Towards a Hamiltonian First Principle Approach Basis Light-Front Quantization

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With

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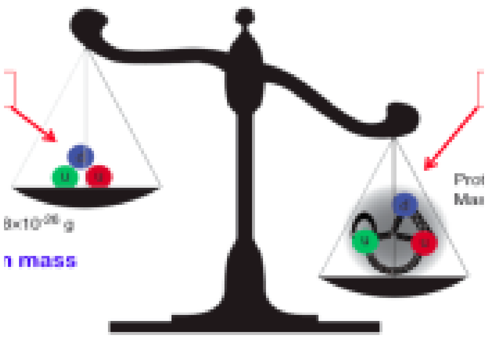
Outline

- **B**asis **L**ight-**F**ront **Q**uantization (**BLFQ**)
  - Light-front Quantization
  - QCD Light-front Hamiltonian
  - BLFQ Procedure
- Application to Proton
  - Form Factors (FFs)
  - Parton Distribution Functions (PDFs)
  - Generalized Parton Distribution Functions (GPDs
- Conclusion and Outlook

Major Questions in Nuclear Physics

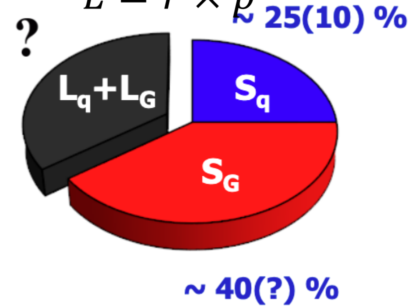
Origin of mass



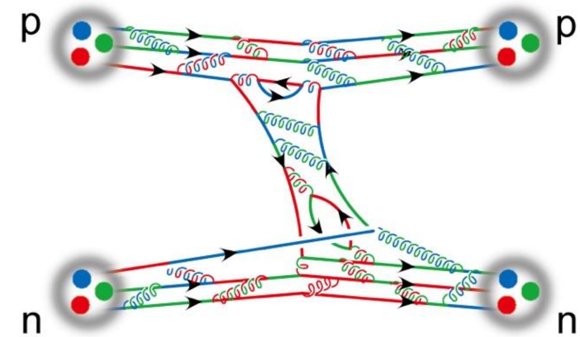
Spin puzzle

Ortibal angular momentum

$\vec{L} = \vec{r} \times \vec{p}$



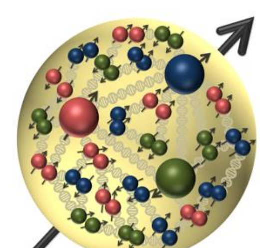
Nuclear force



We need to know the structure of nucleon beyond 1D and how it emerges from QCD

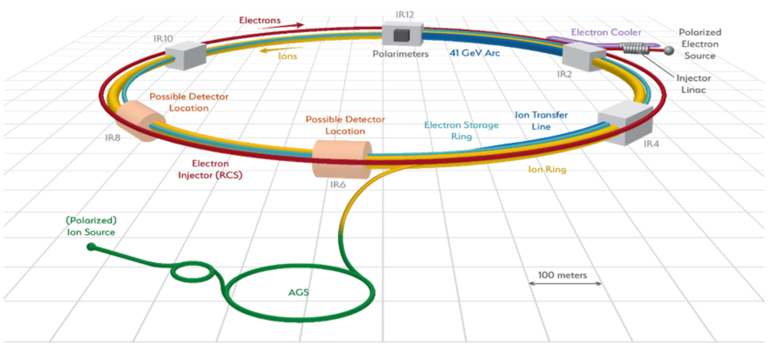
$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q (i\not{D} - m_q)\psi_q \right) - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha}$$

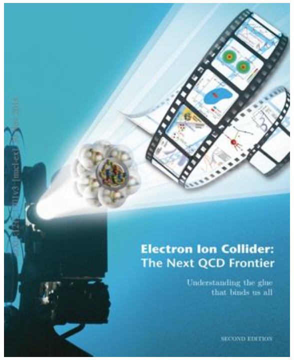
?

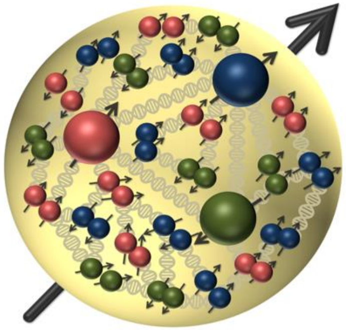


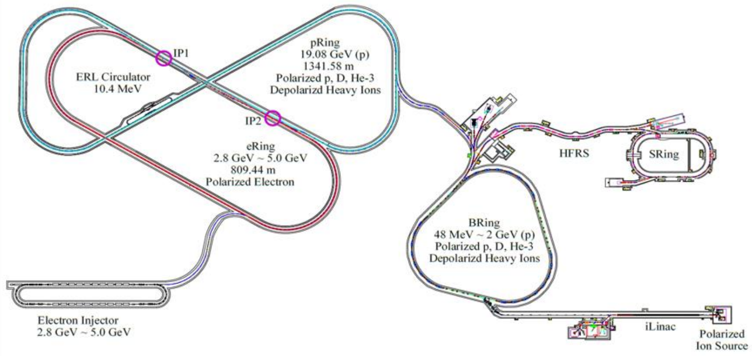
Electron–Ion Colliders

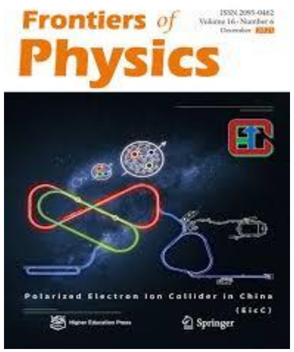
- Electron-Ion colliders with high collision energy and high luminosity











- EIC in the US is under construction by BNL@New York
- EicC in China is been planned by IMPCAS@Huizhou

Complimentarity

Nonperturbative Approach

- Stationary Schrödinger equation universally describes bound-state structure

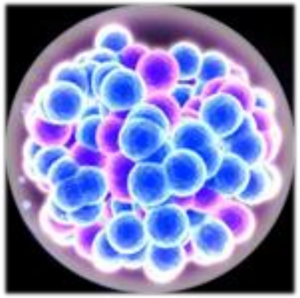
$$H|\psi\rangle = E|\psi\rangle$$

- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic

atom

Nonrelativistic



nucleus

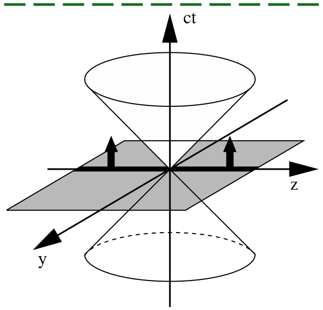
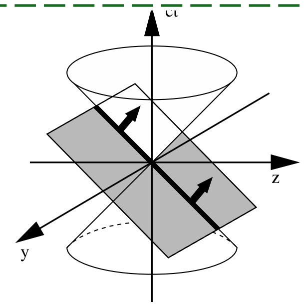
Relativistic

nucleon

- Major challenges from relativity: frame dependence, particle number not conserving…

Light-front Quantization

[Dirac, 1949]

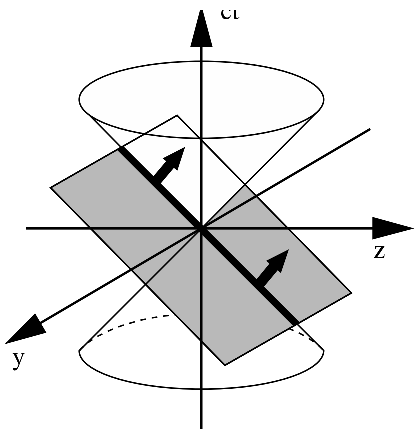
Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Main advantage:

- Frame-independent light-front wave functions

Light-Front Field Theory

➢ Light-Front Coordinates



Light-front variables:

$x^+ = x^0 + x^3$ (light-front time), $\quad x^- = x^0 - x^3, \quad x^\perp = x^{1,2}$

$P^- = P^0 - P^3$(light−front Hamiltonian), $\quad P^+ = P^0 + P^3, \quad P^\perp = P^{1,2}$

Dispersion relation: $P^- = \dfrac{m^2 + P_\wedge^2}{P^+}$ Light-cone gauge: $A^+ = 0$

➢ Eigenvalue equation:

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

- $\boldsymbol{P^-}$: Light-Front Hamiltonian
- $|\boldsymbol{\beta}\rangle$: Eigenstates
- $\boldsymbol{P^-_\beta}$: Eigenvalues for eigenstates

Light-front wave functions → Baryon Structure
Form Factors (FFs),Parton distribution functions (PDFs) ...

- Frame-independent light-front wave functions
- Observables are defined on the light-front
- Light-front wave functions carry parton interpretation

Basis Light-Front Quantization

- Hamiltonian eigenvalue equation: [Vary, et.al, Phys.Rev.C '10]

$$P^{-}|\beta\rangle = P^{-}_{\beta}|\beta\rangle$$

- $\boldsymbol{P^{-}}$: Light-Front Hamiltonian
- $|\boldsymbol{\beta}\rangle$: Eigenstates
- $\boldsymbol{P^{-}_{\beta}}$: Eigenvalues for eigenstates

- Basis setup:

Fock sector expansion: $|\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$

Single particle basis in $|qqq\rangle$: $|\alpha\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$

2-dimension harmonic oscillator	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \|m_i\| + 1) \leq N_{\max}$	$\sum_i k_i^+ = K_{\max}$	$\Lambda = \sum_i (\lambda_i + m_i)$

- Advantages:
  1. Rotational symmetry in transverse plane
  2. Exact factorization between center-of-mass motion and intrinsic motion
  3. Harmonic oscillator basis supplies correct infrared behavior for hadrons

Light-front Hamiltonian

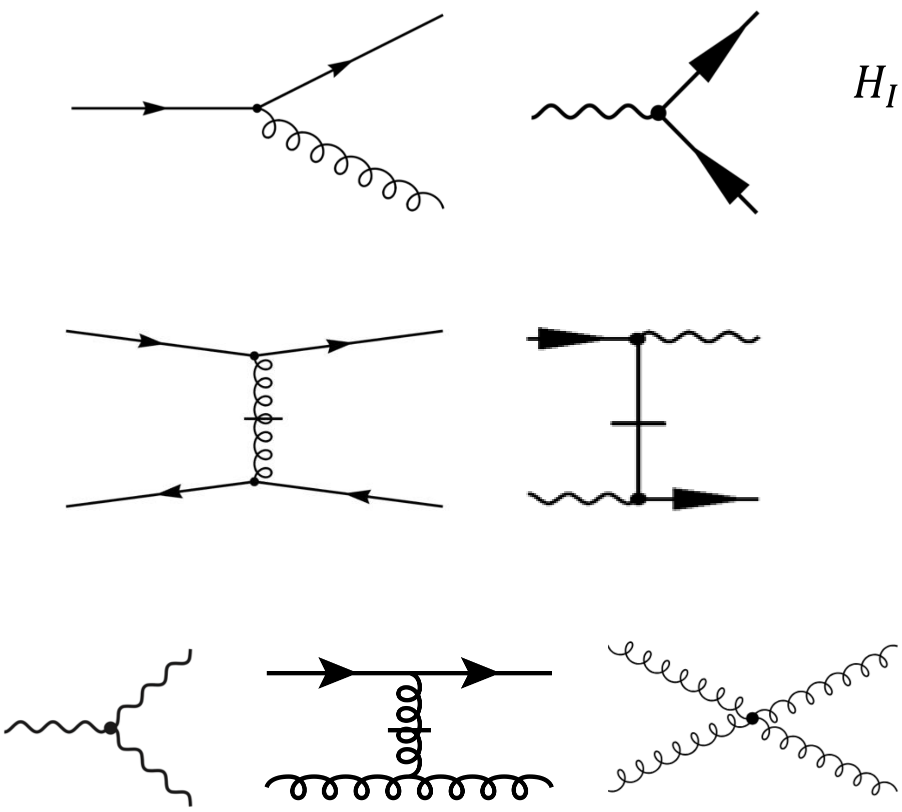
➢ QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} \quad \Longrightarrow \quad P^{-}_{QCD} = H_K + H_I$$

$A^+ = 0$

$$H_K = \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A^i{}_a$$

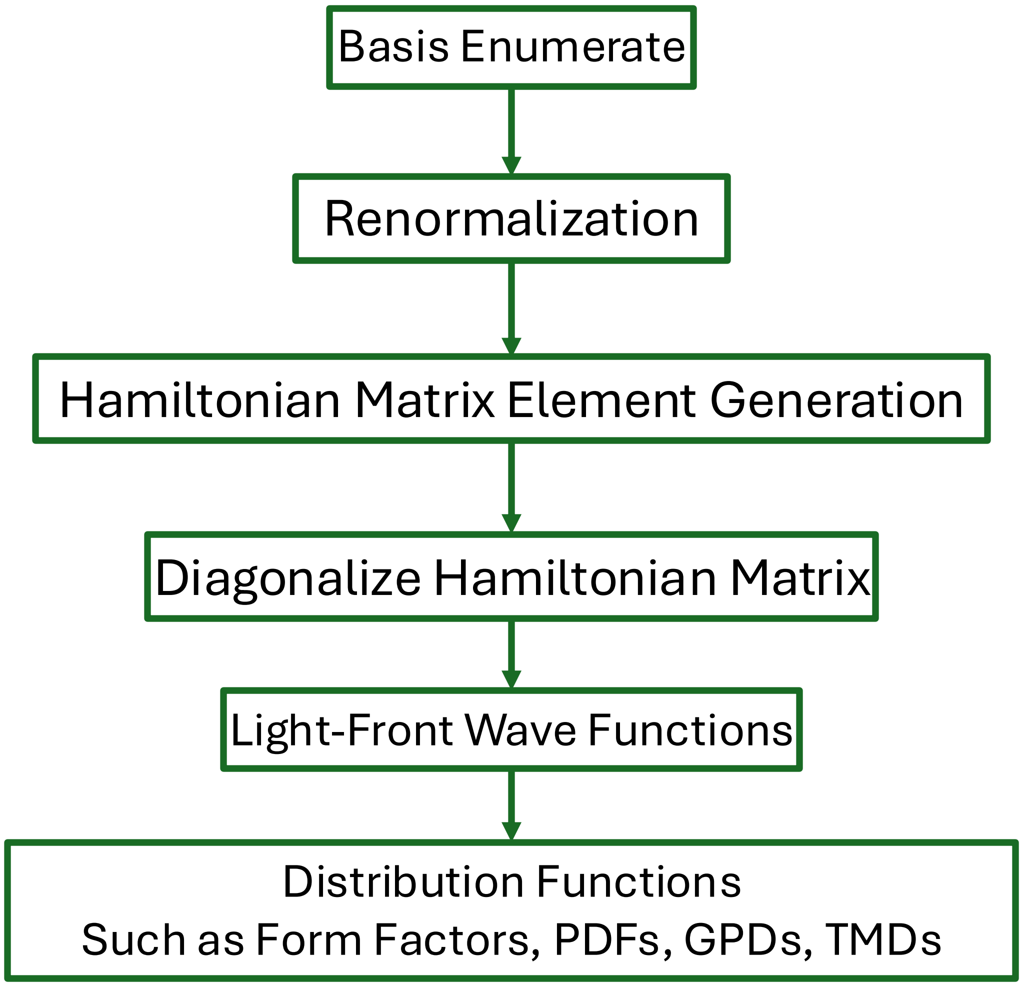
$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\
&- \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\
&+ \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&- \tfrac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c}\frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.
\end{aligned}$$



ψ: quark field operator

A^a_μ: gluon field operator

BLFQ Procedure



Dimension of Basis States

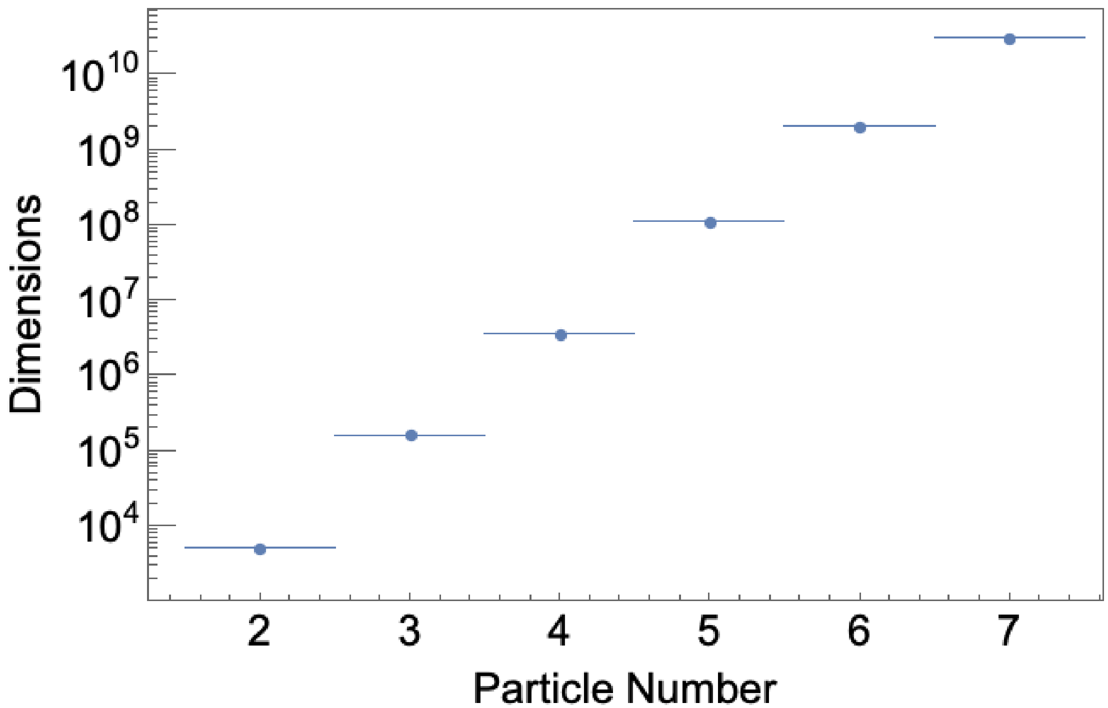
- Expansion in BLFQ basis

$$|\psi_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots$$

$$|\psi_{meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\, q\bar{q}\rangle \ + \cdots$$

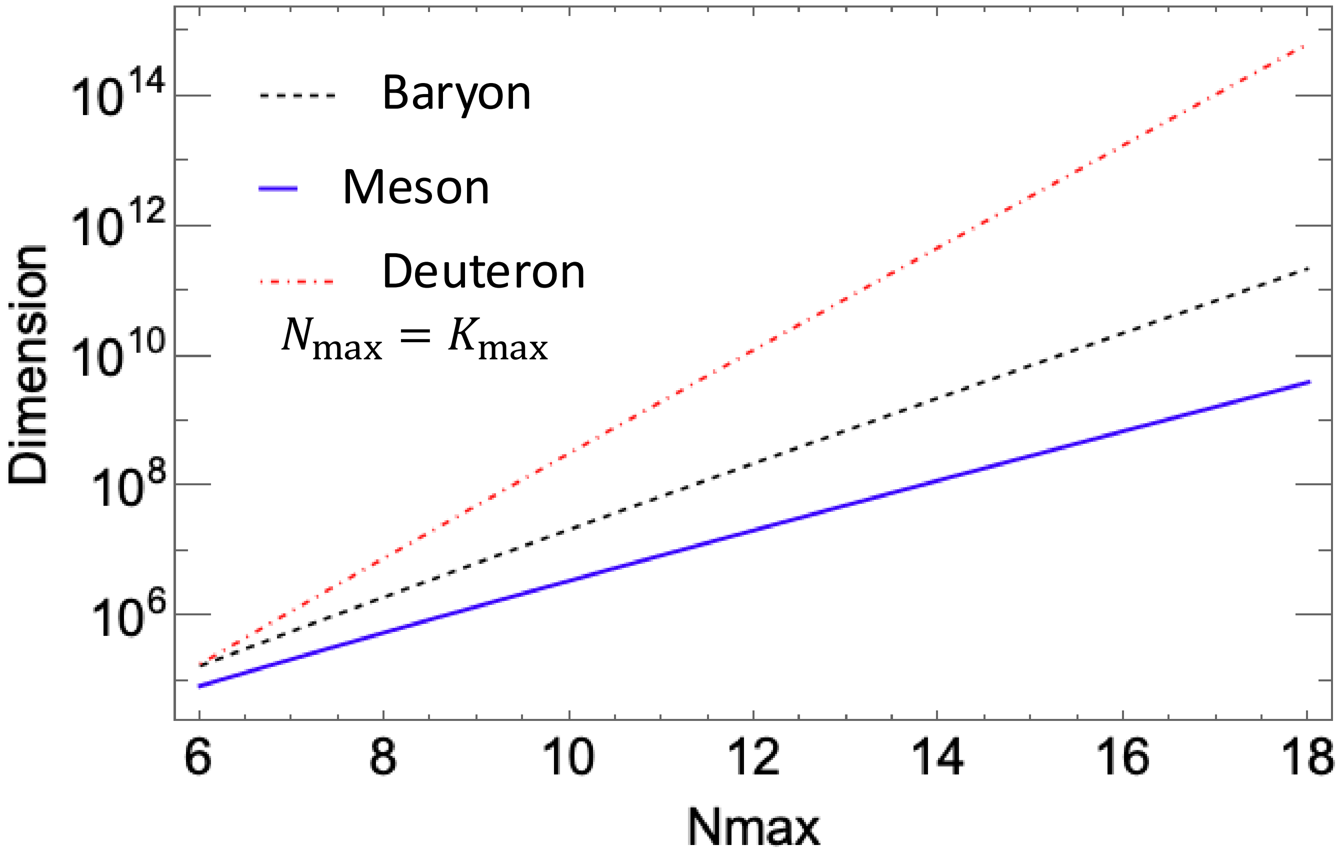
$$|\psi_{deuteron}\rangle = |qqq\, qqq\rangle + |qqq\, qqq\, g\rangle + \cdots$$

For the entire truncation parameters ($N_{\max} = 10$, $K_{\max}$=16), the dimension of the Basis states with different Fock sector expansion



Dimension of Basis States

- Expansion in BLFQ basis



Challenges: The basis size increases exponentially
Parallel computation & GPU acceleration needed

Works on Nucleon

$$|\mathrm{N}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle + \cdots$$

- **Wave Functions:**

  [PRD,102,016008] (2019) [PRD,108 9, 094002] (2023) [arXiv:2408.11298] (2024)

- **GPDs:**

  [PRD,104,094036] (2021) [PLB,847,138305] (2023)

  [PRD,105,094018] (2022) [PRD,110.056027] (2024)

  [PRD,109,014015] (2024) [arXiv:2408.09988] (2024)

  [PLB,855,138809] (2024)

- **TMDs:**

  [PLB,833,137360] (2022) [PLB,855 138831] (2024)

  [PRD,108,036009] (2023)

- **Higher-twist Distribution (GPD,TMD,DPD):**

  [PRD,109,034031] (2024) [PLB,855 138829] (2024)

  [arXiv:2410.11574] (2024)

- **Gravitational Form Factors:**

  [PRD,110,056027] (2024)

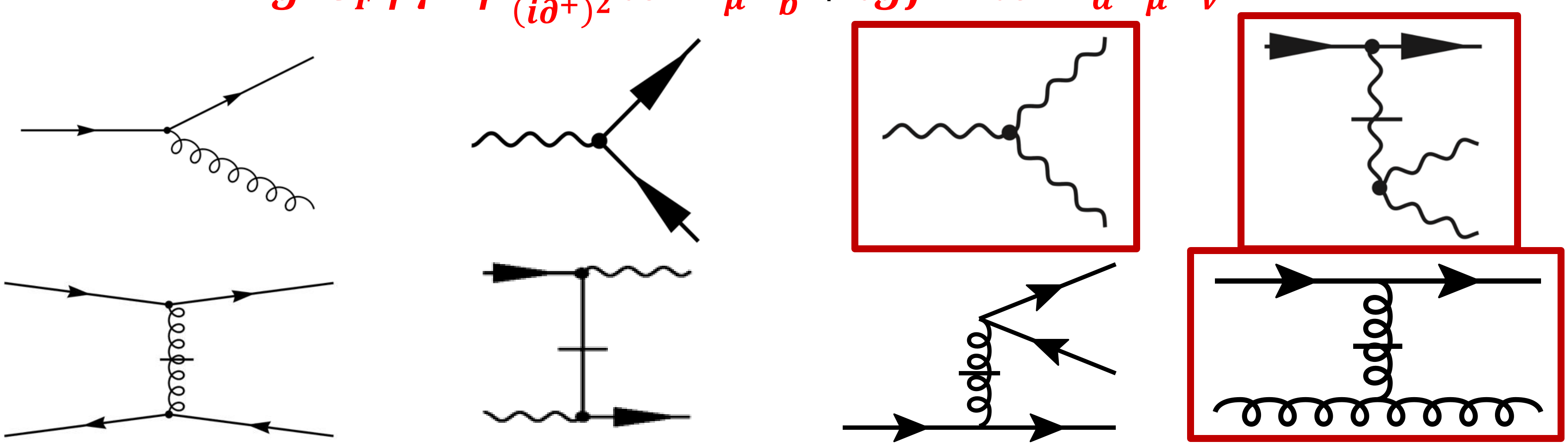
Full BLFQ

$$|N\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqgg\rangle$$

$$P^- = H_{K.E.} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} A_\nu \gamma^\nu \psi$$
$$- g^2 C_F \bar{\psi}\gamma^+\psi \frac{1}{(i\partial^+)^2} i\partial^+ A_\mu^a A_b^\mu + i g f^{abc} i\partial^\mu A_a^\nu A_\mu^b A_\nu^c$$



Fock Sector Decomposition

$$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqgg\rangle$$

$|qqq\ q\bar{q}\rangle \sim 3$ color singlet state

1 singlet ⊗ singlet

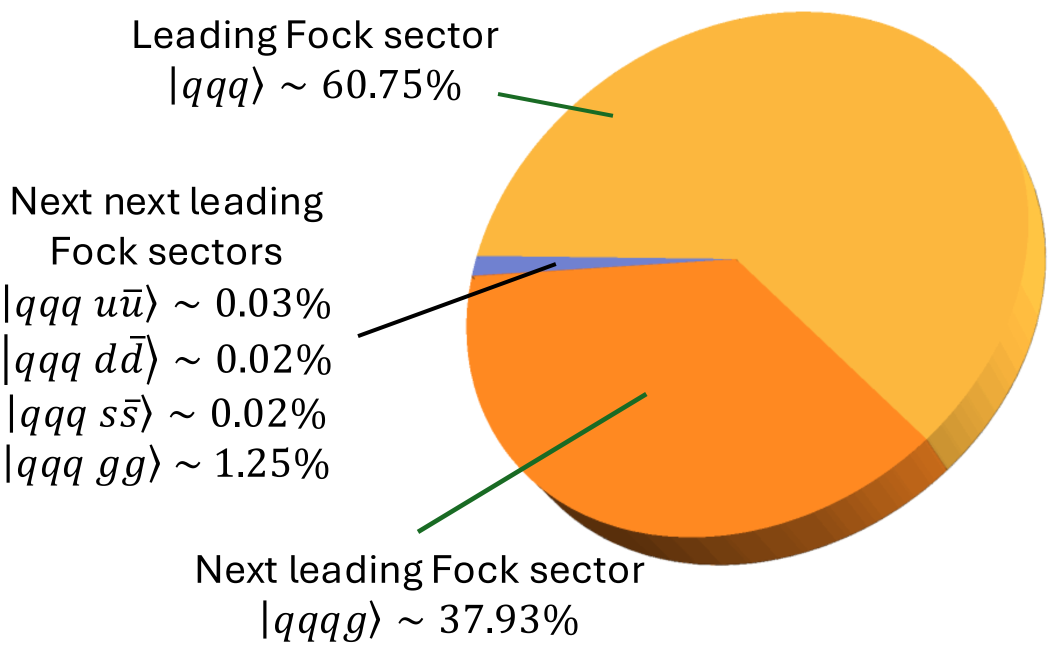
2 octet ⊗ octet

$|qqq\ gg\rangle \sim 6$ color singlet state

1 singlet ⊗ singlet

4 octet ⊗ octet

1 decuplet ⊗ octet ⊗ octet



m_u	m_d	m_s	m_f	g	b	b_{inst}
0.5 GeV	0.45 GeV	0.6 GeV	3.0 GeV	2.5	0.6 GeV	3.0 GeV

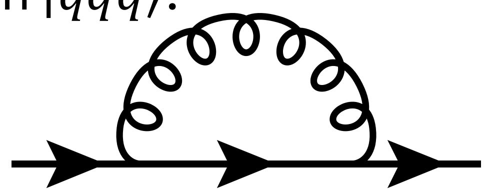
Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 10$

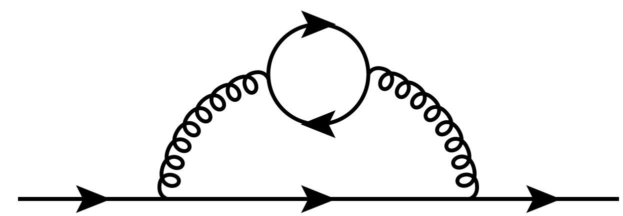
Renormalization Scheme

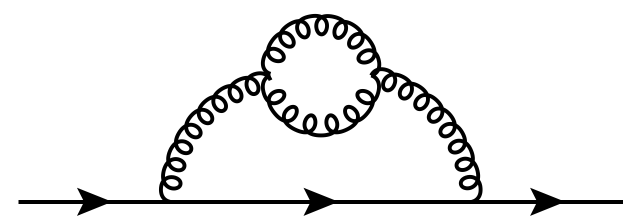
$$\left|P_{baryon}\right\rangle \rightarrow \left|qqq\right\rangle + \left|qqqg\right\rangle + \left|qqqu\bar{u}\right\rangle + \left|qqqd\bar{d}\right\rangle + \left|qqqs\bar{s}\right\rangle + \left|qqqgg\right\rangle$$

Single particle δm Particles in bound state δm On mass shell renormalization

- The mass counter term in $|qqq\rangle$:

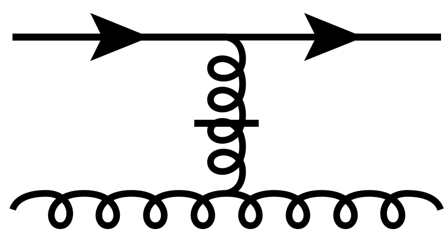


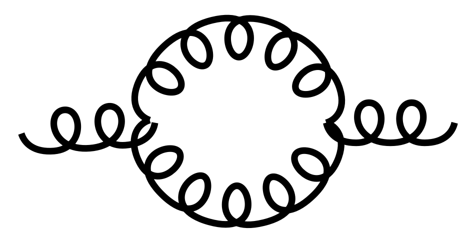




$$M^2 = m_q^2$$

For quarks

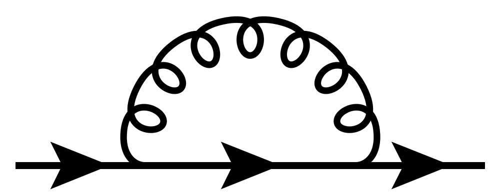


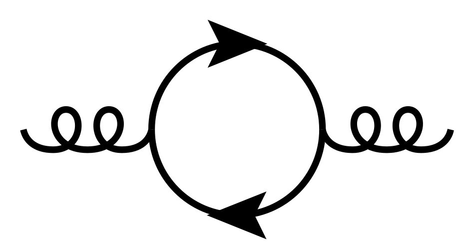


- The mass counter term in $|qqq\ g\rangle$:

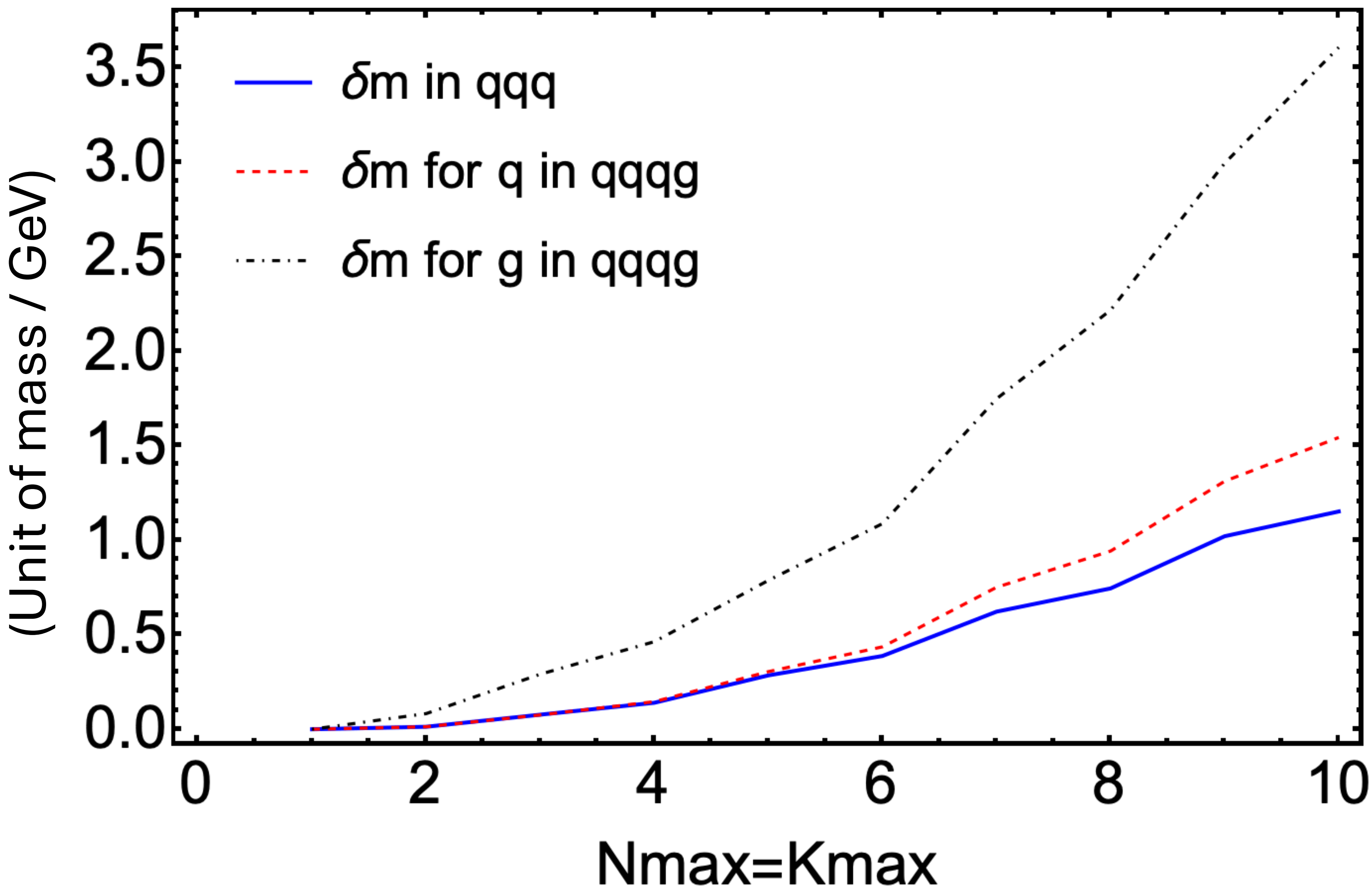
$M^2 = m_q^2$ For quarks

$M^2 = 0$ For gluon





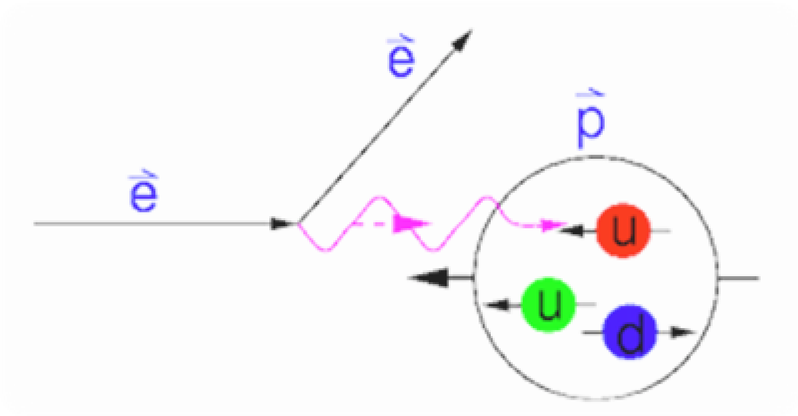
Renormalization Scheme



Including the higher Fock sector, the mass counter term of the valence was suppressed.

Electromagnetic Form Factor

- **Elastic scattering of proton**



[*R. Hofstadter, Nobel Prize 1961*]

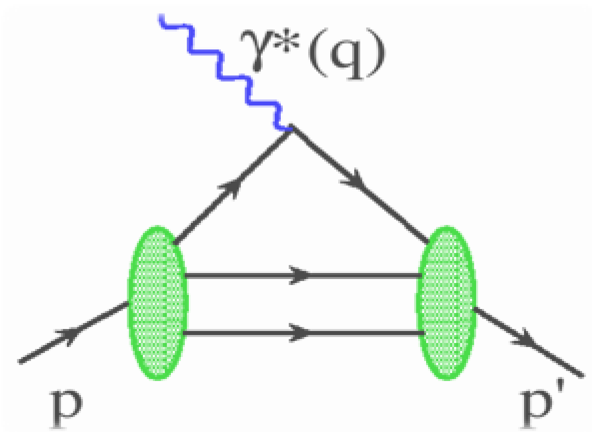
$$e(p) + h(P) \rightarrow e(p') + h(P')$$

- Elastic electron scattering established the extended nature of the proton (proton radius).

The Fourier transformation of these **form factors provide spatial distributions** (***charge and magnetization distributions***).

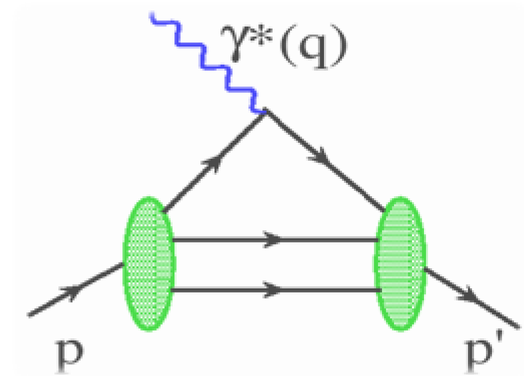
$$\langle N(p')|J^{\mu}(0)|N(p)\rangle =$$
$$\bar{u}(p')\left[\gamma^{\mu} \underbrace{F_1(q^2)}_{\text{Dirac Form Factor}} + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} \underbrace{F_2(q^2)}_{\text{Pauli Form Factor}}\right] u(p)$$

Dirac Form Factor   Pauli Form Factor

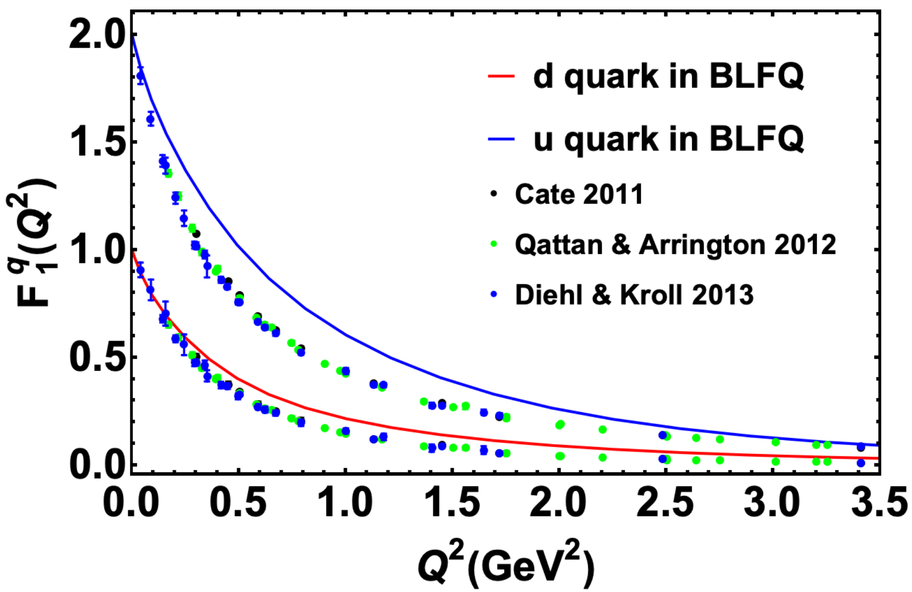


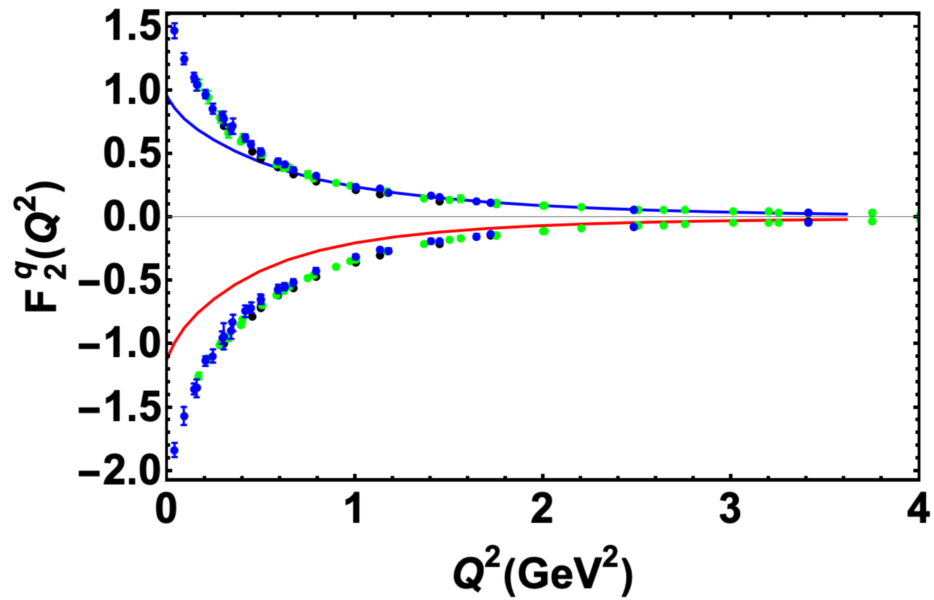
Nucleon Form Factor

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu} \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} \underbrace{F_2(q^2)}\right] u(p)$$



Preliminary results

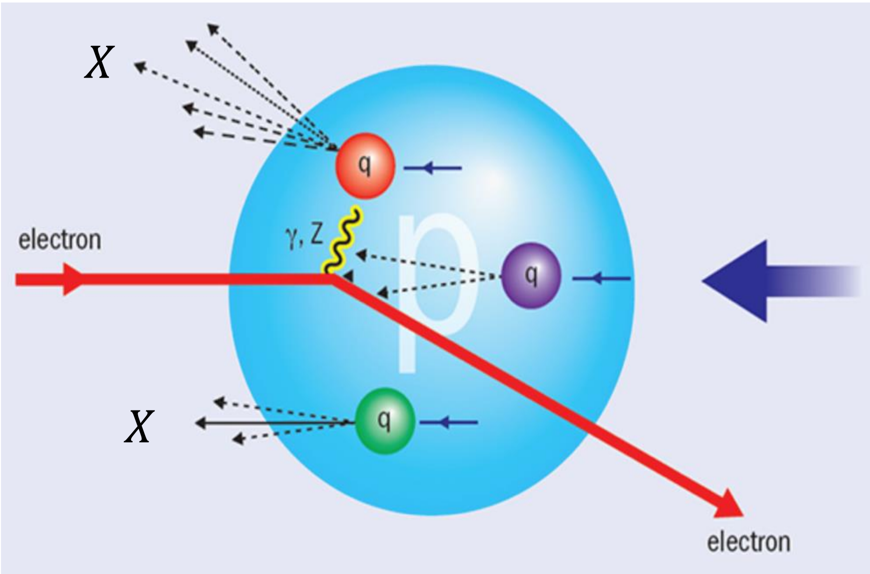




- BLFQ results qualitatively agree with the experimental data for Dirac and Pauli FFs

Parton Distribution Functions (PDF)

➢ **Deep Inelastic Scattering (SLAC 1968)**



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1\ \mathrm{fm}^{-2}$$

$$\Rightarrow \quad \frac{1}{Q} \ll 1\ \mathrm{fm}$$

Discovery of spin ½ quarks and partonic structure

➢ **Parton distribution functions** (PDFs) are extracted from **DIS** processes.

$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi}\, e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(x)\gamma^+\psi(0) | P, \Lambda \rangle$$

PDFs encode the *distribution of longitudinal momentum and polarization* carried by the constituents

Parton Distribution Function

- Parton distribution functions with five Fock sectors

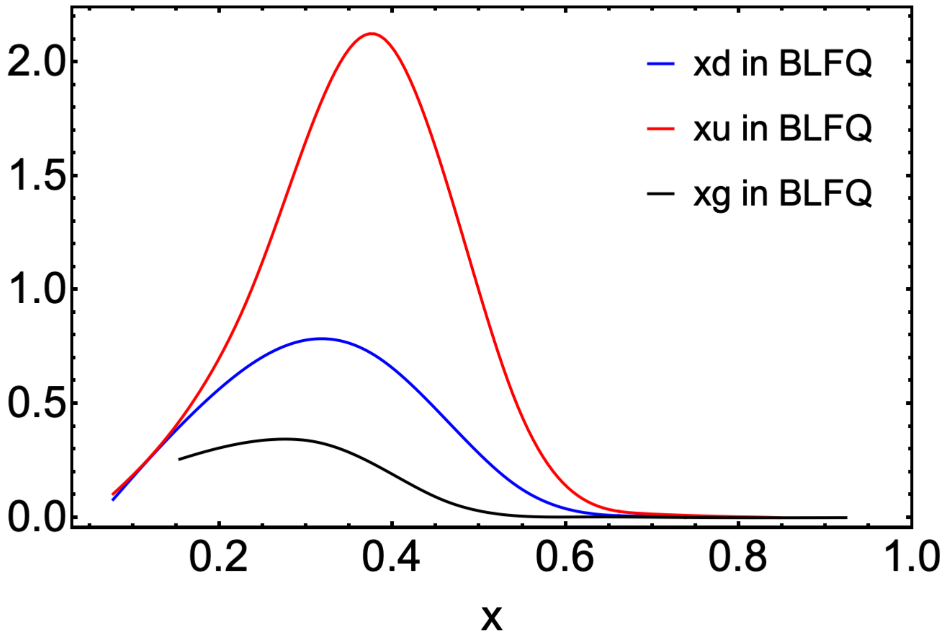
As we include $|qqqgg\rangle$ Fock sector, the endpoint behavior can be improved

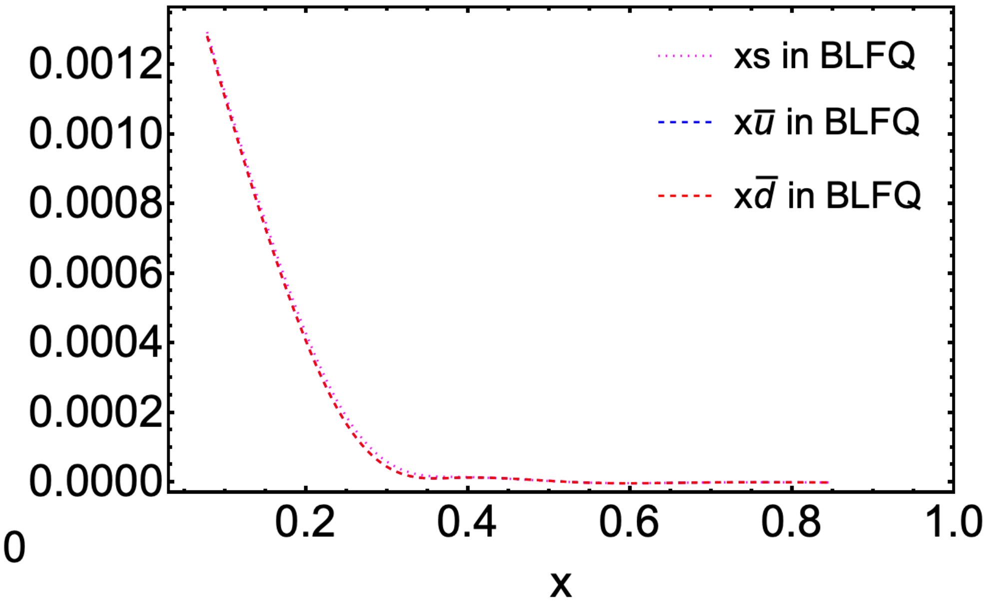
Due to Fock sector truncation (no $qqq\ q\bar{q}\ g, qqq\ ggg$), five-particle contribution too small

Our results qualitative agree with experimental results

Preliminary results

All results at the initial scale





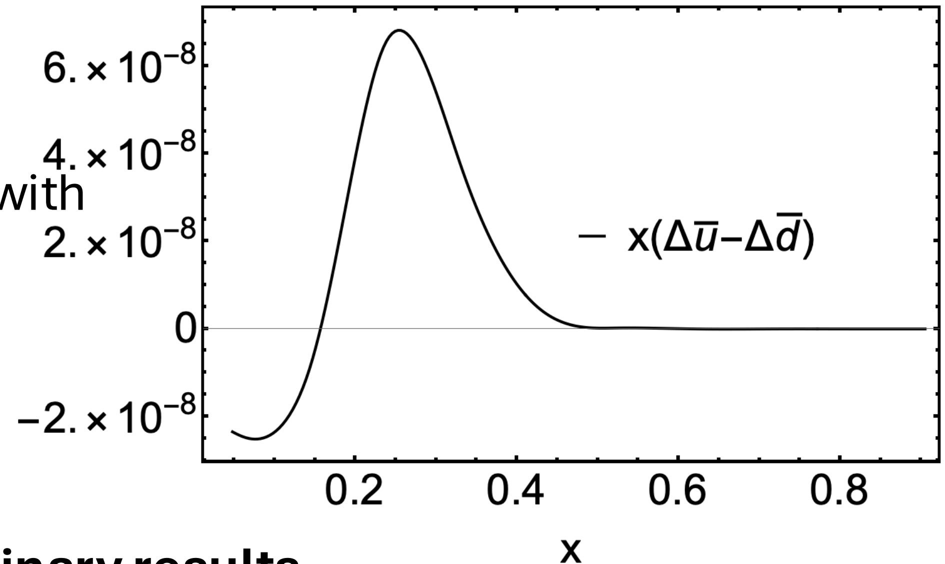
Helicity PDFs

- Helicity PDFs with five particle parton distribution

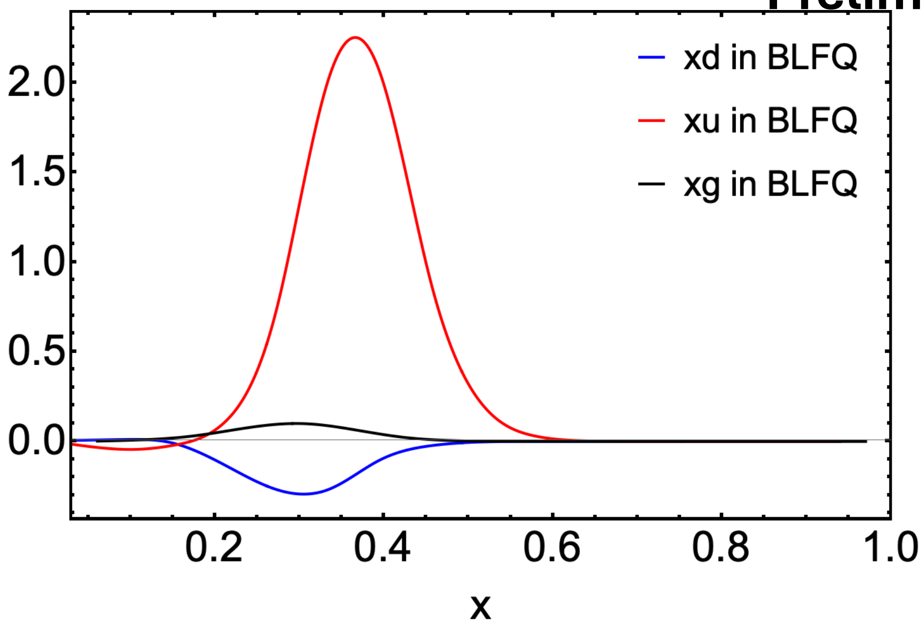
Sea asymmetry qualitatively agrees with JAM results

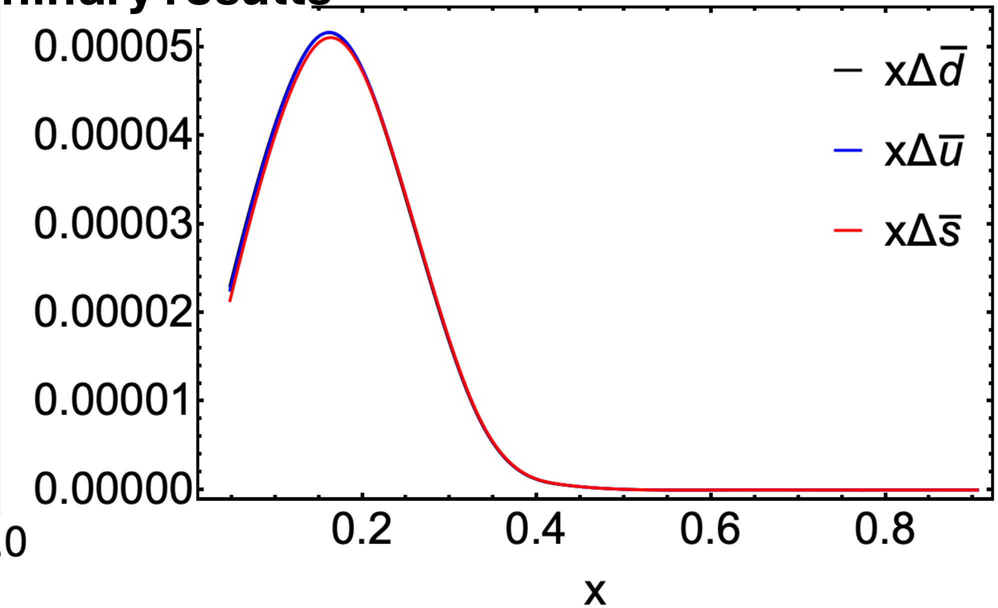
$\Delta\Sigma_u = 0.953 \qquad \Delta\Sigma = 0.743$

$\Delta\Sigma_d = -0.210 \quad \Delta G = 0.081$



Preliminary results





Transversity PDFs

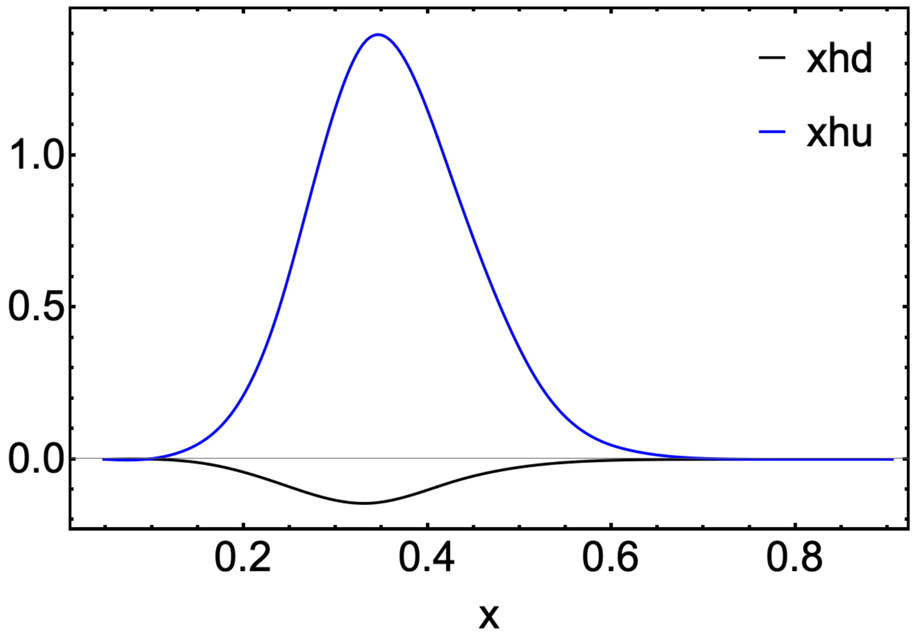
Transversity PDF of u and d has an opposite sign

According to current calculations, there is an asymmetry between $\bar{u}$, $\bar{d}$, and $\bar{s}$

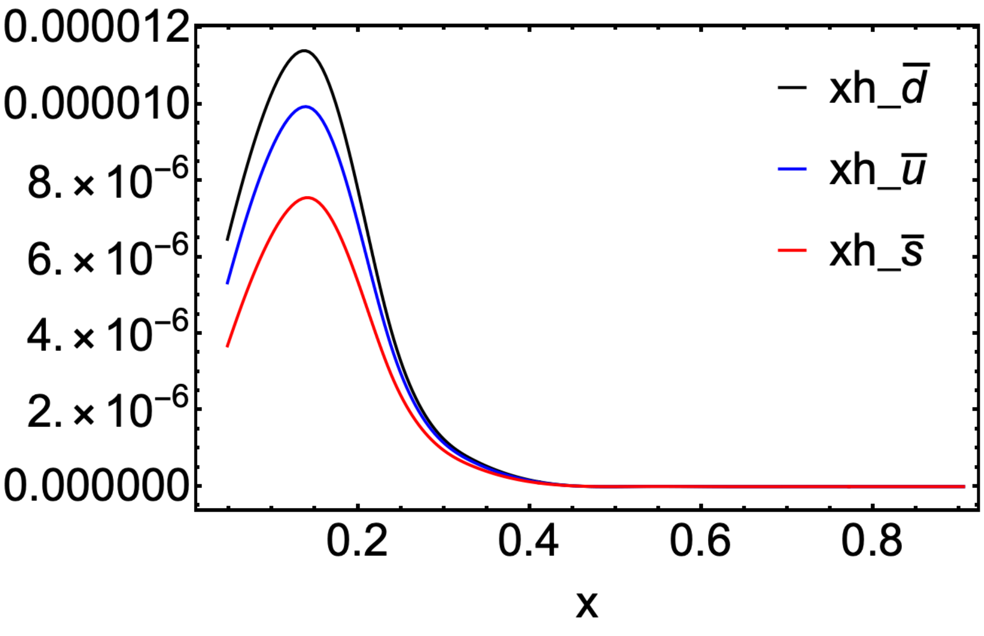
As we increase the truncation parameter, our results approach the experimental data.

Tensor Charge: $\delta u = 0.91$, $\delta d = -0.10$ At initial scale

Preliminary results

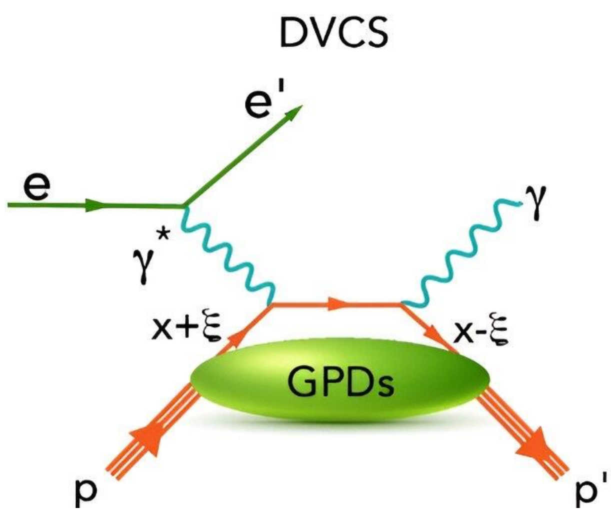


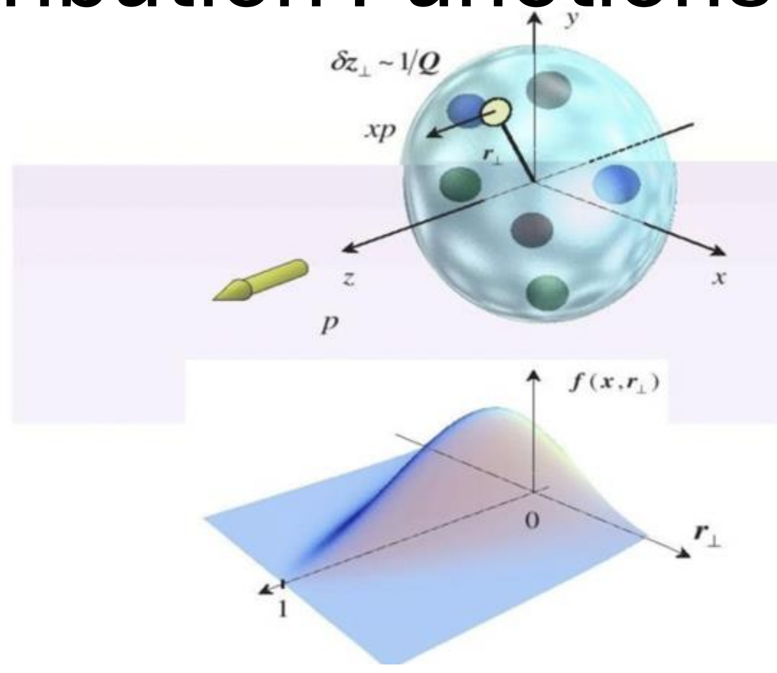
Preliminary results



Generalized Parton Distribution Functions

- Deeply Virtual Compton Scattering (DVCS)

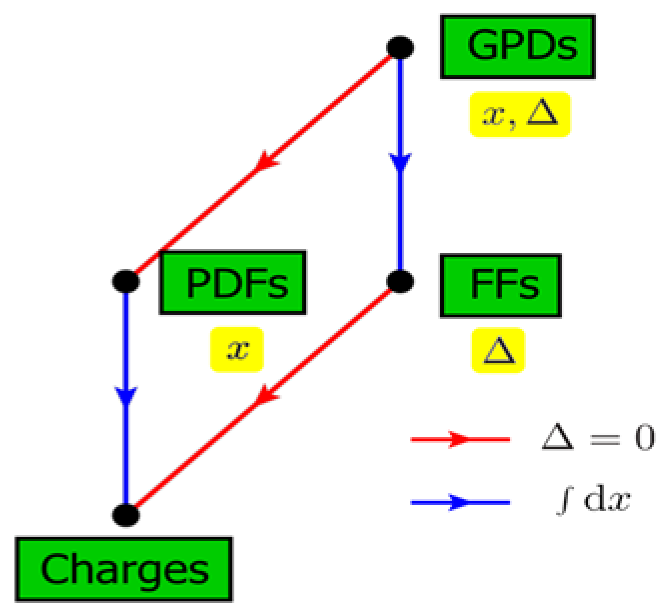




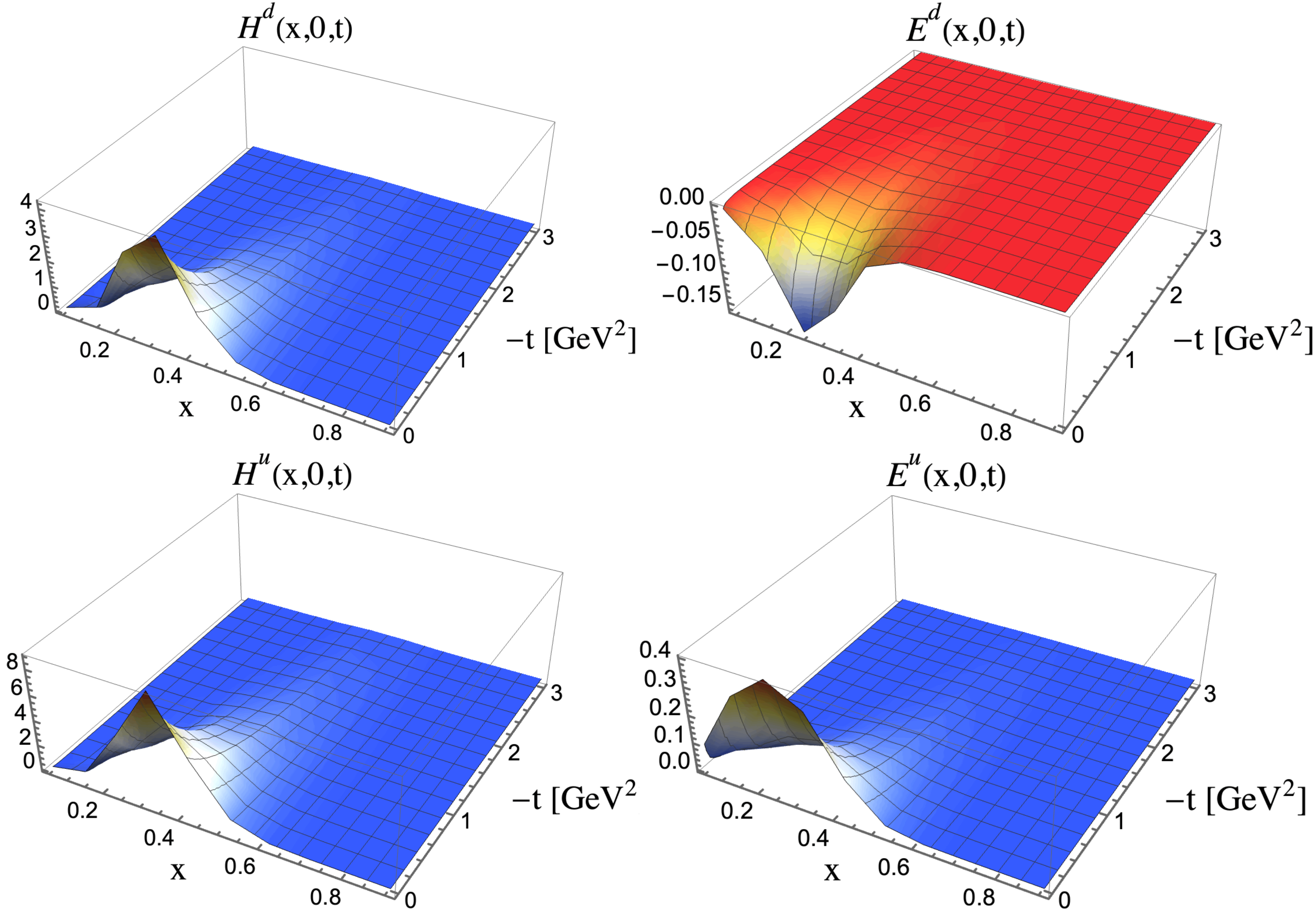
Encode the information about three dimensional spatial structure the spin and orbital angular momentum

With **increasing momentum transfer (t)**, the **peaks of distributions** shift to **larger x**;

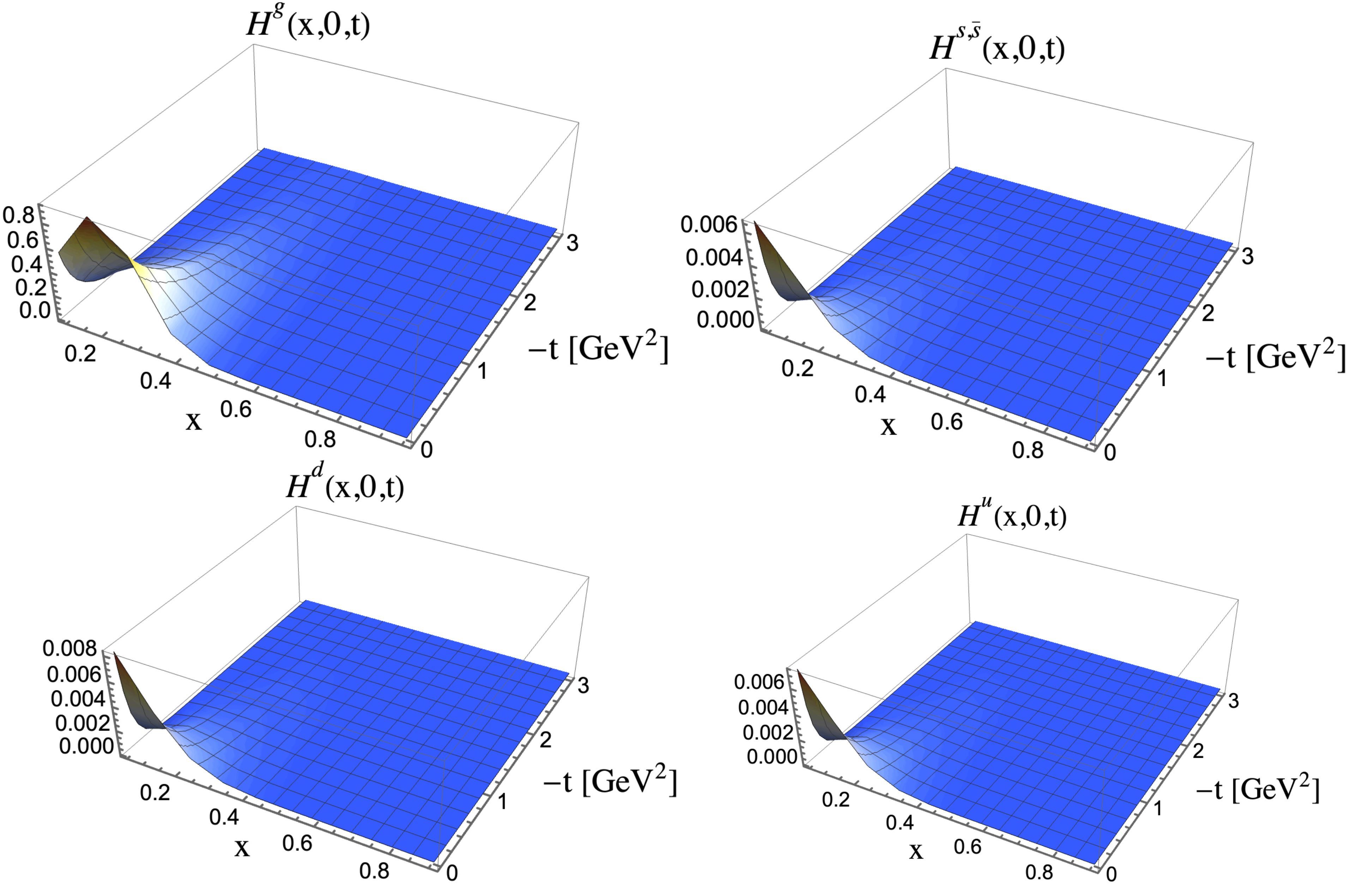
$$t = \Delta^2, x = \frac{k^+}{P^+}, \zeta = \frac{\Delta^+}{P^+} = 0 \qquad b_\perp \xrightarrow{FT} \Delta_\perp$$



Generalized Parton Distribution Function



Generalized Parton Distribution Function



Conclusions

- BLFQ: a non-perturbative approach based on QCD Light-front Hamiltonian
- $|qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle$ Fock sectors have been included
- Include all QCD interactions except four-gluon interactions
- Results qualitatively agree with global fitting
- Working towards a First Principal Approach

Outlook

Current status

Full QCD interaction

Fock sector expansion
$|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$……

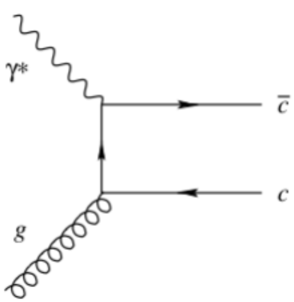
Deutron calculation
$|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$

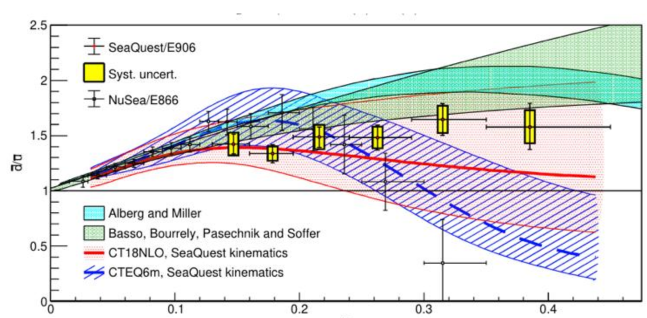
EMC effect

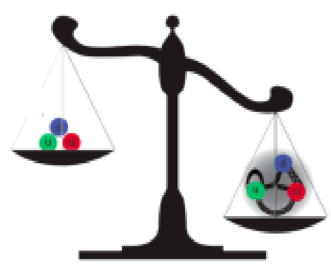
Intrinsic charm

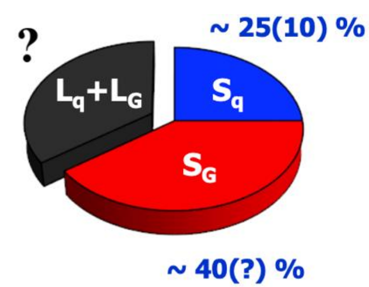
Sea asymmetry

Origin of spin and mass









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November 25-29, 2024
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» Hadron/nuclear structure
» Spin physics
» Relativistic many-body physics
» QCD phase structure
» Light-front field theory
» AdS/CFT and holography
» Nonperturbative QFT methods
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Thank you!
See you in
Huizhou☺