α'-corrections to Near Extremal Dyonic Strings and Weak Gravity Conjecture

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Outline

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2. The problem and our proposal
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4. Dyonic strings in heterotic string on 4-torus
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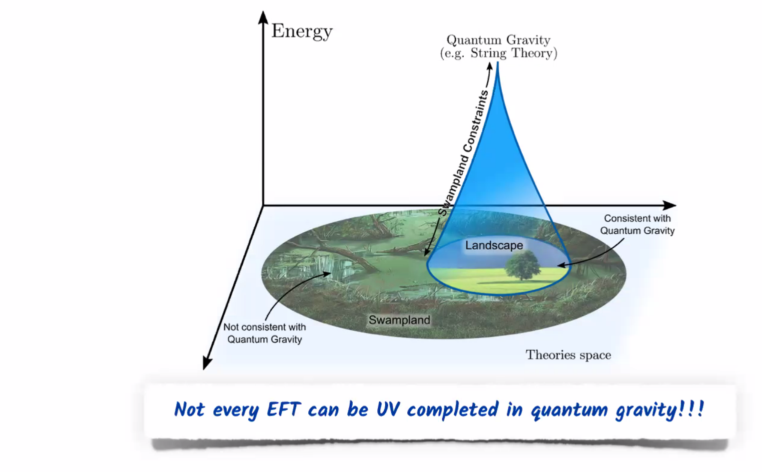
Background & Motivation

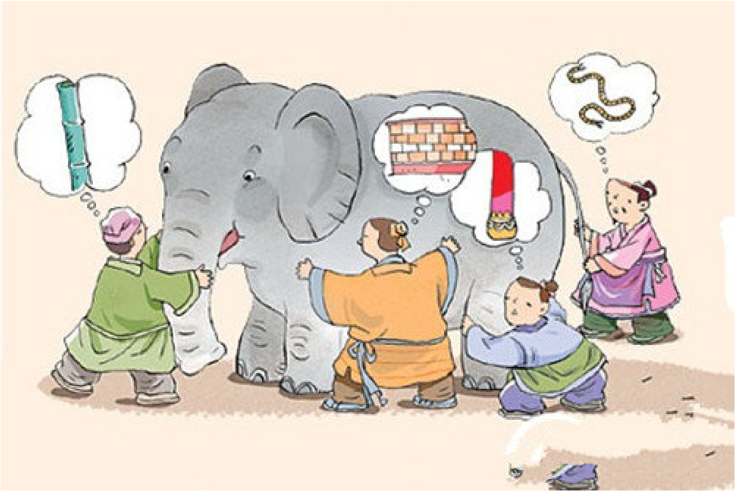


Swampland: Apparently consistent quantum EFTs that cannot be UV completed in quantum gravity

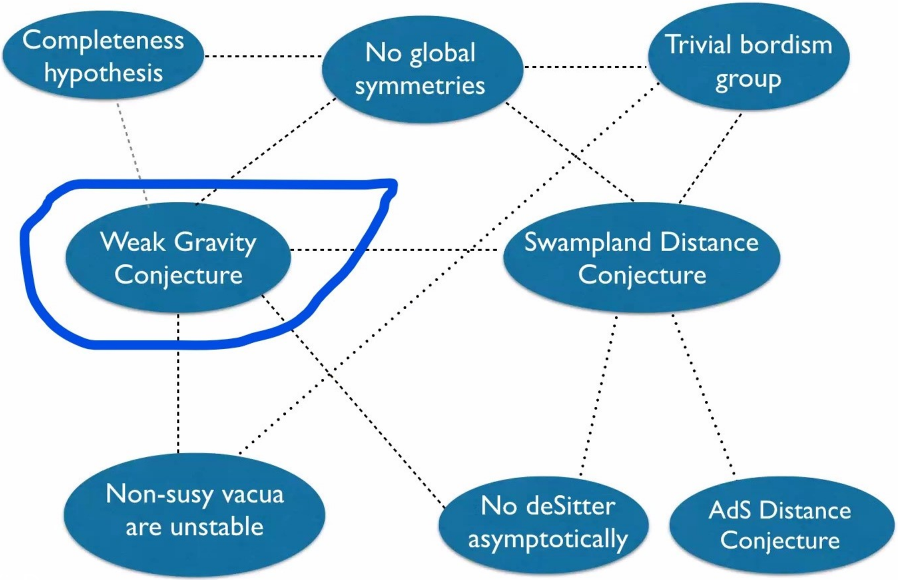
Goal of the Swampland program:

What are the constraints that an effective theory must satisfy to be consistent with quantum gravity?

What distinguishes the landscape from the Swampland?



Swampland Conjectures



Weak Gravity Conjecture (4-dimensions) [Arkani-Hamed, Motl, Nocolis, Vafa, 06']

- (Electric WGC) For a U(1) gauge theory coupled to gravity, there exists at least one charged objects with

$$m/q \leq m/q|_{extremal\ \ BH} \quad \text{in Planck units}$$

so that extremal black holes can decay. Gravity is the *weakest force* for this particle.

$$S = \int d^4X\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{4g^2}F^2 + ...\right]$$

$$m/q|_{extremal\ \ BH} = \sqrt{2}$$

- (Magnetic WGC) The cutoff scale Λ of the effective theory is bounded from above approximately by the gauge coupling

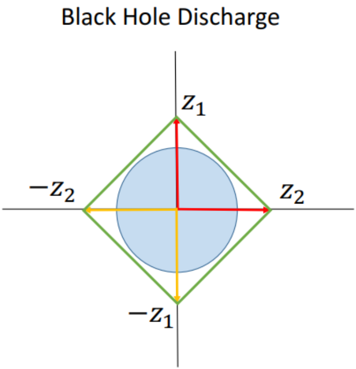
$$\Lambda \leq g M_p$$

so that the magnetic monople is not a black hole.

Some remarks

- The coupling is energy scale dependent
- The charged objects satisfying WGC can be fundamental particles, bound states or even BHs themselves

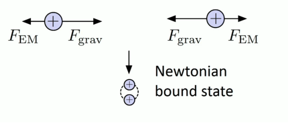
- For Einstein-Maxwell-dilation theory, one needs to distinguish cases where the scalar field mediates force or not. If yes, WGC is replaced by the repulsive force conjecture.
- Generalized to multiple U(1)s (convex hull condition) and *p*-form

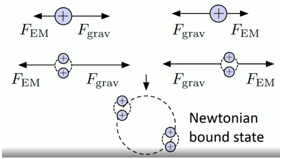


Why WGC (or its variations) should be true?

For two such particles (the one with largest q/m)

$$F_{\text{gravity}} = \frac{m^2}{8\pi M_p^2 r^2}, \quad F_{\text{EM}} = \frac{q^2}{4\pi r^2}$$





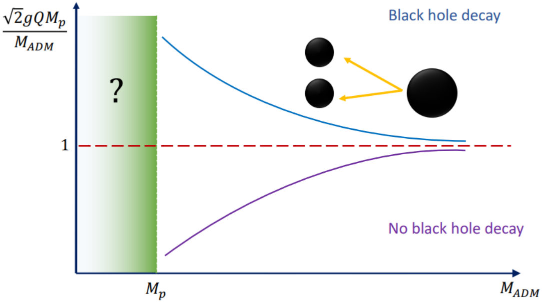
$\Rightarrow$ Infinite number of stable bound states not protected by symmetry. For $N \gg 1$ subextremal Non-relativisitc particles,

$$E_b(\text{binding energy}) \sim -N^3 \Delta m^5 / M_p^4, \quad \Delta m^2 = m^2 - 2q^2 M_p^2$$

There exists N_{crit}, above which the bound states have negative total energy, signalling infinite entropy.

Some evidence of WGC

- Kaluza Klein theories
- Massive string modes in Heterotic string compactified on torus
- GHS black holes in heterotic string with leading α' corrections [Natsuume 94']



Outside the framework of string theory, WGC has been applied to constrain Einstein-Maxwell with 4-derivative corrections [Cheung, Liu, Remmen, 18']

$$\begin{aligned}\Delta\mathcal{L} = \quad & c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu}F^{\mu\nu} + c_5 R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \\ & + c_7 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}.\end{aligned}$$

For an extremal RN black hole to be able to decay to smaller extremal RN black holes

$$\frac{M}{M_p} = \sqrt{2}Q\left(1 + \frac{c}{Q^2} + \cdots\right), \quad c < 0$$

On the other hand, tree-level unitarity of the 4-derivative interactions leads to

$$\Delta S > 0 \quad \text{at fixed } M\,, Q$$

These two yield the same constraint on c_is in the example above, however, they are independent of each other in more general case

The problem

In gravity models preserving supersymmetry, when the extremal limit of charged black holes coincides with BPS limit

$$M = Q$$

regardless of the detailed structure of the higher derivative couplings

Clearly, not all of them can come from quantum gravity. Although

$$\Delta S > 0$$

can give certain constraints.

Our proposal

- We recall that for non-BPS extremal black holes in Einstein-Maxwell theories, the right higher derivative couplings tend to reduce the attractive force, i.e. provide repulsive force
- Slightly moving away from extremality, the contribution from higher derivative couplings should be still repulsive

- These considerations motivated us to look at near extremal (BPS) black holes in supergravity models with higher derivative couplings
- Clearly, at leading order, the attractive force dominates, but what we can focus on the sign of the contribution from the higher derivative couplings

The set up

1) IIA string compactified on K3

2) Heterotic string compactified on 4-torus

In these two cases, the low energy limit is described by $D = 6$ $\mathcal{N} = (1, 1)$ supergravity coupled to 20 vector multiplets where the moduli parameterize the $O(4, 20)/(O(4) \times O(20))$ coset space.

The leading α' corrections are fully supersymmetrized in the NS-NS sector $(g_{\mu\nu}, B_{\mu\nu}, \phi)$ [Bergshoeff, de Roo , 89', Liu, Minasian 13', Novak, Ozkan, YP, Tartaglino-Mazzucchelli 17']

3) We consider static dyonic string solutions that admit regular BPS limit

4) The natural probe is a fundamental string (parallel to the macroscopic string)

Dyonic string solutions in 6D 2-derivative supergravity

The bosonic Lagrangian

$$\mathcal{L}_0 = L(R + L^{-2}\nabla^\mu L \nabla_\mu L - \frac{1}{12} H_{\mu\nu\rho}H^{\mu\nu\rho} + \cdots), \quad H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$$

The static dyonic solution

$$\begin{aligned}
ds_6^2 &= D(r)(-h(r)dt^2 + dx^2) + H_p(r)(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2)\,,\\
B_{(2)} &= 2P\sqrt{1+\frac{\mu}{P}}\omega_2 + \sqrt{1+\frac{\mu}{Q}}A(r)dt\wedge dx\,, \qquad L = \frac{1}{D(r)H_p(r)},\\
D(r) &= A(r) = \frac{r^2}{r^2+Q}, \quad H_p(r) = 1 + \frac{P}{r^2}, \quad h(r) = f(r) = 1 - \frac{\mu}{r^2}
\end{aligned}$$

When $P = 0$, the solution recovers the known solution [Duff, Lu 93']

Thermodynamics

$$
\begin{aligned}
M &= \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P), \\
T &= \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}}, \\
S &= \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q}, \\
Q_{\rm e} &= \frac{\pi}{4}\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\rm e} = \sqrt{\frac{Q}{\mu+Q}}, \\
Q_{\rm m} &= \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\rm m} = \sqrt{\frac{P}{\mu+P}}
\end{aligned}
$$

- $\mu \to 0,\ T \to 0,\ M \to Q_{\rm e} + Q_{\rm m},\ S \to 0$
- $$C_T = \frac{dE}{dT}\Big|_{Q_{\rm e},Q_{\rm m}} = \frac{\sqrt{\mu}f(\mu,P,Q)}{4PQ-\mu^2}, \quad f(\mu,P,Q) > 0$$

  Near extremal, $C_T > 0$ similar to near extremal RN black hole in asymptotically Minkowski.

Probe string

In string frame

$$
\begin{aligned}
S_2 &= -T_2 \int d^2\xi \left(\sqrt{-\det \gamma} + \tfrac{1}{2}\epsilon^{ij}\partial_i X^M \partial_j X^N B_{MN} \right) \\
\gamma_{ij} &= g_{MN}\partial_i X^M \partial_j X^N
\end{aligned}
$$

$$
\xi^0 = t\,, \quad \xi^1 = x
$$

By $x^{\perp} = \text{constant}$, one obtains the static potential

$$
\mathcal{V} = T_2 \left(\sqrt{-g_{tt} g_{xx}} - B_{tx} \right) \,, \quad \mathcal{F} = -\frac{d\mathcal{V}(r)}{dr} \tag{1}
$$

Plugging the solution

$$
\mathcal{V}(r) = \frac{T_2 r^2}{r^2 + Q} (\sqrt{1 - \mu/r} - \sqrt{1 + \mu/Q})
$$

Weak field expansion in the region where $r \gg \max\{\sqrt{\mu}, \sqrt{Q}, \sqrt{P}\}$

$$
\mathcal{V}(r)/T_2 = 1 - \sqrt{1 + \mu/Q} + \frac{\mu}{2} \frac{1 - \sqrt{1 + \mu/Q}}{1 + \sqrt{1 + \mu/Q}} r^{-2} + \mathcal{O}(r^{-4})
$$

In order to see the contributions from different forces, we switch to Einstein frame

$$g_{MN} = L^{-\frac{1}{2}} g^E_{MN}$$

Then the static potential can be rewritten as

$$\mathcal{V} = T_2 \left(L^{-\frac{1}{2}} \sqrt{-g^E_{tt} g^E_{xx}} - B_{tx} \right)$$

In the asymptotic region where $r \gg \max\{\sqrt{\mu}, \sqrt{Q}, \sqrt{P}\}$, we have

$$\begin{aligned}
\sqrt{-g^E_{tt} g^E_{xx}} &= 1 - \frac{P + Q + \mu}{2r^2} + \mathcal{O}(r^{-4}), \\
L^{-\frac{1}{2}}(r) &= 1 + \frac{P - Q}{2r^2} + \mathcal{O}(r^{-4}), \\
B_{tx} &= \sqrt{1 + \mu/Q} - \frac{Q\sqrt{1 + \mu/Q}}{r^2} + \mathcal{O}(r^{-4})
\end{aligned}$$

α'-corrections from IIA string on K3

$$
\begin{aligned}
\Delta S_2\big|_{\rm IIA} \;=\; & \alpha'^3 e^{-2\phi}\Bigg[\frac{\zeta(3)}{3\cdot 2^{11}}\Big(t_8t_8R(\omega_+)^4-\frac{1}{4}\epsilon_8\epsilon_8R(\omega_+)^4-2t_8t_8H^2R(\omega_+)^3 \\
& -\frac{1}{6}\epsilon_9\epsilon_9H^2R(\omega_+)^3+8\cdot 4!\sum_i d_iH^{\mu\nu\lambda}H^{\rho\sigma\tau}\widetilde{Q}^i_{\mu\nu\lambda\rho\sigma\tau}+\cdots\Big)\Bigg] \\
& +\Bigg[\frac{\pi^2}{9\cdot 2^{11}}\alpha'^3 e\Big(t_8t_8R(\omega_+)^4+\frac{1}{4}\epsilon_8\epsilon_8R(\omega_+)^4+\frac{1}{3}\epsilon_9\epsilon_9H^2R(\omega_+)^3-\frac{4}{9}\epsilon_9\epsilon_9H^2(DH)^2R(\omega_+)+\cdots \\
& -\frac{(2\pi)^6}{2}\alpha'^3B\wedge\Big(X_8(R(\omega_+))+X_8(R(\omega_-))\Big)\Bigg],
\end{aligned}
$$

α'-corrections from IIA string on K3

The leading higher derivative correction arises at 1-loop

$$\mathcal{L}_{LR+R^2} = \mathcal{L}_0 + \frac{\alpha'}{16}(\mathcal{L}_{\rm GB} + \mathcal{L}_{\rm Riem^2})$$

$$\begin{aligned}
\mathcal{L}_{\rm GB} &= R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 + \frac{1}{6}RH^2 - R^{\mu\nu}H^2_{\mu\nu} \\
&\quad + \frac{1}{2}R_{\mu\nu\rho\sigma}H^{\mu\nu\lambda}H^{\rho\sigma}{}_{\lambda} + \frac{5}{24}H^4 + \frac{1}{144}\left(H^2\right)^2 - \frac{1}{8}\left(H^2_{\mu\nu}\right)^2 \\
&\quad - \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{\alpha}{}_{\beta}(\omega_+)R_{\lambda\tau}{}^{\beta}{}_{\alpha}(\omega_+)\,, \\
\mathcal{L}_{\rm Riem^2} &= R_{\mu\nu\alpha\beta}(\omega_-)R^{\mu\nu\alpha\beta}(\omega_-) - \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{\alpha}{}_{\beta}(\omega_-)R_{\lambda\tau}{}^{\beta}{}_{\alpha}(\omega_-)
\end{aligned}$$

$$R_{\mu\nu}{}^{\alpha}{}_{\beta}(\omega_\pm) = \partial_\mu\omega^{\alpha}_{\pm\nu\beta} + \omega^{\alpha}_{\pm\mu\gamma}\omega^{\gamma}_{\pm\nu\beta} - (\mu\leftrightarrow\nu)\,, \quad \omega^{\alpha}_{\pm\mu\beta} = \omega^{\alpha}_{\mu\beta} \pm \tfrac{1}{2}H_{\mu}{}^{\alpha}{}_{\beta}$$

- The $\zeta(3)$ term that appears at tree level in the 10D effective "vanishes" upon reduction to 6D [Liu, Minasian 19']

Thus the full ansatz takes the form

$$
\begin{aligned}
ds_6^2 &= D(r)\left(-h(r)dt^2 + dx^2 + 2\omega dt dx\right) + H_p(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right), \\
B_{(2)} &= 2P\sqrt{1+\frac{\mu}{P}}\omega_2 + \sqrt{1+\frac{\mu}{Q}}A(r)dt\wedge dx, \quad L = L(r)
\end{aligned}
$$

$$
\begin{aligned}
L &= L_0 + \delta L, \quad D = D_0 + \delta D, \quad A = D_0 + \delta A \\
f &= f_0 + \delta f, \quad h = f_0 + \delta h, \quad \omega = \delta\omega
\end{aligned}
$$

Thermodynamic quantities with α' corrections

$$
\begin{aligned}
M &= \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P) - \frac{3\pi\mu^2(3\mu+2Q)}{32(\mu+Q)^2(\mu+2Q)}\alpha', \\
T &= \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}} - \frac{\sqrt{\mu}Q(5\mu+4Q)}{4\pi\sqrt{\mu+P}(\mu+Q)^{5/2}(\mu+2Q)}\alpha', \\
S &= \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q} + \frac{\pi^2\sqrt{\mu}\sqrt{\mu+P}Q(5\mu+4Q)}{4(\mu+Q)^{3/2}(\mu+2Q)}\alpha', \\
Q_{\rm e} &= \frac{\pi}{4}\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\rm e} = \sqrt{\frac{Q}{\mu+Q}} + \frac{\mu\sqrt{Q}(5\mu+4Q)}{2(\mu+Q)^{5/2}(\mu+2Q)}\alpha', \\
Q_{\rm m} &= \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\rm m} = \sqrt{\frac{P}{\mu+P}}
\end{aligned}
$$

These satisfy the first law of thermodynamics up to $\mathcal{O}(\alpha'^2)$

$$dM - TdS - \Phi_{\rm e}dQ_{\rm e} - \Phi_{\rm m}dQ_{\rm m} = \mathcal{O}(\alpha'^2)$$

- The BPS limit is still defined at $\mu = 0$

$$T = 0\,, \quad M = Q_{\rm e} + Q_{\rm m}\,, \quad S = 0$$

- Reparametrization degree of freedom at $\mathcal{O}(\alpha')$

$$\mu \to \mu + \alpha' f_1(\mu, P, Q)\,, \quad Q \to Q + \alpha' f_2(\mu, P, Q)\,, \quad P \to P\alpha' f_3(\mu, P, Q)$$

However, the relation among physical quantities are unaffected

- The leading order solution suffices for deriving the thermodynamic quantities at $\mathcal{O}(\alpha')$ [Reall, Santos 19']

$$I_E = I_{E0}(T, \Phi_{\rm e}, Q_{\rm m}) + I_{\rm hd}(T, \Phi_{\rm e}, Q_{\rm m})$$

based on which one can also show

$$\delta S(M,\ Q_{\rm e},\ Q_{\rm m}) = -I_{\rm hd}$$

Here

$$\delta S(M,\ Q_{\rm e},\ Q_{\rm m}) = \frac{\pi^2\sqrt{\mu}\sqrt{\mu + P}(9\mu + 8Q)}{16(\mu + Q)^{3/2}}\alpha'$$

Static potential felt by the probe string

$$\begin{aligned}\mathcal{V}(r)/T_2 &= 1-\sqrt{1+\mu/Q}-\Sigma r^2+\mathcal{O}(1/r^3)\,,\\ \Sigma &= \frac{\mu}{2}+Q\left(1-\sqrt{\frac{\mu+Q}{Q}}\right)-\frac{3\alpha'\,(3\mu+2Q)\,\mu^2}{8(\mu+Q)^2(\mu+2Q)}\end{aligned}$$

The large r expansions of the fields in Einstein frame are given by

$$\begin{aligned}\sqrt{-g^E_{tt}g^E_{xx}} &= 1-\frac{\sigma_g}{r^2}+\mathcal{O}(r^{-4})\,,\quad \sigma_g=\frac{1}{2}\left(P+Q+\mu-\frac{3\alpha'(3\mu+2Q)\mu^2}{8(Q+\mu)^2(2Q+\mu)}\right)\\ L^{-\frac{1}{2}}(r) &= 1-\frac{\sigma_L}{r^2}+\mathcal{O}(r^{-4})\,,\quad \sigma_L=\frac{1}{2}\left(Q-P-\frac{3\alpha'(3\mu+2Q)\mu^2}{8(Q+\mu)^2(2Q+\mu)}\right)\,,\\ B_{tx} &= \sqrt{1+\mu/Q}-\frac{\sigma_B}{r^2}+\mathcal{O}(r^{-4})\,,\quad \sigma_B=Q\sqrt{1+\mu/Q}\end{aligned}$$

$$\Sigma=\sigma_g+\sigma_L-\sigma_B$$

It is clear that if the α' correction $\delta\Sigma$ is positive, it enhances the attractive force, while a negative $\delta\Sigma$ is repulsive.

- α' correction to the static potential expressed in terms of (μ, P, Q) which are not physical quantities and can be redefined with α' dependent terms
- However, the relation among physical quantities should not depend on the reparameterization. We thus need to recast Σ in terms of physical quantities in order to discuss the α'-correction to the static force

  We can first solve for parameters Q and P from

$$Q = \frac{\sqrt{\pi^2\mu^2 + 64Q_{\rm e}^2} - \pi\mu}{2\pi}, \quad P = \frac{\sqrt{\pi^2\mu^2 + 64Q_{\rm m}^2} - \pi\mu}{2\pi}$$

- Express μ in terms of $(T, Q_{\rm e}, Q_{\rm m})$, $(M, Q_{\rm e}, Q_{\rm m})$ or $(S, Q_{\rm e}, Q_{\rm m})$. They correspond to the inclusion of the higher derivative interactions in 3 different physical processes: 1) isothermal process; 2) isoenergetic process; 3) isoentropic process

In case 1, we express μ using $(T, Q_{\rm e}, Q_{\rm m})$ about $T = 0$. We find that

$$\mu = c_0 T^2 + c_2 T^4 + \mathcal{O}(T^6)$$

where

$$\begin{aligned} c_0 &= 64 Q_{\rm e} Q_{\rm m} + 32\pi Q_{\rm m} \alpha' , \\ c_2 &= 512\pi Q_{\rm e} Q_{\rm m} (Q_{\rm e} + Q_{\rm m}) + 128\pi^2 (4 Q_{\rm e} + Q_{\rm m}) Q_{\rm m} \alpha' \end{aligned}$$

We can also perform a low temperature expansion of Σ at fixed electric and magnetic charges

$$\Sigma = 128\pi Q_{\rm e} Q_{\rm m}^2 T^4 + 32\pi^2 Q_{\rm m}^2 T^4 \alpha' + \mathcal{O}(T^6)$$

from which we see that the higher derivative corrections seem to enforce the attractive force.

In the Einstein frame,

$$
\begin{aligned}
\sigma_g &= \frac{2(Q_\mathrm{e}+Q_\mathrm{m})}{\pi} + 64\pi Q_\mathrm{e} T^4 \left(Q_\mathrm{e} Q_\mathrm{m} + Q_\mathrm{m}^2\right) \\
&\quad + 16\pi^2 T^4 \left(4 Q_\mathrm{e} Q_\mathrm{m} + Q_\mathrm{m}^2\right) \alpha' + \mathcal{O}(T^6)\,, \\
\sigma_L &= \frac{2(Q_\mathrm{e}-Q_\mathrm{m})}{\pi} - 64\pi Q_\mathrm{e} T^4 \left(Q_\mathrm{e} Q_\mathrm{m} - Q_\mathrm{m}^2\right) \\
&\quad + 16\pi^2 T^4 \left(Q_\mathrm{m}^2 - 4 Q_\mathrm{e} Q_\mathrm{m}\right) \alpha' + \mathcal{O}(T^6)\,, \\
\sigma_B &= \frac{4 Q_\mathrm{e}}{\pi} + \mathcal{O}(T^6)
\end{aligned}
$$

In case 2, we express μ using $(M\,,Q_{\rm e}\,,Q_{\rm m})$ near the BPS limit in which M approaches $M_0 = Q_{\rm e} + Q_{\rm m}$. Thus we define the expansion parameter as $\delta M = M - M_0$. μ can be expanded in terms of powers of δM

$$\mu = c_1 \delta m + c_2 (\delta m)^2 + \mathcal{O}(\delta m)^3$$

where

$$c_1 = \frac{8}{\pi}\,, \quad c_2 = \frac{-4Q_{\rm e}^2 - 4Q_{\rm e}Q_{\rm m} + 3\pi Q_{\rm m}\alpha'}{\pi Q_{\rm e}^2 Q_{\rm m}}$$

Meanwhile, the leading coefficient in the potential acquires an expansion

$$\begin{aligned} \Sigma &= -\frac{\left(3\pi^2 c_1^2 Q_{\rm e}\alpha' - 4\pi c_1^2 Q_{\rm e}^2\right)}{128 Q_{\rm e}^3}\delta m^2 + \mathcal{O}(\delta m^3)\,, \\ &= \frac{2\delta m^2}{\pi Q_{\rm e}} - \frac{3\delta m^2 \alpha'}{2Q_{\rm e}^2} + \mathcal{O}(\delta m^3) \end{aligned}$$

from which one can see that the higher derivative corrections appear to reduce the attractive force.

The leading falloff coefficients of various fields in Einstein frame, up to first order in α' we have

$$\begin{aligned}
\sigma_g &= \frac{2(Q_\mathrm{e}+Q_\mathrm{m})}{\pi} + \frac{Q_\mathrm{e}+Q_\mathrm{m}}{Q_\mathrm{e}Q_\mathrm{m}}\frac{\delta m^2}{\pi} - \frac{3\delta m^2}{4Q_\mathrm{e}^2}\alpha' + \mathcal{O}(\delta m^3)\,, \\
\sigma_L &= \frac{2(Q_\mathrm{e}-Q_\mathrm{m})}{\pi} + \frac{Q_\mathrm{m}-Q_\mathrm{e}}{Q_\mathrm{e}Q_\mathrm{m}}\frac{\delta m^2}{\pi} - \frac{3\delta m^2}{4Q_\mathrm{e}^2}\alpha' + \mathcal{O}(\delta m^3)\,, \\
\sigma_B &= \frac{4Q_\mathrm{e}}{\pi} + \mathcal{O}(\delta m^3)
\end{aligned}$$

In case 3, we express μ using $(S\,, Q_{\rm e}\,, Q_{\rm m})$ near the BPS limit in which $S \to 0$. To distinguish with the entropy in the thermodynamics, we use s to denote the small entropy expansion parameter. We find that μ can be expanded as

$$\mu = c_0 s^2 + c_2 s^4 + \mathcal{O}(s^6)$$

where up to first order in α

$$c_0 = \frac{1}{4\pi^2 Q_{\rm e} Q_{\rm m}} - \frac{\alpha'}{8\pi Q_{\rm e}^2 Q_{\rm m}}\,, \quad c_2 = -\frac{Q_{\rm e}+Q_{\rm m}}{128\pi^3 Q_{\rm e}^3 Q_{\rm m}^3} + \frac{(4Q_{\rm e}+7Q_{\rm m})}{512\pi^2 Q_{\rm e}^4 Q_{\rm m}^3}\alpha'$$

Meanwhile, the leading coefficient in expansion of the potential acquires a small entropy expansion

$$\begin{aligned}\Sigma &= -\frac{\left(3\pi^2 c_0^2 Q_{\rm e}\alpha' - 4\pi c_0^2 Q_{\rm e}^2\right)}{128 Q_e^3} s^4 + \mathcal{O}(s^6) \\ &= \frac{s^4}{512\pi^3 Q_{\rm e}^3 Q_{\rm m}^2} - \frac{7s^4}{2048\pi^2 Q_{\rm e}^4 Q_{\rm m}^2}\alpha' + \mathcal{O}(s^6)\end{aligned}$$

from which one can see that the higher derivative corrections appear to reduce the attractive force.

The leading falloff coefficients of various fields in Einstein frame, up to first order in α' we have

$$\begin{aligned}
\sigma_g &= \frac{2(Q_{\rm e}+Q_{\rm m})}{\pi} + \frac{Q_{\rm e}+Q_{\rm m}}{1024\pi^3 Q_{\rm e}^3 Q_{\rm m}^3}s^4 - \frac{4Q_{\rm e}+7Q_{\rm m}}{4096\pi^2 Q_{\rm e}^4 Q_{\rm m}^3}s^4\alpha' + \mathcal{O}(s^6)\,,\\
\sigma_L &= \frac{2(Q_{\rm e}-Q_{\rm m})}{\pi} + \frac{Q_{\rm m}-Q_{\rm e}}{1024\pi^3 Q_{\rm e}^3 Q_{\rm m}^3}s^4 - \frac{7Q_{\rm m}-4Q_{\rm e}}{4096\pi^2 Q_{\rm e}^4 Q_{\rm m}^3}s^4\alpha' + \mathcal{O}(s^6)\,,\\
\sigma_B &= \frac{4Q_{\rm e}}{\pi} + \mathcal{O}(s^6)
\end{aligned}$$

α'-corrections from heterotic string on 4-torus

The leading higher derivative correction arises at tree-level

$$\mathcal{L}_{LR+LR^2} = \sqrt{-g}L\left(R + L^{-2}\nabla^\mu L\nabla_\mu L - \frac{1}{12}\widetilde{H}_{\mu\nu\rho}\widetilde{H}^{\mu\nu\rho} + \frac{\alpha'}{8}R_{\mu\nu\alpha\beta}(\omega_+)R^{\mu\nu\alpha\beta}(\omega_+)\right)$$

where the 3-form field strength $\tilde{H}_{\mu\nu\rho}$ includes the Lorentz Chern-Simons term

$$\widetilde{H}_{(3)} = dB_{(2)} + \frac{1}{4}\alpha'\mathrm{CS}_{(3)}(\omega_+)$$

such that it obeys a deformed Bianchi identity

$$d\widetilde{H}_{(3)} = \frac{1}{4}\alpha' R^a{}_b(\omega_+)\wedge R^b{}_a(\omega_+)$$

This theory is related to IIA on K3 by S-duality

$$L^{\mathrm{IIA}} \star H^{\mathrm{IIA}} = \widetilde{H}^{\mathrm{het}}, \quad L^{\mathrm{IIA}} g^{\mathrm{IIA}}_{\mu\nu} = g^{\mathrm{het}}_{\mu\nu}, \quad L^{\mathrm{IIA}} = 1/L^{\mathrm{het}}$$

- Using duality and applying suitable change of coordinates, we obtain the dyonic string solution with α' corrections in heterotic string on 4-torus.

$$
\begin{aligned}
M^{(\mathrm{het})} &= \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P) - \frac{3\pi\mu^2(3\mu+2P)}{32(\mu+P)^2(\mu+2P)}\alpha' \,, \\
T^{(\mathrm{het})} &= \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}} - \frac{\sqrt{\mu}P(5\mu+4P)}{4\pi\sqrt{\mu+Q}(\mu+P)^{5/2}(\mu+2P)}\alpha' \,, \\
S^{(\mathrm{het})} &= \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q} + \frac{\pi^2\sqrt{\mu}\sqrt{\mu+Q}P(5\mu+4P)}{4(\mu+P)^{3/2}(\mu+2P)}\alpha' \,, \\
Q_{\mathrm{e}}^{(\mathrm{het})} &= \frac{1}{4}\pi\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\mathrm{e}}^{(\mathrm{het})} = \sqrt{\frac{Q}{\mu+Q}} \,, \\
Q_{\mathrm{m}}^{(\mathrm{het})} &= \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\mathrm{m}}^{(\mathrm{het})} = \sqrt{\frac{P}{\mu+P}} + \frac{\mu\sqrt{P}(5\mu+4P)}{2(\mu+P)^{5/2}(\mu+2P)}\alpha'
\end{aligned}
$$

which are related to those in the IIA case by interchanging parameters P, Q. At fixed conserved charges $(M,\ Q_{\mathrm{e}},\ Q_{\mathrm{m}})$

$$
\delta S^{\mathrm{het}} = \frac{\pi^2\sqrt{\mu}\sqrt{\mu+Q}(9\mu+8P)}{16(\mu+P)^{3/2}}\alpha' > 0
$$

Static potential felt by the probe string

The probe string in heterotic side is electrically charged, dual to the magnetically charged one in the IIA side we find that the static potential takes the form

$$\mathcal{V}(r)/T_2 = 1 - \sqrt{1+\mu/Q} - \Sigma/r^2 + \mathcal{O}(1/r^3)\,, \quad \Sigma = \frac{\mu}{2} + Q\left(1 - \sqrt{\frac{\mu+Q}{Q}}\right)$$

1) Isothermal

$$\Sigma = 128\pi Q_{\rm e} Q_{\rm m}^2 T^4 + 128\pi^2 Q_{\rm e} Q_{\rm m} T^4 \alpha'$$

2) Isoenergetic

$$\Sigma = \frac{2\delta m^2}{\pi Q_{\rm e}} + \left(\frac{3}{2' Q_{\rm e} Q_{\rm m}^2}\alpha' - \frac{2}{\pi Q_{\rm e}^2} - \frac{2}{\pi Q_{\rm e} Q_{\rm m}}\right)\delta m^3$$

3) Isoentropic

$$\Sigma = \frac{s^4}{512\pi^3 Q_{\rm e}^3 Q_{\rm m}^2} - \frac{s^4}{512\pi^2 Q_{\rm e}^3 Q_{\rm m}^3}\alpha' + \mathcal{O}(s^6)$$

Summary

- We obtained static dyonic string in both IIA on K3 and heterotic on 4-torus with leading α' corrections
- We worked out α' corrected thermodynamics $\Delta S > 0$
- If the inclusion of the correction is an isoentropic process, the higher derivative corrections always tend to reduce the attractive force when the system slightly deviates away from extremality

Future direction

- Strings probing Taub-NUT, adding linear and angular momentum (in progress)
- Check $D = 4\,,5$ examples with embedding in string or M-theory

Thank you for your attention