The light-meson LCDA using the QCD sum rules and its applications

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In collaboration with

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   PI, K, rho, eta, eta’, a1(1260), K0*(1430), ……

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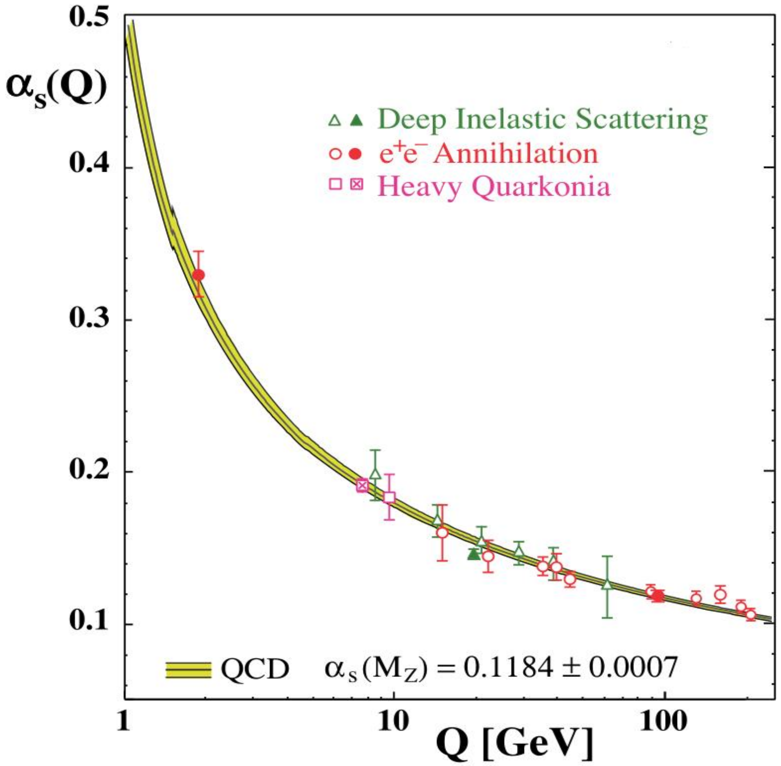
1. The QCD sum rules

The QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{\psi}_q (i \not{D} - m_q) \psi_q$$



微扰论不能解决一切强相互作用问题

$$R_{hadr} \sim 1/\Lambda_{QCD}$$

强子相关过程，因为夸克禁闭束缚态效应

强子相关部分需采用恰当的**非微扰方法**

存在真空中夸克-胶子涨落驱动的束缚效应

QCD求和规则基本思想

Within the sum rule approach, hadrons are represented by their interpolating quark currents taken at large virtualities. The correlation function (Collerator) of these currents is introduced and treated in the framework of the operator product expansion (OPE), where the short-distance and the long-distance quark-gluon interactions are separated.The former are calculated using QCD perturbation theory, whereas the latter are parametrized in terms of universal vacuum condensates (SVZ sum rules) or light-cone distribution amplitudes (LCSR). The result of the QCD calculation is then matched, via dispersion relation, to a sum over hadronic states.

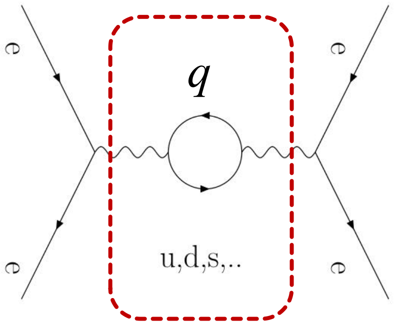
SVZ方法试图从QCD第一原理出发，从小距离来逼近共振态物理。更确切地说，是在QCD算符乘积展开的基础上，考虑了非微扰效应，引入非零的算符真空平均值，借助于色散关系以解决强子共振态中非微扰问题的。

如何做？

QCD求和规则的基础构建恰当关联函数

电磁流守恒 $\partial_\mu j^\mu = 0$

举例 $\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|\, T\{j_\mu(x) j_\nu(0)\}\,|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2)$

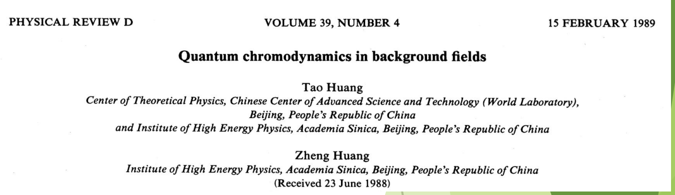


一方面， $Q^2 = \sqrt{-q^2} \to \infty$

可以基于OPE，微扰计算

$$\sum_n C_n(q)\langle 0|O_n|0\rangle$$

真空凝聚项按量纲排列，高维算符有Q^2-幂次压低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Quantum chromodynamics in background fields

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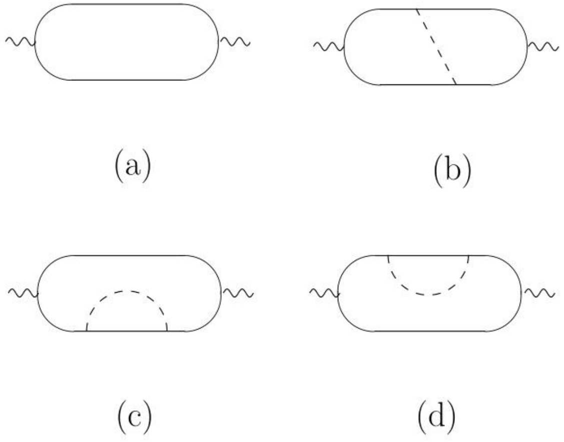
(Received 23 June 1988)

基于背景场易于理解凝聚项

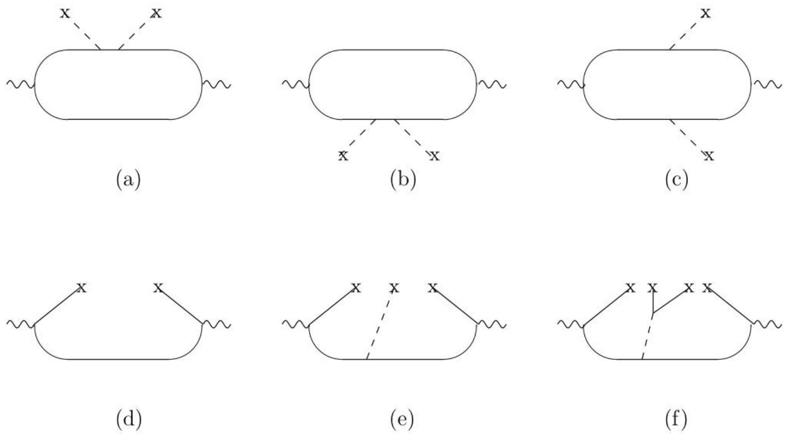
$$A_\mu^a(x) \to A_\mu^a(x) + \phi_\mu^a(x)$$

$$\psi(x) \to \psi(x) + \eta(x)$$

QCD求和规则的基础构建恰当关联函数



Free-quark loop and its NLO corrections

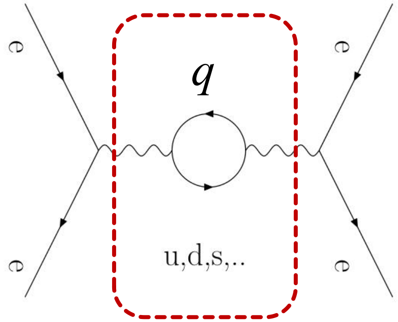


condensates contributions up to dimension-6

$$\Pi(q^2) = -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{-q^2}{4m^2} + \frac{2m\langle\bar{\psi}\psi\rangle}{q^4} + \frac{\alpha_s\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle}{12\pi q^4} + \frac{m^3}{3q^8}\langle g_s\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}\psi\rangle$$
$$+\frac{2\pi\alpha_s}{q^6}\left[\langle(\bar{\psi}\gamma_\mu\gamma_5\frac{\lambda^a}{2}\psi)(\bar{\psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\psi)\rangle + \frac{2}{9}\langle(\bar{\psi}\gamma_\mu\frac{\lambda^a}{2}\psi)(\bar{\psi}\gamma^\mu\frac{\lambda^a}{2}\psi)\rangle\right]$$

QCD求和规则的基础构建恰当关联函数

另一方面 $\sqrt{q^2} \to M_V \text{ with } V = \rho, \omega, \phi, j/\psi, \Upsilon, \cdots$



Using *unitarity relation* obtained by inserting a complete set of intermediate hadronic states

$$2\mathrm{Im}\,\Pi_{\mu\nu}(q) = \sum_n \langle 0|\, j_\mu \,|n\rangle\langle n|\, j_\nu \,|0\rangle \, d\tau_n (2\pi)^4 \delta^{(4)}(q - p_n)$$

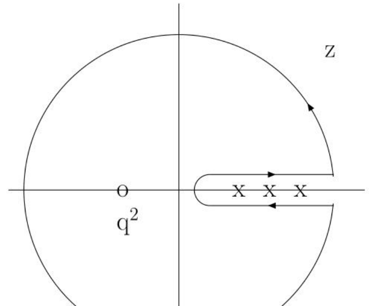
single out the ground-state vector-meson contribution and introduce a compact notation for the rest of contributions

$$\frac{1}{\pi}\mathrm{Im}\;\Pi(q^2) = f_V^2 \delta(q^2 - m_V^2) + \rho^h(q^2)\theta(q^2 - s_0^h)$$

continuum threshold

hadron spectral function

QCD求和规则的基础构建恰当关联函数



Cauchy formula

$$\Pi(q^2) = \frac{1}{2\pi i}\oint_C dz \frac{\Pi(z)}{z-q^2} = \frac{1}{2\pi i}\oint_{|z|=R} dz \frac{\Pi(z)}{z-q^2} + \frac{1}{2\pi i}\int_0^R dz \frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2}$$

Schwartz reflection principle

To link both, using the dispersion relation

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon} \qquad t_{min} = \min\{m_V^2, s_0^h\}$$

$$\Pi(q^2) = \frac{q^2 f_V^2}{m_V^2(m_V^2-q^2)} + q^2\int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s(s-q^2)} + \Pi(0)$$

Doing Borel transformation to suppress unknown contributions

$$\Pi(M^2) \equiv \mathcal{B}_{M^2}\Pi(q^2) = \lim_{\substack{-q^2, n\to\infty \\ -q^2/n = M^2}} \frac{(-q^2)^{(n+1)}}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)$$

QCD求和规则的基础构建恰当关联函数

$$\Pi(M^2) = f_V^2 e^{-m_V^2/M^2} + \int_{s_0^h}^{\infty} ds\, \rho^h(s) e^{-s/M^2}$$

Thus the Borel transformation removes subtraction terms in the dispersion relation and exponentially suppresses the contributions from excited resonances and continuum states heavier than V

quark-hadron duality $$\int_{s_0^h}^{\infty} ds\, \rho^h(s) e^{-s/M^2} \simeq \frac{1}{\pi} \int_{s_0}^{\infty} ds\, \mathrm{Im}\, \Pi^{(pert)}(s) e^{-s/M^2}$$

Then the final sum rule for the rho-meson decay constant is

$$f_\rho^2 = M^2 e^{m_\rho^2/M^2} \left[\frac{1}{4\pi^2} \left(1 - e^{-s_0^\rho/M^2}\right) \left(1 + \frac{\alpha_s(M)}{\pi}\right) + \frac{(m_u + m_d)\langle\bar{\psi}\psi\rangle}{M^4} + \frac{1}{12} \frac{\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \rangle}{M^4} - \frac{112\pi}{81} \frac{\alpha_s \langle\bar{\psi}\psi\rangle^2}{M^6} \right]$$

A simple summary

Several approximations has been for deriving the sum rules, and the accurateness of the QCD sum rules inversely heavily depends on the preciseness of those approximations

- The low limit of M^2 is fixed by demanding that the condensate term with the highest dimension remains a small fraction of sum of all terms, which makes the convergence of condensate expansion under well control
- At too large M^2, the quark-hadron duality approximation cannot be trusted. Therefore, the upper limit on M^2,should make the contribution of the states above s_0 be a small part of the total dispersion integral
- Moreover, the prediction should be flat versus the input parameter of Borel parameter within the allowable Borel window (weak requirement, since it is fixed-dimensional expansion)

2. The sum rules for the light-meson LCDAs

PI, K, rho, eta, eta’, a1(1260), K0*(1430), ……

PI介子twist-2分布振幅

PHYSICAL REVIEW D **90**, 016004 (2014)

Revisiting the pion leading-twist distribution amplitude within the QCD background field theory

Tao Zhong,[1,*] Xing-Gang Wu,[2,†] Zhi-Gang Wang,[3,‡] Tao Huang,[1,§] Hai-Bing Fu,[2] and Hua-Yong Han[2]

$$\phi_\pi(\mu,x) = 6x(1-x)\left[1+\sum_{n=2}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2x-1)\right]$$

$$\langle\xi^n\rangle = \int_0^1 du(2u-1)^n\phi_\pi(u)$$

$$a_2^\pi = \frac{7}{12}(5\langle\xi^2\rangle - 1),$$

$$a_4^\pi = -\frac{11}{24}(14\langle\xi^2\rangle - 21\langle\xi^4\rangle - 1),$$

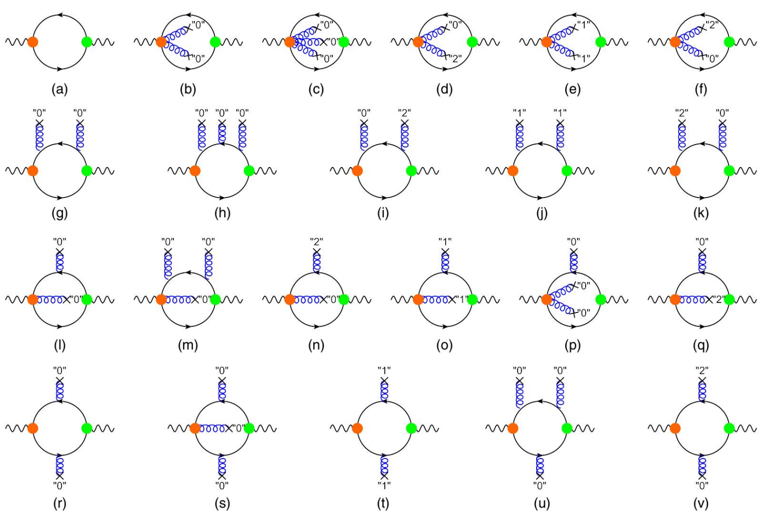
$$a_6^\pi = \frac{5}{64}(135\langle\xi^2\rangle - 495\langle\xi^4\rangle + 429\langle\xi^6\rangle - 5)$$

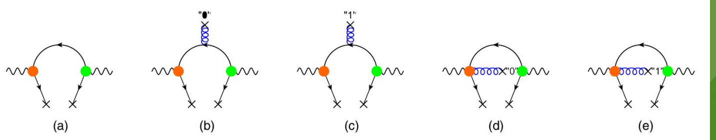
$$\langle 0|\bar{d}(0)\not{z}\gamma_5(iz\cdot\overleftrightarrow{D})^n u(0)|\pi(q)\rangle = i(z\cdot q)^{n+1} f_\pi\langle\xi^n\rangle$$

构建关联函数

$$\Pi_\pi^{(n,0)}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n(x)J_0^\dagger(0)\}|0\rangle = (z\cdot q)^{n+2} I_\pi^{(n,0)}(q^2)$$

$$J_n(x) = \bar{d}(x)\not{z}\gamma_5(iz\cdot\overleftrightarrow{D})^n u(x)$$





$$\langle\xi^2\rangle|_{\mu=1\ \mathrm{GeV}} = 0.338\pm 0.032,$$

$$\langle\xi^4\rangle|_{\mu=1\ \mathrm{GeV}} = 0.211\pm 0.030,$$

$$\langle\xi^6\rangle|_{\mu=1\ \mathrm{GeV}} = 0.163\pm 0.030,$$

与后来值相比稍偏大

$$\langle\xi^n\rangle_{2;\pi}|_\mu = \frac{(\langle\xi^n\rangle_{2;\pi}|_\mu\langle\xi^0\rangle_{2;\pi}|_\mu)|_{\text{From Eq. (7)}}}{\sqrt{\langle\xi^0\rangle_{2;\pi}^2|_\mu|_{\text{From Eq. (10)}}}}$$

零阶矩归一化问题

Eur. Phys. J. C (2016) 76:390

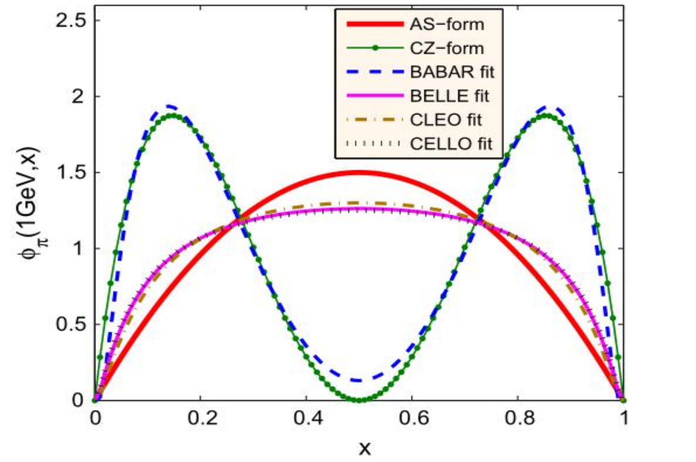


Fig. 9 The pion DAs with the parameters listed in Table 1, which are fit from the TFF data of BABAR, BELLE, CLEO, and CELLO Collaborations, respectively. As a comparison, we also present the asymptotic-DA [6] and the CZ-DA [36] in the figure

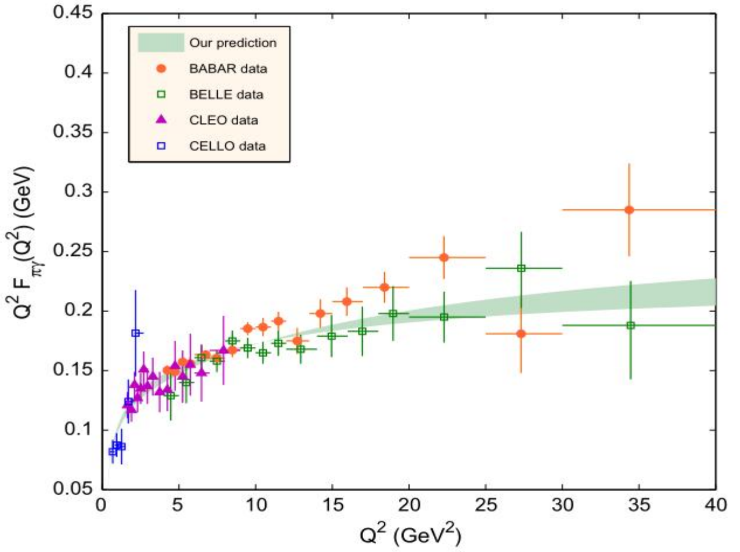


Fig. 10 The predicted pion–photon TFF $Q^2 F_{\pi\gamma}(Q^2)$ by using the parameters determined from the BELLE and the CLEO data. The BABAR, the BELLE, the CLEO, and the CELLO data are also presented as a comparison

PI介子twist-2分布振幅

PHYSICAL REVIEW D **104**, 016021 (2021)

Improved light-cone harmonic oscillator model for the pionic leading-twist distribution amplitude

Tao Zhong,[1,*] Zhi-Hao Zhu,[2] Hai-Bing Fu,[1,3,†] Xing-Gang Wu,[3,4,‡] and Tao Huang[5,§]

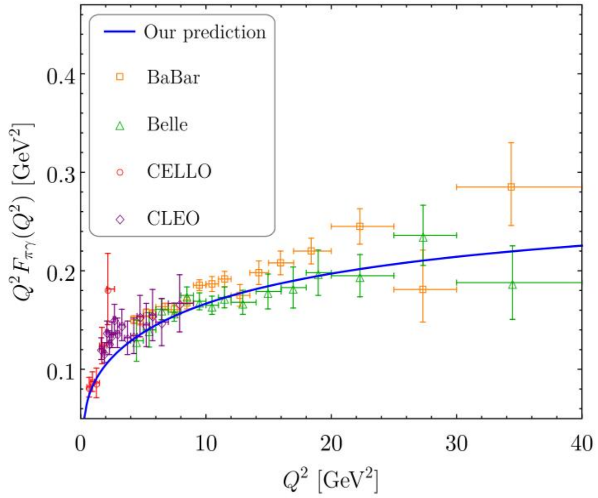


TABLE III. Our predictions for the first five nonvanishing moments and inverse moment of the pion DA, compared to other theoretical predictions. The values obtained using the formula combining Eqs. (7) and (9) are also shown.

	μ[GeV]	$\langle\xi^2\rangle_{2;\pi}\|_\mu$	$\langle\xi^4\rangle_{2;\pi}\|_\mu$	$\langle\xi^6\rangle_{2;\pi}\|_\mu$	$\langle\xi^8\rangle_{2;\pi}\|_\mu$	$\langle\xi^{10}\rangle_{2;\pi}\|_\mu$	$\langle x^{-1}\rangle\|_\mu$
BFTSR (this work)	1	0.271(13)	0.138(10)	0.087(6)	0.064(7)	0.050(6)	3.95
BFTSR (this work)	2	0.254(10)	0.125(7)	0.077(6)	0.054(5)	0.041(4)	3.33
Asymptotic	∞	0.200	0.086	0.048	0.030	0.021	3.00
LF holographic ($B = 0$) [54]	1,2	0.180, 0.185	0.067, 0.071	…	…	…	2.81,2.85
LF holographic ($B \gg 1$) [54]	1,2	0.200, 0.200	0.085, 0.085	…	…	…	2.93,2.95
LF holographic [55]	~1	0.237	0.114	…	…	…	4.0
Platykurtic [56]	2	$0.220^{+0.009}_{-0.006}$	$0.098^{+0.008}_{-0.005}$	…	…	…	$3.13^{+0.14}_{-0.10}$
LF quark model [57]	~1	0.24(22)	0.11(9)	…	…	…	…
Sum rules [58]	1	0.24	0.11	…	…	…	…
Renormalon model [59]	1	0.28	0.13	…	…	…	…
Instanton vacuum [60,61]	1	0.22, 0.21	0.10,0.09	…	…	…	…
NLC sum rules [62]	2	$0.248^{+0.016}_{-0.015}$	$0.108^{+0.05}_{-0.03}$	…	…	…	3.16(9)
Sum rules [5]	2	0.343	0.181	…	…	…	4.25
Dyson-Schwinger [RL,DB] [14]	2	0.280, 0.251	0.151, 0.128	…	…	…	5.5,4.6
Lattice [63]	2	0.28(1)(2)	…	…	…	…	…
Lattice [64]	2	0.2361(41)(39)	…	…	…	…	…
Lattice [65]	2	0.27(4)	…	…	…	…	…
Lattice [66]	2	0.2077(43)	…	…	…	…	…
Lattice [67]	2	0.234(6)(6)	…	…	…	…	…
Lattice [68]	2	0.244(30)	…	…	…	…	…
Eq. (7) + Eq. (9)	1	0.303(19)	0.179(21)	0.128(16)	0.098(14)	0.082(20)	…

类似SR计算步骤，η/η’介子twist-2分布振幅

Eur. Phys. J. C (2022) 82:12
https://doi.org/10.1140/epjc/s10052-021-09958-0
THE EUROPEAN PHYSICAL JOURNAL C
Regular Article - Theoretical Physics

$\eta^{(\prime)}$-meson twist-2 distribution amplitude within QCD sum rule approach and its application to the semi-leptonic decay $D_s^+ \to \eta^{(\prime)}\ell^+\nu_\ell$

Dan-Dan Hu[1], Hai-Bing Fu[1,3,a], Tao Zhong[1], Long Zeng[2,3], Wei Cheng[4], Xing-Gang Wu[2,3,b]

Eur. Phys. J. C (2024) 84:15
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THE EUROPEAN PHYSICAL JOURNAL C
Regular Article - Theoretical Physics

Properties of the η_q leading-twist distribution amplitude and its effects to the $B/D^+ \to \eta^{(\prime)}\ell^+\nu_\ell$ decays

Dan-Dan Hu[1,a], Xing-Gang Wu[1,b], Hai-Bing Fu[2,c], Tao Zhong[2,d], Zai-Hui Wu[2,e], Long Zeng[1,f]

Using the simple quark-flavor basis

$$|\eta\rangle = \cos\varphi|\eta_q\rangle - \sin\varphi|\eta_s\rangle \qquad |\eta_q\rangle = |\bar{u}u+\bar{d}d\rangle/\sqrt{2}$$
$$|\eta'\rangle = \sin\varphi|\eta_q\rangle + \cos\varphi|\eta_s\rangle \qquad |\eta_s\rangle = |\bar{s}s\rangle$$

$$D_s^+ \to \eta^{(\prime)}\ell^+\nu_\ell$$

$$f_+^{B(D)\to\eta} = \cos\phi f_+^{B(D)\to\eta_q}$$
$$f_+^{B(D)\to\eta'} = \sin\phi f_+^{B(D)\to\eta_q},$$

正在考虑更复杂的混合，胶子成分，η_c成分

类似SR计算步骤，ρ介子纵向twist-2分布振幅

PHYSICAL REVIEW D **94,** 074004 (2016)

ρ-meson longitudinal leading-twist distribution amplitude within QCD background field theory

Hai-Bing Fu,[1] Xing-Gang Wu,[2,*] Wei Cheng,[2] and Tao Zhong[3]

构建关联函数

$$\Pi_\rho^{(n,0)}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n(x)J_0^\dagger(0)\}|0\rangle = (z\cdot q)^{n+2}I^{(n,0)}(q^2) \qquad J_n(x) = \bar{d}(x)\not{z}(iz\cdot\overleftrightarrow{D})^n u(x)$$

$$\langle 0|\bar{d}(0)\not{z}(iz\cdot\overleftrightarrow{D})^n u(0)|\rho(q,\lambda)\rangle = (e^{(\lambda)*}\cdot z)(q\cdot z)^n m_\rho f_\rho^\parallel \langle\xi_{n;\rho}^\parallel\rangle$$

	$a_{2;\rho}^\parallel$	$a_{4;\rho}^\parallel$	$\langle\xi_{2;\rho}^\parallel\rangle$	$\langle\xi_{4;\rho}^\parallel\rangle$	$\langle x^{-1}\rangle$
Our predictions	0.119(82)	−0.035(100)	0.241(28)	0.109(10)	3.30(34)
NLCSR [9]	0.047(58)	−0.057(118)	0.216(21)	0.089(9)	2.97(39)
BB [10]	0.150(70)	···	0.251(24)	···	3.45(21)
Lattice QCD [11]	0.197(158)	···	0.268(54)	···	3.60(48)
BS [12]	0.111	0.036	0.238	0.115	3.44
AdS/QCD [13,14]	0.104	0.053	0.236	0.115	3.47
LFQM [15]	0.014	−0.005	0.205	0.088	3.03
IM [16]	−0.010	−0.033	0.196	0.080	2.87

类似SR计算步骤，a1(1260)-介子twist-2分布振幅

$$\langle 0|\bar{q}_1(z)\gamma_\mu\gamma_5 q_2(-z)|a_1(q,\lambda)\rangle = i m_{a_1} f_{a_1}^{\parallel}\int_0^1 dx e^{i(xz\cdot q-\bar{x}z\cdot q)} q_\mu \frac{e^{*(\lambda)}\cdot z}{q\cdot z}\phi_{2;a_1}^{\parallel}(x,\mu)$$

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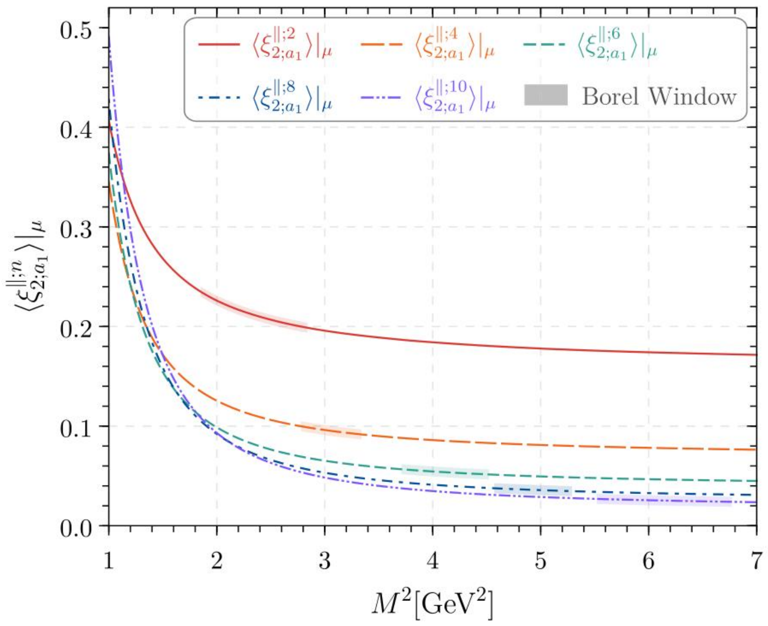
https://doi.org/10.1140/epjc/s10052-022-10555-y

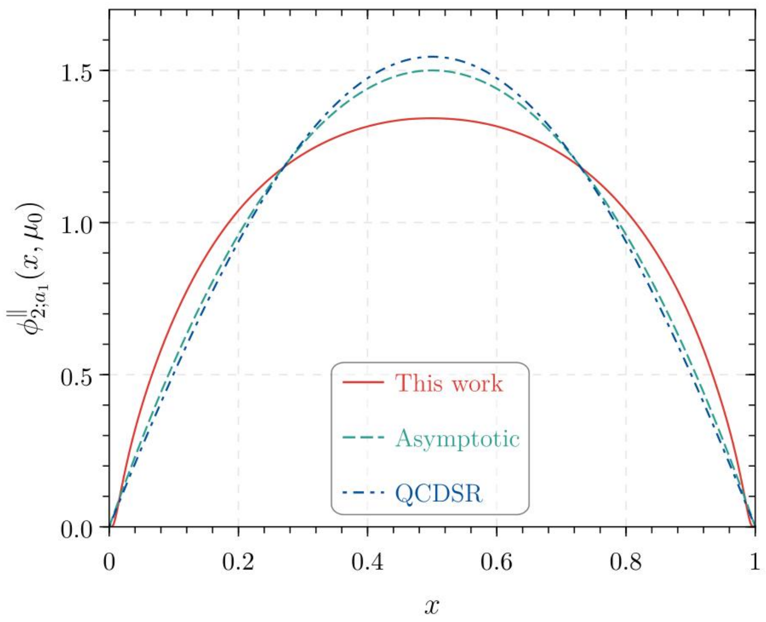
THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

$a_1(1260)$-meson longitudinal twist-2 distribution amplitude and the $D \to a_1(1260)\ell^+\nu_\ell$ decay processes

Dan-Dan Hu[1], Hai-Bing Fu[1,a], Tao Zhong[1,b], Zai-Hui Wu[1], Xing-Gang Wu[2,c]





类似SR计算步骤，K0*(1430)-介子twist-2分布振幅

$$\langle 0 \left| \bar{s}(z)\gamma_\mu u(-z) \right| K_0^{*+} \rangle = \bar{f}_{K_0^*} p_\mu \int_0^1 du e^{i(2u-1)p\cdot z} \phi_{2;K_0^*}(u,\mu)$$

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

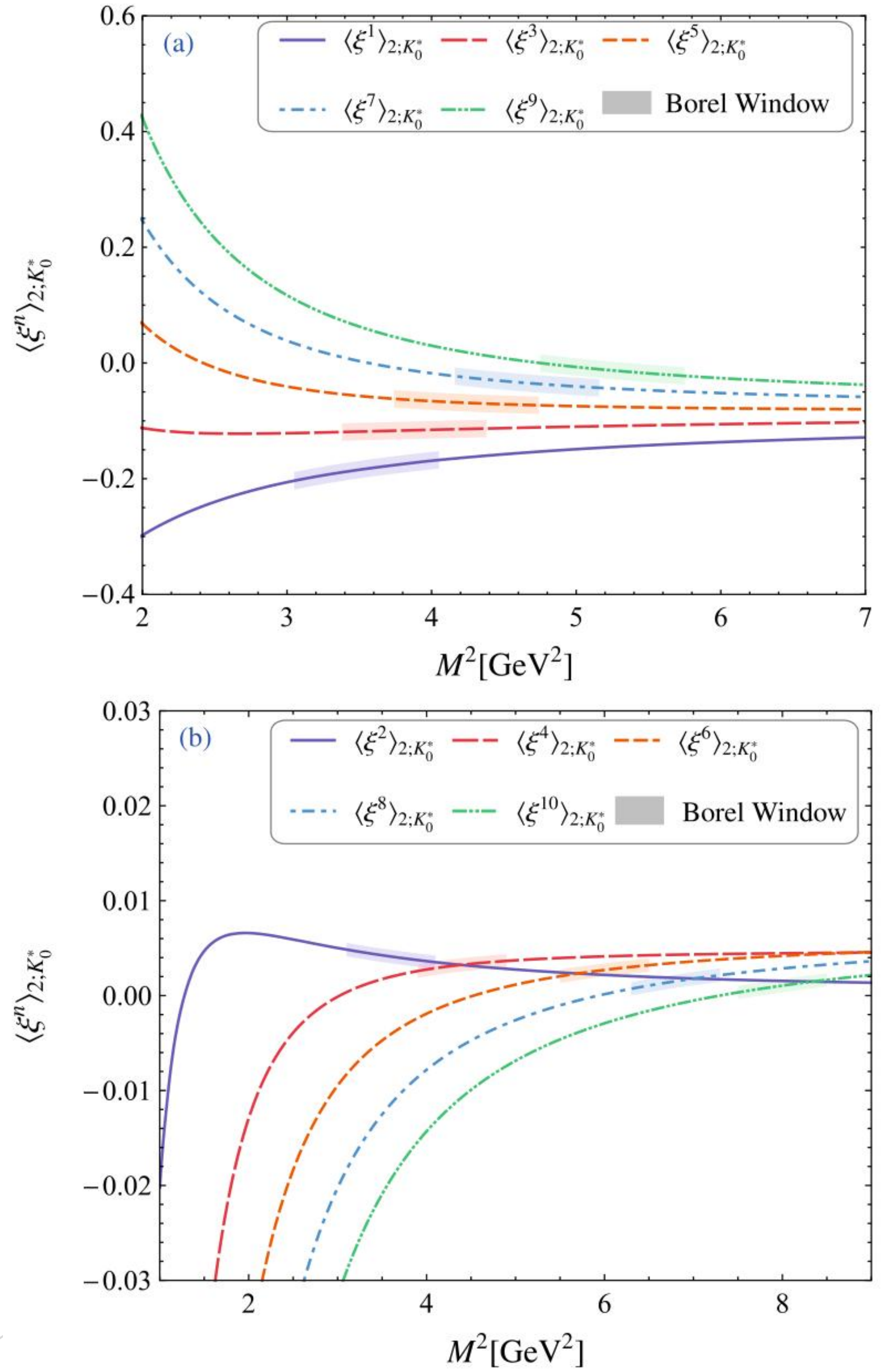
$K_0^*(1430)$ twist-2 distribution amplitude and $B_s, D_s \to K_0^*(1430)$ transition form factors

Dong Huang[1], Tao Zhong[1,a], Hai-Bing Fu[1,2,b], Zai-Hui Wu[1], Xing-Gang Wu[2,c], Hong Tong[1]

构建关联函数

$$\Pi_{2;K_0^*}(z,q) = i \int d^4x e^{iq\cdot x} \langle 0|T\{J_n(x), \hat{J}_0^\dagger(0)\}|0\rangle = (z\cdot q)^{n+1} I_{2;K_0^*}(q^2)$$

$$J_n(x) = \bar{s}(x) \not{z} (iz\cdot \overleftrightarrow{D})^n u(x) \quad \hat{J}_0^\dagger(0) = \bar{u}(0)s(0)$$



3. Their applications to heavy-to-light meson decays

早期采用PQCD方法
考虑轻介子波函数以及sudakov压低端点

$$
\begin{aligned}
F_+^{B\to P}(q^2) &= \frac{\pi C_F}{N_c} f_P f_B M_B^2 \int d\xi dx \int b_B db_B\, b_P db_P\, \alpha_s(t) \times \exp(-S(x,\xi,b_P,b_B;t)) \\
&\times S_t(x)S_t(\xi)\Big\{\Big[\Psi_P(x,b_P)\left((x\eta+1)\Psi_B(\xi,b_B) - \bar{\Psi}_B(\xi,b_B)\right) \\
&+ \frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\left((1-2x)\Psi_B(\xi,b_B) + \left(x+\frac{1}{\eta}-1\right)\bar{\Psi}_B(\xi,b_B)\right) \\
&- \frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\cdot\left(\left(1+2x-\frac{2}{\eta}\right)\Psi_B(\xi,b_B) - (1+x-\frac{1}{\eta})\bar{\Psi}_B(\xi,b_B)\right) \\
&+ \frac{m_0^p}{M_B}\Psi_\sigma(x,b_P)\left(\Psi_B(\xi,b_B) - \frac{\bar{\Psi}_B(\xi,b_B)}{2}\right)\Big] h_1(x,\xi,b_P,b_B) \\
&- (1+\eta+x\eta)\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_P,b_B) \\
&+ \Big[\Psi_P(x,b_p)\left(-\xi\bar{\eta}\Psi_B(\xi,b_B) + \frac{\Delta(\xi,b_B)}{M_B}\right) + 2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot \\
&\quad \left((1-\xi)\Psi_B(\xi,b_B) + \xi(1-\frac{1}{\eta})\bar{\Psi}_B(\xi,b_B) + 2\frac{\Delta(\xi,b_B)}{M_B}\right)\Big] h_1(\xi,x,b_B,b_P)\Big\}.
\end{aligned}
$$

只适用于大反冲区域，而后单极点或双极点延拓
但精度有限——逐渐放弃

$$
\begin{aligned}
F_0^{B\to P}(q^2) &= \frac{\pi C_F}{N_c} f_P f_B M_B^2 \int d\xi dx \int b_B db_B\, b_P db_P\, \alpha_s(t) \times \exp(-S(x,\xi,b_P,b_B;t)) \\
&\times S_t(x)S_t(\xi)\Big\{\Big[\Psi_P(x,b_P)\eta\left((x\eta+1)\Psi_B(\xi,b_B) - \bar{\Psi}_B(\xi,b_B)\right) \\
&+ \frac{m_0^p}{M_B}\Psi_p(x,b_P)((2-\eta-2x\eta)\Psi_B(\xi,b_B) - (1-\eta-x\eta)\bar{\Psi}_B(\xi,b_B)) \\
&- \frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\cdot(\eta(2x-1)\Psi_B(\xi,b_B) - (1+x\eta-\eta)\bar{\Psi}_B(\xi,b_B)) \\
&+ \eta\frac{m_0^p}{M_B}\Psi_\sigma(x,b_P)\left(\Psi_B(\xi,b_B) - \frac{\bar{\Psi}_B(\xi,b_B)}{2}\right)\Big] h_1(x,\xi,b_P,b_B) \\
&- [3-\eta-x\eta]\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_P,b_B) \\
&+ \Big[\Psi_P(x,b_P)\eta\left(\xi\bar{\eta}\Psi_B(\xi,b_B) + \frac{\Delta(\xi,b_B)}{M_B}\right) \\
&+ 2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\left((\eta(1+\xi)-2\xi)\Psi_B(\xi,b_B) - (\eta\xi-\xi)\bar{\Psi}_B(\xi,b_B)\right. \\
&+ 2(2-\eta)\frac{\Delta(\xi,b_B)}{M_B}\Big)\Big] h_1(\xi,x,b_B,b_P)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
F_T^{B\to P}(q^2) &= \frac{\pi C_F}{N_c} f_P f_B M_B^2 \int d\xi dx \int b_B db_B\, b_P db_P\, \alpha_s(t) \times \exp[-S(x,\xi,b_P,b_B;t)] \\
&\times S_t(x)S_t(\xi)\Big\{\Big[\Psi_P(x,b_P)\left(\Psi_B(\xi,b_B) - \bar{\Psi}_B(\xi,b_B)\right) + \frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot \\
&\left(\frac{1}{\eta}\bar{\Psi}_B(\xi,b_B) - x\Psi_B(\xi,b_B)\right) + \frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\left(\frac{x\eta+2}{\eta}\Psi_B(\xi,b_B)\right. \\
&\left.-\frac{1}{\eta}\bar{\Psi}_B(\xi,b_B)\right) + \frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}\Psi_B(\xi,b_B)\Big] h_1(x,\xi,b_P,b_B) - \frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6} \\
&[M_B\Delta(\xi,b_P)]h_2(x,\xi,b_P,b_B) + \Big[\Psi_P(x,b_P)\left(\frac{\Delta(\xi,b_B)}{M_B} - \xi\Psi_B(\xi,b_B)\right) + \\
&2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\left(\Psi_B(\xi,b_B) - \frac{\xi}{\eta}\bar{\Psi}_B(\xi,b_B)\right)\Big] h_1(\xi,x,b_B,b_P)\Big\}, \qquad (7)
\end{aligned}
$$

随后采用LCSR方法
但关联流采用多种选择，并未严格按介子的量子数来构造夸克流
手征流方法——优点降低高扭度算符贡献，缺点引入未知连续态贡献（例如引入新基态）
关键是代价究竟值不值得——长久争议

correlation with chiral current

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle K(p)|T\{\bar{s}(x)\gamma_\mu(1+\gamma_5) \times b(x), \bar{b}(0)i(1+\gamma_5)d(0)\}|0\rangle = \Pi_+[q^2,(p+q)^2]p_\mu + \Pi_-[q^2,(p+q)^2]q_\mu.$$

可消除不确定性较大的twist-3项贡献

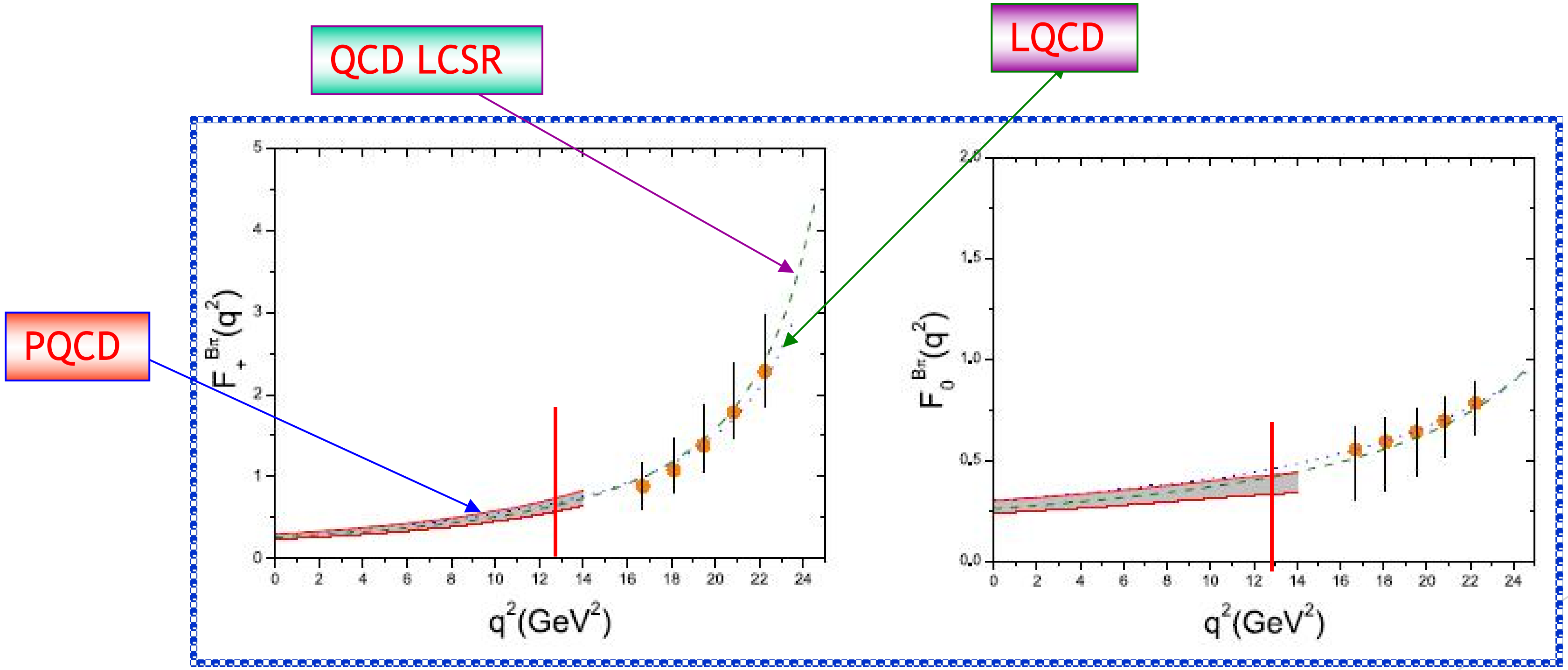
新思路：从求解B介子分布振幅出发，并完成演化方程

PHYSICAL REVIEW D **71,** 034018 (2005)

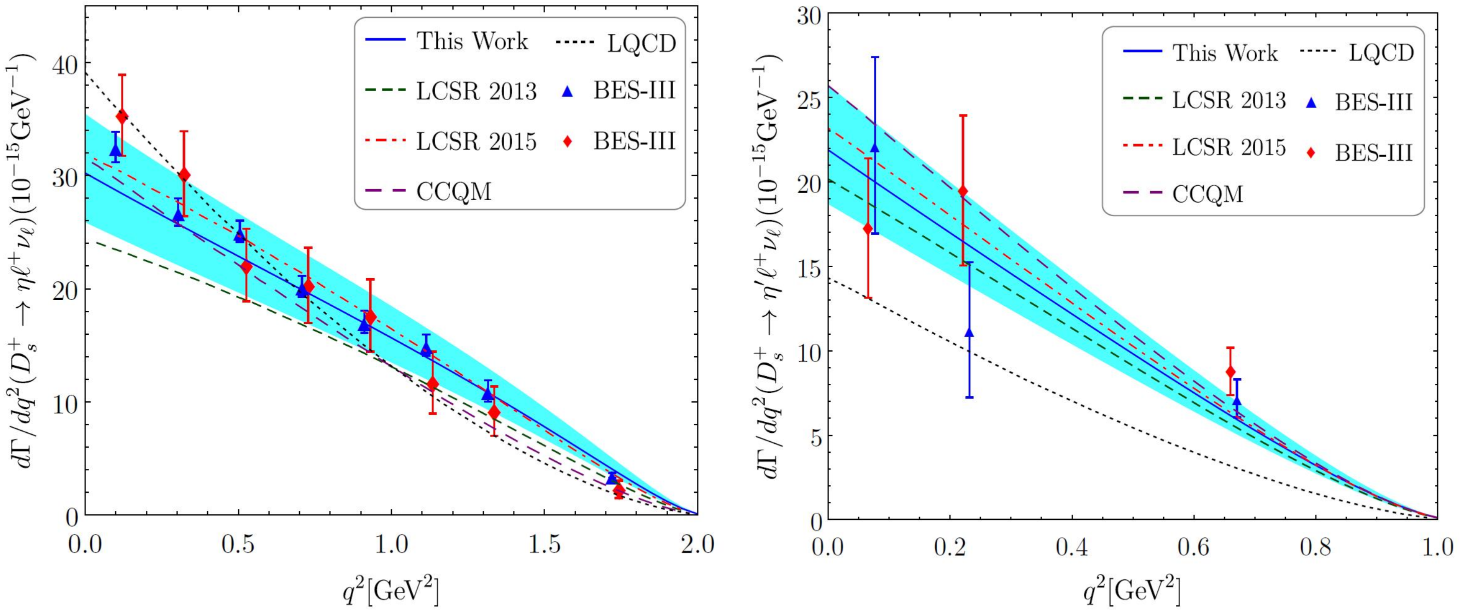
Consistent analysis of the $B \to \pi$ transition form factor in the whole physical region

Tao Huang[1,2,*] and Xing-Gang Wu[2,†]

三种方法结果互补，QCD求和规则居中起到承上启下作用



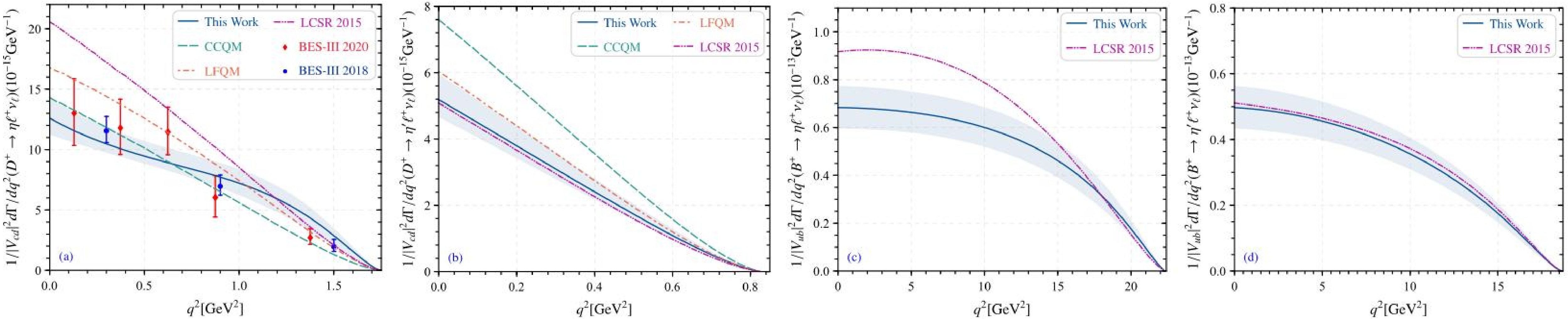
$D_s \to \eta^{(\prime)}\ell\nu$ decays



Mode	$\mathcal{B}(D_s{}^+ \to \eta e^+\nu_e)$	$\mathcal{B}(D_s{}^+ \to \eta\mu^+\nu_\mu)$	$\mathcal{B}(D_s{}^+ \to \eta' e^+\nu_e)$	$\mathcal{B}(D_s{}^+ \to \eta'\mu^+\nu_\mu)$
This work (LCSR)	$2.346^{+0.418}_{-0.331}$	$2.320^{+0.413}_{-0.327}$	$0.775^{+0.137}_{-0.114}$	$0.756^{+0.134}_{-0.111}$
BESIII [6, 7]	$2.323 \pm 0.063 \pm 0.063$	$2.42 \pm 0.46 \pm 0.11$	$0.824 \pm 0.073 \pm 0.027$	$1.06 \pm 0.54 \pm 0.07$
PDG [93]	2.32 ± 0.08	2.4 ± 0.5	0.80 ± 0.07	1.1 ± 0.5
CLEO [4]	$2.48 \pm 0.29 \pm 0.13$	-	$0.91 \pm 0.33 \pm 0.05$	-
CLEO [3]	$2.28 \pm 0.14 \pm 0.19$	-	$0.68 \pm 0.15 \pm 0.06$	-
CCQM [32]	2.24	2.18	0.83	0.79
LFQM [27]	2.26 ± 0.21	2.22 ± 0.20	0.89 ± 0.09	0.85 ± 0.08
QCD SR-I [16]	2.6 ± 0.7	-	0.89 ± 0.34	-
QCD SR-II [16]	2.3 ± 0.4	-	1.0 ± 0.2	-
LCSR [17]	2.40 ± 0.28	-	0.79 ± 0.14	-
LCSR [57]	2.00 ± 0.32	-	0.75 ± 0.23	-

$B(D)^+ \to \eta^{(\prime)}\ell^+\nu_\ell$ decays

Mode	Channels	$\lvert V_{cd}\rvert$
This work	$D^+ \to \eta e^+ \nu_e$	$0.236^{+0.017}_{-0.017}$
This work	$D^+ \to \eta \mu^+ \nu_\mu$	$0.228^{+0.017}_{-0.017}$
This work	$D^+ \to \eta' e^+ \nu_e$	$0.253^{+0.028}_{-0.032}$
BESIII 2020 [28]	$D^+ \to \eta \mu^+ \nu_\mu$	$0.242 \pm 0.028 \pm 0.033$
BESIII 2013 [16]	$D^+ \to \mu^+ \nu_\mu$	$0.221 \pm 0.006 \pm 0.005$
BaBar 2014 [17]	$D^0 \to \pi^- e^+ \nu_e$	$0.206 \pm 0.007 \pm 0.009$
CLEO 2009 [18]	$D \to \pi e^+ \nu_e$	$0.234 \pm 0.009 \pm 0.025$
HFLAV [19]	$D \to \pi \ell \nu_\ell$	$0.225 \pm 0.003 \pm 0.006$
LQCD 2019 [20]	$D \to \pi \ell \nu$	0.233 ± 0.137
PDG [24]	$D \to \pi \ell \nu$	$0.233 \pm 0.003 \pm 0.013$
	Channels	$\lvert V_{ub}\rvert \times 10^{-3}$
This work	$B^+ \to \eta \ell^+ \nu_\ell$	$3.752^{+0.373}_{-0.351}$
This work	$B^+ \to \eta' \ell^+ \nu_\ell$	$3.888^{+0.688}_{-0.787}$
CLEO 2007 [15]	$B^0 \to \pi^- \ell^+ \nu$	$3.60 \pm 0.4 \pm 0.2$
HFLAV [19]	$B \to \pi \ell \nu_\ell$	$3.70 \pm 0.10 \pm 0.12$
BaBar 2011 [21]	$\bar{B} \to X_u \ell \bar{\nu}$	$4.33 \pm 0.24 \pm 0.15$
Belle [22]	$B \to X_u \ell \nu$	$4.09 \pm 0.39 \pm 0.18$
PDG [24]	$B \to \pi \ell \bar{\nu}$	3.82 ± 0.20
PDG [24]	$B \to X_u \ell \bar{\nu}$	$4.13 \pm 0.12 \pm 0.18$
LQCD 2015 [105]	$B \to \pi \ell \nu$	3.72 ± 0.16



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https://doi.org/10.1140/epjc/s10052-020-7758-4

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Regular Article - Theoretical Physics

The $D \to \rho$ semileptonic and radiative decays within the light-cone sum rules

Hai-Bing Fu[1,a], **Long Zeng**[1,b], **Rong Lü**[1,c], **Wei Cheng**[2,d], **Xing-Gang Wu**[3,e]

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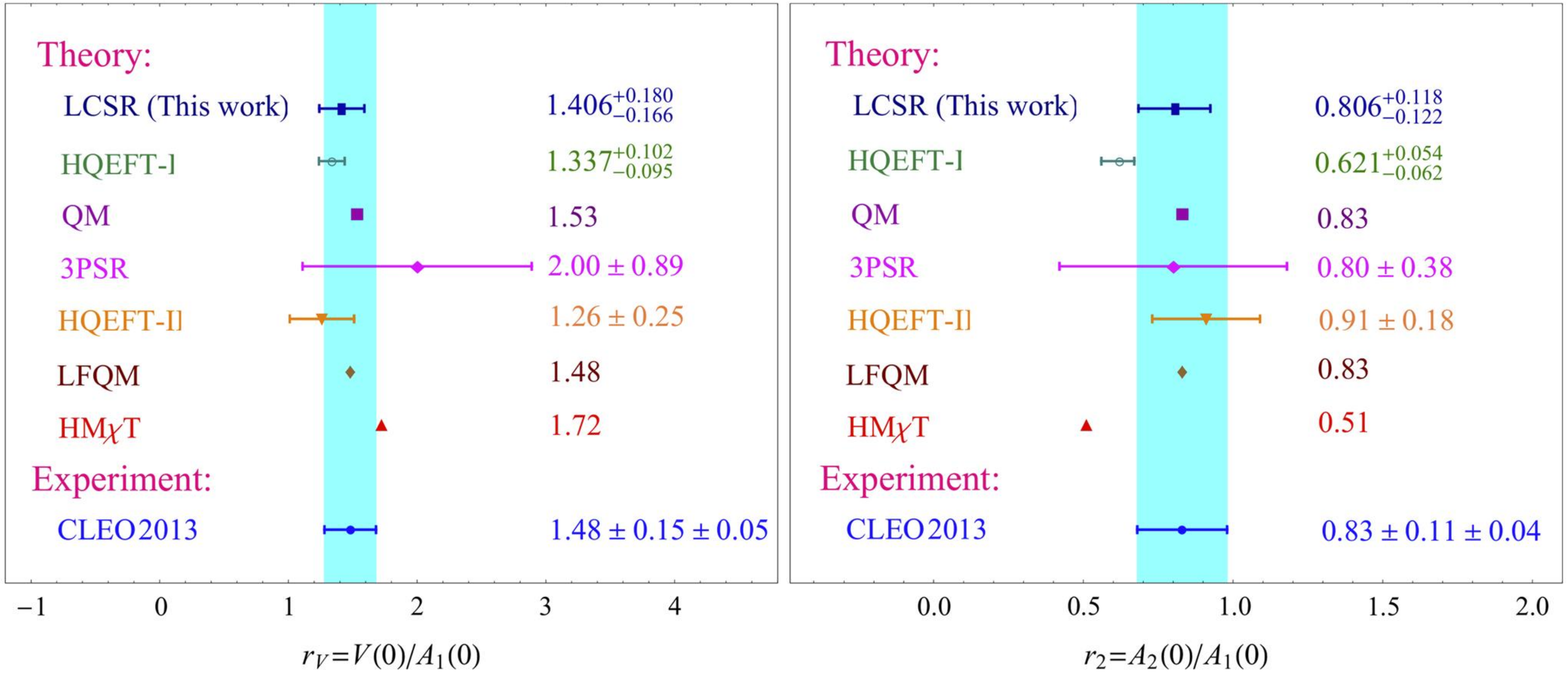
[2] State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China
[3] Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

Decay Mode	$D^0 \to \rho^- e^+ \nu_e$	$D^+ \to \rho^0 e^+ \nu_e$
This paper	$1.749^{+0.421}_{-0.297} \pm 0.006$	$2.217^{+0.534}_{-0.376} \pm 0.015$
CLEO2005 [2]	$1.94 \pm 0.39 \pm 0.13$	$2.1 \pm 0.4 \pm 0.1$
CLEO2013 [3]	$1.77 \pm 0.12 \pm 0.10$	$2.17 \pm 0.12^{+0.12}_{-0.22}$
3PSR [6]	0.5 ± 0.1	–
HQEFT [7]	1.4 ± 0.3	–
NWA [58]+HQEFT [8]	1.67 ± 0.27	2.16 ± 0.36
NWA [58]+LFQM [12]	1.73 ± 0.07	2.24 ± 0.09
FK [13]	2.0	2.5
ISGW2 [59]	1.0	1.3



decays $B_s, D_s \to K_0^*(1430)\ell\nu_\ell$

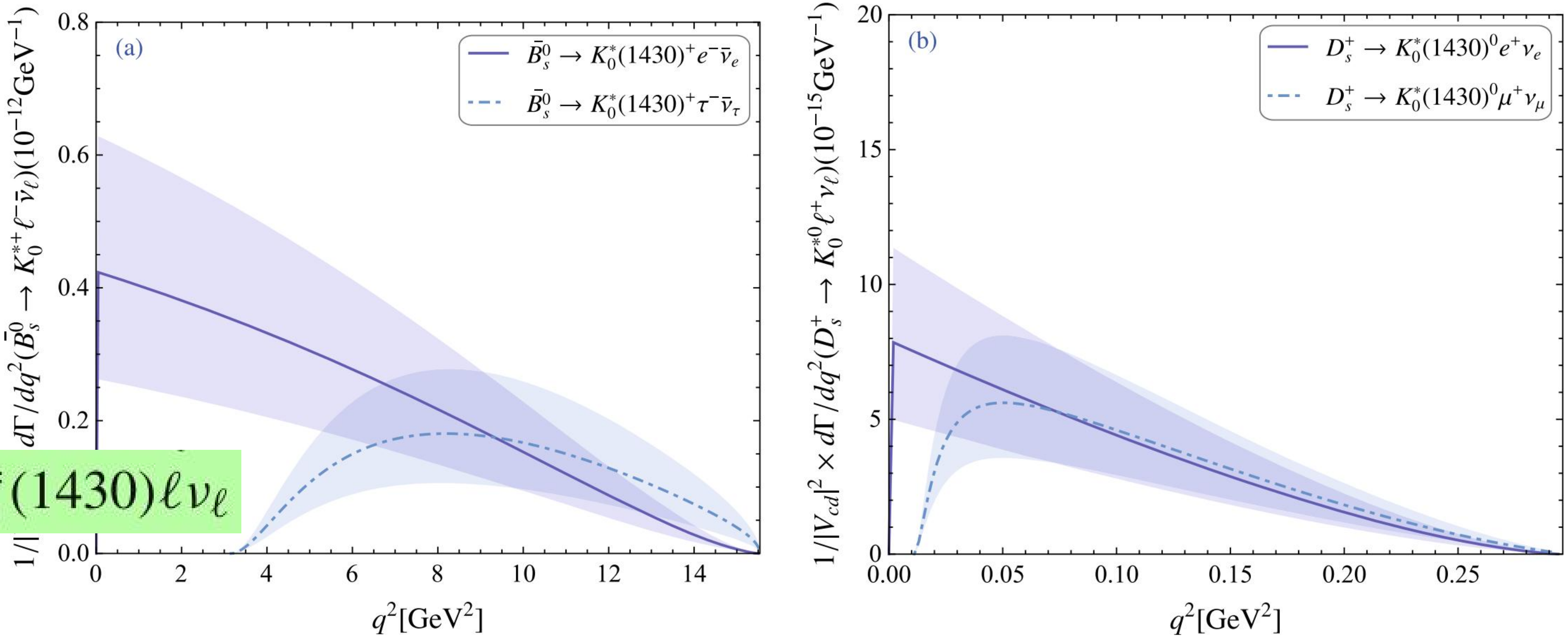
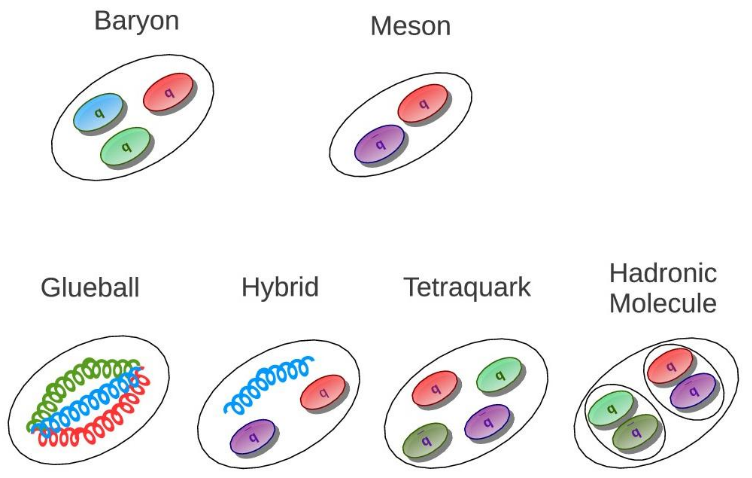


TABLE VIII: Branching fractions ($\times 10^4$) of the semileptonic decays $\overline{B}_s^0 \to K_0^*(1430)^+ \ell^- \bar{\nu}_\ell$ with $\ell = e, \mu$ and τ, respectvely.

	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ e^- \bar{\nu}_e)$	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ \mu^- \bar{\nu}_\mu)$	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ \tau^- \bar{\nu}_\tau)$
This work	$1.13^{+0.74}_{-0.51}$	$1.13^{+0.74}_{-0.51}$	$0.50^{+0.40}_{-0.25}$
Ref. [2]	$0.71^{+0.14}_{-0.14}$	$0.71^{+0.14}_{-0.14}$	$0.21^{+0.04}_{-0.04}$
Ref. [3]	$2.45^{+1.77}_{-1.05}$	$2.45^{+1.77}_{-1.05}$	$1.09^{+0.82}_{-0.47}$
Ref. [8](I)	–	$0.99^{+0.89}_{-0.37}$	$0.49^{+0.33}_{-0.17}$
Ref. [8](II)	–	$1.67^{+1.32}_{-0.53}$	$0.71^{+0.57}_{-0.26}$
Ref. [8](III)	–	$1.90^{+1.48}_{-0.63}$	$0.65^{+0.55}_{-0.24}$
Ref. [9]	$1.3^{+1.3}_{-0.4}$	$1.3^{+1.2}_{-0.4}$	$0.52^{+0.57}_{-0.18}$
Ref. [11]	$1.27^{+0.36}_{-0.36}$	$1.27^{+0.36}_{-0.36}$	$0.54^{+0.16}_{-0.16}$

4. Summary and Outlook



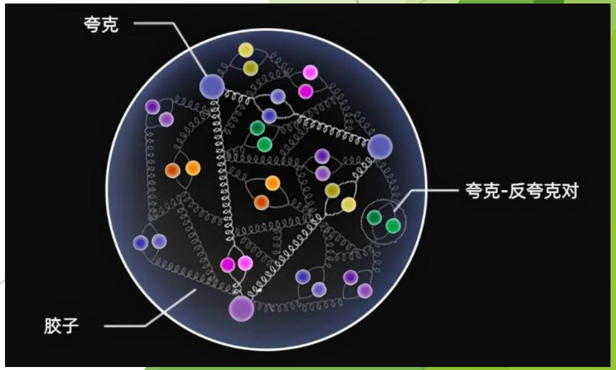
各种方法都有其优缺点，不轻易放弃，也不轻易相信
做到极致，或者有新天地

QCD求和规则可起重要作用，可用于更广阔情形，例如新强子态

为获得高精度结果，需要仔细考虑高维算符、微扰修正等效应

同时，求和规则精度仍然有限，需进一步改进，以期提高精度

- 考虑双Borel变换
- 深入讨论基态贡献、连续态贡献、激发态贡献
- 高维度算符贡献
- 高扭度分布振幅贡献
- 高阶微扰效应
- 基于价夸克模型，对于复杂情形如何做？



谢 谢

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