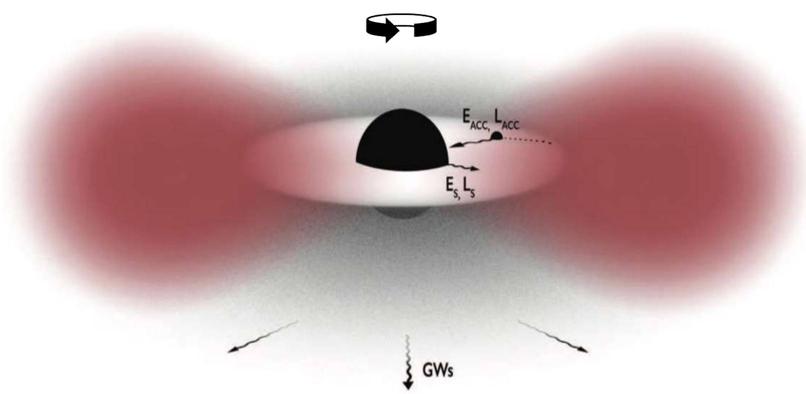
Axion Bose-Einstein Condensate



Hong Zhang （张宏）

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2023/03/16 USTC

Outline

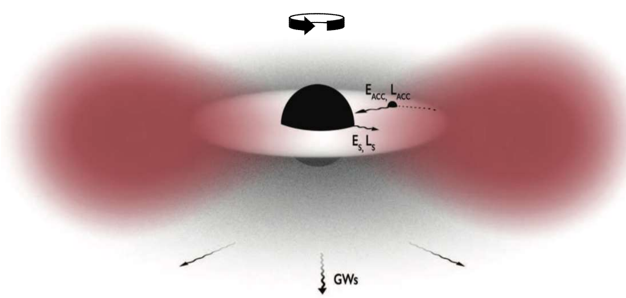
- ✧ **Axions**
- ✧ **Dilute and Dense Axion Stars**

  Properties & Radio Signals

- ✧ **Black Hole Superradiance**

  Properties & GW Signals

- ✧ **Summary**



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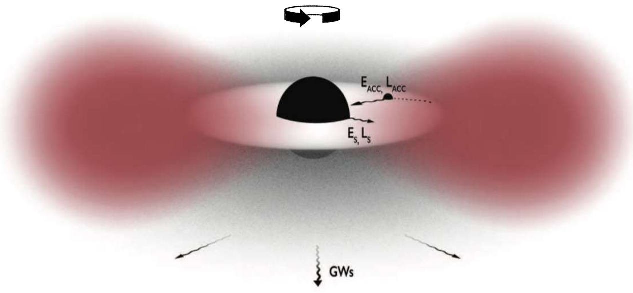


Strong CP Problem

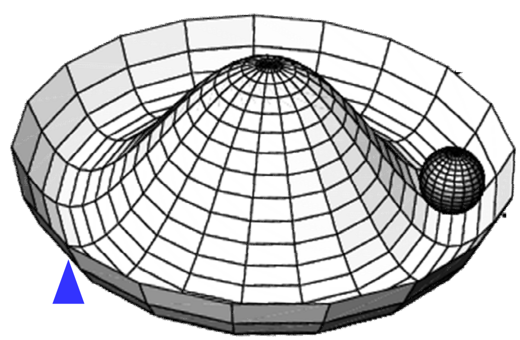
- Strong CP-violating term $\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$

  Neutron electric dipole moment measurement: $\theta \lesssim 10^{-10}$

  Surprisingly small because:
  - High-energy physics breaks CP
  - "Anthropic boundary" : $\theta \lesssim 10^{-3}$

- Peccei-Quinn Mechanism

$$\underbrace{\theta}_{\text{Unnaturally small parameter}} \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} \longrightarrow \underbrace{\phi}_{\text{Dynamical field: Axion.}} \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$$



The potential is **tilted** by quark condensate
The axion field **slides** down to $\phi = 0$
Restore the CP symmetry

Peccei & Quinn (1977)
Weinberg (1978), Wilczek (1978)

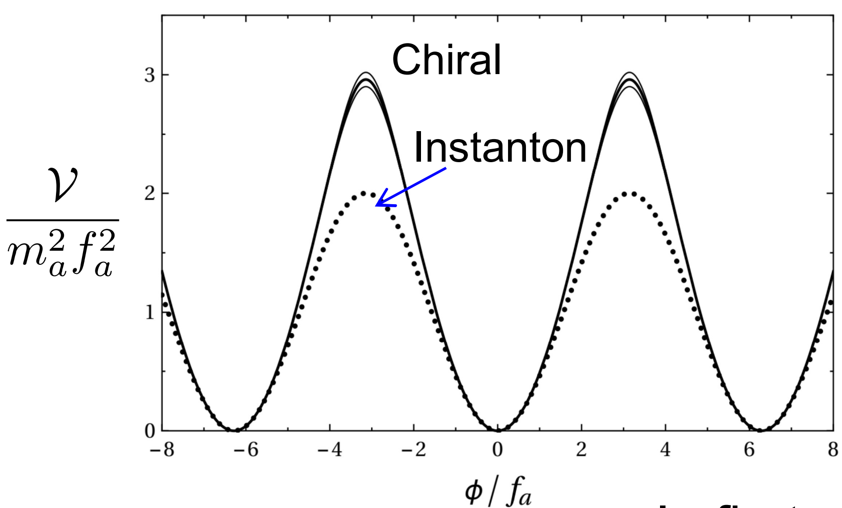
Relativistic Axions

Real pseudoscalar field Temperature below 1GeV

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathcal{V}(\phi)$$

Attractive force when ϕ is small

- Instanton $\mathcal{V}(\phi) = m_a^2 f_a^2 \left[1 - \cos(\phi/f_a)\right] = \frac{1}{2} m_a^2 \phi^2 - \frac{m_a^2}{4! f_a^2}\phi^4 + \cdots$
- Chiral $\mathcal{V}(\phi) = m_\pi^2 f_\pi^2 \left(1 - \left[1 - \frac{4z}{(1+z)^2}\sin^2(\phi/2f_a)\right]^{1/2}\right)$ $z = \frac{m_u}{m_d} \approx 0.48$



Periodic $\mathcal{V}(\phi) = \mathcal{V}(\phi + 2\pi f_a)$

For QCD axion: $m_a f_a = (80 \text{ MeV})^2$

Constraints from astro. & cosmology

$$10^8 \text{ GeV} < f_a < 10^{13} \text{ GeV}$$

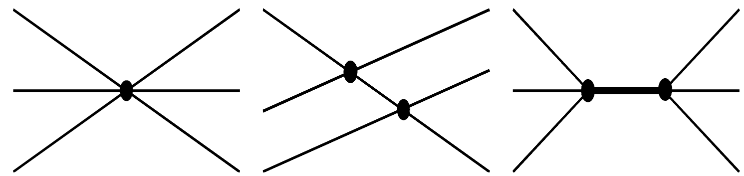
$$10^{-6} \text{ eV} < m_a < 10^{-2} \text{ eV}$$

In first part of this talk, I choose $m_a = 10^{-4}$ eV.

Couplings

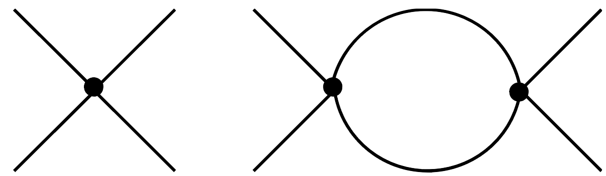
- Self-interaction: vertices with $2n$ axions ($n \geq 2$)

  e.g. Instanton model:



$$\mathcal{V}(\phi) = m_a^2 f_a^2 \left[1 - \cos(\phi/f_a)\right]$$

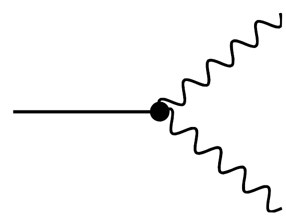
  Each loop is suppressed by



$$(m_a/f_a)^2 \sim 10^{-48}$$

  Classical Field Theory!

- Very weak coupling to photons



$$\mathcal{L}_{\rm em} = \frac{c_{\rm em}\alpha}{16\pi f_a}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\phi.$$

Decay rate: $$\Gamma_a = \frac{c_{\rm em}^2\alpha^2 m_a^3}{256\pi^3 f_a^2}.$$

Axion lifetime ~ 10^{36} years

Age of Universe ~ 10^{10} years

Photon energy: $m_a/2 \sim 10\ \mathrm{GHz}$

Radio frequency

Axion Cosmology

- Cold dark matter axions are produced abundantly at QCD phase transition scale T ~ 1 GeV

Non-thermal axion production mechanism

For more details, see Lect. Notes Phys. 741 (2008)

Mostly non-relativistic

- Vacuum misalignment — Coherent — Preskill, Wise & Wilczek (1983); Abbot & Sikivie (1983); Dine & Fischler (1983)
- Cosmic string decay — Incoherent $\longrightarrow$ Coherent — Davis (1986); Hararie & Sikivie (1987)

Axion Dark Matter

- spin-0 non-relativistic boson

  with extremely small mass $6 \times 10^{-6}\ \mathrm{eV} \lesssim m_a \lesssim 2 \times 10^{-3}\ \mathrm{eV}$

  and extremely small self-coupling and coupling to SM particles

  (suppressed by $3 \times 10^{9}\ \mathrm{GeV} \lesssim f_a \lesssim 10^{12}\ \mathrm{GeV}$)

  and lifetime much longer than the age of our universe

Good candidate for dark matter!

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- Different from other cold dark matter.

  Take $m_a = 10^{-4}$ eV , de Broglie wave length ~ 2 mm

  Use local density 0.4 GeV/cm^3, $n \times \lambda_{dB}^3 \sim 3 \times 10^{10}$

  Huge occupation number!

- In coherence ⟹ the axions are in BEC!

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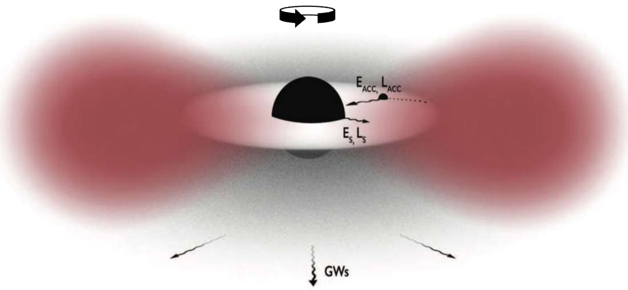


- ✧ Summary

Warmup: 1-d S-G Oscillon (breather)

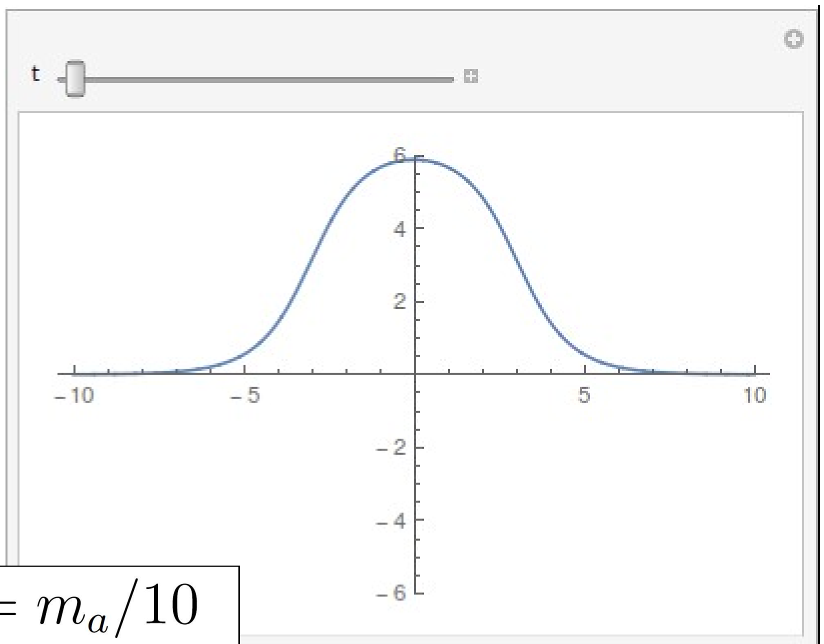
EOM (1-d Sine-Gordon eq., no gravity)

Ablowitz et.al., PRL (1973)

$$\frac{\partial^2}{\partial t^2}\phi(t,x) - \frac{\partial^2}{\partial x^2}\phi(t,x) + m_a^2 f_a \sin\frac{\phi(t,x)}{f_a} = 0$$

Analytic solution ($0 < \omega < 1$ is the frequency)

$$\phi(t,x) = 4 f_a \arctan\left[\sqrt{\omega^{-2}-1}\,\mathrm{sech}(\sqrt{1-\omega^2}\, m_a x)\cos(m_a \omega t)\right]$$



$\omega = m_a/10$
$m_a = f_a = 1$

Features:

- Periodic
- Shape changes slightly
  --Dominated by ω
  --Small components with $3\omega, 5\omega$ ...
- Exponentially small at infinity (no radiation)
- Stable against perturbation

Non-relativistic EFT (Part I)

- Full Lagrantian for real scalars

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathcal{V}(\phi)$$

- Naïve non-relativistic reduction

Chavanis, PRD (2011),Chavanis, Delfini, PRD (2011)
Braaten, Mahapatra, **HZ**, PRD (2016)

$$\phi(\mathbf{r},t) = \frac{1}{\sqrt{2m_a}}\left[\psi(\mathbf{r},t)e^{-im_a t} + \psi^*(\mathbf{r},t)e^{+im_a t}\right]$$

Complex scalar

For systematic treatment, see Braaten, Mahapatra, **HZ**, PRD (2016), PRD(2018)
Namjoo, Guth, Kaiser, PRD (2018)

- Ignore all terms with rapid oscillating phase

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2}i\left(\psi^*\dot{\psi} - \dot{\psi}^*\psi\right) - \frac{1}{2m_a}\nabla\psi^*\cdot\nabla\psi - \mathcal{V}_{\text{eff}}$$

$$\mathcal{V}_{\text{eff}} = m_a\psi^*\psi - \frac{1}{16}\frac{(\psi^*\psi)^2}{f_a^2} + \cancel{\frac{1}{288}\frac{(\psi^*\psi)^3}{m_a f_a^4} + \ldots}$$

Attractive interaction!

Dilute limit

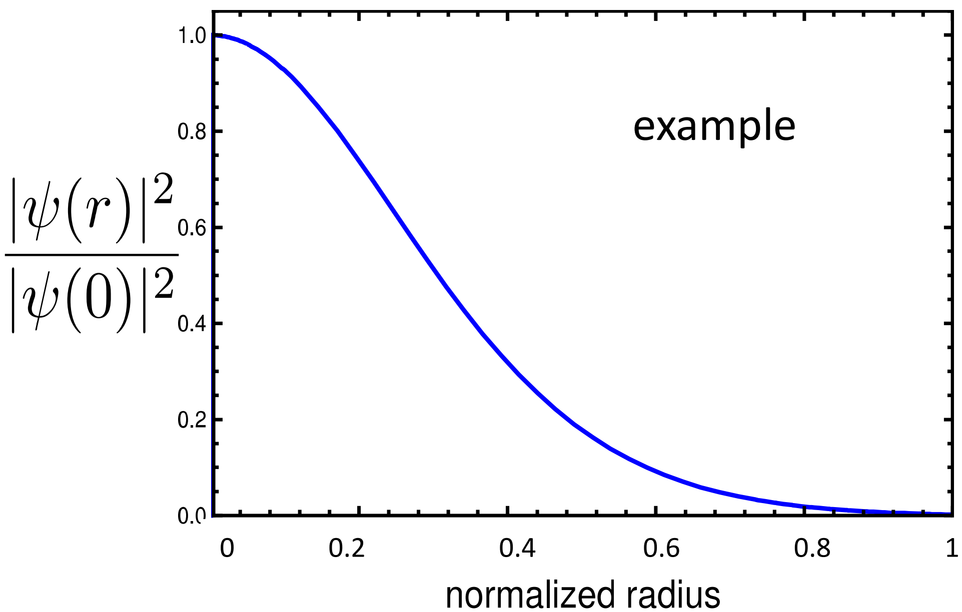
Expand by $\dfrac{\psi^*\psi}{m_a f_a^2}$

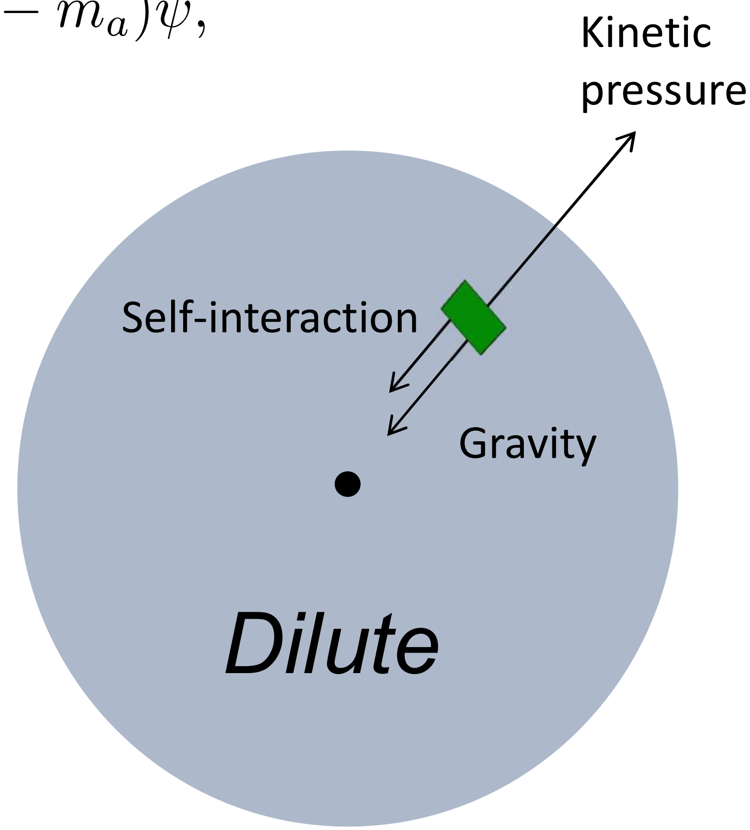
Dilute Axion Stars

Assume:
- Truncated potential, dilute axion limit
- Newtonian gravity
- Spherically symmetric

$$-\frac{\nabla^2}{2m_a}\psi + \left[(\mathcal{V}'_{\rm eff}(\psi^*\psi) + m_a\Phi\right]\psi = (\mu - m_a)\psi,$$

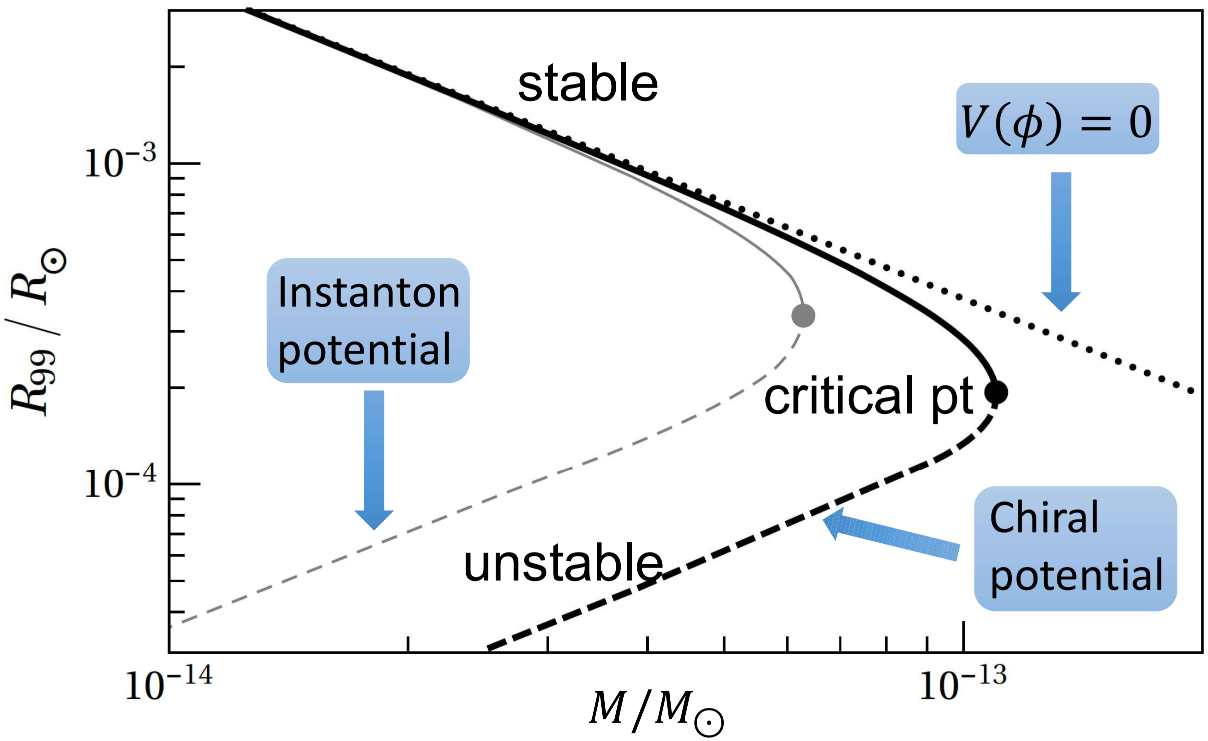
$$\nabla^2\Phi = 4\pi G m_a \psi^*\psi.$$





Dilute Axion Star: M vs R

- Heavier dilute axion stars have smaller radii.
- Critical mass: beyond which the kinetic pressure cannot balance the attractive self-interaction and gravity



$$M_* \sim 10^{-13} M_{\odot}$$

$$R_* \sim 10^{-4} R_{\odot} \sim R_{☽}$$

- Grow by attracting surrounding axions.
- Collapse when heavier than the critical mass.

Non-relativistic EFT (Part II)

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2} i \left(\psi^* \dot{\psi} - \dot{\psi}^* \psi \right) - \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi - \mathcal{V}_{\rm eff}$$

- Dilute axion field

$$\mathcal{V}_{\rm eff} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \cancel{\frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4} + \ldots}$$

Dilute limit

- In dense axion field $(\psi^* \psi) \sim m_a f_a^2$, must keep all orders

Both instanton and chiral potential can be summed to all orders

e.g. Instanton potential:

$$\mathcal{V}_{\rm eff}(\psi^* \psi) = \tfrac{1}{2} m_a \psi^* \psi + m_a^2 f_a^2 \left[1 - J_0(2\psi^* \psi / m_a f_a^2) \right]$$

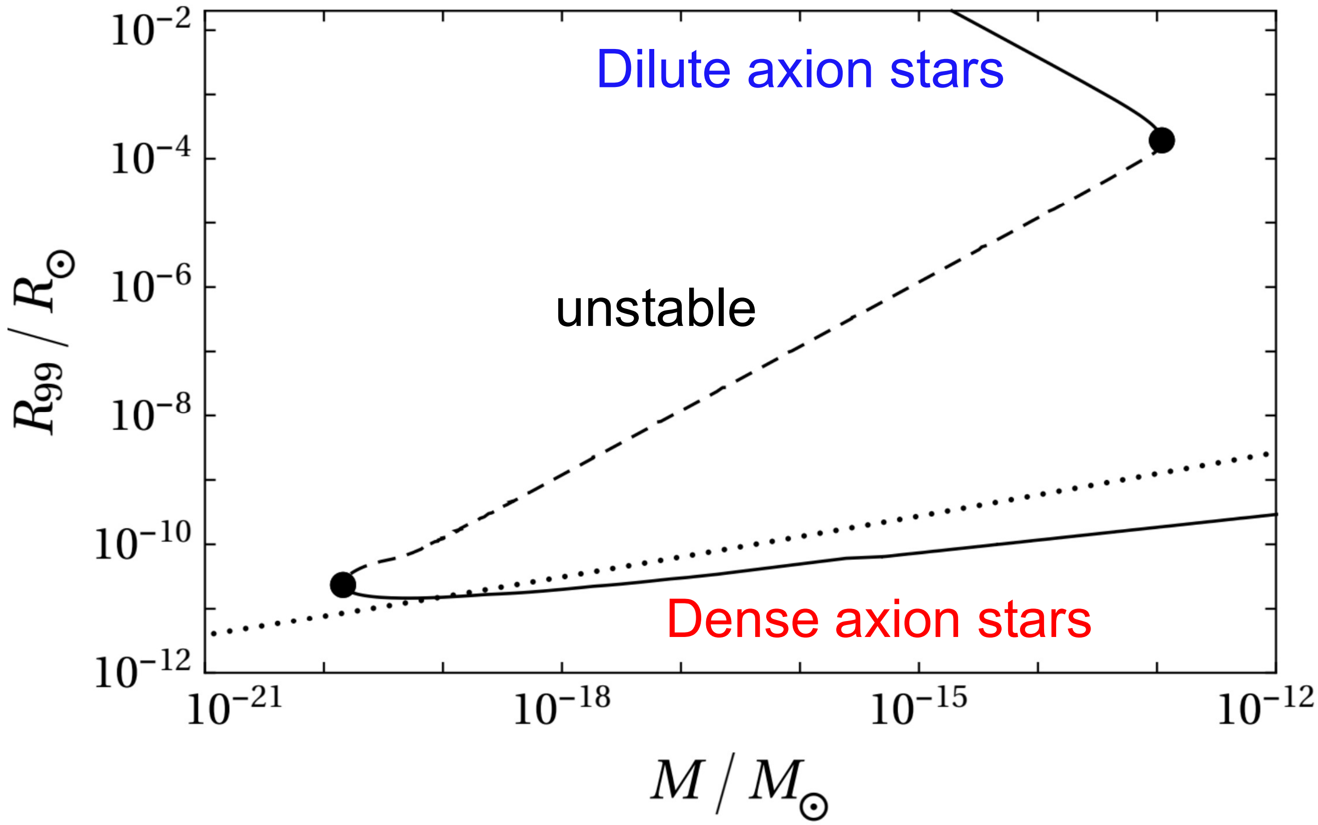
Eby, Suranyi, Vaz, Wijewardhana (2015)
Braaten, Mahapatra, **HZ**, PRD (2016), PRD (2018)

Dense Branch

With untruncated potential, a new dense branch is found.

Assume: • NREFT • Newtonian gravity • Isotropic

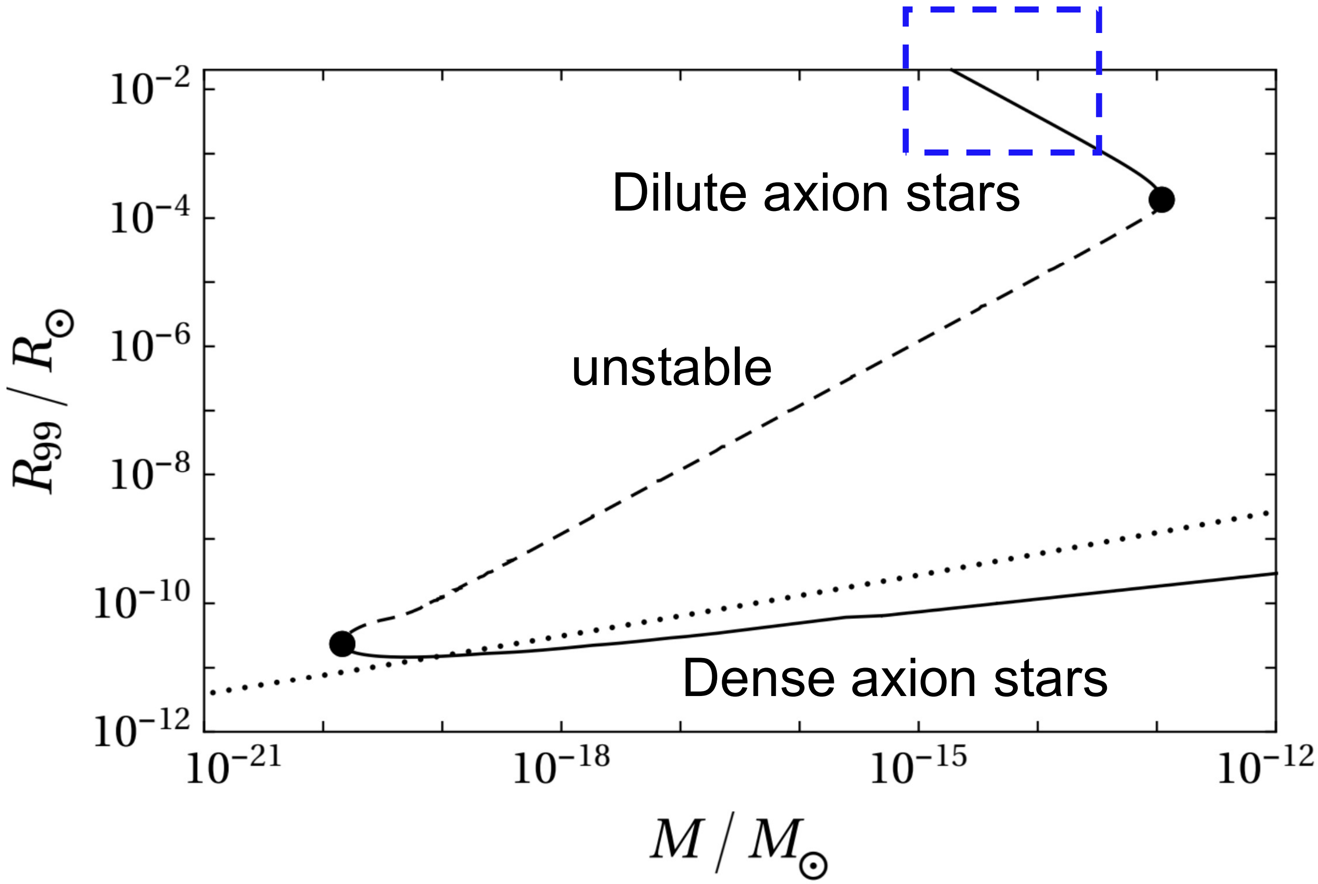
May form as a remnant of the dilute axion star collapse.



Braaten, Mahapatra, **HZ,** PRL (2016), Braaten, **HZ,** RMP (2019)

Axion Stars

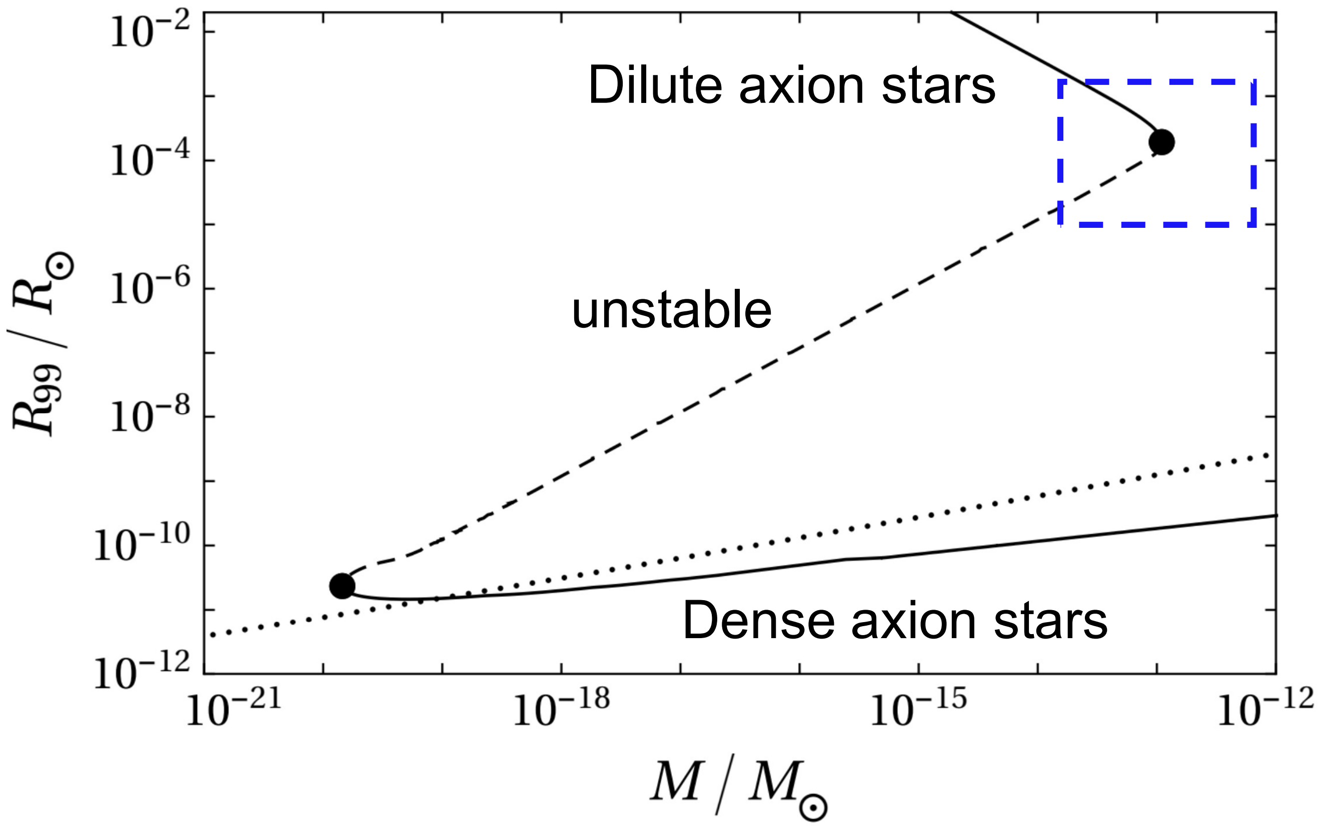
Quantum pressure balances gravity,
All self-interactions can be ignored!



Braaten, Mahapatra, **HZ,** PRL (2016), Braaten, **HZ,** RMP (2019)

Axion Stars

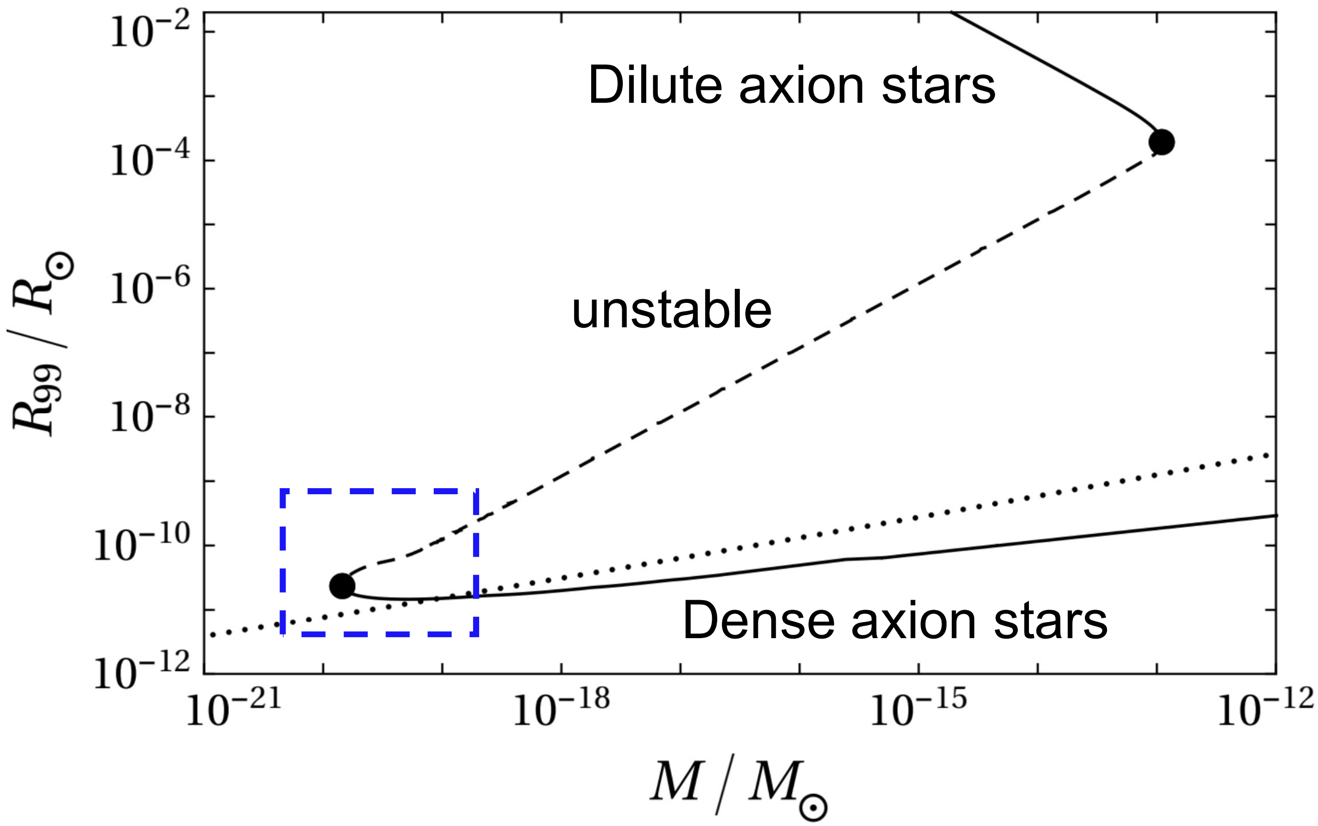
Quantum pressure balances (gravity + ϕ^4 interaction),
Attractive ϕ^4 interaction causes the turning over.



Braaten, Mahapatra, **HZ,** PRL (2016), Braaten, **HZ,** RMP (2019)

Axion Stars

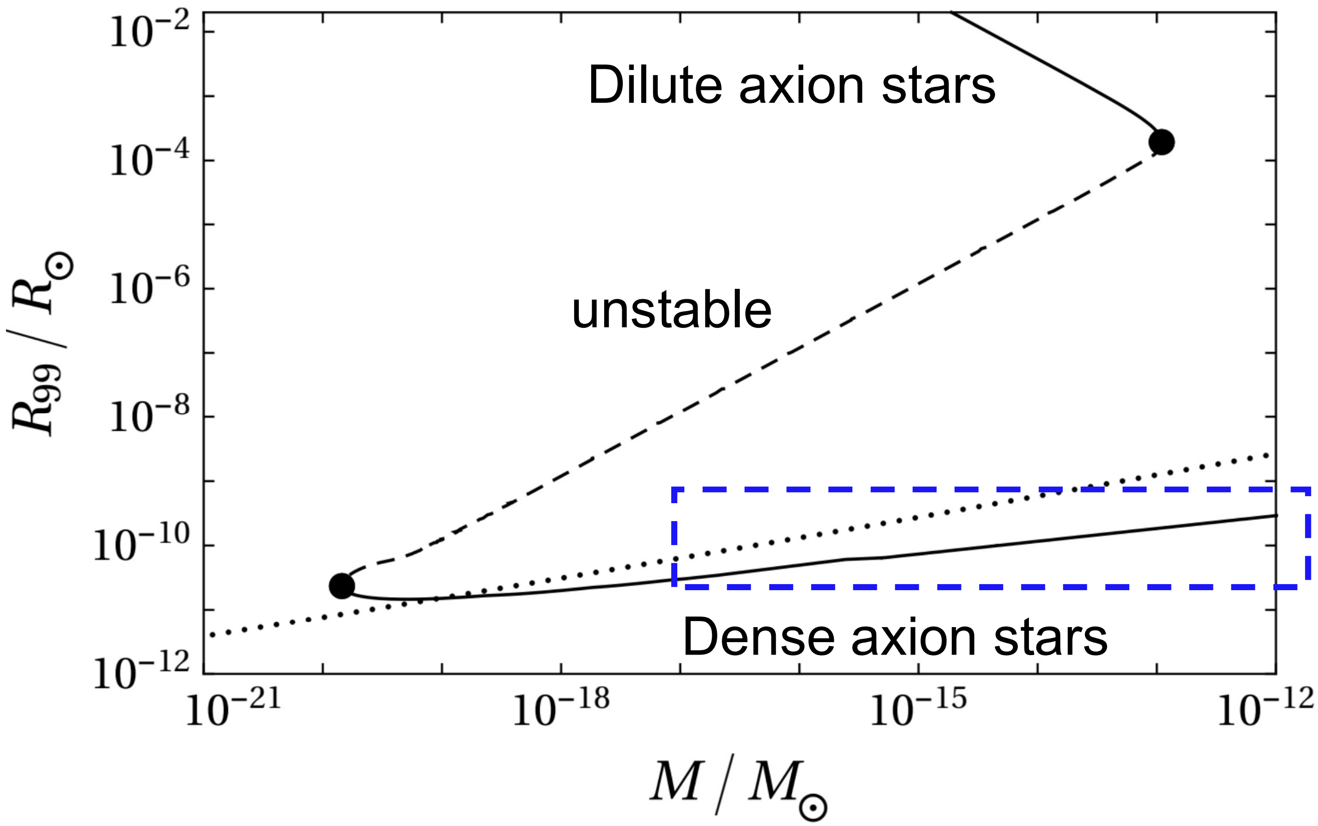
Higher orders in the potential become important.
Quantum pressure balances full potential.
Gravity can be ignored! Same results are obtained without gravity.



Braaten, Mahapatra, **HZ,** PRL (2016), Braaten, **HZ,** RMP (2019)

Axion Stars

Gravity is important at large mass.
Newtonian gravity is not accurate anymore.

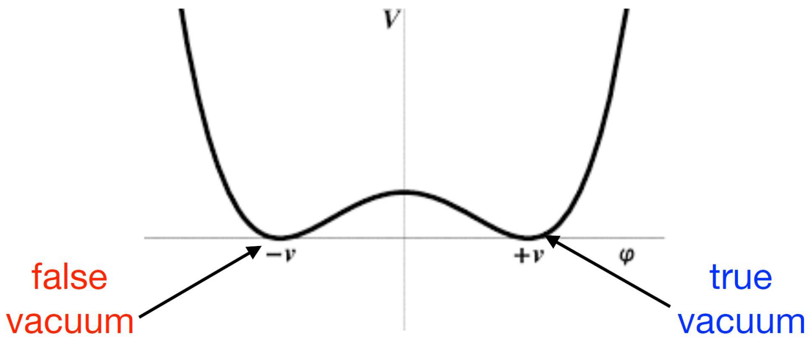


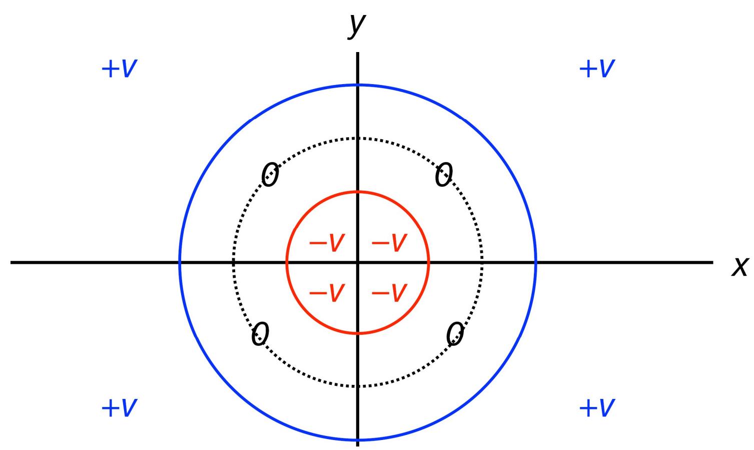
Braaten, Mahapatra, **HZ,** PRL (2016), Braaten, **HZ,** RMP (2019)

Detour: Oscillons

- Real scalar field with 3-d isotropic double-well potential

Bogolubsky & Makhankov (1976)





Inside: false vacuum
Outside: true vacuum

- Oscillon ~ dense axion star without gravity

Detour: Time Evolution of Oscillons

Three stages found in some numerical calculation

1. relaxation

Bogolubsky & Makhankov (1976)

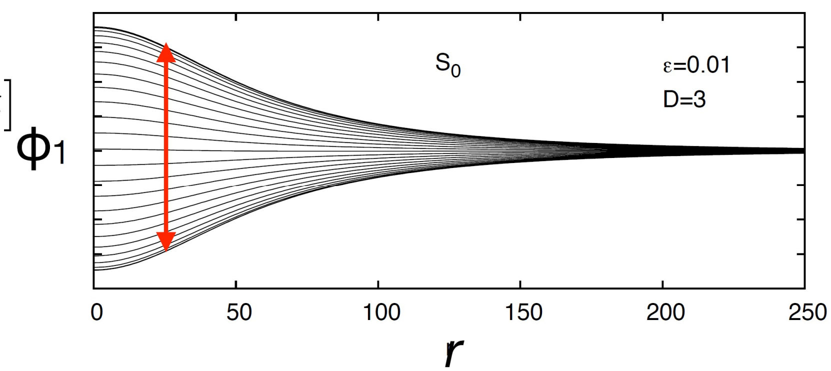
From a given initial profile, radiate a large fraction of energy into outgoing waves

2. **oscillon**!

Localized oscillating configuration stabe for many oscillations, slowly radiates outgoing waves.

$$\phi(r,t) = \sum_{n=1}^{\infty} \phi_{2n+1}(r) \cos[(2n+1)\omega t]$$

$$(\omega \approx m_a)$$



3. Sudden collapse

Configuration suddenly become unstable, disappear into outgoing waves.

Detour: Time Evolution of Oscillons

Three stages found in some numerical calculation

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From a given initia
into outgoing wave

2. **oscillon**!

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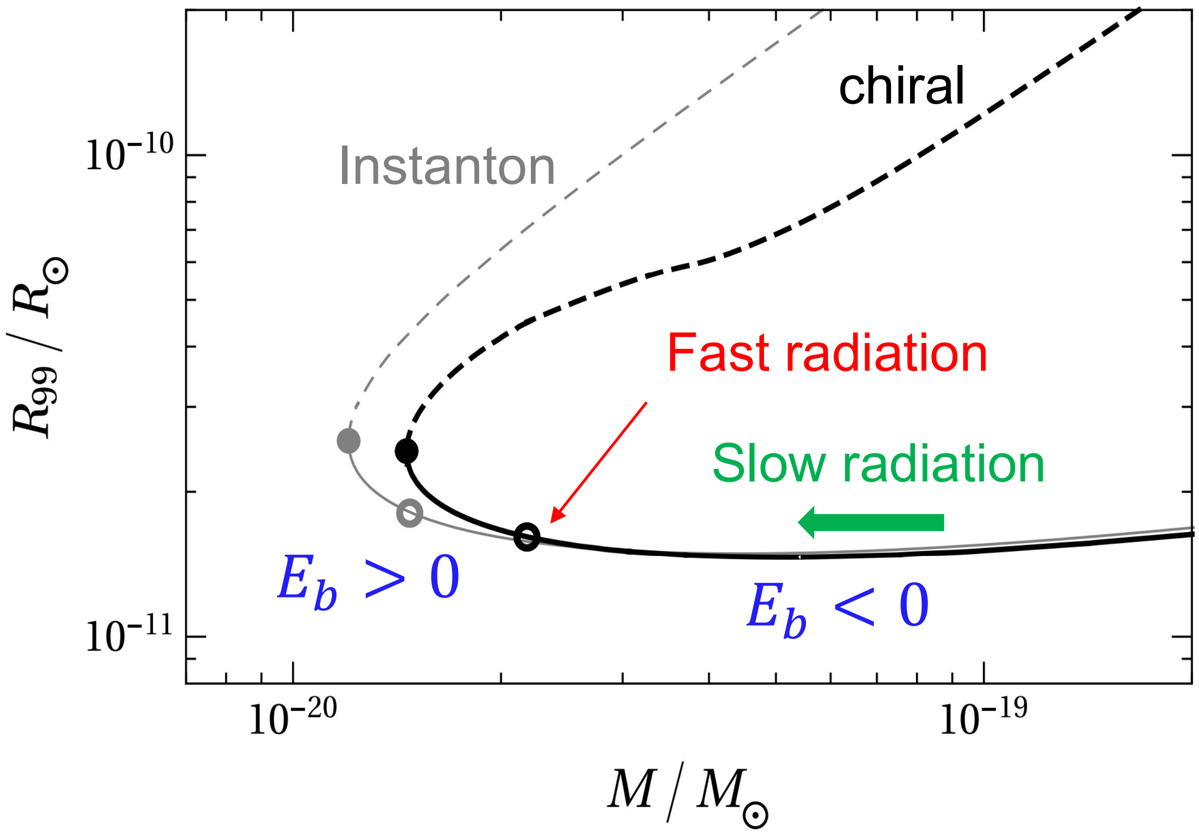


Observation of Dense Axion BEC

- The radiation power of dense axion star

  Heaviest dense axion star luminosity ~ 40W

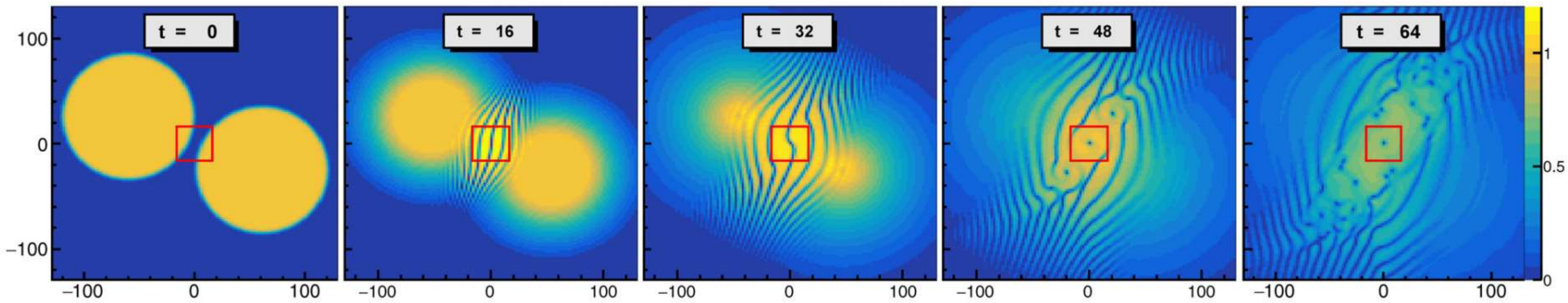
  Too weak!

  Catastrophic phenomenon: collision of two axion stars,
  collision of an axion star with a neutron star …

- Catastrophic process is complicated

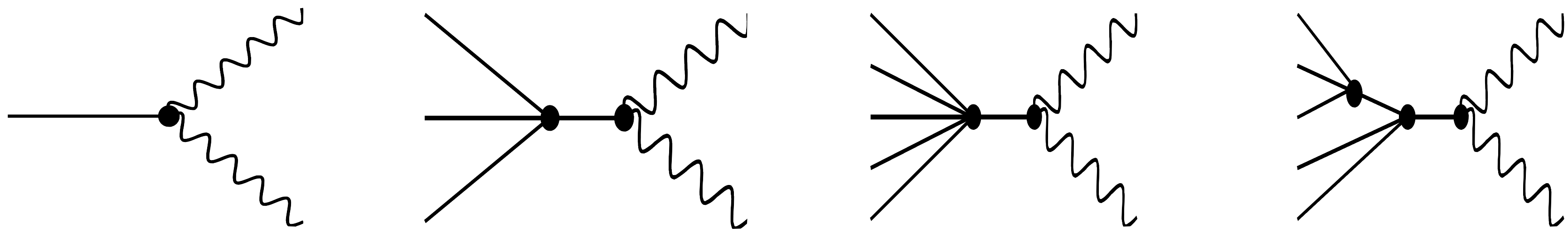
  Collision of two 2-d axion BEC with only ϕ^4 interaction

  Orbital angular mom. localized to vortices.



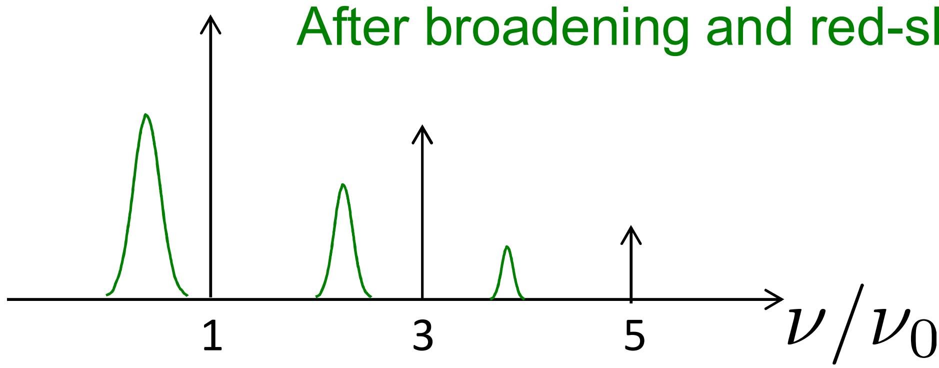
Deng, Wang and **HZ**, Physics Fluids (2022)

Odd-integer Harmonics



$$\nu_0 = \frac{m_a}{2} \qquad \nu = 3\frac{m_a}{2} \qquad \nu = 5\frac{m_a}{2} \qquad \cdots$$

- Odd-integer harmonics of the fundamental radio frequency.



Unique feature of axions !!

Braaten, Mohapatra, HZ, arXiv:1609.05182

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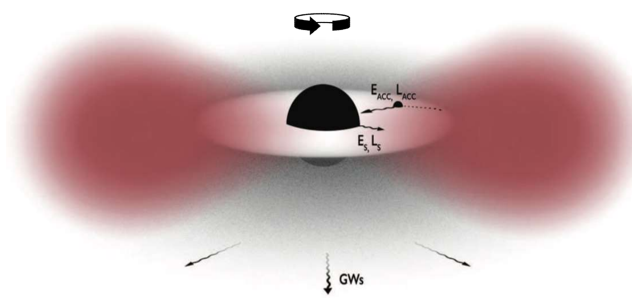

- ✧ **Summary**

Black Hole Bomb

- Because of the wave nature, the ultralight scalar cannot fall into the black hole as point particles.
- The wave equation should be solved with the Einstein equation. When the field is weak, its feedback to the metric can be ignored.
- When scattered by a **Schwarzschild BH**, the phase shift has a nonzero imaginary part, corresponding to the absorption of the scalar field by the BH.
- When scattered by a **Kerr BH**, the incoming wave can be amplified by Penrose process.

Figure from internet

Published: 28 July 1972

Floating Orbits, Superradiant Scattering and the Black-hole Bomb

WILLIAM H. PRESS & SAUL A. TEUKOLSKY

Nature **238**, 211–212 (1972) | Cite this article

Massive Scalar in Kerr Metric

- Bound states: a natural "mirror"

  Free scalar field with mass μ: $(\nabla^{\nu}\nabla_{\nu} + \mu^2)\Phi = 0$

- The radial and angular parts can be factorized

$$\phi(t, \vec{r}) = \sum_{l,m} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{lm}(r) S_{lm}(\theta) + \text{c.c.} \right]$$

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Spheroidal harmonics,
similar to spherical harmonics

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Spheroidal harmonics,
similar to spherical harmonics

- $\omega_{n\ell m}$ is solved from the radial equation

  Complex eigen-energy: $\omega_{n\ell m} = \omega_{n\ell m}^{(R)} + i\omega_{n\ell m}^{(I)}$

  Three "quantum" numbers: (n, l, m)

  $\omega_{n\ell m}^{(R)} \approx \mu$, but $\omega_{n\ell m}^{(I)}$ is 7 orders of magnitude smaller.

  Numerical calculation requires extremely high precision.

Detweiler's Method

Detweiler, PRD 22 (1980) 2323

(1) find the large-r asymptotic wavefunction, then find its small-r limit

$$\frac{(2\kappa)^{l'}\Gamma(-2l'-1)}{\Gamma(-l'-\lambda)}r^{l'}+\frac{(2\kappa)^{-l'-1}\Gamma(2l'+1)}{\Gamma(l'+1-\lambda)}r^{-l'-1}$$

(2) find the small-r asymptotic wavefunction, then find its large-r limit

$$\frac{(2b)^{-l'}\Gamma(2l'+1)}{\Gamma(l'+1)\Gamma(l'+1-2ip)}r^{l'}+\frac{(2b)^{l'+1}\Gamma(-2l'-1)}{\Gamma(-l'-2ip)\Gamma(-l')}r^{-l'-1}$$

$\kappa, l', \lambda, b, p$ are functions of M, μ, ω, a

(3) At $\alpha \equiv M\mu \ll 1$, the two wavefunctions have an overlapped region. The ratios of the two coefficients must be the same.

(4) The small quantity $\delta\lambda$ for perturbation: $l' + 1 - \lambda = -n - \delta\lambda \;\; (n = 0,1 \dots)$

$\omega = \omega_0 + \omega_1 \delta\lambda$ with ω_0, ω_1 real functions of M, μ, n, l

$$\delta\lambda^{(0)} = -\mathbf{2}\, ip\,(4\kappa b)^{2l+1}\frac{(n+2l+1)!(l!)^2}{n!\,[(2l)!(2l+1)!]^2}\prod_{j=1}^{l}(j^2+4p^2),$$

Detweiler's result has an extra factor of 2,
due to mistreatment of Γ functions with negative argument.

NLO Solution

Bao, Xu and **HZ,** PRD (2020)

- LO analytical result is inconsistent with the numerical solution

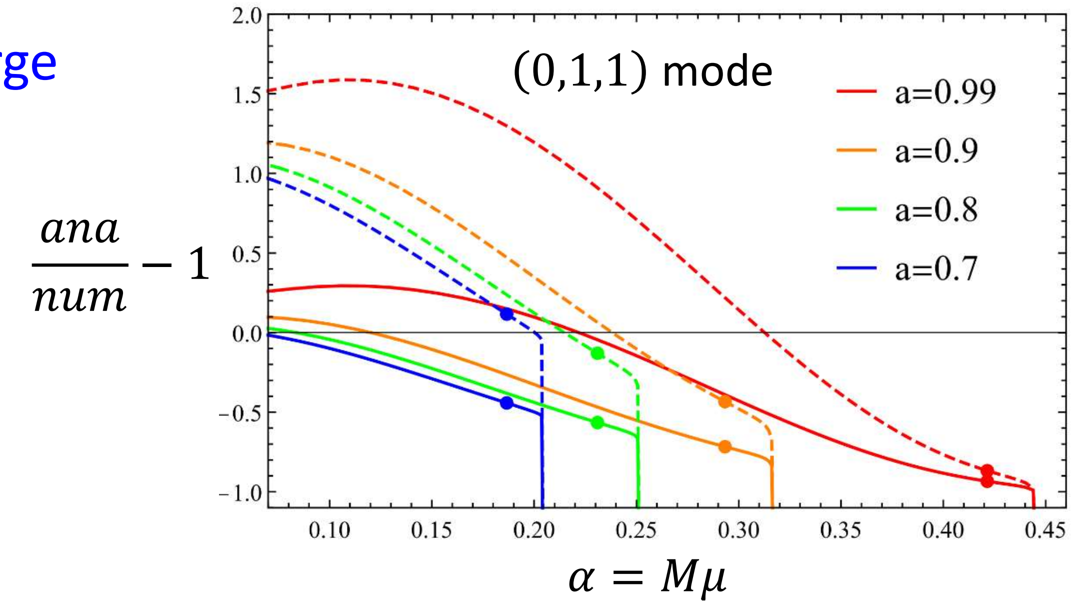
Analytical result should converge to numerical result at small α.

At small α with $a = 0.99$,

Err. of original result $\sim$ 150%

Err. of corrected result $\sim$ 30%

Error increases with a.



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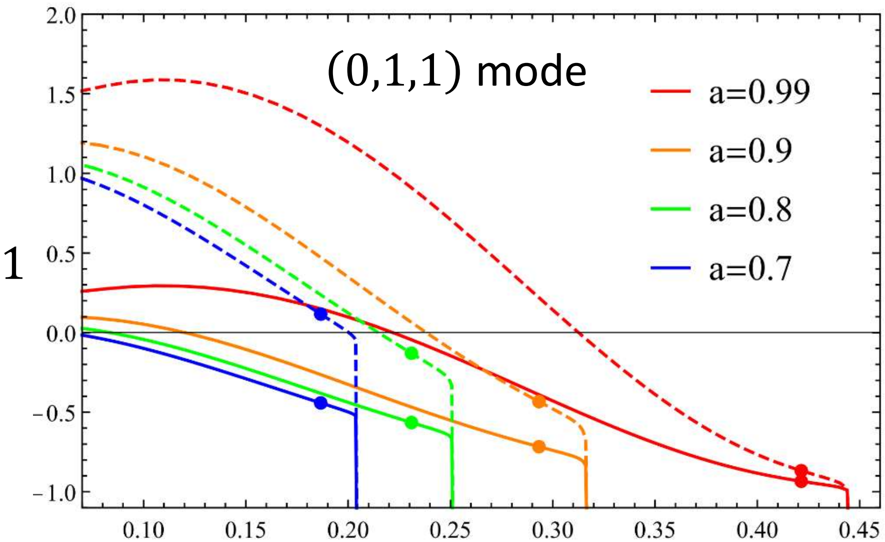
$$\frac{ana}{num} - 1$$

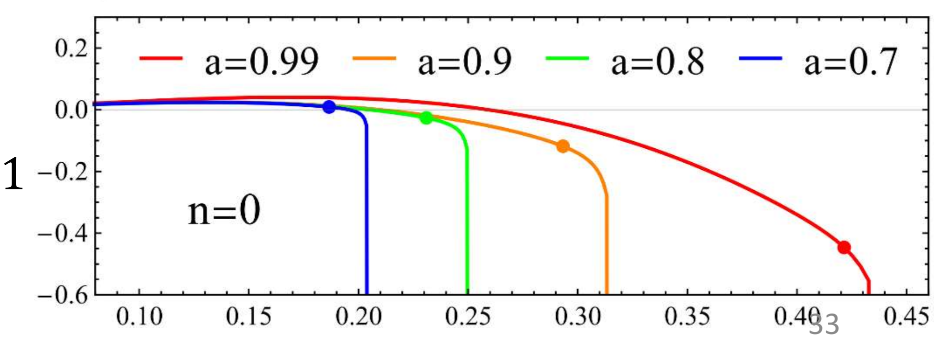


$\alpha = M\mu$

- There is a NLO term proportional to $\dfrac{\alpha^2}{\sqrt{1-a^2}}$, important at large a
- Other NLO contributions are also added.

At small α, **err.** $<$ **5%**

$$\frac{ana}{num} - 1$$



$\alpha = M\mu$

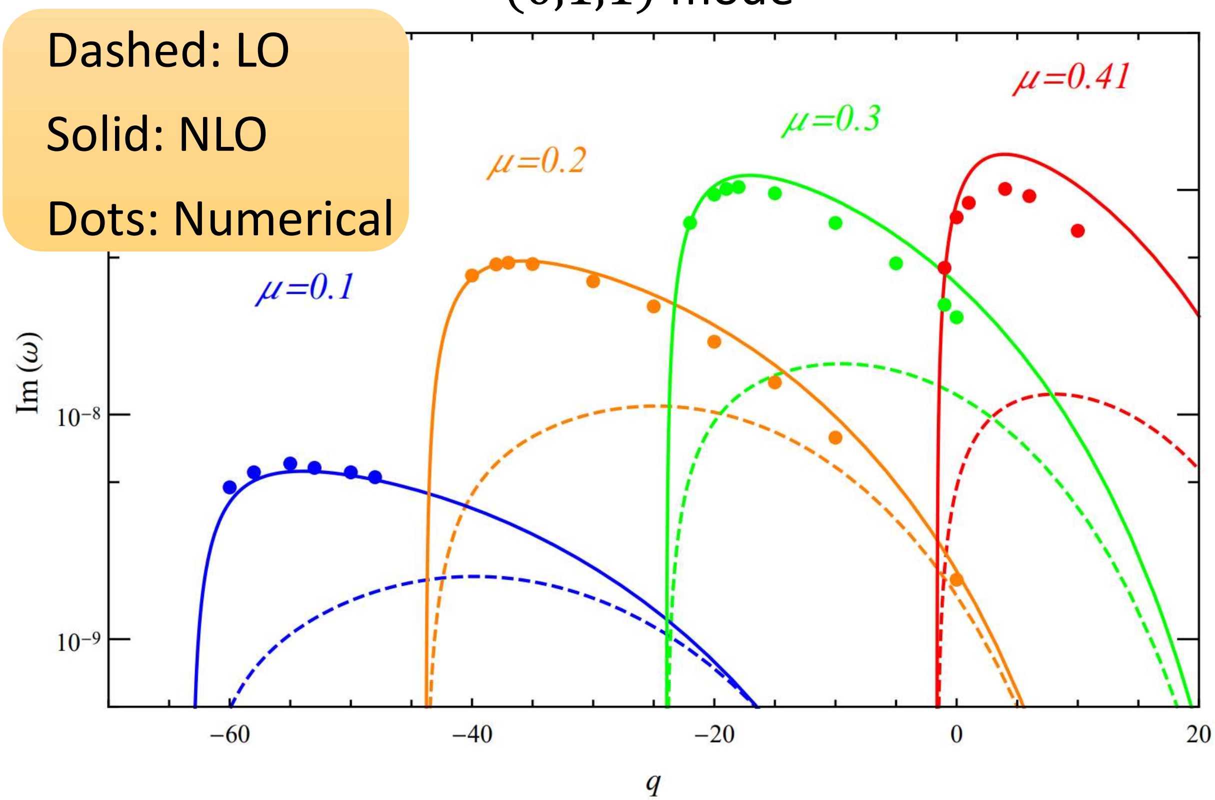
NLO Sol. of KNBH

Bao, Xu and **HZ,** arXiv:2301.05317

- NLO solution greatly improves the precision

BH mass is normalized to 1, BH charge $Q = 0.02$

$(0,1,1)$ mode



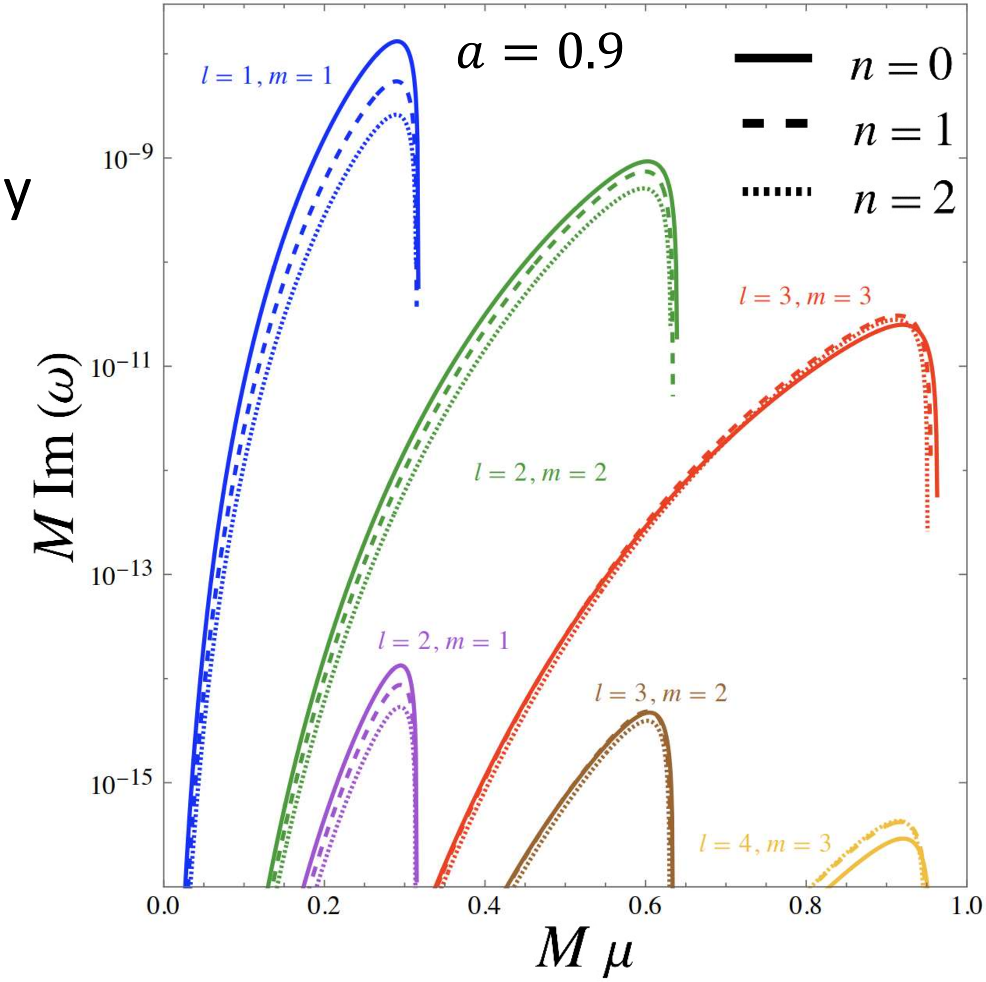
- In the rest of the talk, I focus only on Kerr BH.

Superradiance Rate of Kerr BH

- Three indices (n, l, m)
- Cloud mass rises exponentially

$$\boxed{\omega_I \rightarrow \dot{M}_s = 2M_s\omega_I}$$

- Dominant mode: $(n = 0, l, m = l)$
- Subdominant mode: $(1, l, m = l)$
- Modes with $m < l$ are unimportant.



- The right edge is because of the **superradiance condition:**

$$\omega_R < m\,\Omega_H$$

$\Omega_H = a/2r_+$ is the angular velocity of the BH horizon.

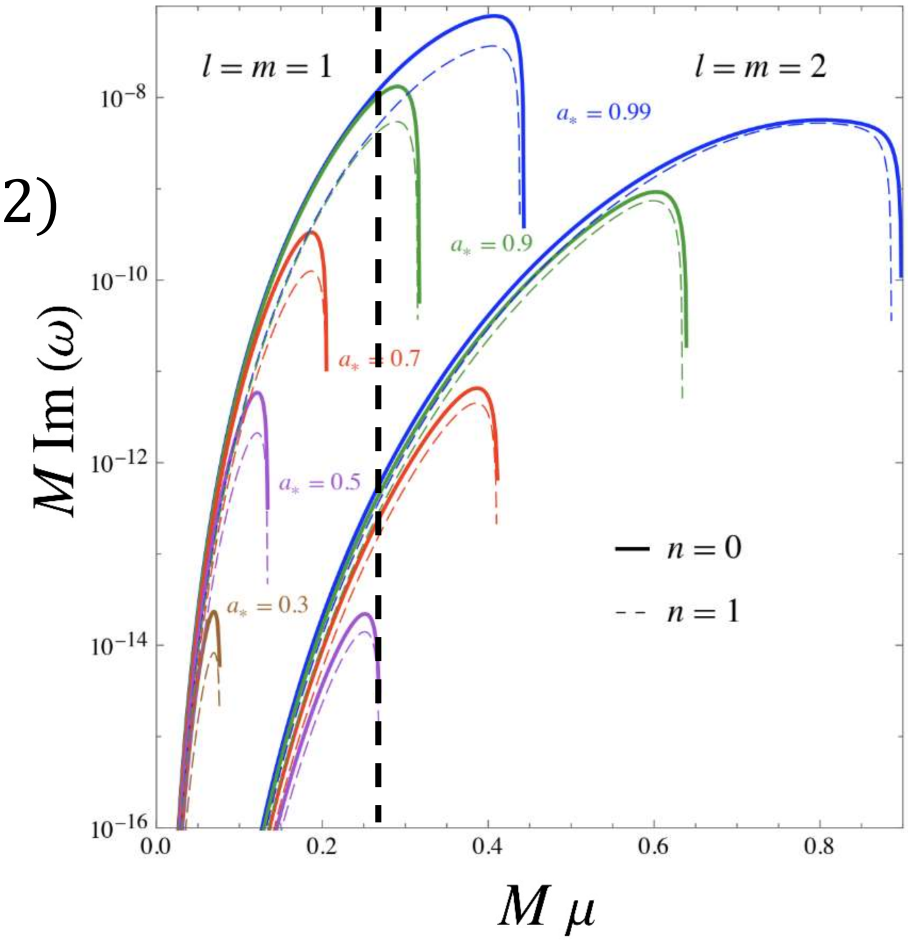
Superradiance Rate of Kerr BH

- Three indices (n, l, m)
- Consider modes $(0,1,1)$ and $(0,2,2)$
- Fixing $M\mu$, reducing BH spin a

Superradiance rate decreases;

There is a critical value of a_C^{011} where the superradiance of $(0,1,1)$ mode stops;

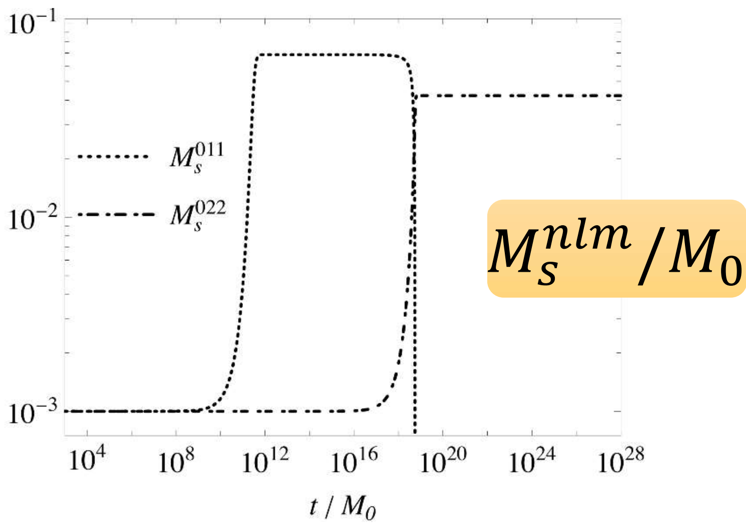
The $(0,2,2)$ mode is still extracting BH spin J

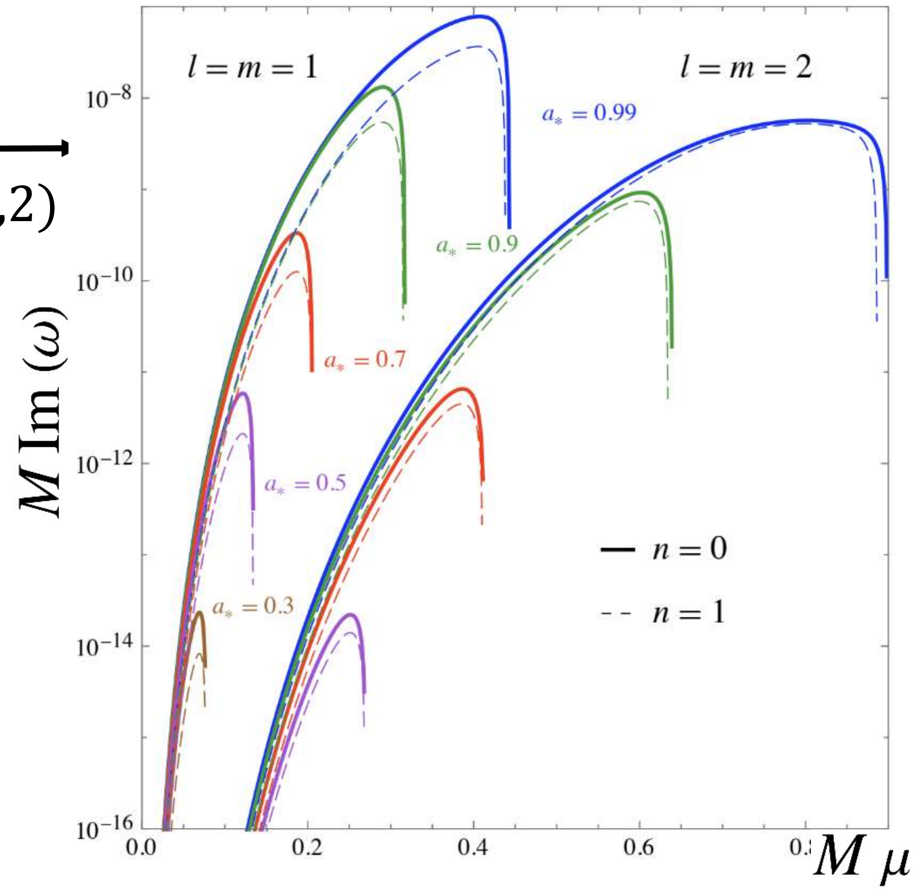


When BH spin is below a_C^{011}, the $(0,1,1)$ mode returns J to the BH. BH spin is a_C^{011} until $(0,1,1)$ mode is depleted.

Time Evolution I

- Only dominant modes: (0,1,1) and (0,2,2)
- No accretion or GW emission





- The (0,1,1) mode grows faster due to larger value of superradiance rate.
- The (0,1,1) mode depletes while the (0,2,2) mode rises.

Guo, Bao and **HZ,** arXiv:2212.07186

Time Evolution I

- Only dominant modes: (0,1,1) and (0,2,2)
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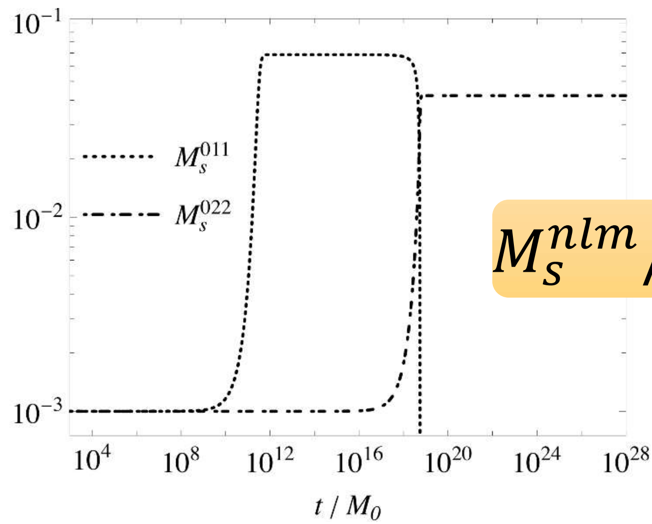

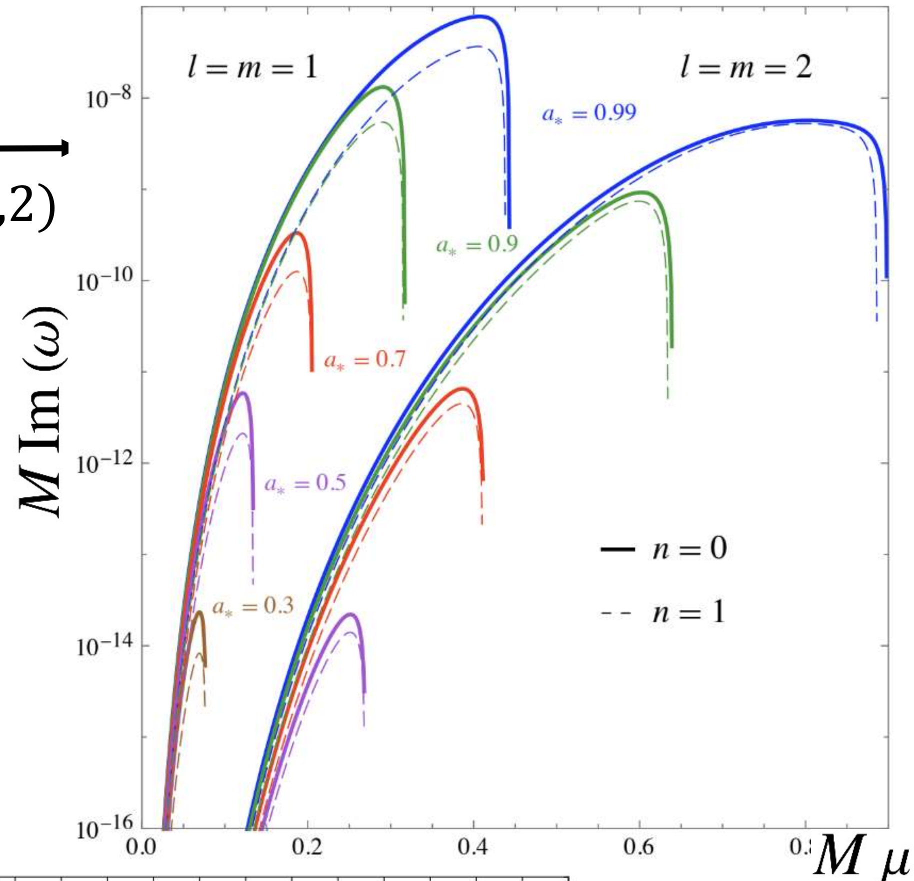

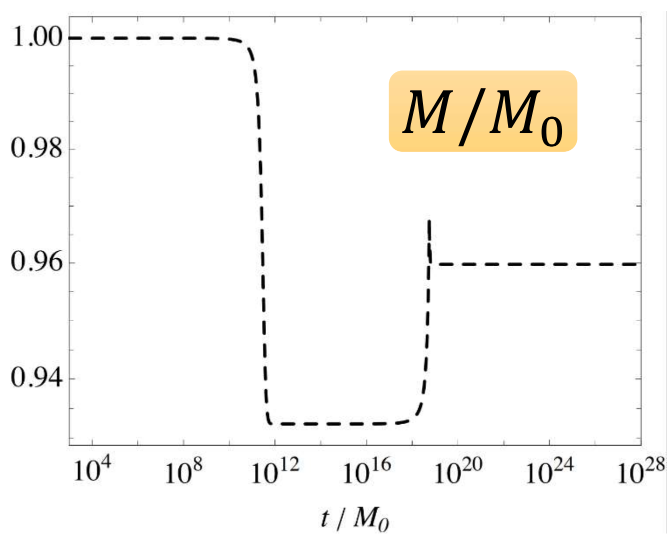

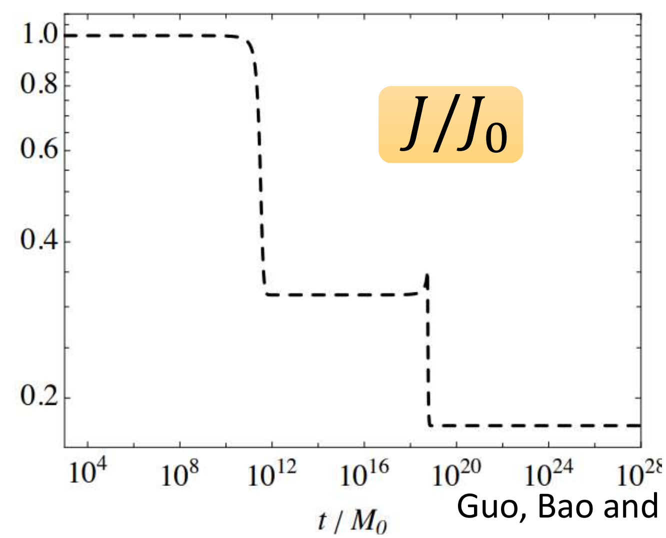

Guo, Bao and **HZ,** arXiv:2212.07186

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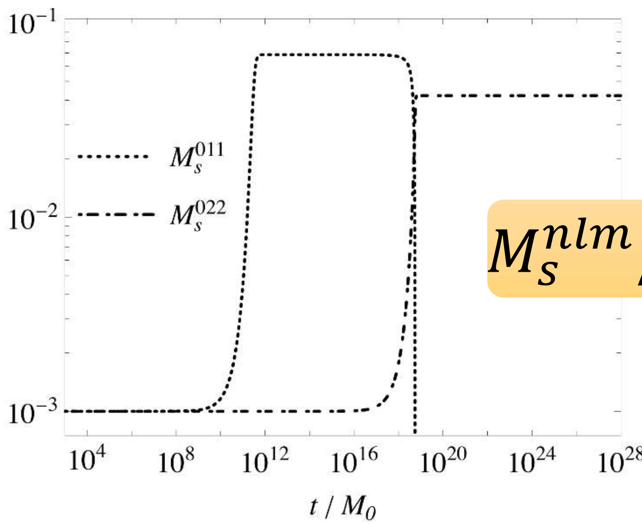
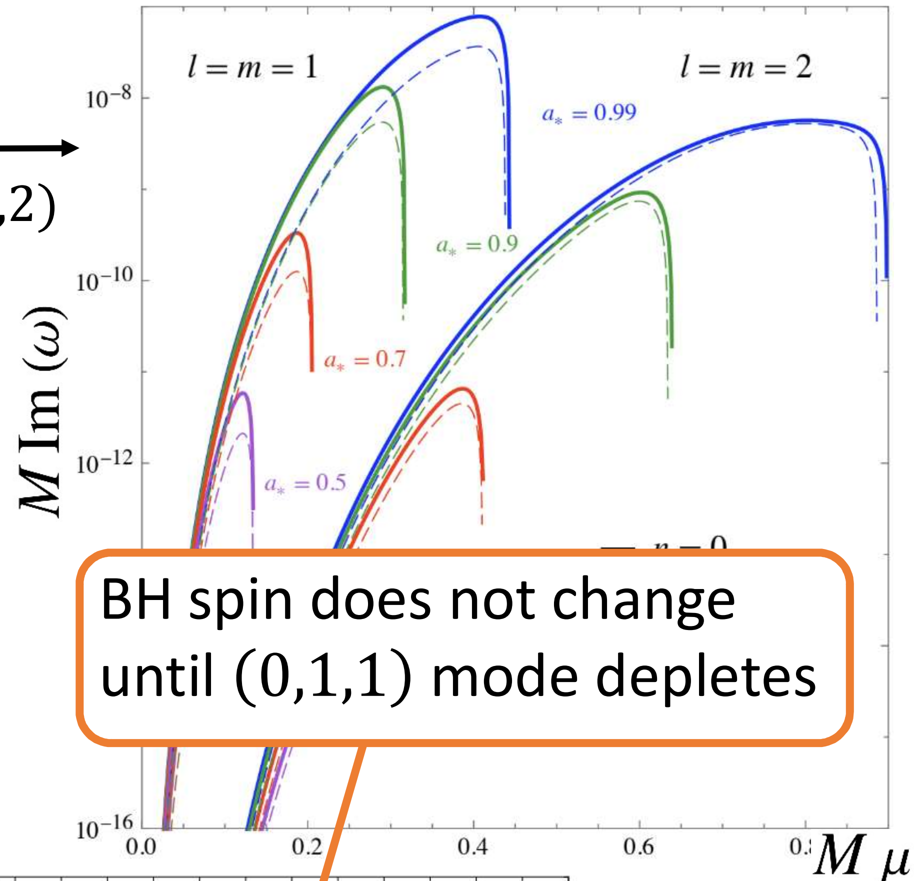
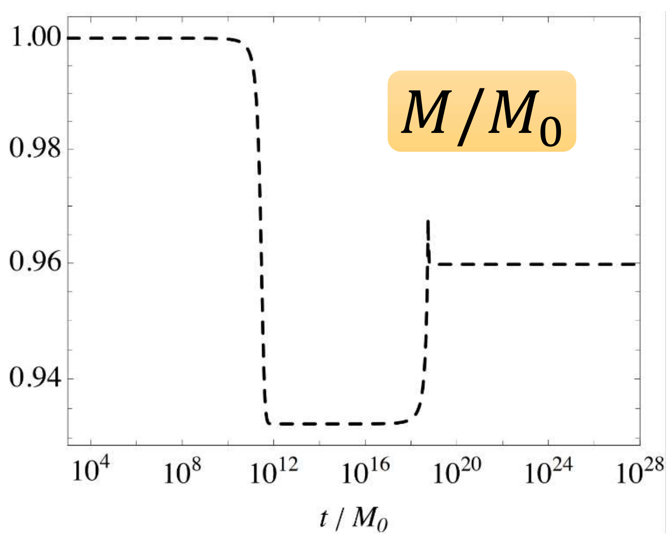
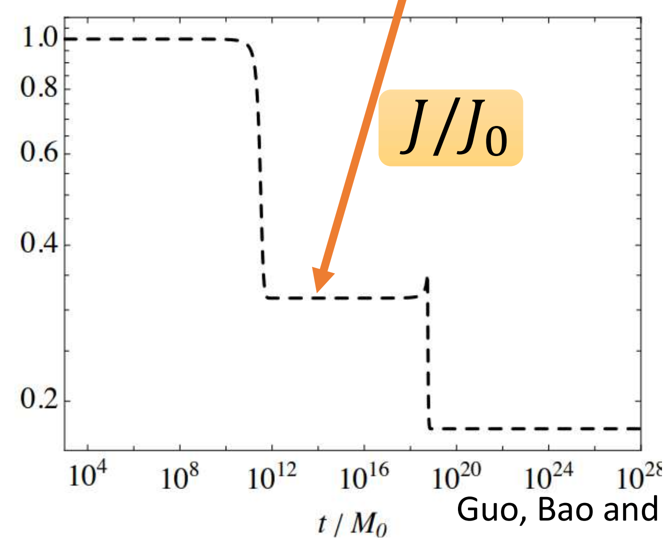




BH spin does not change until (0,1,1) mode depletes

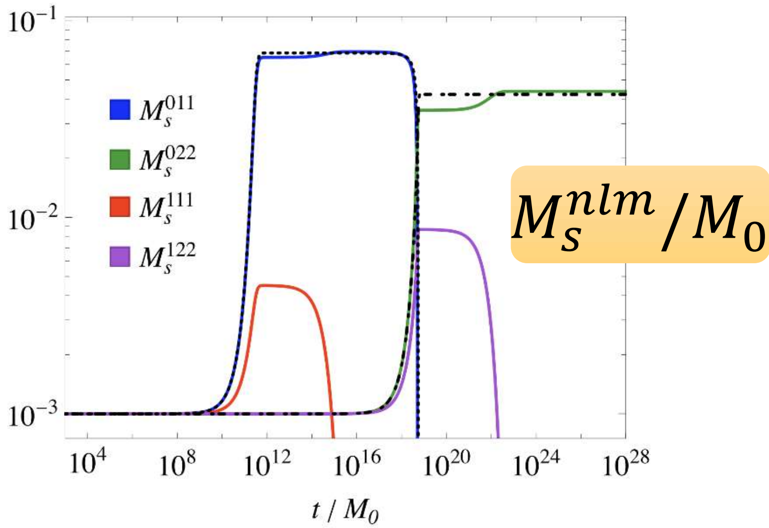


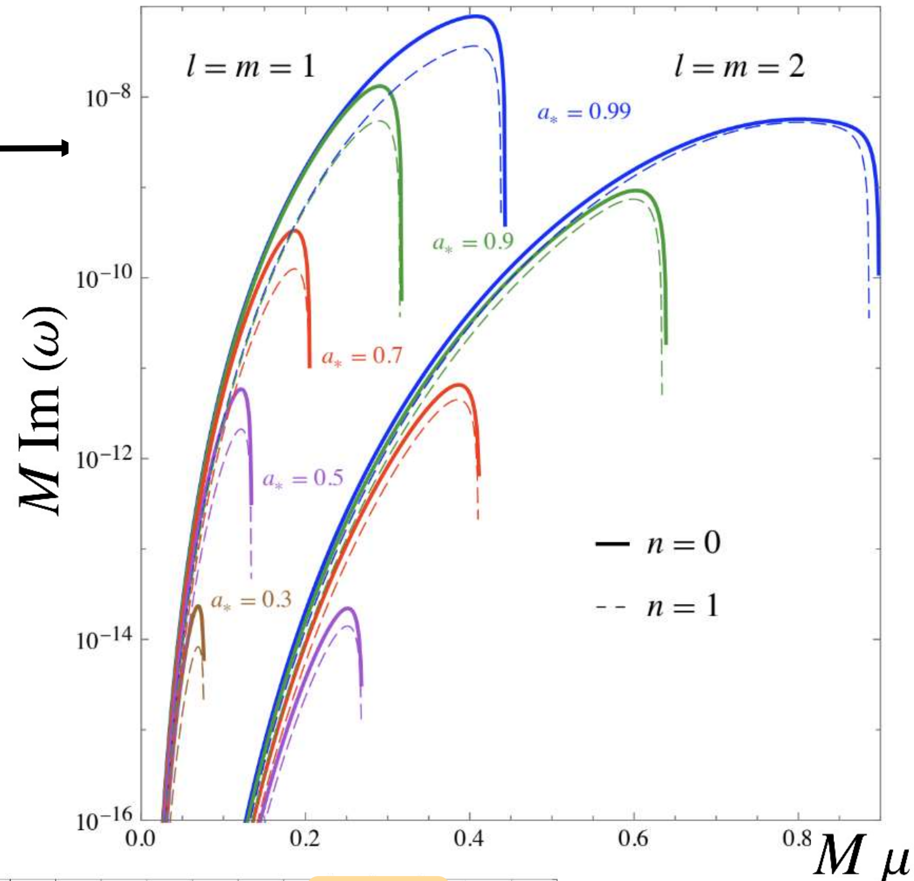


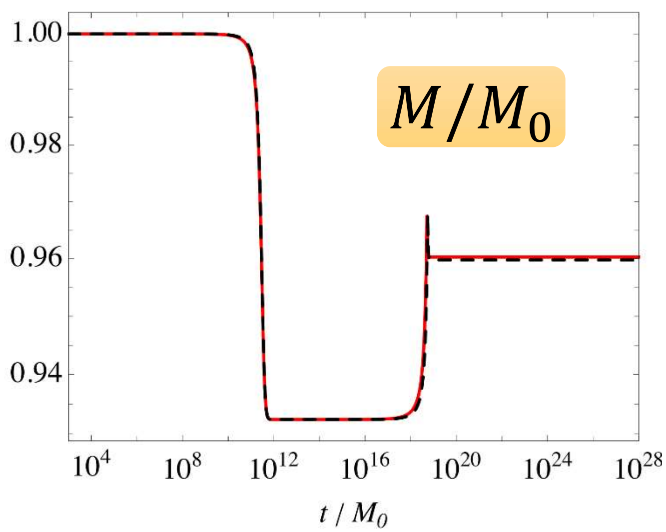
Guo, Bao and **HZ,** arXiv:2212.07186

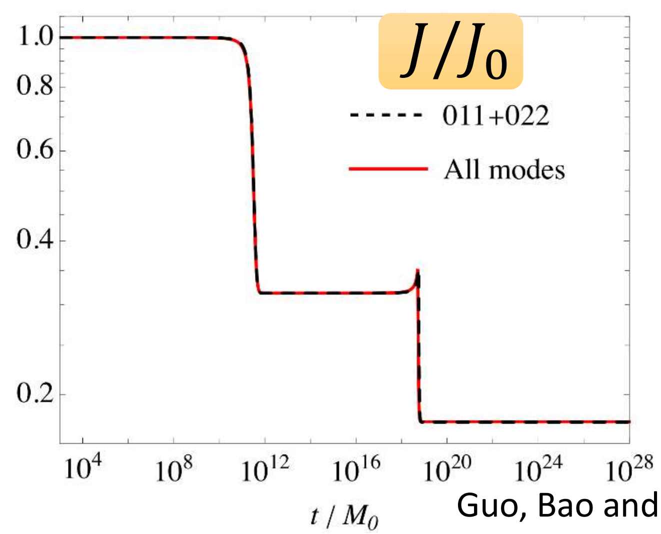
Time Evolution II

- (0,1,1), (0,2,2), (1,1,1), (1,2,2)
- No accretion or GW emission





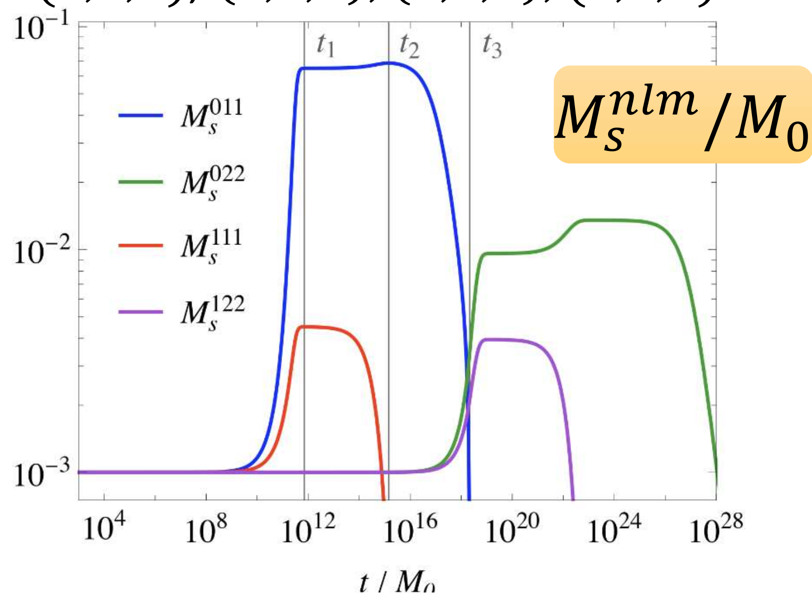


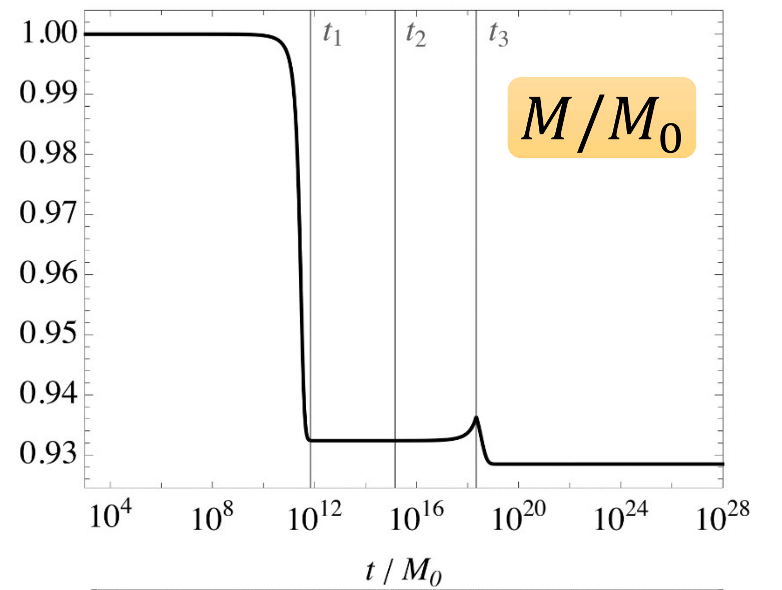


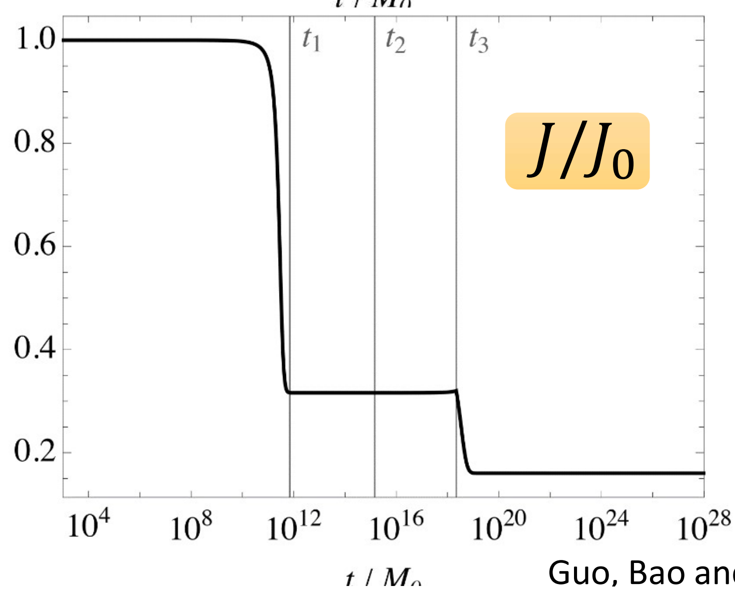
Guo, Bao and **HZ,** arXiv:2212.07186

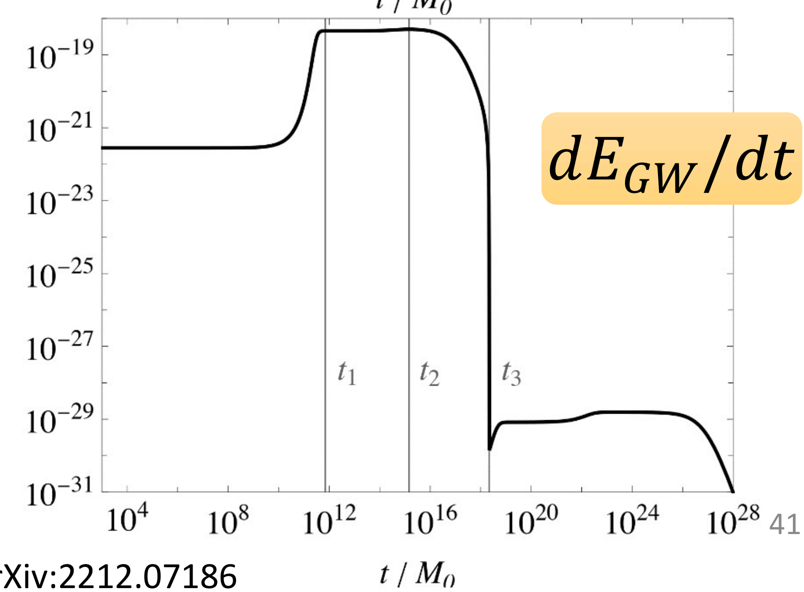
Time Evolution III

- (0,1,1), (0,2,2), (1,1,1), (1,2,2)
- With GW emission, no accretion







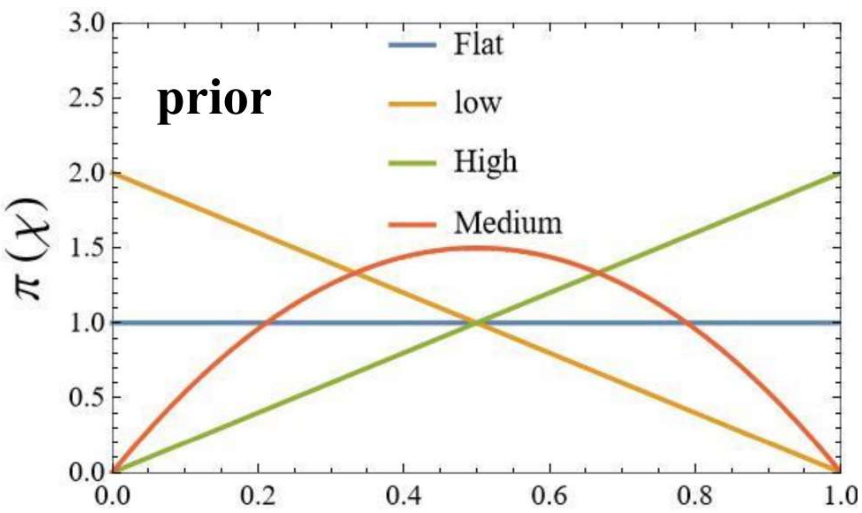


Guo, Bao and **HZ,** arXiv:2212.07186

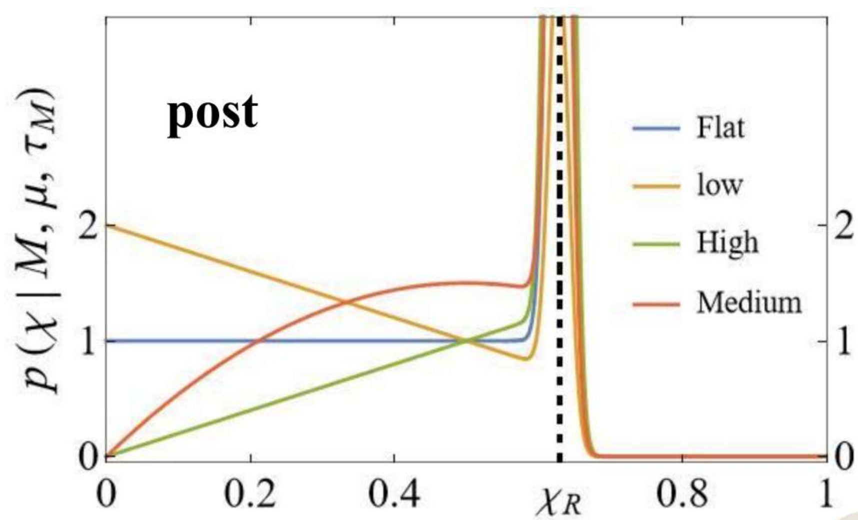
BH Regge Trajectory

Cheng, Bao and **HZ,** arXiv:2201.11338

- Superradiance modifies the BH spin distribution



BH spin a



BH spin a

- High spin BHs quickly lose the spin to the axion clouds

- Consider 3 scenarios: **high**, **flat**, **low** to estimate the effect of the initial BH spin.

Constrain Axion Mass

Cheng, Bao and **HZ,** arXiv:2201.11338

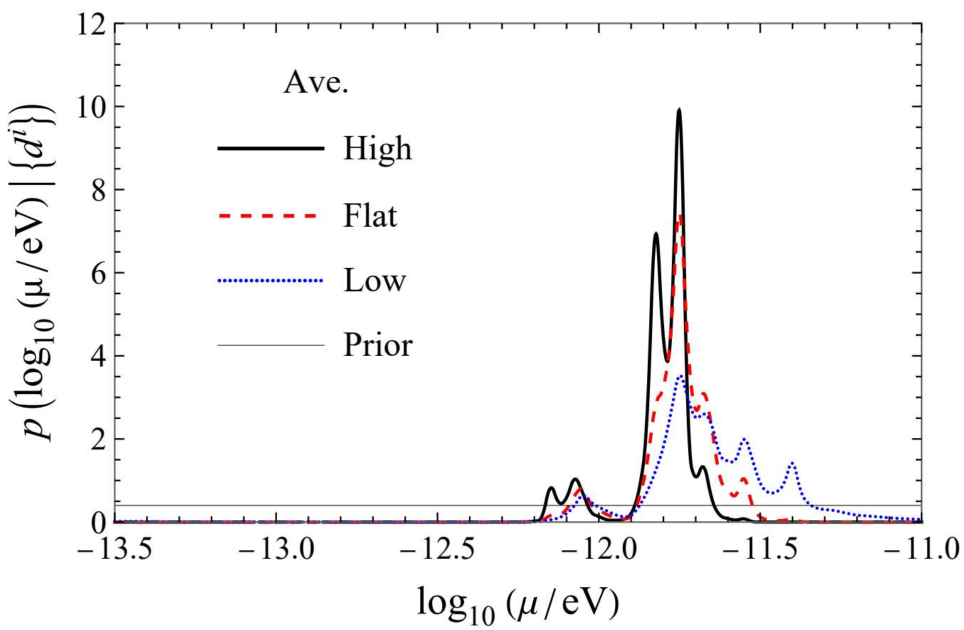
- Data & Assumptions

Include all BBHs in three phases of GTWC data reported by LVK collaboration, only excluding the events with neutron.

Axion mass prior is log-uniform between $10^{-13.5}$ to 10^{-11} eV.

Lifetime of BHs distributes log-uniformly between 10^{6} to 10^{10} years

Approximate the **initial BH spin** distributions with 3 scenarios.

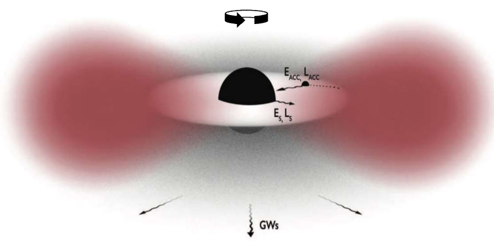


Two slightly favored ranges are identified, but evidence is weak.

GW Emission

Guo, Bao and **HZ,** arXiv:2212.07186

- Previous calculation only consider the $(n = 0, l = 1, m = 1)$ mode



Monochromatic, constant energy flux,
Cannot distinguish from neutron stars

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Guo, Bao and **HZ,** arXiv:2212.07186

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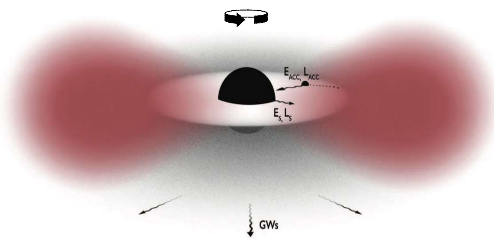

Monochromatic, constant energy flux,
Cannot distinguish from neutron stars

- Different modes have slightly different angular speeds

$$\phi(t,\vec{r}) = \sum_{l,m} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{lm}(r) S_{lm}(\theta) + \text{c.c.} \right]$$

$$\omega_R^{nlm} \approx \mu \left[1 - \frac{\alpha^2}{2(n+l+1)^2} \right] + O(\alpha^4)$$

e.g. $\cos[(\omega + \Delta\omega)t] + \cos[(\omega - \Delta\omega)t] = 2\cos(\Delta\omega\, t)\cos(\omega t)$

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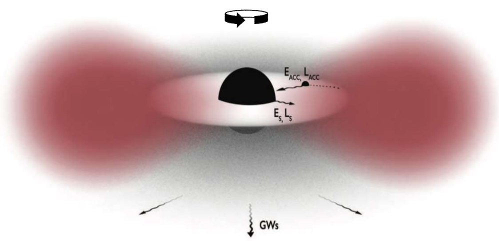

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e.g $\cos[(\omega + \Delta\omega)t] + \cos[(\omega - \Delta\omega)t] = 2\boxed{\cos(\Delta\omega\, t)}\cos(\omega t)$

Modulation of amp. and energy flux. **Beat!**

GW Emission

Guo, Bao and **HZ,** arXiv:2212.07186

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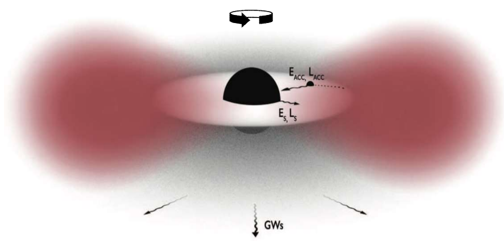

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- Strength of the beat signal.

Two (0,1,1) axions ⟹ graviton: Amp.$\propto N_{011}$, freq. $= 2\omega^{011}$

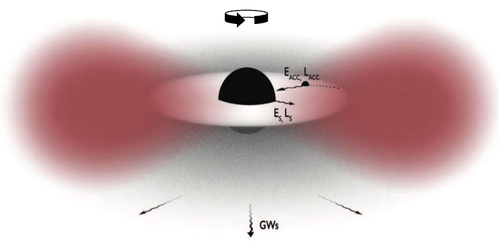
(0,1,1) + (1,1,1) ⟹ graviton: Amp.$\propto \sqrt{N_{011} N_{111}}$, freq. $= \omega^{011} + \omega^{111}$

Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}$

GW Emission

Guo, Bao and **HZ,** arXiv:2212.07186

- Previous calculation only conside

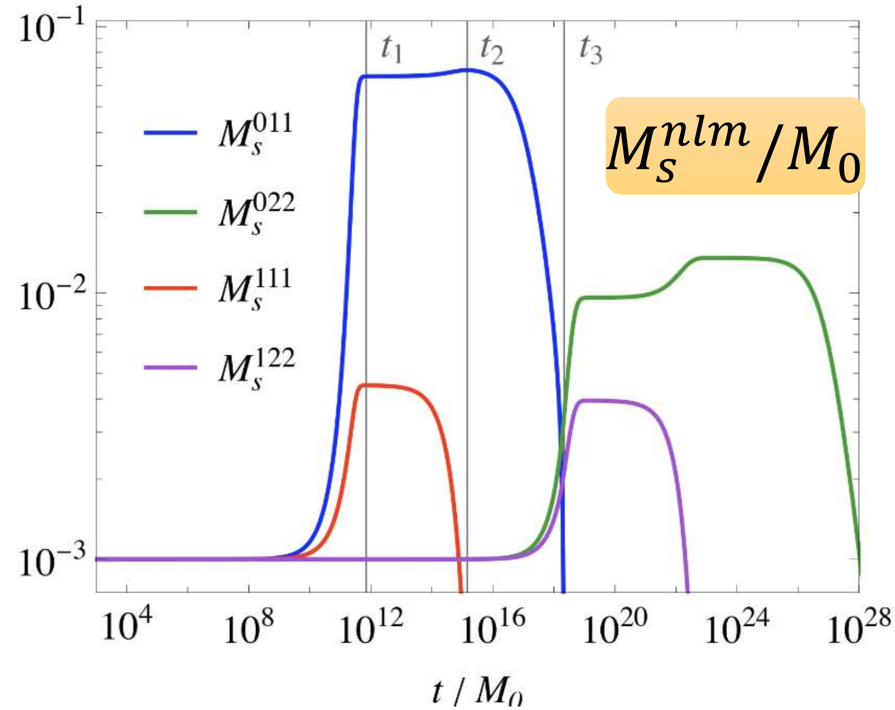


Mo
Can

- Different modes have slightly dif

$$\phi(t,\vec{r}) = \sum_{l,m} \int d\omega \left[e^{i(m\varphi-\omega t)} R_{lm}(r) S_{lm}\right.$$

e.g $\cos[(\omega+\Delta\omega)t] + \cos[($



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Guo, Bao and **HZ,** arXiv:2212.07186

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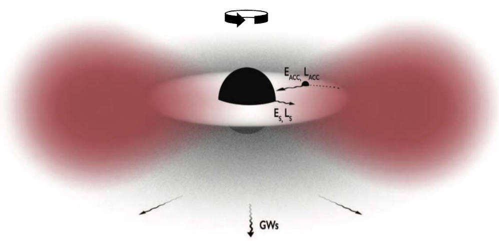

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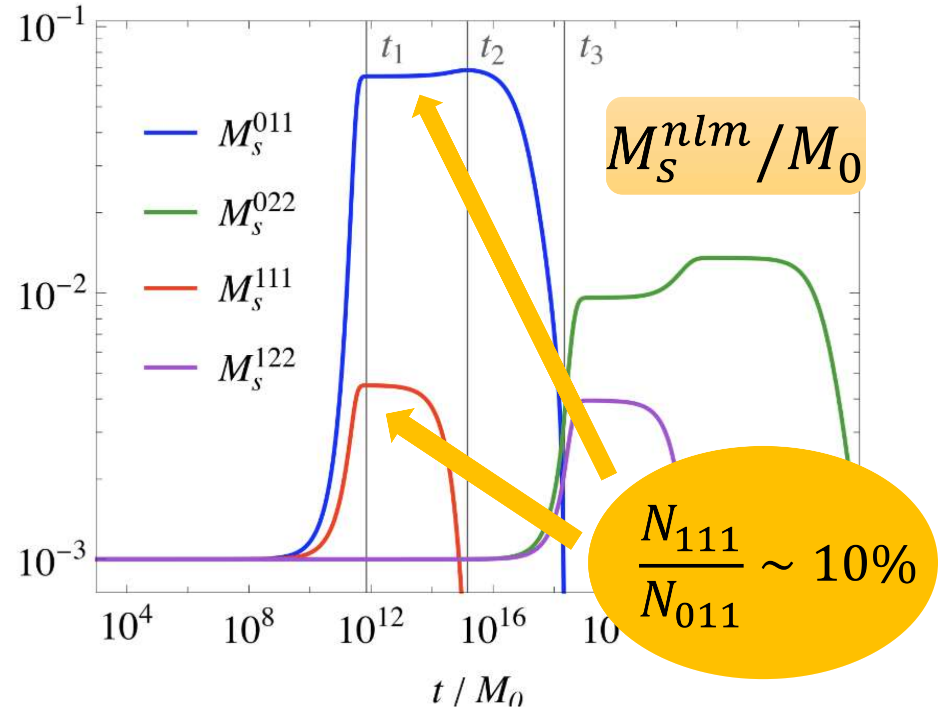

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GW Emission

Guo, Bao and **HZ,** arXiv:2212.07186

- Use Teukolsky formalism to calculate the beat signal

LO

NLO

$\sqrt{N_{111}/N_{011}}$

suppressed

$$
\begin{aligned}
\frac{dE_{\rm GW}}{dt} = & \frac{1}{8\pi}\sum_{\tilde{l}}\Bigg\{\boxed{\frac{N_{011}^2}{\omega^{(011)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_1)}\right|^2}{\tilde{\omega}_1^2}} + \frac{N_{111}^2}{\omega^{(111)^2}}\frac{\left|U_{l2}^{(\tilde{\omega}_2)}\right|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde{\omega}_3}\right|^2}{\tilde{\omega}_3^2} \\
& \boxed{+4\sqrt{\frac{N_{011}^3 N_{111}}{\omega^{(011)^3}\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_3)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right]} \\
& +2\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|}{\tilde{\omega}_1\tilde{\omega}_2}\cdot\cos\left[2\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_2)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right] \\
& +4\sqrt{\frac{N_{011}N_{111}^3}{\omega^{(011)}\omega^{(111)^3}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_2\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*)-\phi_{\tilde{l}2}^{(\tilde{\omega}_2)}+\phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\right]\Bigg\}.
\end{aligned}
$$

$$
\tilde{\omega}_1 \equiv 2\omega^{(011)},\ \tilde{\omega}_2 \equiv 2\omega^{(111)},
$$
$$
\tilde{\omega}_3 \equiv \omega^{(011)} + \omega^{(111)},\ \tilde{\omega}_4 \equiv \omega^{(111)} - \omega^{(011)}
$$

GW Emission

Guo, Bao and **HZ,** arXiv:2212.07186

- Use Teukolsky formalism to calculate the beat signal

LO

$$\frac{dE_{\rm GW}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\left\{ \frac{N_{011}^2}{\omega^{(011)^2}}\frac{\left|U_{l2}^{(\tilde\omega_1)}\right|^2}{\tilde\omega_1^2} + \frac{N_{111}^2}{\omega^{(111)^2}}\frac{\left|U_{l2}^{(\tilde\omega_2)}\right|^2}{\tilde\omega_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde\omega_3}\right|^2}{\tilde\omega_3^2}\right.$$

NLO

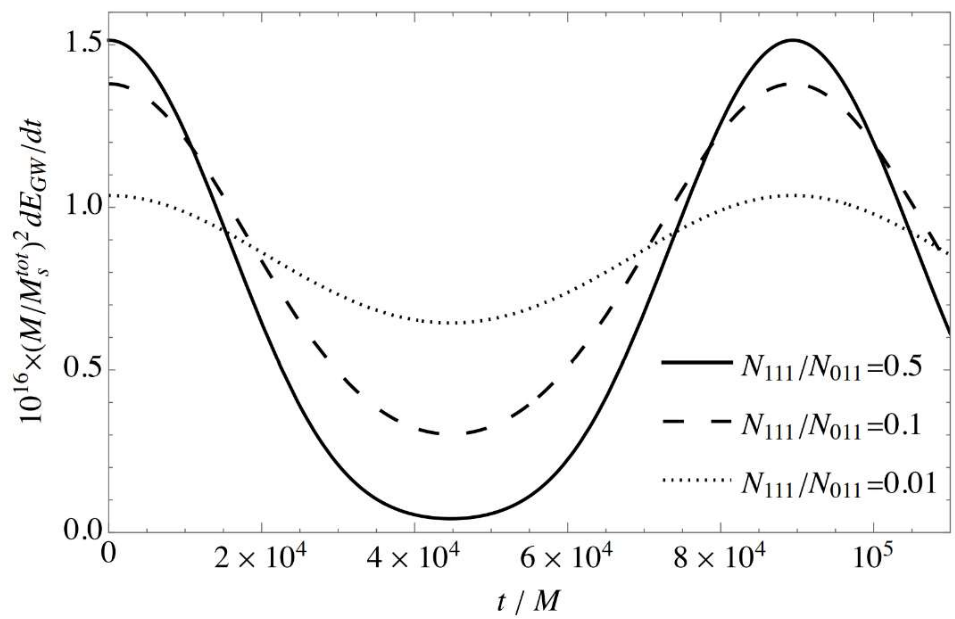
$\sqrt{N_{111}/N_{011}}$ suppressed

$$+4\sqrt{\frac{N_{011}^3 N_{111}}{\omega^{(011)^3}\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde\omega_1)}\right|\left|U_{\tilde{l}2}^{(\tilde\omega_3)}\right|}{\tilde\omega_1\tilde\omega_3}\cdot\cos\left[\tilde\omega_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde\omega_3)} + \phi_{\tilde{l}2}^{(\tilde\omega_1)}\right]$$

$+ 2 \ \ldots \ \phi_{\tilde{l}2}^{(\tilde\omega_1)}\Big]$

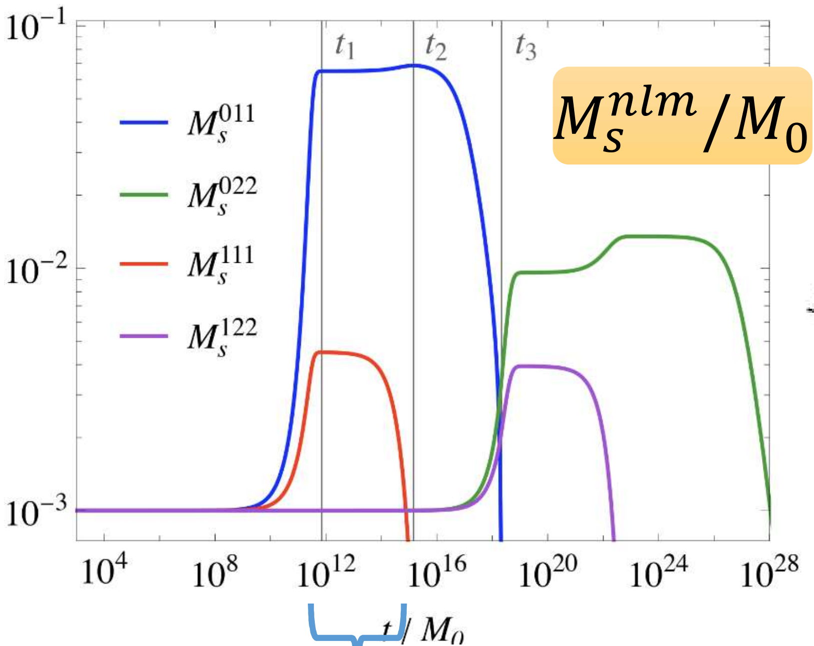
$+ 4 \ \ldots \ + \phi_{\tilde{l}2}^{(\tilde\omega_3)}\Big]\Big\}.$

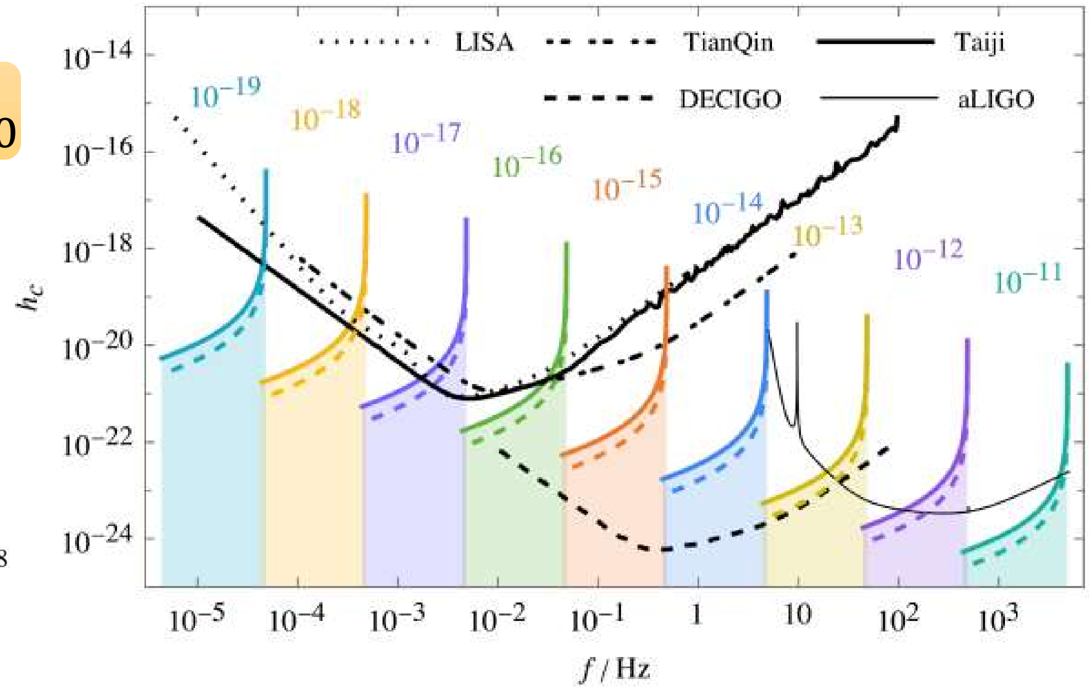
.1)



GW Beat: Observation

Guo, Bao and **HZ,** arXiv:2212.07186





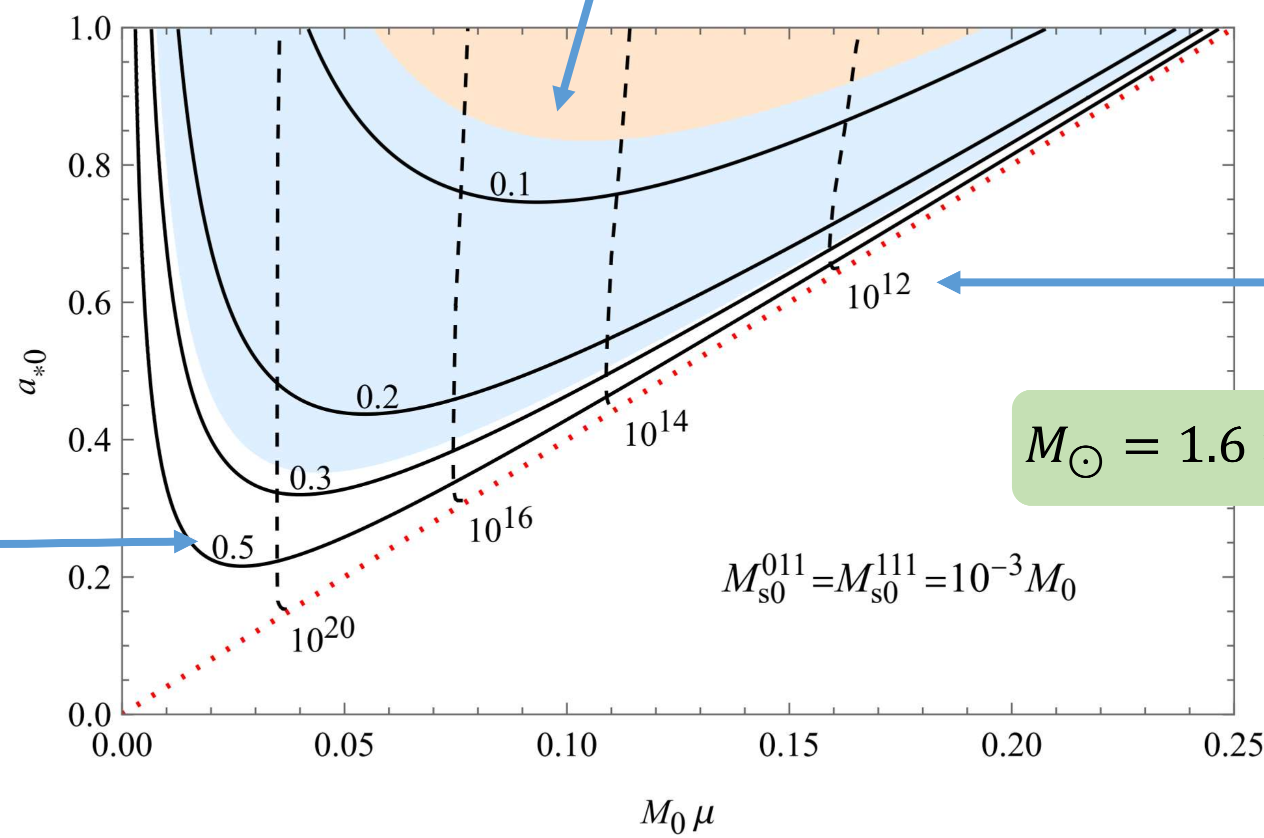
The BH spin here is determined by $M\mu$

- Parameters: $M\mu = 0.17$ (so $a_C = 0.6$), $M_s/M = 0.1,\ N_{111}/N_{011} = 0.1$
- The red shift ranges from 0.001 to 10
- The current and future GW telescope can cover a large range of axion mass.

GW Beat: Observation

Guo, Bao and **HZ,** arXiv:2212.07186

M_s/M_0 = 0.05 (orange) and 0.01 (blue)



$\frac{N_{111}}{N_{011}}$

$\frac{\Delta t}{M_0}$

$M_\odot = 1.6\times 10^{-13}$yr

- **Three factors** to consider for observation
  total signal strength, beat signal strength, beat duration.

GW Beat: Observation

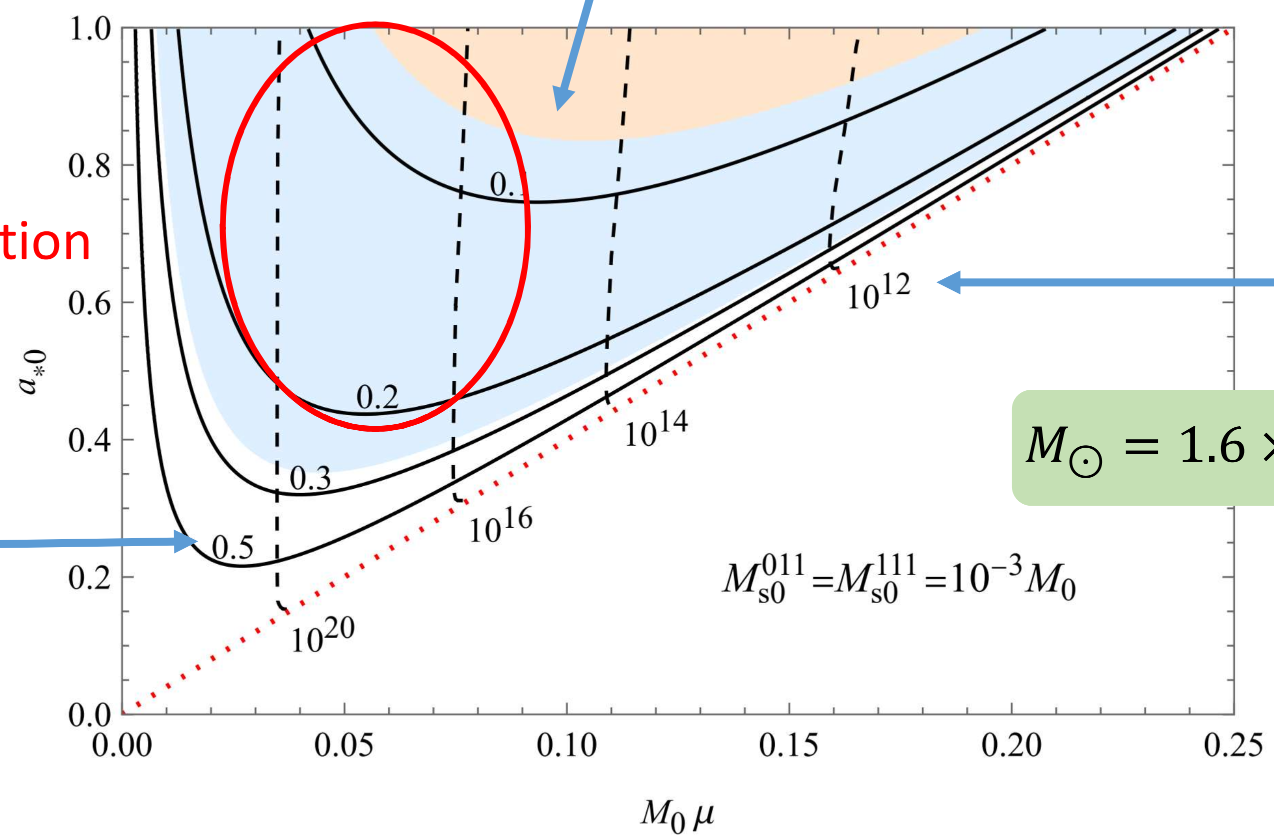
Guo, Bao and **HZ,** arXiv:2212.07186

M_s/M_0 = 0.05 (orange) and 0.01 (blue)

Best region for observation

$\frac{\Delta t}{M_0}$

$\frac{N_{111}}{N_{011}}$



$M_\odot = 1.6 \times 10^{-13}$yr

- **Three factors** to consider for observation
  total signal strength, beat signal strength, beat duration.

Summary

- If dark matter (DM) consists of ultralight scalars, they would exist in BEC state in the universe, different from the heavy DM candidates.
- For the QCD axion, a large portion could exist in form of axion stars, either dilute or dense.
- The photons in odd-integer harmonics of a fundamental radio frequency are a unique signature of the QCD axion.
- For even lighter axion-like particles (ALPs), they could form BEC around rotating black holes (BHs) by superradiance.
- The observed BHs cannot have high spin if the ALP has a proper mass, which is used to constrain the ALP mass.
- The gravitational wave emitted by ALP condensates around rotating BHs have unique "beat" signal.
- Non-minimal ALP models are much richer in phenomenology.

Summary

- If dark matter (DM) consists of ultralight scalars, they would exist in BEC state in the universe, different from the heavy DM candidates.
- For the QCD axion, a large portion could exist in form of axion stars, either dilute or dense.
- The photons in odd-integer harmonics of a fundamental radio frequency are a uni CD axion.

Thank you!

- For even lighter axio they could form BEC around rotating black holes (BHs) by superradiance.
- The observed BHs cannot have high spin if the ALP has a proper mass, which is used to constrain the ALP mass.
- The gravitational wave emitted by ALP condensates around rotating BHs have unique “beat” signal.
- Non-minimal ALP models are much richer in phenomenology.