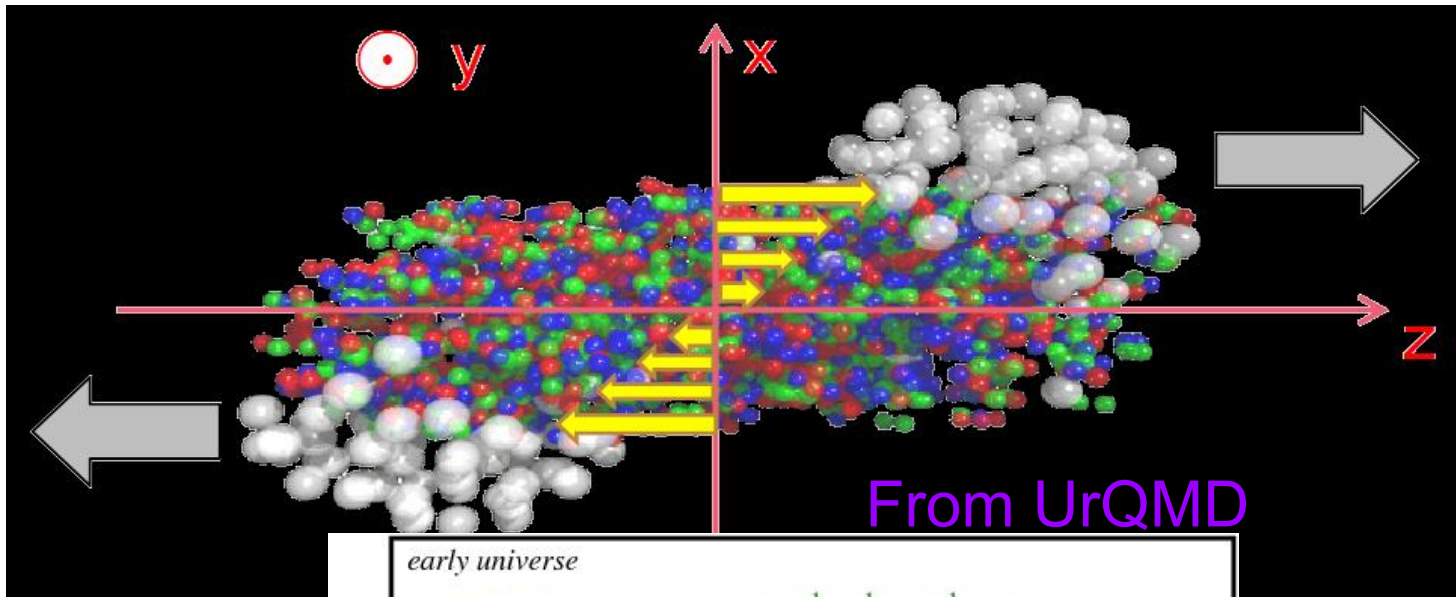


(a)

Chen et al., JHEP(2018)  
Chen et al., JHEP(2023)

## **II. QCD phase transition under rotation**

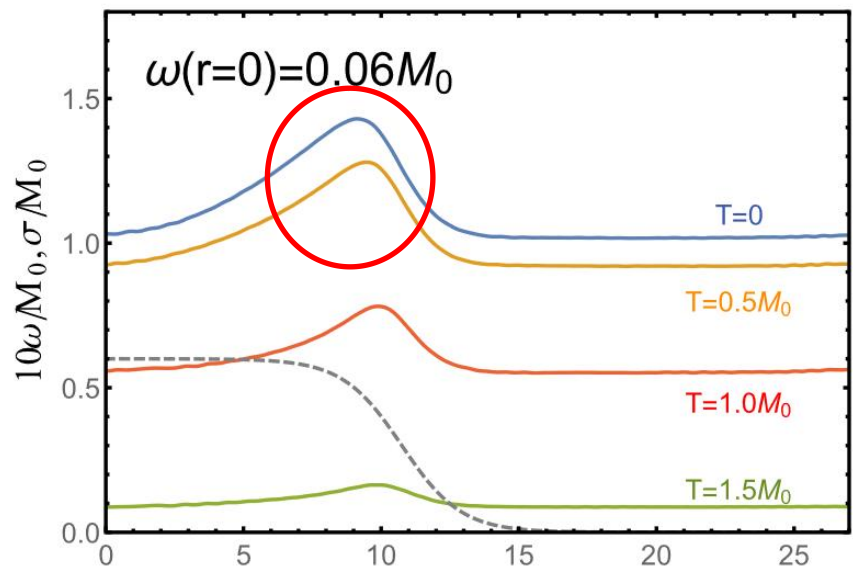
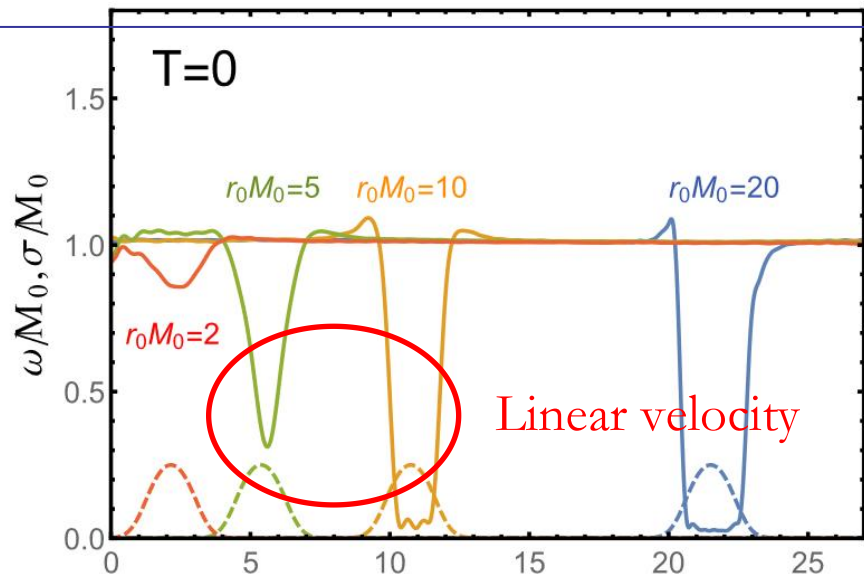
# Heavy Ion Collision



**L. Adamczyk, et al. (STAR),  
Nature 548 (2017) 62.**



# Models

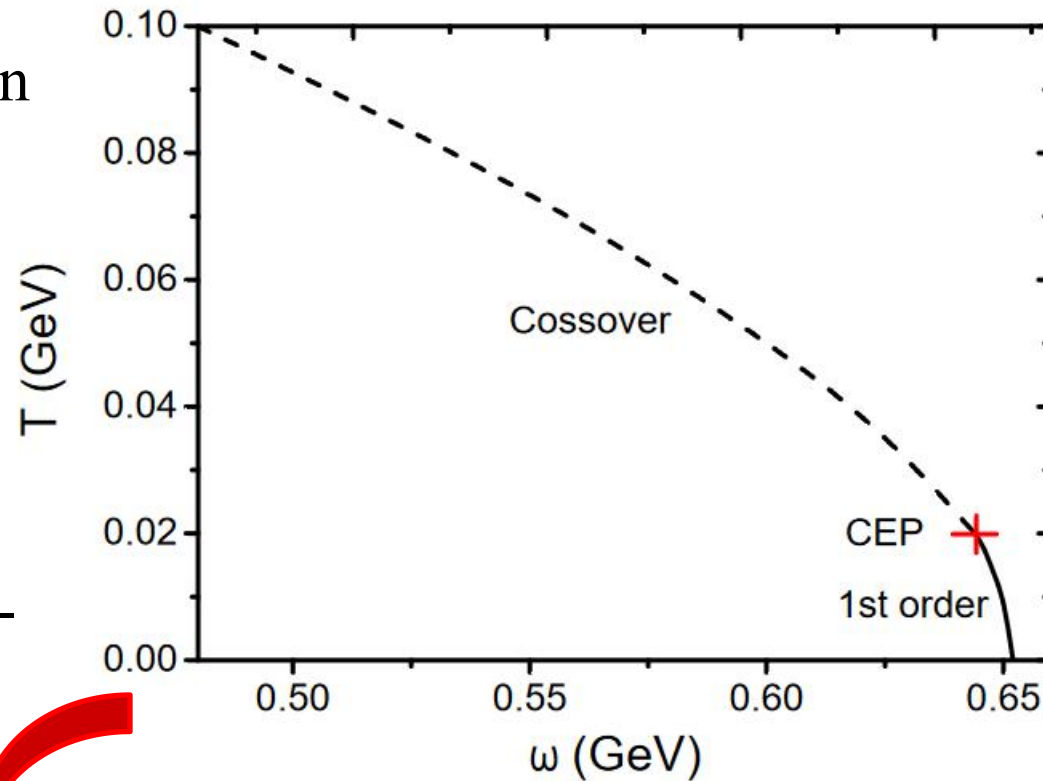


**L. Wang, et al., Phys.Rev.C (2019)**

Suppression

Centrifugal-like effect

$T - \Omega$  phase diagram



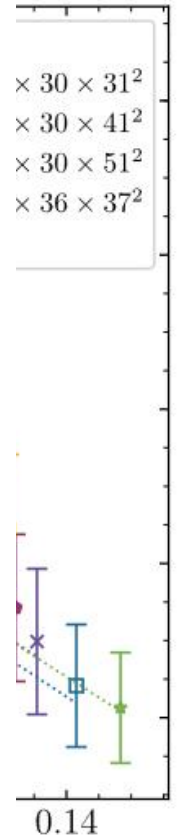
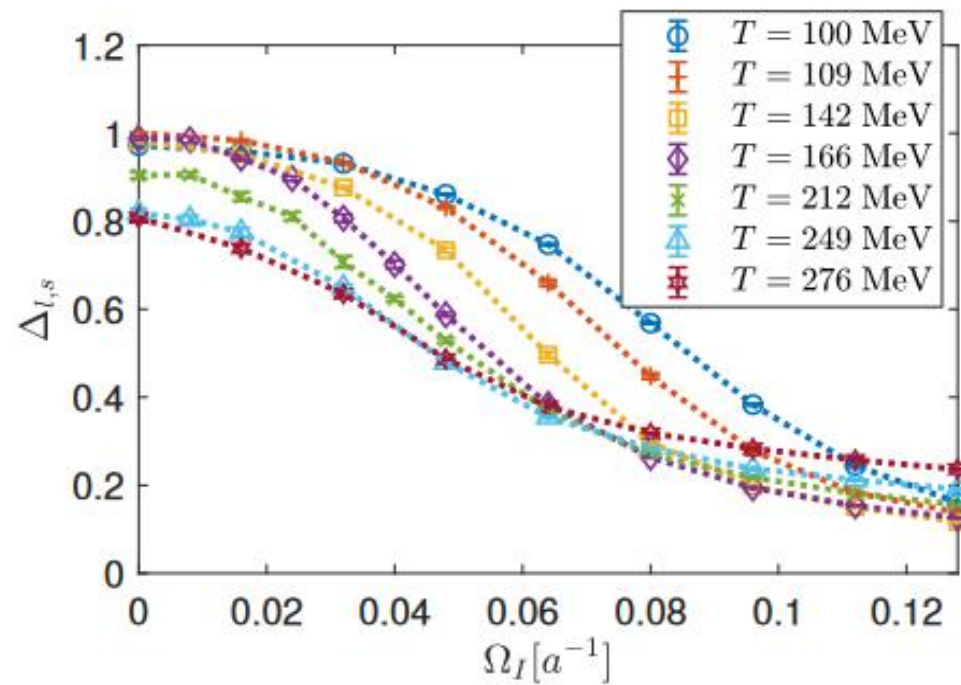
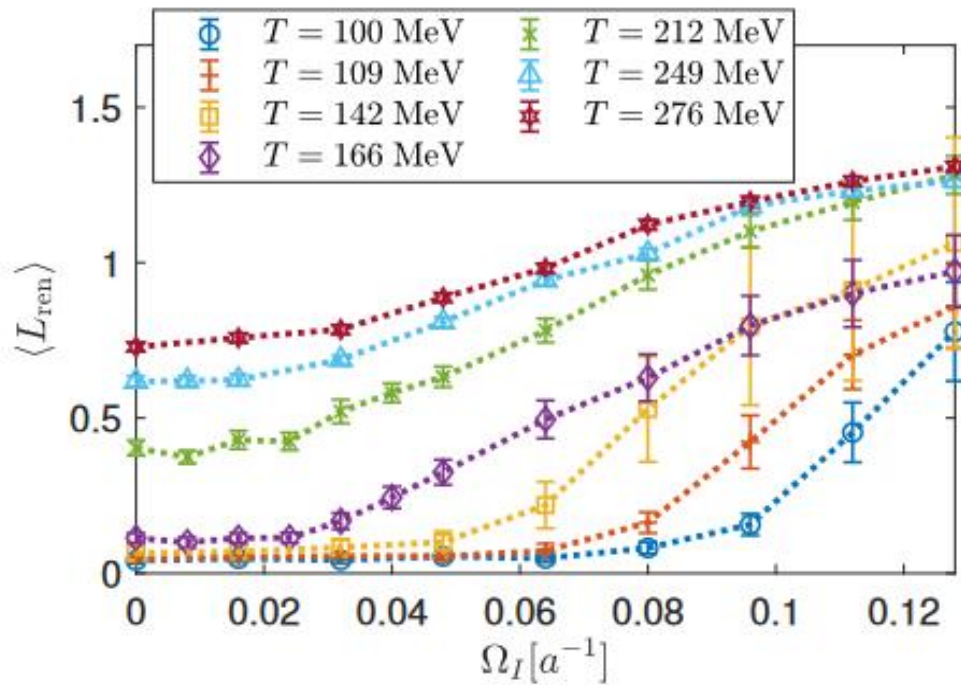
**Y. Jiang, et al., Phys.Rev.Lett (2016)**

Similar to chemical potential  $^6\mu$



# Lattice QCD

T  
C  
S  
F  
V  
C  
i



**J. Yang, X. Huang, arXiv: 2307.05755**

FIG. 4. The Polyakov loop and chiral condensate as functions of  $\Omega_I$ .

] a

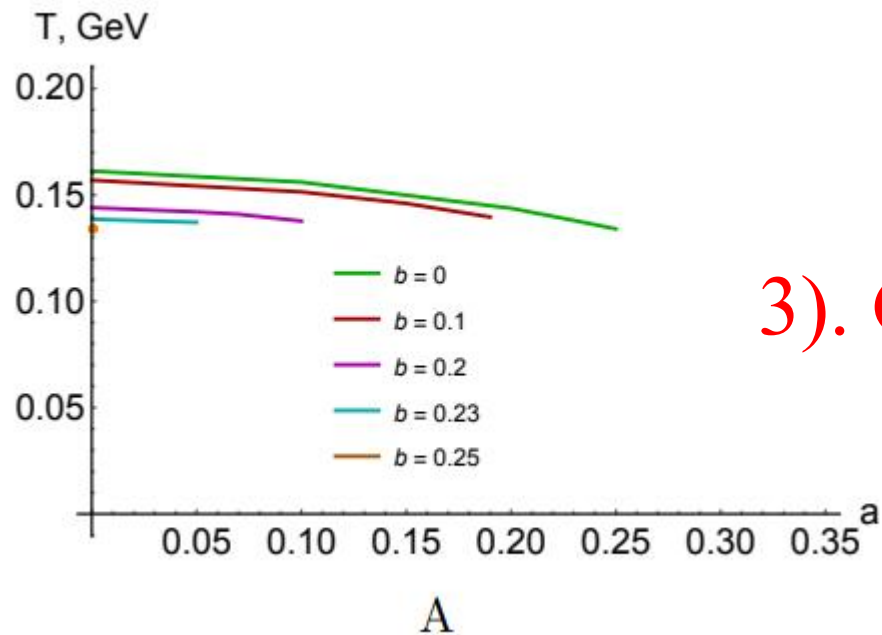
analytically continued to real angular velocity.

**V.V. Braguta, et al., Phys.Rev.D (2021)**

# Holographic results

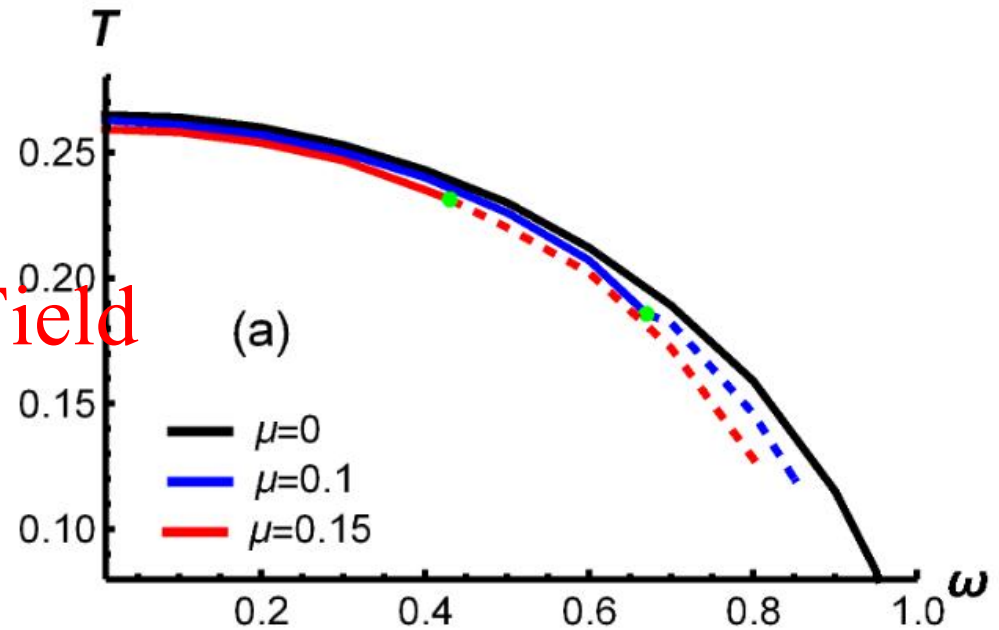
## Deconfinement phase transition

1). Kerr-AdS black hole



**I.Y. Aref'eva, et al., JHEP (2021)**

2). Doing a local Lorentz boost

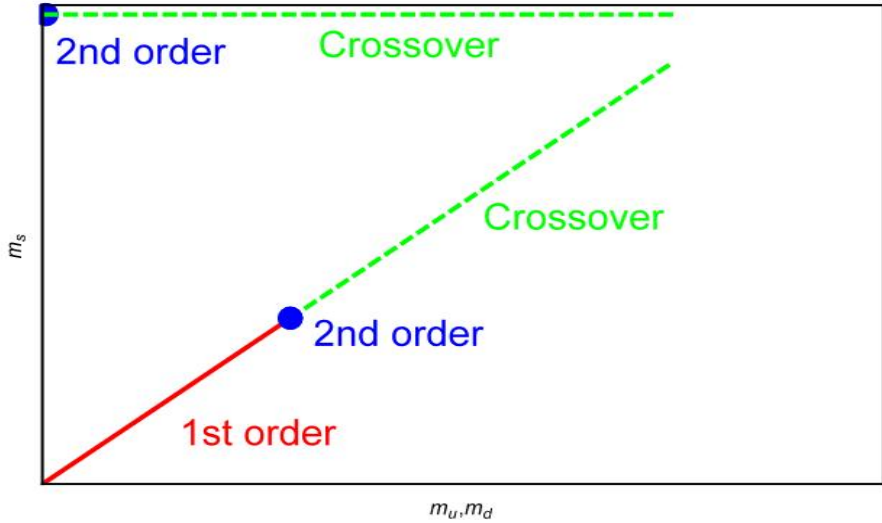


**X. Chen, et al., JHEP (2021)**

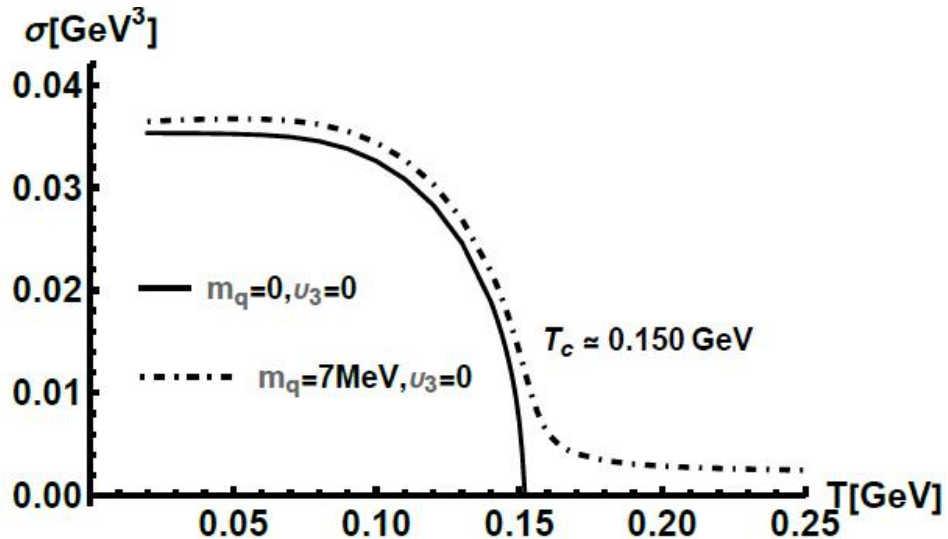
3). Gauge Field

# Chiral Condensation

Chelabi et al., Phys.Rev.D (2016)  
Chelabi et al., JHEP (2016)



( a )



( b )

5D action

$$S = - \int d^5x \sqrt{-g} e^{-\Phi} \text{Tr}(D_m X^\dagger D^m X + V_X(|X|)).$$

$$V(\chi) \equiv \text{Tr}(V_X(|X|)) = -\frac{3}{2}\chi^2 + v_3\chi^3 + v_4\chi^4.$$

$$\Phi(z) = -\mu_1 z^2 + (\mu_1 + \mu_0) z^2 \tanh(\mu_2 z^2),$$

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	$p$	$\Delta$	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A_{L\mu}^a$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A_{R\mu}^a$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$(2/z) X^{\alpha\beta}$	0	3	-3

Conformal AdS<sub>5</sub>

$$ds^2 = \frac{L^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu),$$

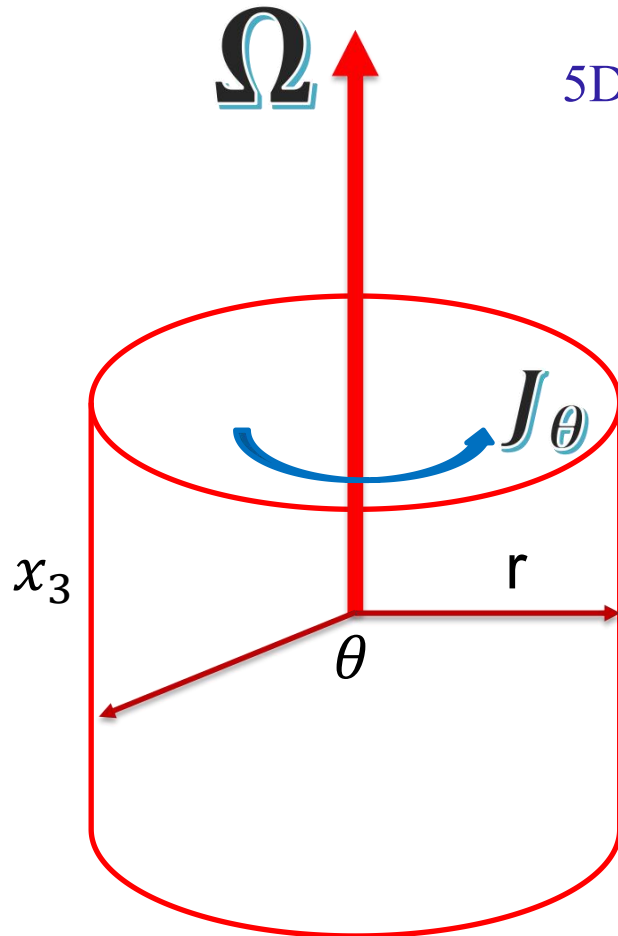
# Model

Y. Chen, Danning Li and M. Huang, Phys.Rev.D (2022)

Metric  $ds^2 = \frac{L^2}{z^2} [-f(z)dt^2 + \frac{dz^2}{f(z)} + dr^2 + r^2d\theta^2 + dx_3^2], \quad f(z) = 1 - (\frac{z}{z_h})^4$

5D action  $S_M = - \int d^5x \sqrt{-g} e^{-\Phi(z)} \left\{ \text{Tr}[(D^M X)^\dagger (D_M X) + V_X(|X|)] + \frac{1}{4} F_{MN} F^{MN} \right\},$

The only nonzero component of the gauge field is  $A_\theta$ , which is dual to the polar current operator  $\langle \bar{q} \gamma_\theta q \rangle$  and described an effective polarization term  $\vec{\Omega} \cdot \vec{J}$  in the dual field theory with angular momentum  $\vec{J}$ .



$$D_M X = \partial_M X - i A_M X$$

AdS Boundary conditions

$$\chi|_{z=0} = m_q \zeta,$$

$$A_\theta|_{z=0} = \Omega(r) r^2,$$

# Inhomogeneous condensation

Y. Chen, Danning Li and M. Huang, Phys.Rev.D (2022)

Parameters  $(m_q, v_3, v_4) = (0, -3, 8)$ ,  $\mu_0 = (0.43 \text{ GeV})^2$ ,  $\mu_1 = (0.83 \text{ GeV})^2$ ,  $\mu_2 = (0.176 \text{ GeV})^2$

$$T_c \simeq 174 \text{ MeV}$$

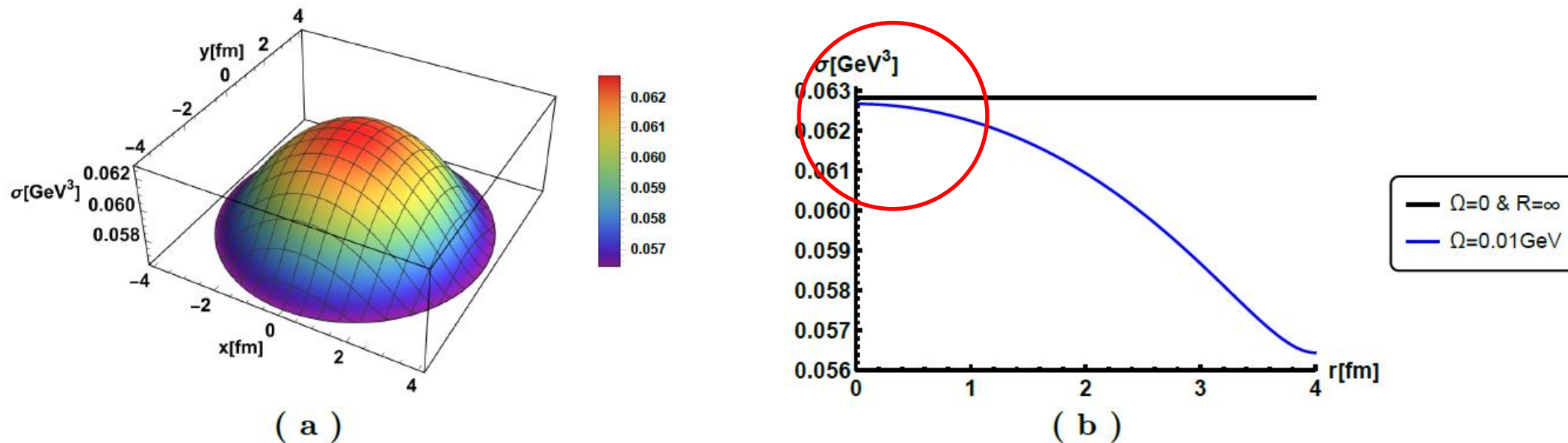


Figure 2. 3D and 2D plots of chiral condensation as a function of radial  $r$  at  $T = 170 \text{ MeV}$  and  $\Omega = 0.01 \text{ GeV}$  with NBC and  $(m_q, v_3, v_4) = (0, -3, 8)$ . In Fig.(b), the black line indicates the value of condensation at the same temperature without rotation and finite size.



# Tolman-Ehrenfest effect

Y. Chen, Danning Li and M. Huang, Phys.Rev.D (2022)

Different angular velocity distribution

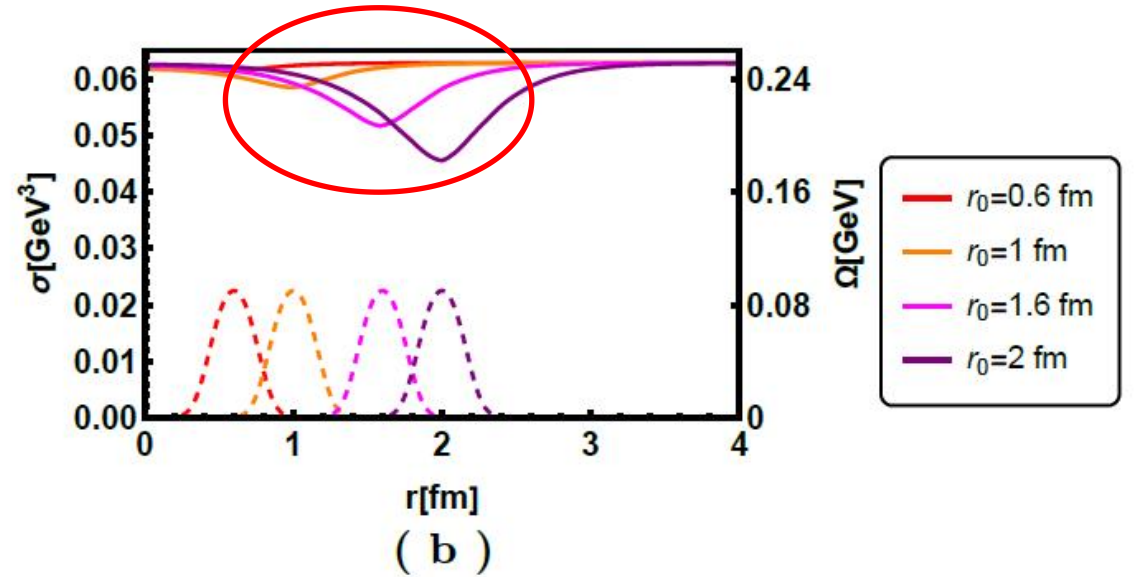
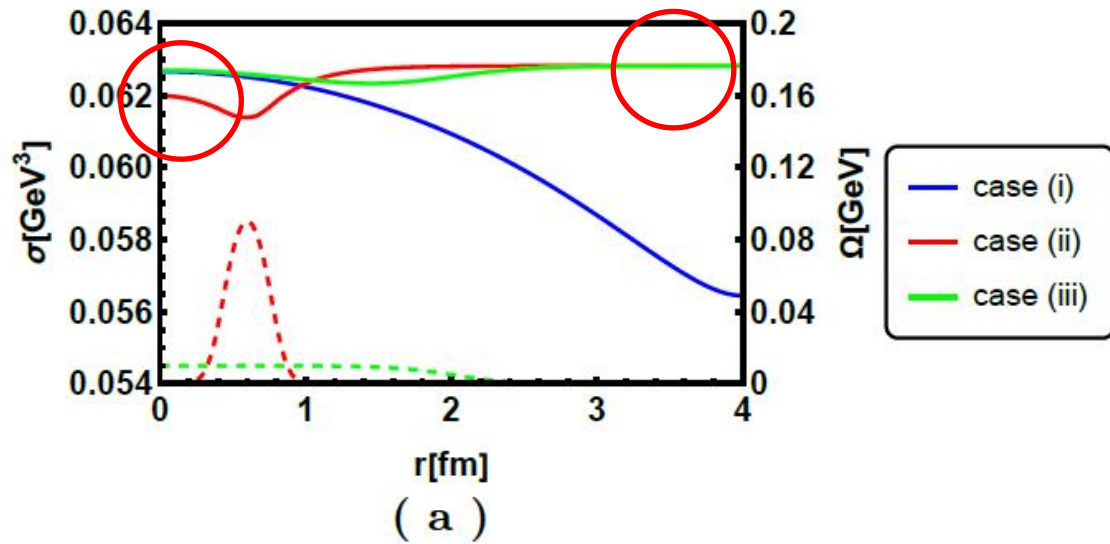


Figure 3. The chiral condensation as a function of radial  $r$  at  $T = 170$  MeV with NBC and  $(m_q, v_3, v_4) = (0, -3, 8)$ , where the solid and dashed lines denote the profile of the condensation and the distribution of the angular velocity, respectively. In (a), the three cases of angular velocity distribution are: (i)  $\Omega = 0.01$ , (ii)  $\Omega(r) = 0.18(\exp[1.5(r - 10)^2] + 1)^{-1}$  and (iii)  $\Omega(r) = 0.01(\exp[(r - 10)] + 1)^{-1}$ . Fig.(b) represents case (ii)  $\Omega(r) = 0.18(\exp[1.5(r - r_0)^2] + 1)^{-1}$  with  $r_0 = 0.6$  fm, 1 fm, 1.6 fm and 2 fm.

$$\begin{aligned}
 & \partial_r A_\theta - \chi^2 A_\theta = 0 \\
 & \partial_r \chi^2 - \frac{A_\theta^2}{r^2} \chi = 0
 \end{aligned}
 \xrightarrow{A_\theta \sim \Omega r^2}
 \frac{A_\theta^2}{r^2} \sim (\Omega r)^2 \sim (v)^2$$

# Phase diagram

Y. Chen, Danning Li and M. Huang, Phys.Rev.D (2022)

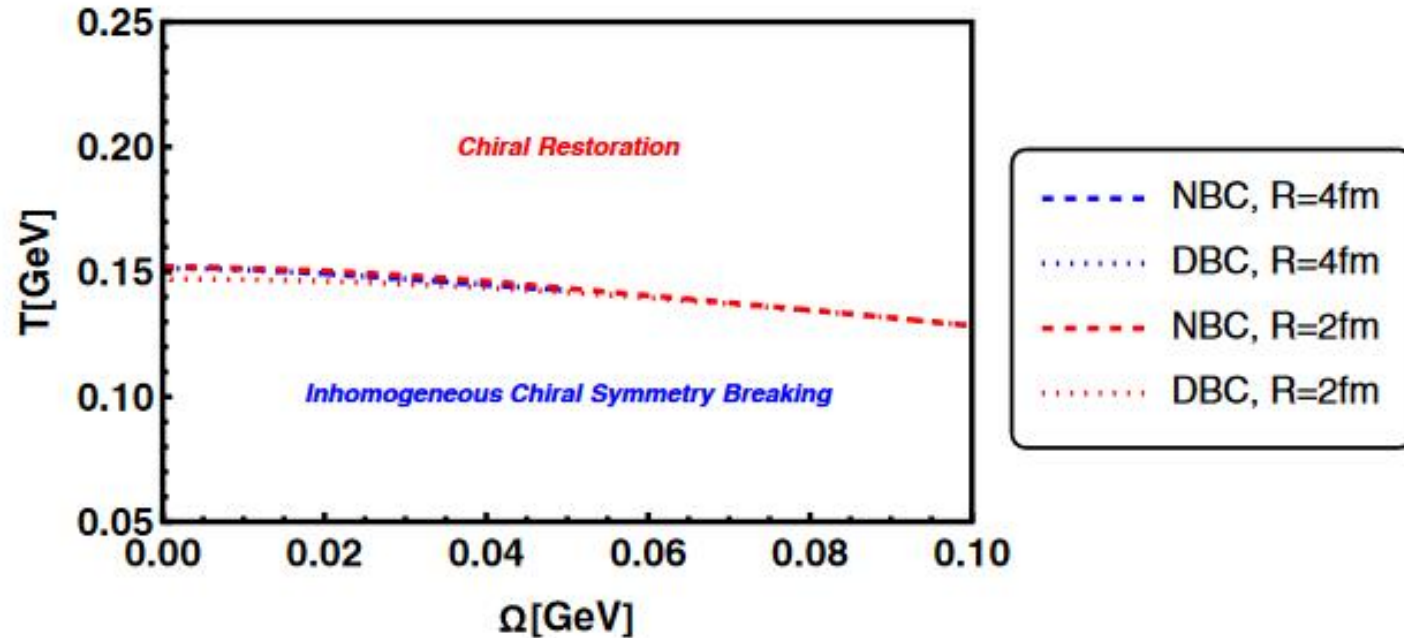


FIG. 13. The  $T - \Omega$  phase diagram at fixed radius  $R$  is shown in the figure. Here, the red and blue lines correspond to radii of  $R = 2$  and  $4$  fm, respectively, and the dashed and dotted lines represent NBC and DBC, respectively.

## **II. (a) Anisotropic background and deconfinement phase transition**



# DHQCD Model

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386

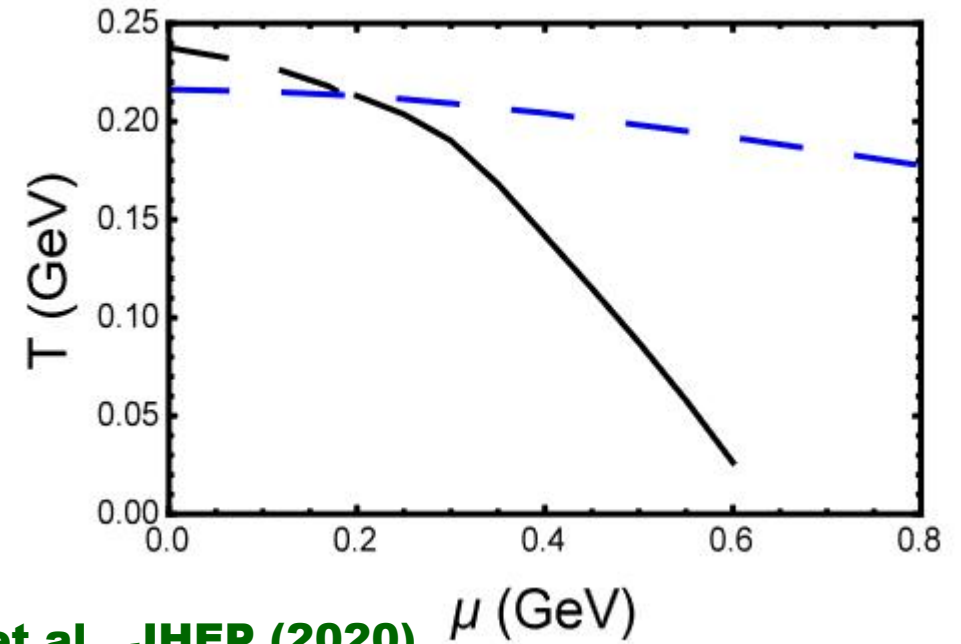
The dynamical holographic QCD (DHQCD) model with chemical potential is described by the Einstein-Maxwell-dilaton system. The action in the string frame can be written as

$$\begin{aligned} S_{\text{tot}}^s &= S_G^s + S_M^s, \\ S_G^s &= \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^s} e^{-2\Phi} \left[ R^s + 4\partial_M \Phi \partial^M \Phi - V^s(\Phi) - \frac{h(\Phi)}{4} e^{\frac{4\Phi}{3}} F_{MN} F^{MN} \right], \\ S_M^s &= - \int d^5x \sqrt{-g^s} e^{-\Phi} \text{Tr} \left[ \nabla_M X^\dagger \nabla^M X + V_X(|X|, F_{MN} F^{MN}) \right], \end{aligned}$$

potential reconstruction method

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_G^4 z^2 / \mu_G^2).$$

The DHQCD model can successfully describe hadron spectra, QCD phase transition, thermodynamical properties, transport properties of QCD matter.



X. Chen, et al., JHEP (2020)

# Model

The action in the Einstein frame can be written as

$$S_G^e = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^e} \left[ R^e - \frac{1}{2} \partial_M \Phi \partial^M \Phi - V^e(\Phi) - \frac{h(\Phi)}{4} F_{MN} F^{MN} \right],$$

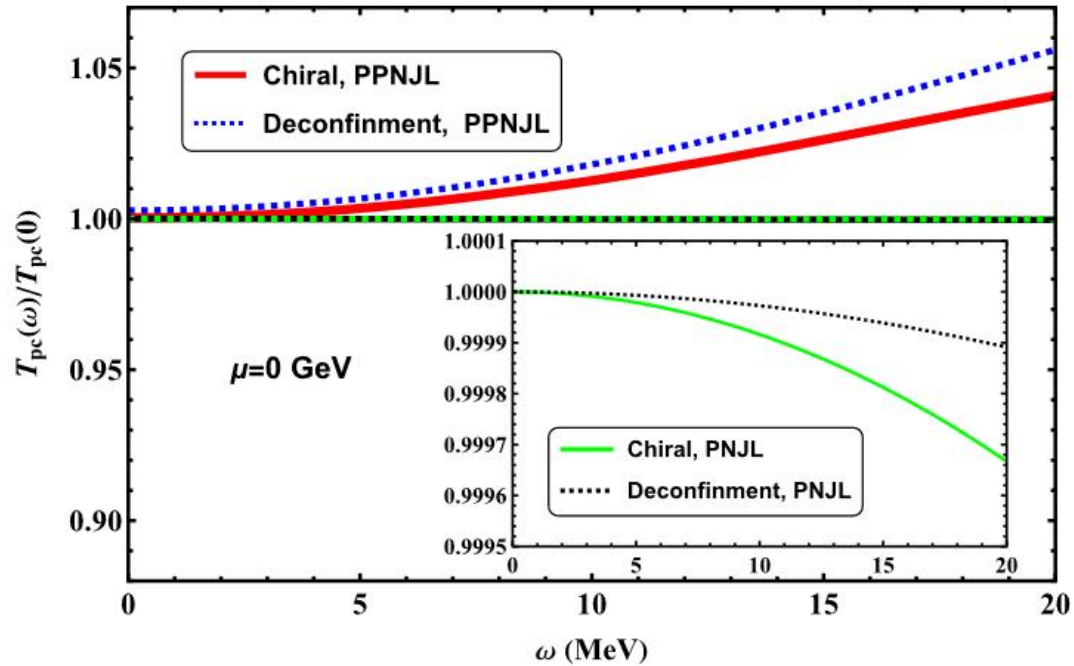
The rotation is introduced by a non-zero polar angle component of the gauge field. It can be expanded as  $A_\theta \sim \Omega r^2 + \rho_\theta$ . **Near the center**, the current  $\rho_\theta$  can be neglected and the gauge field is approximated as

$$A_M = (A_t, 0, 0, A_\theta, 0), \quad A_\theta = \Omega r^2,$$

And the background metric of the Einstein frame in the **cylindrical** coordinate system is

$$ds^2 = \frac{L^2 e^{2A_e(z)}}{z^2} \left[ -f(z) dt^2 + \frac{dz^2}{f(z)} + e^{B(z)} dr^2 + r^2 e^{B(z)} d\theta^2 + e^{-2B(z)} dx_3^2 \right],$$

# Polarized dilaton



polarized Polyakov-loop potential in polarized-PNJL model

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T, \omega)}{T^4} = -Cf(T, \omega) \left(\frac{T}{T_0}\right)^2 \Phi \bar{\Phi} - \frac{1}{3} (\Phi^3 + \bar{\Phi}^3) + C^{-1} f^{-1}(T, \omega) \left(\frac{T}{T_0}\right)^{-2} \Phi^2 \bar{\Phi}^2,$$

F. Sun, et al., Phys.Rev.D (2024)

We considering the **polarization** of the gluo-dynamics induced by the rotation

$$\Phi(z, \Omega) = \mu_G(\Omega)^2 z^2 \tanh(\mu_{G^2}(\Omega)^4 z^2 / \mu_G(\Omega)^2).$$

$\mu_G(\Omega)$  can be expanded as

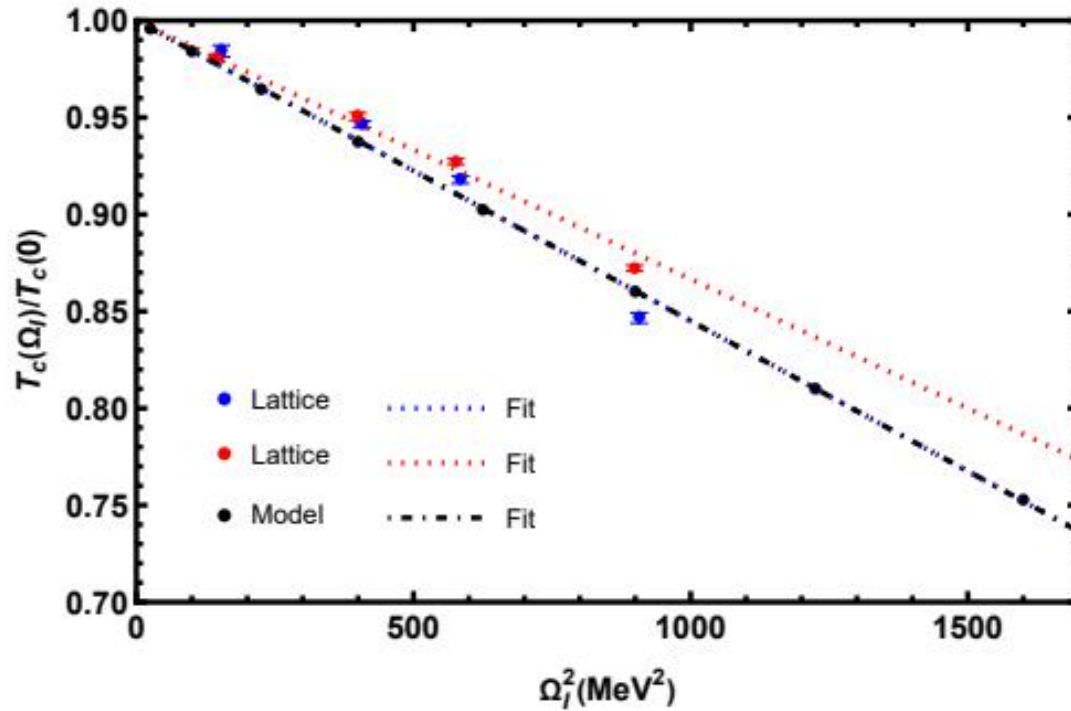
$$\mu_G + \mu_\Omega \Omega^2 + \mu_{\Omega^2} \Omega^4 + \dots$$

$$\Phi = (\mu_G + \mu_\Omega \Omega^2)^2 z^2 \tanh(\mu_{G^2}^4 z^2 / (\mu_G + \mu_\Omega \Omega^2)^2).$$

Y. Chen, et al., arXiv:2405.06386 27

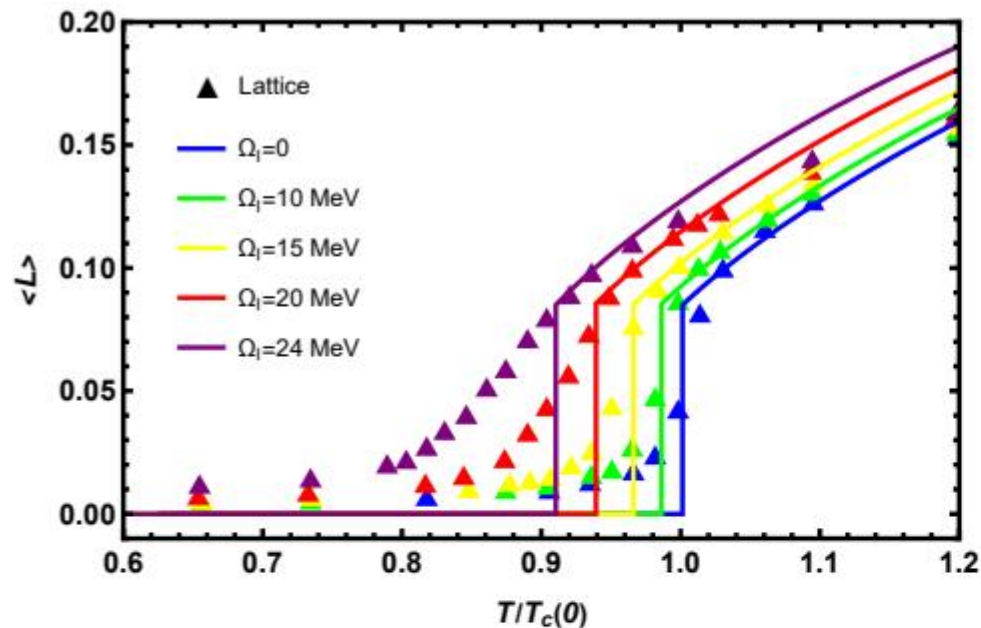
# Results

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386

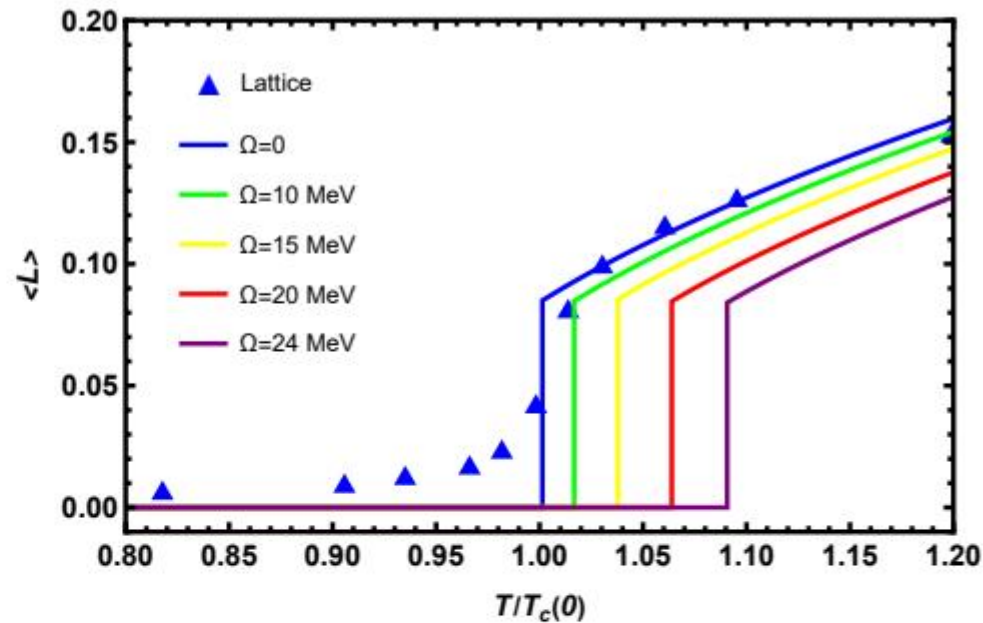


The only parameter  $\mu_\Omega$  to be determined in the DHQCD model is based on the relationship between the phase transition temperature  $T_c(\Omega_I)$  and the imaginary angular velocity predicted by lattice QCD.

$$T_c(\Omega_I)/T_c(0) = 1 - C_2\Omega_I^2$$

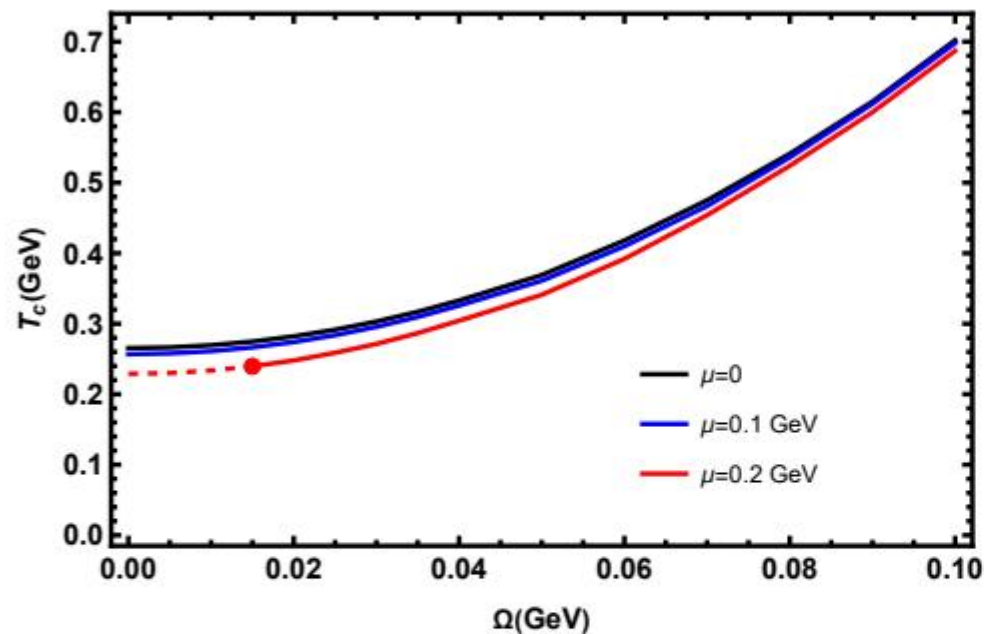


(a) imaginary rotation

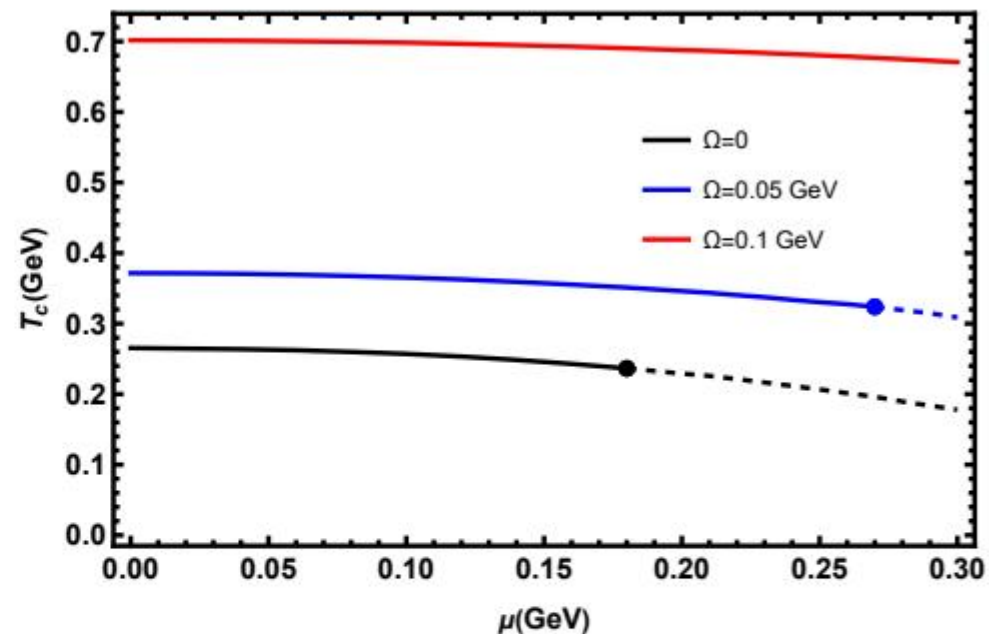


(b) real rotation

FIG. 4. The expectation value of the Polyakov loop  $\langle L \rangle$  obtained from the DHQCD model for pure gluon system and lattice as a function of temperature under both real and imaginary rotation. The triangles in the figure represent the lattice QCD data [100] and the solid lines are the model calculations.



(a)



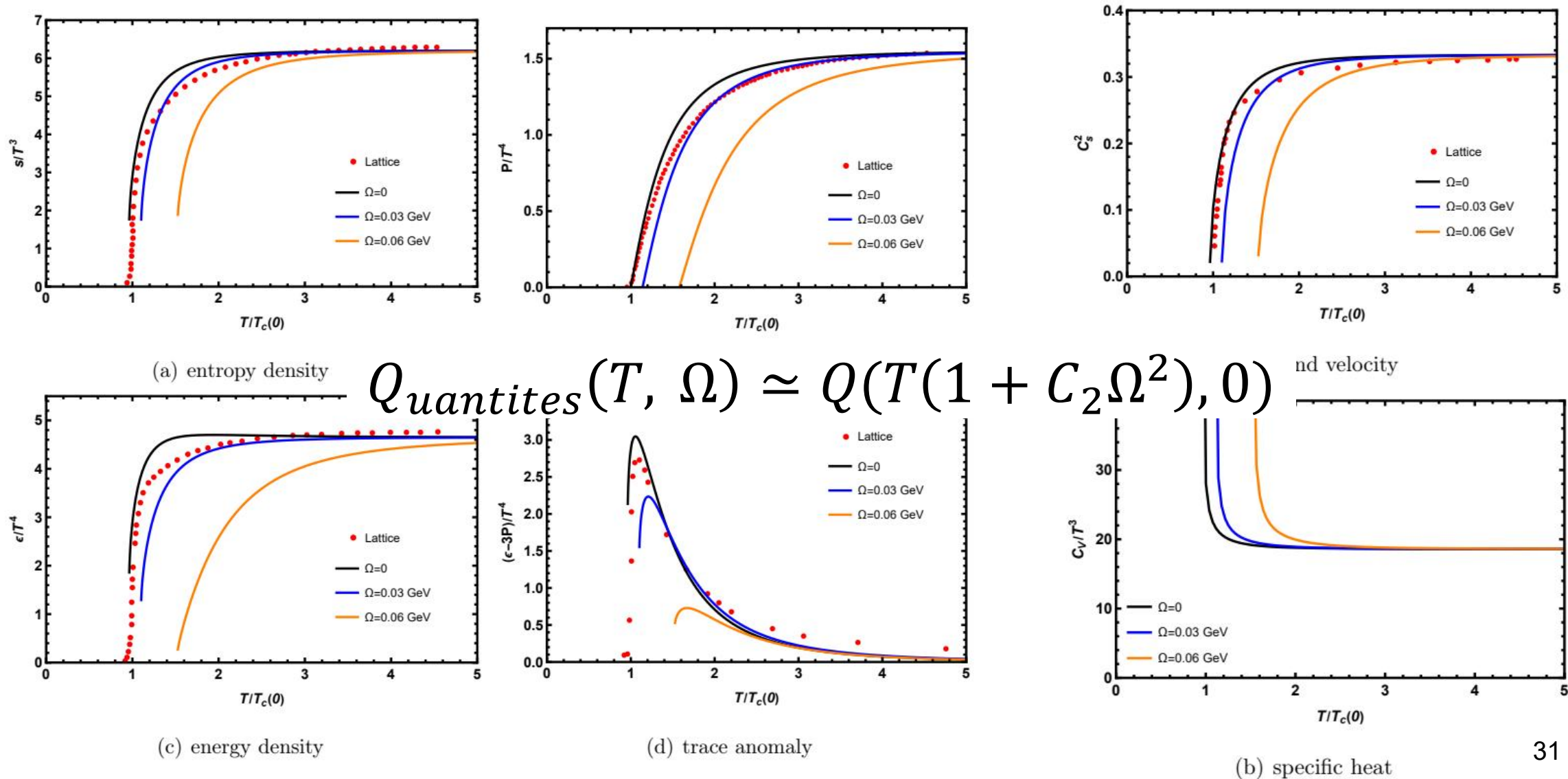
(b)

FIG. 5. The  $T - \Omega$  and  $T - \mu$  phase diagrams of deconfinement phase transition for the pure gluon system.



# Results

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386



## **II. (b) Chiral phase transition under rotation**



# DHQCD Model

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386

The DHQCD model can be written as

$$D_M X = \partial_M X - iA_M X$$



$$S_{\text{tot}}^s = S_G^s + S_M^s,$$

$$S_G^s = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^s} e^{-2\Phi} \left[ R^s + 4\partial_M \Phi \partial^M \Phi - V^s(\Phi) - \frac{h(\Phi)}{4} e^{\frac{4\Phi}{3}} F_{MN} F^{MN} \right],$$

$$S_M^s = - \int d^5x \sqrt{-g^s} e^{-\Phi} \text{Tr} [\nabla_M X^\dagger \nabla^M X + V_X(|X|, F_{MN} F^{MN})],$$

The potential of action is considered as the following form

$$V_X(|X|, F_{MN} F^{MN}) = \left(\frac{m_5^2}{2} + \lambda_2 F^2\right) \chi^2 + (v_4 + \lambda_4 F^2) \chi^4 + (v_6 + \lambda_6 F^2) \chi^6,$$

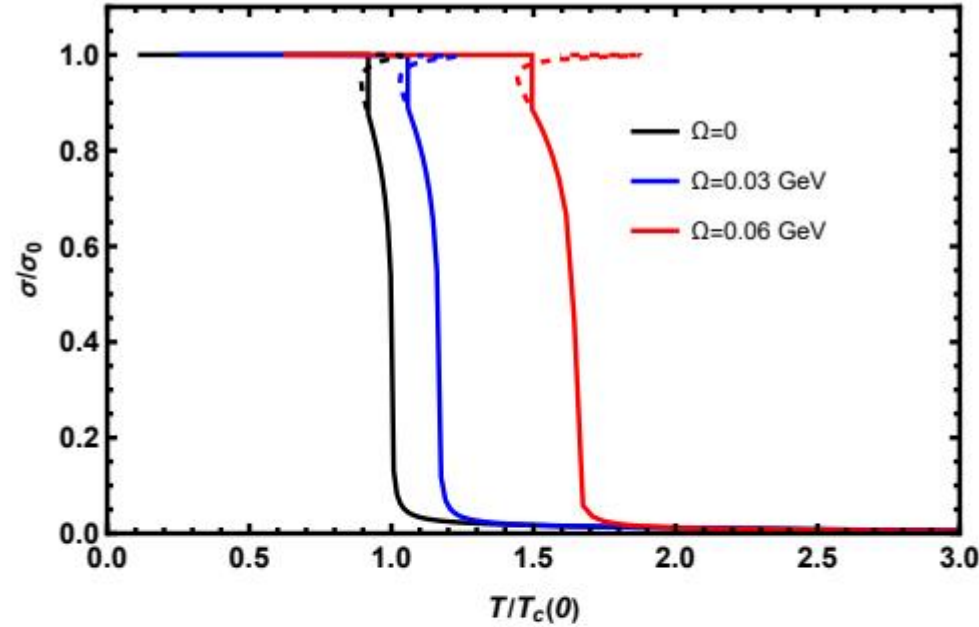


The equation of motion of scalar field can be obtained as

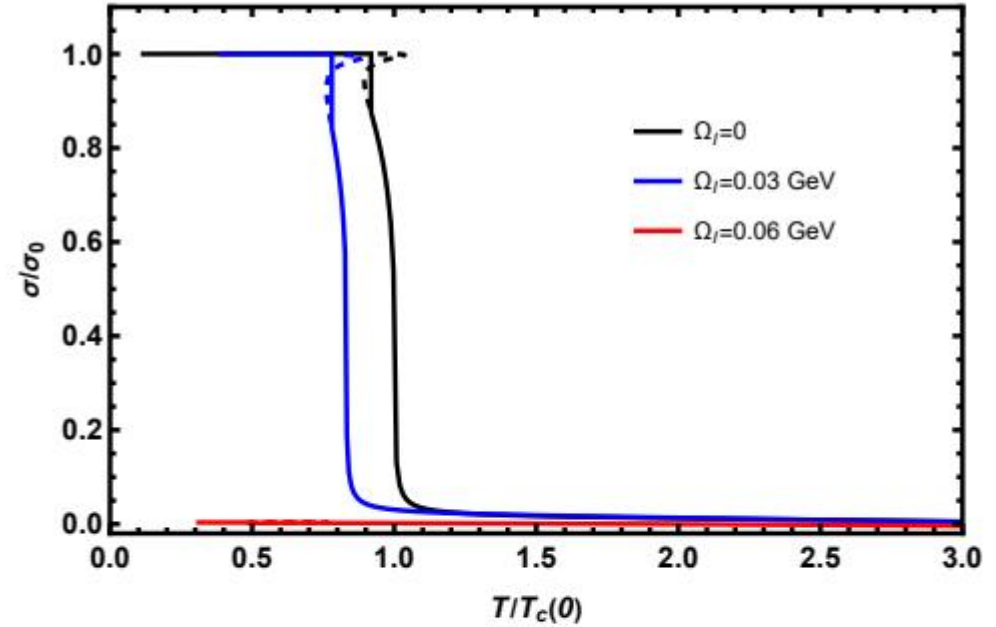
$$\begin{aligned} & - \frac{8e^{-2(A_s+B)}}{f} \chi \Omega^2 z^2 (\lambda_2 + \lambda_4 \chi^2 + \lambda_6 \chi^4) + \frac{2e^{-2A_s}}{f} \chi A_t'^2 z^2 (\lambda_2 + \lambda_4 \chi^2 + \lambda_6 \chi^4) \\ & - \frac{e^{2A_s}}{f z^2} \chi (-3 + 4v_4 \chi^2 + 6v_6 \chi^4) + \left( 3A_s' + \frac{f'}{f} - \Phi' - \frac{3}{z} \right) \chi' + \chi'' = 0, \end{aligned}$$

# Results

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386



(a) real rotation

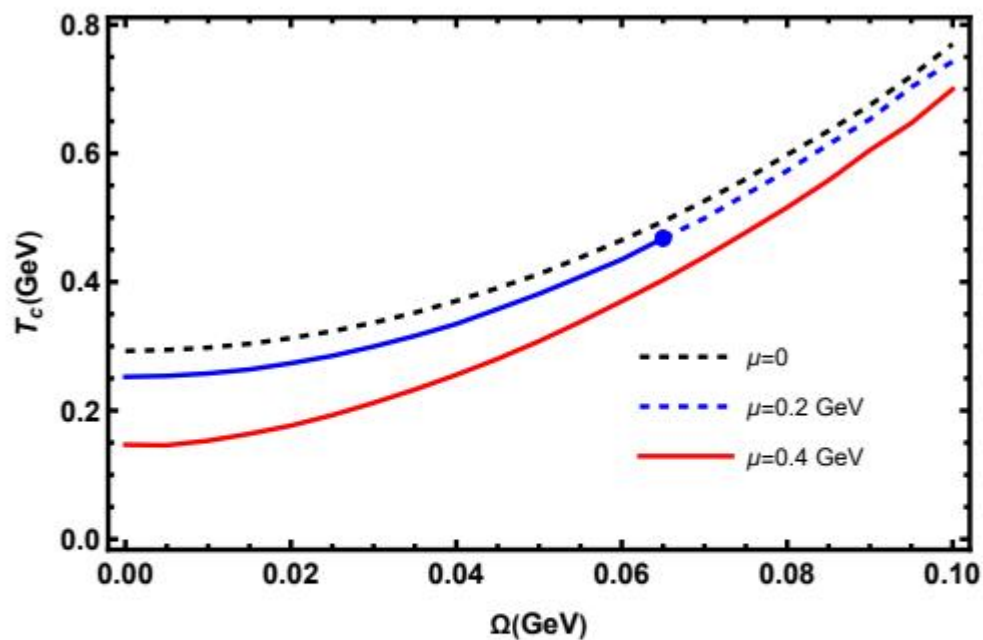


(b) imaginary rotation

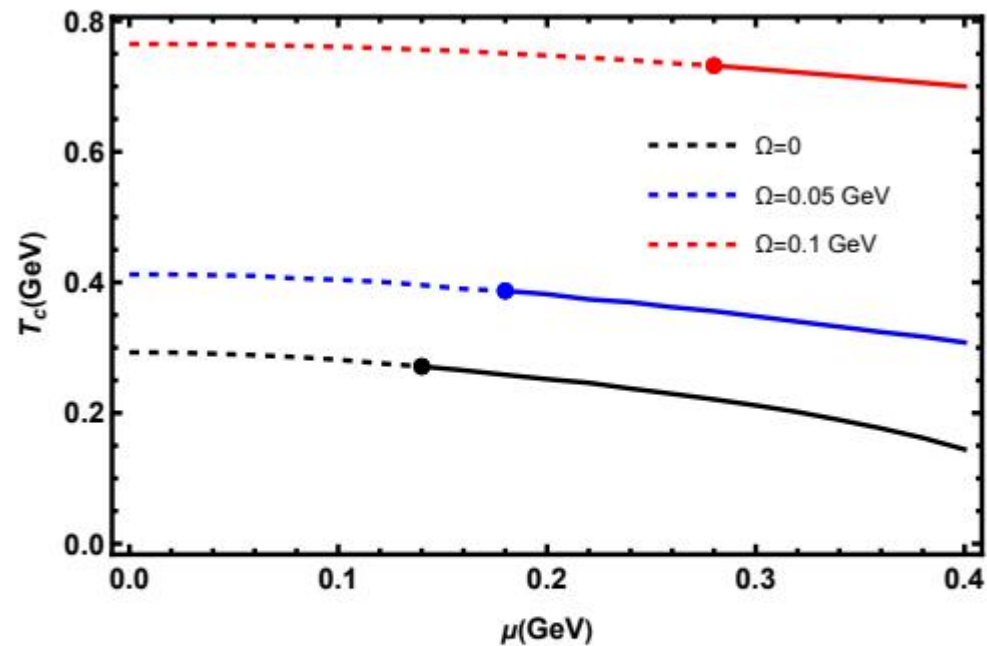
FIG. 6. The chiral condensate for 2-flavor system as a function of temperature at different angular velocities and imaginary angular velocities with chemical potential  $\mu = 0.12$  GeV. In the figure  $\sigma_0$  is the maximum value of condensation and  $T_c(0)$  is the critical temperature with zero angular velocity.

# Results

Y. Chen, X. Chen, D. Li and M. Huang, arXiv:2405.06386



(a)

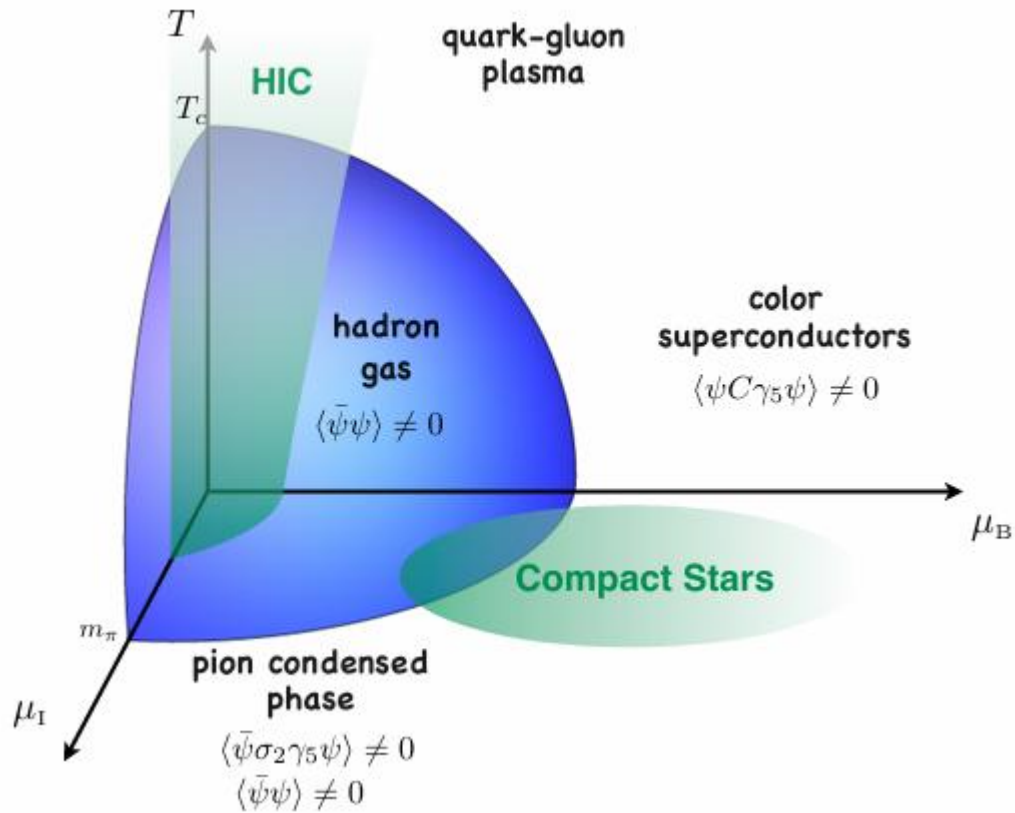


(b)

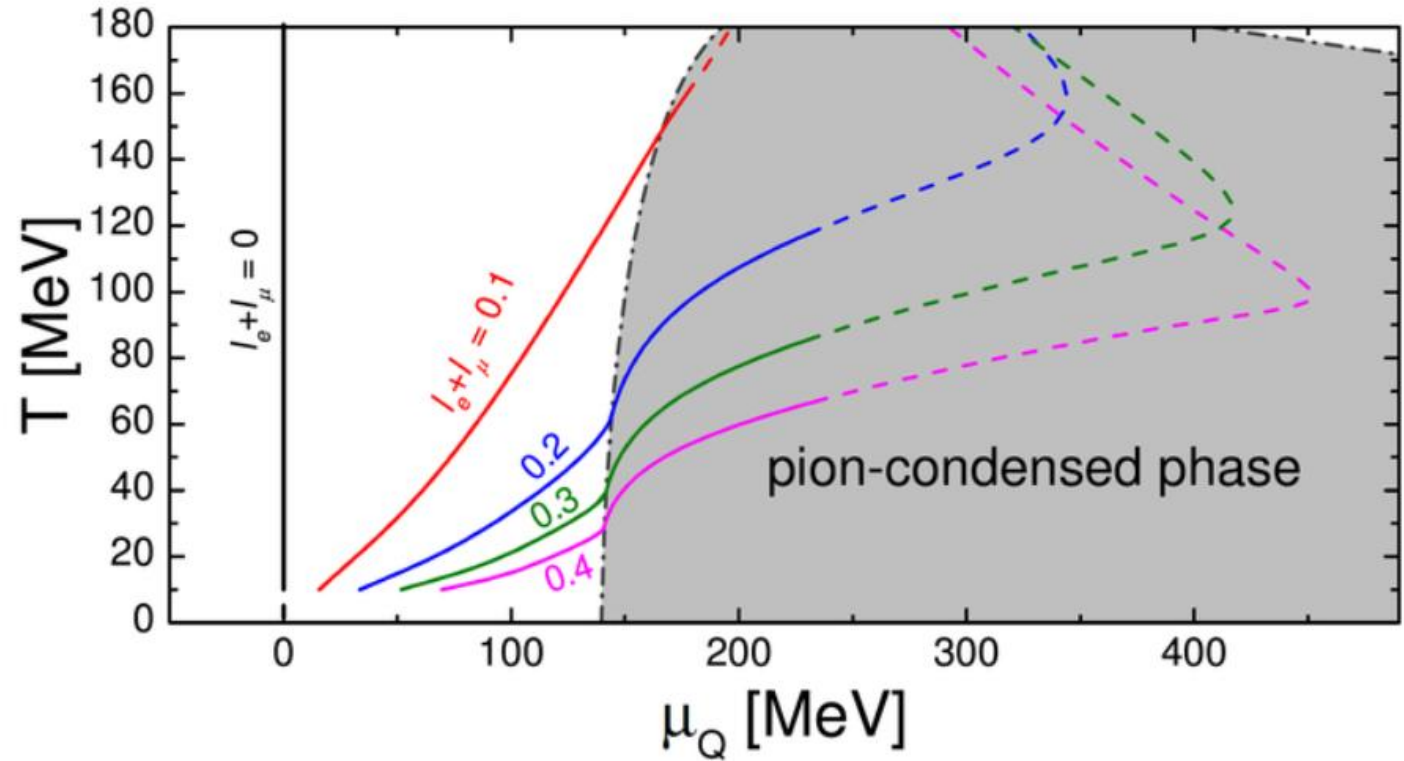
FIG. 7. The  $T - \Omega$  and  $T - \mu$  phase diagrams of chiral phase transition for 2-flavor system.

# **III. Pion Condensation**

# Phase Diagram



**Mannarelli, Particles (2019)**



**Vovchenko et al., PRL126, 012701 (2021)**

The holographic QCD model with  $N_f = 2$  is constructed from  $SU(2)_L \times SU(2)_R \equiv SU(2)_V \times SU(2)_A$  flavor symmetry. The probe action is given as

$$S_{KKSS} = - \int d^4x \int_0^{z_h} dz \sqrt{-g} e^{-\Phi} \text{Tr} \left[ |DX|^2 + m_5^2(z) |X|^2 + \lambda |X|^4 + \frac{1}{4g_5^2} (F_V^2 + F_A^2) \right]$$

The vacuum of scalar corresponds to sigma condensate and pion condensate can be given as  $X = \frac{1}{2} (\Sigma t^0 + \Pi^a t^a)$ . The nonzero gauge field of model is  $V_0^{(3)} = \mu_I - \rho_I z^2$ , with isospin chemical potential and isospin density. And the background metric is AdS-black hole

$$ds^2 = \frac{L^2}{z^2} \left[ -f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]$$

we have considered two possible forms of the five-dimensional mass squared as follows:

$$m_5^2(z) = -3 - \mu_c^2 z^2, \quad (\text{SW-I})$$

$$m_5^2(z) = -3 \left( 1 + \gamma_m \tanh [\kappa \Phi(z)] \right), \quad (\text{SW-II})$$

The equations of motion of the holographic model are obtained as

$$\frac{\lambda\Sigma^3}{2z^2} + \frac{v\Pi a_2}{f} + \left( -\frac{a_2^2}{f} + \frac{\lambda\Pi^2 + 2m_5^2(z)}{2z^2} \right) \Sigma + \left( -f' + f \left( \frac{3}{z} + \Phi' \right) \right) \Sigma' - f\Sigma'' = 0, \quad (22)$$

$$\frac{\lambda\Pi^3}{2z^2} + \frac{v\Sigma a_2}{f} + \left( -\frac{v^2}{f} + \frac{\lambda\Sigma^2 + 2m_5^2(z)}{2z^2} \right) \Pi + \left( -f' + f \left( \frac{3}{z} + \Phi' \right) \right) \Pi' - f\Pi'' = 0, \quad (23)$$

$$\frac{g_5^2\Sigma}{z^2} (v\Pi - a_2\Sigma) + \left( -\frac{f}{z} - f\Phi' \right) a_2' + f a_2'' = 0, \quad (24)$$

$$\frac{g_5^2\Pi}{z^2} (a_2\Sigma - v\Pi) + \left( -\frac{f}{z} - f\Phi' \right) v' + f v'' = 0. \quad (25)$$

For the (pseudo-)scalar fields, the expansions are

$$\Sigma \rightarrow m_q \zeta z + \frac{\langle \sigma \rangle}{\zeta} z^3 + \mathcal{O}(z^3), \quad \Pi^a \rightarrow \frac{\langle \pi^a \rangle}{\zeta} z^3 + \mathcal{O}(z^3),$$



# Condensation

Y. Chen, M. Ding, K. Bitaghsir Fadafan,  
D. Li and M. Huang, arXiv: 2408.17080

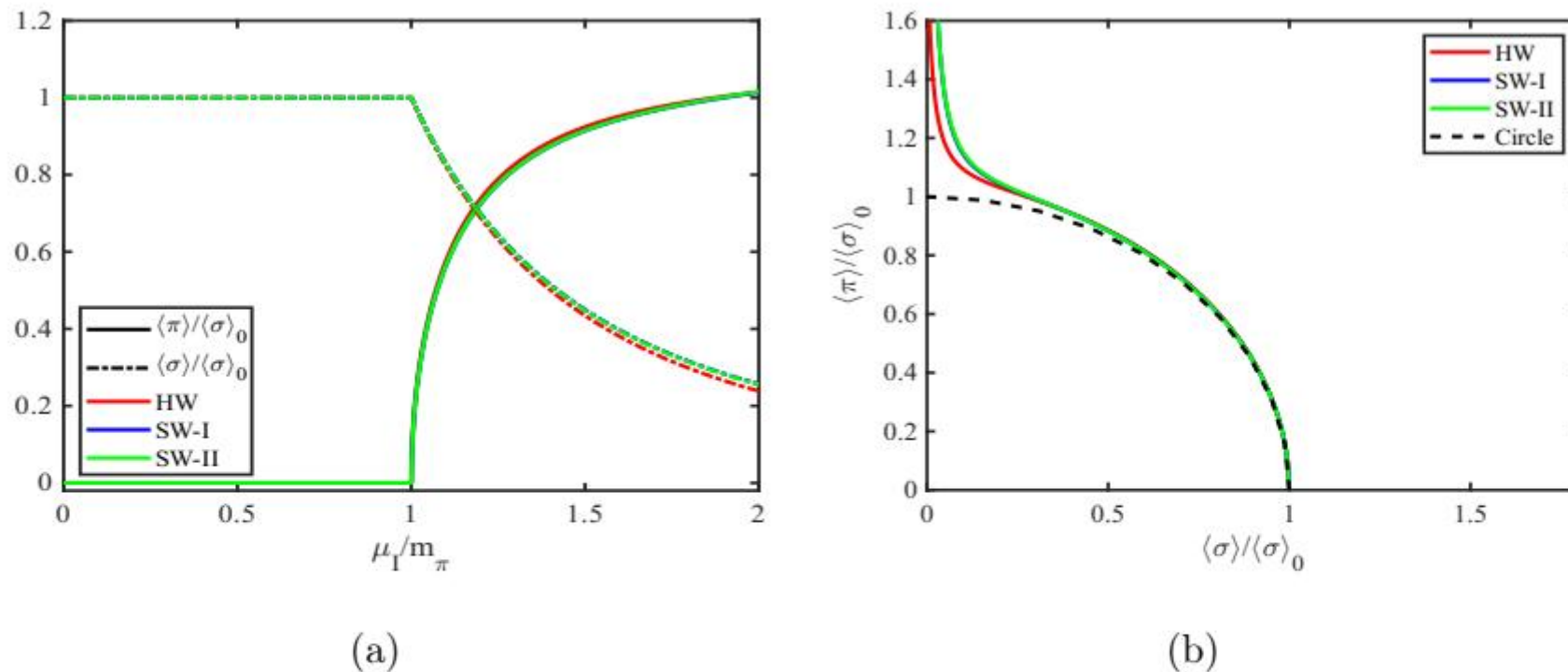


FIG. 1. Panel (a) illustrates how the sigma and pion condensates vary with the isospin chemical potential  $\mu_I$  in both the HW and SW models. Panel (b) reveals the interrelationship between the sigma and pion condensates, where the black dashed line represents the unit circle.



# Condensation

Y. Chen, M. Ding, K. Bitaghsir Fadafan,  
D. Li and M. Huang, arXiv: 2408.17080

According to  $\chi$ PT, the expressions for isospin density and axial-vector condensation can be simplified to

$$n_I = f_\pi^2 \mu_I \left( 1 - \frac{m_\pi^4}{\mu_I^4} \right) \Theta(\mu_I - m_\pi),$$

$$\langle \sigma \rangle_A = \frac{f_\pi^2 m_\pi^2}{\mu_I} \sqrt{1 - \frac{m_\pi^4}{\mu_I^4}} \Theta(\mu_I - m_\pi),$$

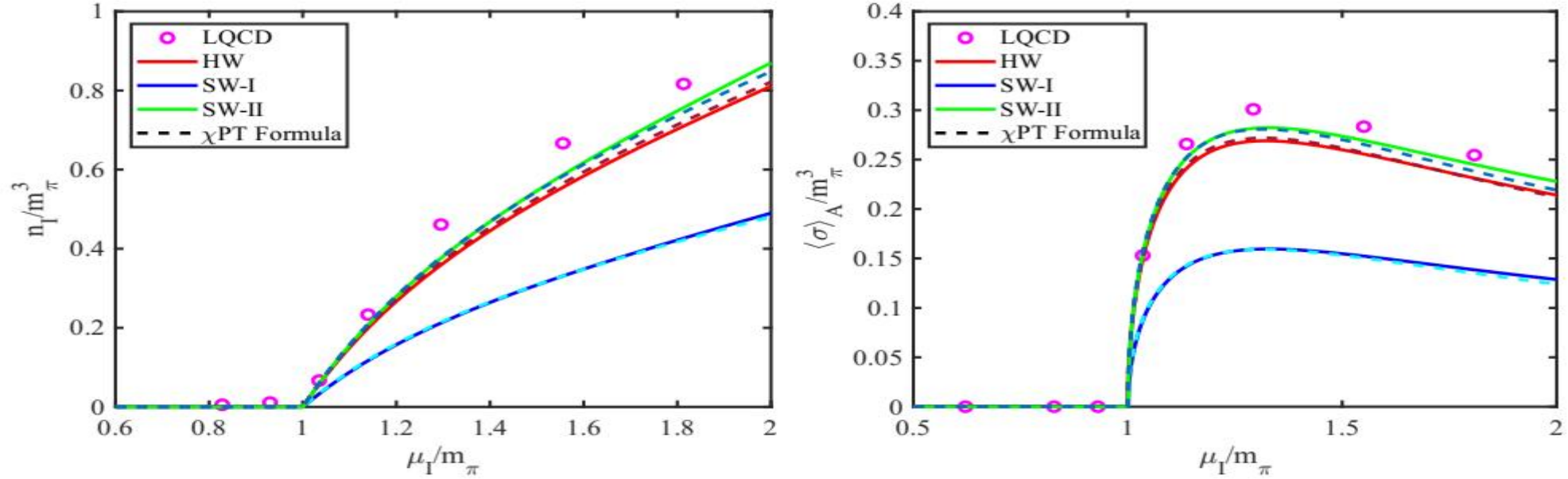


FIG. 2. The isospin density and axial vector condensate as a function of isospin chemical potential  $\mu_I$  in HW and SW model. The black dashed lines are calculations from  $\chi$ PT formulas (29) and (30).

At zero temperature, the pionic pressure  $p$  and energy density  $\epsilon$  can be given by thermodynamic relations

$$p = \frac{\log Z_{\text{QCD}}}{V} = \int_0^{\mu_I} d\mu'_I n_I(\mu'_I), \quad \epsilon = -p + \mu_I n_I.$$

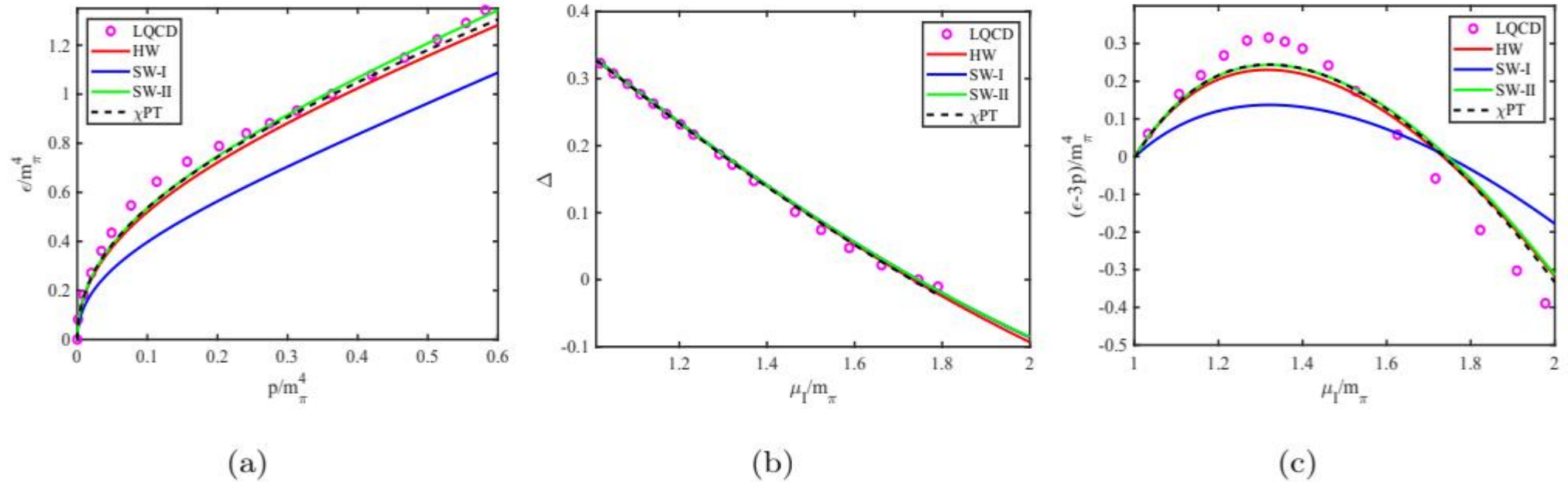


FIG. 3. The equation of state, normalized trace anomalies  $\Delta = 1/3 - p/\epsilon$  and  $(\epsilon - 3p)/m_\pi^4$  in HW and SW model. The magenta circles represent lattice QCD data [7]. The black dashed lines are calculations from  $\chi$ PT [22].

The square of the speed of sound and adiabatic index is given as are defined as

$$c_s^2 = \frac{\partial p}{\partial \epsilon},$$

$$\gamma = \frac{d \log p}{d \log \epsilon} = \frac{\epsilon}{p} c_s^2.$$

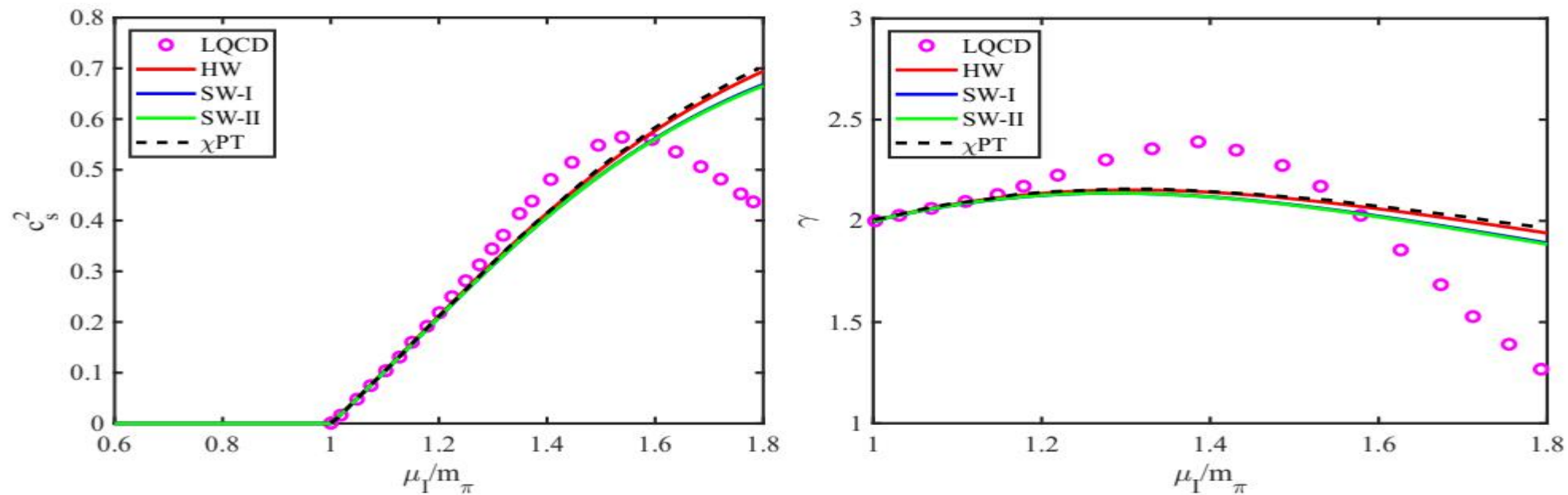


FIG. 4. The speed of sound and the adiabatic index as a function of the isospin chemical potential in the HW and SW models. The black dashed lines are calculations from  $\chi$ PT and the magenta circles are the lattice QCD data from Ref. [22].

The form of the TOV equations are as follow

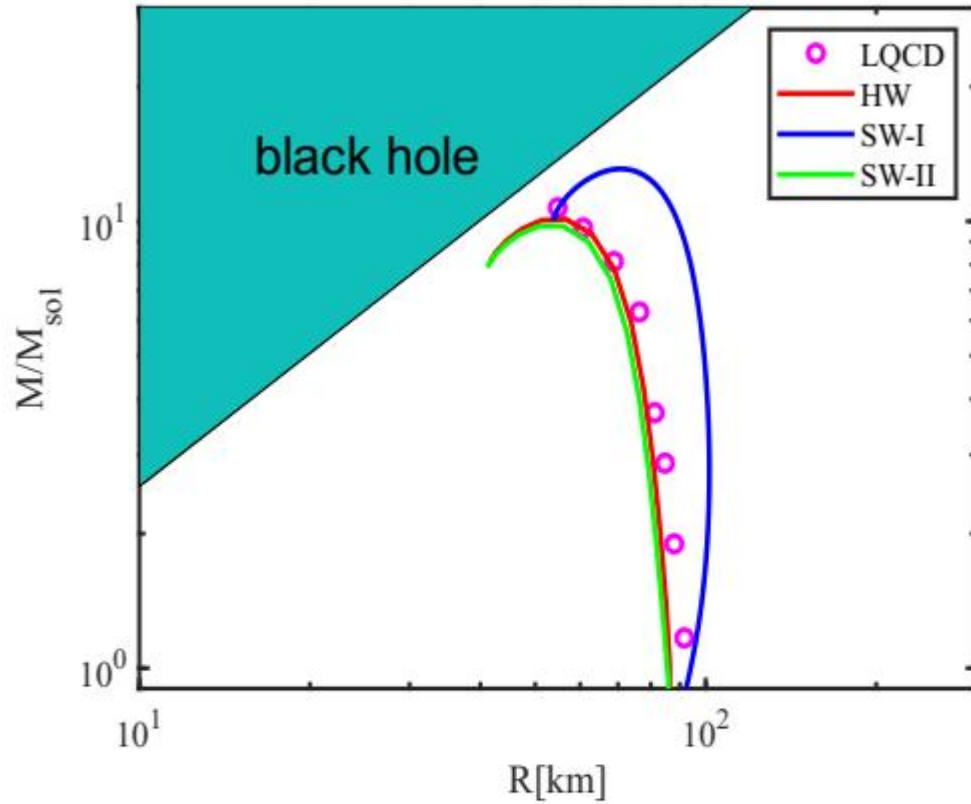
$$\frac{d}{dr}p(r) + [p(r) + \epsilon(r)] \frac{G [m(r) + 4\pi r^3 P(r)]}{r^2 \left[1 - 2\frac{Gm(r)}{r}\right]} = 0,$$
$$\frac{d}{dr}m(r) - 4\pi r^2 \epsilon(r) = 0,$$

The dimensionless tidal deformability  $\Lambda$

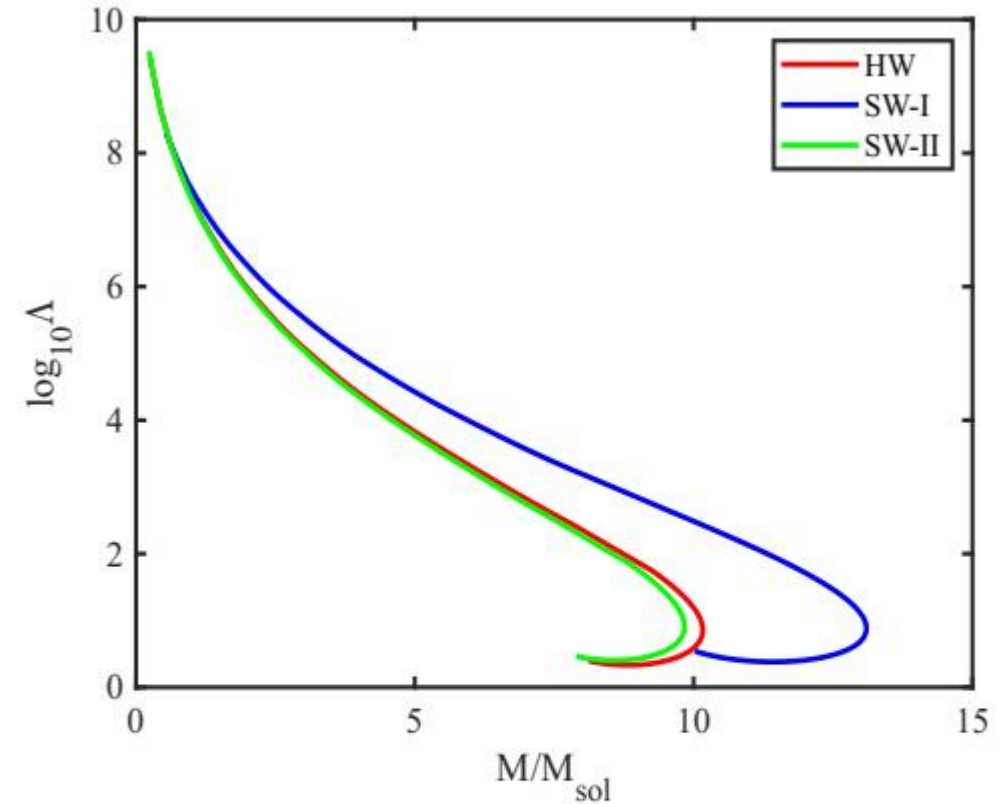
$$\Lambda = \frac{2k_2}{3C^5},$$

where

$$k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y - 1) - y] \{2C[6 - 3y + 3C(5y - 8)]$$
$$+ 4C^3 [13 - 11y + C(3y - 2) + 2C^2(1 + y)]$$
$$+ 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \ln(1 - 2C)\}^{-1},$$



(a)



(b)

FIG. 5. The mass-radius relation and tidal deformability of pion stars. The magenta circles are the mass-radius relation obtained through the equation of state from lattice QCD data Ref. [22].

## **IV. Conclusion and discussion**

# Summary

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- 1) We use a new approach to investigate the effect of rotation on deconfinement and chiral phase transitions in the dynamical holographic QCD model. Our holographic calculations are consistent with lattice QCD data.
  - ① the non-trivial term  $A_\theta = \Omega r^2$  in the gauge field
  - ② **polarization of the gluo-dynamics** induced by the rotation (the  $\Omega$  dependent dilaton field)



# Summary

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2) In the DHQCD model, the effect of rotation on the deconfinement phase transition and the chiral restoration phase transition shows consistency. Both the critical temperatures decrease/increase with

imaginary/real angular velocity as  $\frac{T}{T_c} \sim 1 - C_2 \Omega_I^2$  and  $\frac{T}{T_c} \sim 1 + C_2 \Omega^2$ ,

which is consistent with lattice QCD results.

3) We found that the effect of rotation on the entropy density, pressure, square of the speed of sound, and specific heat can be approximated by

the relationship  $Q(T, \Omega) \simeq Q(T(1 + C_2 \Omega^2), 0)$

# Summary

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4) The results from the holographic models show good agreement with lattice QCD in terms of **isospin density, axial-vector condensation, EoS, and normalized trace anomaly**. However, discrepancies were found in the calculations of **the sound speed and adiabatic index**.

5) Further analysis shows that the results from the holographic models are very similar to those from  **$\chi$ PT**.

**Thanks !**