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Gravitational Wave

- Gravity is weak between common objects
- Need to look in the universe for strong gravity
- Need to rotate celestial very fast, better close to speed of light, to generate strong gravitational wave
- Gravitational wave has been detected when two blackholes merge
- LIGO has detected many cases
 - Frequency up to more than 100Hz



Gravitons

- Always assumed by physics community to exist
- $E = hf$
- Need high frequency to observe quantum properties
 - At least visible light frequency
 - Better $f > 10^{14}$ Hz
- Impossible for celestial objs.
- Need 10^{19} GeV for particles
 - LHC 10^4 GeV

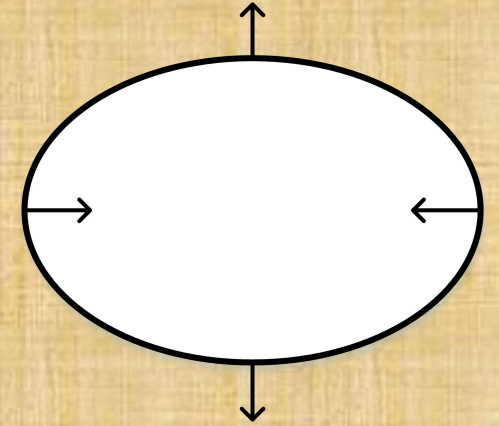


Gravitational Wave and Gravitons

- Plane wave

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$h_{\mu\nu} = e_{\mu\nu} e^{ikx} + e^*_{\mu\nu} e^{-ikx}$$

- Classical gravitational field is measured on $e_{\mu\nu}$
- $e_{\mu\nu}$ much smaller when frequency goes higher
- **Single graviton energy higher**



- Polarization $k_\mu = (\omega, 0, 0, \omega)$

$$e_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & e_{11} & e_{12} & 0 \\ 0 & e_{12} & -e_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Stress Energy Tensor

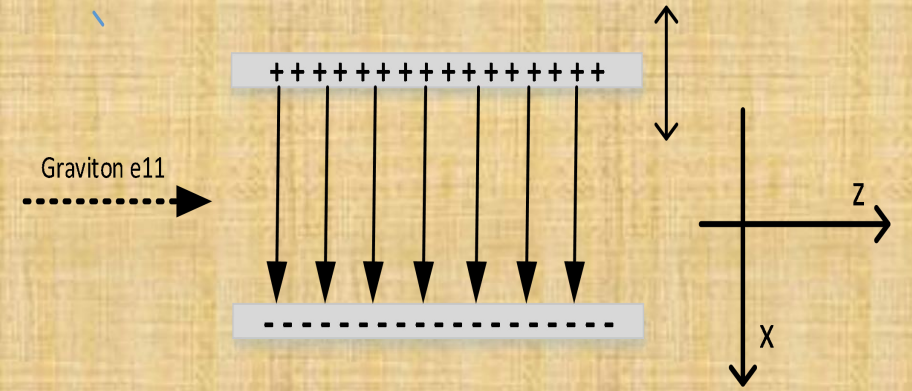
$$\langle t_{\mu\nu} \rangle = \frac{k_\mu k_\nu}{8\pi G} (|e_{11}|^2 + |e_{12}|^2)$$

From Gravitons to Photons

- EM Action with gravity

$$L = \int d^4x \sqrt{g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + g^{\mu\nu} j_\mu(x) A_\nu \right]$$

$$D_\mu A_\nu - D_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}$$



$$L(h, A) = \int d^4x \left(\frac{1}{8} h_\rho{}^\rho \bar{F}_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \bar{F}_{\mu\rho} F_\nu{}^\rho h^{\mu\nu} - \frac{1}{2} h_\nu{}^\nu j_\mu A^\mu + j_\mu A_\nu h^{\mu\nu} \right)$$

- Background field help to generate EM radiation, charge doesn't

$$L(h, A) = -2\text{Re} \int d^4x [\bar{E} E_1 e_{11} + \bar{E} E_2 e_{12}] e^{i\omega(z-t)}$$

- Background electric or magnetic field perpendicular to gravitational wave propagation generate EM radiation, e_{11} and e_{12} generate different polarization

From Photon to Graviton

- The stress-energy tensor of electro-magnetic field:

$$T_{EM\mu\nu} = F_{\mu\lambda}F^{\lambda}_{\nu} - \frac{1}{4}\eta_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

- Background field breaks angular momentum conservation

- Plane wave energy tensor

$$T_{EM\mu\nu} = \begin{pmatrix} -\omega^2 A^2 & 0 & 0 & \omega^2 A^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega^2 A^2 & 0 & 0 & \omega^2 A^2 \end{pmatrix}$$

- With a back ground electric field in x direction, and EM wave polarized in x direction

$$\begin{pmatrix} -\omega\bar{E}A\cos(\omega z - \omega t) & 0 & 0 & \omega\bar{E}A\cos(\omega z - \omega t) \\ 0 & -\omega\bar{E}A\cos(\omega z - \omega t) & 0 & 0 \\ 0 & 0 & \omega\bar{E}A\cos(\omega z - \omega t) & 0 \\ \omega\bar{E}A\cos(\omega z - \omega t) & 0 & 0 & \omega\bar{E}A\cos(\omega z - \omega t) \end{pmatrix}$$

- With a back ground electric field in x direction, and EM wave polarized in y direction, the linear term of stress energy tensor is:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega\bar{E}A\cos(\omega z - \omega t) & 0 \\ 0 & -\omega\bar{E}A\cos(\omega z - \omega t) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Computation

- The effective current:

$$\begin{aligned}j_x &= 2\omega\bar{E}|e_{11}|\cos(\omega z - \omega t) \\j_y &= 2\omega\bar{E}|e_{12}|\cos(\omega z - \omega t)\end{aligned}$$

- Use Green's Function

$$A_x(r, \vec{k}, t) \approx \int_{inside} d^3x' \frac{\omega\bar{E}|e_{11}|}{2\pi r} \cos\left[\omega\left(z' - t + r - \vec{k} \cdot \vec{x}'\right)\right]$$

- Probability for a graviton turning into photon in background electric field

$$\epsilon_{G-EM} = P_{EM}/P_G = 4\pi G\epsilon_0\bar{E}^2 L^2 / c^4$$

- In background magnetic field

$$\epsilon_{G-EM} = P_{EM}/P_G = 4\pi GL^2\bar{B}^2 / \mu_0 c^4$$

Analysis

- As a graviton travels in background EM field, it slowly turns into a photon, the probability amplitude is proportional to distance, the probability is proportional to distance square.

- In the lab and in the universe, it is easier to have a stronger magnetic field than electric field.

$$\epsilon_{G-EM} = 8.2 \times 10^{-38} \left(\frac{BL}{T \cdot m} \right)^2$$

- A photon travels in the background EM field will have the same probability to turn into graviton. Given a long enough distance, there will be oscillation

- The key is that both photon and graviton travels in the speed of light

- If we make a graviton detector with 30T magnetic field and 10km length

$$\epsilon_{G-EM} = 7.2 \times 10^{-27}$$

- for 1W wave of $\omega = 10^{14} Hz$, we will get about 2 events a month.

- For non-constant magnetic field, when wave length small compares to the distance scale of variation

$$\epsilon_{G-EM} = 4\pi G / \mu_0 c^4 \left[\left(\int B_x dl \right)^2 + \left(\int B_y dl \right)^2 \right]$$

QED 1-loop Correction

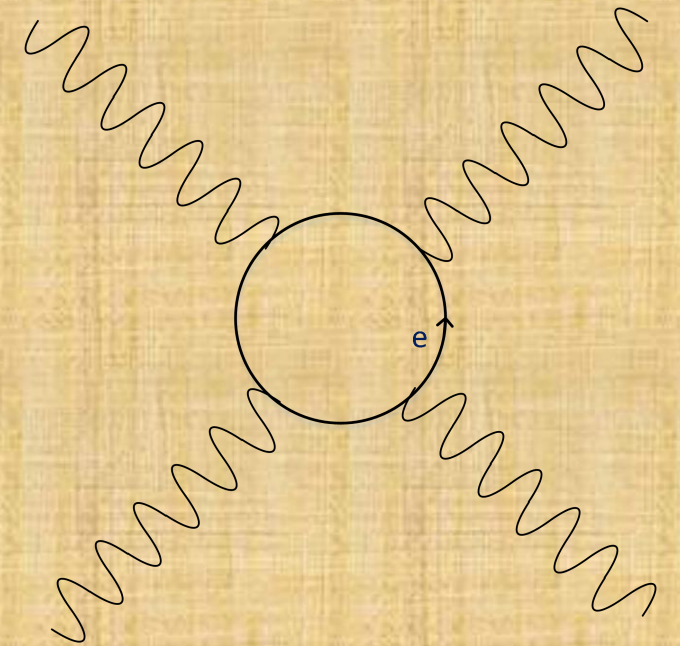
- Quantum field theory effects makes EM non-linear
- Euler-Heisenberg Lagrangian:

$$L_{1QED} = -\frac{2\alpha^2}{45m_e^4} \int d^4x \left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E} \cdot \vec{B})^2 \right]$$

- Consider background field, $A_\nu \rightarrow \bar{A}_\nu + A_\nu$, with a background magnetic field

$$F_{23} = F^{23} = -F_{32} = -F^{32} = \bar{B}$$

$$L_{QED}(\bar{B}) = -\int d^4x \left[\left(\frac{1}{4} - \frac{2\alpha^2 \bar{B}^2}{45m_e^4} \right) F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2 \bar{B}^2}{45m_e^4} (8B_1^2 + 14E_1^2) \right]$$



- For non-constant magnetic field, when wave length small compares to the distance scale of variation

$$\epsilon_{G-EM} = 4\pi G/\mu_0 c^4 \left[\left(\int \mathbf{B}_x dl \right)^2 + \left(\int \mathbf{B}_y dl \right)^2 \right]$$

QED 1-loop Correction

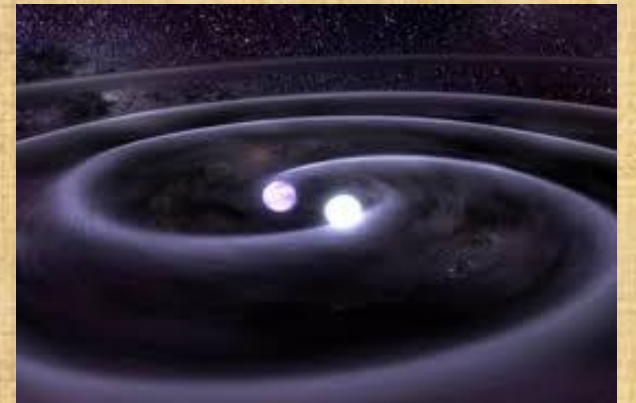
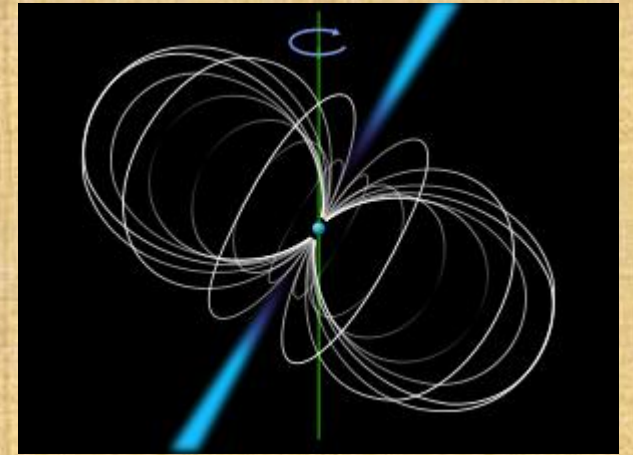
- Speed of light is changed under background EM field.

$$v_{x-photon} = 1 - \frac{14\alpha^2 \bar{B}^2}{45m_e^4}$$
$$v_{y-photon} = 1 - \frac{8\alpha^2 \bar{B}^2}{45m_e^4}$$
$$\frac{\alpha^2 \bar{B}^2}{m_e^4} \rightarrow \frac{\alpha^2 \hbar^3 \bar{B}^2}{\mu_0 c^5 m_e^4} = 3 \times 10^{-23} \left(\frac{\bar{B}}{T}\right)^2$$

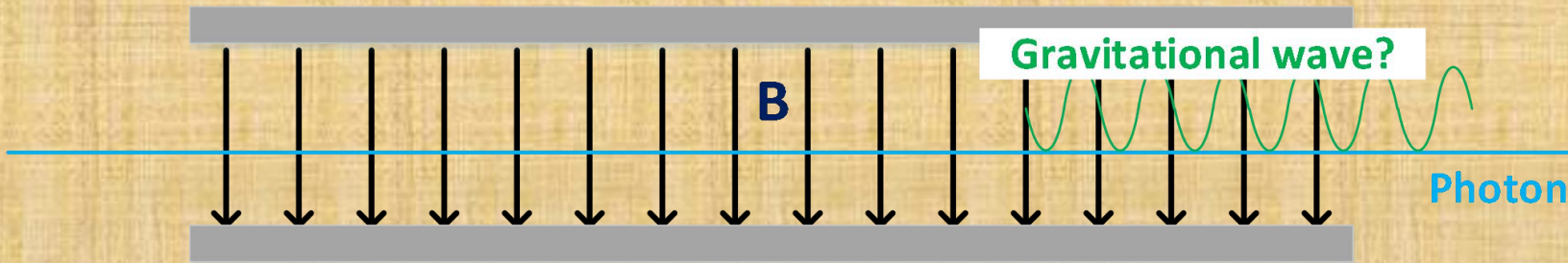
- QED effect has no impact on ground experiment
- QED significantly impact Magnetars and Neutron stars

Implications

- There are more gravitons in the universe than we believed. a typical galaxy has a size of 100000 light years ($10^{21}m$) and an average magnetic field of $10^{-9}T$, and most of the field are not turbulent, the switching probability is on the order of 10^{-14} (versus $\sim 10^{-40}$ thermal radiation).
- Neutron stars and Magnetars
- Creating/detecting gravitons in the lab
 - 60's Hertz Experiment considered
 - Quantum optics from 80-90's
 - Receive gravitons from the universe
 - Fixed direction, background problems
 - **Detecting missing photons**
 - Relative low cost



Is Gravitational Field a Quantum Field?



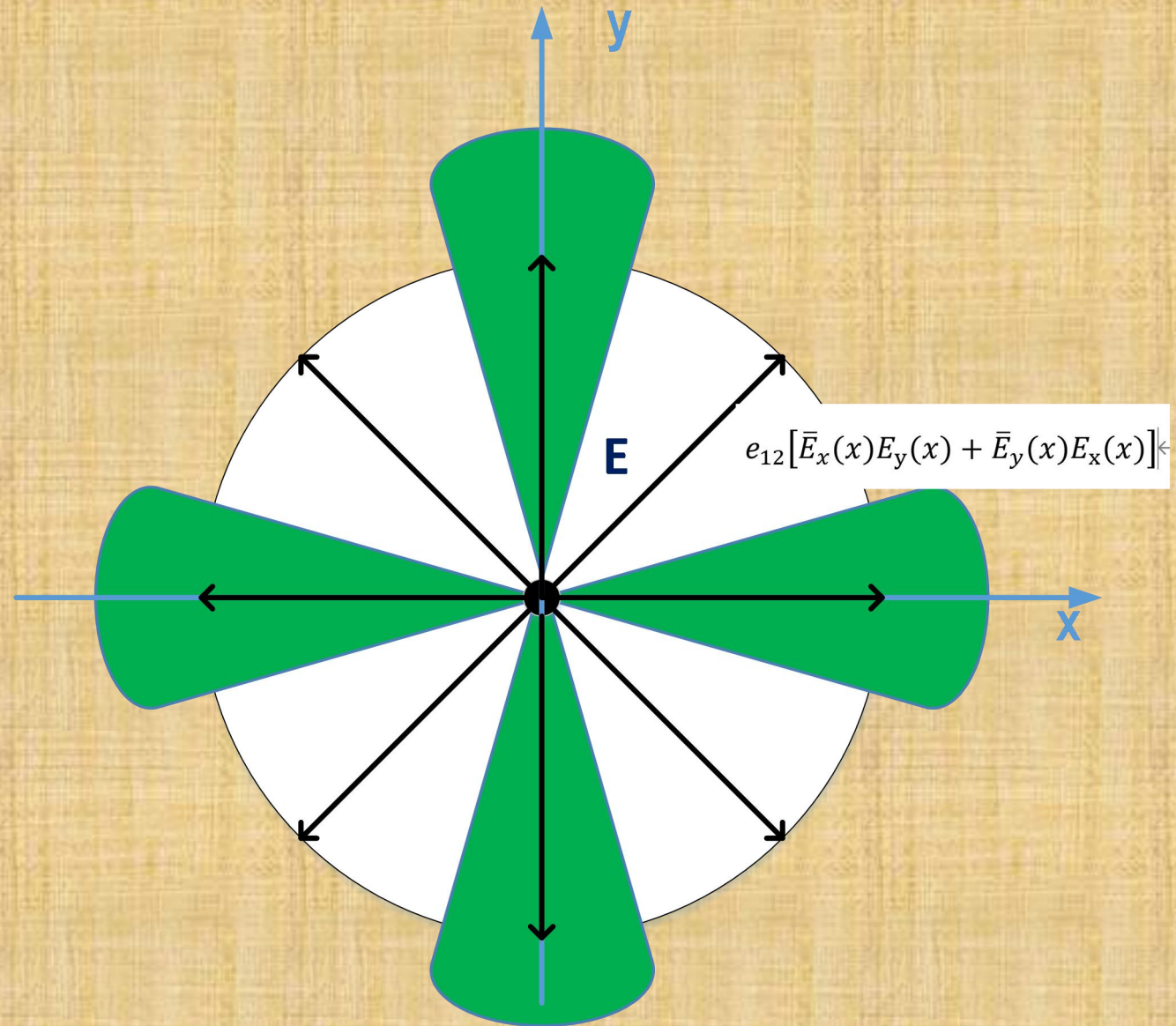
Can gravitational field remain classic while others are quantized?

Blackbody radiation?
Weak

- From this phenomenon, we know that classical general relativity + Quantum Electro-Magnetism is not consistent
- Quantum general relativity is not consistent so far
- This is THE experiment that can prove the existence of gravitons!
- More convincing than seeing photon from gravitons

On-going Research

- Graviton-Atom interaction
- Crystal
- Ordinary matters
- Neutrino Experiment



Conclusion

- **In gravity, experiments lags behind theory for ~100 years**
 - **Newton's gravitational law – Cavendish experiment**
 - **Einstein general relativity – discovery of gravitational wave**
- **Quantum gravity phenomenology and experiment can make important progress in this century**
- **No need to go to Planck Energy**