非微扰方法
及其在高能物理中的应用
专题研讨会

利用格点QCD研究ρ介子

Study ρ meson on Lattice QCD

Jia-Jun Wu(吴佳俊, 国科大)

合作者：**Zhengli Wanga(王政力), Derek B. Leinweber, Chuan Liu(刘川), Liuming Liu(刘柳明), Peng Sun(孙鹏), Anthony W. Thomas, Hanyang Xing(邢瀚洋), Kang Yu(余康)**

Paper is preparing …

中国科学技术大学　　2024. 10. 27　　合肥

Outline

- Motivation
- Lattice QCD introduction
- ρ meson spectra on LQCD
- Summary

Motivation

Resonance

(Mass , Width, Pole position, Coupling)

Partial Wave Analysis
With various Model:
Such as Breit-Wigner Form, Flatte Form, Coupled Channel Form, and so on

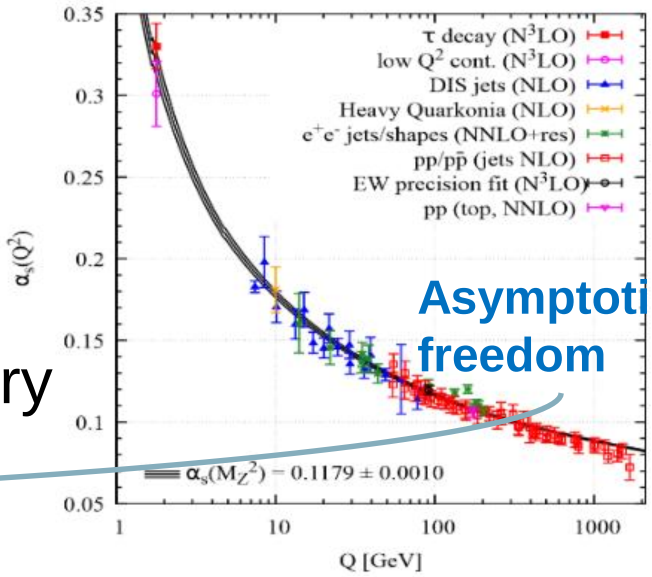
Confinement

?

Non-perturbative

Experimental Observable

QCD theory



Asymptotic freedom

Perturbative

Motivation: Spectrum & Scattering

- Spectrum: Table of the energy of the physical states, can reflect the inner structure information.
- Scattering: The line shapes of Cross section or T-matrix show the properties of the states and interaction.

- These observables can constrain the models.

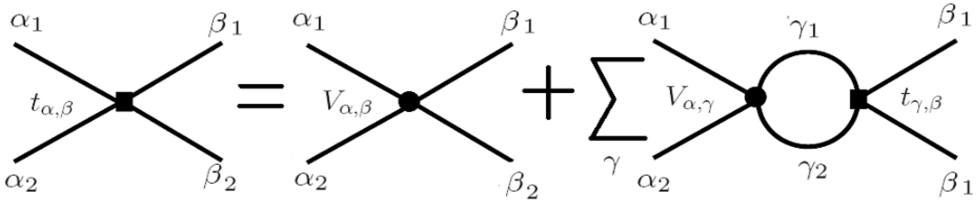
Powerful Parameters, Powerful Model

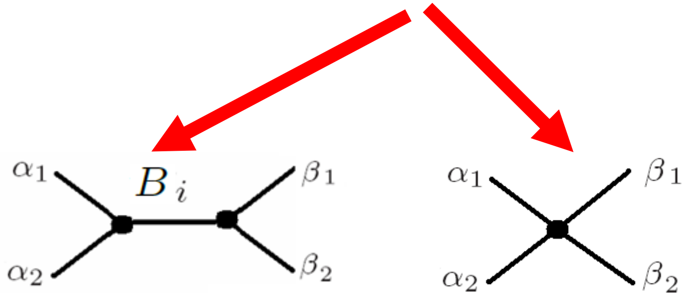
Motivation: Scattering

Lattice Spectrum !!!

The dimension of m_π

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$g_{i,\alpha}^*\dfrac{1}{E-m_i}g_{i,\beta}$ $\qquad$ $V_{\alpha,\beta}$

- $T^{-1}(E)\sim E-m_i-\Sigma(\mathrm{E})$
  $= E-m_i-\mathrm{Re}(\Sigma(\mathrm{E}))-\mathrm{iIm}(\Sigma(\mathrm{E}))$
- $T^{-1}(E)\sim E-m_i-\mathrm{Re}(\Sigma(\mathrm{E}))+i\Gamma(E)/2$
- $m_{phys}=m_i+\mathrm{Re}(\Sigma(\mathrm{E}))$
  $=m_0+am_\pi^2+\cdots+\mathrm{Re}(\Sigma(\mathrm{E}))$

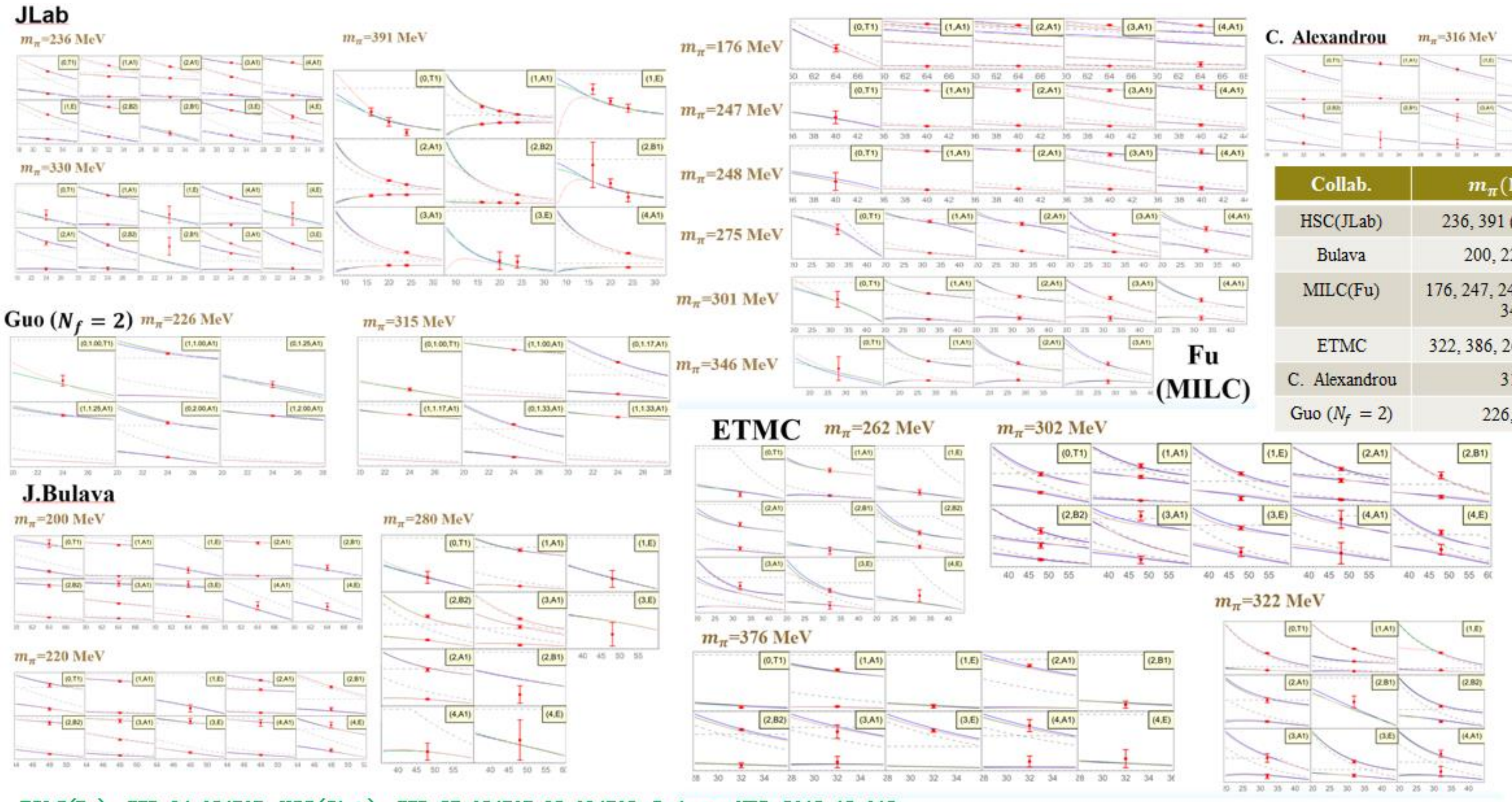
$$\Sigma_{ij}(\mathrm{E})=\sum_\alpha\int k^2dk\frac{g_{i,\alpha}^*G_{i,\alpha}}{E-\sqrt{m_{\alpha1}^2+k^2}-\sqrt{m_{\alpha2}^2+k^2}+i\epsilon}$$

Λ

$$\mathrm{G}_{i,\alpha}=g_{i,\alpha}+\sum_\beta\int\frac{k^2dk\tilde{t}_{\alpha\beta}g_{j,\beta}}{E-\sqrt{m_{\beta1}^2+k^2}-\sqrt{m_{\beta2}^2+k^2}+i\epsilon}\qquad \tilde{t}_{\alpha\beta}=v_{\alpha\beta}+\sum_\gamma\int\frac{k_\gamma^2dk_\gamma\,v_{\alpha\gamma}\tilde{t}_{\gamma\beta}}{E-\sqrt{m_{\gamma1}^2+k_\gamma{}^2}-\sqrt{m_{\gamma2}^2+k_\gamma{}^2}+i\epsilon}$$

Motivation: π mass dependence

- **The ρ is the best one!**
- **The largest lattice data of ρ.**
- **The lowest state can decay through strong interaction.**
- **We almost clearly understand ρ.**



Collab.	m_π(MeV)
HSC(JLab)	236, 391 (330, 283)
Bulava	200, 220, 280
MILC(Fu)	176, 247, 248, 275, 301, 346
ETMC	322, 386, 262, 302, 376
C. Alexandrou	316
Guo (N_f = 2)	226, 315

MILC(Fu): PRD.94.034505; HSC(Jlab): PRD.87.034505, 92.094502; Bulava: NPB.2018.12.018;
ETMC: EPJA56,61; Guo: PRD.94.034501; C.Alex:PRD.96.034525

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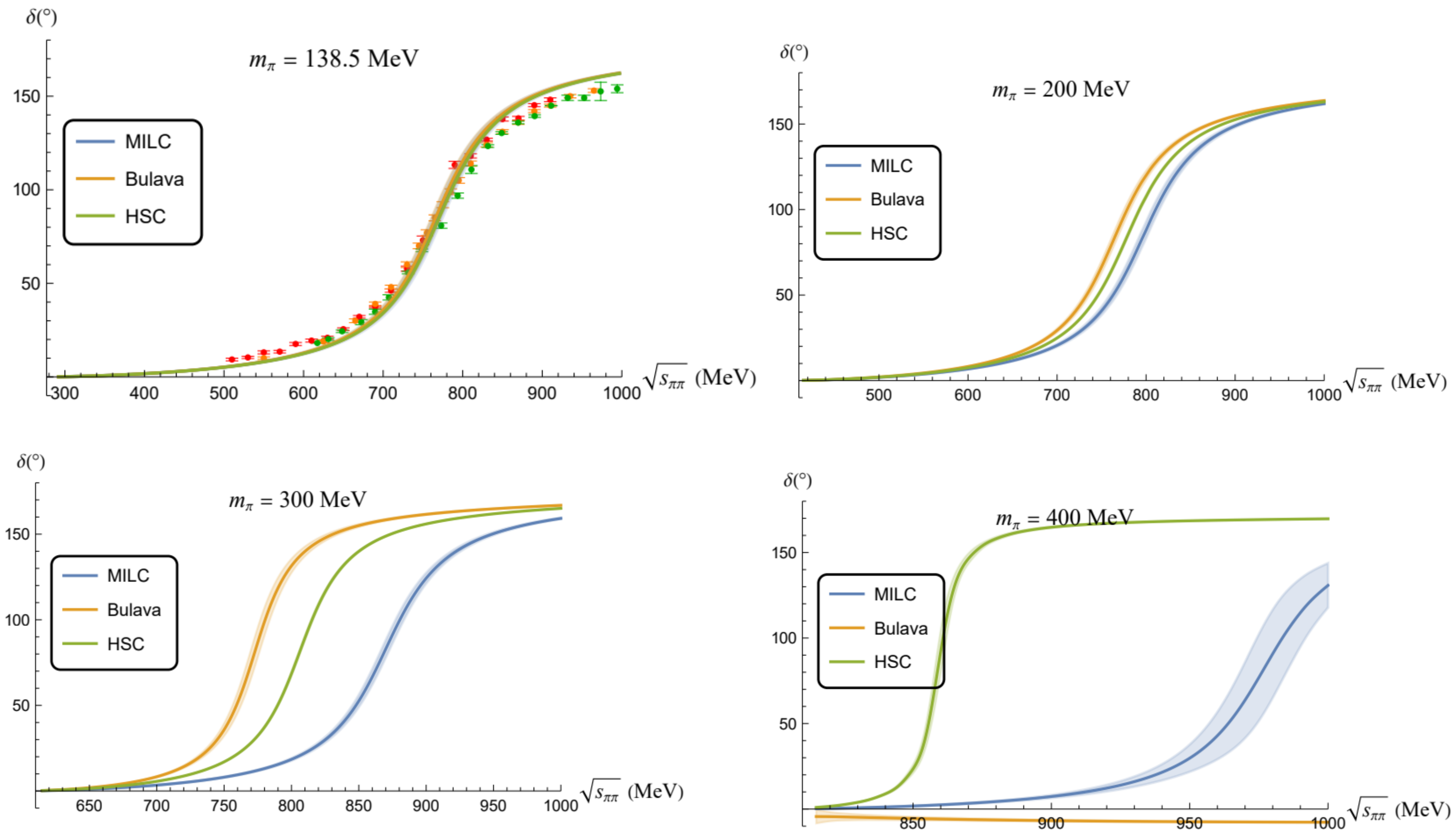


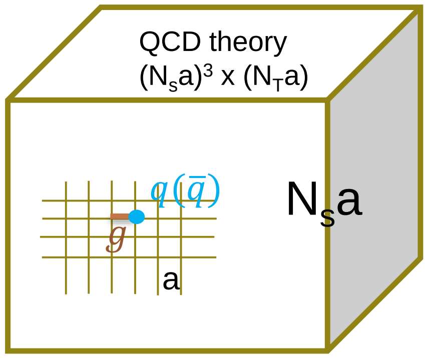
However

Lattice spacing effect,
Fermion action effect…

We need a systematic Lattice data group to study ρ !

Lattice QCD introduction

1. QCD theory: on a box in the Euclid four space



2. a -> UV cutoff, $N_s a$ -> Infrared truncation

3. Lattice QCD -> a model of statistical physics.

$$\langle O\rangle = \int D\phi O[\phi]P[\phi] \qquad P[\phi] = \frac{1}{Z}e^{-S[\phi]} \qquad Z = \int D\phi e^{-S[\phi]}$$

Φ: field quantity, *S*[*ϕ*]: Action, *O*[*ϕ*]:physical quantity

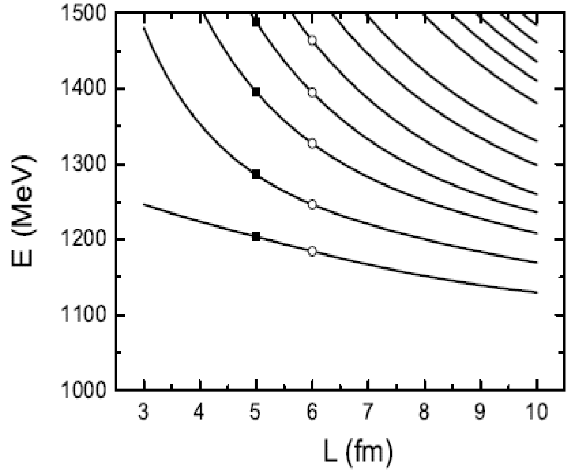
4. Monte Carlo method

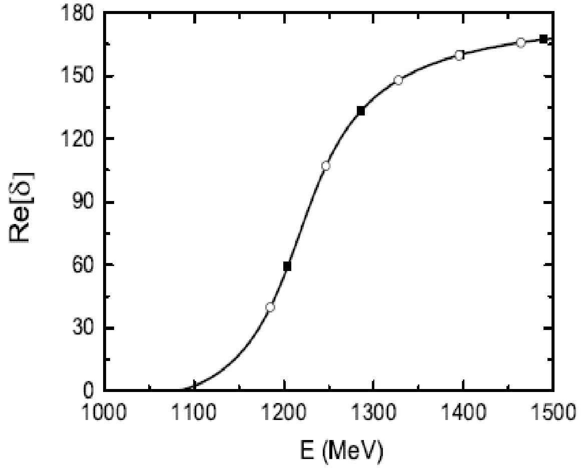
5. Three steps for Lattice QCD to real world

a, Configuration

b, Measurement $\sum_{(\vec{y}-\vec{x})\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})}\left\langle T\left(\psi(t;\vec{y}),\psi^\dagger(t;\vec{y})\right)\right\rangle \sim \sum_{\Gamma,i} Z_i^\Gamma e^{-E_i^\Gamma t}$

c, Transformation





L —— E —— $\delta_{\pi\pi}(E)$

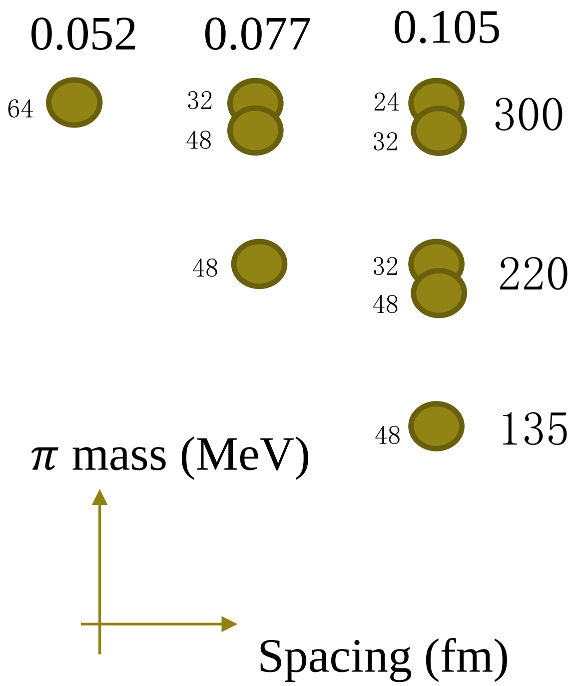
$\delta(k) = \Delta(L) \bmod \pi$

M. Luscher, NPB 354, 531 (1991).

Amplitude to Resonance information

ρ meson spectra on LQCD -- Configuration

Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C24P34	$24^3 \times 64$	$0.10530 fm$	6.20	340	4.38
C24P29	$24^3 \times 72$			292	3.75
C32P29	$32^3 \times 64$			292	5.01
C32P23	$32^3 \times 64$			228	3.91
C48P23	$48^3 \times 96$			225	5.79
C48P14	$48^3 \times 96$			135	3.56
F32P30	$32^3 \times 96$	$0.07746 fm$	6.41	303	3.81
F48P30	$48^3 \times 96$			303	5.72
F32P21	$32^3 \times 64$			210	2.67
F48P21	$48^3 \times 96$			207	3.91
H48P32	$48^3 \times 144$	$0.05187 fm$	6.72	321	4.06
H64P32	$64^3 \times 128$			321	5.41



CLQCD, PRD109 (2024) 5, 054507

ρ meson spectra on LQCD -- Operator

$[000]T_1^-$	$[001]A_1$	$[001]E_2$	$[011]A_1$
$\rho[000]$	$\rho[001]$	$\rho[001]$	$\rho[011]$
$\pi[001]\pi[00{-1}]$	$\pi[000]\pi[001]$	$\pi[0{-1}0]\pi[011]$	$\pi[000]\pi[011]$
$\pi[011]\pi[0{-1}{-1}]$	$\pi[0{-1}0]\pi[011]$	$\pi[0{-1}{-1}]\pi[111]$	$\pi[{-1}00]\pi[111]$
$\pi[111]\pi[{-1}{-1}{-1}]$	$\pi[{-1}{-1}0]\pi[111]$		$\pi[01{-1}]\pi[002]$
	$\pi[00{-1}]\pi[002]$		

$[011]B_1$	$[011]B_2$	$[111]A_1$	$[002]A_1$
$\rho[011]$	$\rho[011]$	$\rho[111]$	$\rho[002]$
$\pi[010]\pi[001]$	$\pi[{-1}00]\pi[111]$	$\pi[000]\pi[111]$	$\pi[000]\pi[002]$
$\pi[110]\pi[{-1}01]$	$\pi[110]\pi[{-1}01]$	$\pi[100]\pi[011]$	
$\pi[0{-1}1]\pi[002]$		$\pi[200]\pi[{-1}11]$	

$$\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}\Gamma^P u - \bar{d}\Gamma^P d)$$

$$\rho^{0\dagger} = \frac{1}{\sqrt{2}}(\bar{u}\bar{\Gamma}^P u - \bar{d}\bar{\Gamma}^P d)$$

$$\pi^+ = \bar{d}\Gamma^k u, \quad \pi^- = \bar{u}\Gamma^k d$$

$$\pi\pi = \frac{1}{\sqrt{2}}(\pi^+(k_1)\pi^-(k_2) - \pi^-(k_1)\pi^+(k_2))$$

$$= \frac{1}{\sqrt{2}}(\bar{d}\Gamma^{k_1}u\bar{u}\Gamma^{k_2}d - \bar{u}\Gamma^{k_1}d\bar{d}\Gamma^{k_2}u)$$

$$(\pi\pi)^\dagger = \frac{1}{\sqrt{2}}(\bar{d}\bar{\Gamma}^{k_2}u\bar{u}\bar{\Gamma}^{k_1}d - \bar{u}\bar{\Gamma}^{k_2}d\bar{d}\bar{\Gamma}^{k_1}u)$$

ρ meson spectra on LQCD -- correlation function

$$C_{ij} = \left\langle 0 \left| \mathcal{O}_i \mathcal{O}_j^\dagger \right| 0 \right\rangle = \sum_n Z_i^n Z_j^{n^*} e^{-E_n t}$$

Computed based on wick contraction and quark propagator calculated based on the configuration.

To parameterize the correlation function !

Energy level in the finite volume

- **Solve the generalized eigenvalue problem(GEVP)**

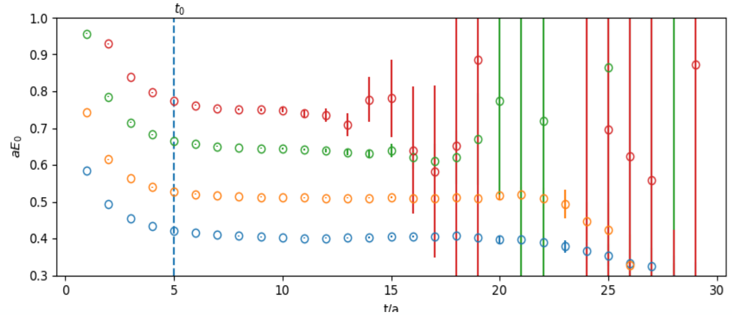
$$\mathcal{G}_{ij}(t_0 + dt) u_j^\alpha = \mathrm{e}^{-m_\alpha dt} \mathcal{G}_{ij}(t_0) u_j^\alpha,$$

$$[\mathcal{G}^{-1}(t_0)\mathcal{G}(t_0 + dt)]_{ij} u_j^\alpha = c^\alpha u_i^\alpha$$

$$v_i^\alpha [\mathcal{G}(t_0 + dt)\mathcal{G}^{-1}(t_0)]_{ij} = c^\alpha v_j^\alpha,$$

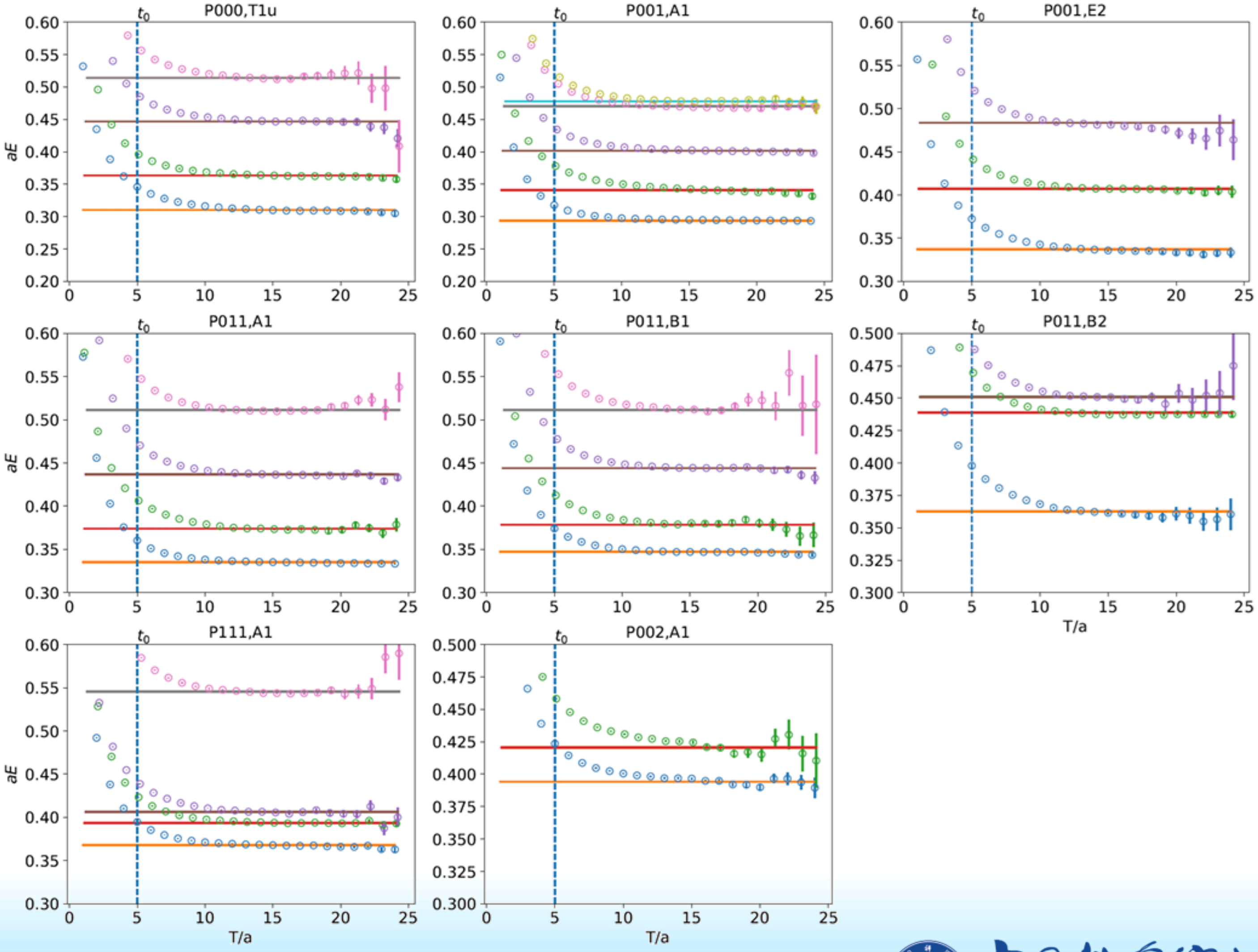
$$\mathcal{G}^\alpha(t) = v_i^\alpha \mathcal{G}_{ij}(t) u_j^\alpha,$$

$$E^\alpha(t) = \frac{1}{n} \log \frac{\mathcal{G}^\alpha(t)}{\mathcal{G}^\alpha(t+n)},$$



Kiratidis, et al., PRD 91, 094509 (2015)

ρ meson spectra on LQCD -- Spectra



Configuration: F48P30

There are 8 different spectra with different total momentum and irreps, including following irreducible representations.:

$p^2 = 0 \; : \; T_1^-,$

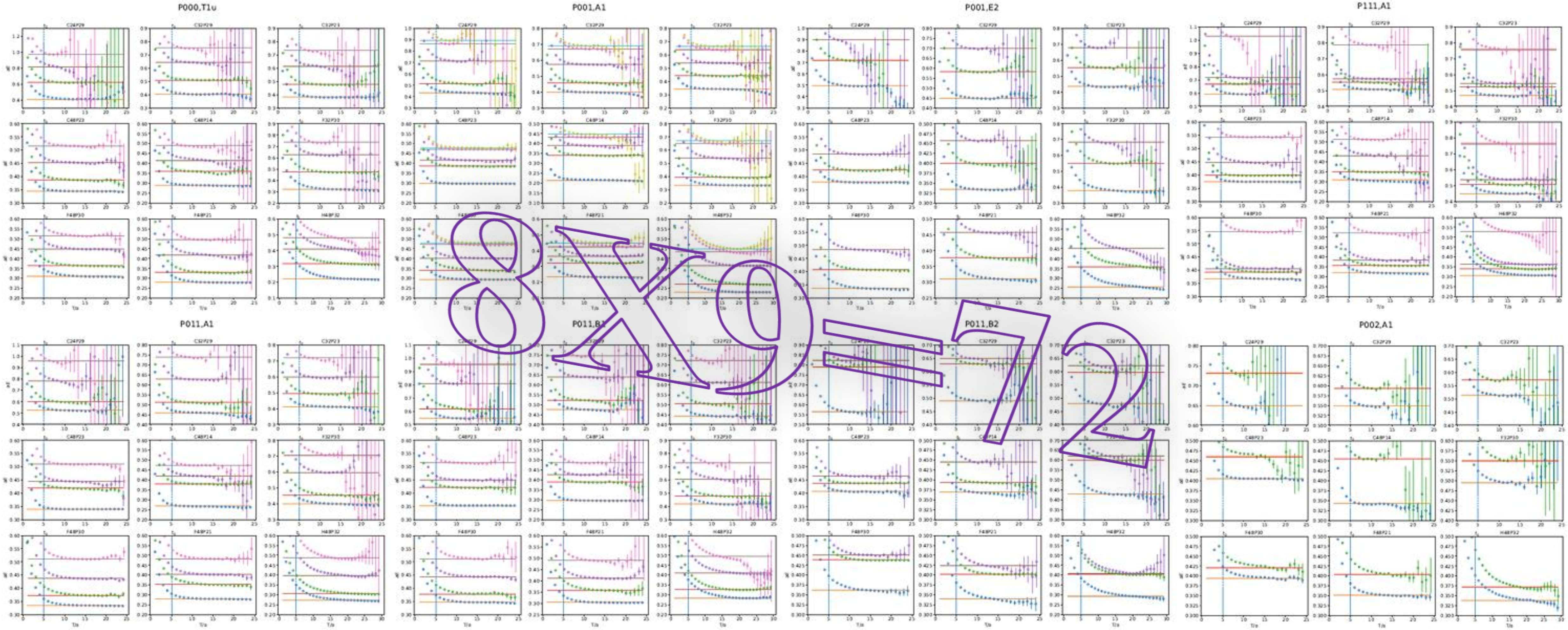
$p^2 = 1 \; : \; A_1$ and $E_2,$

$p^2 = 2 \; : \; A_1, B_1$ and $B_2,$

$p^2 = 3 \; : \; A_1,$

$p^2 = 4 \; : \; A_1.$

ρ meson spectra on LQCD -- Spectra



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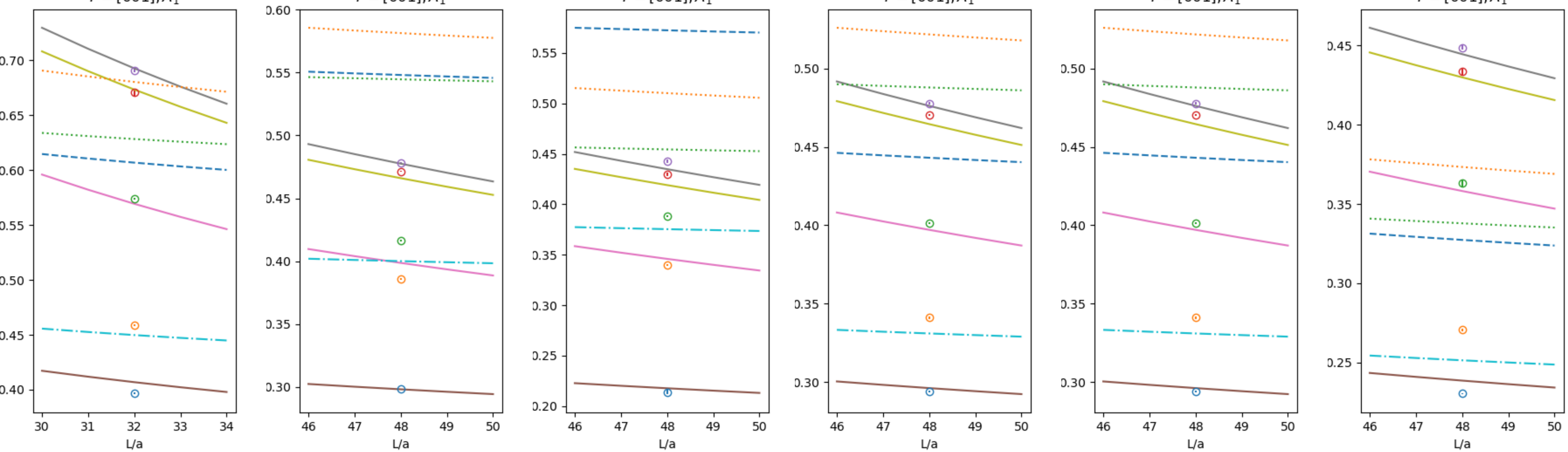
University of Chinese Academy of Sciences

ρ meson spectra on LQCD -- Spectra

C32P29	C48P23	C48P14	F48P30	F48P21	H48P32
m_π ~ 300 MeV	~ 220 MeV	~ 135 MeV	~ 300 MeV	~ 220 MeV	~ 300 MeV



a ~ 0.105 fm (C32P29, C48P23, C48P14)

a ~ 0.077 fm (F48P30, F48P21)

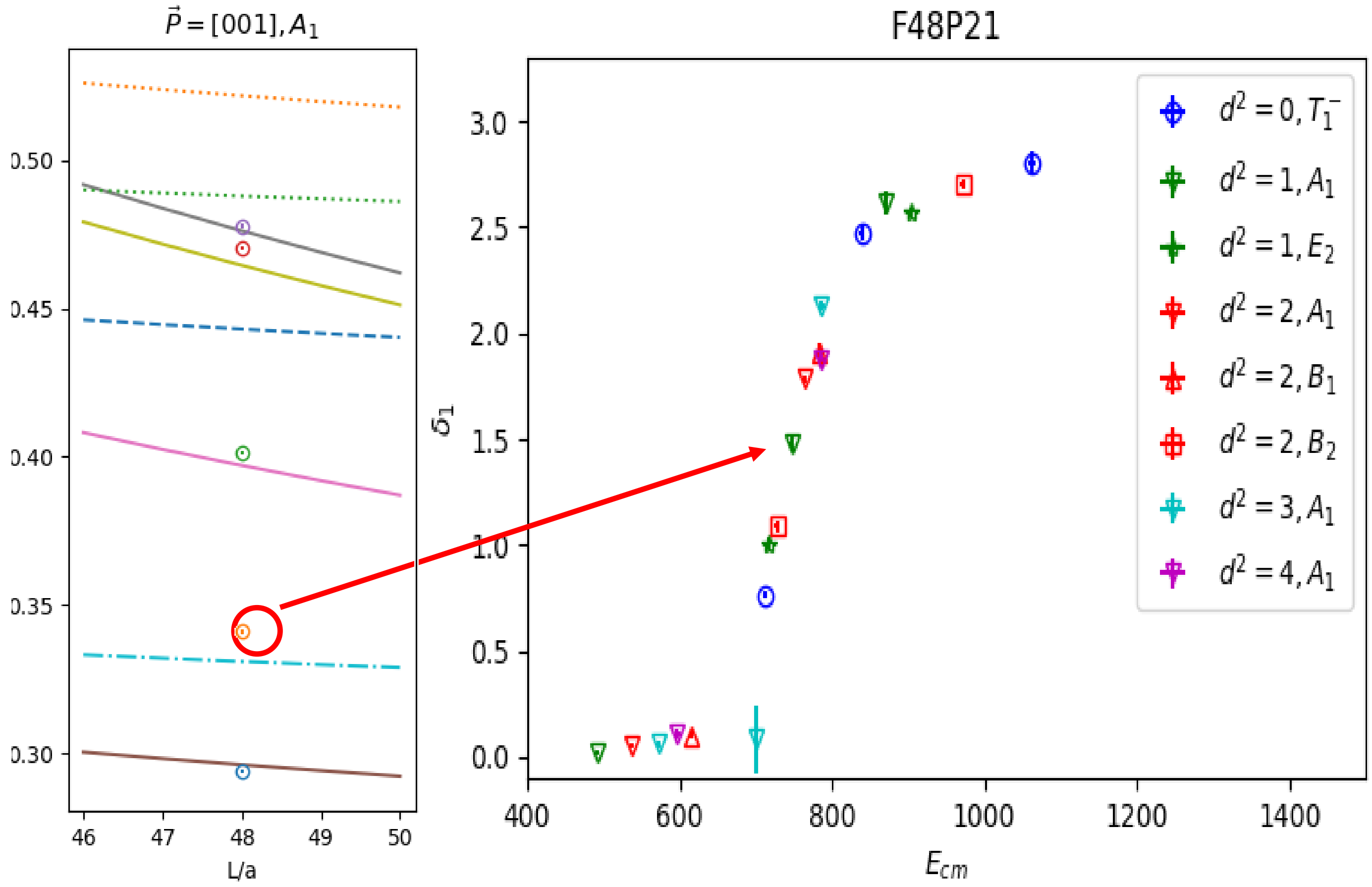
a ~ 0.052 fm (H48P32)

ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

d^2	Λ, μ	$M_{11,11}^{(\vec{P},\Lambda,\mu)}$
0	$T_1^-, 2$	$w_{0,0}$
1	$A_1, 1$	$w_{0,0} + 2w_{2,0}$
1	$E_2, 1$	$w_{0,0} - w_{2,0}$
1	$E_2, 2$	$w_{0,0} - w_{2,0}$
2	$A_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} + i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} - i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_2, 1$	$w_{0,0} - w_{2,0} + \sqrt{6}w_{2,2}$
3	$A_1, 1$	$w_{0,0} - i2\sqrt{6}w_{2,2}$

$$\det\left(M_{ln,l'n'}(k) - \delta_{ll'}\delta_{nn'}\cot(\delta_l)\right) = 0, \quad \delta_1 = \operatorname{arccot} M_{11,11}^{(\vec{P},\Lambda,\mu)},$$

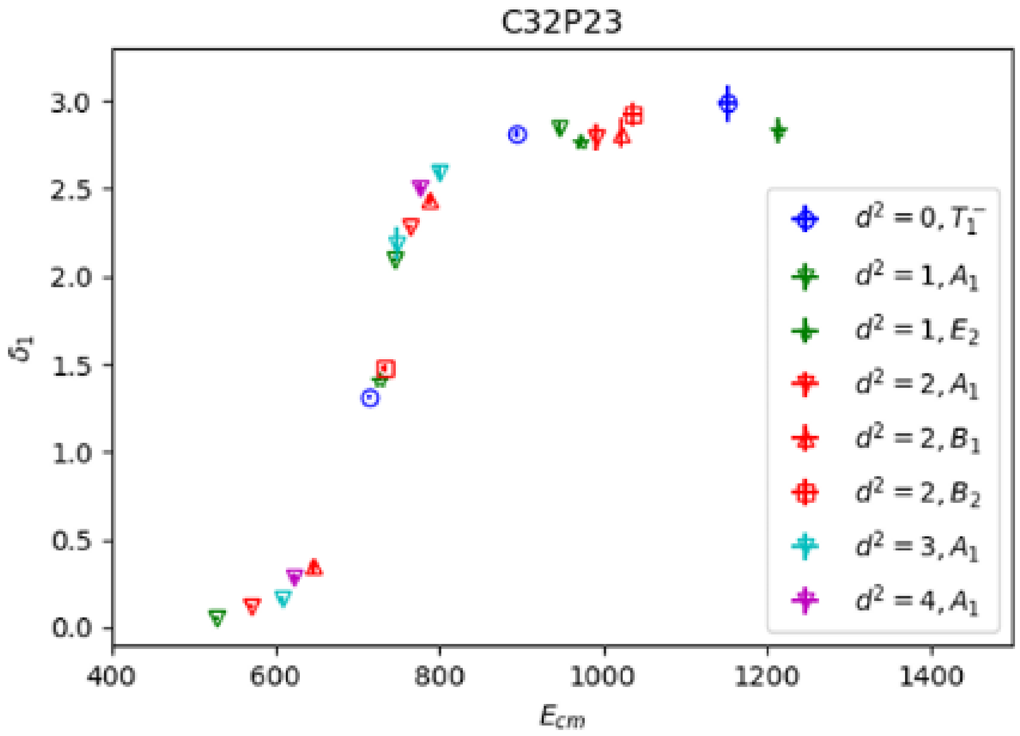
$$w_{j,s} = \frac{\mathcal{Z}_{js}(1,q^2)}{\pi^{3/2}\sqrt{2j+1}\gamma q^{j+1}}, \quad q = \frac{kL}{2\pi}, \quad \delta_{l\geq 3} \text{ are negligible}$$

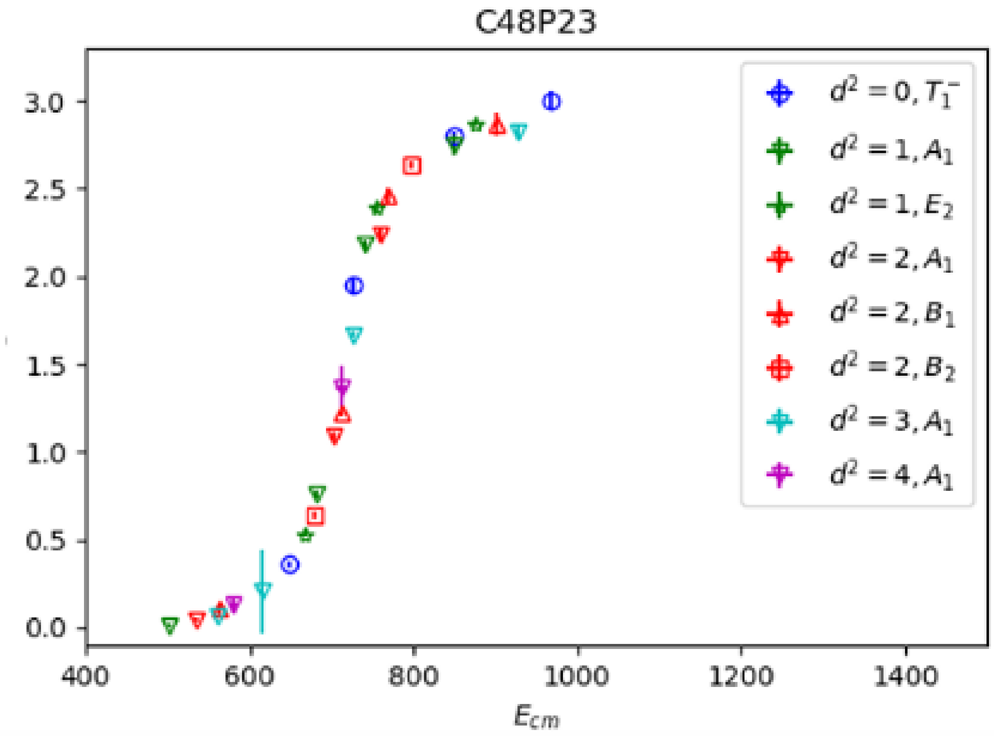


$$E_{cm} = \sqrt{E_{001}^2 - 4\pi^2/L^2} = \sqrt{(0.34/a)^2 - 4\pi^2/(48a)^2} = 771 \text{ MeV}$$

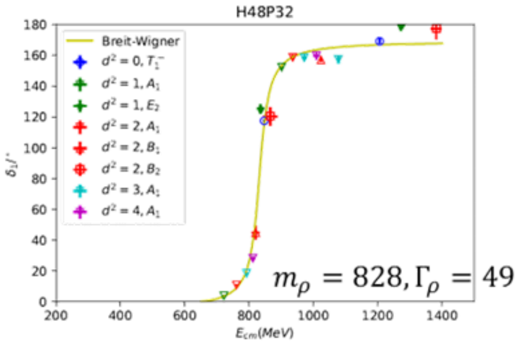
ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

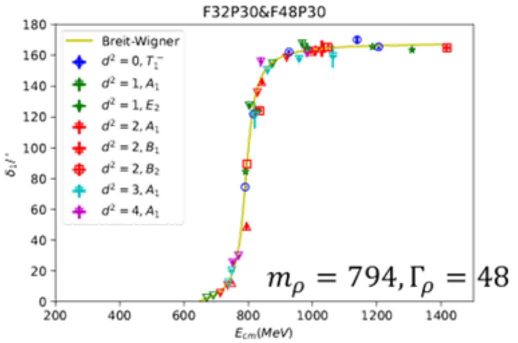
Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C32P23	$32^3 \times 64$	$0.10530fm$	6.20	228	3.91
C48P23	$48^3 \times 96$			225	5.79

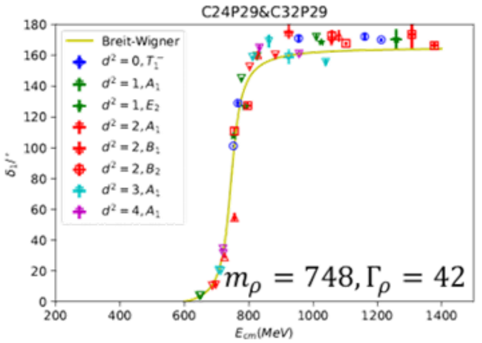




ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

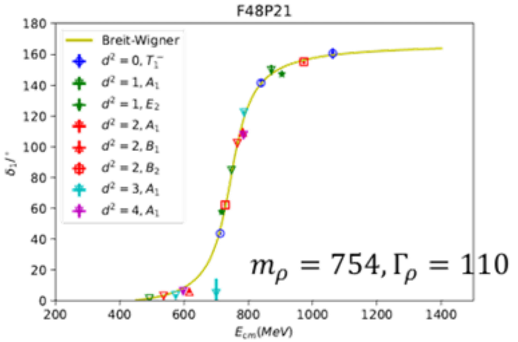


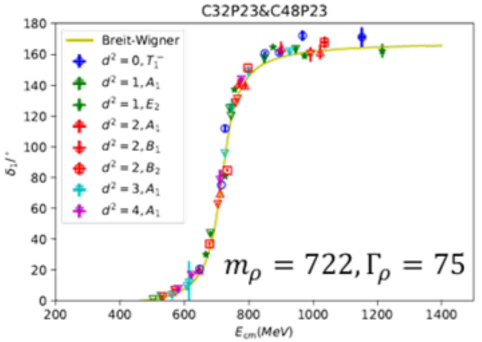




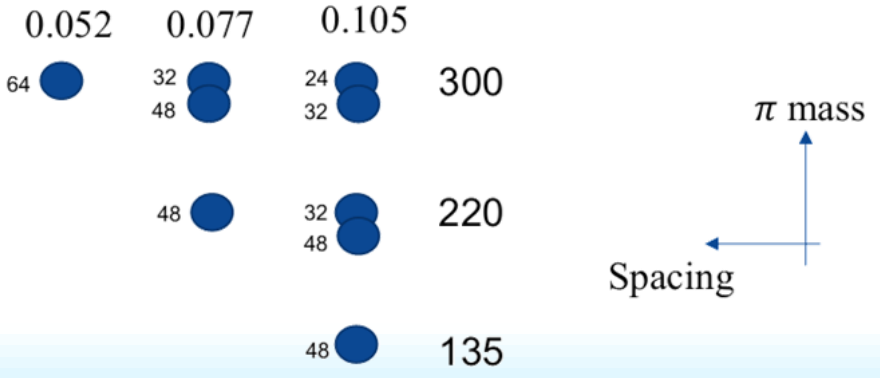
$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)} \sin\delta(s),$$

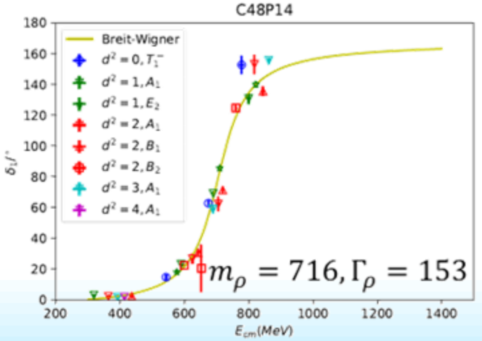
$$\Gamma(s) = \frac{p^{*3}}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}.$$



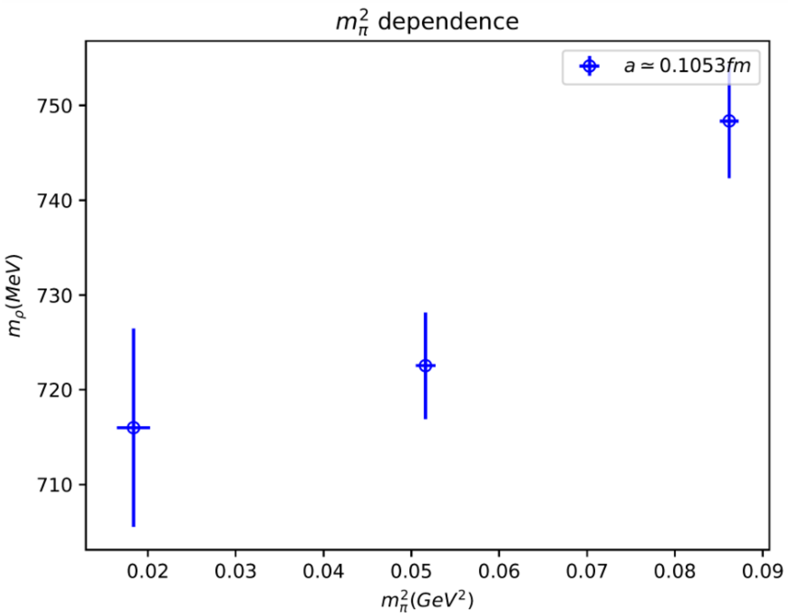


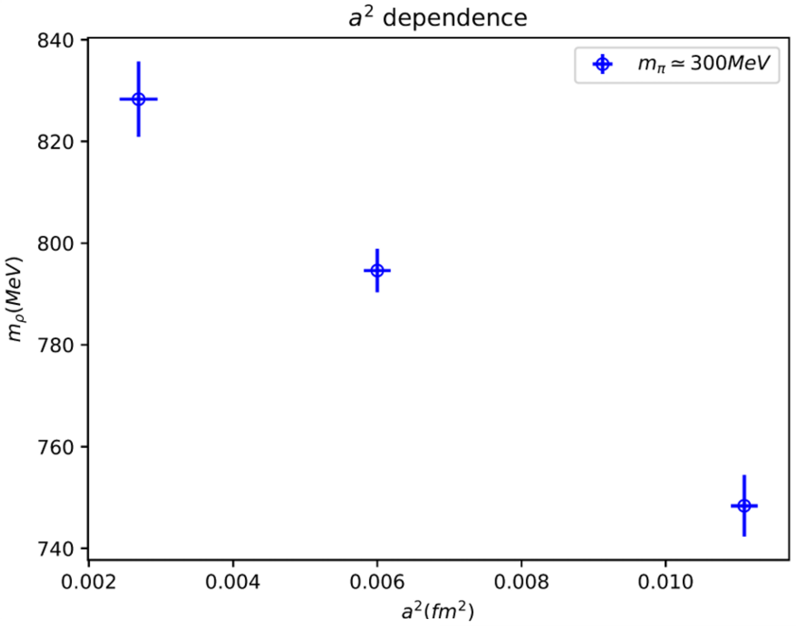
1. The lineshape of the phase shift from large lattice size are even more smooth since $m_\pi L$ is larger
2. There exists the jump of phase shift from 0 degree to 180 degrees





ρ meson spectra on LQCD -- M & Γ of ρ meson





0.052	0.077	0.105	
828/49	794/48	748/42	300
	754/110	722/75	220
		716/153	135

Spacing (fm)

$$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2$$

$c_1 = 0.7611(85)\,\text{GeV},$

$c_2 = 0.807(81)\,\text{GeV}^{-1},$

$c_3 = -7.13(79)\,\text{GeV} \cdot \text{fm}^{-2},$

$\chi^2/d.o.f \ = \ 0.76$

$$g = \tilde{c}_0 + \tilde{c}_1 m_\pi^2 + \tilde{c}_2 a^2$$

$\tilde{c}_1 = 6.37(11),$

$\tilde{c}_2 = -3.35(87)GeV^{-2},$

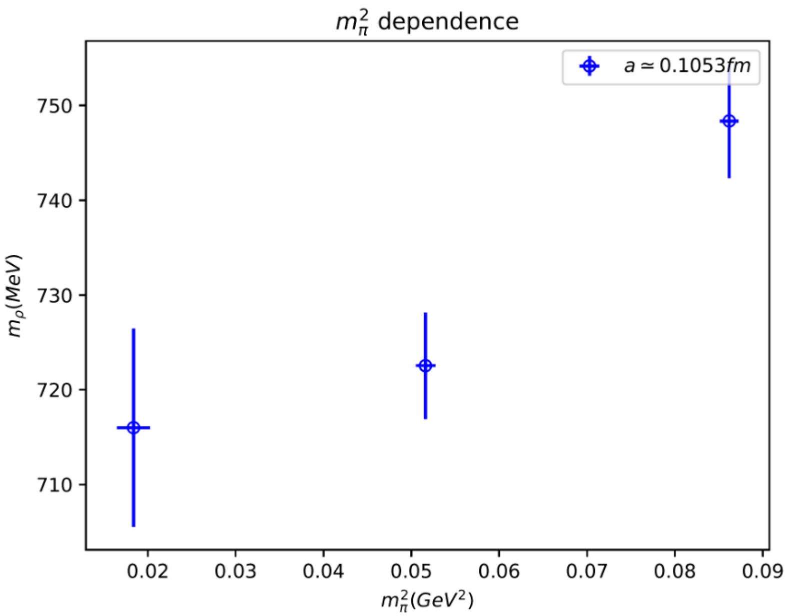
$\tilde{c}_3 = -16.8(6.9)fm^{-2},$

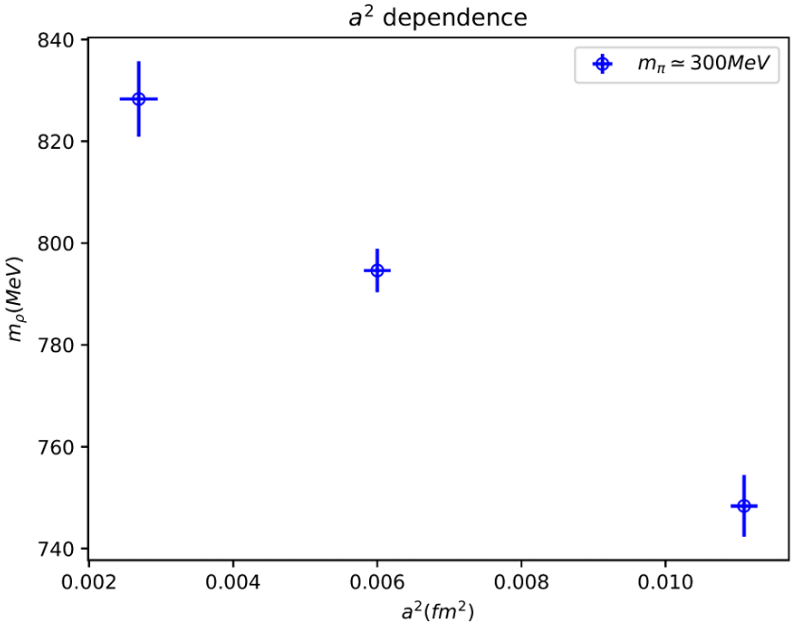
$\chi^2/d.o.f = 0.26$

Table 4: Fit results for m_ρ, $g_{\rho\pi\pi}$ and Z_{pole}

	C24P29 & C32P29	C32P23 & C48P23	C48P14	F32P30 & F48P30	F48P21	H48P32
m_ρ	748.38(6.0)	722.55(5.6)	716.0(10.46)	794.62(4.24)	751.3(2.98)	828.3(7.4)
$g_{\rho\pi\pi}$	5.249(79)	5.32(14)	6.04(44)	6.09(58)	6.80(38)	6.44(45)
Z_{pole}	744.94(6.0) - 19.89(1.5)i	715.23(5.6) - 38.47(2.8)i	703.89(10.4) - 70.77(8.5)i	789.65(4.3) - 24.71(1.0)i	743.53(3.0) - 52.46(1.7)i	824.51(7.4) - 22.93(0.9)i

ρ meson spectra on LQCD -- M & Γ of ρ meson





$\rho(770)$ T-MATRIX POLE $\sqrt{s}$	$(761-765)-i(71-74)$ MeV
$\rho(770)$ **MASS**	
NEUTRAL ONLY, e^+e^-	775.26 ± 0.23 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.2 ± 0.9 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}
$\rho(770)$ **WIDTH**	
NEUTRAL ONLY, e^+e^-	147.4 ± 0.8 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	$151.5^{+1.9}_{-2.1}$ MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 MeV (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2$ Here we take $m_{\pi-phy} = m_{\pi^0} \sim 135$ MeV

$$m_\rho = 775.8(8.5)\text{MeV}, \quad \Gamma_\rho(m_\rho) = 160(10)\text{MeV},$$

$$Z_{\text{pole}} = 766.7(8.6) - i76.5(4.4)\text{MeV}.$$

ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

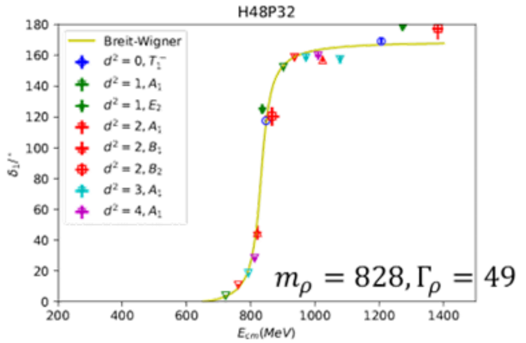
$$T^{I=1}_{\pi\pi\to\pi\pi}(z) = \frac{\left|V_{\rho\pi\pi}(\bar{k}(z))\right|^2}{z - m_\rho^B - \Sigma(z)},$$

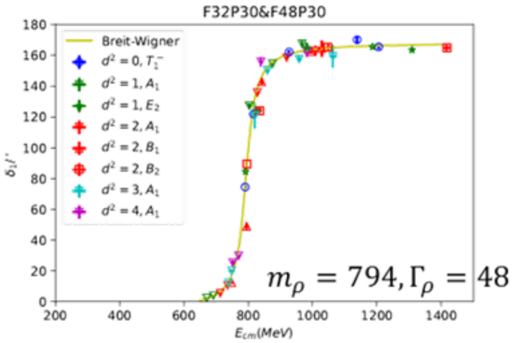
$$\Sigma(z) = \int q^2 dq \frac{\left|V_{\rho\pi\pi}(q)\right|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}.$$

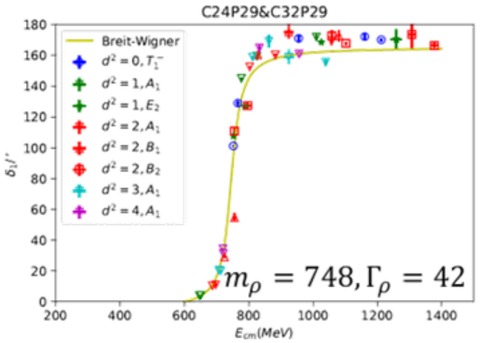
$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)}\sin\delta(s),$$

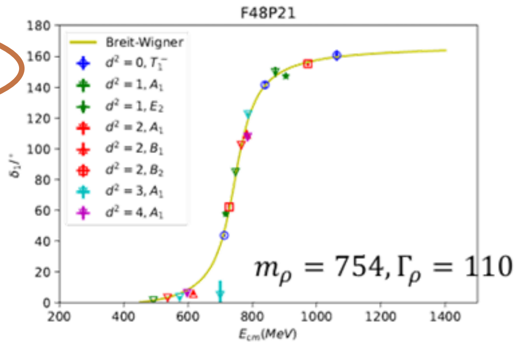
$$\Gamma(s) = \frac{p^{*3}}{s}\frac{g^2_{\rho\pi\pi}}{6\pi}.$$

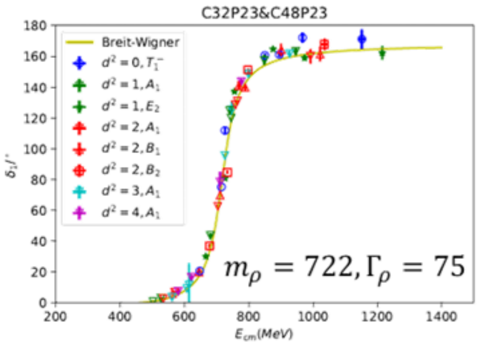
HEFT

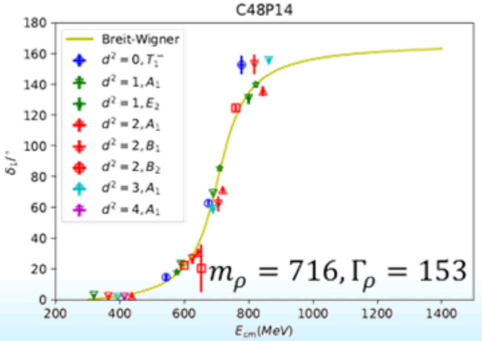




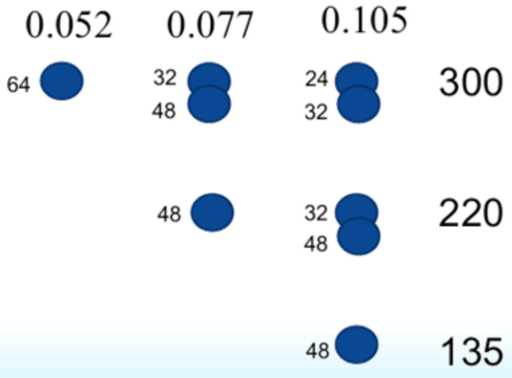


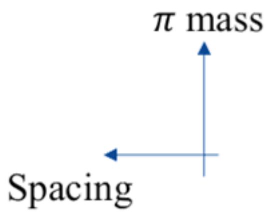






1. The lineshape of the phase shift from large lattice size are even more smooth since $m_\pi L$ is larger
2. There exists the jump of phase shift from 0 degree to 180 degrees





Introduction of HEFT

J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206
Y. Li etc. PRD 101(2020), 114501
PRD 103(2021), 094518

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

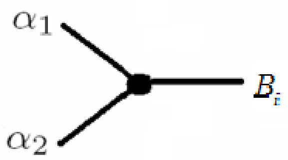
$|B_i>$ bare state， bare mass m_i

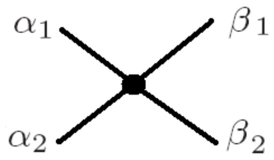
$|\alpha(k_\alpha)>$ non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

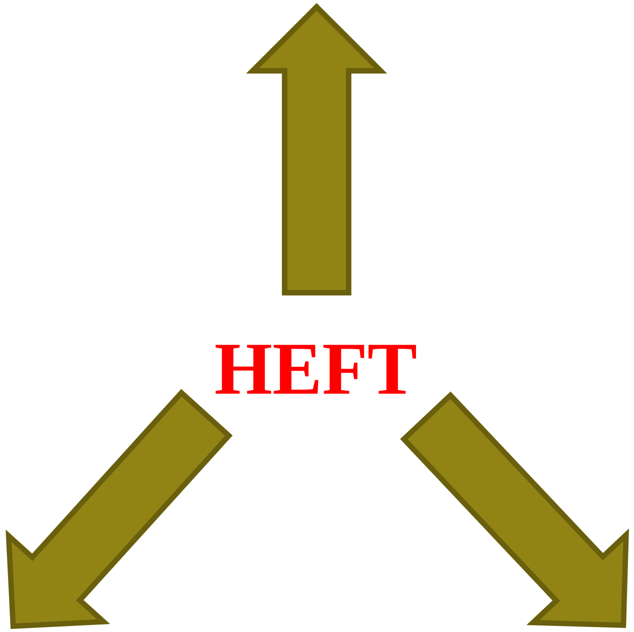
$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$





Resonance
(Mass , Width, Pole position, Coupling)



HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

ρ meson spectra on LQCD -- Phase shift of $\pi\pi$

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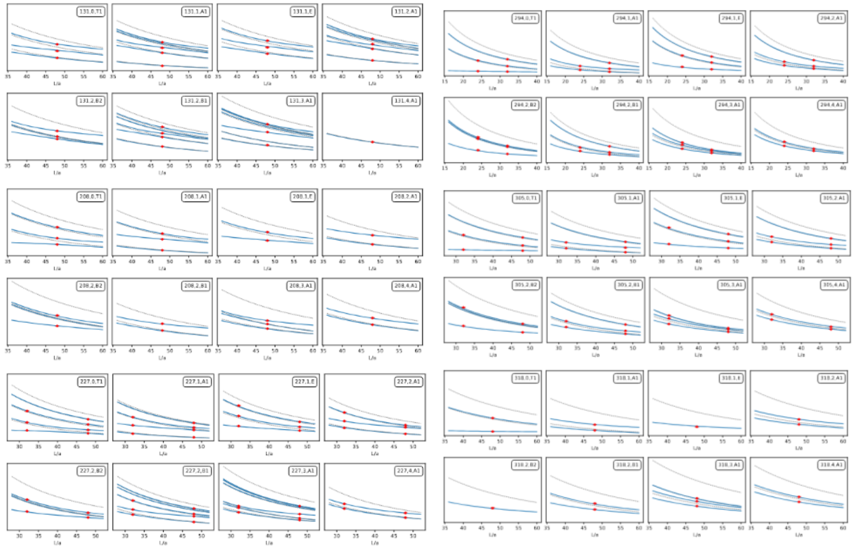
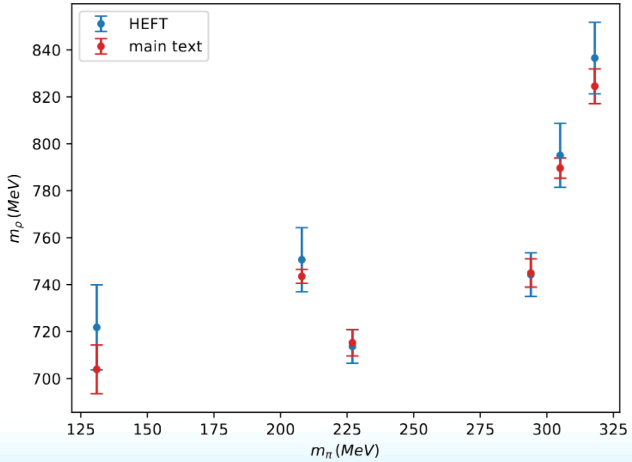
$$\Sigma(z) = \int q^2 dq \frac{\left|V_{\rho\pi\pi}(q)\right|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}.$$

$$V_{\rho\pi\pi}(k) = \frac{g_{\rho\pi\pi} k}{\sqrt{m_\rho^B}\sqrt{k^2 + m_\pi^2}} \left(\frac{\Lambda^2_{\rho\pi\pi}}{k^2 + \Lambda^2_{\rho\pi\pi}}\right)^2$$



- Three free parameters, $m_\rho^B, g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}$
- Here we take $g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}$ as constant for different m_π and a.
- But m_ρ^B is pion mass dependence.

$$m_\rho^B(m_\pi, a) = c_0 + c_1 m_\pi^2 + \xi a^2,$$



Resonance

(Mass , Width, Pole position, Coupling)

HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

ρ meson spectra on LQCD -- Phase shift of ππ

- Three free parameters, $m_\rho^B, g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}$
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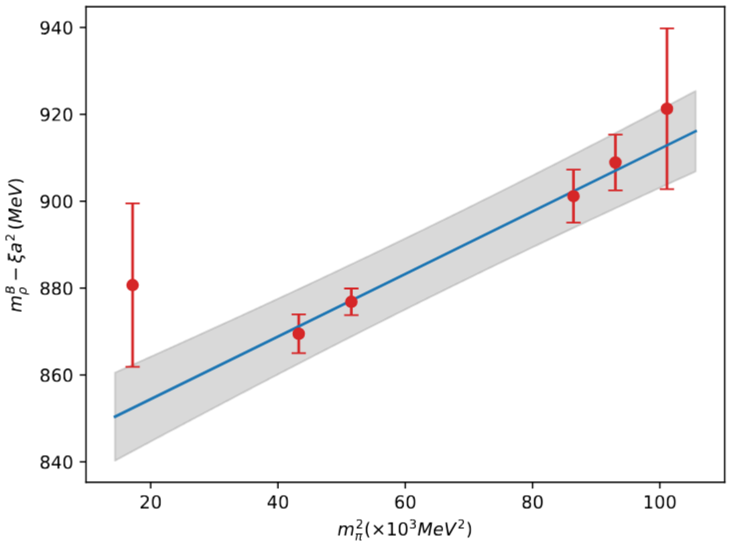

$$(c_0, c_1, \xi) = (840,\ 0.72 \times 10^{-3},\ -7912),$$

$$\mathrm{cov}(c_0, c_1, \xi) = \begin{pmatrix} 125 & -0.88 \times 10^{-3} & -7474 \\ & 0.013 \times 10^{-6} & 11.7 \times 10^{-3} \\ & & 7.39 \times 10^5 \end{pmatrix},$$

$$c_1 = 0.7611(85)\,\mathrm{GeV},$$
$$c_2 = 0.807(81)\,\mathrm{GeV}^{-1},$$
$$c_3 = -7.13(79)\,\mathrm{GeV} \cdot \mathrm{fm}^{-2},$$

$$m_\rho^B(m_\pi^{\mathrm{phy}}, 0) = 853 \pm 10\ \mathrm{MeV}$$

$$m_\rho^{\mathrm{pole,ext}} = (792 \pm 11 \pm 4) - i(64 \pm 4 \pm 4),$$

$$Z_{\mathrm{pole}} = 766.7(8.6) - i76.5(4.4)\mathrm{MeV}.$$

$\rho(770)$ T-MATRIX POLE $\sqrt{s}$ $(761 - 765) - i(71 - 74)$ MeV

Summary

We using CLQCD configuration measure the rho meson mass and width at the physical pion mass and continuum limit with the highest precision up to now!

$$m_\rho = 775.8(8.5)\mathrm{MeV}, \quad \Gamma_\rho(m_\rho) = 160(10)\mathrm{MeV},$$

$$Z_{\mathrm{pole}} = 766.7(8.6) - i76.5(4.4)\mathrm{MeV}.$$

Thanks for attention!