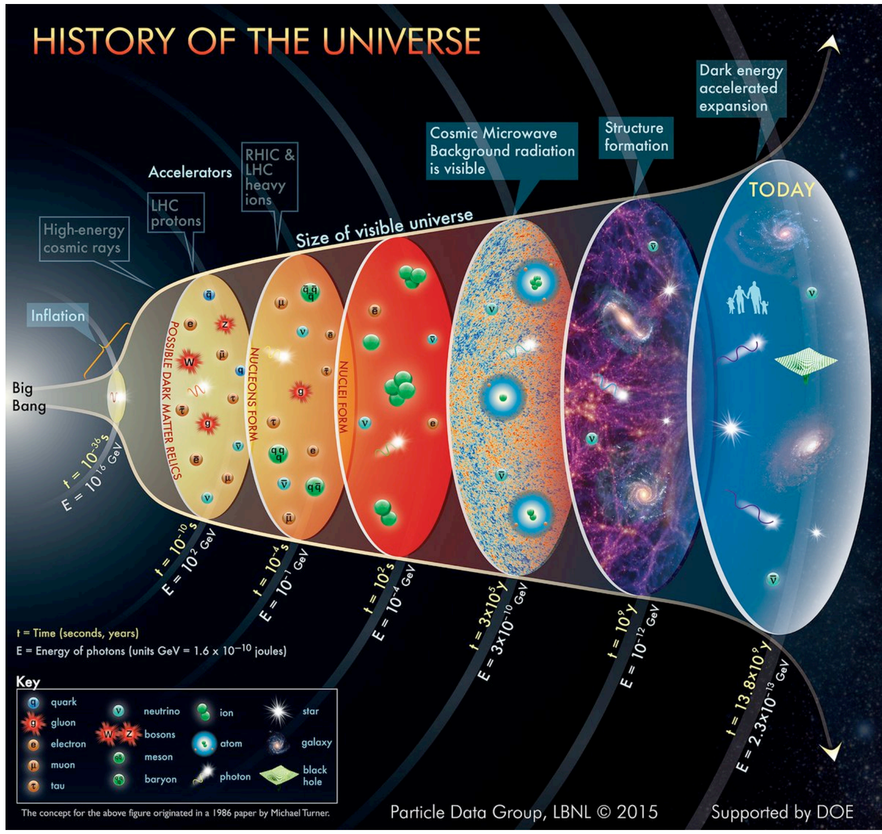
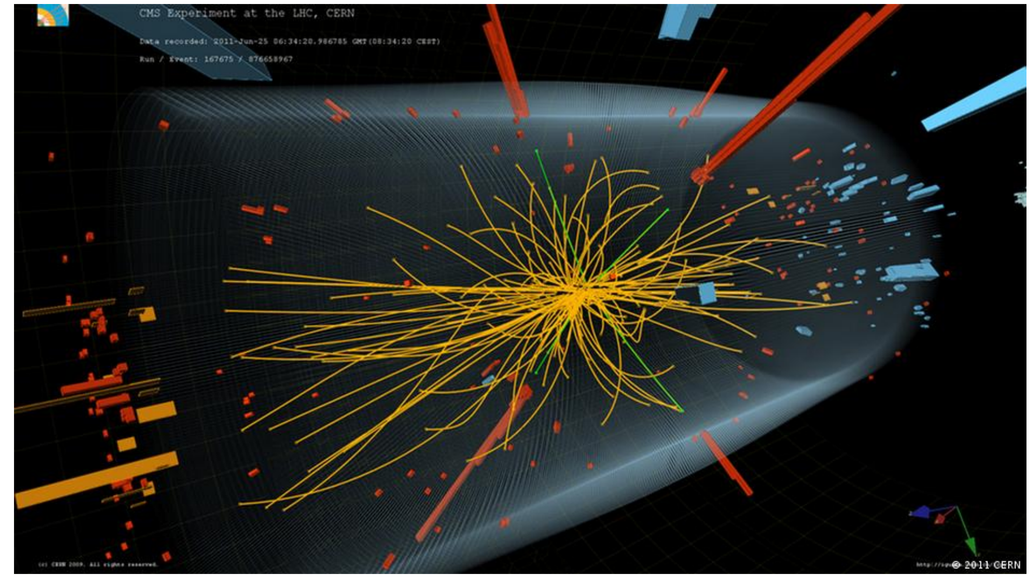
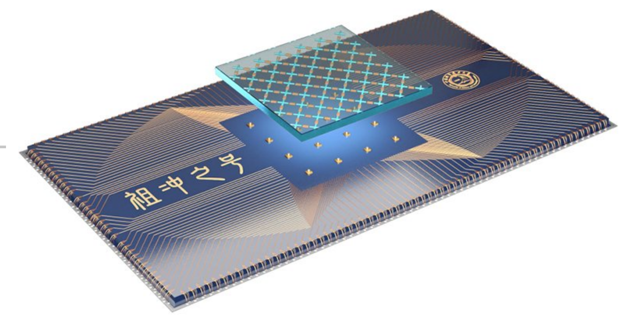






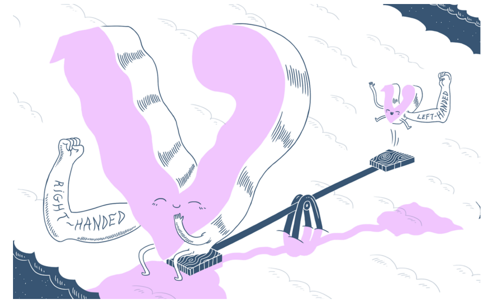
中国科学技术大学
University of Science and Technology of China

Ying-Ying Li
李英英

arXiv:2402.16780, PRL.129, 051601
in collaboration with
Marcela Carena, Henry Lamm, Wanqiang Liu

Gauge-redundant digitizations of lattice field theories on a Quantum Computer

Feb. 29, 2024

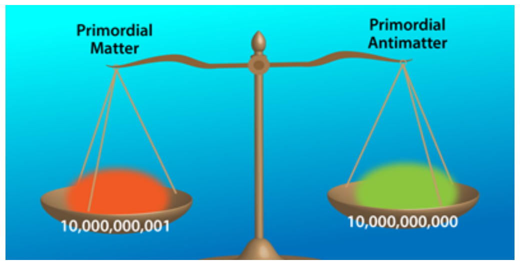


neutrino mass

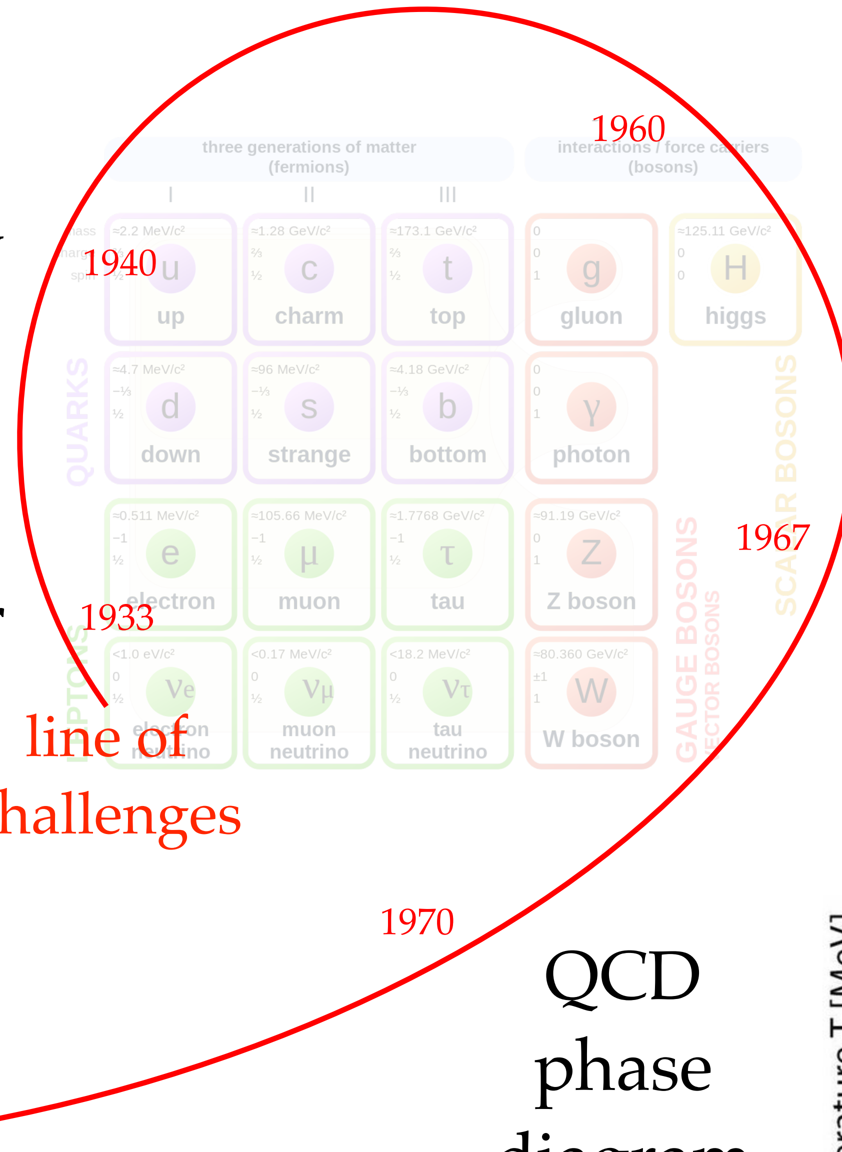
$$\mathscr{L} \supset -\frac{1}{4}G^2 + \frac{\theta g_s^2}{32\pi^2} G\tilde{G}$$

$$\overline{\theta} \lesssim 10^{-10}$$

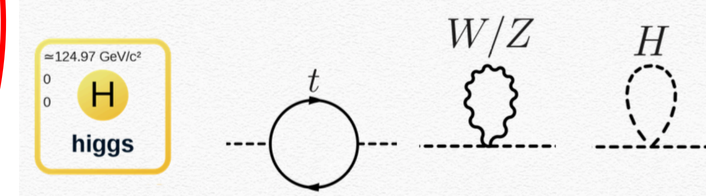
strong CP problem



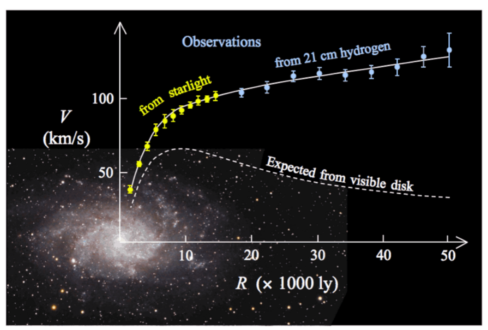
baryon asymmetry



$$\delta m_{SM}^2 \sim -\frac{3y_t^2\Lambda^2}{16\pi^2} + \frac{3g^2\Lambda^2}{16\pi^2} + \frac{3\lambda_{SM}\Lambda^2}{16\pi^2}$$

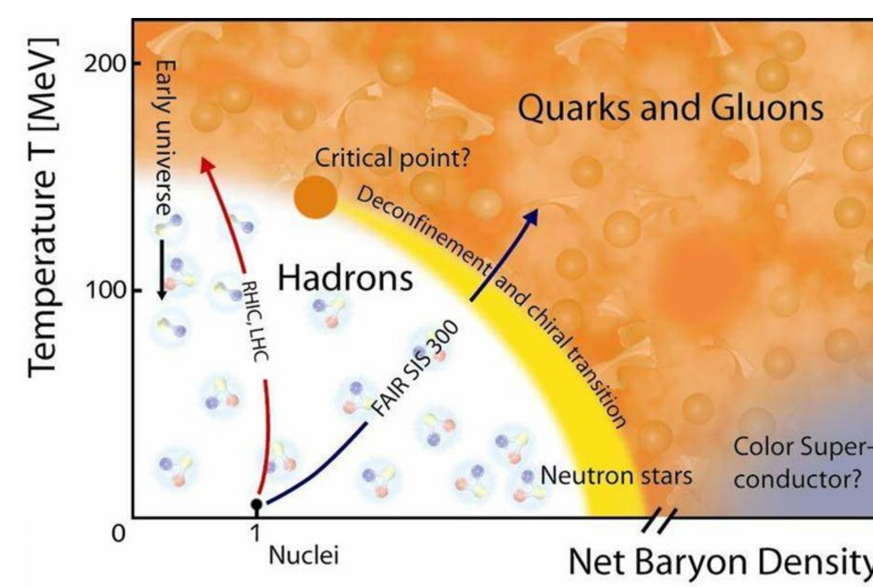


naturalness



dark matter

QCD phase diagram



now

new physics?

computational efforts?

quantum gravity?

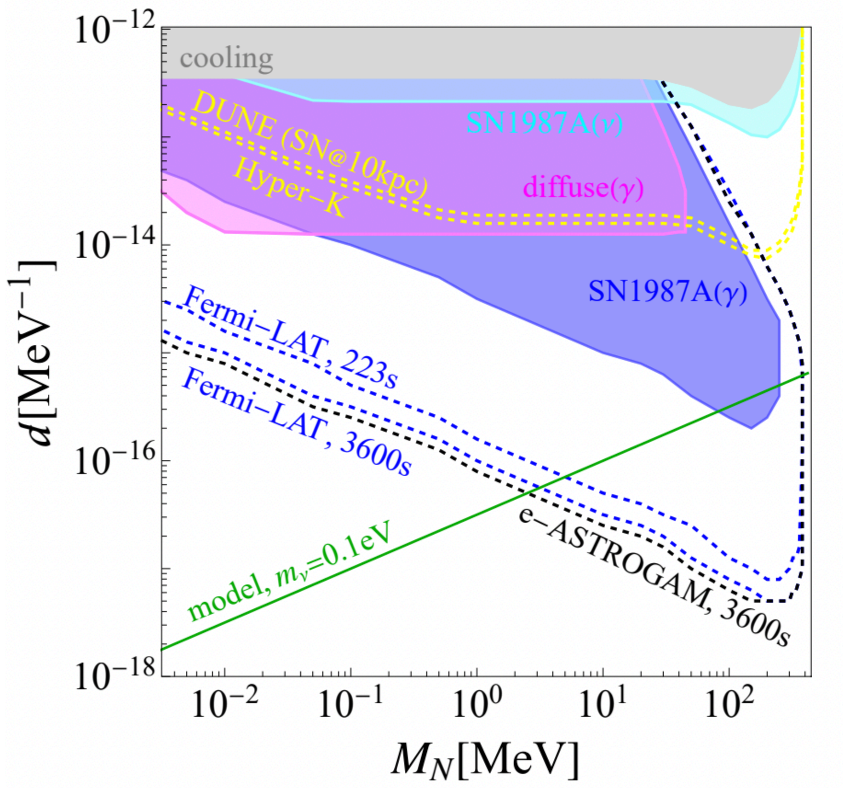
new physics → computational efforts

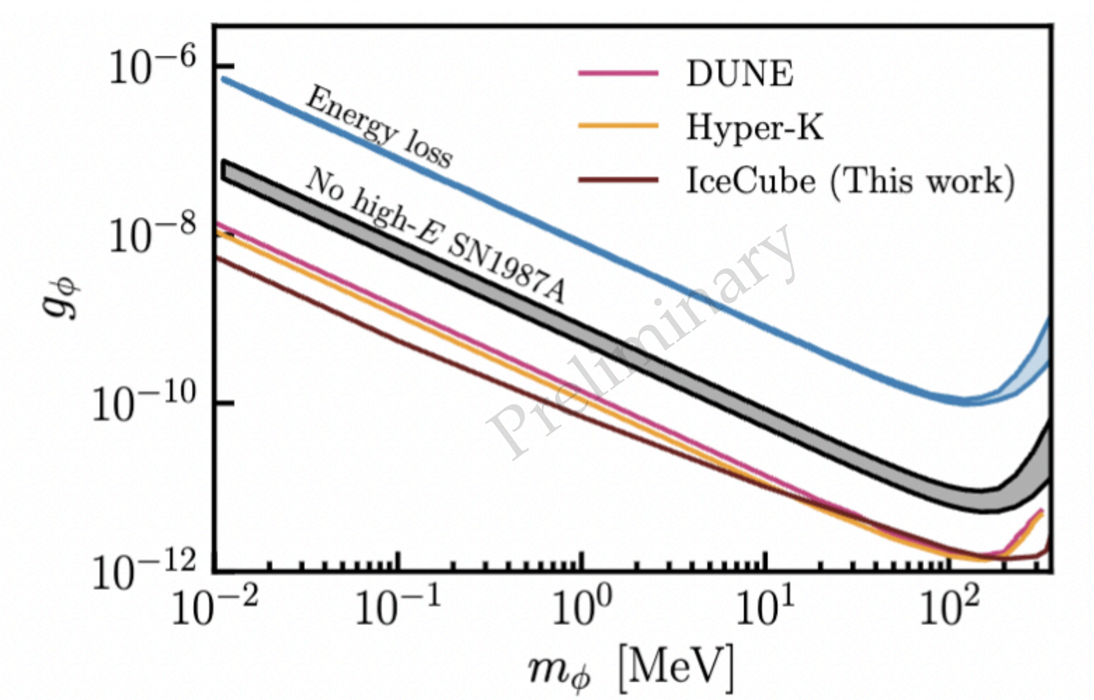
weakly-interacting particles

strongly interacting many-body system

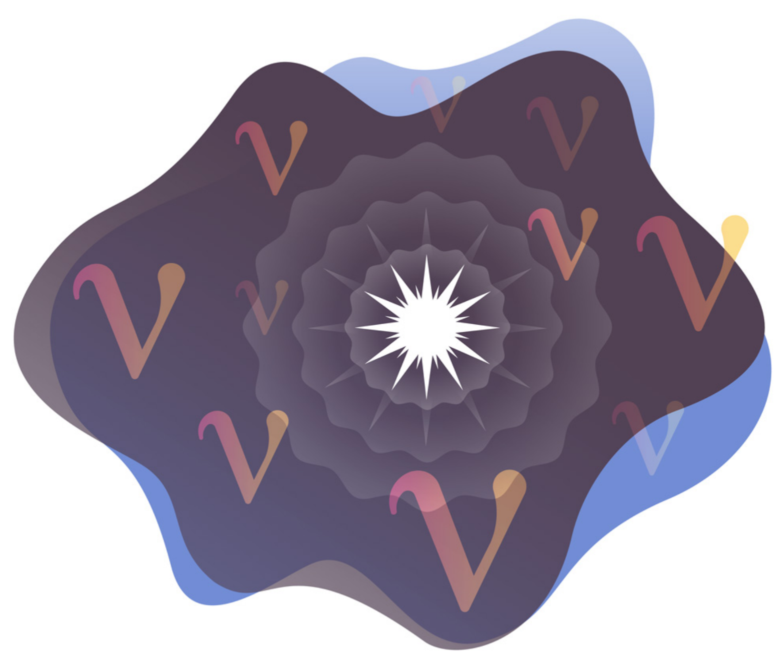
$\nu - \nu/N$ interactions

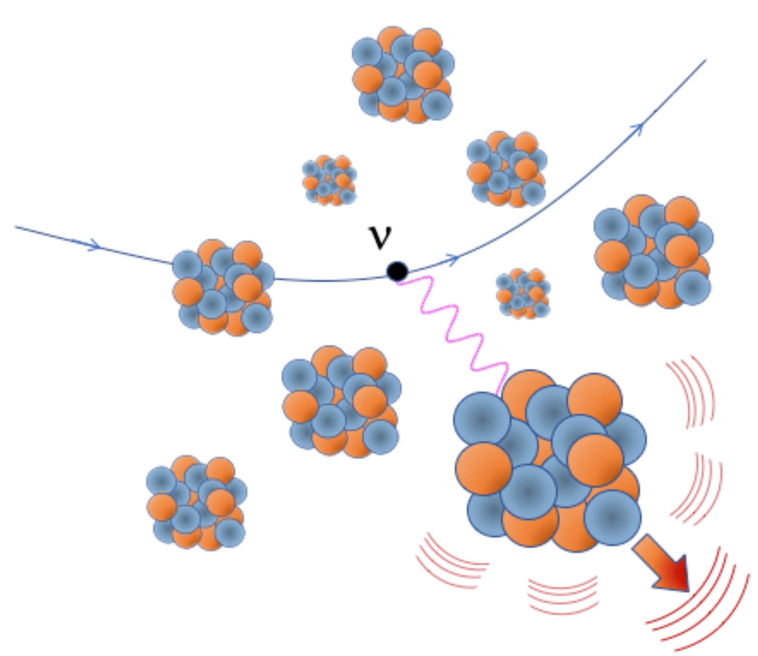
V. Brdar, A. de Gouvêa, **YYL**, P. A. N. Machado, arXiv:2302.10965





C. A. Argüelles, V. Brdar, J. Lazar, **YYL**, in prep

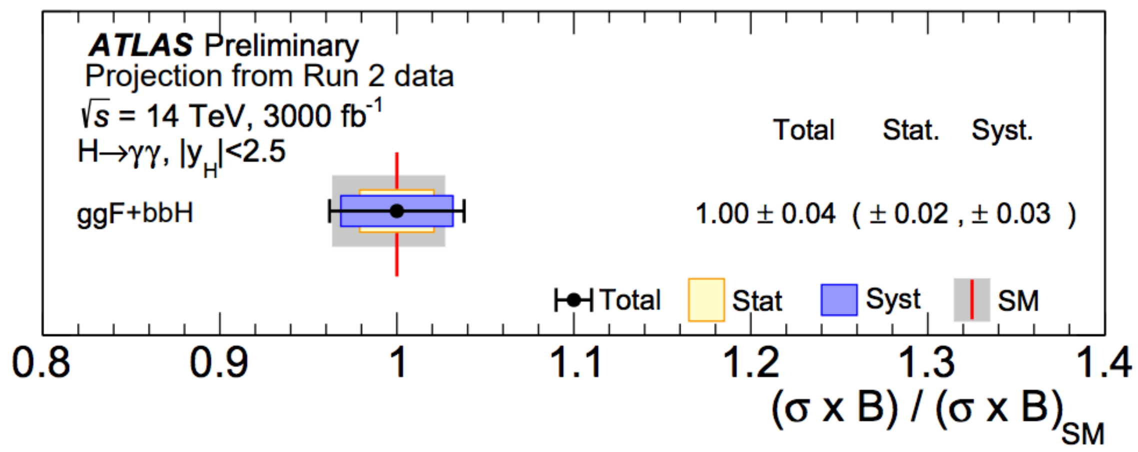


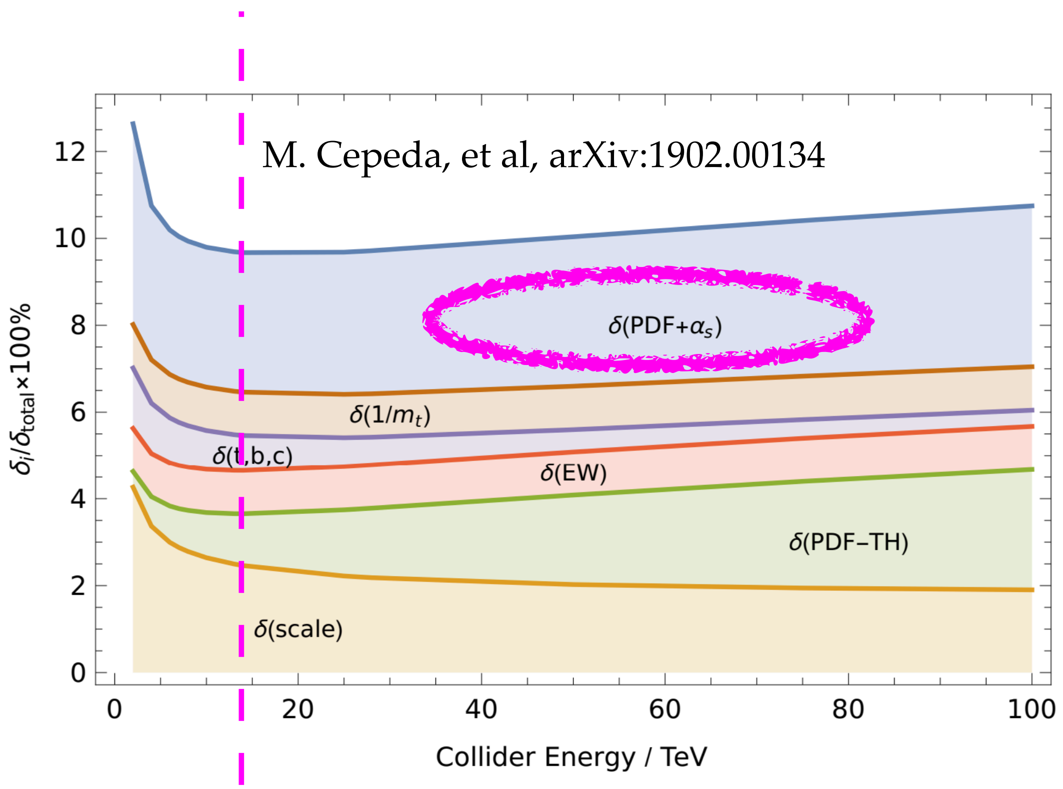


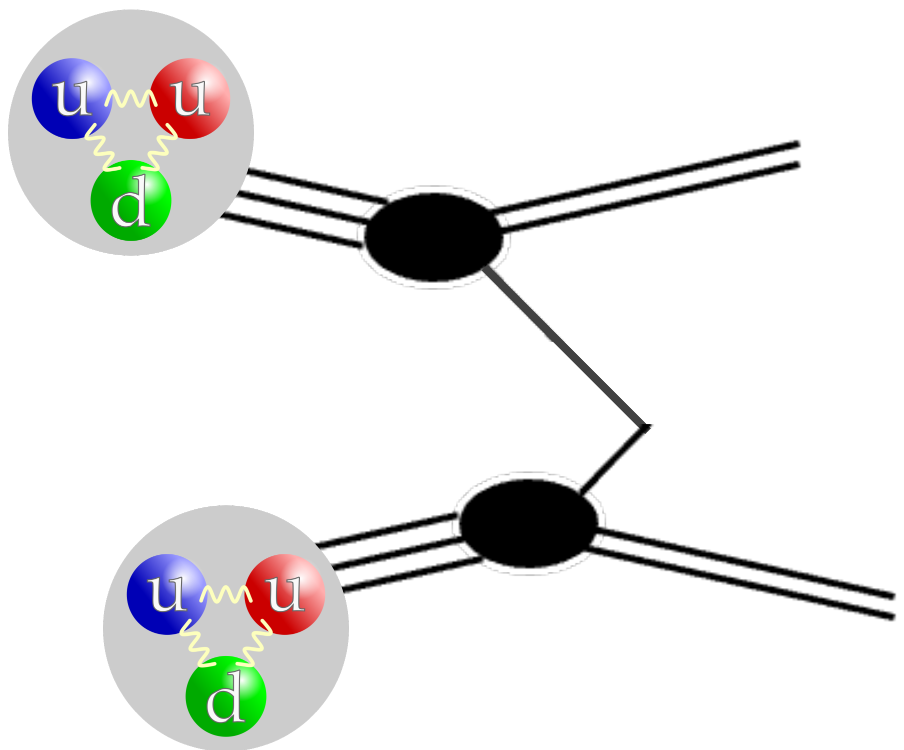
new physics → computational efforts

collider precisions

real-time strong dynamics

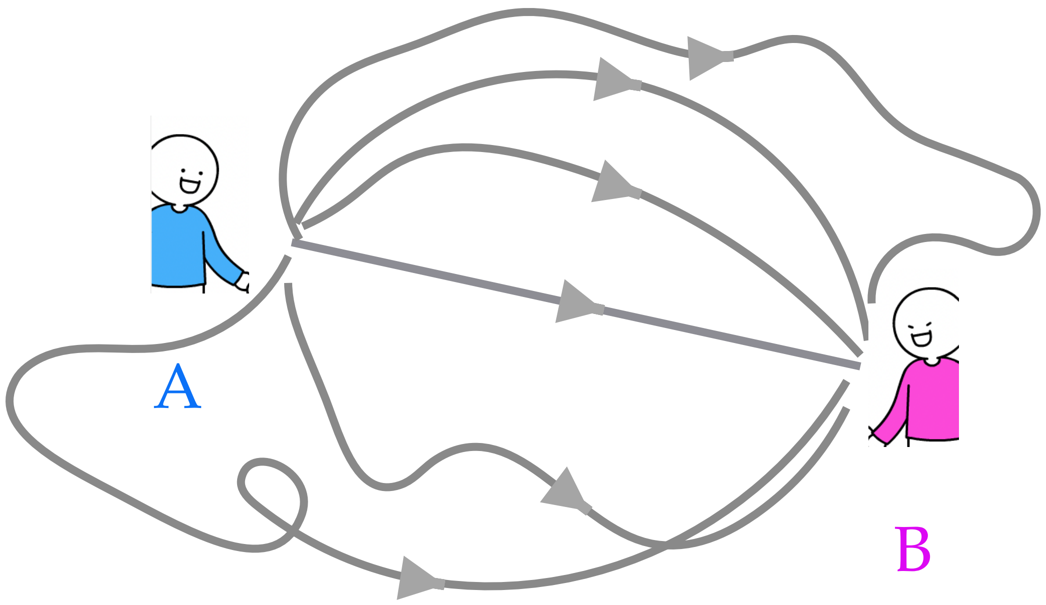


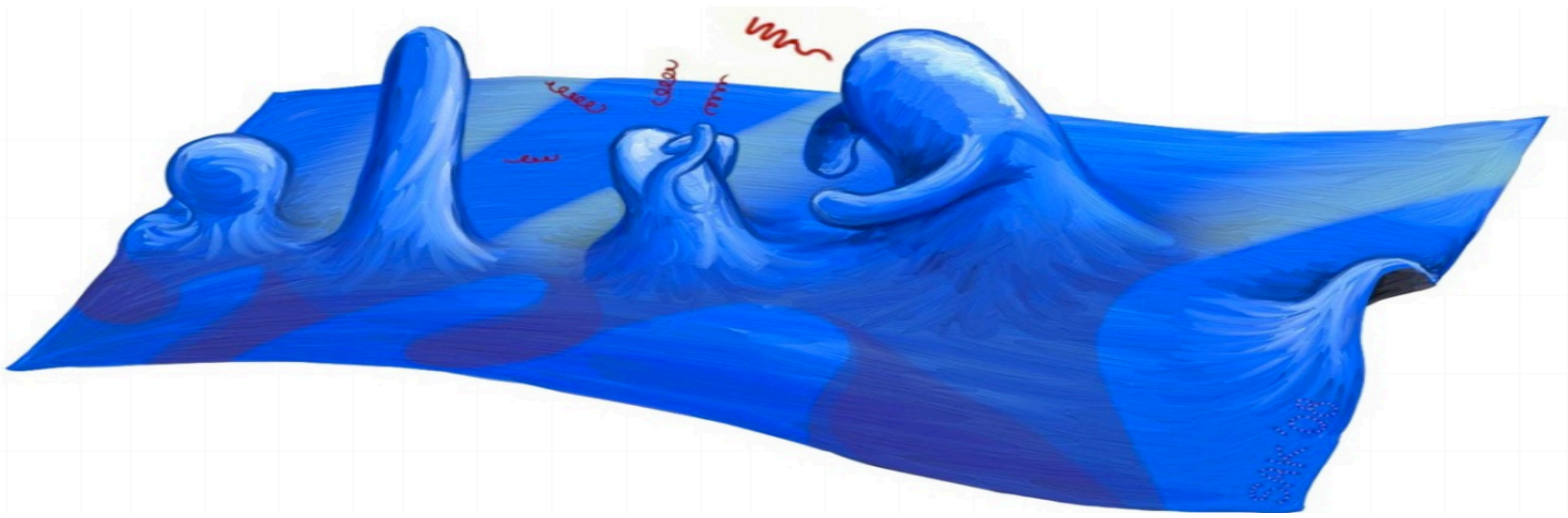




Parton Distribution Function (PDF)
``the probability of getting partons
with certain momentum out of proton"

Relativistic Quantum Mechanics → Quantum Field Theory (QFT)

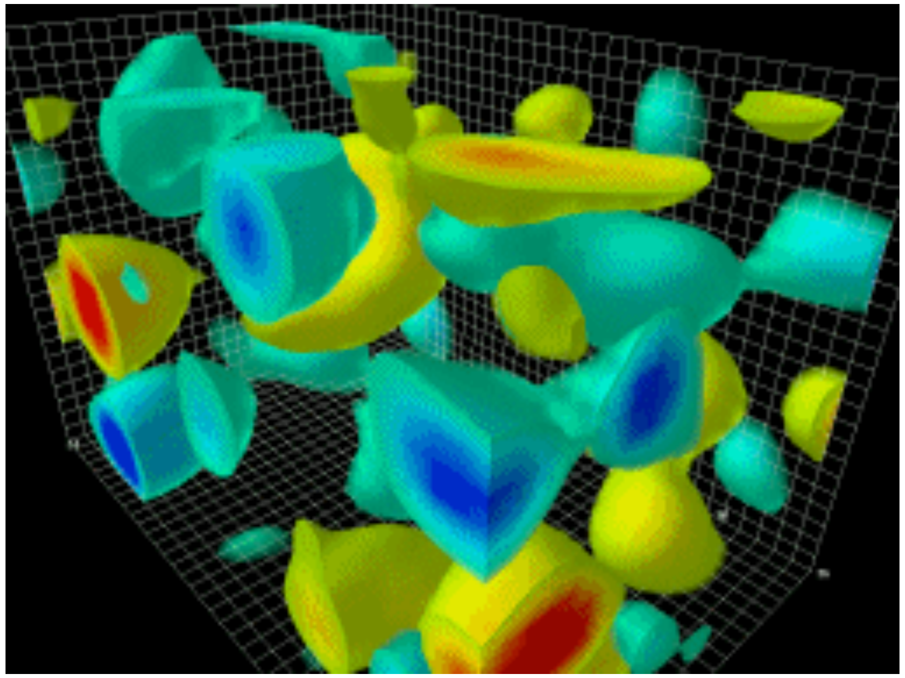




path integral on the background of field configurations

Lattice QCD - Euclidean Spacetime

remains the only tool for
precise, controllable,
first principle calculations



field configurations
$\mathcal{C}$ on lattice

path integral in
Euclidean spacetime

Monte Carlo
sampling of lattice
field configurations

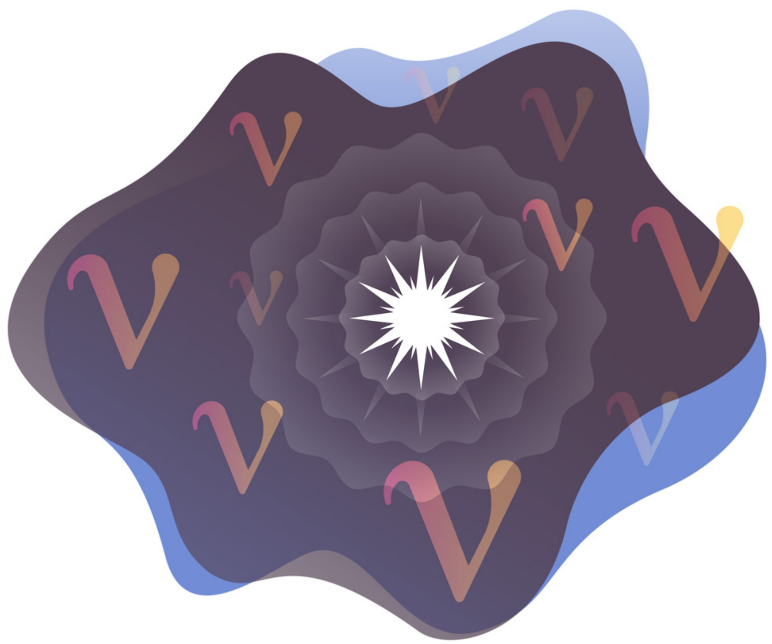
→

Euclidean
correlations and
physical observables

$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$

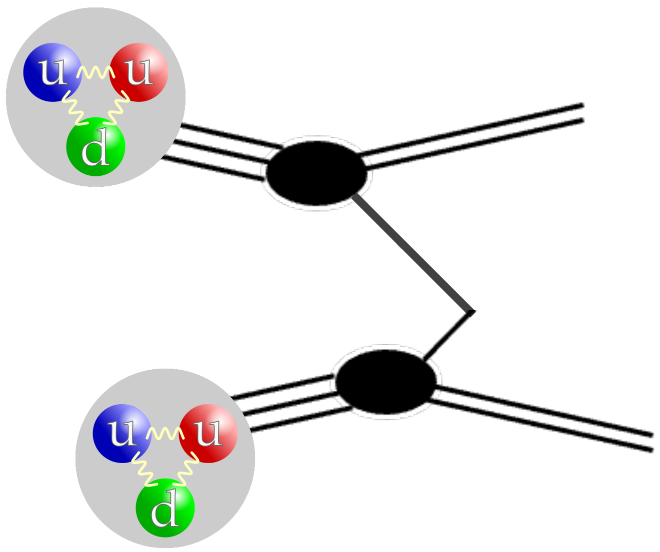
finite density



$$S \to S + iS_1$$

real-time dynamics

PDF



$$\int \mathcal{D}\phi e^{iS}$$

complex $S(\mathcal{C})$

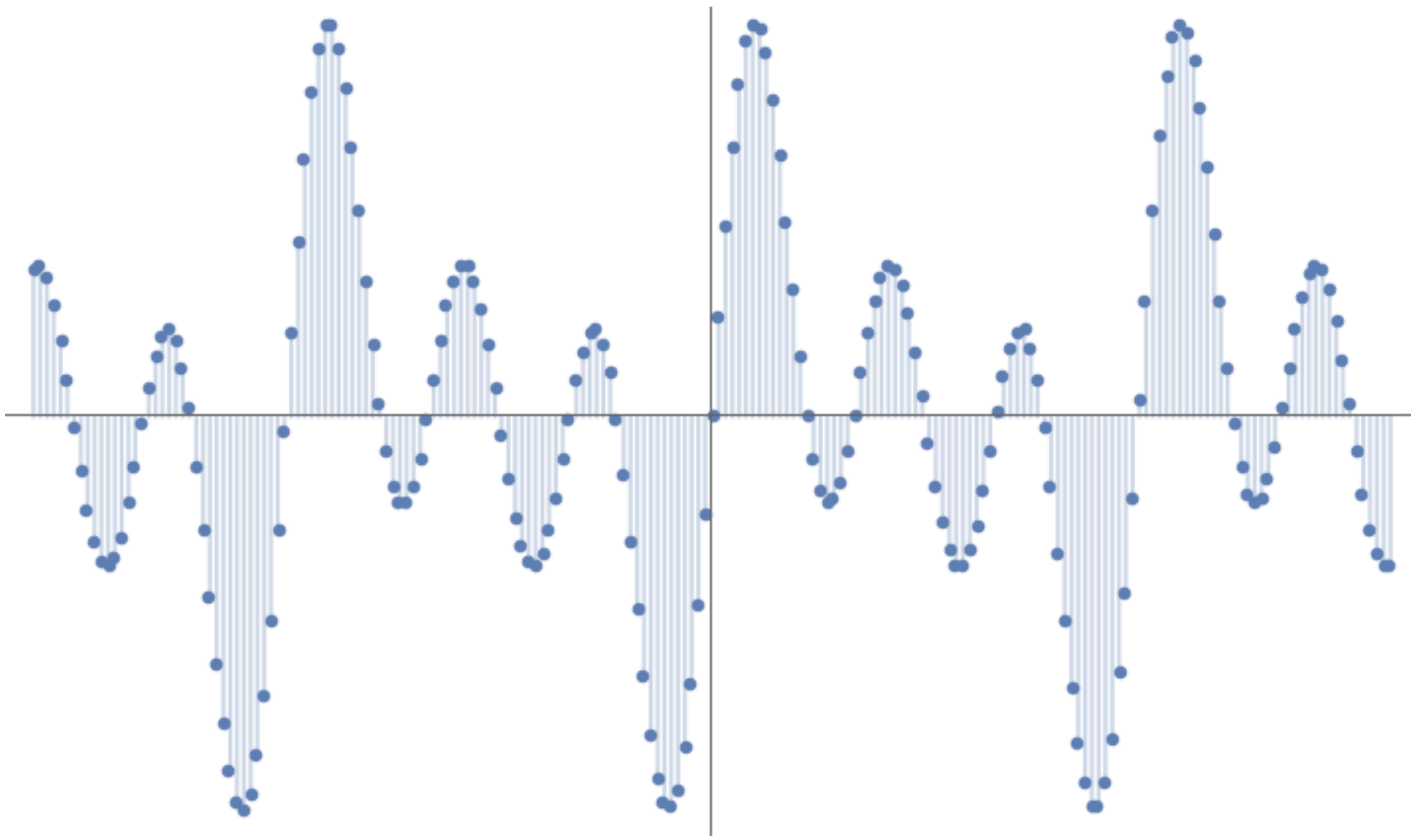
Lattice QCD - Euclidean Spacetime

Sign Problem

complex $S(\mathcal{C})$

$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

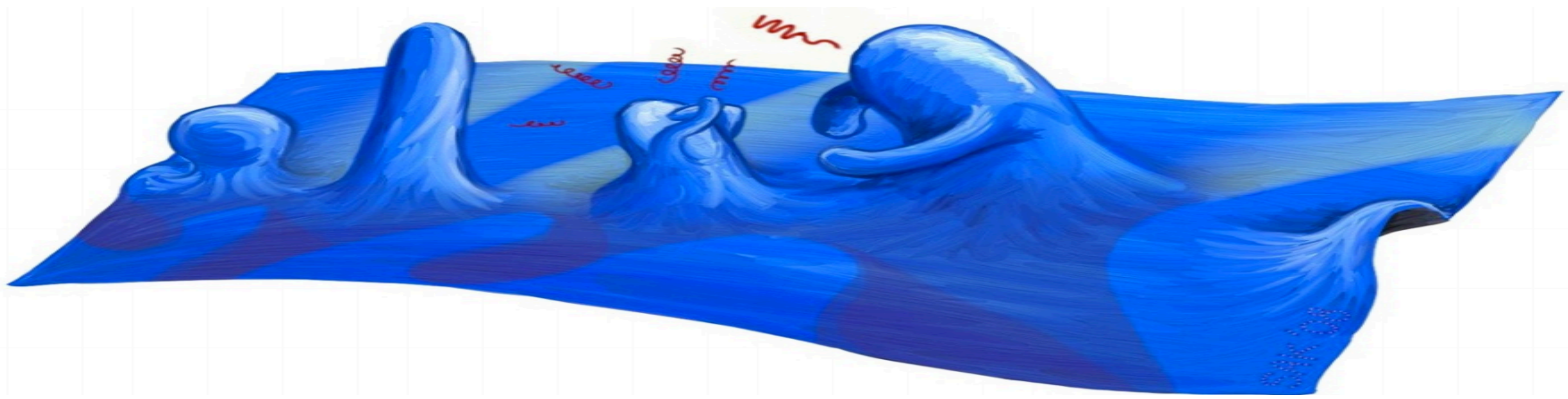
$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$



configuration space $\mathcal{C}$ is exponentially large in system size

system size N_V : number of lattice sites

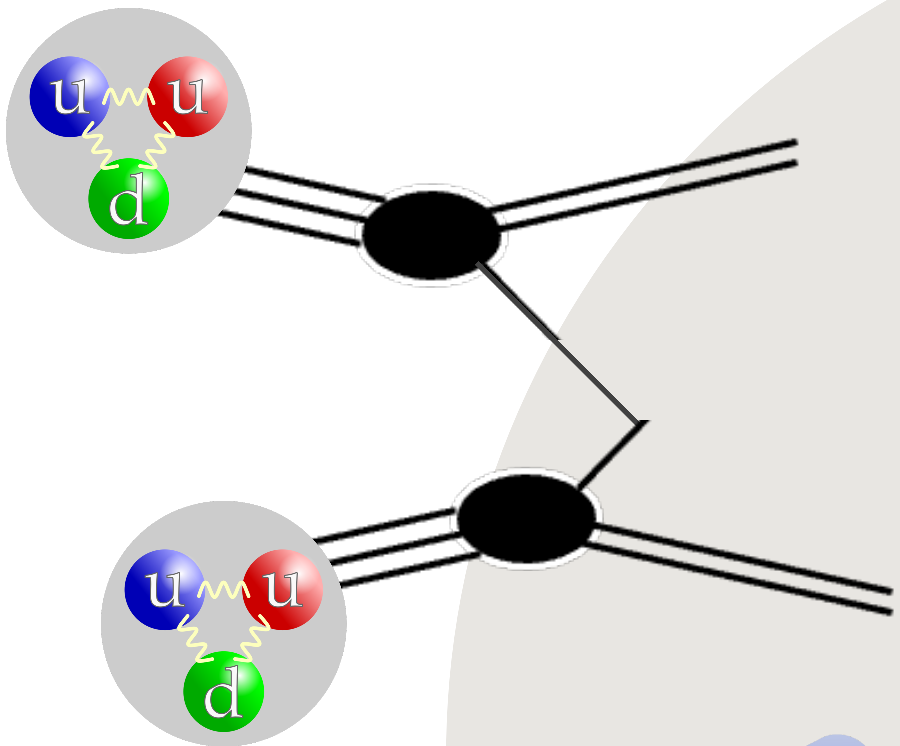
$$\int \mathcal{D}\phi e^{iS} = \langle x| \, e^{-iHt} \, |y\rangle$$



$$\text{dim}H \propto |G|^{N_V}$$

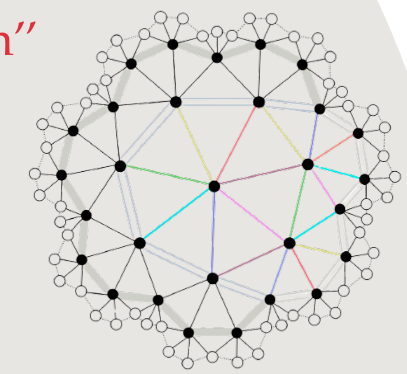
system size N_V : number of lattice sites

exponentially large number
of classical bits in system size

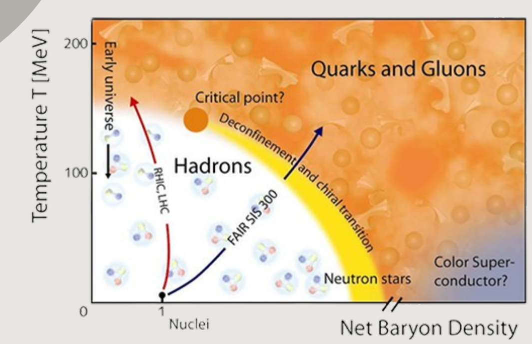


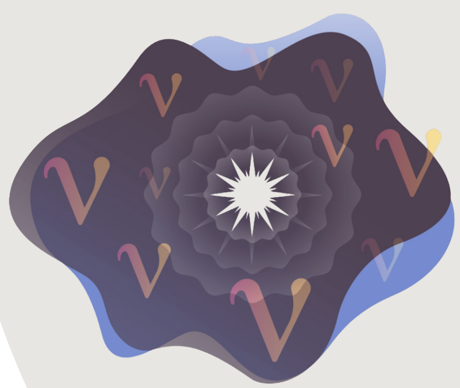
High Energy Physics

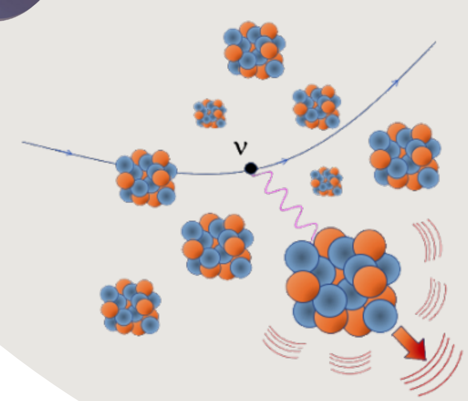
real-time dynamics
finite density
quantum interference
out-of equilibrium
``strongly interacting many-body system"

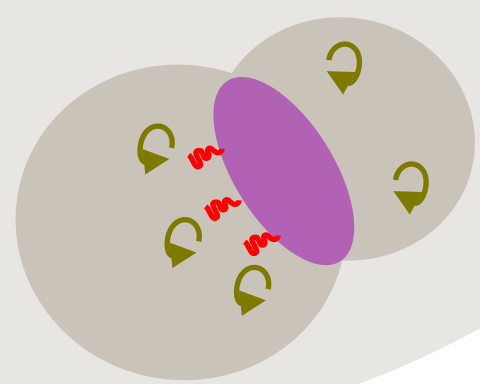


CLASSICAL
EASY
polynomial time









``a computing system that scales well with the system size?"

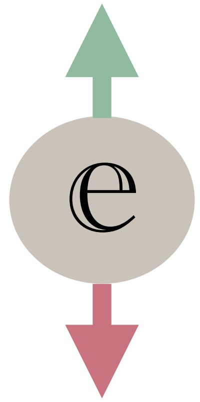
Quantum Computing

1982 "nature isn't classical"

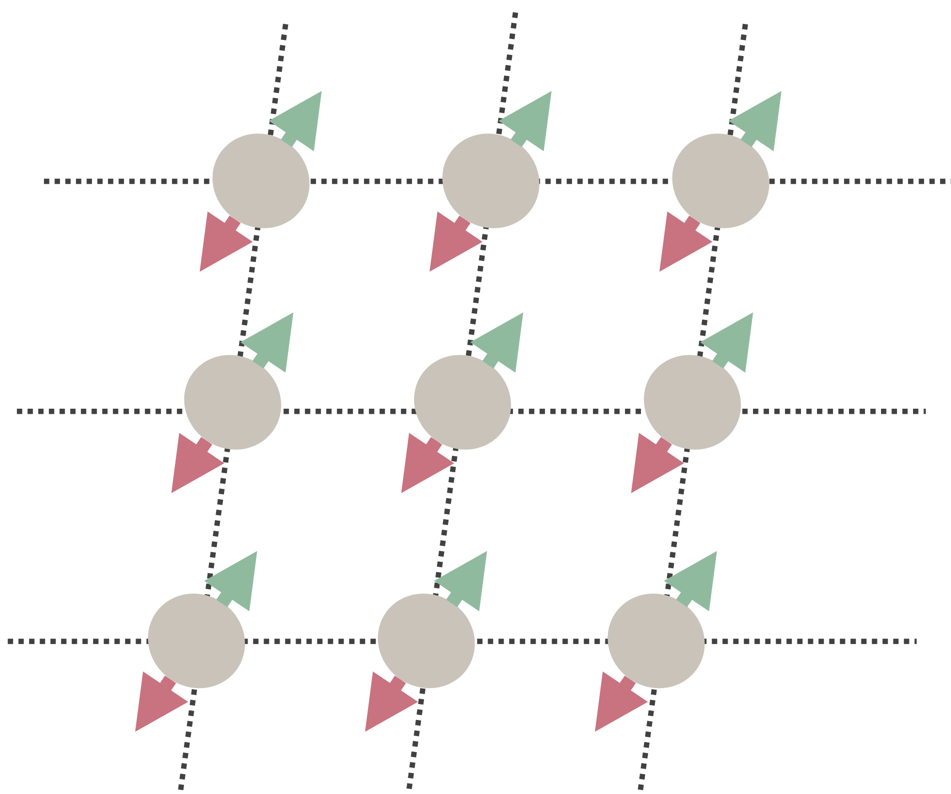
R. P. Feynman

" if you want to make a simulation of Nature, you'd better make it quantum mechanical"

Quantum Computing



$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$



$$N_q = 300$$

2^{300} classical bits are needed to describe the system of 300 qubits

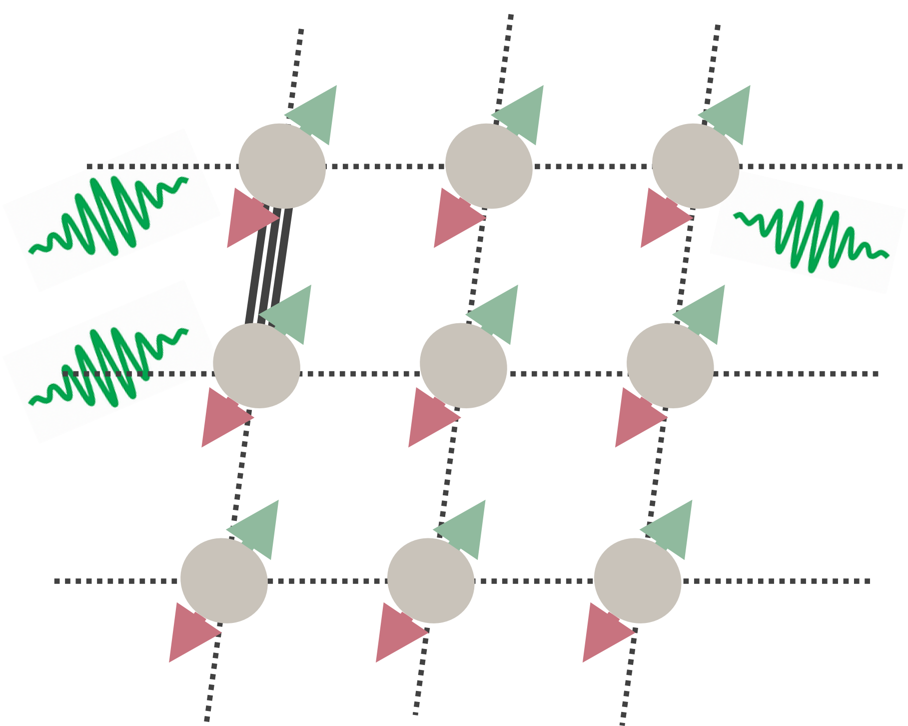
``a computer that uses qubits"

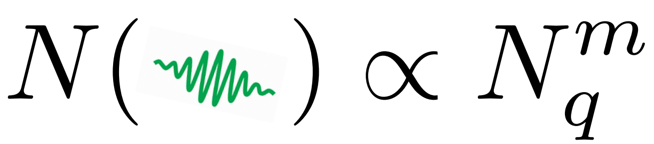
- 1996 - Seth LIoyd: efficient simulation of LOCAL Hamiltonians

Universal Quantum Simulators

Seth Lloyd

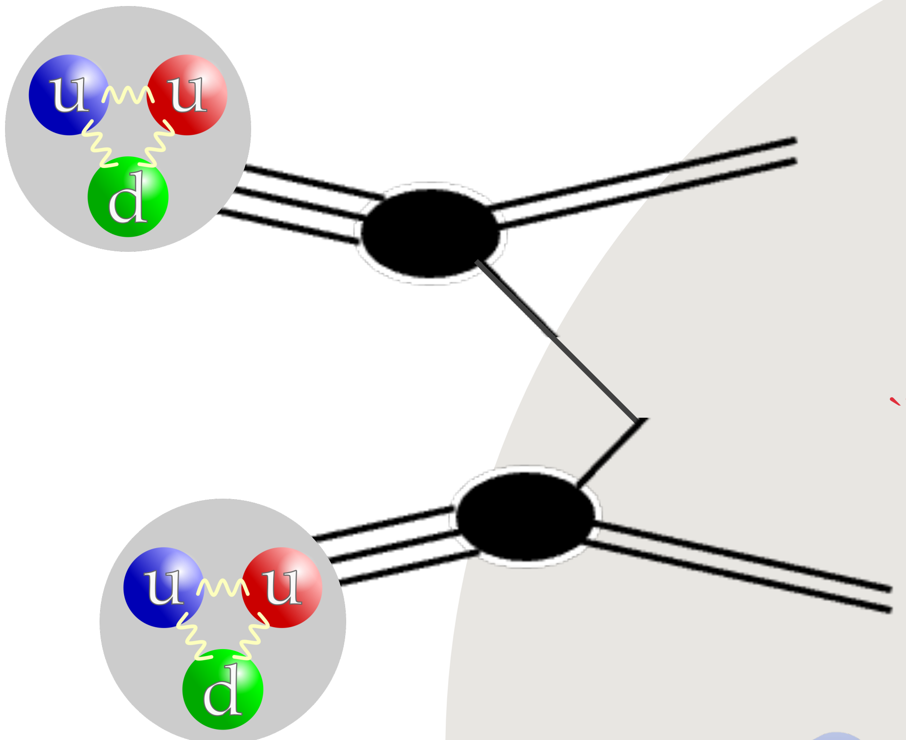
Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.





Polynomial Time Complexity

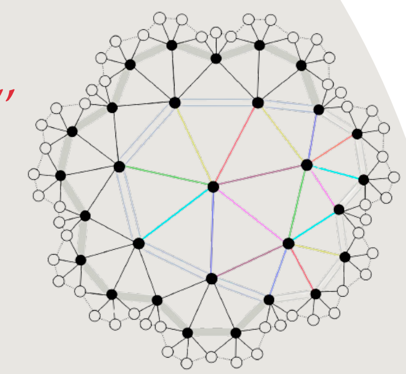
How Powerful It is?



QUANTUM EASY

High Energy Physics

real-time dynamics
finite density
quantum interference
out-of equilibrium
``strongly interacting many-body system''

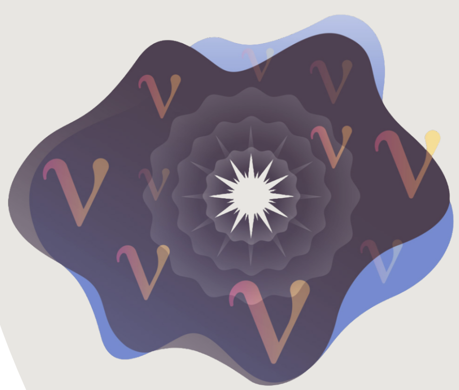


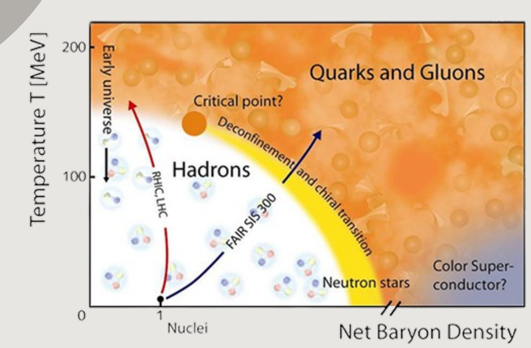
CLASSICAL
EASY

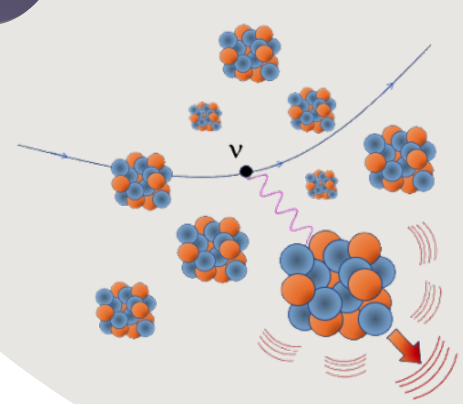
polynomial time

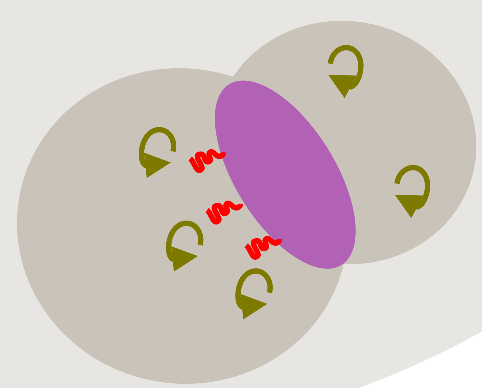
QUANTUM HARD

e.g. traveling salesmen problem





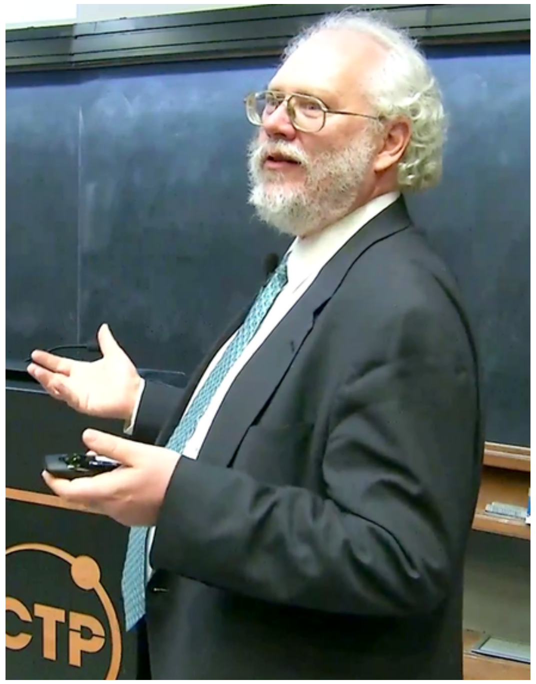




Quantum Computing

- 1990s - error-correcting codes and fault-tolerant methods

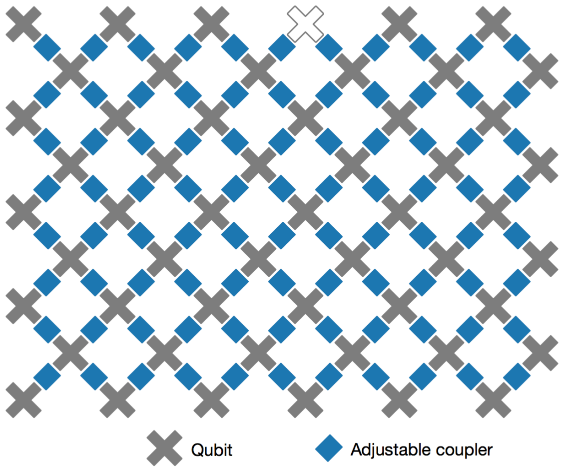
  Quantum Threshold Theorem



P. Shor

- 2019 - Google: Quantum Computational Supremacy

  The quantum hardware is producing meaningful results

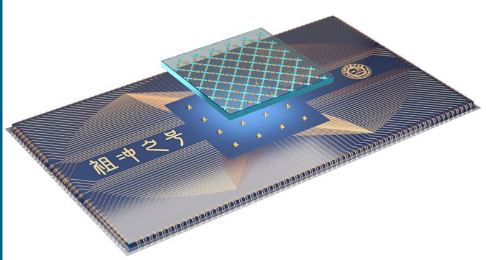


Quantum Computing

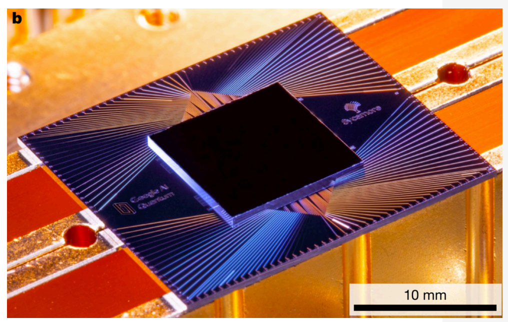
Now - Noisy Intermediate Scale Quantum (NISQ) era

more than 50 well controlled qubits, not error-corrected yet

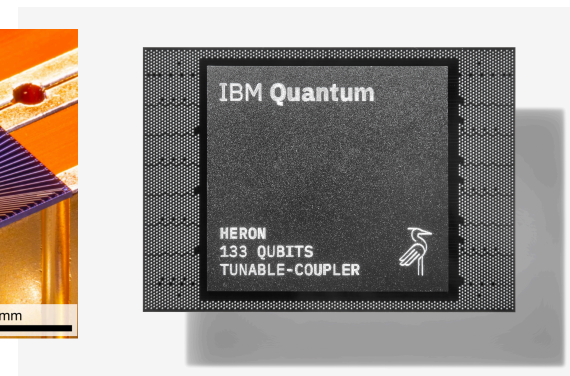
superconducting processor



176 qubits



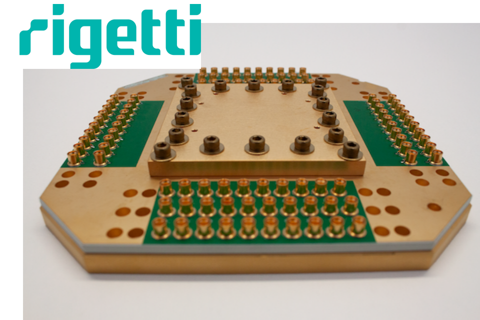
54 qubits



1121 qubits
access to 133 qubits

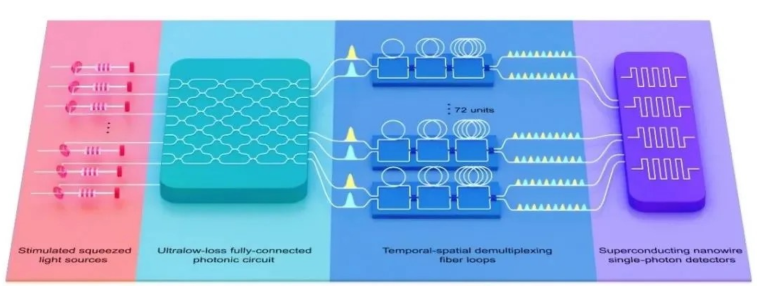
multi-chip quantum processor

rigetti



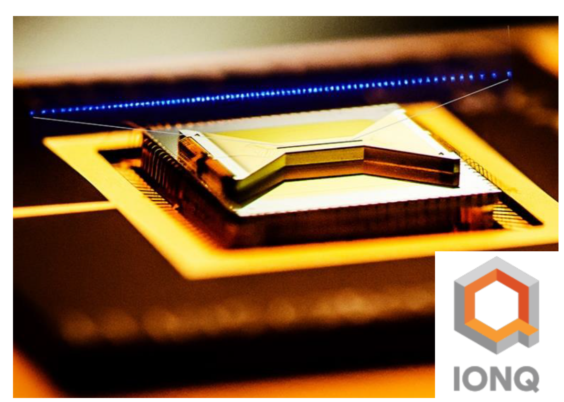
80 qubits

photon qubits



Jiuzhang - 255 qubits

trapped ion qubits



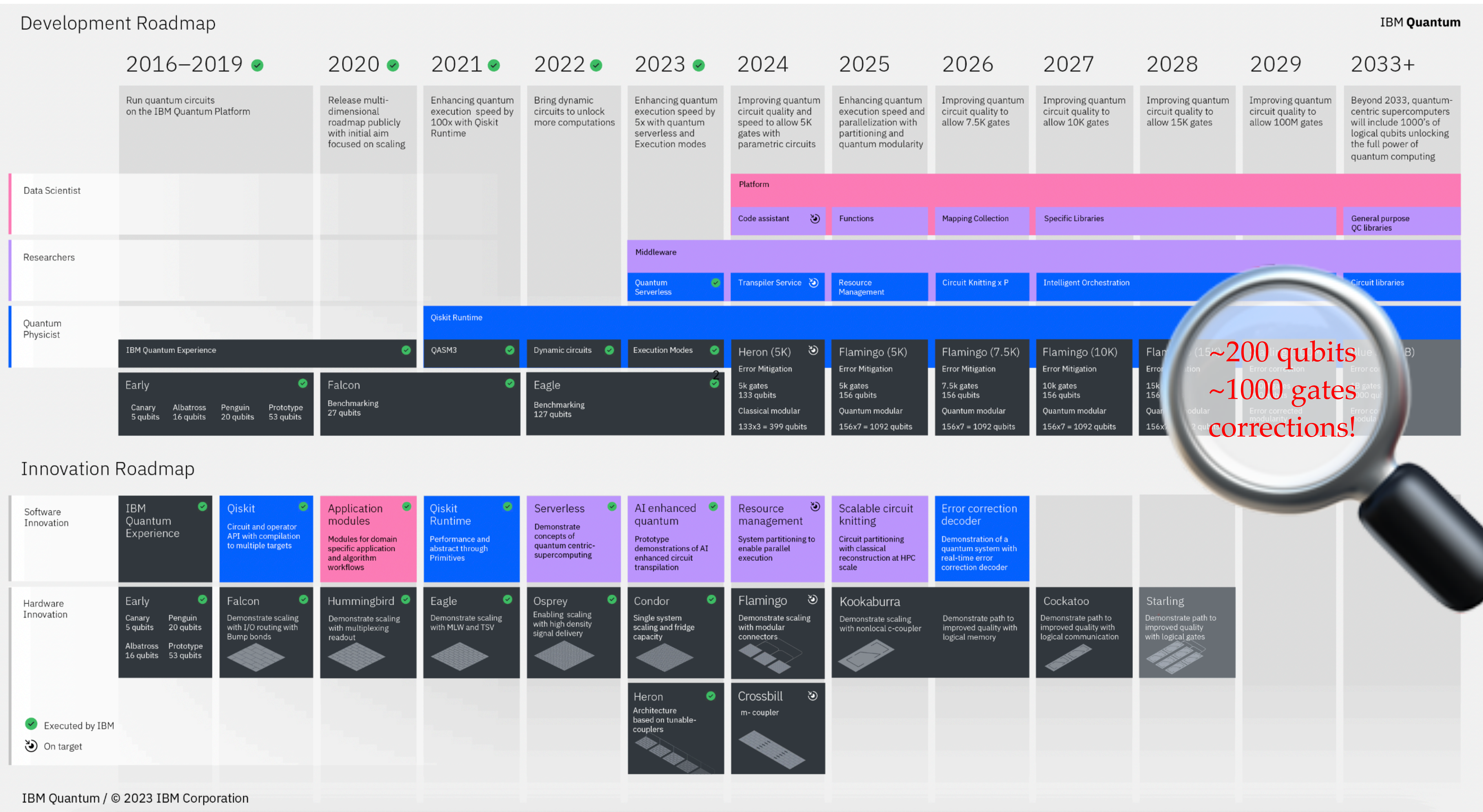
IONQ

22 qubits

48 logical qubits

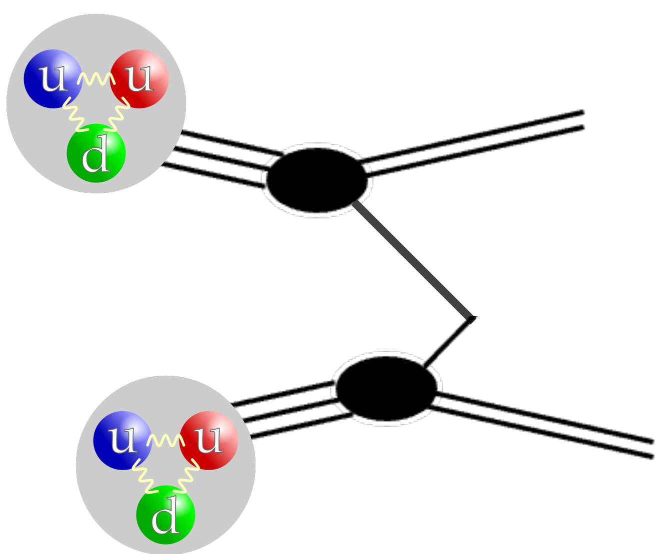
Quantum Computing

Next decades



Quantum Computing for HEP

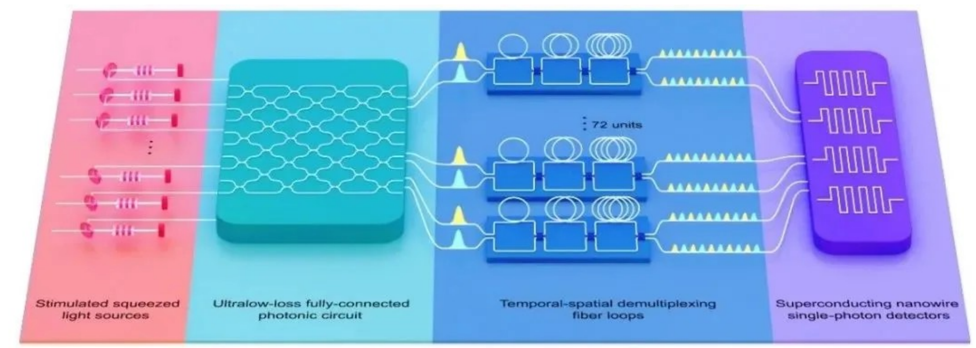
$$\int \mathcal{D}\phi e^{iS} = \langle x| e^{-iHt} |y\rangle$$

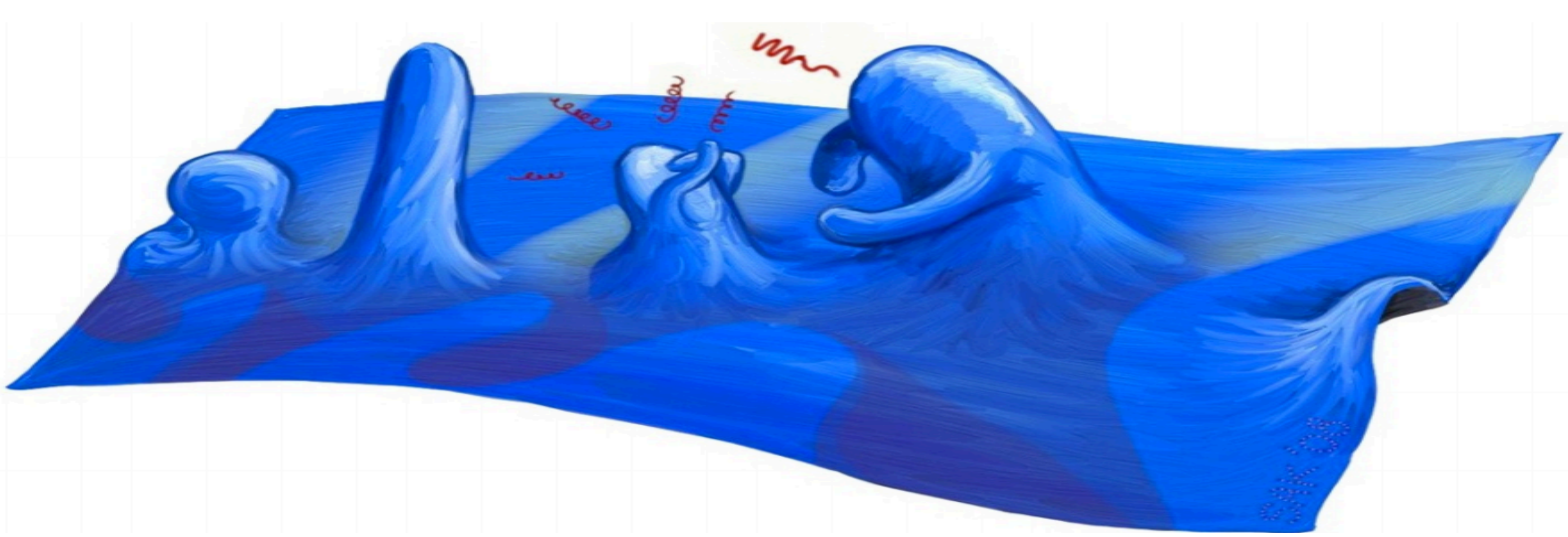


mapping

DOF to qubits

time evolution to quantum gates





non-trivial vacuum,
composite initial state,
bosonic and fermionic DOF,
symmetries, …

(2010s) galactic algorithms

(2020s) pocket of methods for every steps,
continuum limits,
error corrections

(2030s) ?

$|q\rangle^N \to |G\rangle$

↓

$\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$

↓

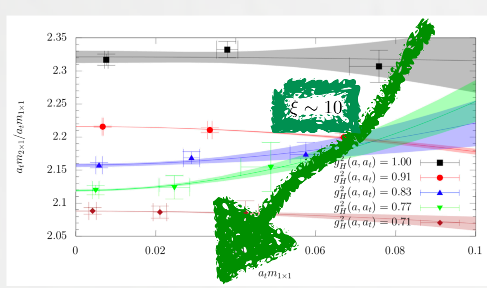
$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

↓

$\langle \mathcal{O} \rangle$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

various methods



2011-

2020 -

2030s -

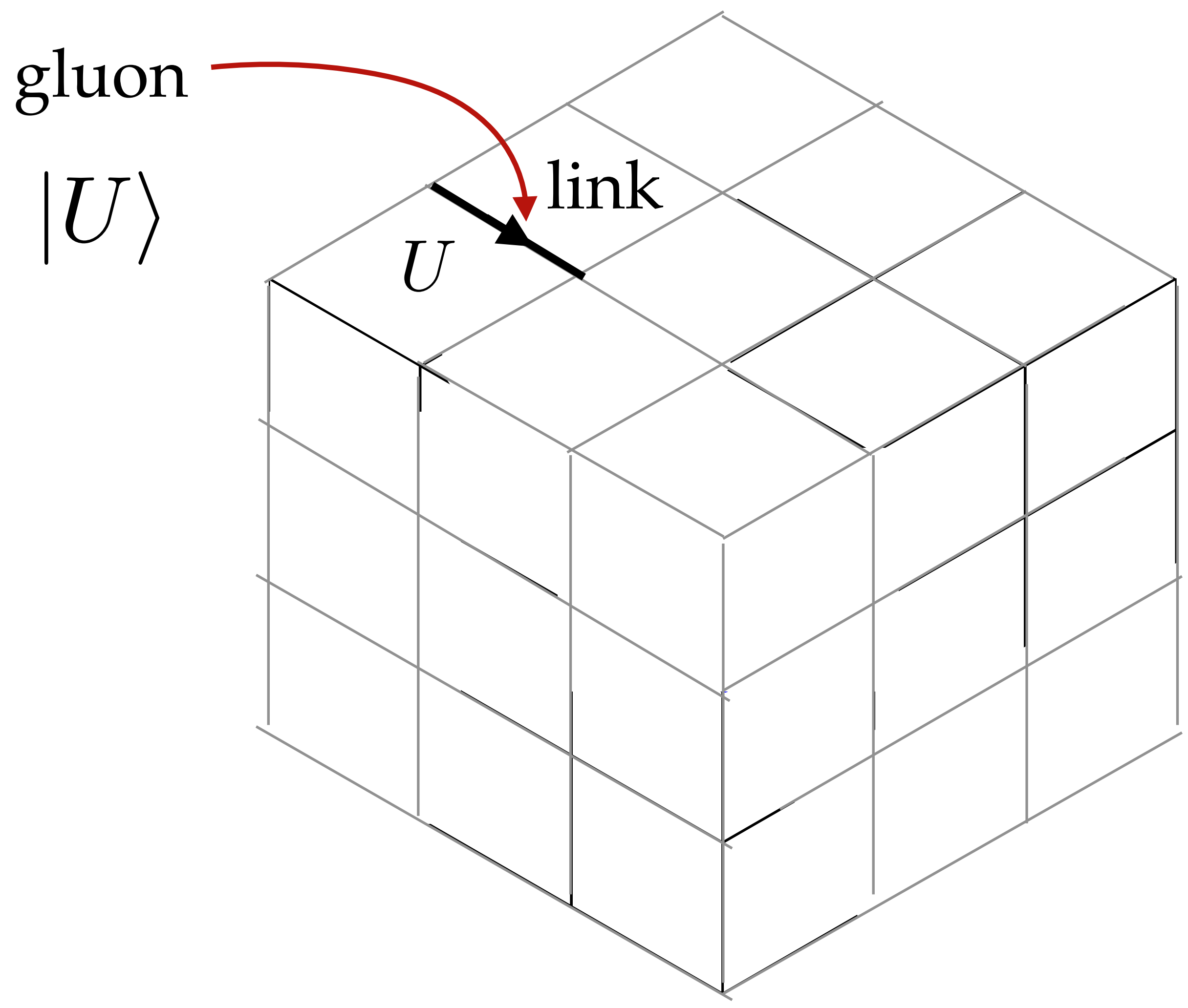
Gauge Symmetries in Quantum Simulations

- Gauge transformation
  - Hamiltonians
  - redundant Hilbert space
- Gauge redundancy utilized for error corrections
- Error threshold for gauge redundant encodings
- Time-evolution with gauge redundant encodings

M. Carena, H. Lamm,**YYL**, W. Liu, arXiv:2402.16780

M. Carena, H. Lamm,**YYL**, W. Liu, PRL. **129**, 051601

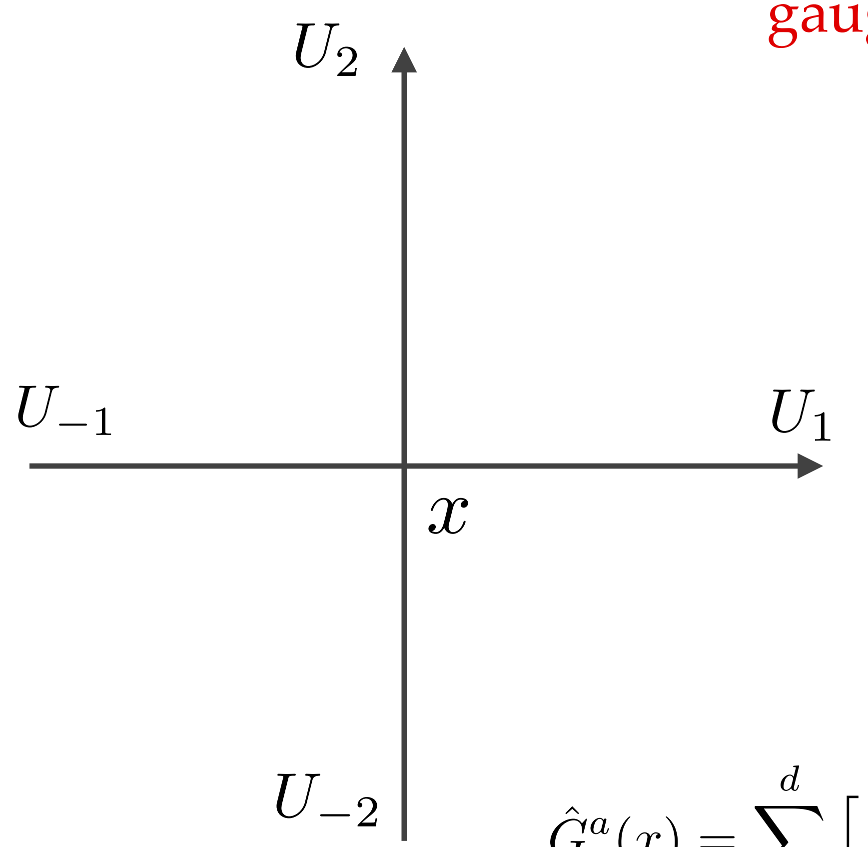
Gauge transformations

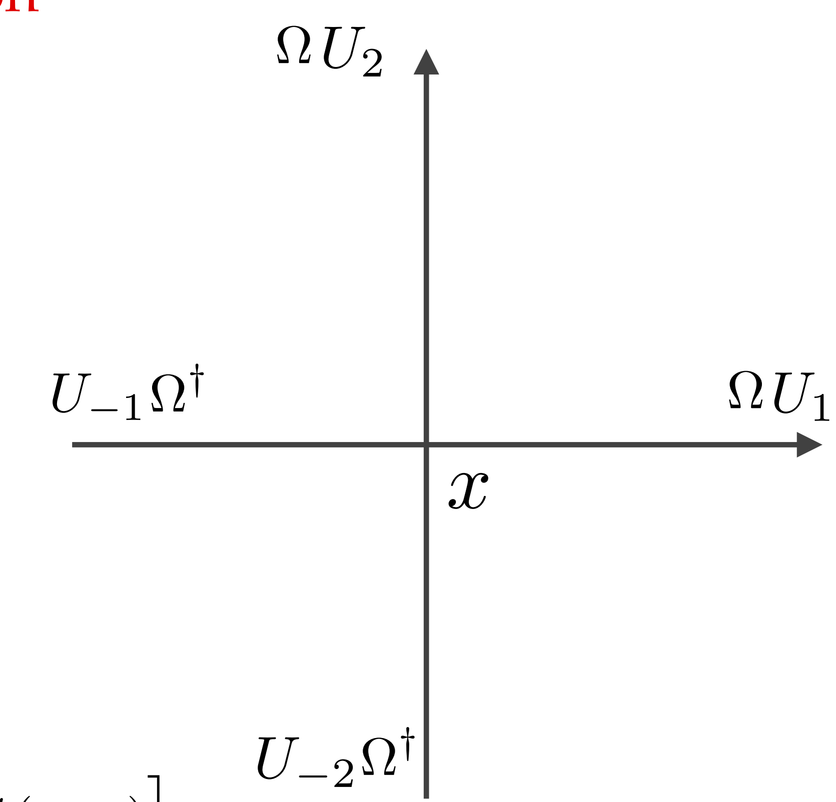


Gauge transformations

$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

gauge transformation



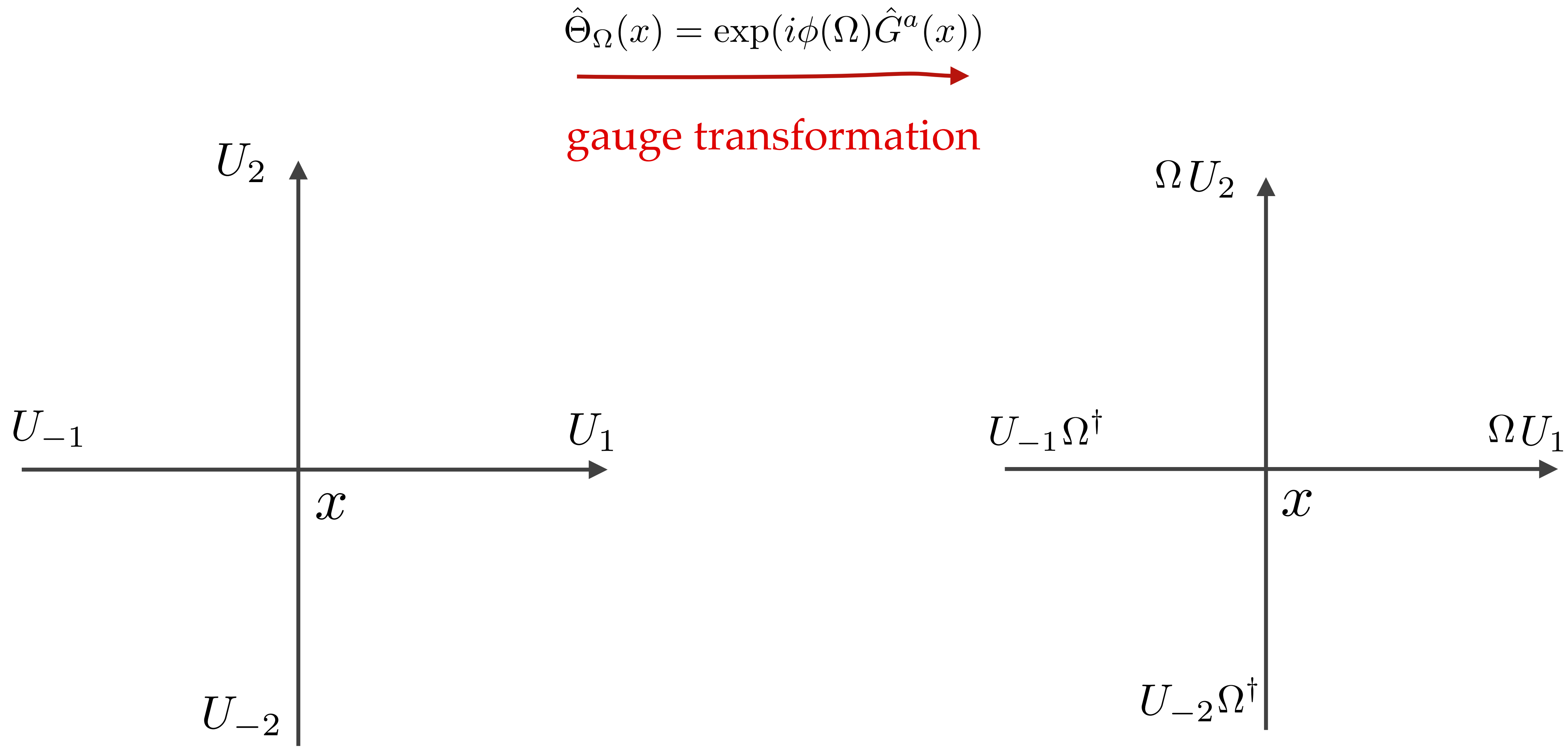


$$\hat{G}^a(x) = \sum_{i=1}^{d} \left[\hat{E}^a_R(x - e_i, e_i) - \hat{E}^a_L(x, e_i)\right]$$

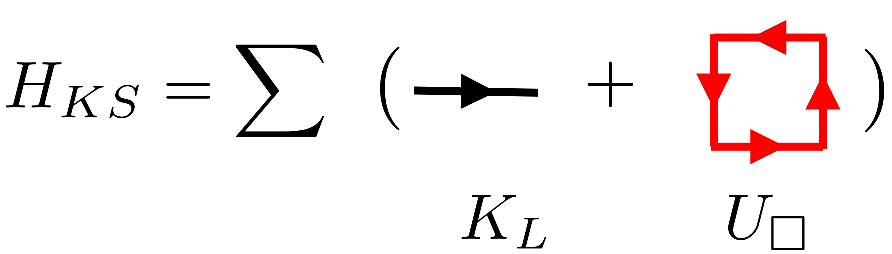
lattice analog of covariant
divergence of chromo-electric field

$$\text{quadratic Casmir}: \quad \hat{E}^2 \left|jm_L m_R\right\rangle = j(j+1)\left|jm_L m_R\right\rangle \qquad \left|jm_L m_R\right\rangle \overset{\text{FT}}{\leftrightarrow} \left|U\right\rangle$$

Gauge transformations

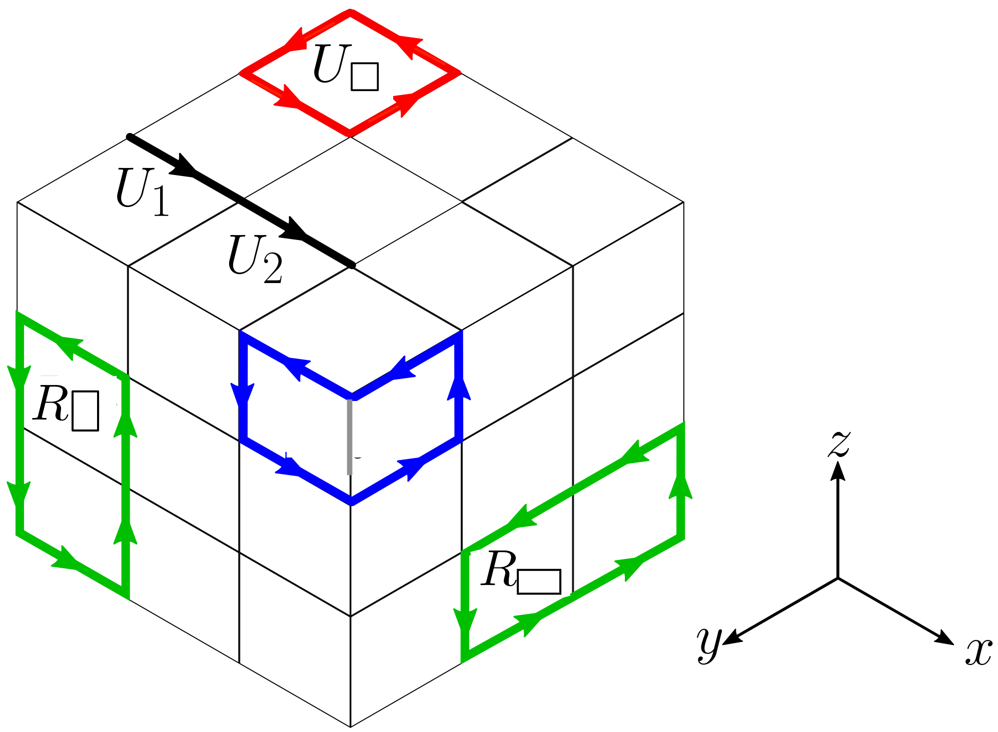


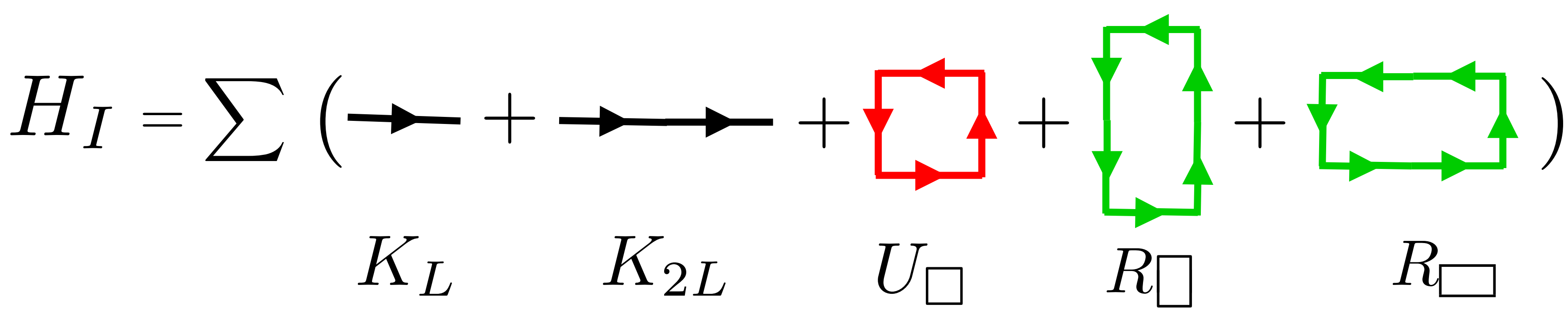
gauge invariant Hamiltonian



quadratic Casimir

Gauge transformations



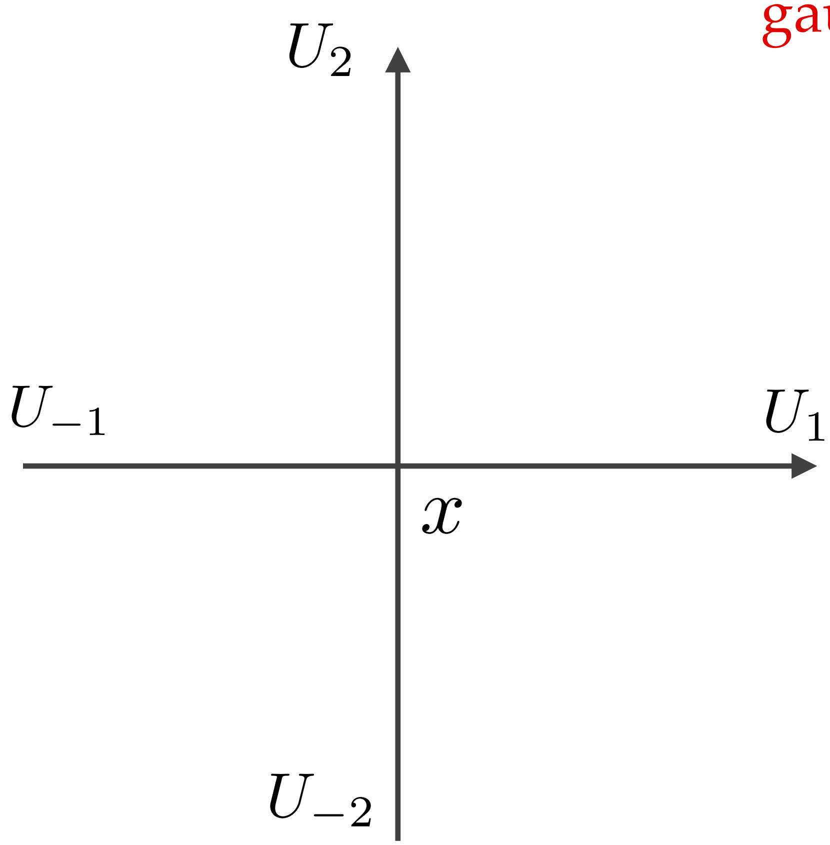


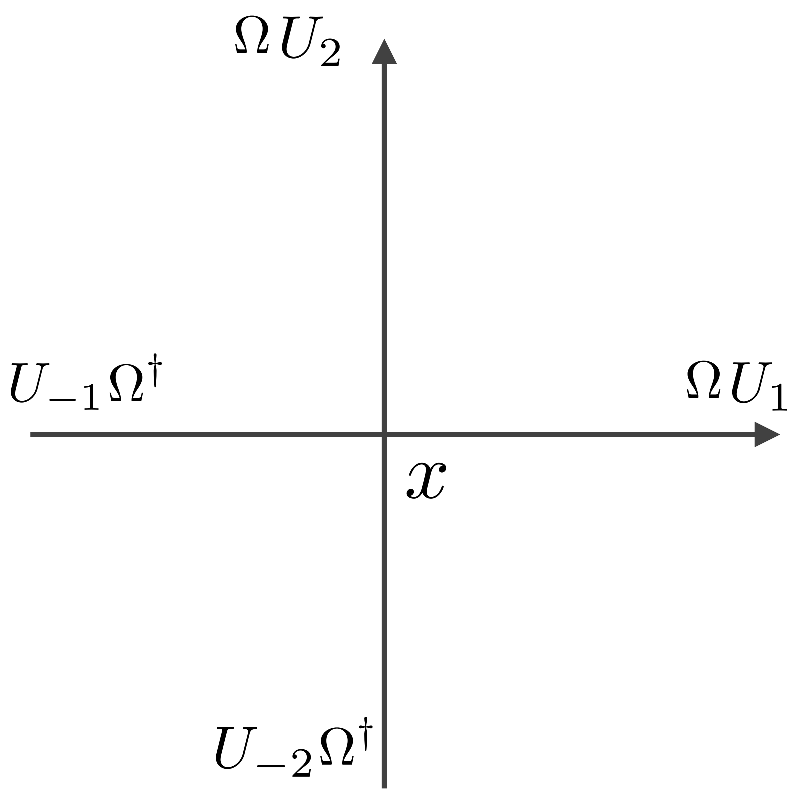
improved Hamiltonian

Gauge transformations

$$\hat{\Theta}_{\Omega}(x) = \exp(i\phi(\Omega)\hat{G}^{a}(x))$$

gauge transformation





gauge equivalent states

$$\hat{\Theta}_{\Omega}(x)\left|U_{-1}U_{1}U_{-2}U_{2}\right\rangle = \left|U'_{-1}U'_{1}U'_{-2}U'_{2}\right\rangle$$

Gauge transformations

gauge invariant states

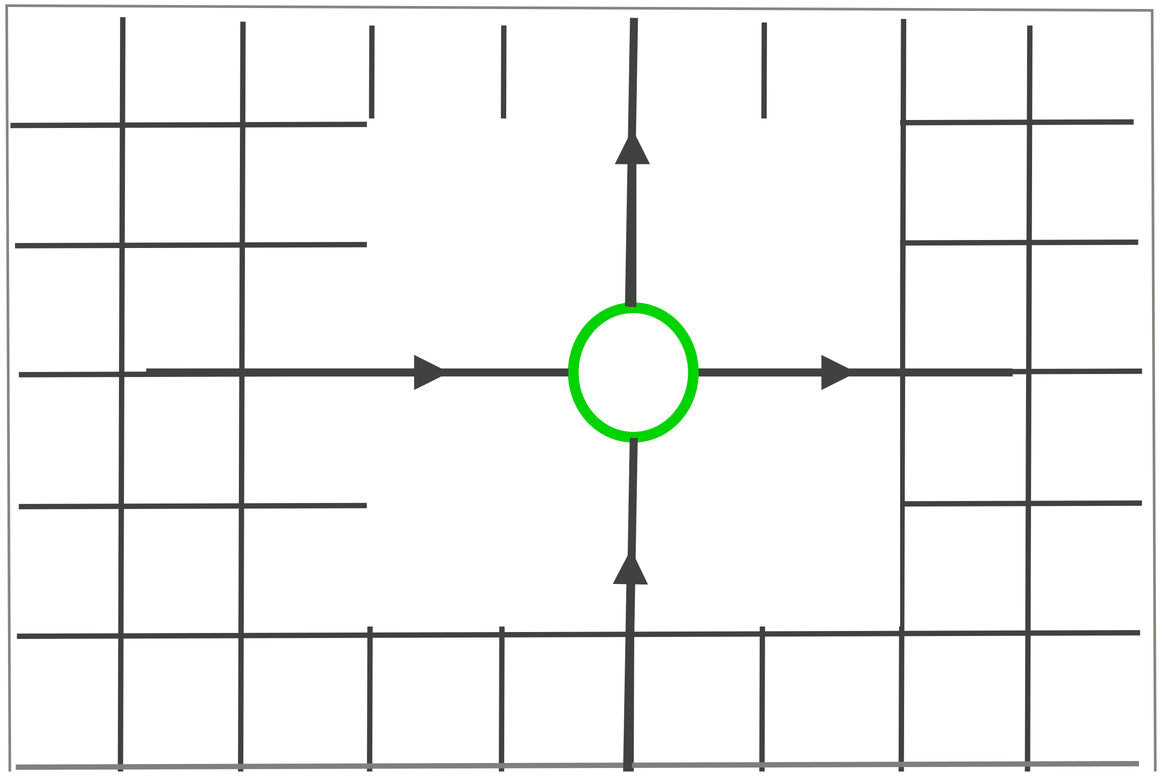
$$\hat{\Theta}_\Omega(x)\,|\psi_{\rm phys}\rangle = |\psi_{\rm phys}\rangle$$

neutral charge

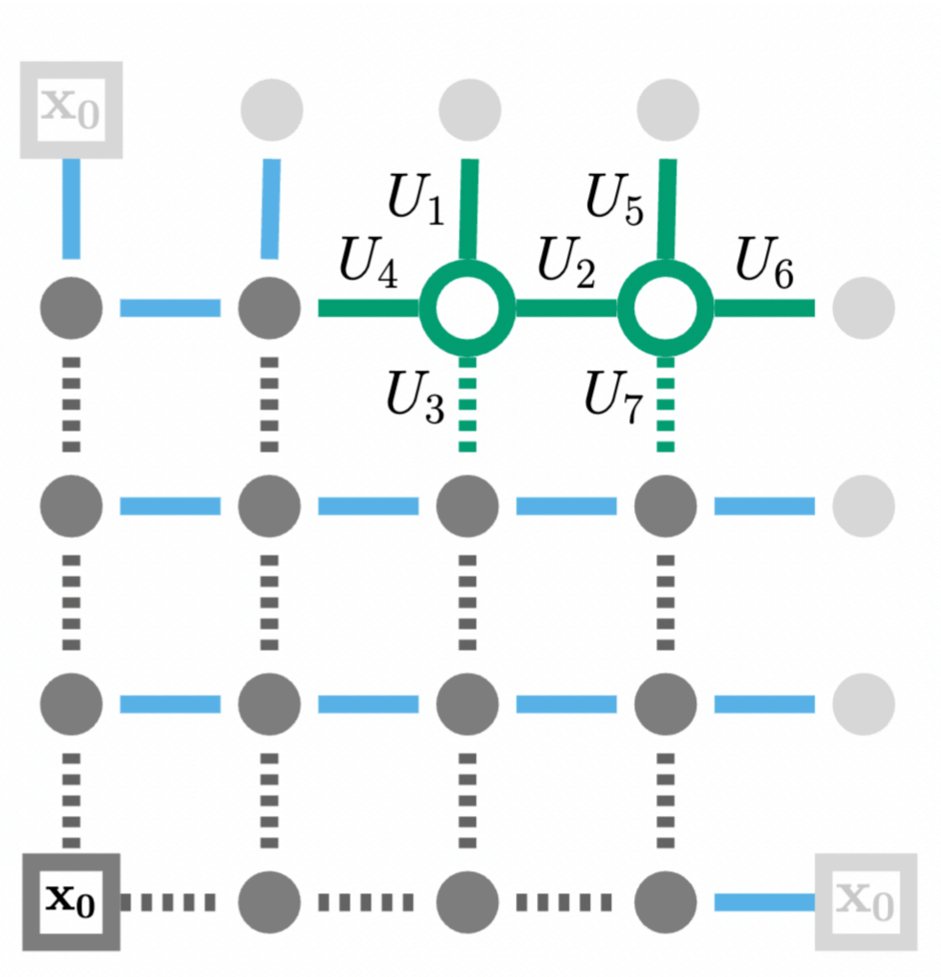
$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

$$\hat{G}^a(x) = \sum_{i=1}^{d}\left[\hat{E}^a_R(x-e_i,e_i) - \hat{E}^a_L(x,e_i)\right]$$

$$\hat{G}^a(x)\,|\psi_{\rm phys}\rangle = 0$$



Gauge transformations



$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

gauge redundant

$$\hat{G}^a(x)\,|\psi\rangle = 0$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

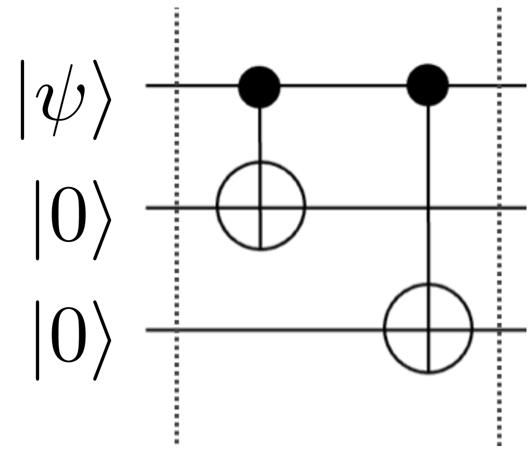
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

Gauge redundancy utilized for error corrections

quantum error corrections

$|\psi\rangle$

$\alpha|0\rangle + \beta|1\rangle$



$|\Psi\rangle = \alpha|000\rangle + \beta|111\rangle$

undetectable errors

$\mathcal{H}_{\text{full}}$:

$|001\rangle, |010\rangle, |100\rangle$

$|011\rangle, |110\rangle, |101\rangle$

quantum errors

detectable errors

$\mathcal{H}_{\text{code}} : |111\rangle, |000\rangle$

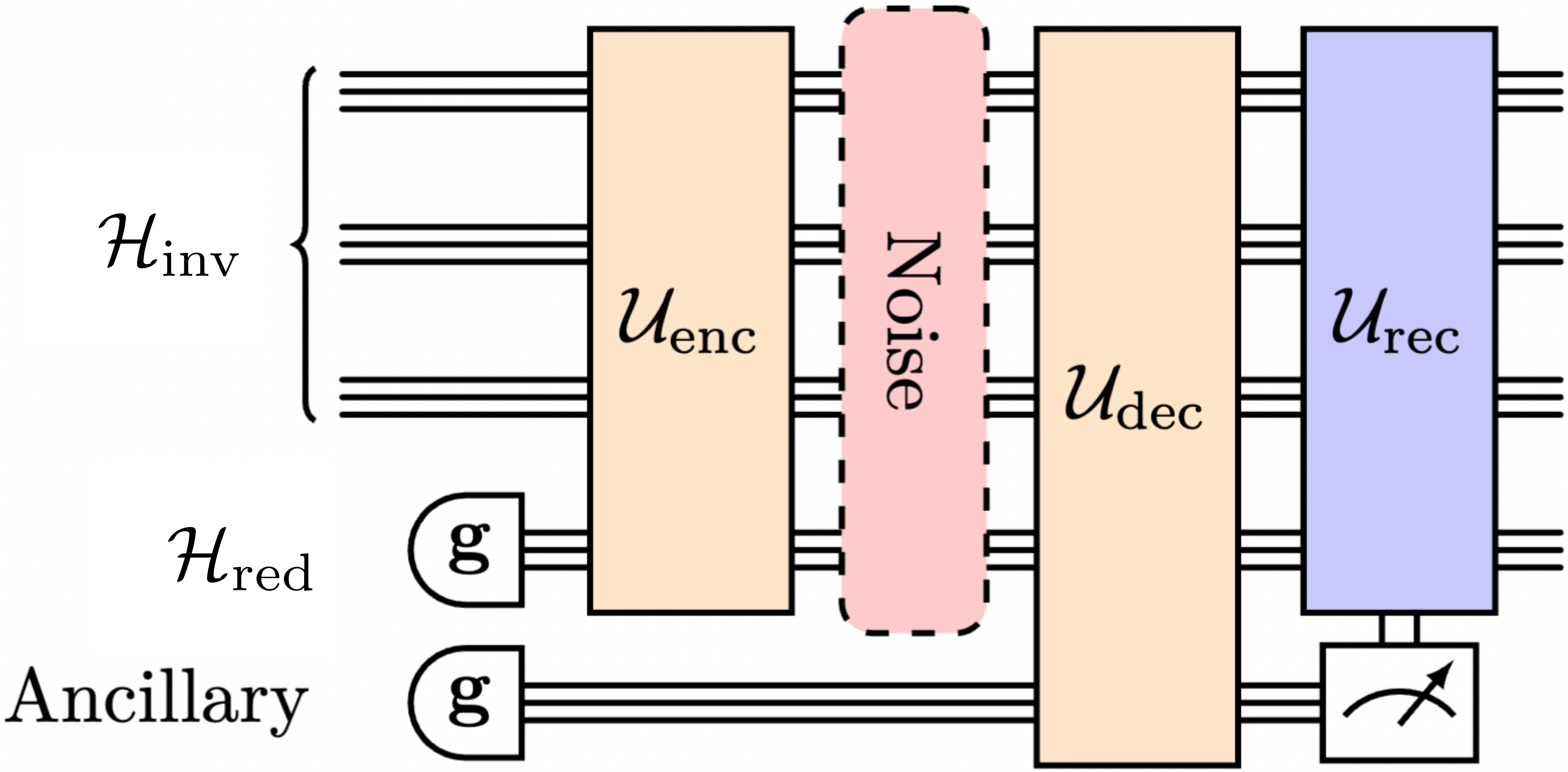
$\hat{\Theta}_{\mathcal{S}}(x)|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$

$$\hat{\Theta}_{\mathcal{S}} = \{I, Z_1 Z_2, Z_2 Z_3, Z_1 Z_3\}$$

Gauge redundancy utilized for error corrections

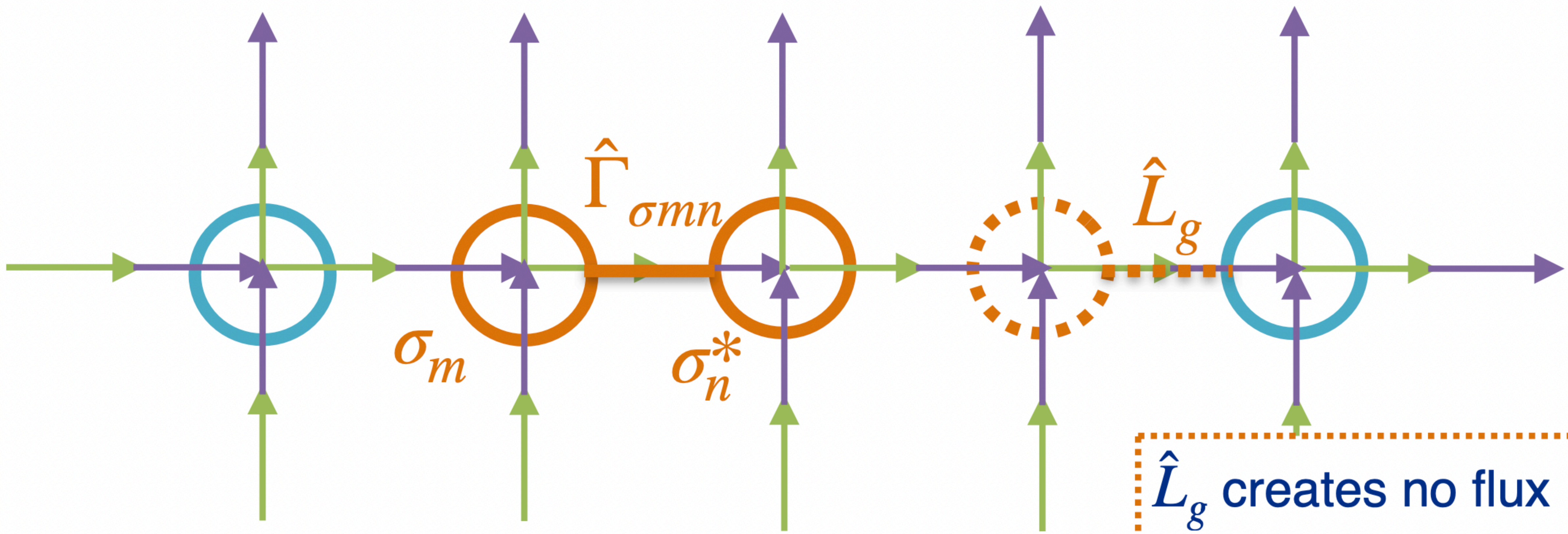
state preparation:
encoding Gauss's law

charge
computation



Gauge redundancy utilized for error corrections

correctable errors



Gauss's law:
0 net flux

$\hat{\Gamma}_\sigma$ creates flux pair
Correctable

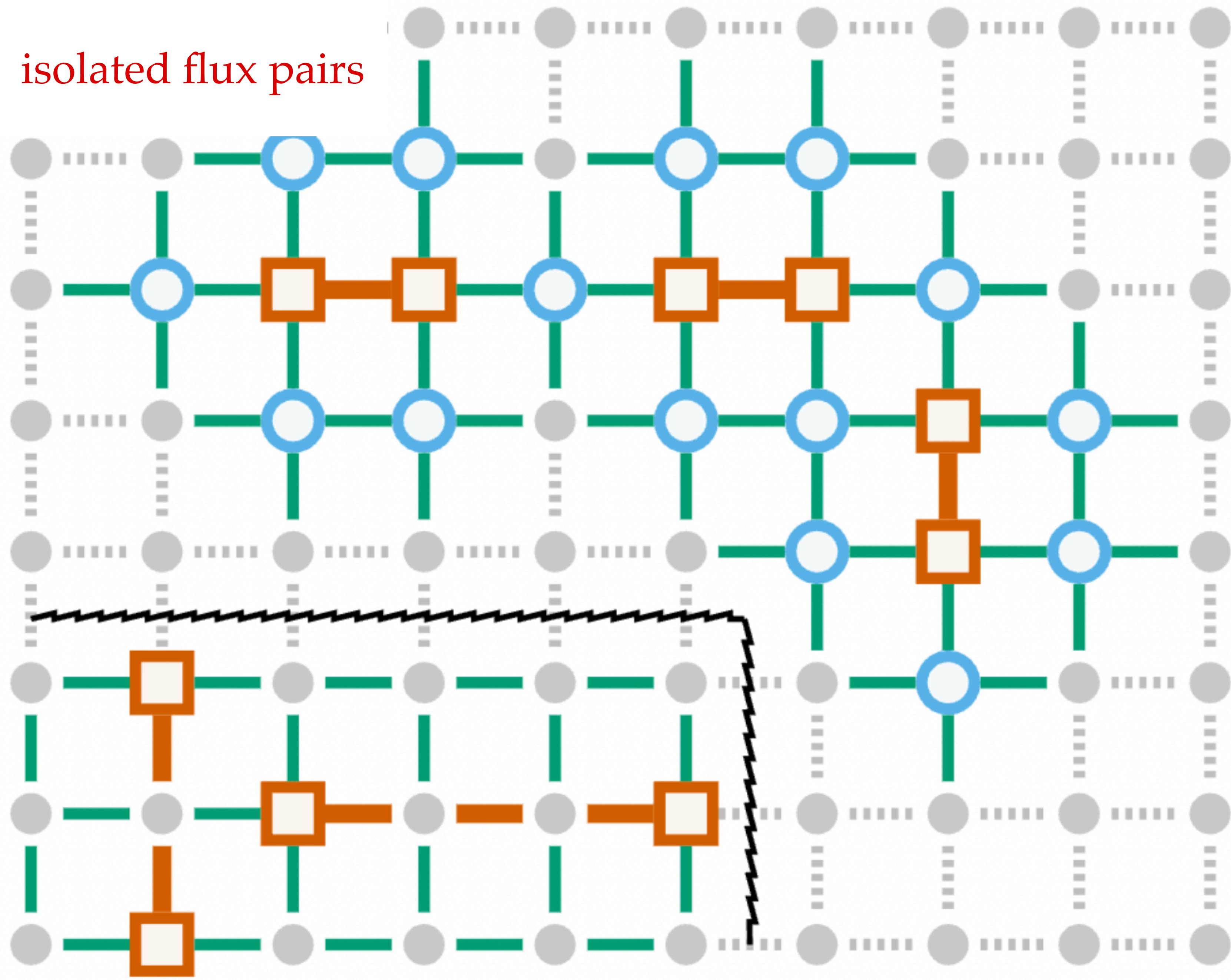
$\hat{L}_g$ creates no flux
(Abelian center)
or 1 flux (non-Abelian)
Not Correctable

Gauge redundancy utilized for error corrections

correctable errors

isolated flux pairs

KL condition



Error threshold for gauge redundant encodings

worthwhile to keep the redundancy?

resource requirements?

easy implementation or
Hamiltonian complexity?

resilience to errors?

Error threshold for gauge redundant encodings

resource requirements?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$N_q = N_L \log |G|$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$N_q = (N_L - N_V + 1) \log |G|$$

Error threshold for gauge redundant encodings

Hamiltonian complexity?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$H_{KS} = \sum \left(\rightarrow + \square \right)$$

K_L $U_\square$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

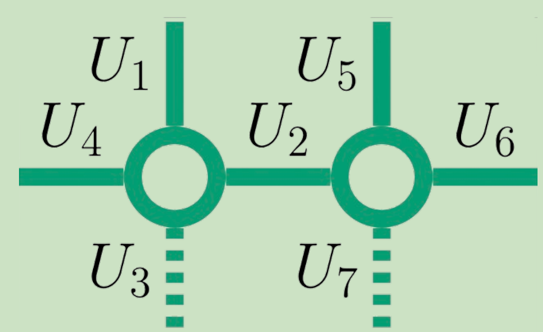
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$H_{KS} = \sum \left(\rightarrow + \square \right)$$

K_L $U_\square$

kinetic terms for U_3, U_7

depend on other links



Error threshold for gauge redundant encodings

resilience to errors?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

+ error correction

single link correctable error rate ϵ_c

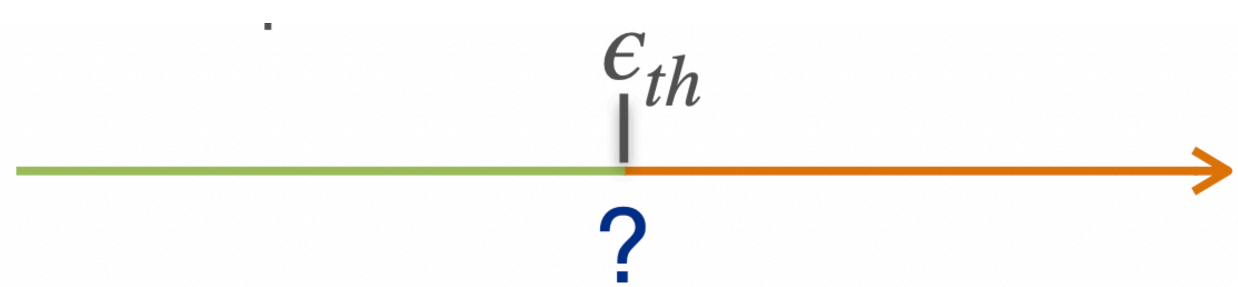
$$F_{\text{restored}} \geq \sum_{n=0}^{N_L} Q_n \epsilon_c^n (1-\epsilon)^{N_L - n}$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

not correctable

$$F_{\text{inv}} \geq (1-\epsilon)^{N_L - N_V + 1}$$

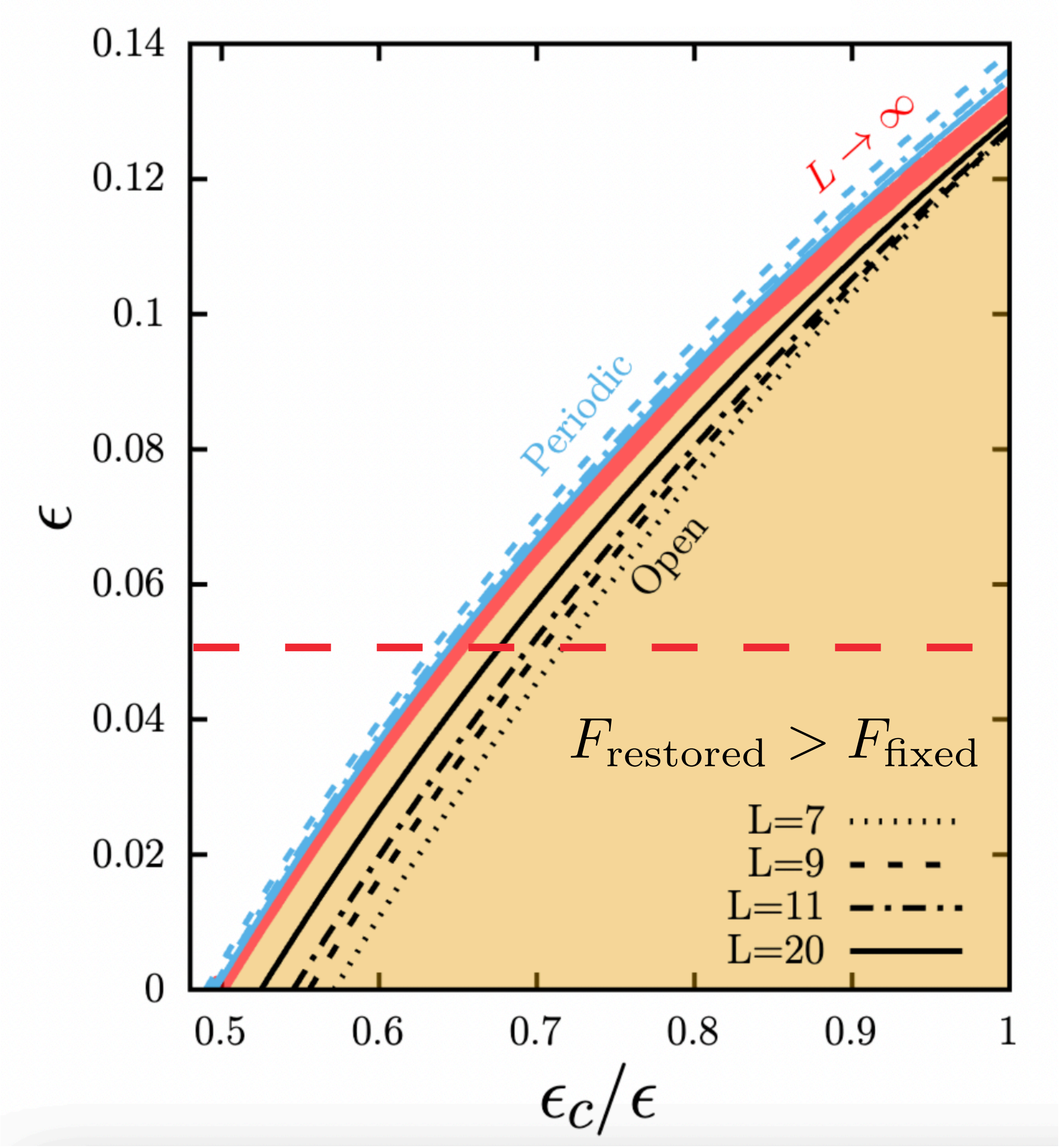


ϵ single link error rate

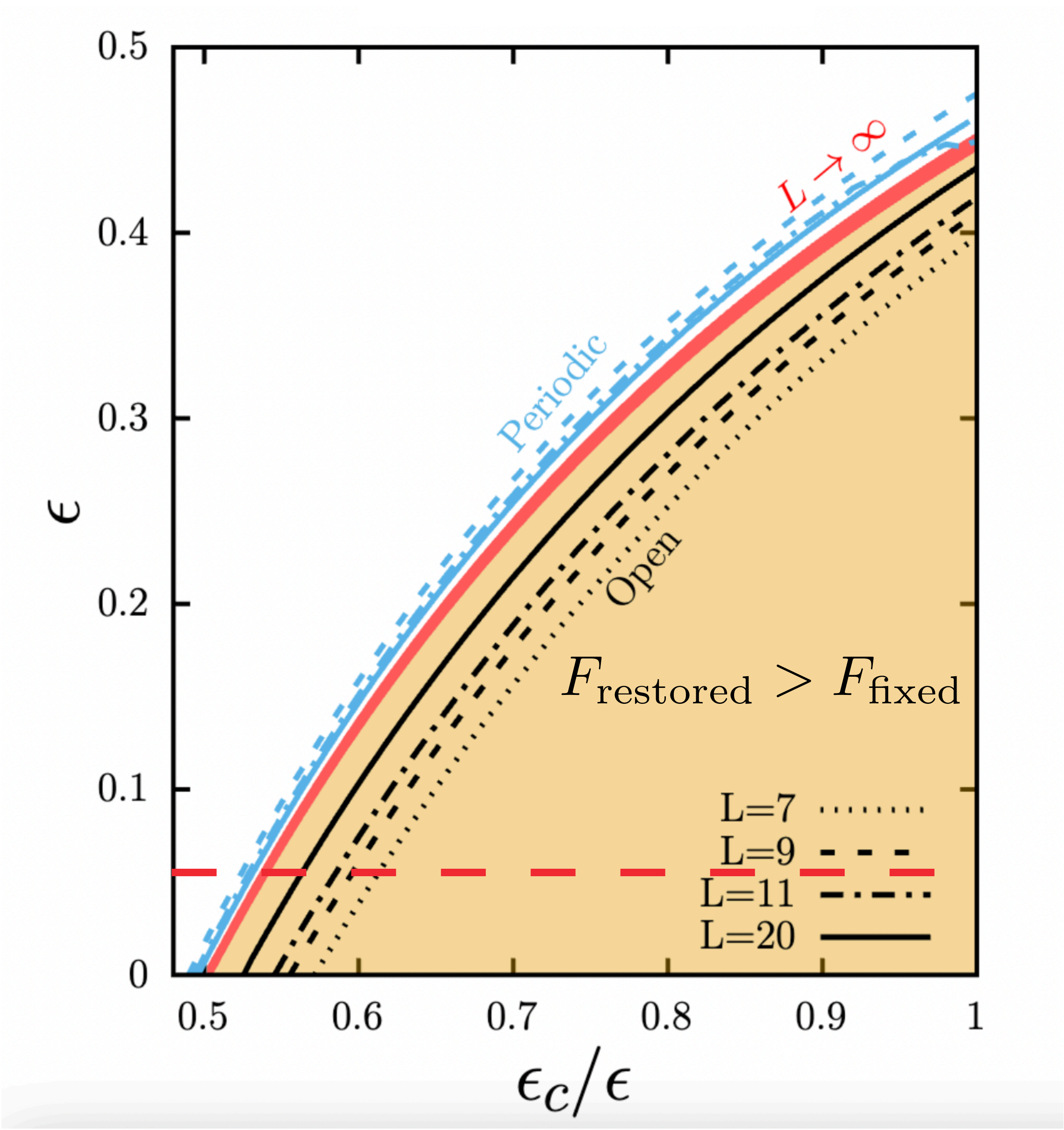
Error threshold for gauge redundant encodings

2d lattice

isolated flux pairs



KL condition



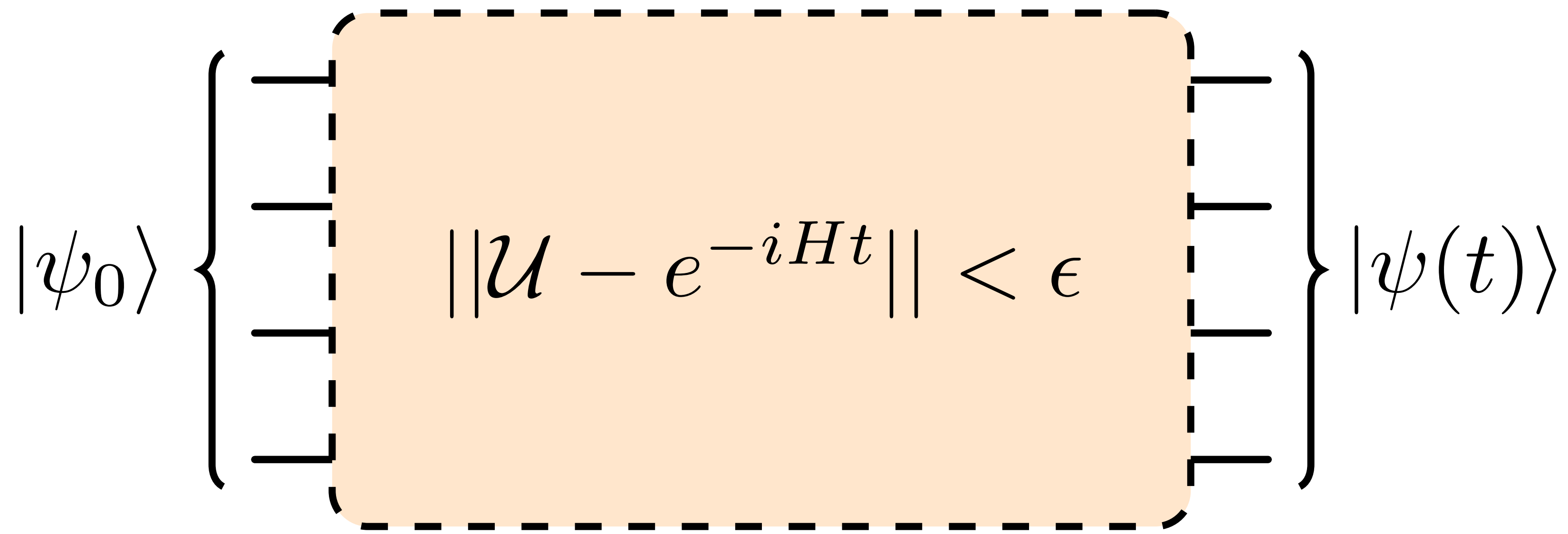
Near-term hardwares are reaching such error threshold!

applying to various methods of
encoding gauge field

including charged matters

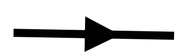
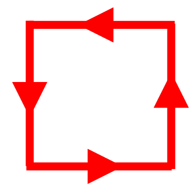
Propagation

digital quantum computer



time-evolution with gauge redundant encodings

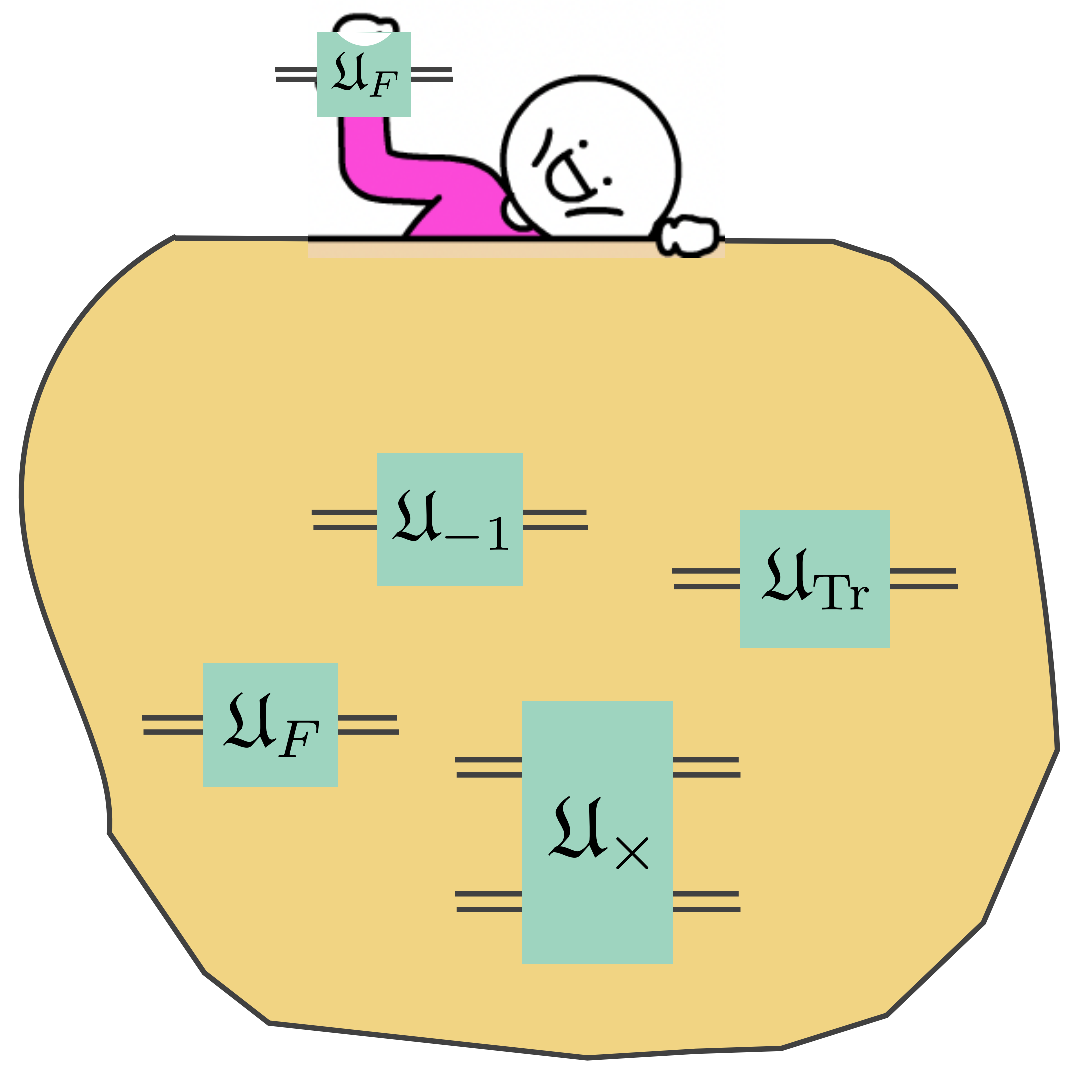
Propagation with gauge redundant encodings

$H_{KS} = \sum \Big($  $+$  $\Big)$

K_L $\qquad$ $U_\square$

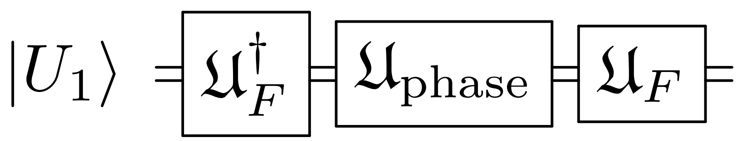
$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_L} e^{-i\delta t U_\square}\right]^{t/\delta t}$$

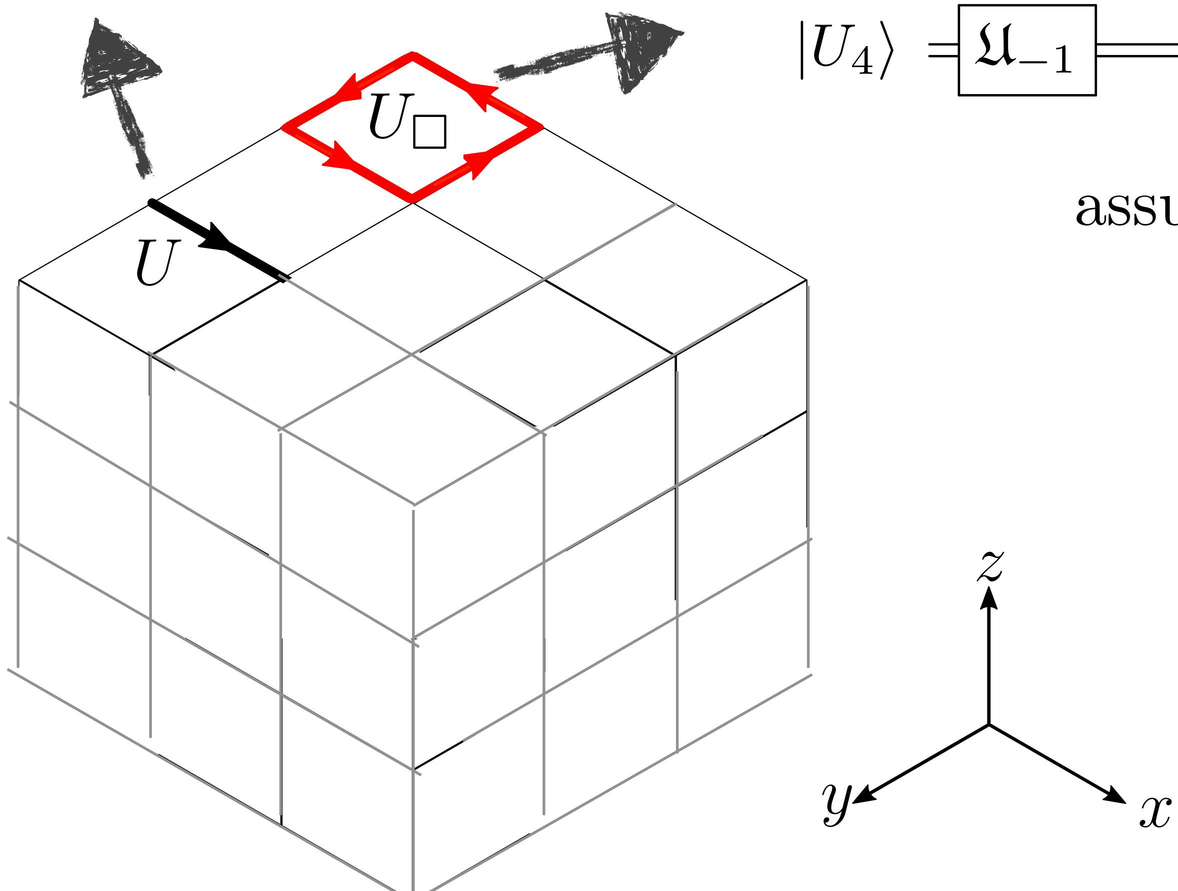
$G-$register : $|U\rangle$



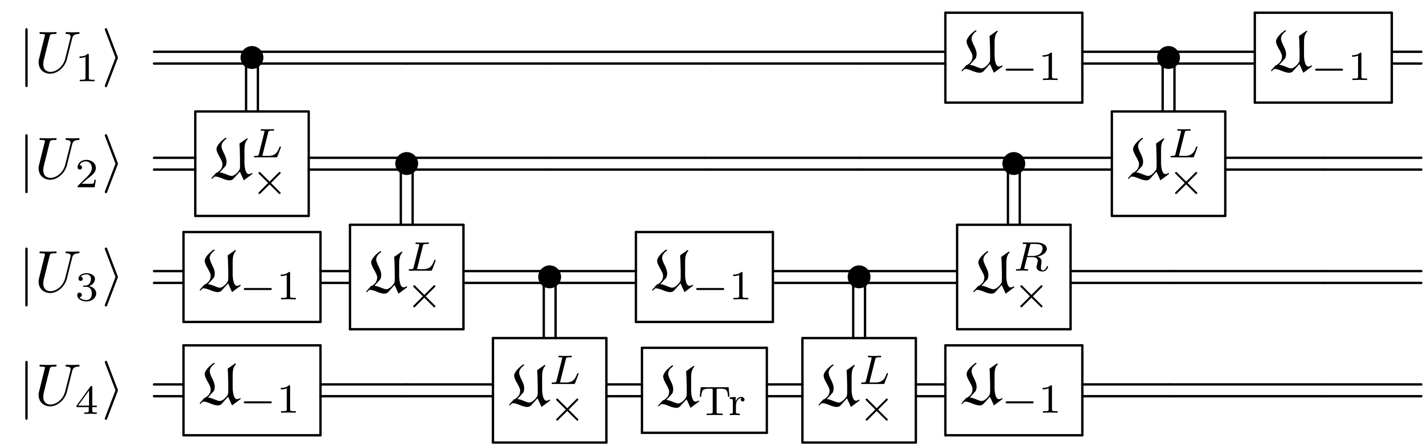
Propagation with gauge redundant encodings

$$e^{-i\delta t K_L}$$



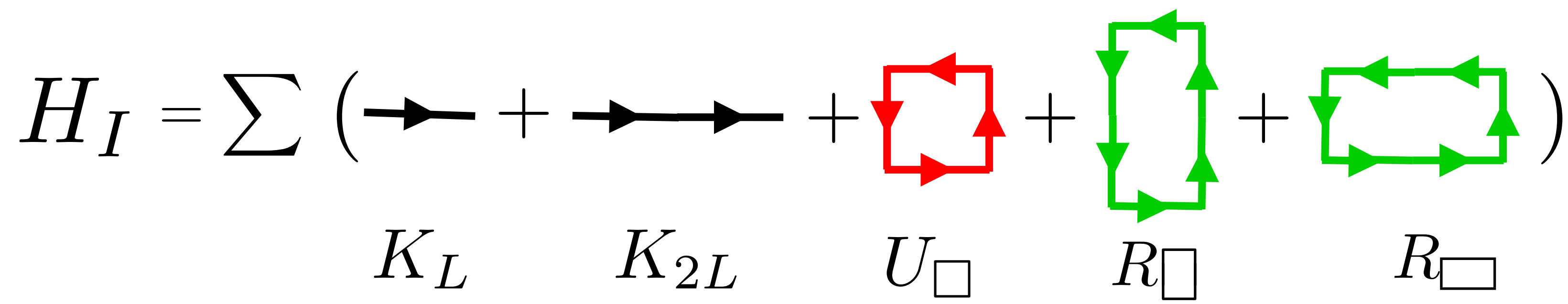


$$e^{-i\delta t U_\square}$$

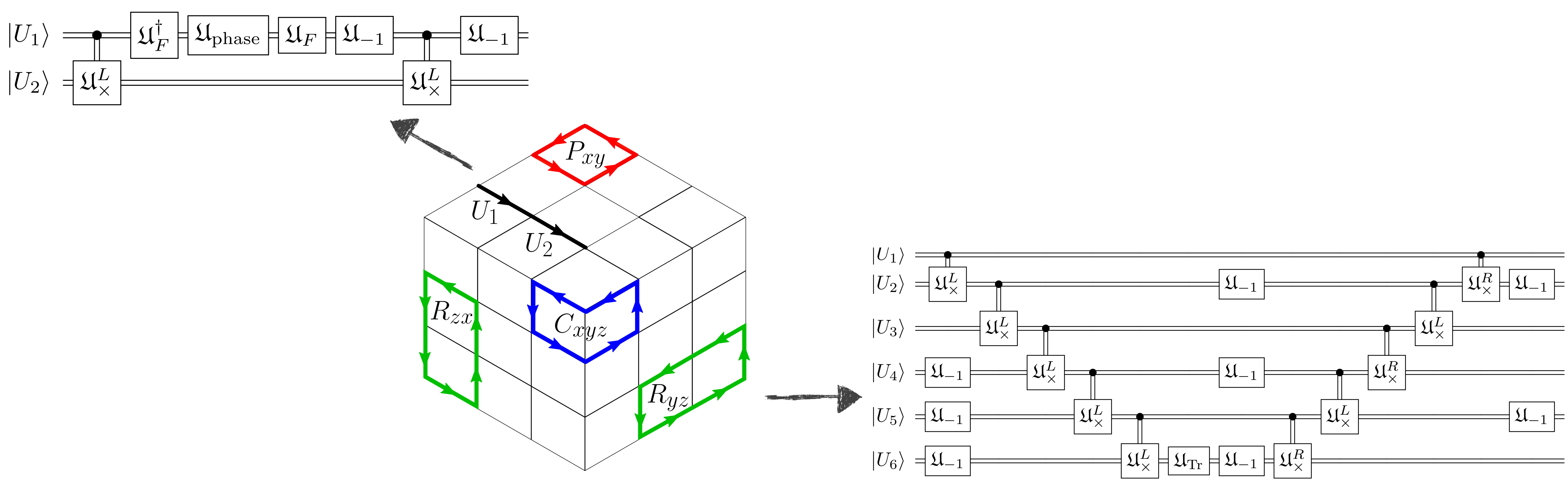


assuming linear register connectivity

Propagation with gauge redundant encodings



$$\langle U_1', U_2' | \mathcal{U}_{K_{2L}} | U_1, U_2\rangle = \delta_{U_1'U_2', U_1U_2} \langle U_1'| e^{i\theta K_{L_1}} |U_1\rangle$$



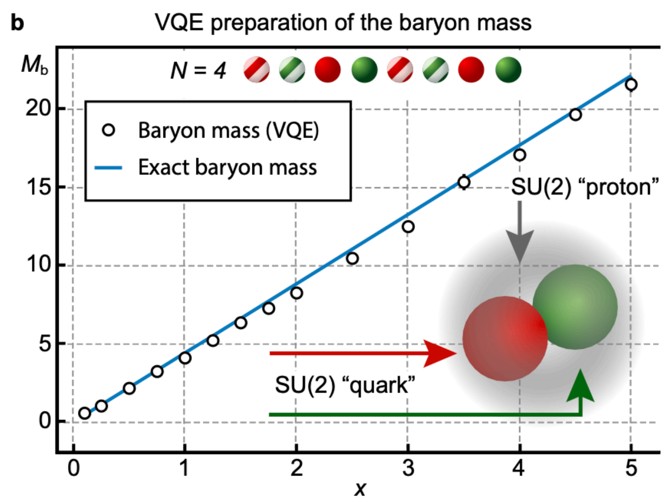
Demonstration of improved Hamiltonian is allowed in the near future

Physics Benchmarks for Quantum Computing

in the NISQ era

Physics Benchmarks for Quantum Computing

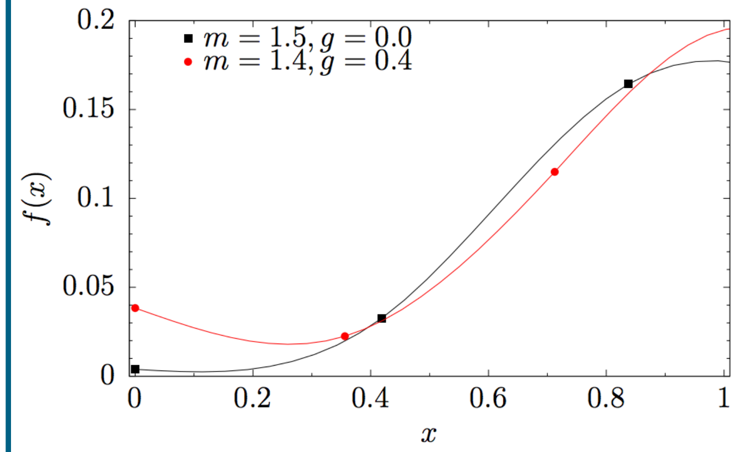
proton state preparation



Atas et al, Nat Commun 12, 6499 (2021)

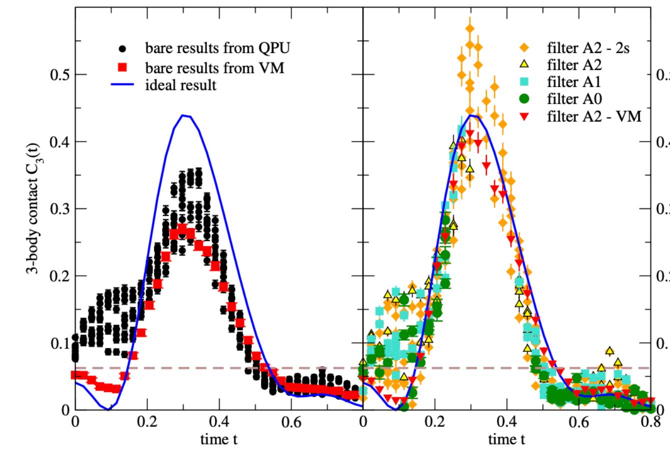
PDF

$$\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{\partial}{\partial\epsilon_t}\frac{\partial}{\partial\epsilon_0}\langle\Psi|e^{-iHt}e^{-i\mathcal{O}\epsilon_t}e^{iHt}e^{i\mathcal{O}\epsilon_0}|\Psi\rangle$$



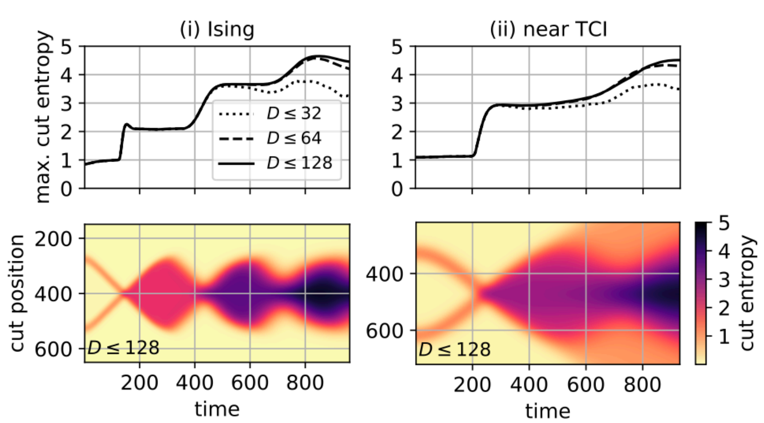
Lamm, et al., T. Li, et al,

neutrino-nucleus scattering



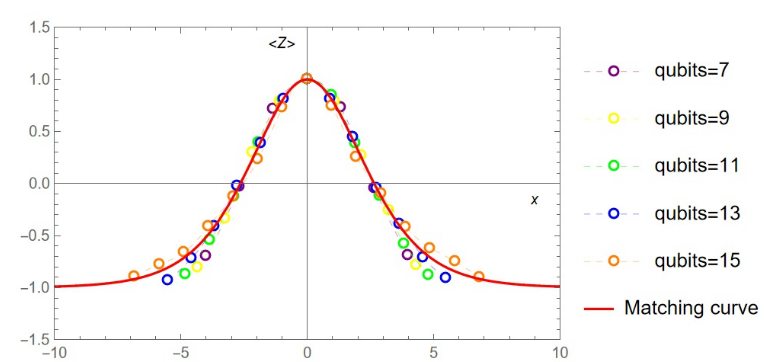
[A. Roggero et al., arXiv:1911.06368]

bubble collisions



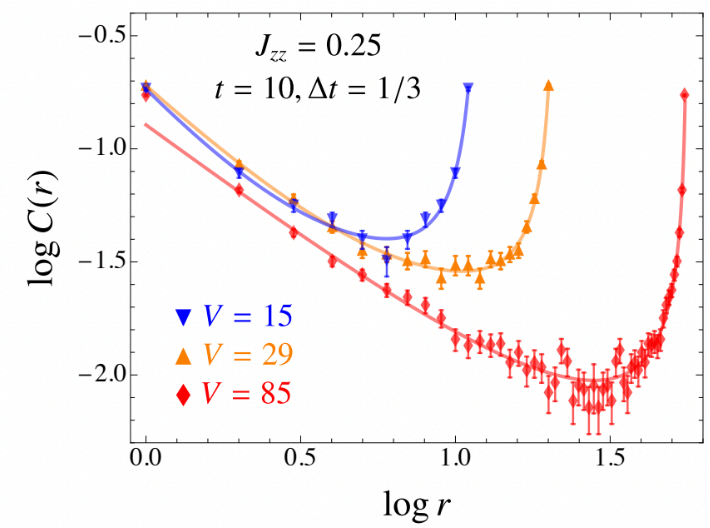
Milsted, et al, PRXQuantum.3.020316

topological objects preparation



M. Huang, **YYL**, L.-T. Wang, H. Zhang, in preparation

holography



YYL, et al., arXiv:2312.10544

Summary and Outlook

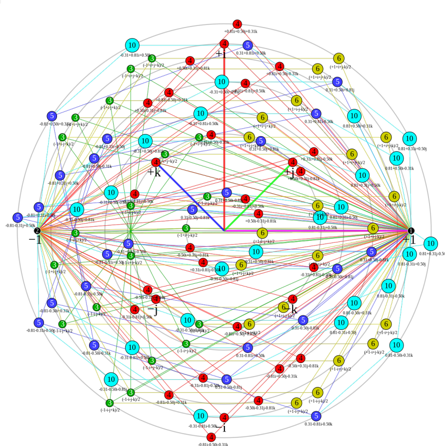
- **Gauge redundancy as quantum error correction codes**

  quantum error threshold for gauge-redundant digitization, with the error rate achievable for near-term quantum devices.

- **Techniques on real-time simulation of lattice field theory**

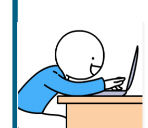
  improved Hamiltonian: matrix elements for the improved terms, circuits designed

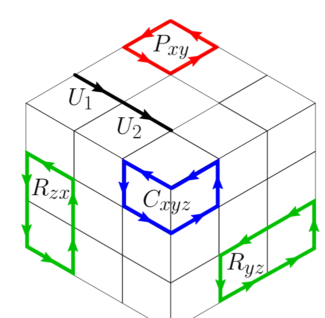
discrete subgroup redundancies



improved Hamiltonian demonstration with redundancies

$$|\langle H_{KS}(a) - H\rangle| \sim |\langle H_I(2a) - H\rangle|?$$

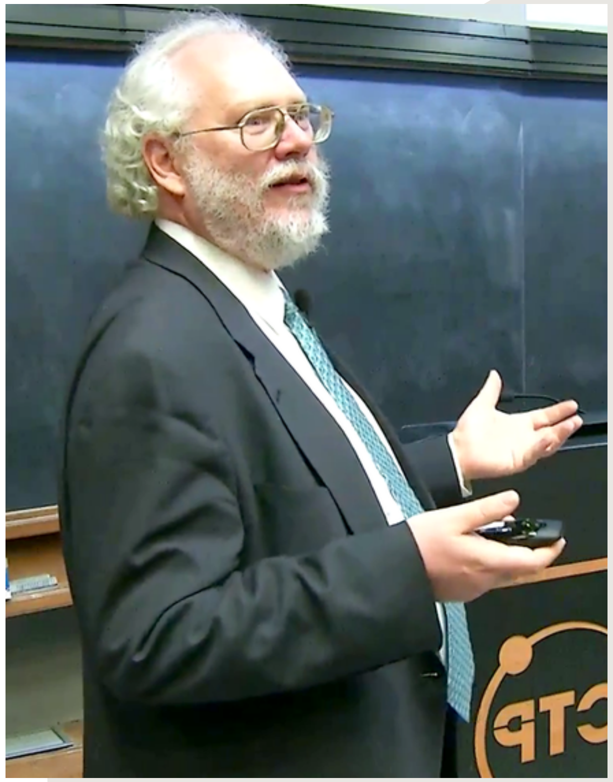




Thank you

BACK UP

How Powerful It is?



QUANTUM EASY

factoring problem

CLASSICAL EASY

polynomial time

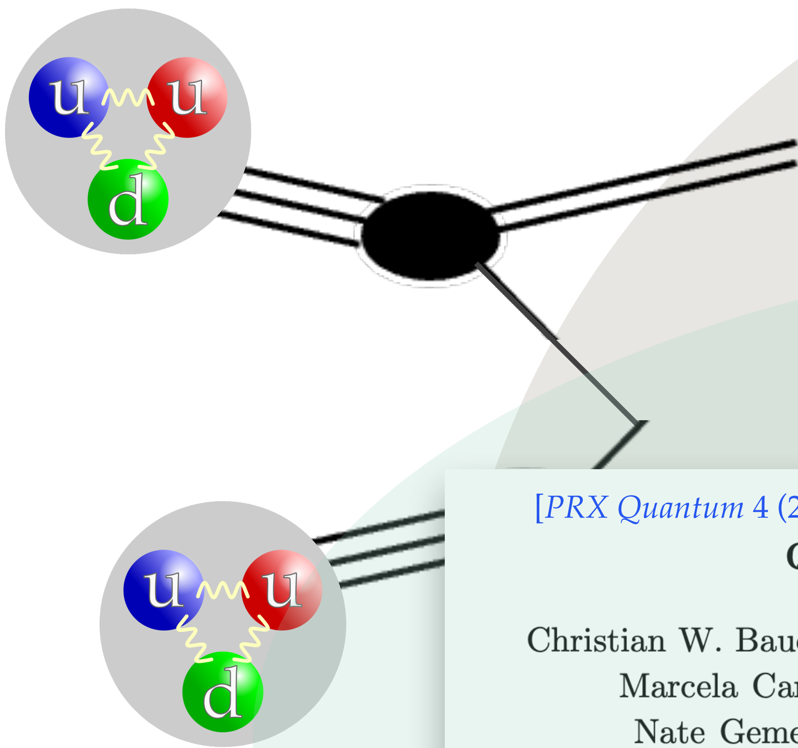
complexity
strong interactions

many-body entanglement,
quantum spacetime,
molecular chemistry,
novel materials,
phases of quantum matter
...

QUANTUM HARD

e.g. traveling salesmen problem

How Powerful It is?



QUANTUM EASY

QUANTUM HARD

High Energy Physics

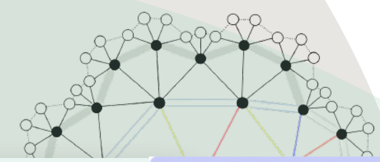
real-time dynamics

finite density

quantum interference

out-of equilibrium

``strongly interacting many-body system''

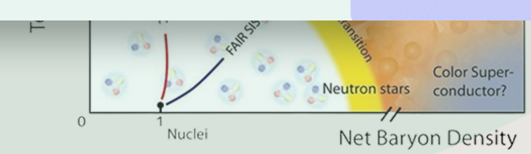


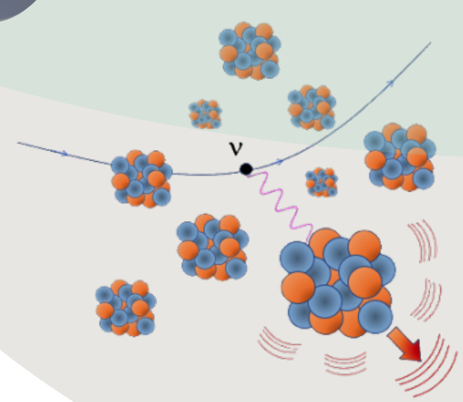
[*PRX Quantum* 4 (2023) 2, 027001]

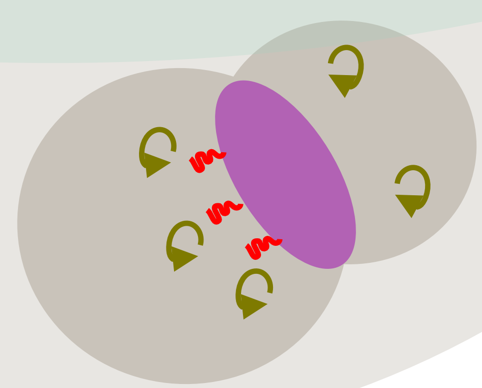
Quantum Simulation for High Energy Physics

Christian W. Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4]
Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9]
Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5]
Ying-Ying Li,[5] Junyu Liu,[14,15] Mikhail Lukin,[16] Yannick Meurice,[17]
Christopher Monroe,[18,19,20,21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23]
Enrico Rinaldi,[24,25,26] Alessandro Roggero,[27,28] David I. Santiago,[29,30]
Martin J. Savage,[31] Irfan Siddiqi,[29,30,32] George Siopsis,[33] David Van Zanten,[5]
Nathan Wiebe,[34,35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

- Collider Phenomenology
- Matter in and out of Equilibrium
- Neutrino (Astro)physics
- Early Universe and Cosmology
- Quantum Gravity



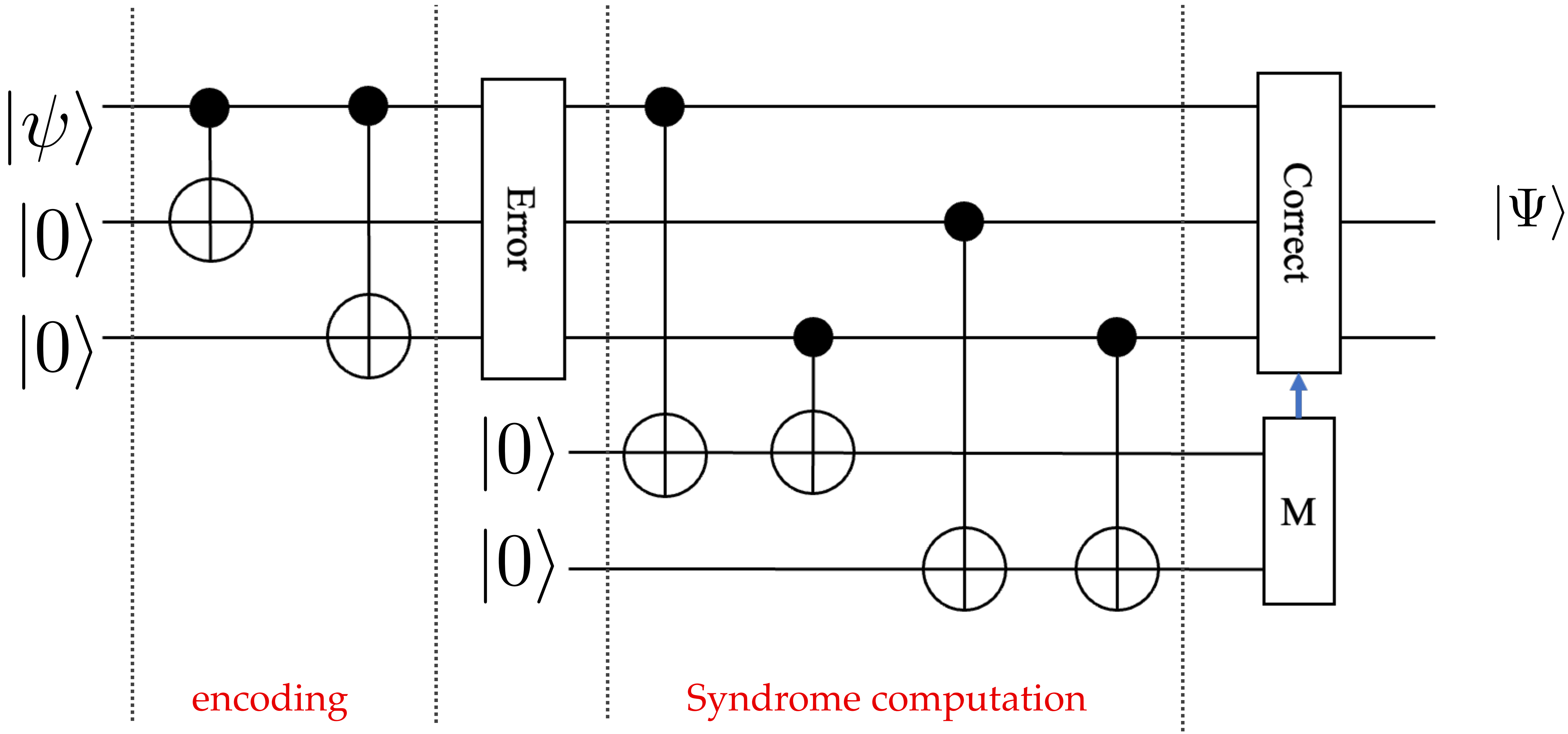




Gauge redundancy utilized for error corrections

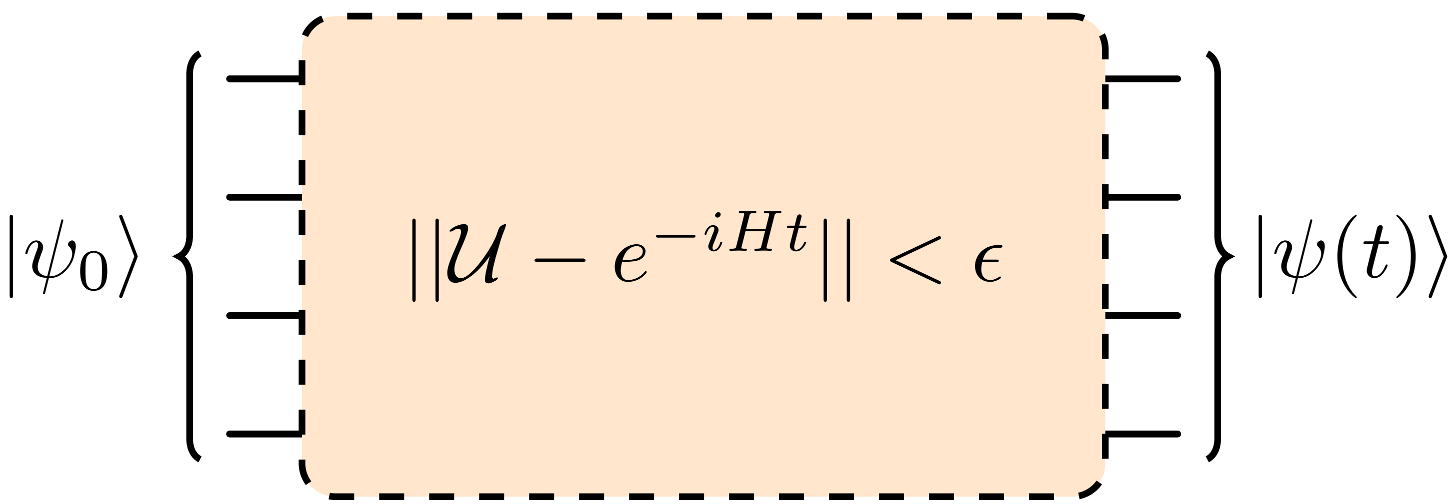
quantum error corrections

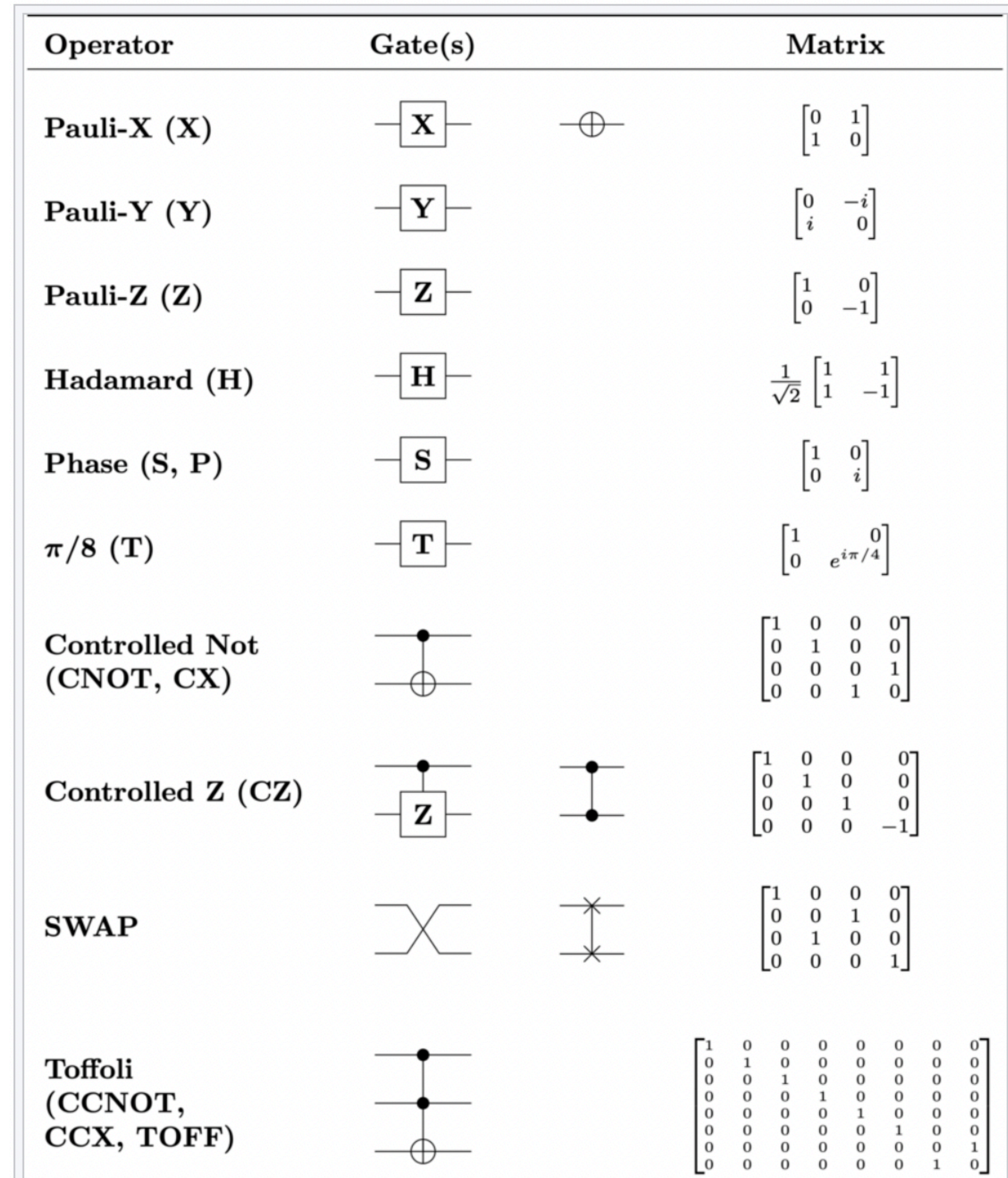
$$|\Psi\rangle = \alpha\,|000\rangle + \beta\,|111\rangle \qquad \hat{\Theta}_{\mathcal{S}}(x)\,|\Psi\rangle \stackrel{?}{=} |\Psi\rangle$$



Propagation with gauge redundant encodings

DIGITAL

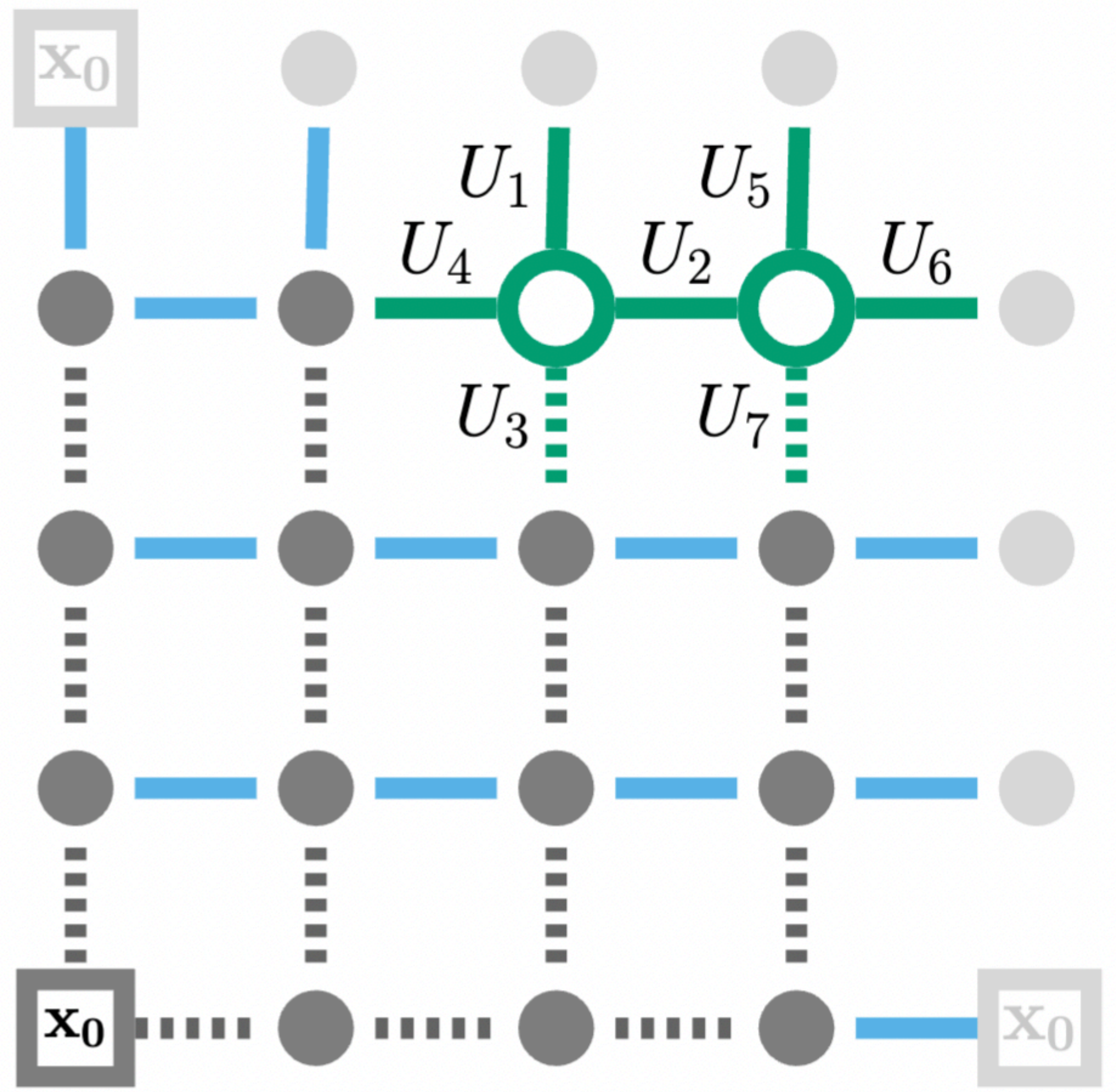




Operator	Gate(s)	Matrix
Pauli-X (X)	X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Gauge redundancy utilized for error corrections

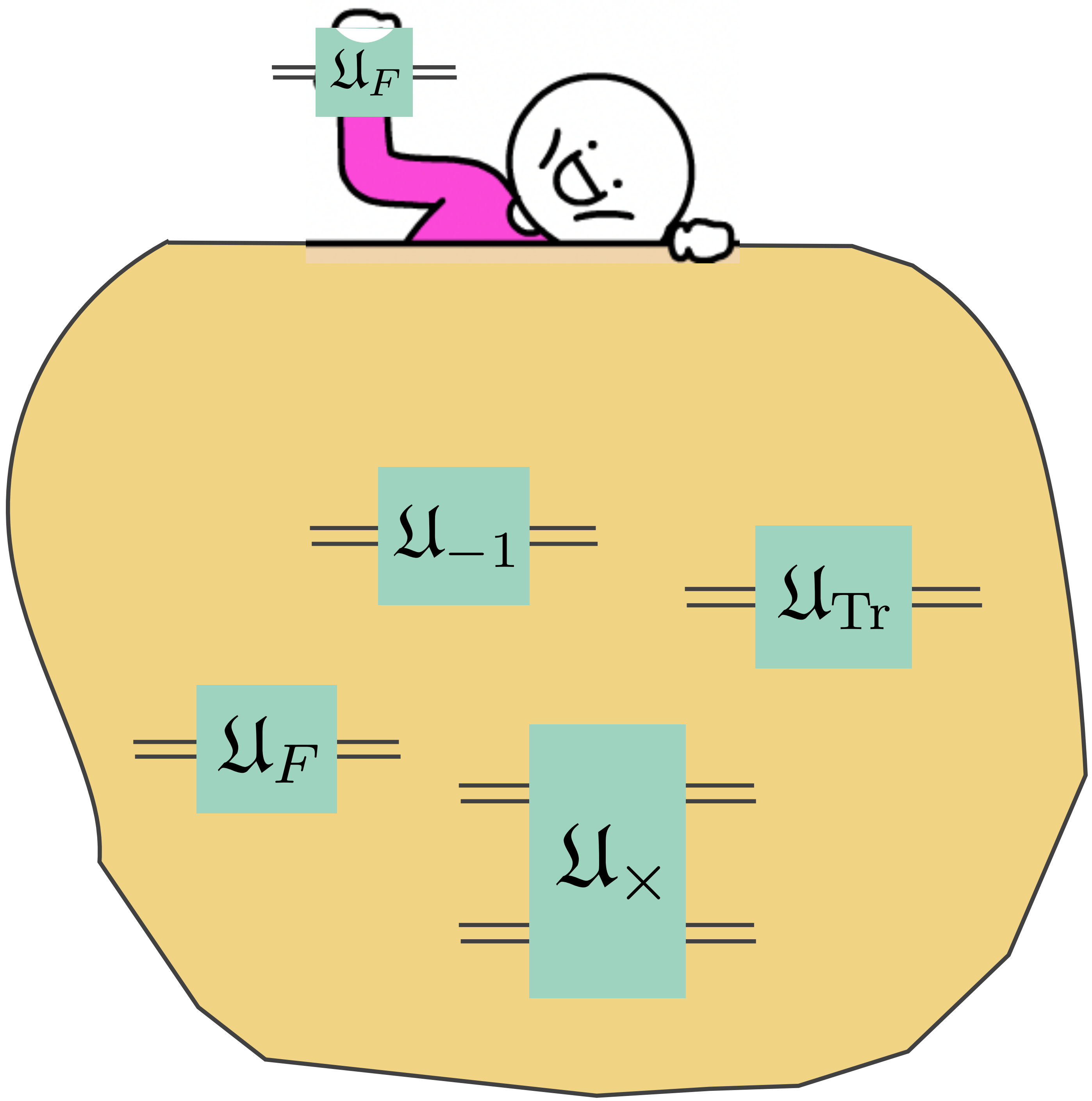
$\mathcal{H}_{\mathrm{red}}$



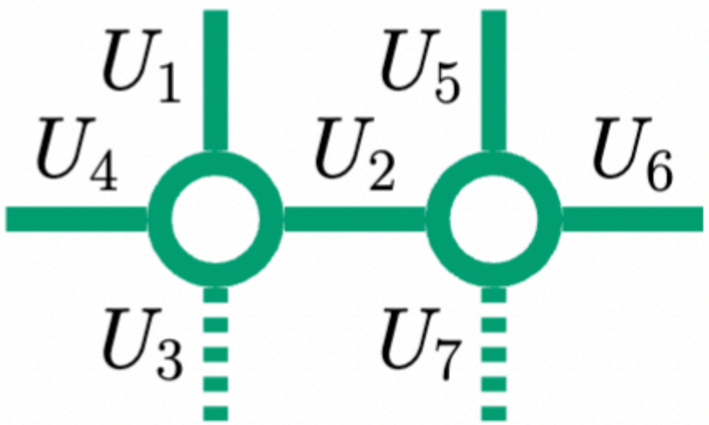
Maximal Tree Gauge

Gauge redundancy utilized for error corrections

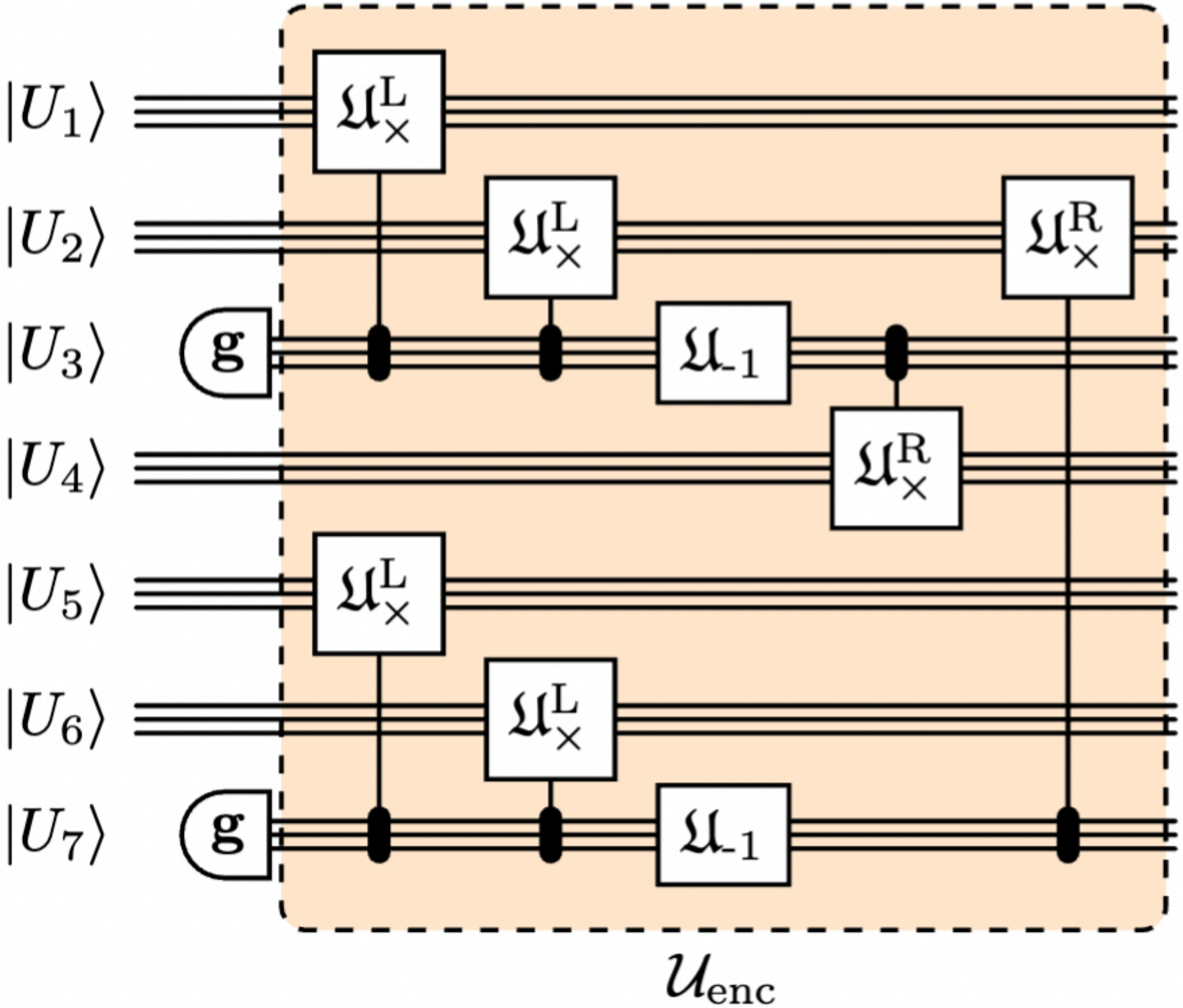
$$G\text{−register} : |U\rangle \quad =\!=$$



Gauge redundancy utilized for error corrections

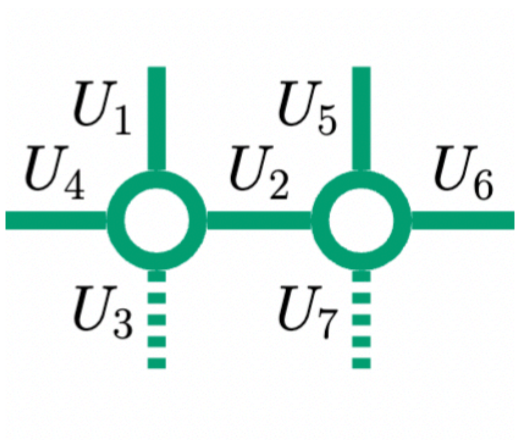


Encoding
Gauss's Law

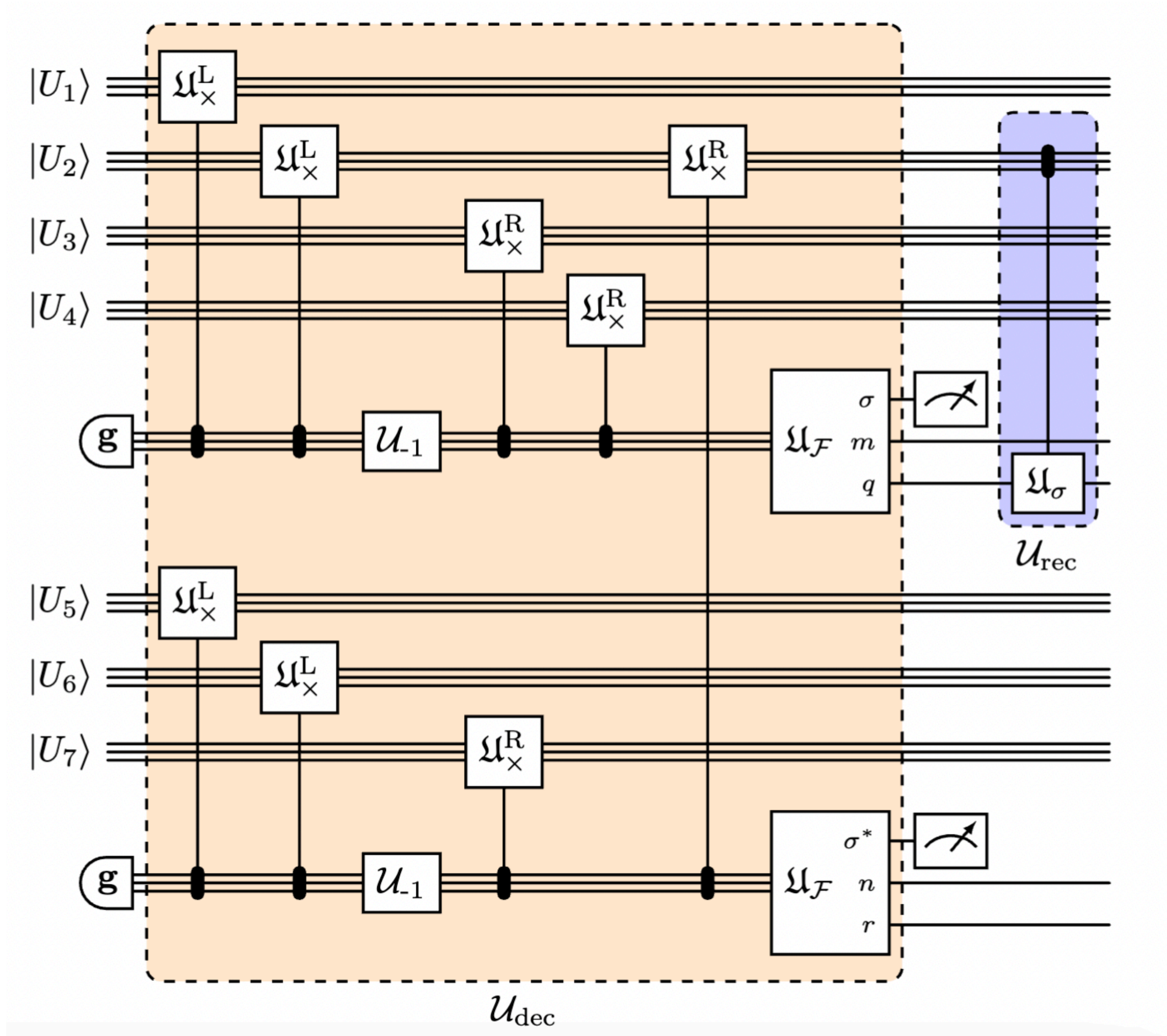


M. Carena, H. Lamm, **YYL**, W. Liu, to appear

Gauge redundancy utilized for error corrections



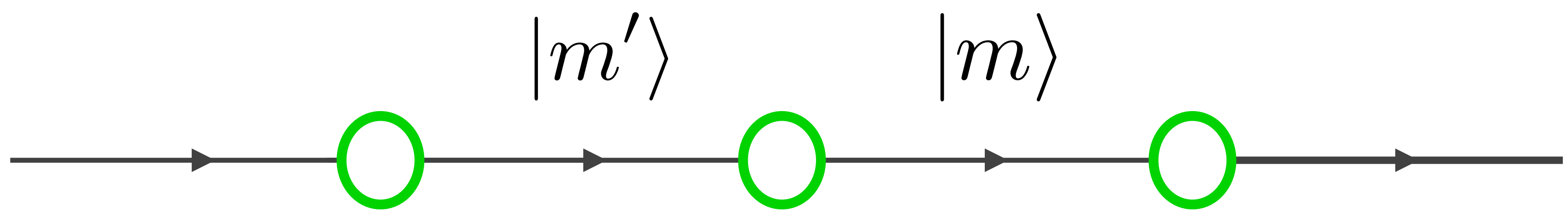
charge
computation



M. Carena, H. Lamm, **YYL**, W. Liu, to appear

Gauge redundancy utilized for error corrections

Z_N $\quad |n\rangle : e^{i2\pi n/N}$ $\qquad |n\rangle = \sum_m e^{i2\pi nm/N} |m\rangle$

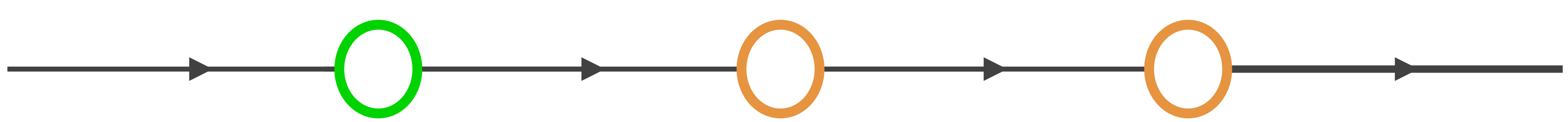


$\hat{G}^a(x)\,|\psi\rangle = 0 \longrightarrow m' = m$

Case 1 $\quad \hat{L}_g = (\hat{\chi})^g = \sum_{n=0}^{N-1} |n\rangle \langle (n+g) \bmod N|$

preserves gauge symmetry
not detectable

Case 2 $\quad \hat{\Gamma}_\sigma = (\hat{\nu})^\sigma = \sum_{n=0}^{N-1} e^{2\pi i n\sigma/N} |n\rangle \langle n|$ $\qquad |m\rangle \to |m-\sigma\rangle$



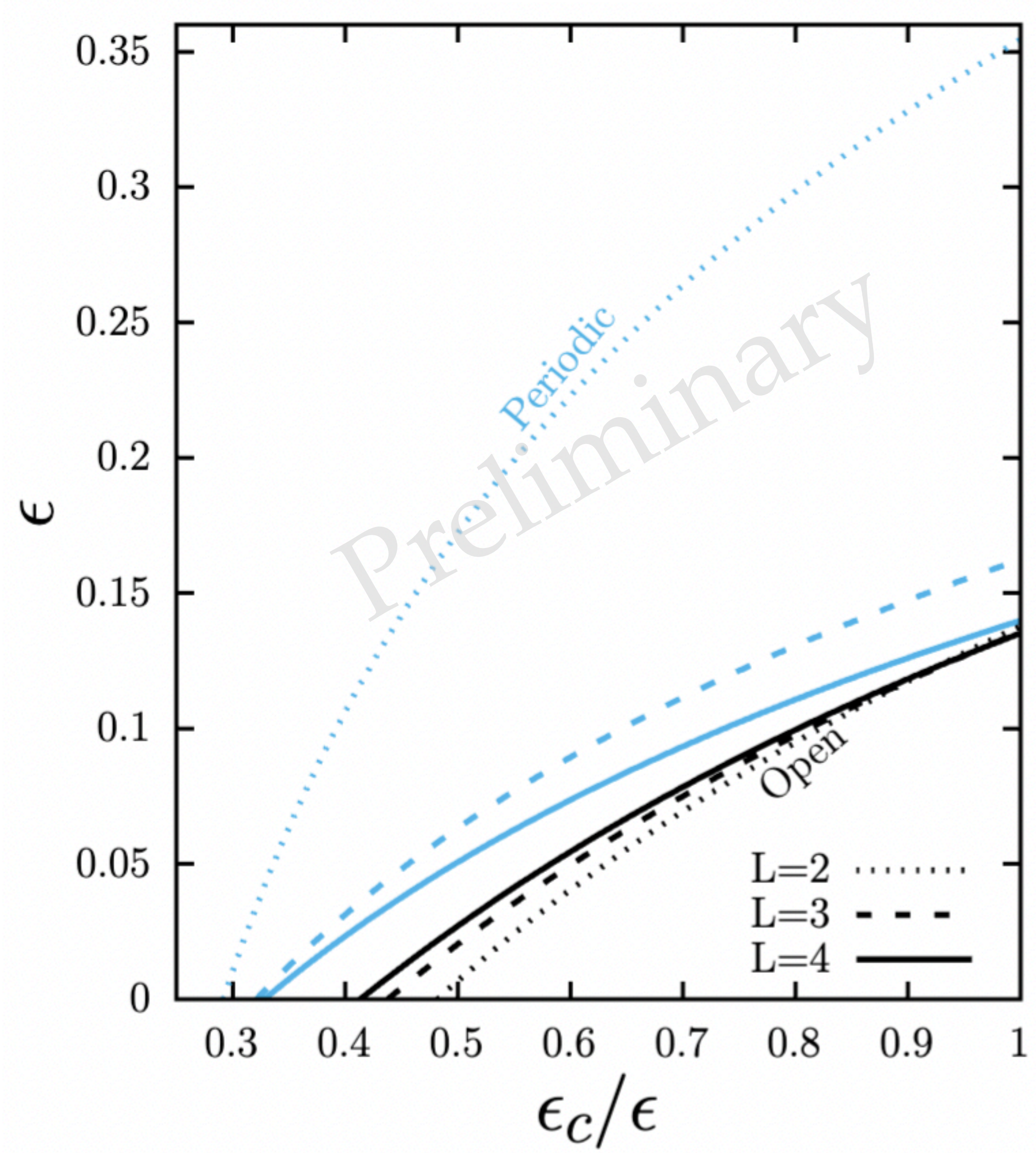
correctable

M. Carena, H. Lamm, **YYL**, W. Liu, to appear

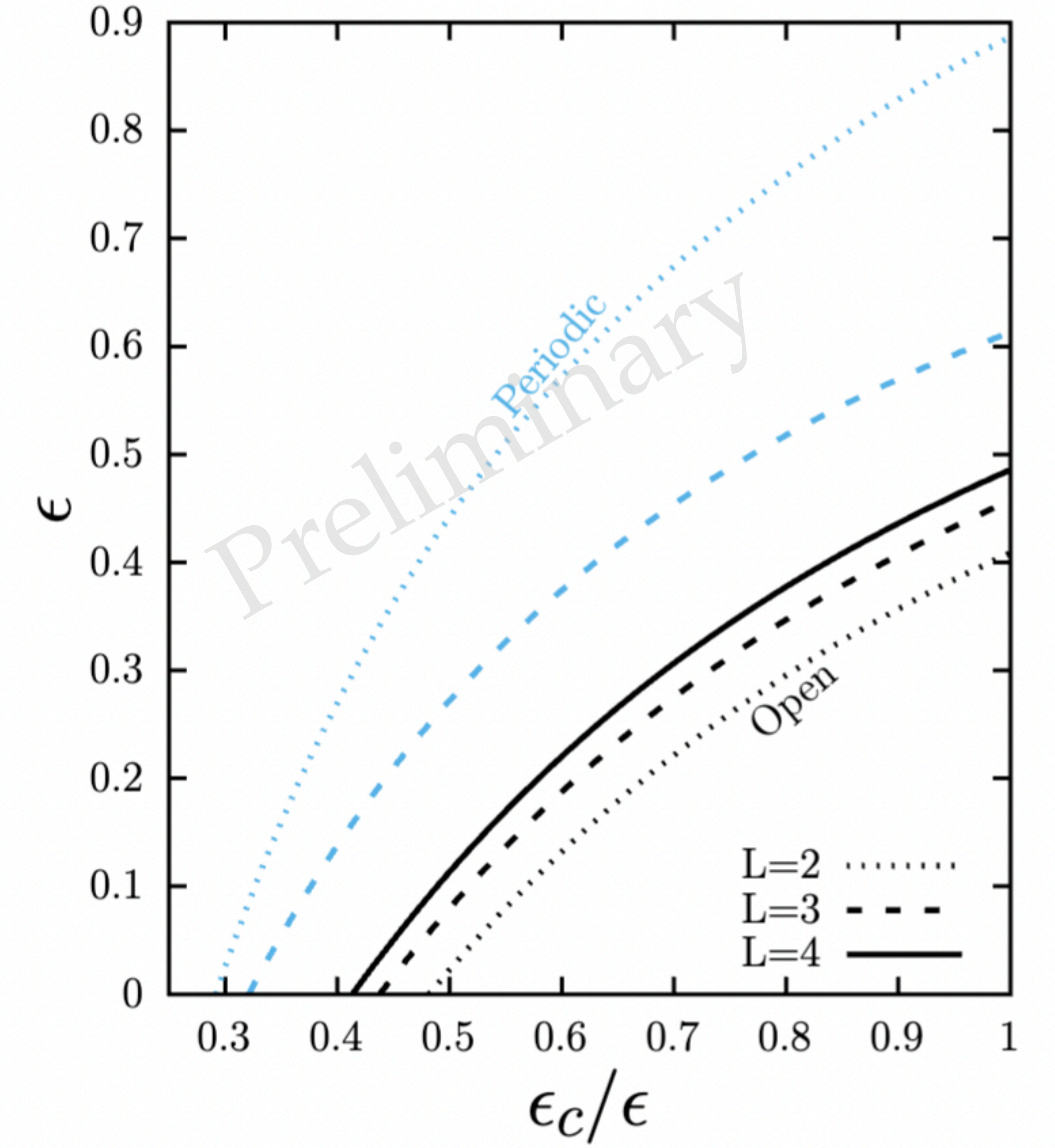
Error threshold for gauge redundant encodings

3d lattice

isolated
flux pairs



KL
condition



M. Carena, H. Lamm,**YYL**, W. Liu, to appear