NANJING UNIVERSITY

Mauricio N. Ferreira

Non-perturbative methods and their application to high energy physics

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Emergent gluon mass: Foundations and lattice signals

Dynamical mass generation in QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \bar{\psi}^{i}_{f} \left(i\gamma^{\mu} D_{\mu} - m_{f} \right)_{ij} \psi^{j}_{f} + \frac{1}{2\xi} (\partial^{\mu} A^{a}_{\mu})^{2} - \bar{c}^{a} \partial^{\mu} D^{ab}_{\mu} c^{b}$$

- At the level of the Lagrangian:
  - **Gluons** are massless;
  - **Quarks** have **current masses**, but **far smaller than the hadrons they constitute**
- Vast majority of the observable mass is **generated by the nonperturbative QCD dynamics**.
- Perturbation theory cannot generate mass at any finite order
- To study **dynamical mass generation**, we look at the behavior of the nonperturbative QCD Schwinger functions (propagators and vertices):

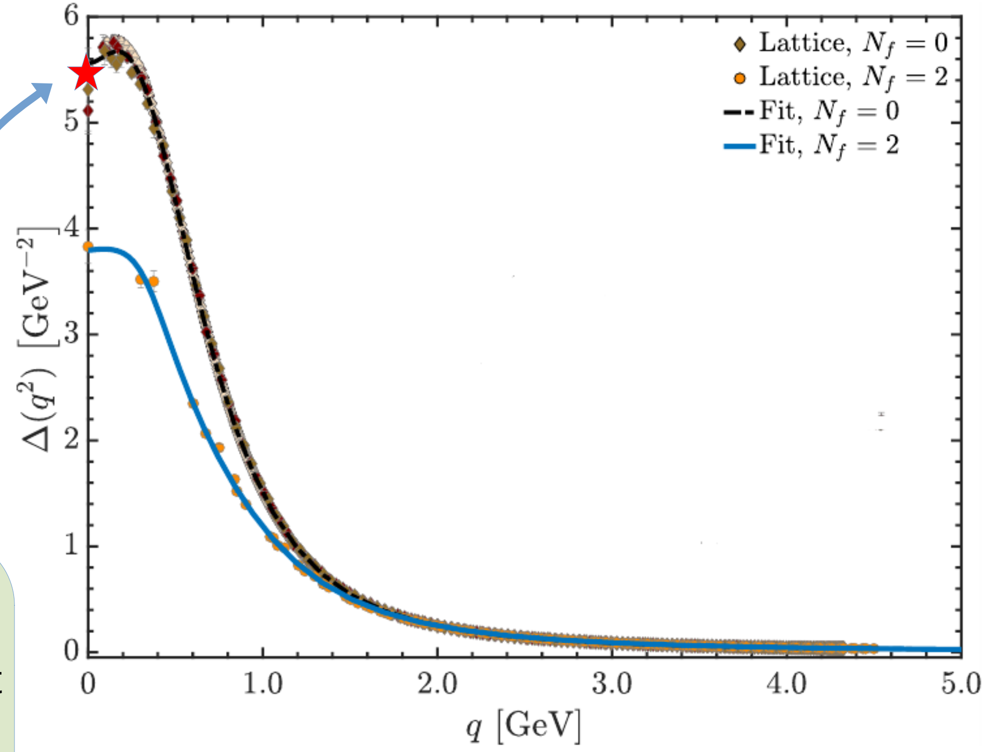
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).
J. Papavassiliou, Chin. Phys. C 46, no.11, 112001 (2022).

Mass generation leaves **distinctive signals in the infrared** momentum region of the Schwinger functions.

Gluon propagator and its mass gap

Gluon self-interaction can dynamically generate a mass gap.
J. M. Cornwall, Phys. Rev. D26, 1453 (1982).

Lattice QCD: The Landau gauge gluon propagator saturates at the origin.
A. Cucchieri and T. Mendes, PoS **LATTICE2007**, 297 (2007).
I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
O. Oliveira and P. J. Silva, Phys. Rev. D **86**, 114513 (2012).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Rev. D **104**, no.5, 054028 (2021).



- **Unequivocal signal of gluon mass scale generation.**
- **Eliminates many infrared divergences** that are present at the perturbative level;
- Makes QCD a well-defined theory in the infrared;
- One of the pillars of Emergent Hadron Mass.

M. Ding, C. D. Roberts and S. M. Schmidt, Particles 6, 57-120 (2023).
J. Papavassiliou, Chin. Phys. C 46, no.11, 112001 (2022).
C. D. Roberts, Symmetry 12, no.9, 1468 (2020).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

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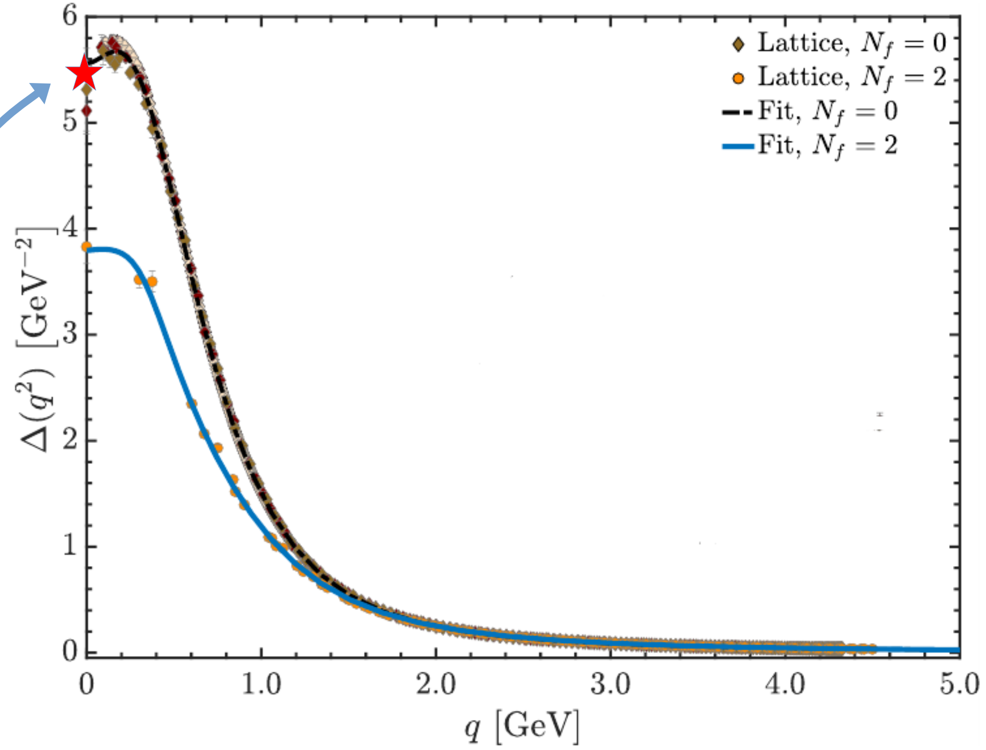


- Seen with or without quarks

A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D **86**, 074512 (2012).
D. Binosi, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **95**, no.11, 114009 (2017).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, J. Rodríguez-Quintero and S. Zafeiropoulos, Eur. Phys. J. C **80**, no.2, 154 (2020).

Gluon mass generation mechanism must be driven by gauge sector dynamics: truly, mass from nothing

We can focus on pure Yang-Mills theory

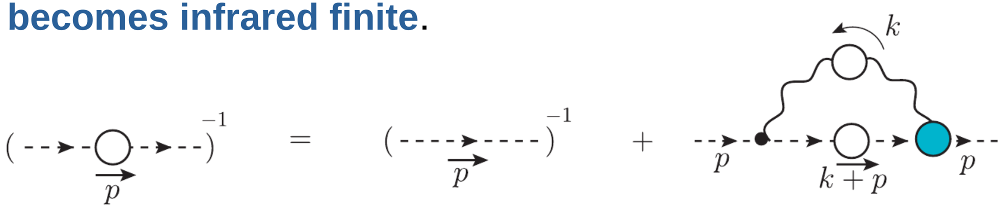
Implications: Infrared finite ghost dressing function

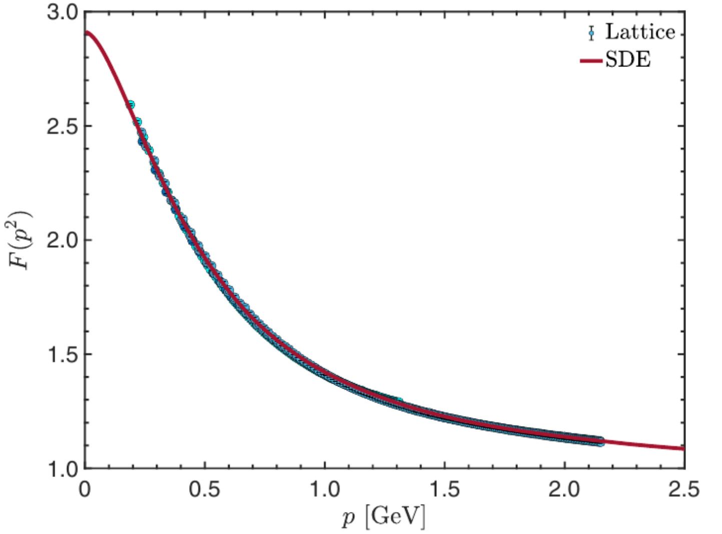
The generation of a gluon mass gap leaves distinctive imprints in other Schwinger functions. For example:

- The **ghost propagator,** $D(q^2)$**, remains massless.**
- But its **dressing function**, $F(q^2)$, given by

$$D(q^2) = \frac{iF(q^2)}{q^2}$$

becomes infrared finite.





A. Cucchieri and T. Mendes, PoS **LATTICE2007**, 297 (2007).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).
P. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, JHEP **06**, 099 (2008).
I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. B **676**, 69-73 (2009).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D **94**, 054005 (2016).
M. Q. Huber, Phys. Rept. **879**, 1-92 (2020).

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou, J. Rodriguez-Quintero, Phys. Rev. D **104** no.5, 054028, (2021).

Implications: No Landau pole in QCD

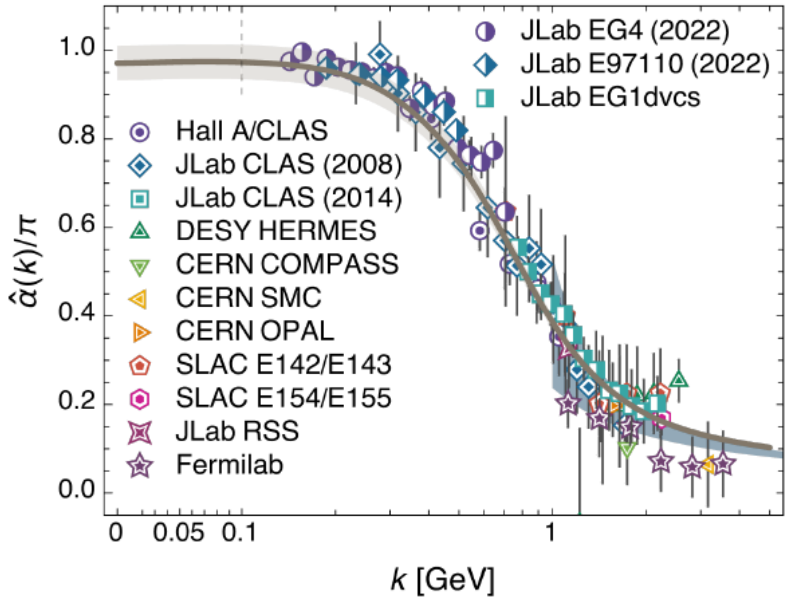
Together, the infrared finiteness of $\Delta(q^2)$ and $F(q^2)$ imply that:

- There is **no Landau pole in QCD**;
- QCD is a well-defined theory in the infrared.
- Leads to the construction of a **Renormalization Group Invariant** and **Process Independent** **effective charge**, $\hat{\alpha}(q^2)$, analogous to the Gell-Mann Low charge of QED.

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodríguez-Quintero, Phys. Rev. D **96**, no.5, 054026 (2017).
Z. F. Cui, et al, Chin. Phys. C **44**, no.8, 083102 (2020).

- $\hat{\alpha}(q^2)$ is a key ingredient in various studies of the hadron structure.

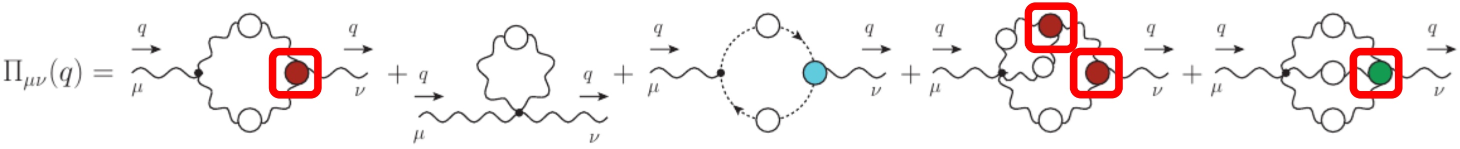
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).



Implications: effects on the gluonic vertices

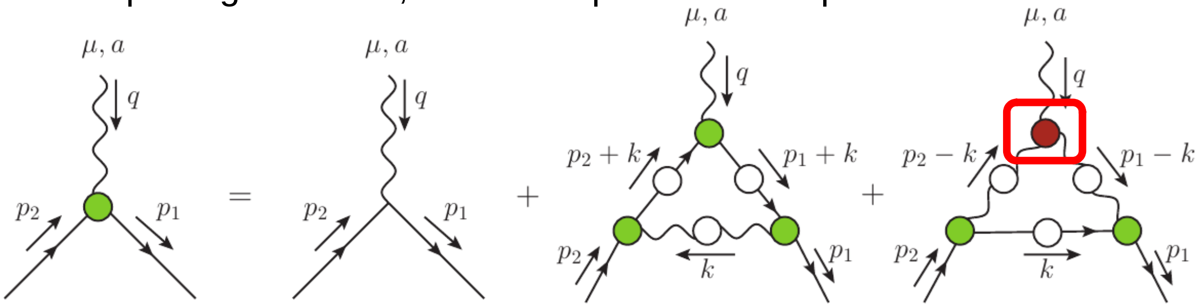
The generation of a gluon mass gap also has important implications for the **three- and four-gluon vertices**. These:

1) Appear as ingredients in DSEs, such as the gluon propagator. Necessary for a self consistent treatment.



2) Important phenomenologically:

- Affect the quark-gluon vertex, a key ingredient in Chiral Symmetry Breaking. Indeed, the three-gluon vertex appears in the quark-gluon DSE, where its quantitative impact must be taken into account:



A. C. Aguilar, M. N. F., G. T. Linhares, B. M. Oliveira and J. Papavassiliou, arXiv:2408.15370.
F. Gao, J. Papavassiliou and J. M. Pawlowski, Phys. Rev. D 103, no.**9**, 094013 (2021).
A. K. Cyrol, M. Mitter, J. M. Pawlowski and N. Strodthoff, Phys. Rev. D **97**, no.5, 054006 (2018).
A. L. Blum, R. Alkofer, M. Q. Huber and A. Windisch, EPJ Web Conf. **137**, 03001 (2017).

Implications: effects on the gluonic vertices

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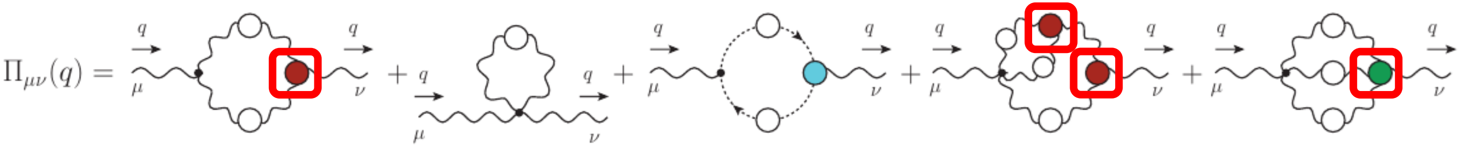
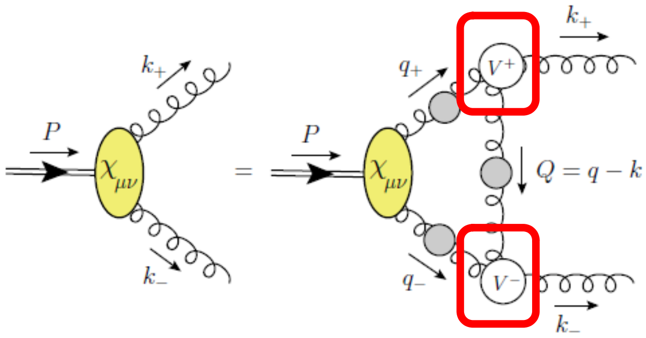


2) Important phenomenologically:

- Also, key ingredients for continuum studies of glueballs and hybrids.

  Example, the 0^{-+} glueball Bethe-Salpeter equation:



J. Meyers and E. S. Swanson, Phys. Rev. D **87**, no.3, 036009 (2013).
S. S. Xu, Z. F. Cui, L. Chang, J. Papavassiliou, C. D. Roberts and H. S. Zong, Eur. Phys. J. A **55**, no.7, 113 (2019).
E. V. Souza, M. N. F., A. C. Aguilar, J. Papavassiliou, C. D. Roberts, S.-S. Xu, Eur. Phys. J. A **56**, no.1, 25 (2020).
M. Q. Huber, C. S. Fischer and H. Sanchis-Alepuz, Eur. Phys. J. C **80**, no.11, 1077 (2020).
J. M. Pawlowski, C. S. Schneider, J. Turnwald, J. M. Urban and N. Wink, Phys. Rev. D **108**, no.7, 076018 (2023).

Implications: effects on the gluonic vertices

The generation of a gluon mass gap also has important implications for the **three- and four-gluon vertices**.

The study of these vertices has been difficult in the past due to:

1) Complicated tensor structures;
2) Depend on many variables;
3) Need for high statistics (in the case of lattice studies).

In recent years, continuum and lattice studies have overcome these issues, delivering a clear picture of the gauge sector in Landau gauge.

Transversely projected three- and four-gluon vertices

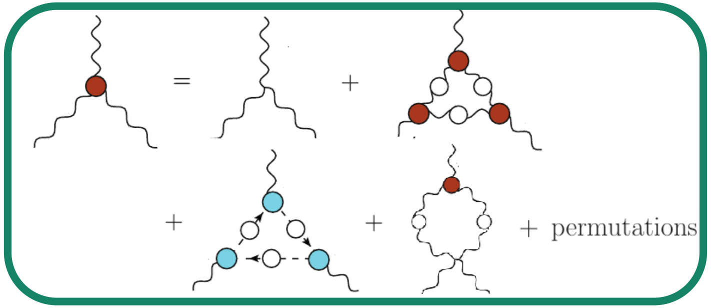
We focus on the **Landau gauge**, where, due to the transversality of the gluon propagator,

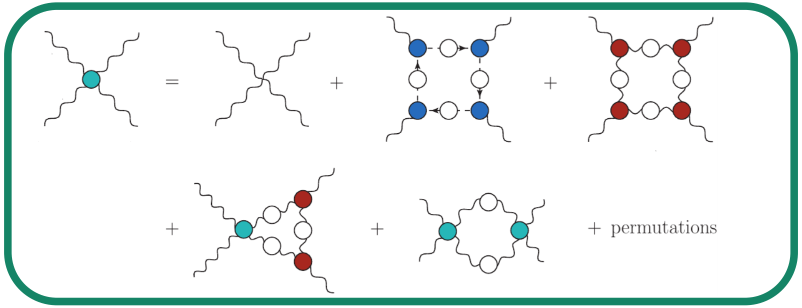
$$\Delta_{\mu\nu}(q) = P_{\mu\nu}(q)\Delta(q)\,, \qquad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,,$$

most quantities depend only on the transversely projected vertices,

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) := P_\alpha^{\alpha'}(q)P_\mu^{\mu'}(r)P_\nu^{\nu'}(p)\Gamma_{\alpha'\mu'\nu'}(q,p,r)$$
$$\overline{\Gamma}^{abcd}_{\alpha\mu\nu\rho}(q,r,p,u) := P_\alpha^{\alpha'}(q)P_\mu^{\mu'}(r)P_\nu^{\nu'}(p)P_\rho^{\rho'}(u)\Gamma^{abcd}_{\alpha'\mu'\nu'\rho'}(q,p,r,u)$$

- This already **provides some simplification of the tensor structures**.
- Later we will see that l**ongitudinal tensor structures are important for the gluon mass gap**.
- Then, we compute the transverse three- and four-gluon vertices, through lattice as well as DSEs.



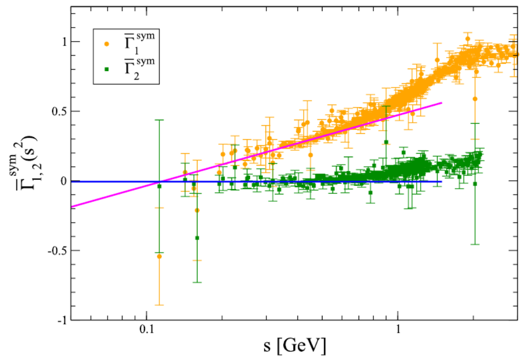


Transversely projected three- and four-gluon vertices

1) Complicated tensor structures;
Solution: Classical form factor is completely dominant.

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)\Gamma_1(q,r,p)$$

G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D **89**, 105014 (2014).
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D **93**, no. 3, 034026 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).

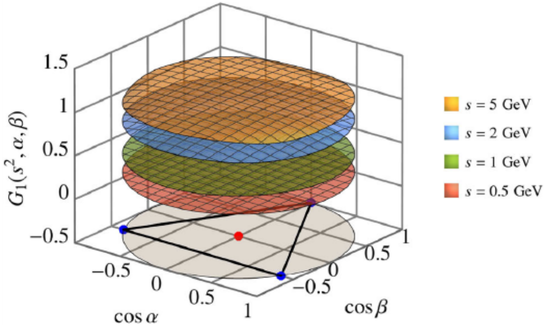


2) Depend on many variables;
Solution: **Planar degeneracy**, form factors are accurately approximated as functions of a single Bose-symmetric variable,

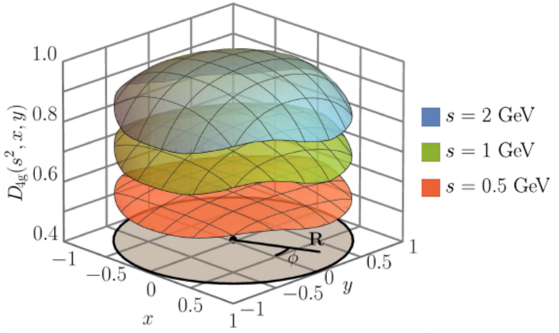
$$\Gamma_1(q,r,p) \approx L_{sg}(s) \qquad s^2 = (q^2+r^2+p^2)/2$$

G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D **89**, 105014 (2014).
F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **84**, no.7, 676 (2024).
F. Pinto-Gómez, F. De Soto and J.~Rodríguez-Quintero, Phys. Rev. D **110**, no.1, 014005 (2024)

three-gluon



four-gluon



3) Need for high statistics (in the case of lattice studies).

Solution: exploit planar degeneracy, average over the same s^2

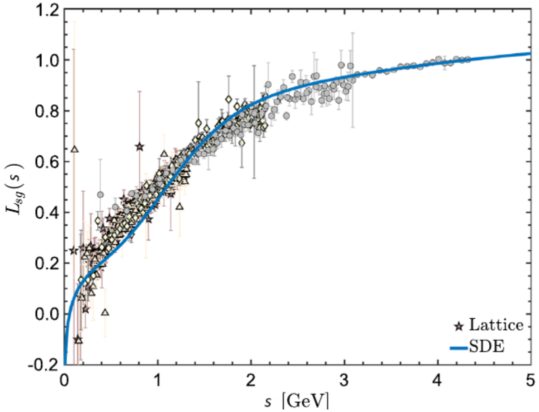
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, J. Rodríguez-Quintero and L. R. Santos, arXiv:2408.06135.

Transversely projected three- and four-gluon vertices

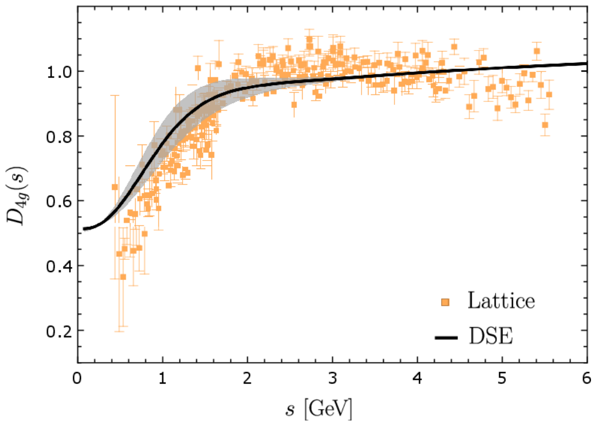
As a result, we now have a compact and accurate description of the three- and four-gluon vertices, which are characterized by two single-variable functions:

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)L_{sg}(s)$$

$$\overline{\Gamma}^{abcd}_{\alpha\mu\nu\rho}(q,r,p,u) \approx \overline{\Gamma}^{0\,abcd}_{\alpha\mu\nu\rho}(q,r,p,u)D_{4g}(s)$$

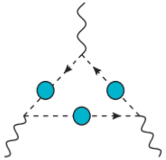


A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **84**, no.7, 676 (2024).



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, J. Rodríguez-Quintero and L. R. Santos, arXiv:2408.06135.

- Both are considerably **suppressed in the infrared**;
- **Gluon mass gap** makes the **four-gluon classical form factor infrared finite**;
- For the **three-gluon**, **massless ghosts** cause a **logarithmic divergence at the origin**.



$$\sim \ln\left(\frac{r^2}{\mu^2}\right)$$

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D **89**, no. 8, 085008 (2014).

Origin of the gluon mass gap

Together, these results for the gauge sector Schwinger functions provide a solid framework for practical calculations. But this leaves us with a **question**:

How can the gluon acquire a mass gap?

- Gauge symmetry must be explicitly preserved;
- No associated mass term, $m^2 A^2$, in Lagrangian;
- No elementary scalar field for a Higgs mechanism.

Answer:

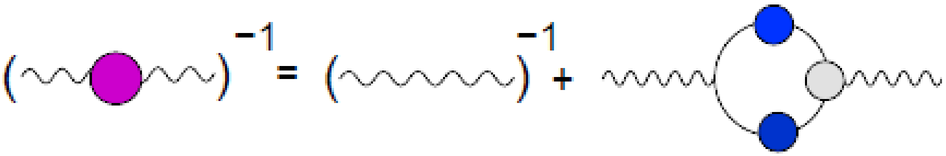
Through the Schwinger mechanism

Schwinger mechanism

"A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer."

J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

Dyson-Schwinger equation for gauge boson propagator



$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

If, for some reason

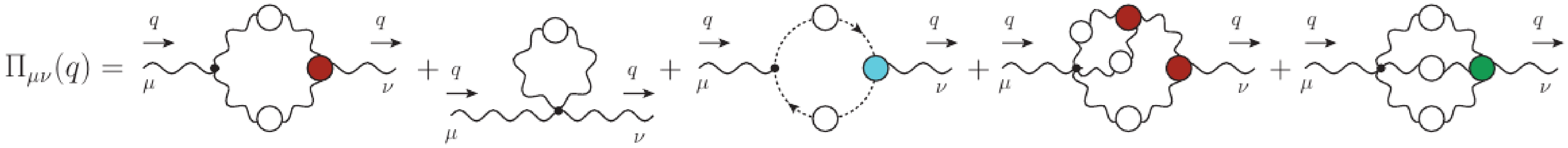
$$\lim_{q\to 0} \Pi(q^2) = \frac{c}{q^2}\,, \qquad c > 0$$

$$\Delta^{-1}(0) = c > 0$$

But how can the vacuum polarization acquire such a pole?

Vertex irregularities

From the gluon Schwinger-Dyson equation,



it has been shown in numerous works that:

Pole in the vacuum polarization hinges on the presence of vertex irregularities

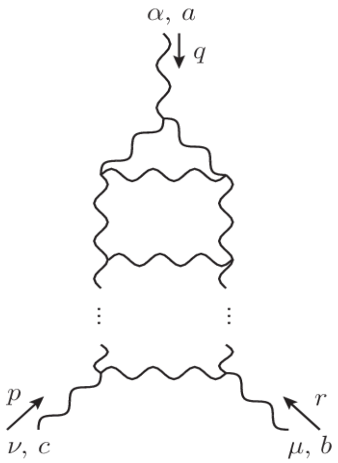
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D **94**, 054005 (2016).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016)
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

- Usual logarithmic divergences of perturbative QCD are not enough. Instead:

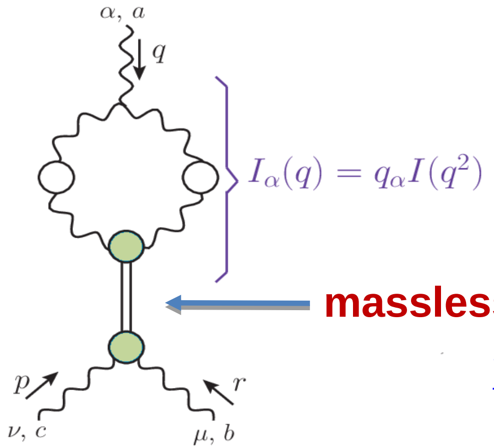
Vertices can develop poles at zero momentum through the formation of **massless bound states**.

Massless bound state formalism

If the interaction is sufficiently strong → formation of **massless bound states**



$q \to 0$



Longitudinally coupled

massless colored excitation

$$\frac{i\delta^{ab}}{q^2}$$

Vertices of the theory acquire **longitudinally coupled poles at zero gluon momentum, e.g.:**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\,\mathbb{C}(r^2) + \dots$$

pole-free — Schwinger pole

Residue functions

E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

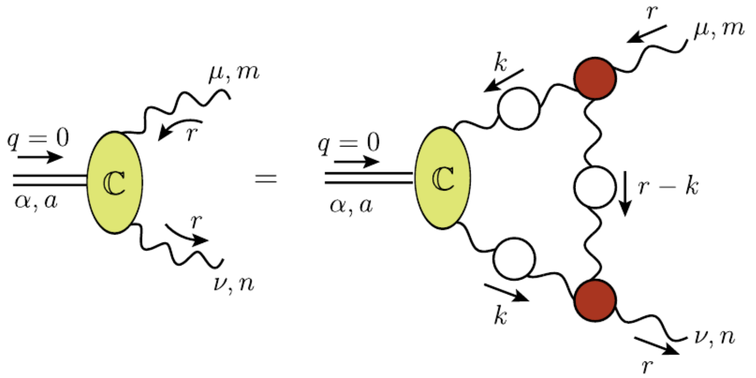
Bethe-Salpeter equation

The formation of a massless bound state is **dynamical and governed by a Bethe-Salpeter equation.**

Recalling:

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2) + \ldots$$

The function $\mathbb{C}(r^2)$ satisfies the equation



BS amplitude

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

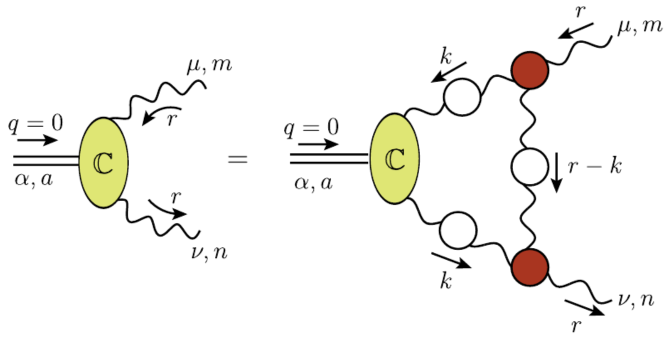
Bethe-Salpeter equation

The Bethe-Salpeter equation admits **nontrivial solutions compatible with lattice ingredients** for the:

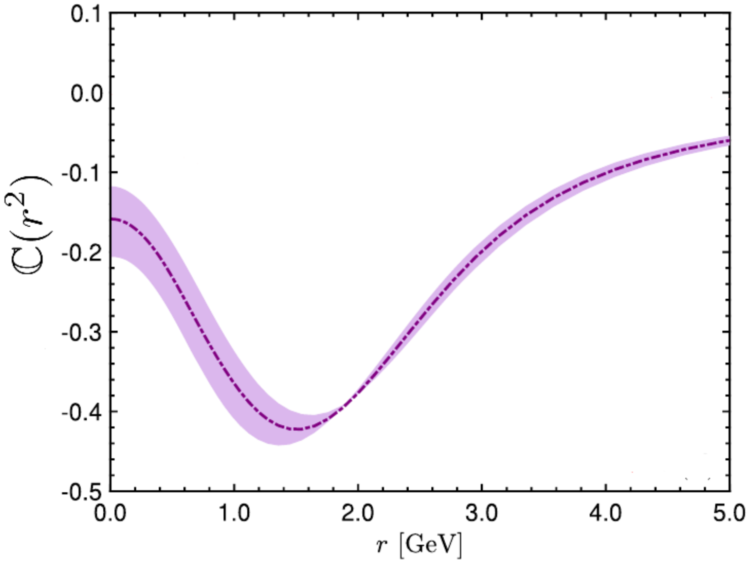
- Propagator;
- Vertex;
- and, value of the coupling

$$\alpha_s \approx 0.3 \ \ @ \ \ \mu = 4.3 \text{ GeV}$$

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
D. Binosi and J. Papavassiliou, Phys. Rev. D **97**, no.5, 054029 (2018).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C **78**, no.3, 181 (2018).
M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).



BS amplitude

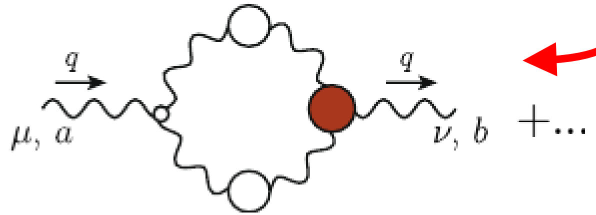


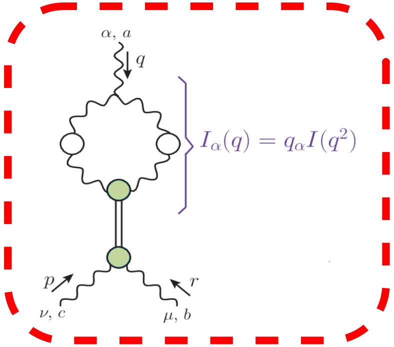
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Massless bound state formalism

Massless poles in the three-gluon vertex lead to pole in the gluon vacuum polarization:

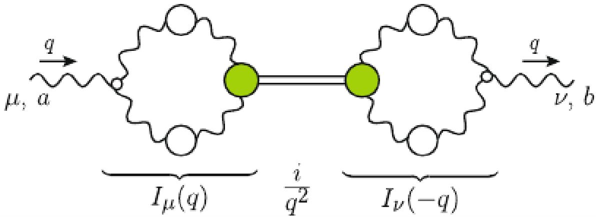
$$\Delta^{-1}(q^2) = q^2 \quad +$$

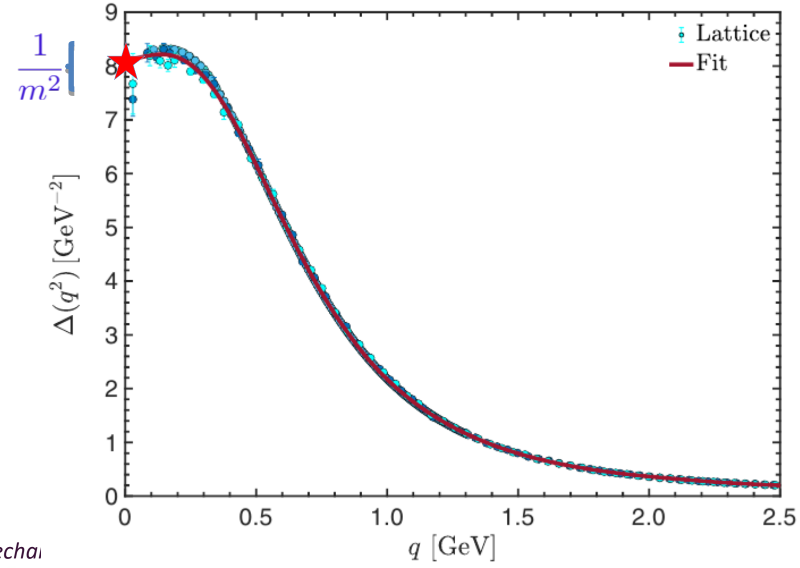




$q \to 0$

$$m^2 =$$





After careful renormalization, agreement with lattice saturation value.

M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

Schwinger poles in lattice results?

Now, the **lattice can also compute the three-gluon vertex. Can we see longitudinal poles in it?**

Unfortunately, no!

The Schwinger **poles are longitudinally coupled**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \underbrace{\Gamma_{\alpha\mu\nu}(q,r,k)}_{\text{pole-free}} + \underbrace{\frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2)}_{\text{Schwinger pole}} + \dots$$

But **lattice simulations only access transverse tensor structures.**

→ **Lattice extracts the pole-free part of the vertex.**

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B **761**, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).

A smoking gun signal?

Question:

Is there a smoking-gun signal of the massless bound state poles, which can be tested with lattice inputs?

Answer:

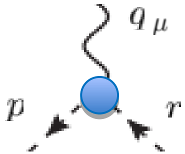
Yes, the **displacement of the Ward identities** satisfied by the vertices.

- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- If there is a massless bound state pole, the **propagators and pole-free parts of the vertices must change in shape to accommodate the pole contribution to the Ward identities**.

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

A toy example: scalar QED

Schwinger mechanism **off**



Ward-Takahashi identity

$$q^\mu \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$, $p \to -r$ — Taylor expansion

Textbook Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Schwinger mechanism **on**

$$\mathrm{I\Gamma}_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Ward-Takahashi identity does not change

$$q^\mu \mathrm{I\Gamma}_\mu(q,r,p) = q^\mu \Gamma_\mu(q,r,p) + C(q,r,p)$$
$$= D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$ — Taylor expansion

Displaced Ward identity

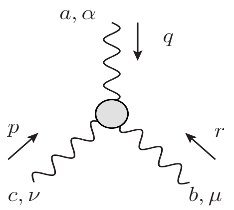
$$\underbrace{\Gamma_\mu(0,r,-r)}_{\text{pole-free}} = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

Displacement = BS amplitude

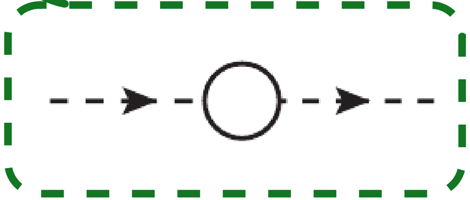
Ward identity displacement in QCD

The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**:

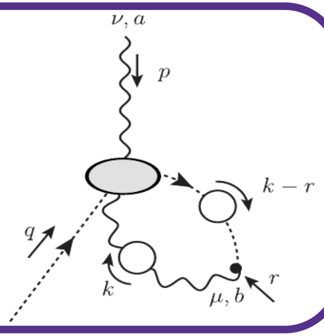
$$q^\alpha \mathrm{I}\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$



$$:= \mathrm{I}\Gamma^{abc}_{\alpha\mu\nu}(q,r,p)$$



$$H_{\mu\nu}(r,q,p) :=$$



Then, assume the three-gluon vertex has a massless bound state pole:

$$\mathrm{I}\Gamma_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2) + \ldots$$

And expand around *q = 0*

Ward identity displacement in QCD

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

★ **Ingredients can (mostly) be computed with lattice simulations.**

★ **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

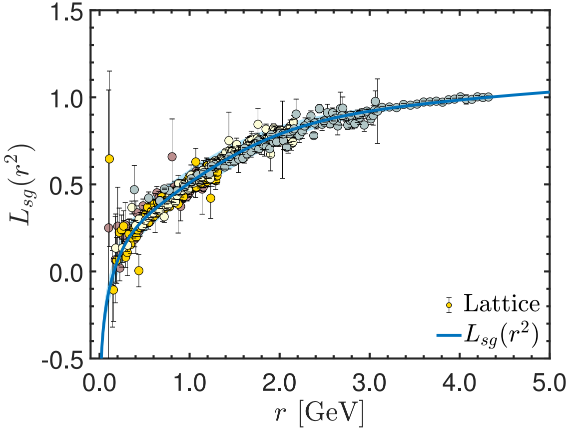
Ward identity displacement in QCD

$$q^{\alpha}\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ ↓ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$



Soft-gluon form factor of the three-gluon vertex

$$P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(r)\Gamma_{\alpha\mu'\nu'}(0,r,-r) = 2L_{\rm sg}(r^2)r_{\alpha}P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_{\mu}q_{\nu}/q^2$$

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

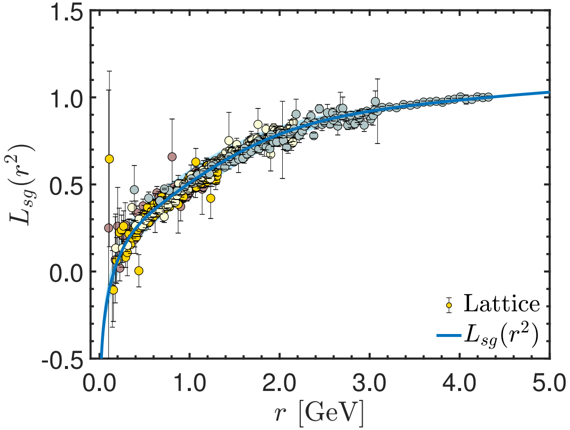
Ward identity displacement in QCD

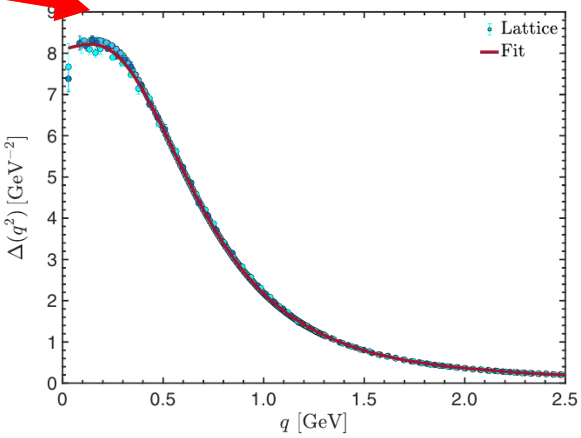
$$q^{\alpha}\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$





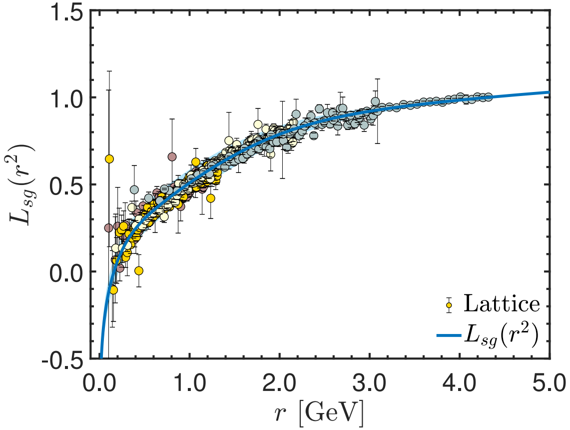
Ward identity displacement in QCD

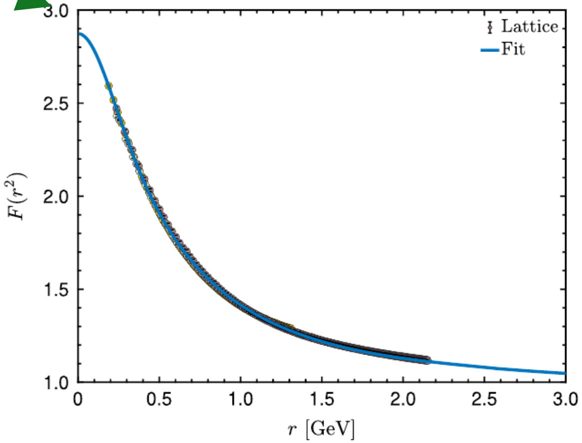
$$q^{\alpha}\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

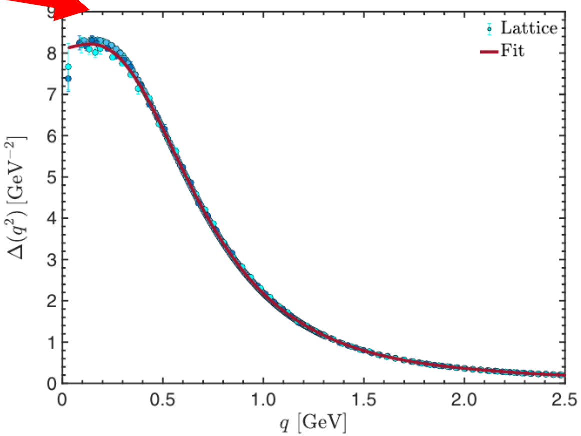
$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$







Ward identity displacement in QCD

$$q^\alpha \mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

★ **Only one ingredient not yet determined directly by lattice simulations.**

Ward identity displacement in QCD

$$q^\alpha \mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Isolate classical tensor structure

Ward identity

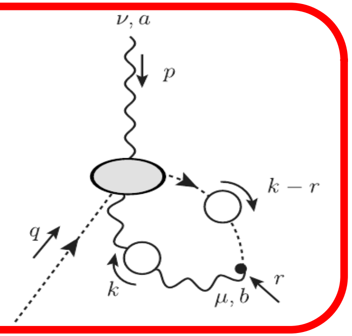
$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Partial derivative of the ghost-gluon kernel:

$$\mathcal{W}(r^2) = -\frac{1}{3} r^\alpha P^{\mu\nu}(r)\left[\frac{\partial H_{\nu\mu}(r,q,p)}{\partial q^\alpha}\right]_{q=0}$$

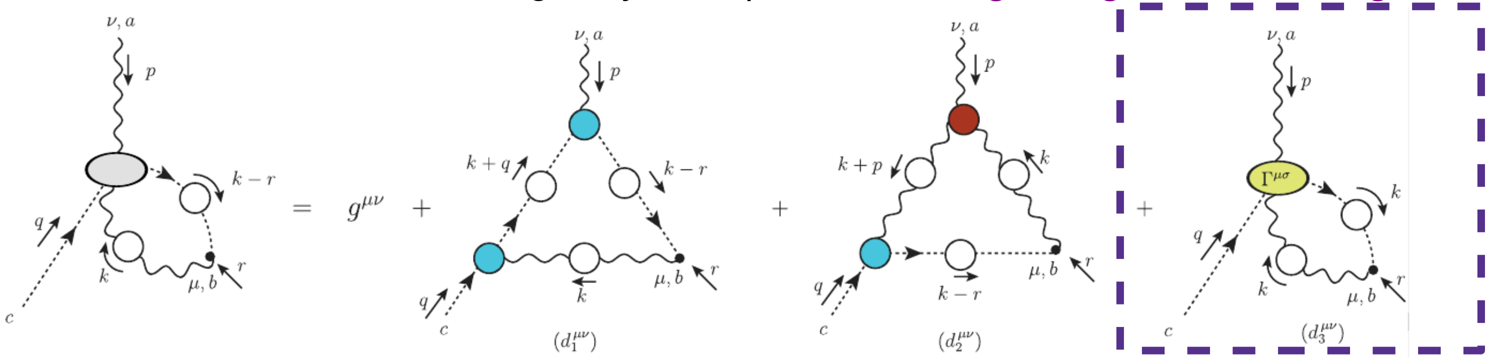
- **In principle, computable on the lattice, but not currently available.**
  A. C. Aguilar, M. N. F. and J. Papavassiliou, Eur. Phys. J. C **81**, no.1, 54 (2021).
- **Resort to a lattice-driven SDE analysis.**

$H_{\mu\nu}(r,q,p) :=$



Lattice driven Schwinger-Dyson calculation

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

Depends on:

1) Gluon and ghost propagators.

2) Four-point function probably subleading. Will be omitted.

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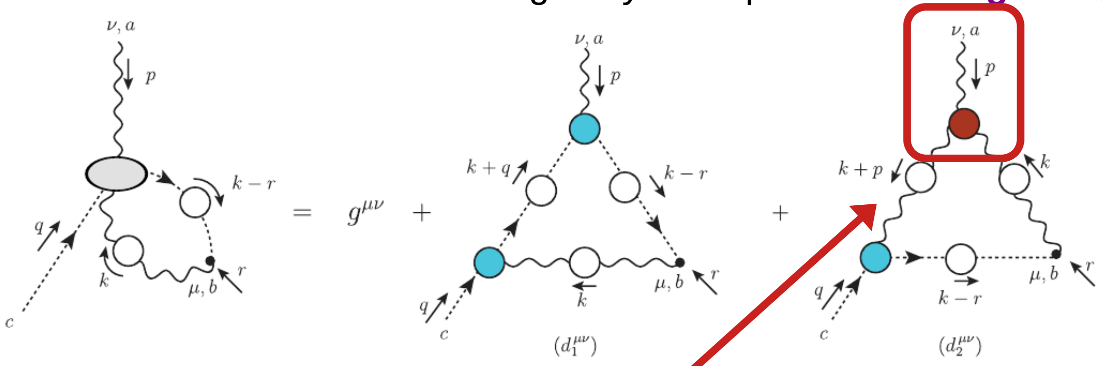

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Depends on:

1) **Gluon and ghost propagators.**
2) **Four-point function probably subleading. Will be omitted.**
3) **General kinematics three-gluon vertex.**
   By now well-determined by continuum and lattice studies.

G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016).
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).
F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).
F. Pinto-Gómez, F. De Soto and J. Rodríguez-Quintero, Phys. Rev. D **110**, no.1, 014005 (2024).

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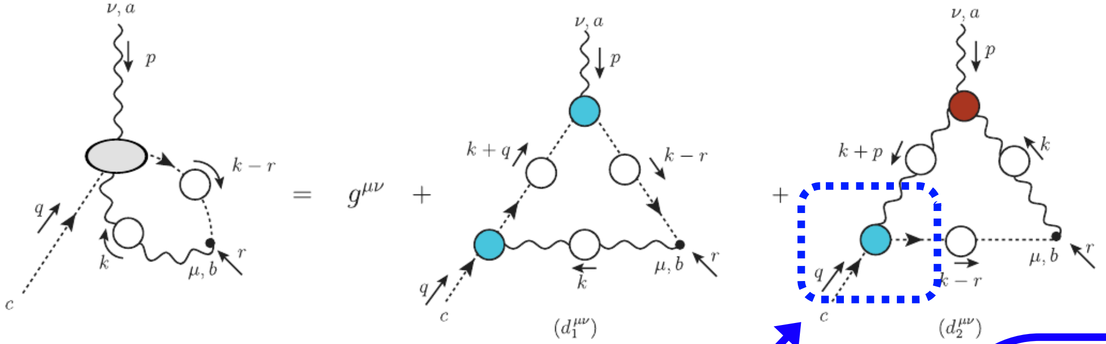


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

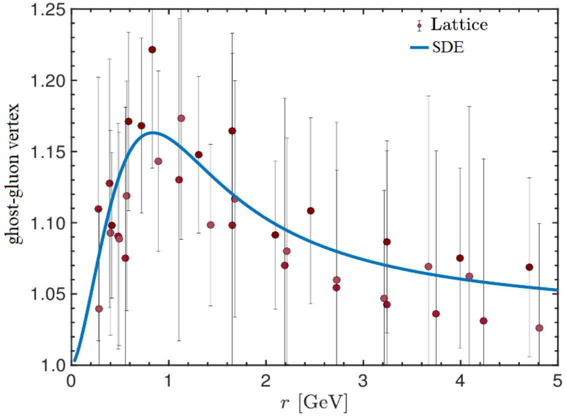
Depends on:

1) **Gluon and ghost propagators.**
2) **Four-point function probably subleading. Will be omitted.**
3) **General kinematics three-gluon vertex.**
4) **General kinematics ghost-gluon vertex;**
   Determined self-consistently through same SDE plus STI:

$$\mathbb{\Gamma}_\nu(r,q,p) = r^\mu H_{\mu\nu}(r,q,p)$$

M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

Reproduces available SU(3) lattice results:
E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al.
Braz. J. Phys. 37, 193 (2007).



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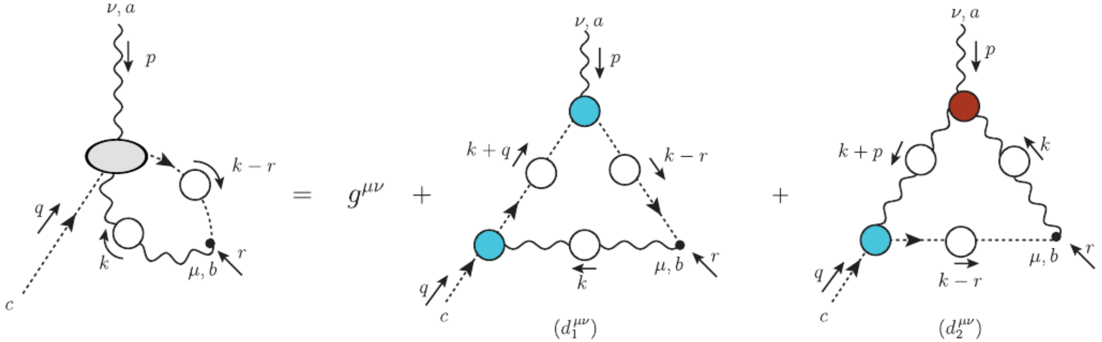

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

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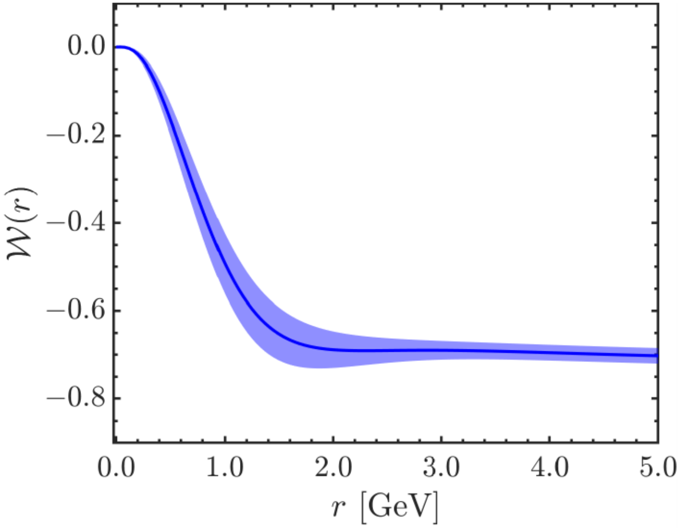
With these ingredients at hand, we compute $\mathcal{W}(r^2)$

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

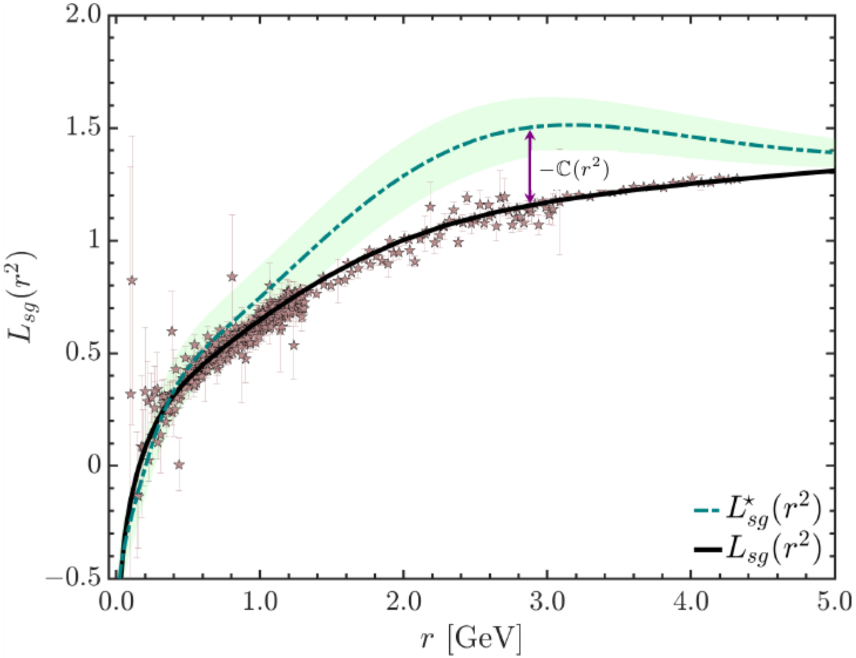


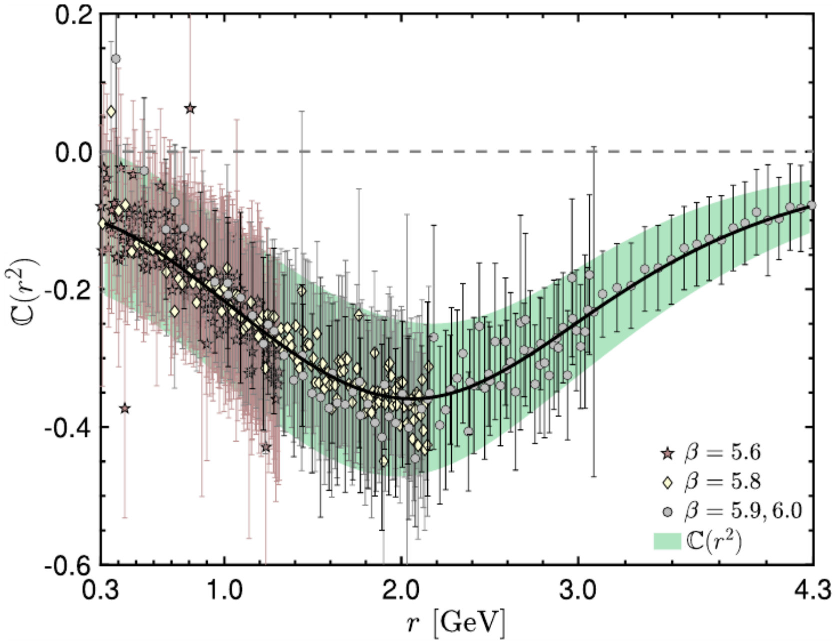
Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$





A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

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$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

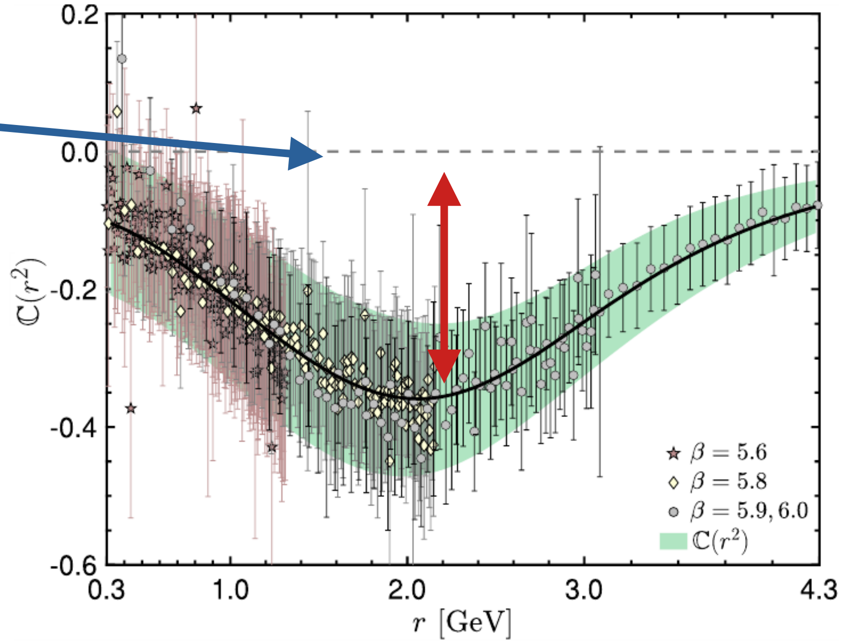
- $\mathbb{C}(r^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

***p*-value of null hypothesis is tiny:**

$$P_{\mathbb{C}_0} = \int_{\chi^2=2\,630}^{\infty} \chi^2_{\text{PDF}}(515, x)dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the 5σ level.



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

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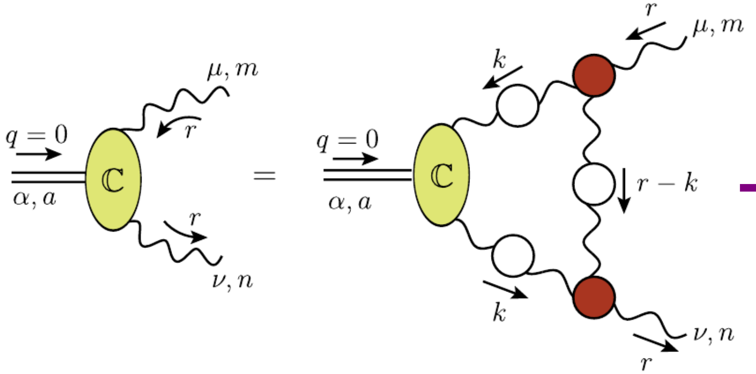
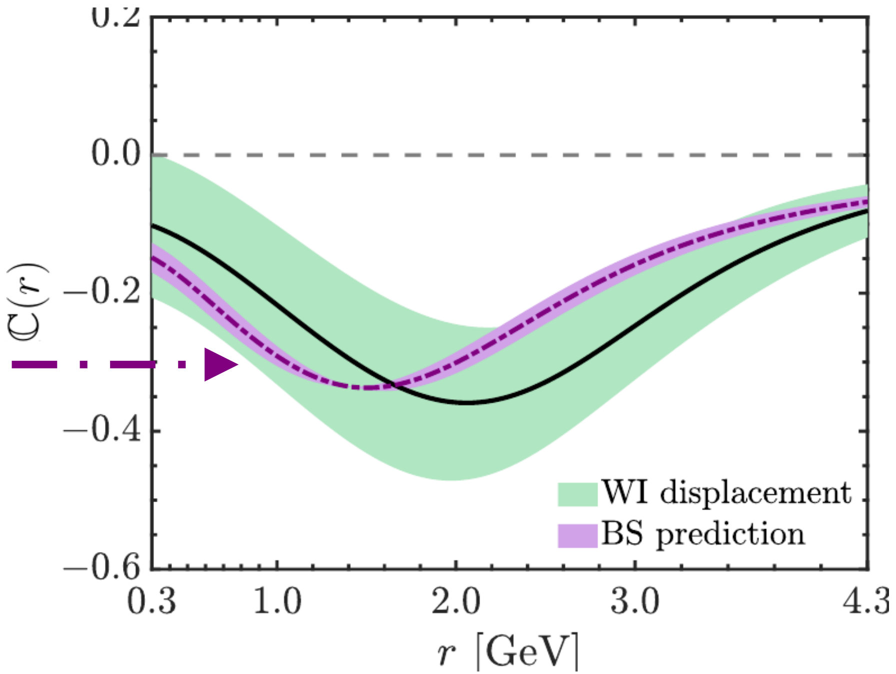
$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

- Moreover, we find good agreement with the BSE prediction.





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

Conclusions

- **Gluon self-interactions** generate a **gluon mass gap through the Schwinger mechanism**, manifest by the **saturation of the gluon propagator at the origin**.

- **Eliminates several infrared divergences**, most notably the Landau pole of perturbative QCD.

- **Schwinger mechanism** is triggered by the formation of **massless bound state poles in the three-gluon vertex.**

- Leads to **displacement of the Ward identity, whose amplitude coincides with BS amplitude of the massless bound state.**

- The **occurrence of this displacement is confirmed**, by combining state-of-the-art **lattice and Dyson-Schwinger results** for the propagators and vertices.

- **We obtain a clear displacement which agrees with the Bethe-Salpeter prediction.**

- **In the future**, understand role of **poles in other vertices** and **truncation error** in the analysis.

Backup slides

Massless bound state formalism

Important: These bound states are not *glueballs*!

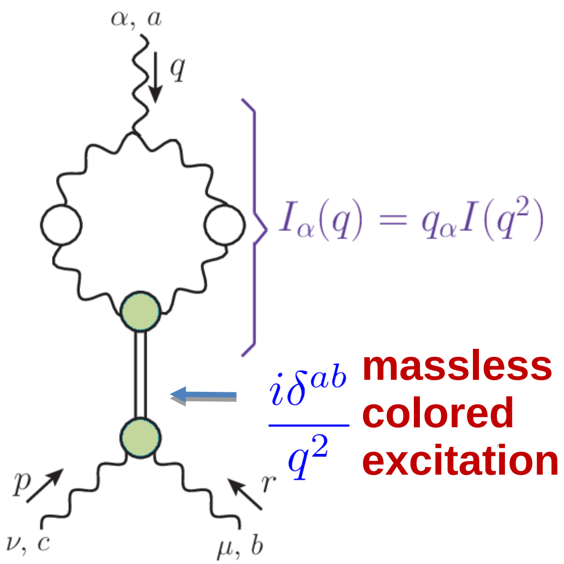
Glueballs:

- Color singlets.
- Massive.
- Appear in the spectrum.

V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E 18, 1-49 (2009).

Schwinger mechanism poles:

- Colored states.
- Massless.
- Do not appear in the spectrum (would-be Goldstone boson, eaten to generate the gluon mass)



J. Smit, Phys. Rev. D **10**, 2473 (1974).
E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254-3279 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).

Ward identity displacement in QCD

$$q^\alpha \mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

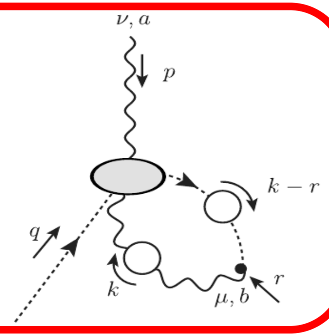
$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Partial derivative of the ghost-gluon kernel:

$$\mathcal{W}(r^2) = -\frac{1}{3}r^\alpha P^{\mu\nu}(r)\left[\frac{\partial H_{\nu\mu}(r,q,p)}{\partial q^\alpha}\right]_{q=0}$$

$H_{\mu\nu}(r,q,p) :=$ 

- **In principle, computable on the lattice, but not currently available.**
  A. C. Aguilar, M. N. F. and J. Papavassiliou, Eur. Phys. J. C **81**, no.1, 54 (2021).

- **Resort to a lattice-driven SDE analysis.**

Schwinger mechanism poles do not show in lattice results

A typical vertex form factor on the lattice is given by:

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$

$$\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\text{poles}}$$

Given that the poles are longitudinally coupled:

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)V^{\alpha\mu\nu}(q,r,p) = 0$$

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

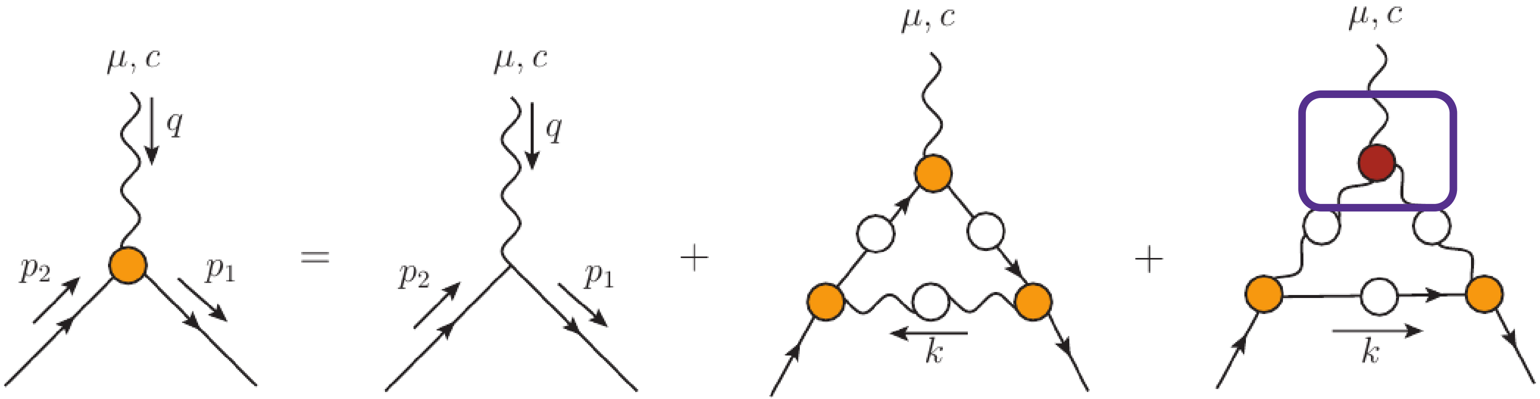
Lattice extracts the pole-free part of the vertex.

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodríguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

Poles in other vertices: including dynamical quarks

The Dyson-Schwinger equations couple vertices of different species and number of external legs.

- If a **longitudinally coupled pole** is generated **in the three-gluon vertex**, it tends to **spread out to other vertices as well**.
- In particular, the **quark-gluon vertex picks up a longitudinally coupled pole**:



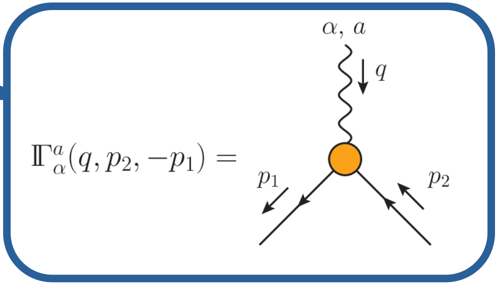
This allows additional tests of the Schwinger mechanism, and studying the role of dynamical quarks.

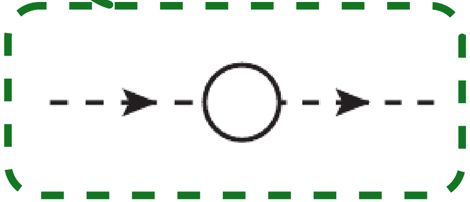
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

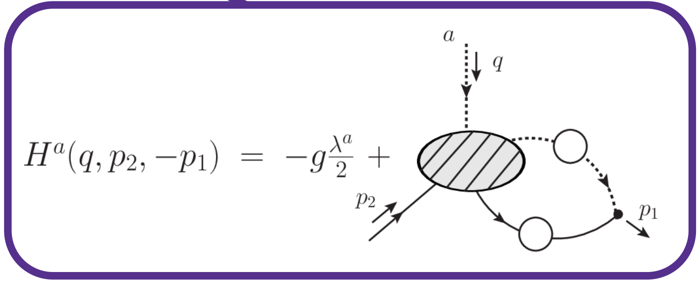
Ward identity displacement of the quark-gluon vertex

The same idea of Ward identity displacement applies to the quark-gluon vertex. We start with the STI

$$q^\alpha \mathrm{I\Gamma}_\alpha(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$







Again, assume that the vertex has a massless bound state pole:

$$\mathrm{I\Gamma}_\alpha(q,p_2,-p_1) = \Gamma_\alpha(q,p_2,-p_1) + \frac{q_\alpha}{q^2}\not{q}\,Q_3(p_2^2) + \dots$$

BS amplitude

And expand around $q = 0$

Ward identity displacement of the quark-gluon vertex

$$q^\alpha \mathrm{I\Gamma}_\alpha(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).

$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

$$\lambda_1^\star(p^2)$$

Displacement = BS amplitude

★ **Ingredients can be computed using lattice results.**

O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **99**, no.9, 094506 (2019).
A. Kizilersü, O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **103**, no.11, 114515 (2021).

★ **Combine ingredients and determine if there is a nontrivial displacement.**

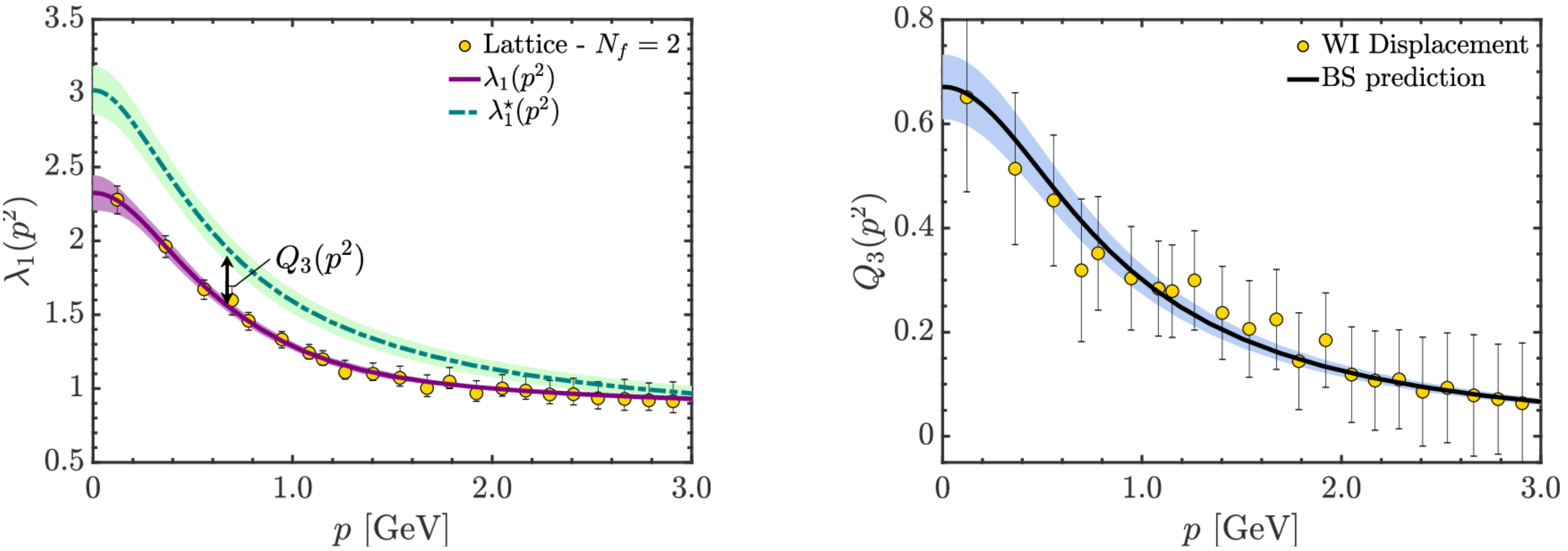
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1 + 4p^2 K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

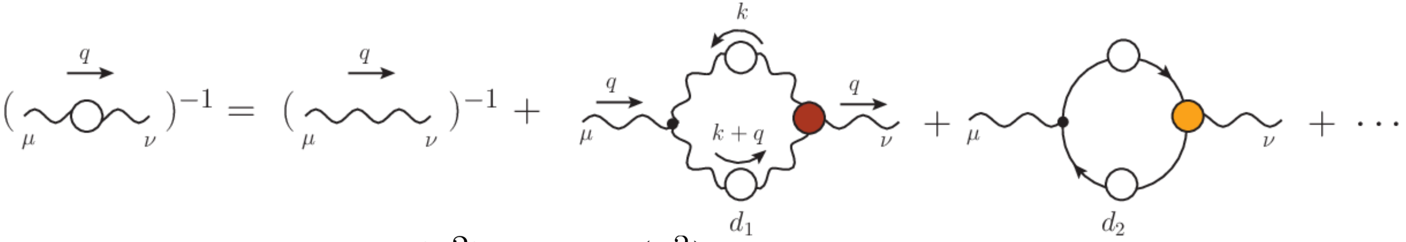
$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$



A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

Gluon self-interaction is dominant in generation of gluon mass gap

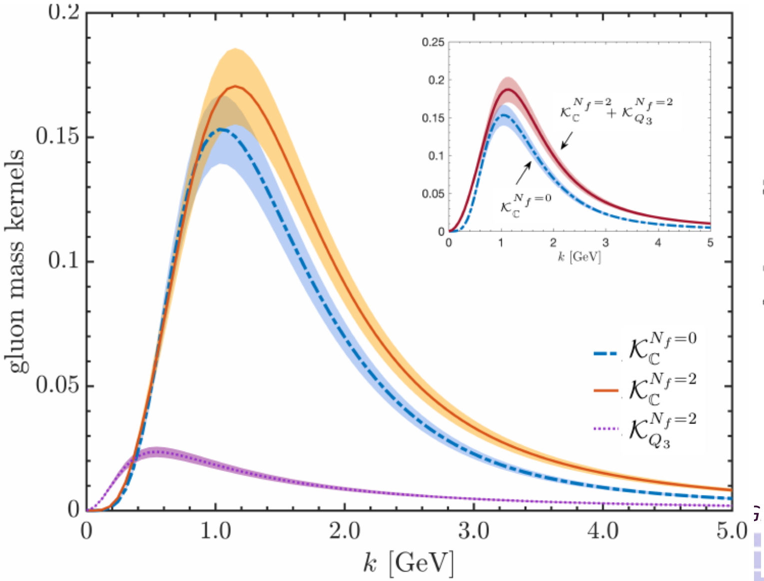
From the gluon SDE:

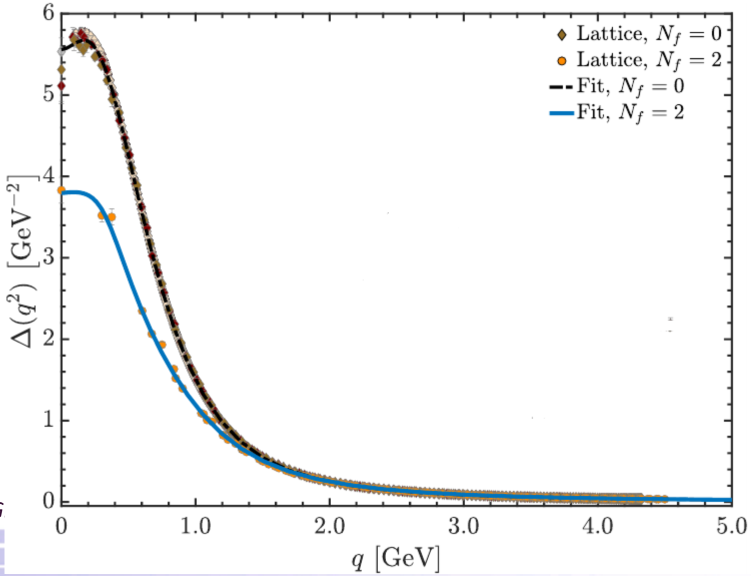


one finds an expression for the mass gap in terms of $\mathbb{C}(p^2)$ and $Q_3(p^2)$:

$$m^2 = \int_0^\infty dy\, \mathcal{K}_{\mathbb{C}}^{N_f}(y) + \int_0^\infty dy\, \mathcal{K}_{Q_3}^{N_f}(y)$$

three-gluon **quark-gluon**





- Unquenched gluon mass gap is larger, in agreement with lattice.
- Three-gluon is the biggest contribution.
- Gluon self-interaction drives the Schwinger mechanism in QCD.

Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

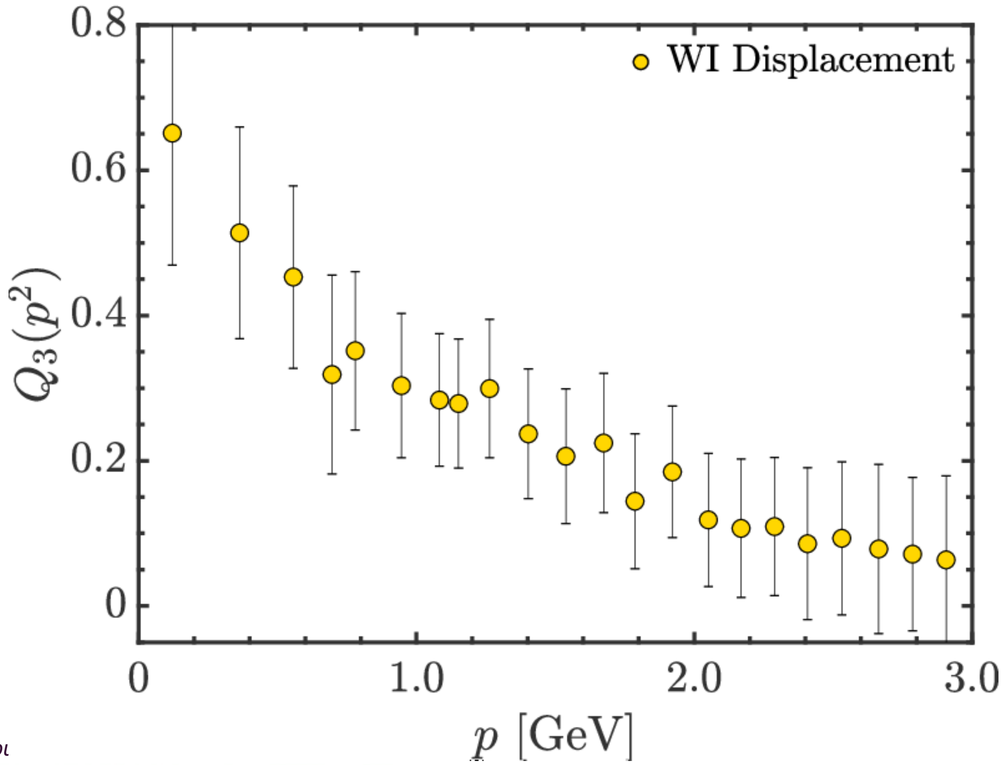
- $Q_3(p^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$Q_3(p^2) = Q_3^0(p^2) := 0$$

***p*-value of null hypothesis is very small:**

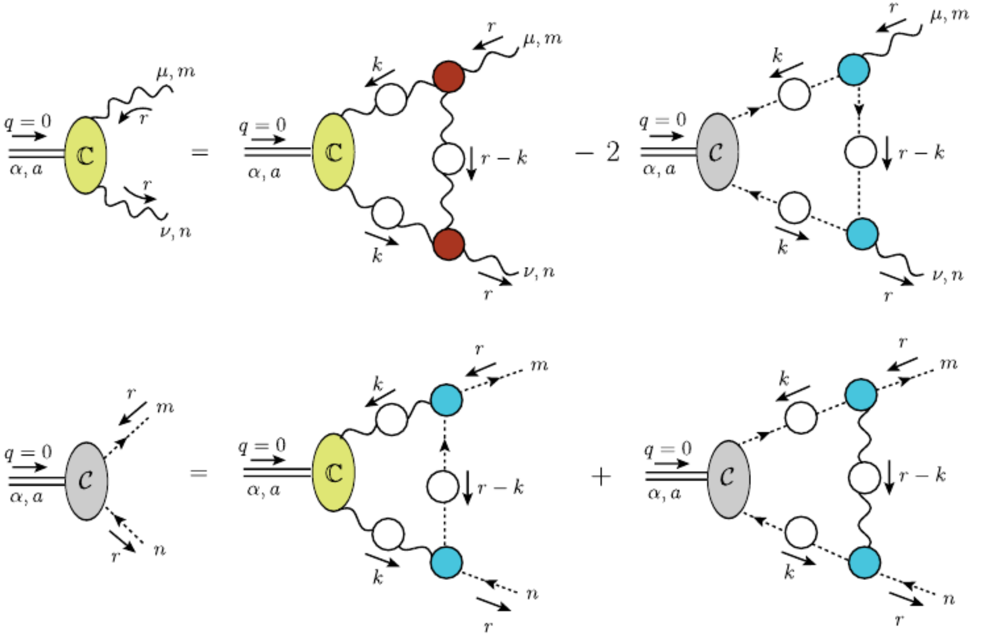
$$P_{Q_3^0} = \int_{\chi^2=119}^{\infty} \chi^2_{\rm PDF}(18,x)dx = 6.5\times 10^{-17}$$

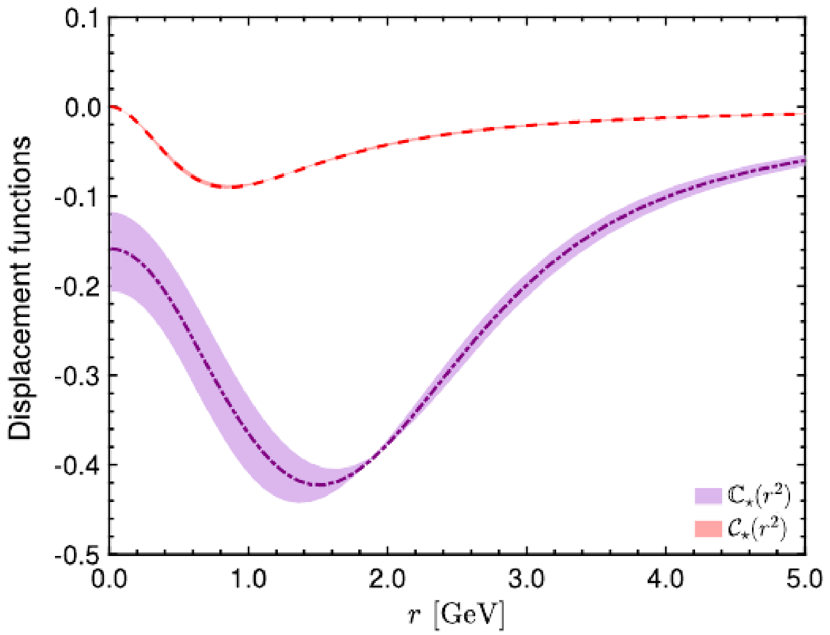
- Excludes the null hypothesis at the 8σ level.



Pole of the ghost-gluon vertex

The Dyson-Schwinger equation for the displacement amplitude $\mathbb{C}(r^2)$ can be coupled to a pole also in **ghost-gluon vertex**



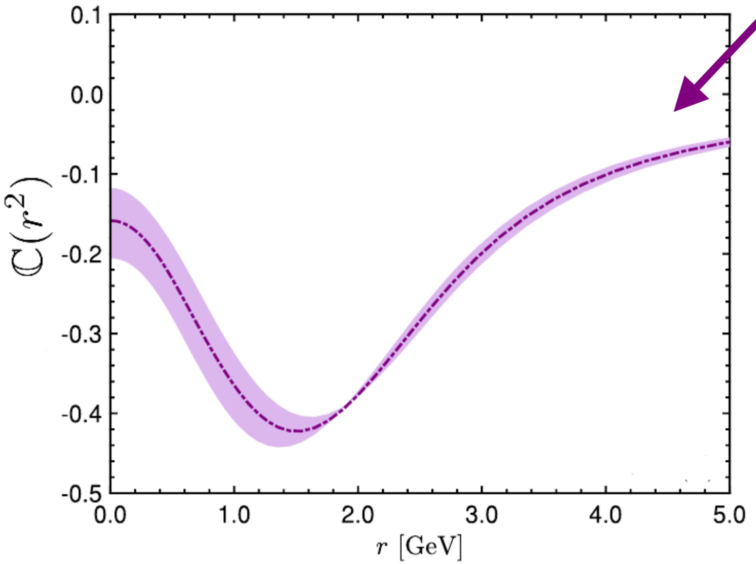


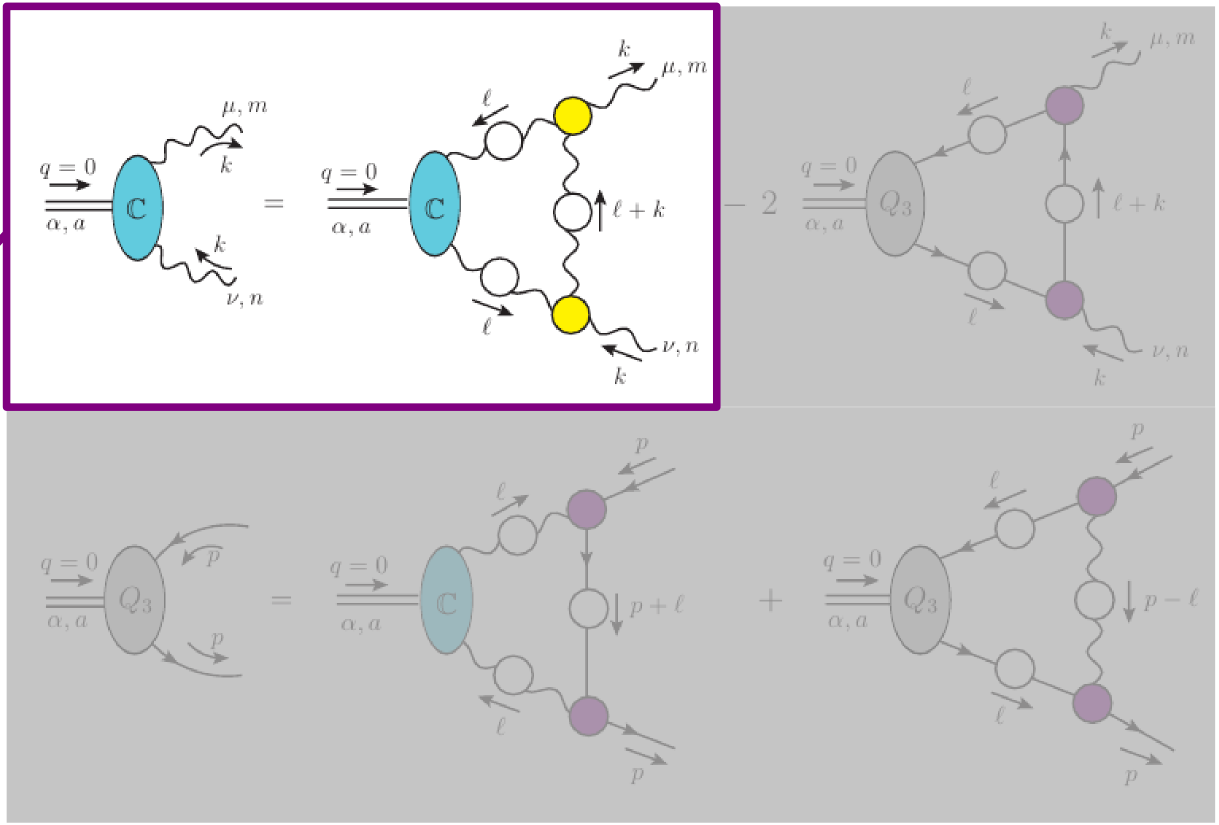
Effect on $\mathbb{C}(r^2)$ is negligible because ghost-gluon pole amplitude, $\mathcal{C}(r^2)$, is subleading.

A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.

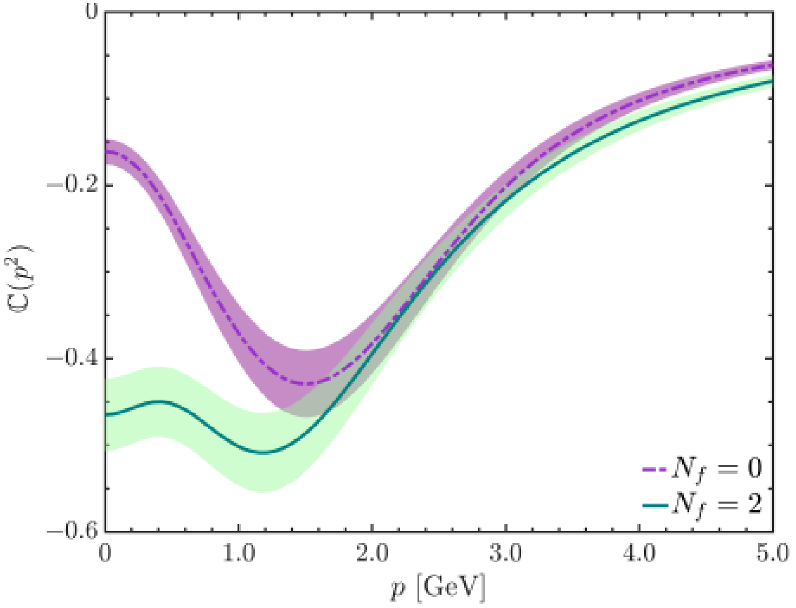


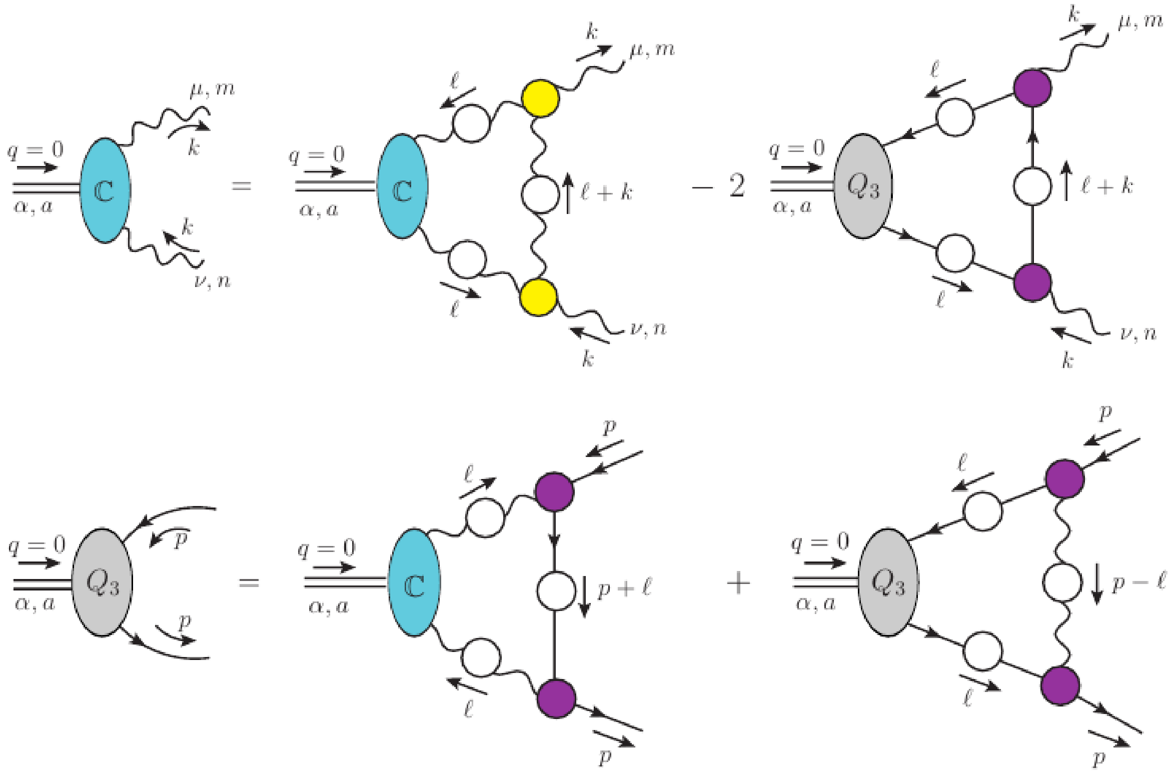


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.

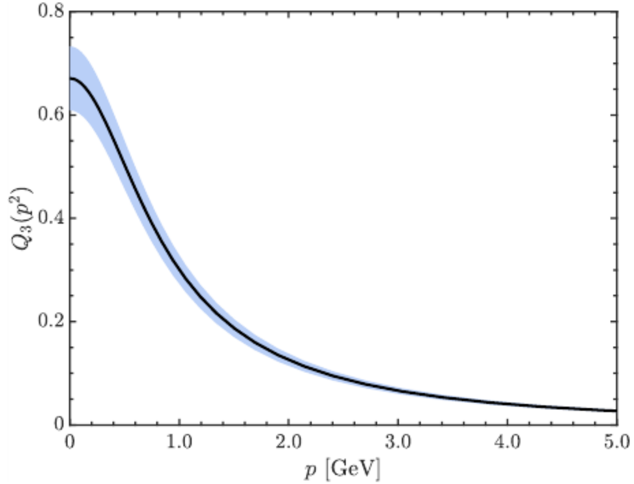


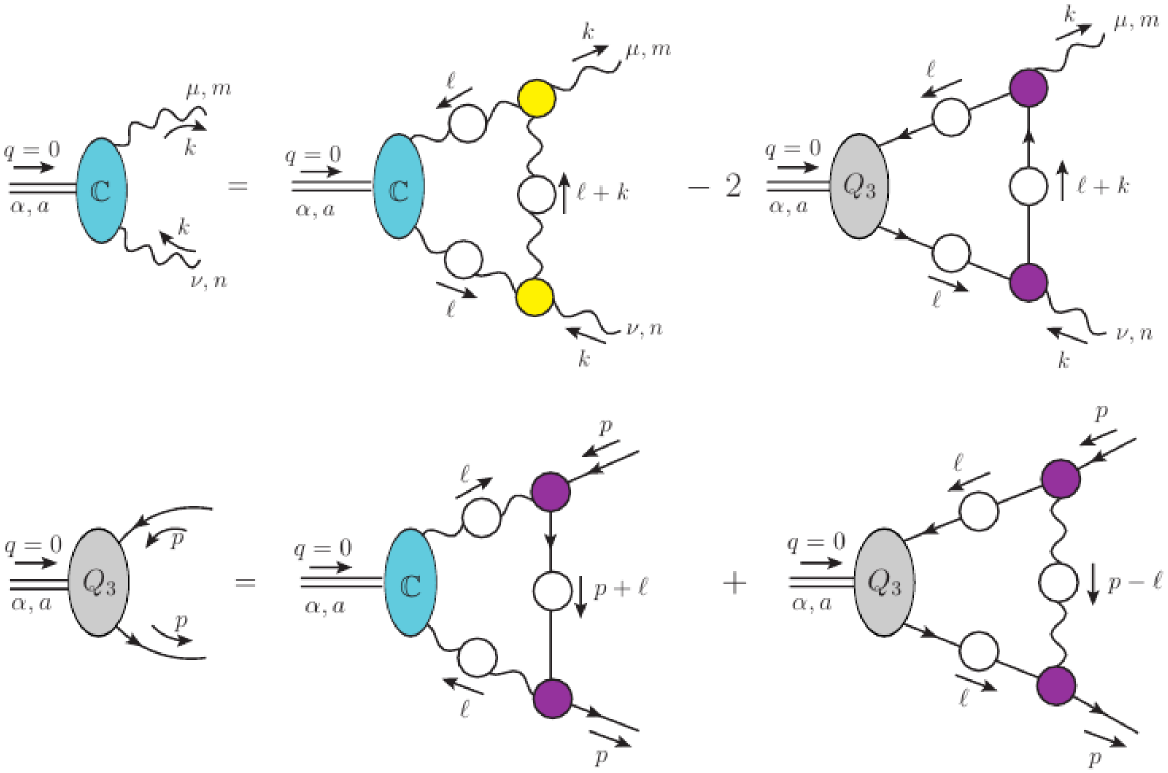


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
- But also generates a **pole in the quark-gluon vertex**, with amplitude $Q_3(p^2)$.



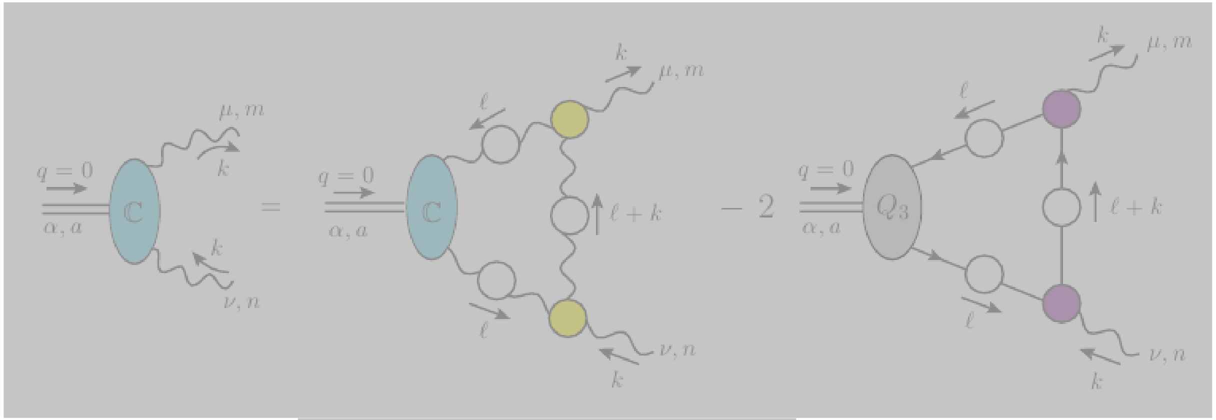


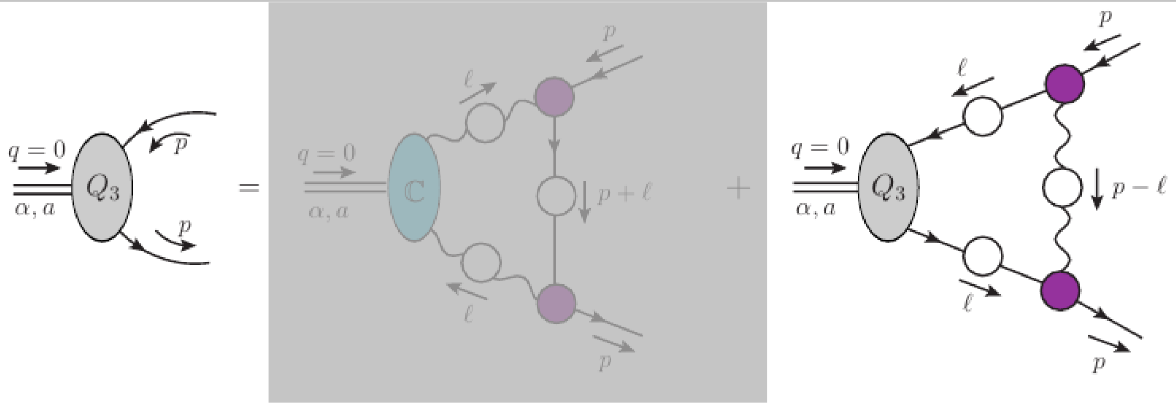
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Gluon self-interaction is dominant in generation of gluon mass gap

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
-
- But also generates a **pole in the quar-gluon vertex**, with amplitude $Q_3(p^2)$.
- **But turning off the three-gluon pole, no solution is found!**

Gluon self-interaction drives gluon mass generation





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

Ward identity displacement of the quark-gluon vertex

$$q^{\alpha}\Pi_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$ — Isolate classical tensor structure

Ward identity

$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

Partial derivative of the quark-ghost kernel

$$\left.\frac{\partial H(q,p,-q-p)}{\partial q^{\mu}}\right|_{q=0} = \gamma_{\mu}K_1(p^2) + 4p_{\mu}\not{p}K_2(p^2) + 2p_{\mu}K_3(p^2) + 2\widetilde{\sigma}_{\mu\nu}p^{\nu}K_4(p^2)$$

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

Ward identity displacement of the quark-gluon vertex

$$q^{\alpha}\Gamma_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

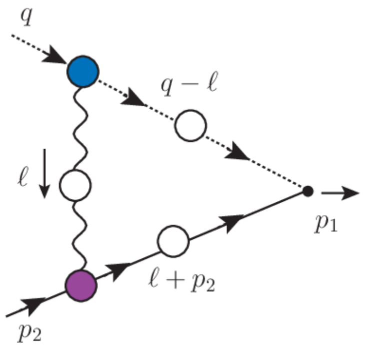
$q \to 0$ — Isolate classical tensor structure

Ward identity

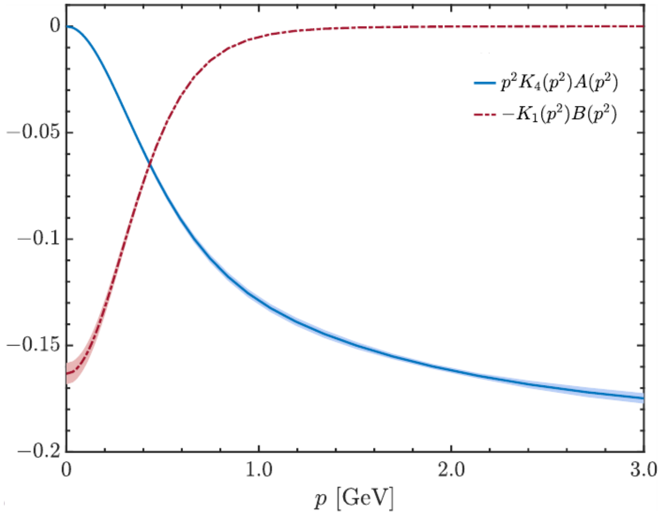
$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2 K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

Computed through a lattice driven Dyson-Schwinger analysis

$$H^a(q,p_2,-p_1) = -g\frac{\lambda^a}{2} + $$

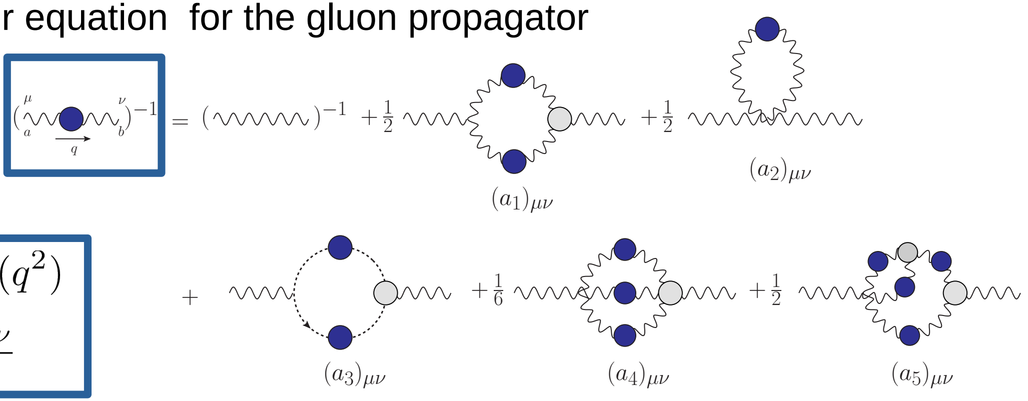


$+ \ldots$



Seagull cancellation

- The **gluon mass gap generation must occur without violating gauge symmetry**.
- Recalling the Dyson-Schwinger equation for the gluon propagator



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$
$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

It can be shown that

Gauge symmetry + Regular vertices at $q^2 = 0$ → $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass gap is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

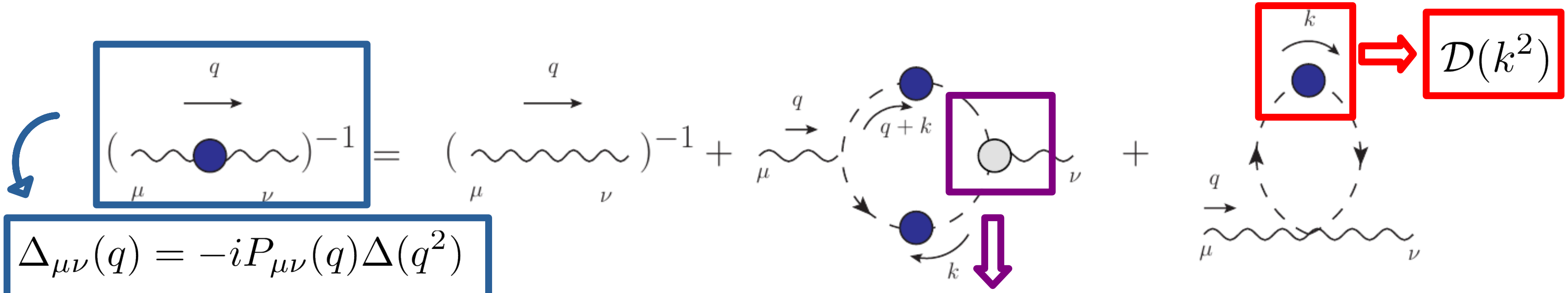
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Seagull cancellation

To understand **how gauge fields can become massive by the Schwinger mechanism**, let us first recall how gauge symmetry ***usually*** implies their masslessness.

A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

To this end, consider the **Dyson-Schwinger equation** for the scalar QED **photon propagator**



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Gamma_\nu(q, k, -q-k)$$

At *q* = 0, we obtain:

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

Seagull cancellation

Now, **gauge symmetry** implies the **Ward identity**:

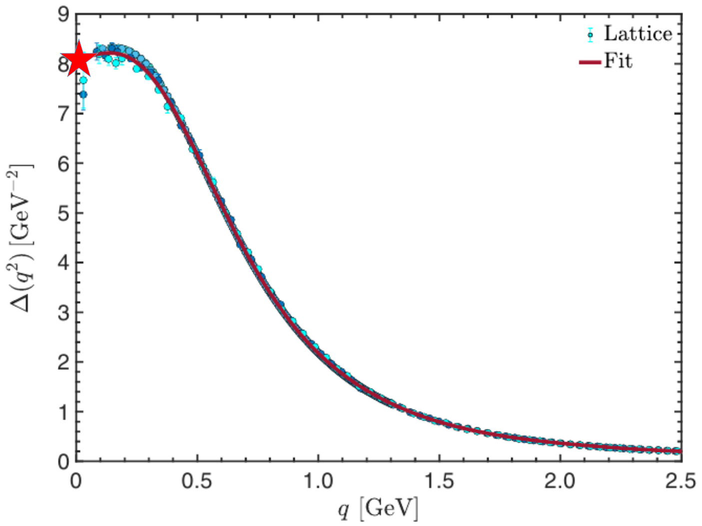
$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2) \quad \xrightarrow{q=0} \quad \Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^\mu}$$

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2) k^\mu \Gamma_\mu(0,k,-k) - 2ie^2 \int_k \mathcal{D}(k^2)$$

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\left[\int_k k^2 \frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k^2)\right] = 0$$

Seagull identity (integration by parts in *d* dimensions).

Then, how can we have saturation?



A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

Evading the seagull cancellation

Suppose the vertex has a **pole at q=0, coupled longitudinally to q**, i.e.

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

$$\Gamma_\mu(q,r,p) \to \mathbb{\Gamma}_\mu(q,r,p) = \frac{q_\mu}{q^2}C(q,r,p) + \Gamma_\mu(q,r,p)$$

Does not contribute explicitly to $\Delta(q^2)$ because it is longitudinal.

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

However, now the regular part satisfies a "displaced" Ward identity:

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Gamma_\mu(0,r,-r) = \frac{\partial\mathcal{D}^{-1}(k^2)}{\partial k^\mu} - 2r_\mu\mathcal{C}(r^2)$$

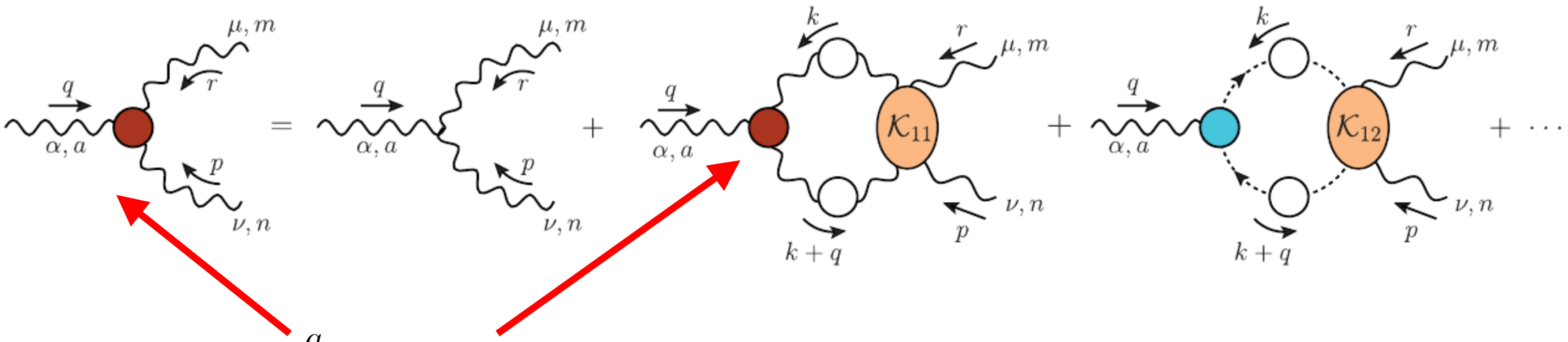
$$\mathcal{C}(r^2) := \left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\int_k k^2\mathcal{D}^2(k^2)\mathcal{C}(k^2)$$

Derivation of the Schwinger pole Bethe-Salpeter equation

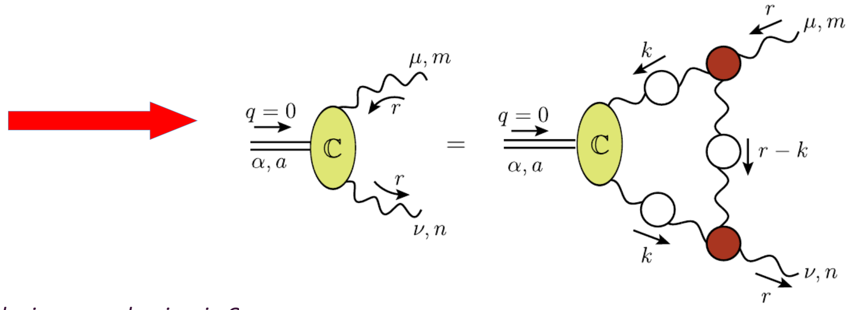
We start with the Dyson-Schwinger (or more generaly nPI) equation for the vertex and assume the presence of a massless pole:



$$\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \ldots$$

Now multiply by q^2 and take $q = 0$. Only terms containing poles remain:

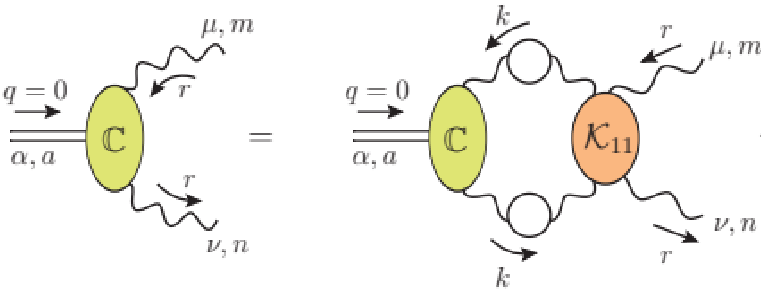
- **Inhomogeneous Dyson-Schwinger** equation becomes a **Homogeneous Bethe-Salpeter** equation.



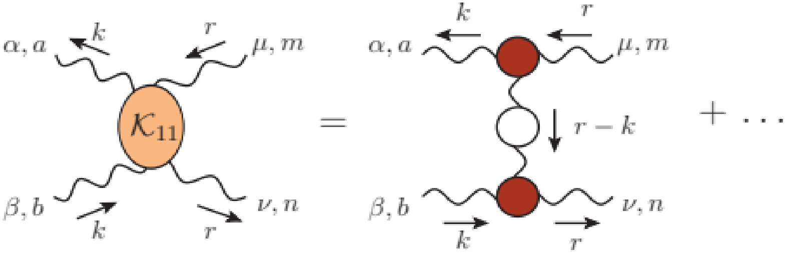
A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

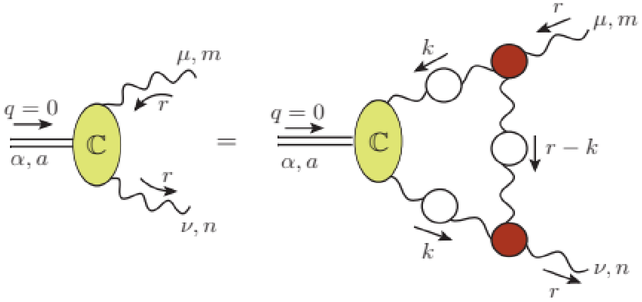
One-gluon exchange approximation

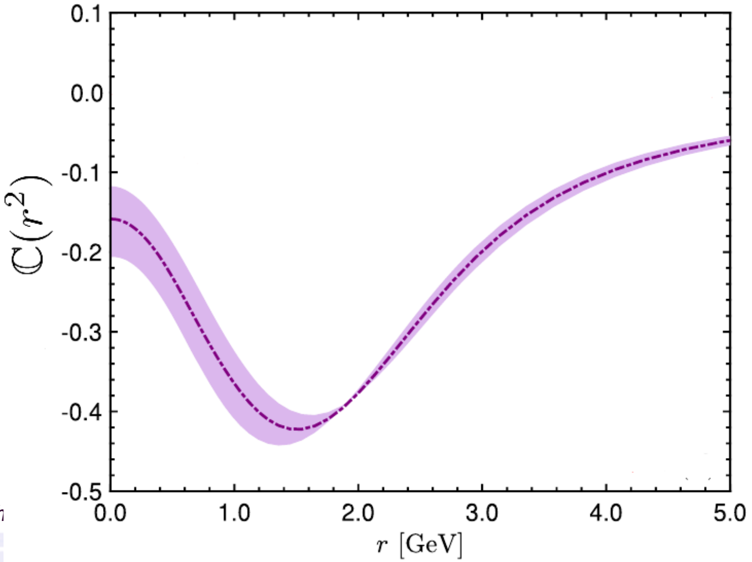
From the Bethe-Salpeter equation, we can



Truncation







A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

Method 2): Planar degeneracy

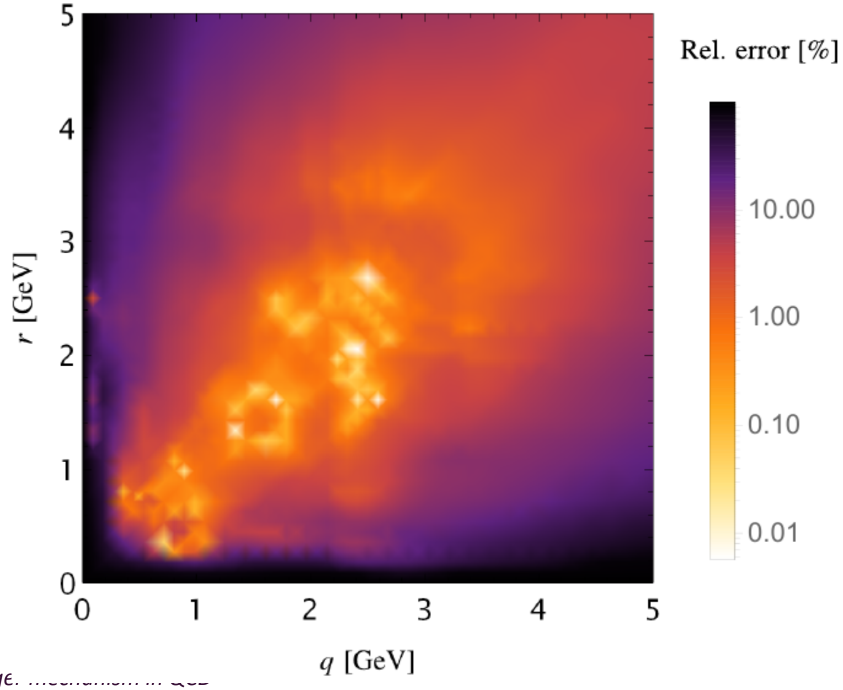
To quantify the accuracy of the approximation it is convenient to define

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^{0}_{\mathcal{W}}(q^2,r^2,p^2)}$$

Planar degeneracy →

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\text{sg}}(s^2)$$

Then we can measure the relative difference between $L_{\text{sg}}(s^2)$ and $\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)$



- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

Errors are propagated from known error of the planar degeneracy approximation.

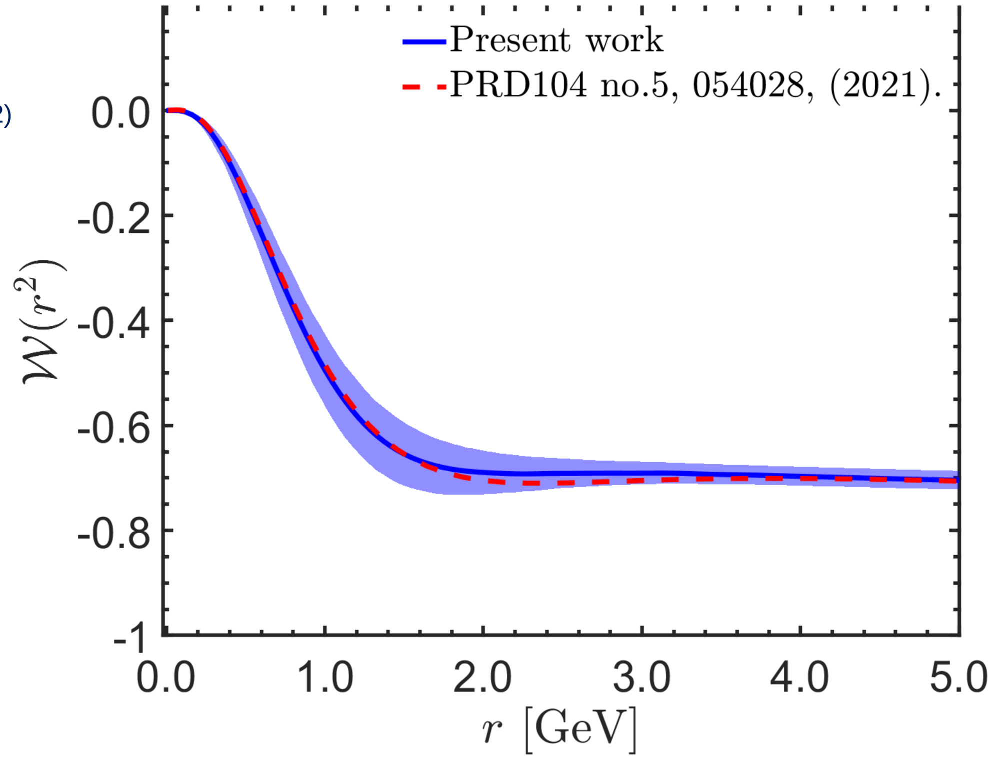
- **Result agrees well with previous calculation**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022)

- Previous result employed a particular *Ansatz* for three-gluon vertex.

- **New result stringently constrained by lattice simulation of the three-gluon vertex.**

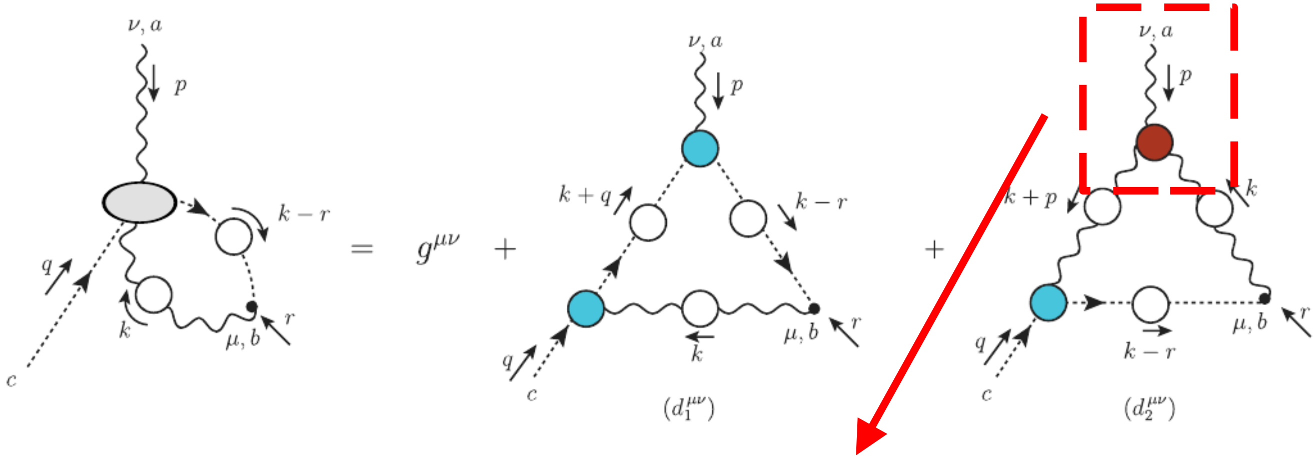
Impact of three-gluon vertex under control

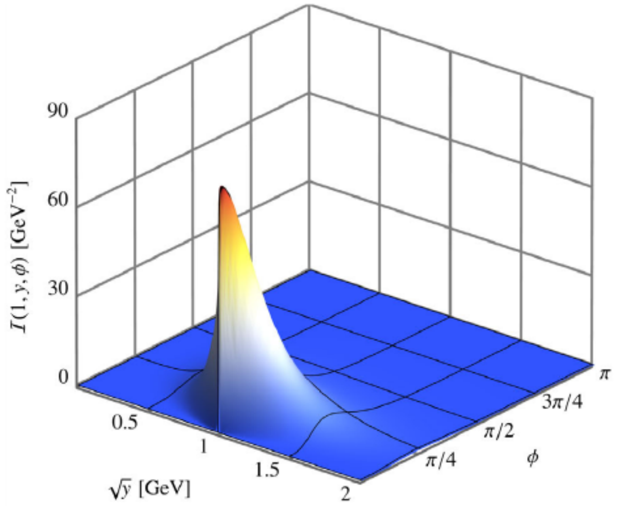


A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Truncation error

The full Dyson-Schwinger equation for $\mathcal{W}(r^2)$ is





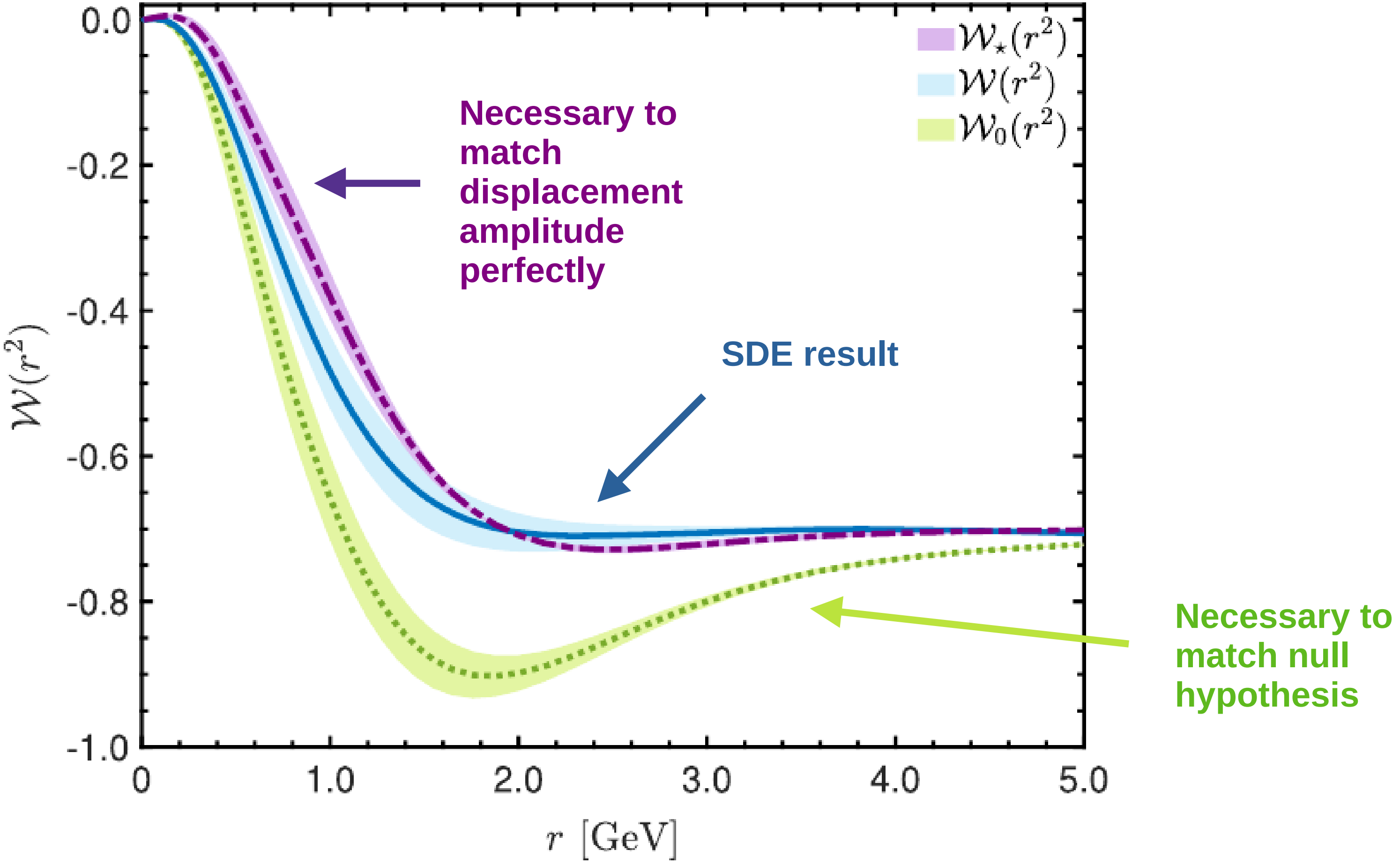
- Three-gluon vertex is a complicated object, with 14 tensor structures.

A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.9, 094010 (2019).
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980). [erratum: Phys. Rev. D **23**, 3085 (1981)].

- But $\mathcal{W}(r^2)$ integrand is sharply peaked, and is sensitive only to the particular projection $L_{\rm sg}(r^2)$ which is well determined by **lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

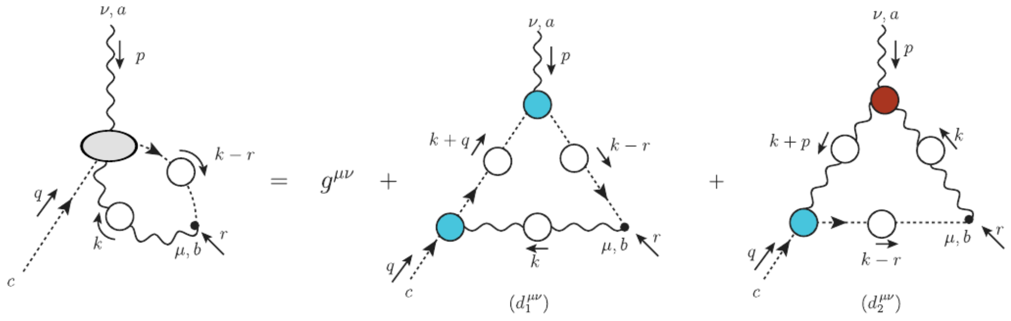
Truncation error

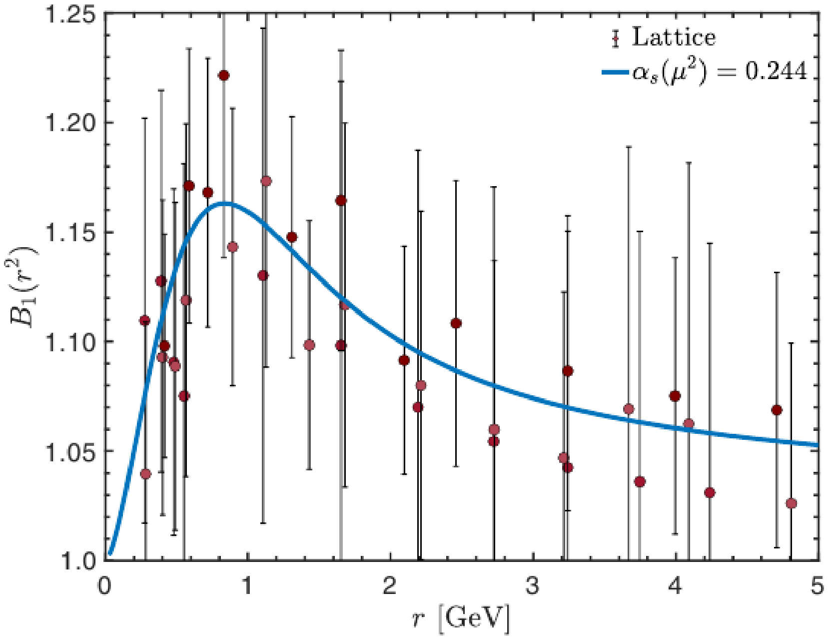


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error

The same truncation used to determine $\mathcal{W}(r^2)$, reproduces the available lattice data for the ghost-gluon vertex:





A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Lattice data from: A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).

Inputs

The parametrizations to lattice data used were of the form:

$$\Delta^{-1}(r^2) = r^2\left[\frac{d}{1+(r^2/\kappa^2)}\ln\left(\frac{r^2}{\mu^2}\right) + A^{\delta}(r^2)\right] + \nu^2 R(r^2),$$

$$F^{-1}(r^2) = A^{\gamma}(r^2) + R(r^2),$$

where

$$A(r^2) := 1 + \omega \ln\left(\frac{r^2+\eta^2(r^2)}{\mu^2+\eta^2(r^2)}\right),$$

$$\eta^2(r^2) = \frac{\eta_1^2}{1+r^2/\eta_2^2},$$

$$R(r^2) = \frac{b_0 + b_1^2 r^2}{1+(r^2/b_2^2)+(r^2/b_3^2)^2} - \frac{b_0 + b_1^2\mu^2}{1+(\mu^2/b_2^2)+(r^2/b_3^2)^2}.$$

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).