

Towards High Precision Investigation of Heavy Quark Physics

Wei Wang

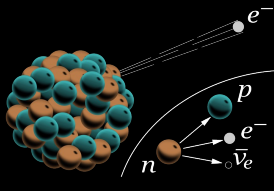
Shanghai Jiaotong University

2024/09/20

Indirect search *for* New Physics



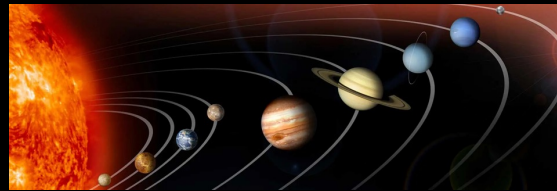
Higgs boson



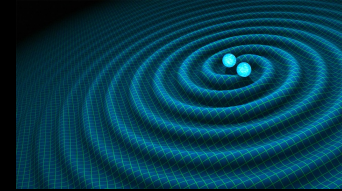
neutrino



Charm



Neptune in Solar



Gravitational wave



Black hole

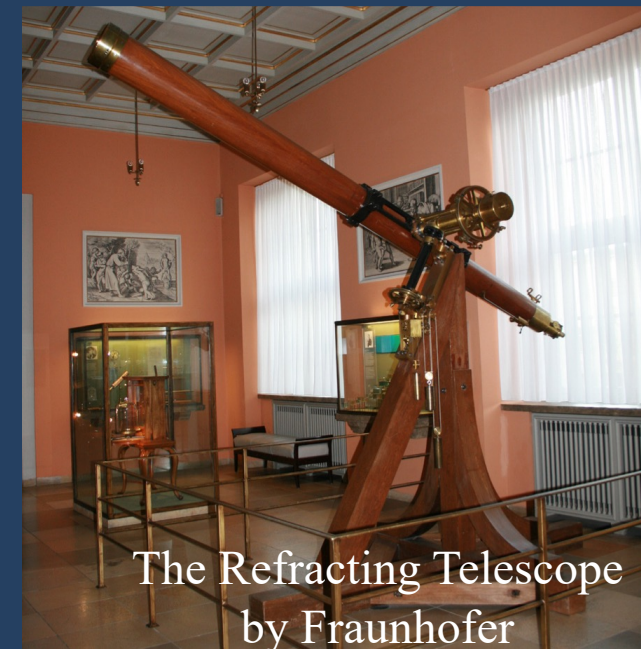
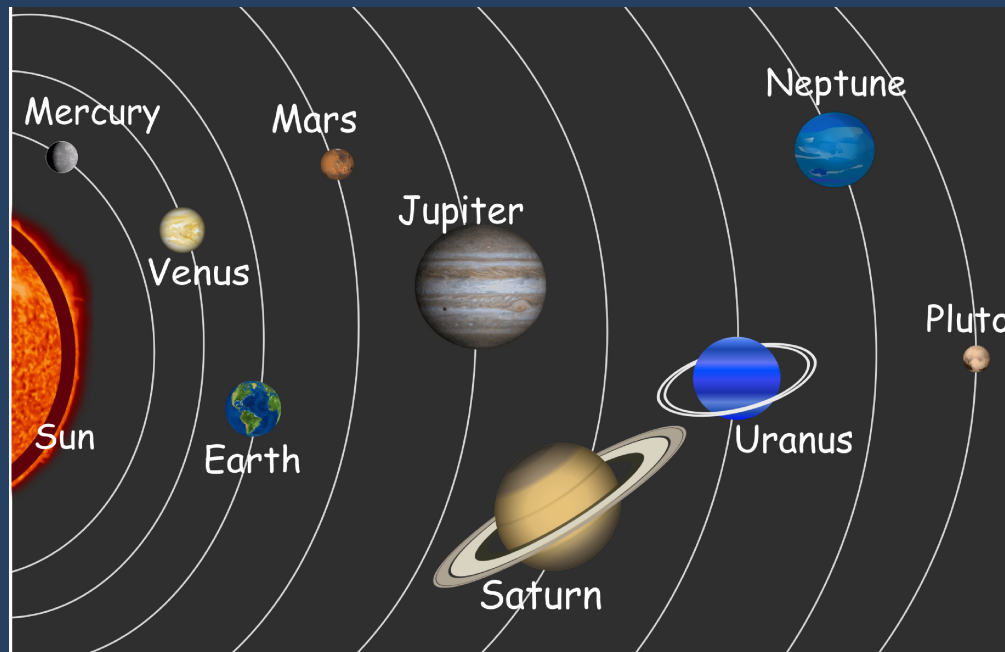
The discovery of Neptune

Orbital Anomalies: In early 19th century, astronomers noticed anomalies in Uranus's orbit that could not be explained by the theory.

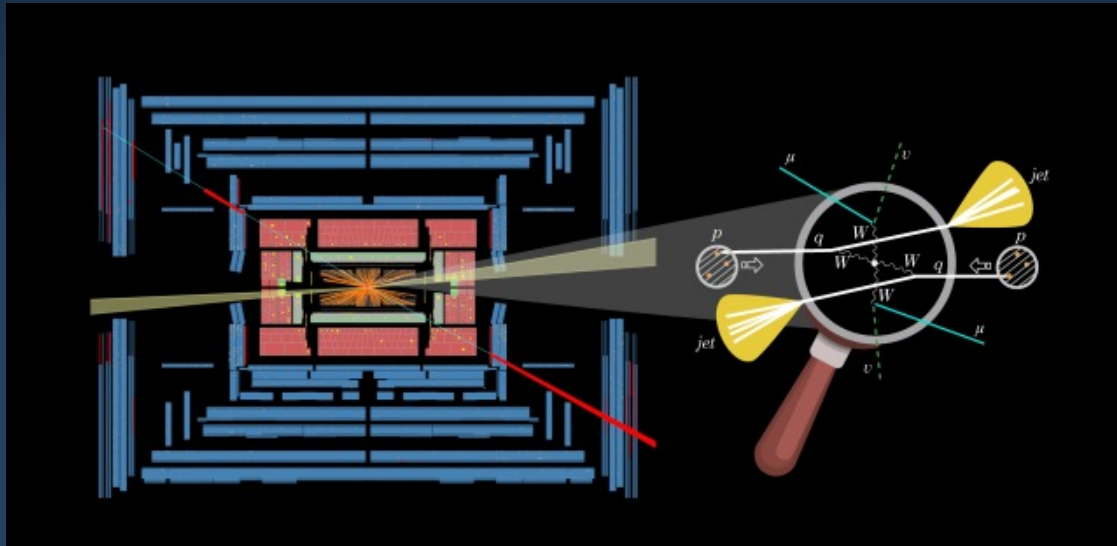
Mathematical Prediction: In 1845, Adams (British) and Le Verrier (French) independently calculated the possible position of an eighth planet.

Discovery: On September 23, 1846, Galle (German) observed Neptune at Berlin Observatory.

Early Observations: In 1612, Galileo observed Neptune but mistakenly identified it as a star.

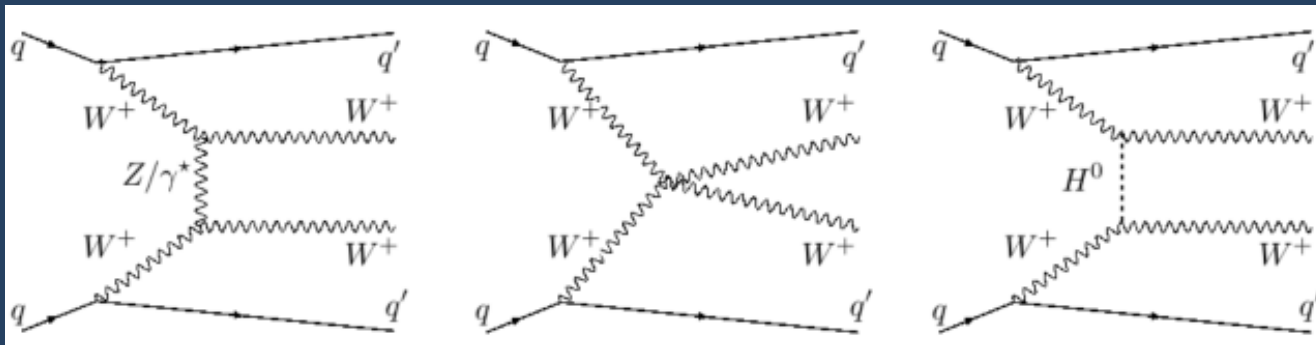


The Refracting Telescope
by Fraunhofer



$$1 + 2 \rightarrow \lim_{s \rightarrow \infty} \sigma$$

$$1 + 2 + 3 \rightarrow \lim_{s \rightarrow \infty} \sigma$$



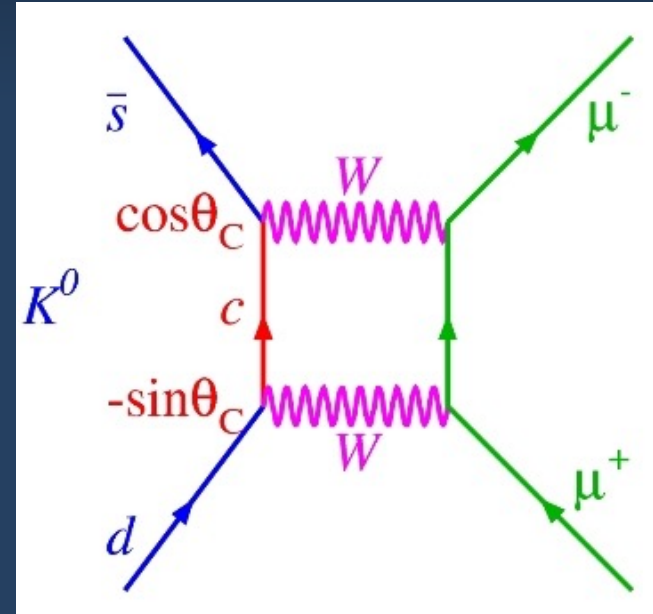
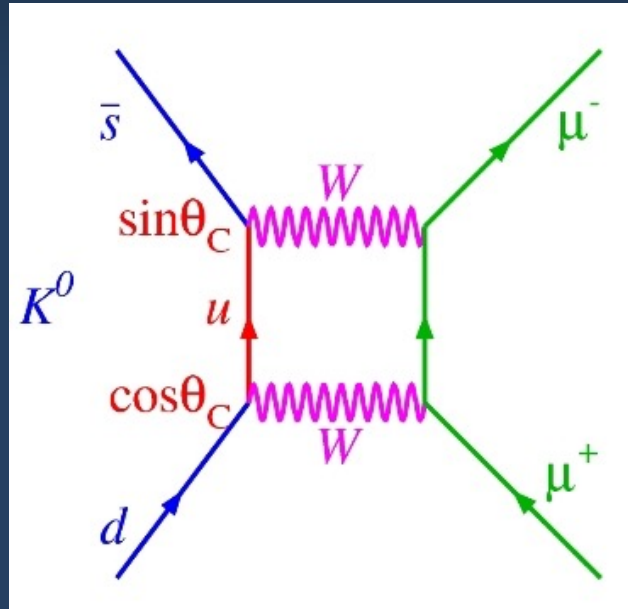
1

2

3

Unitarity is restored by the Higgs boson.

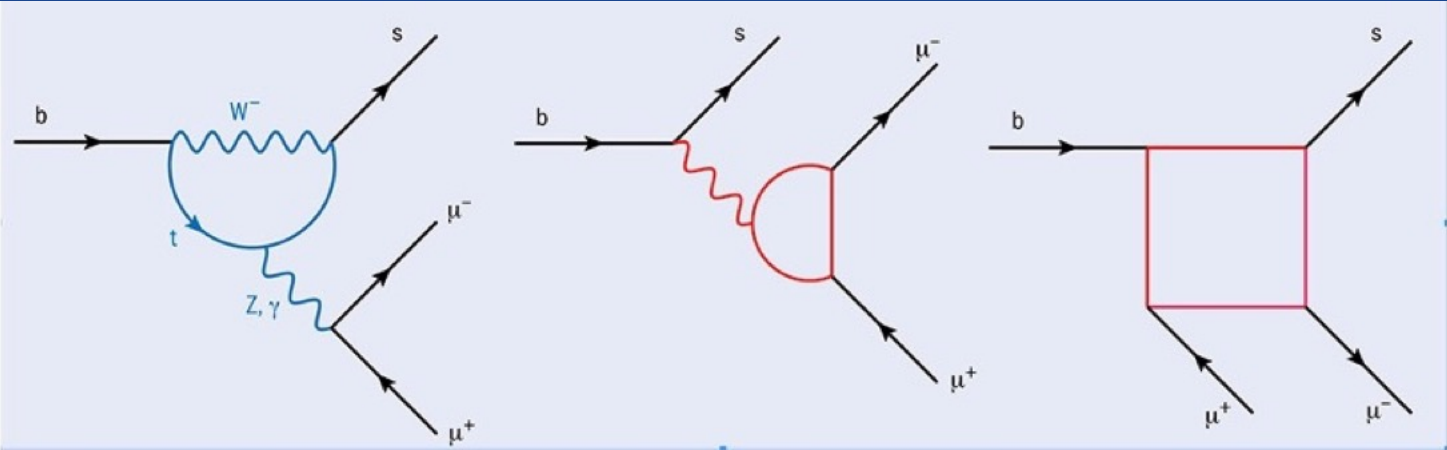
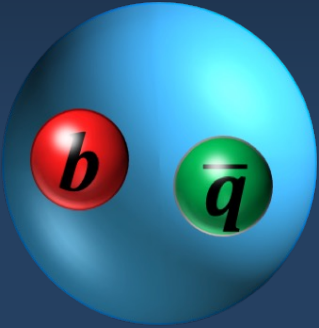
Charm: Where have all the K-zeroes gone ?



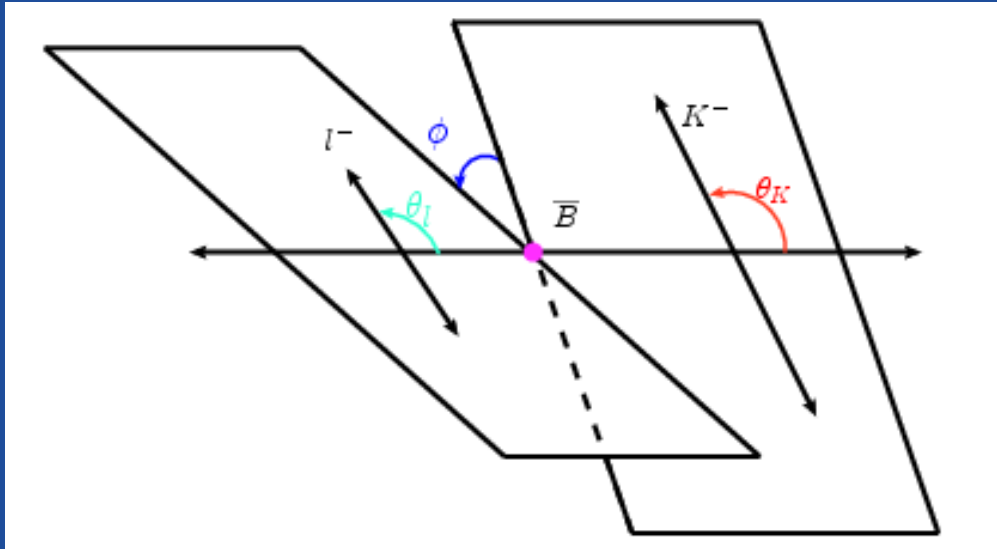


Heavy Quark Physics and Indirect search

Heavy Quark Physics and Indirect Search



$$B \rightarrow K^* l^+ l^- \rightarrow K \pi l^+ l^-$$



Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays #3

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 • e-Print: [1705.05802](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [datasets](#)

↻ 828 citations

Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb^{-1} of integrated luminosity #3

LHCb Collaboration • Roel Aaij (CERN) et al. (Dec 14, 2015)

Published in: *JHEP* 02 (2016) 104 • e-Print: [1512.04442](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [datasets](#)

↻ 717 citations

Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for $B \rightarrow K^{(*)} \ell^+ \ell^-$ #10

Belle Collaboration • J.-T. Wei (Taiwan, Natl. Taiwan U.) et al. (Apr, 2009)

Published in: *Phys.Rev.Lett.* 103 (2009) 171801 • e-Print: [0904.0770](#) [hep-ex]

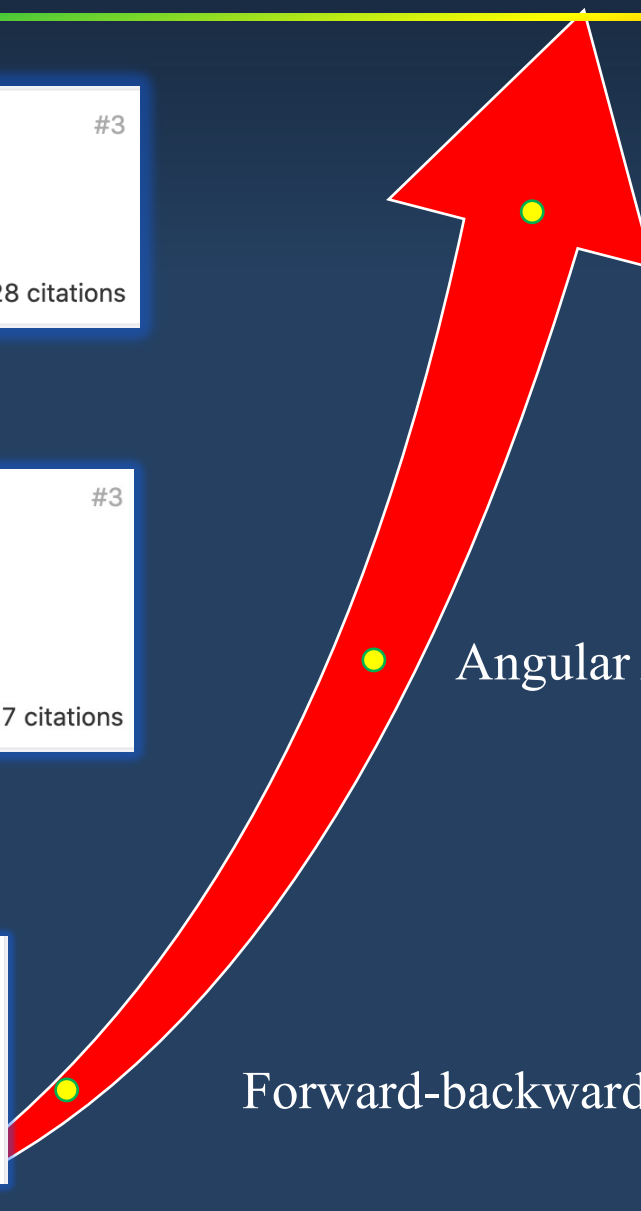
[pdf](#) [DOI](#) [cite](#)

↻ 521 citations

Lepton Flavor Universality

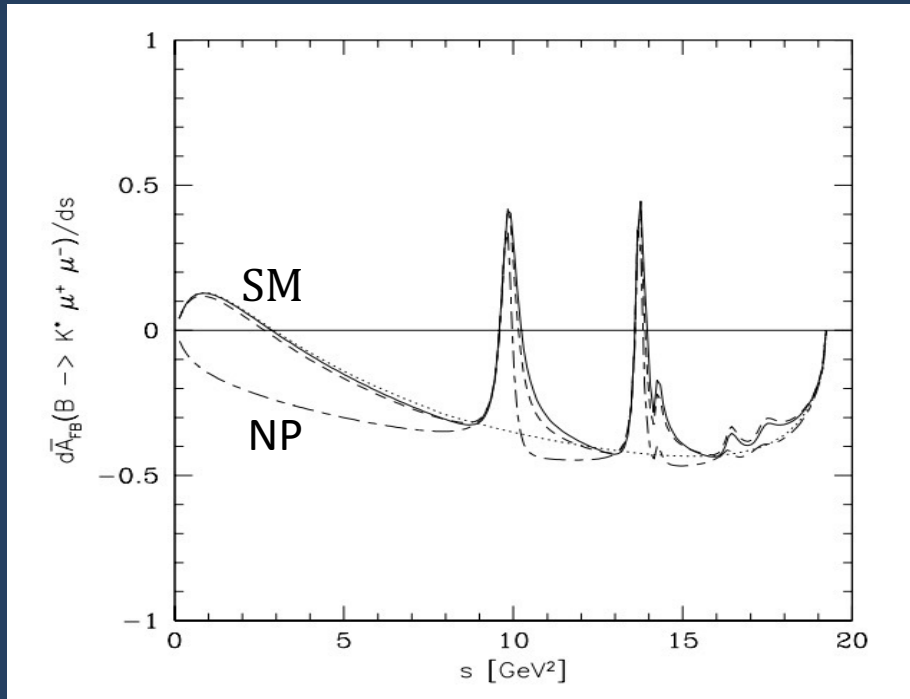
Angular Analysis and P'_5

Forward-backward Asymmetry

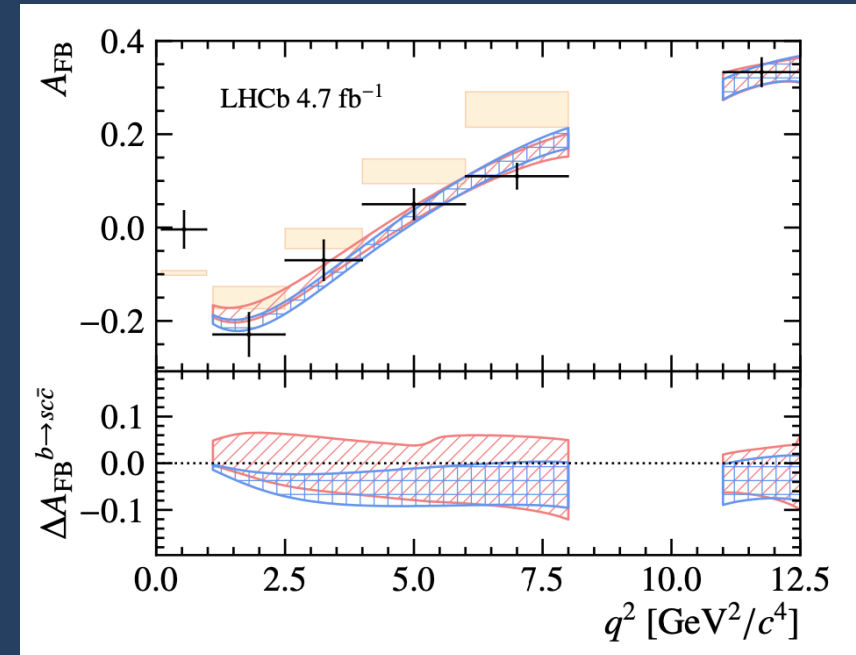


FCNC: Forward-backward Asymmetry in $B \rightarrow K^* ll$

$$\frac{dA_{FB}}{dq^2} = \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta_l} / \Gamma$$

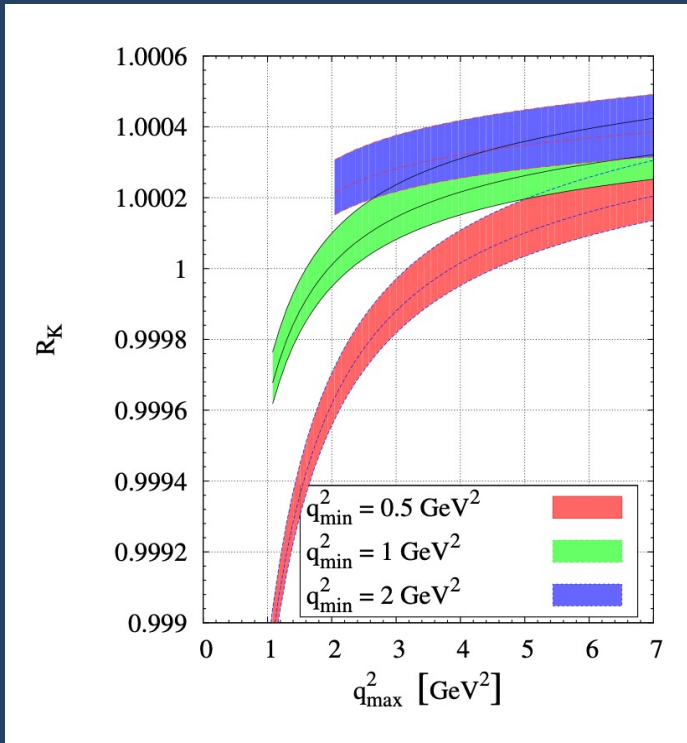


Ali, et.al, PRD61,074024 (2000)

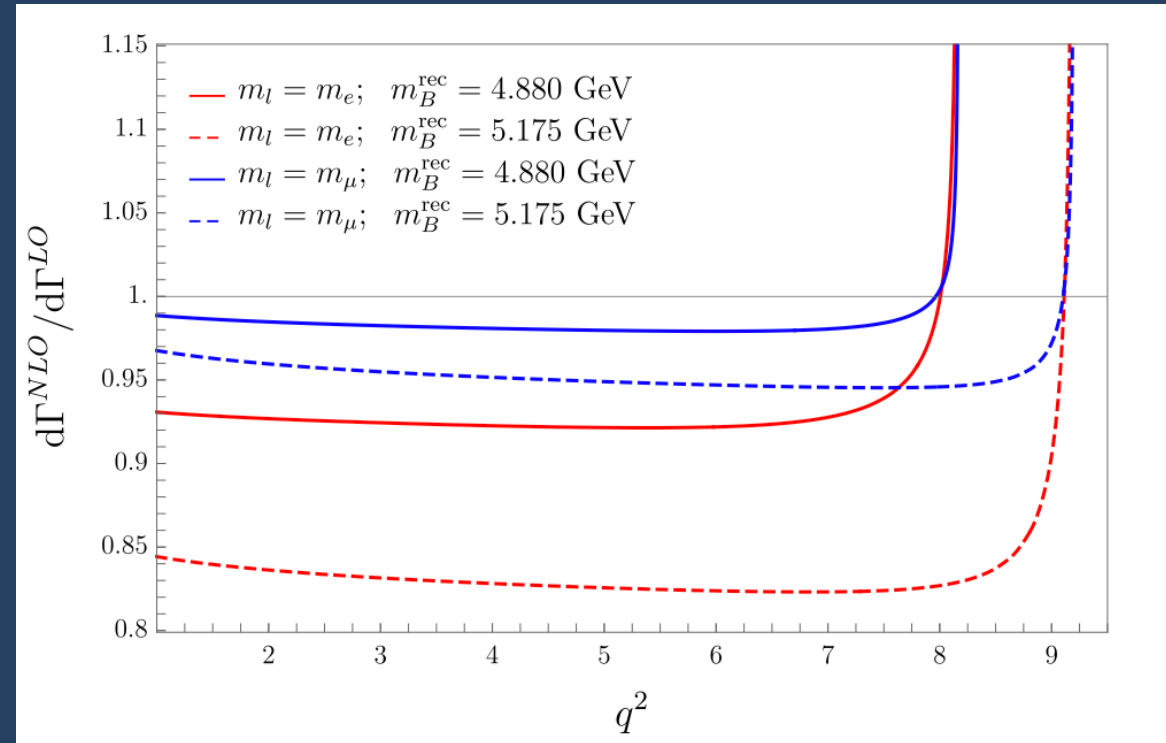


LHCb, PRD 109, 052009 (2024)

$$R_K = \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B^+ \rightarrow K^+ \mu^+ \mu^-]}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B^+ \rightarrow K^+ e^+ e^-]}{dq^2} dq^2}$$



C. Bobeth et al., JHEP 12 (2007) 040



M. Bordone et al., EJPC 76 (2016) 440

$$R_K = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst})$$

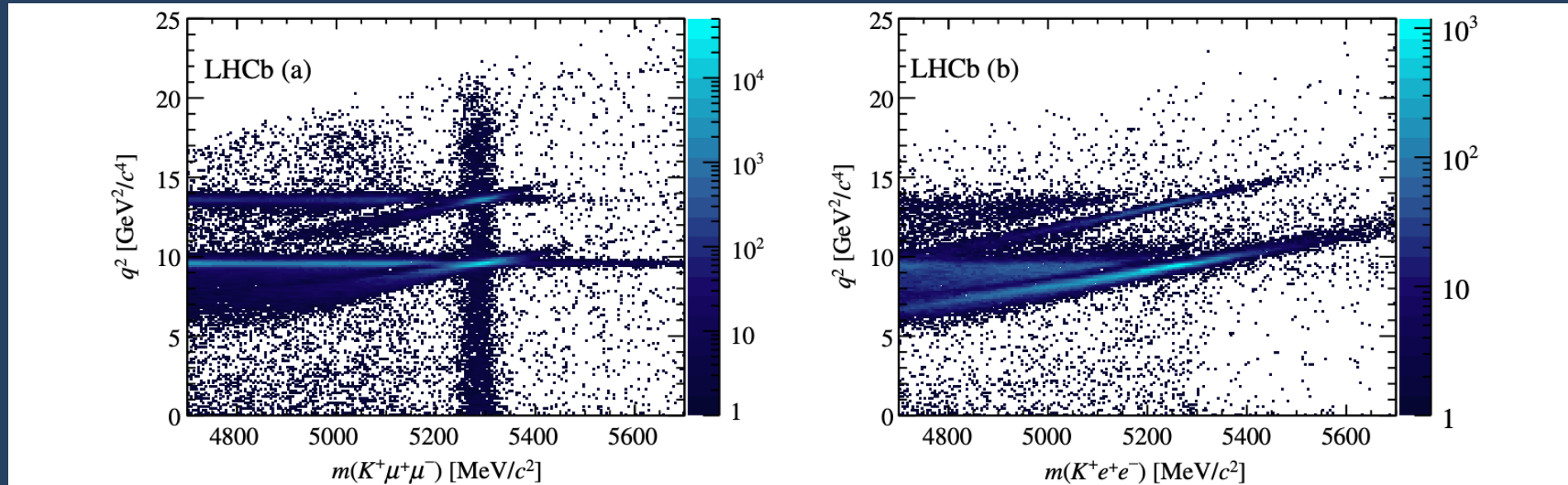
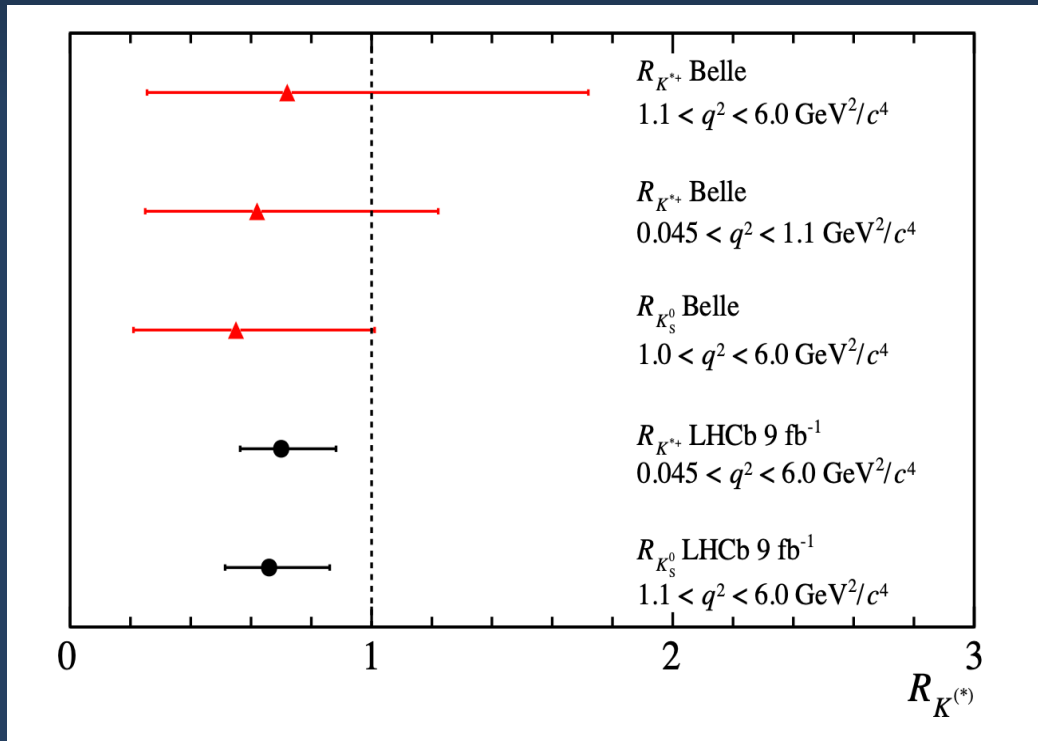
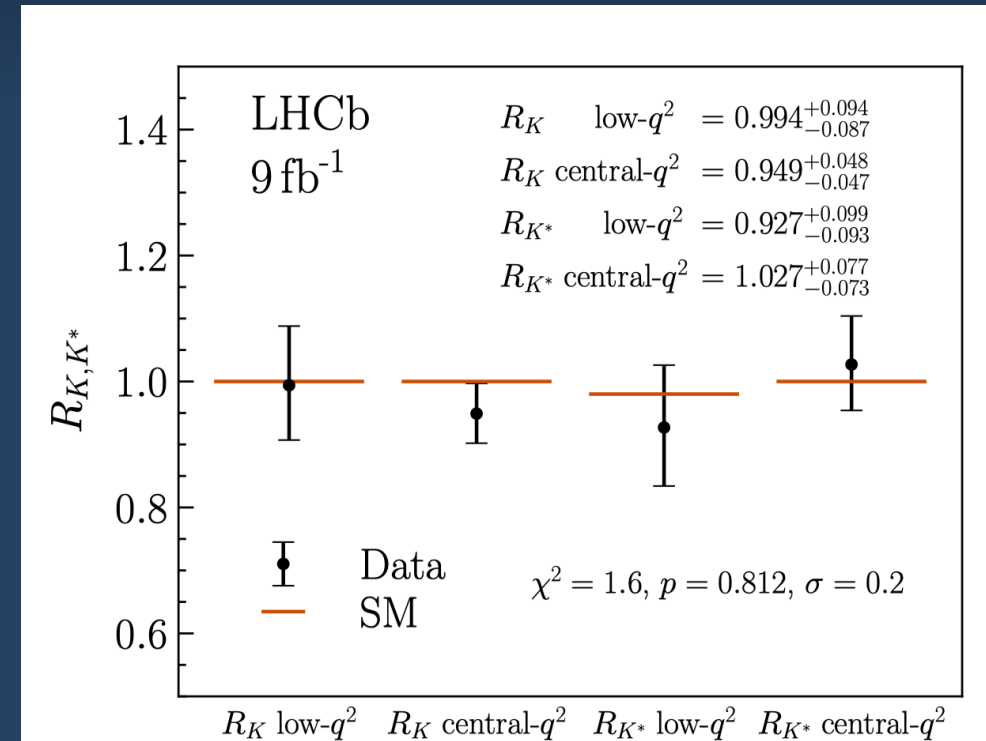


FIG. 1 (color online). Dilepton invariant mass squared q^2 as a function of the $K^+\ell^+\ell^-$ invariant mass, $m(K^+\ell^+\ell^-)$, for selected (a) $B^+ \rightarrow K^+\mu^+\mu^-$ and (b) $B^+ \rightarrow K^+e^+e^-$ candidates. The radiative tail of the J/ψ and $\psi(2S)$ mesons is most pronounced in the electron mode due to the larger bremsstrahlung and because the energy resolution of the ECAL is lower compared to the momentum resolution of the tracking system.

LHCb Collaboration: PRL 113 (2014) 151601 (Selected for a Viewpoint in Physics)

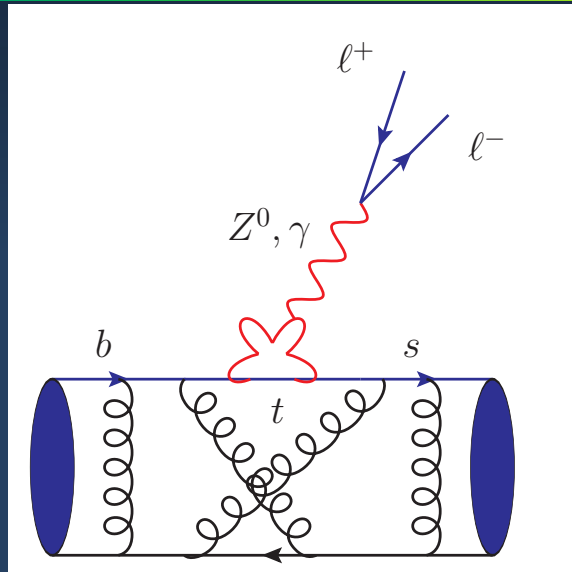


LHCb, PRL 128 (2022) 191802



LHCb, PRD 108 (2023) 032002

Review: Rep. Prog. Phys. 87 (2024) 077802, <https://doi.org/10.1088/1361-6633/ad4e65>



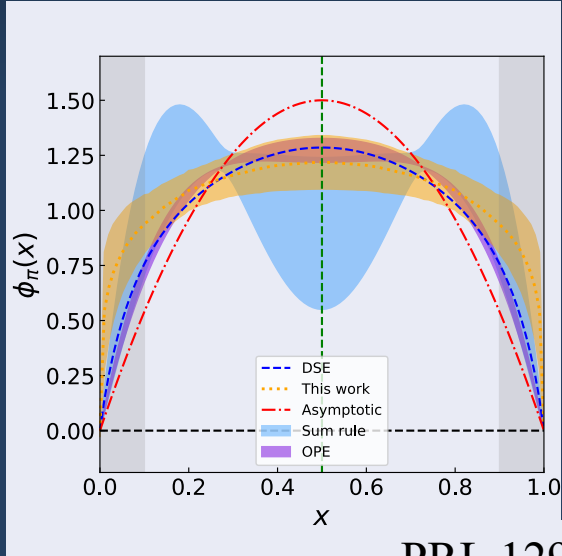
Ali, Kramer, Zhu, EPJC47, 625(2006)

$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2) \zeta_a(q^2) + \sum_{\pm} \int_0^{\infty} \frac{d\omega}{\omega} \phi_{\pm}^B(\omega) \int_0^1 du \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2)$$

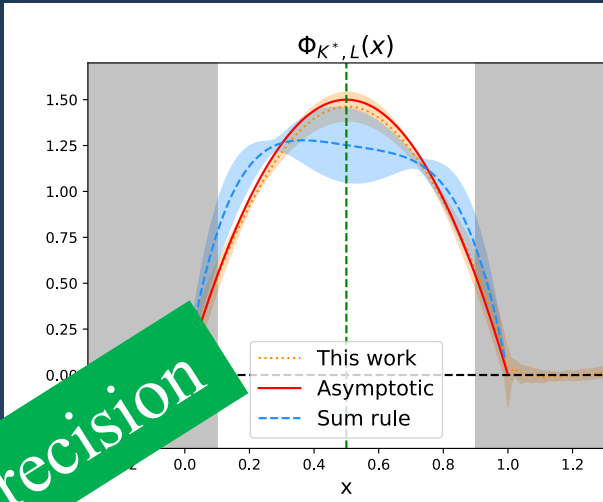
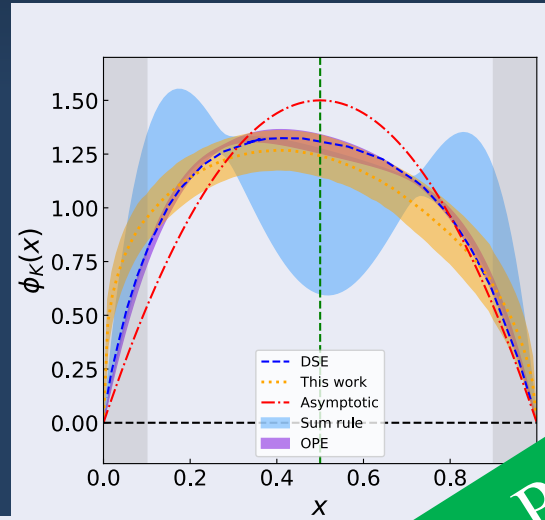
LCSR

B-meson LCSR

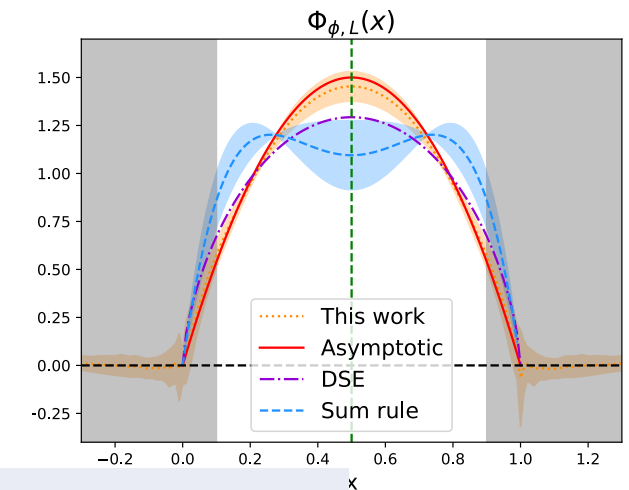
Without reliable (precise) knowledge on LCDAs, it is hard to probe NP



PRL 129, 132001 (2022)

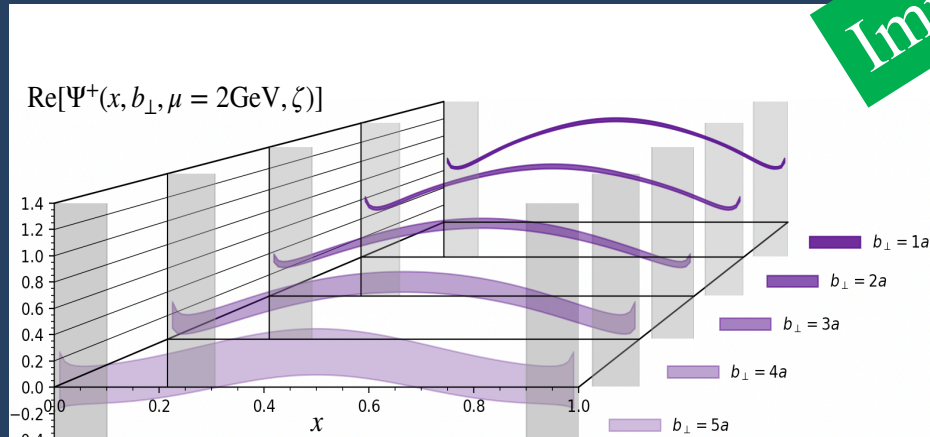


PRL 127, 062002 (2021)

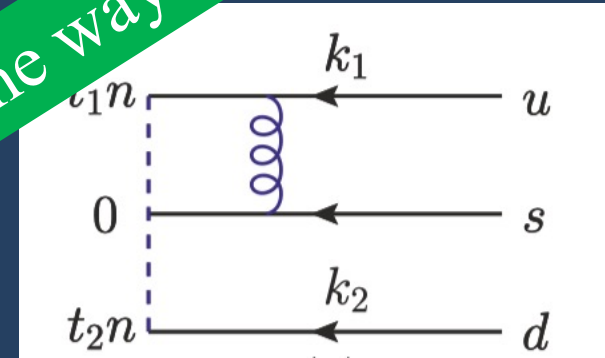


Improve Precision

Lattice on the way

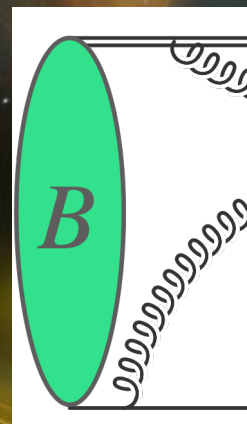


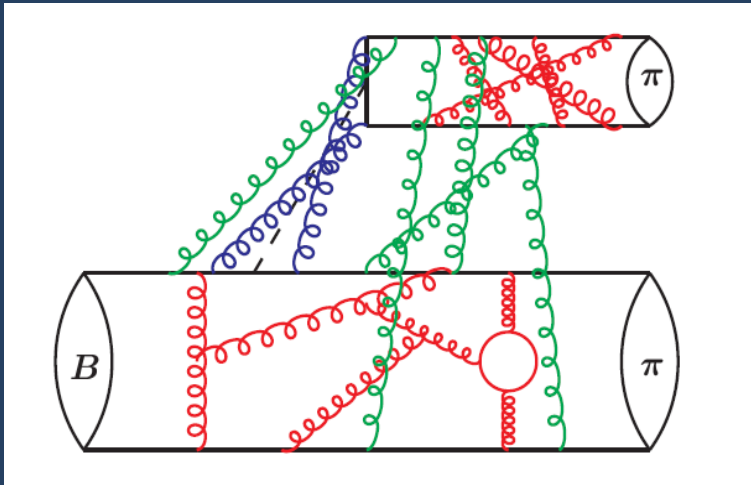
PRD109, L091503 (2024)



JHEP 07 (2023) 191; JHEP 12 (2023) 044

Heavy Meson LCDAs





$$\langle \pi(p') \pi(q) | Q_i | \bar{B}(p) \rangle = f^{B \rightarrow \pi}(q^2) \int_0^1 dx T_i^I(x) \phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y)$$

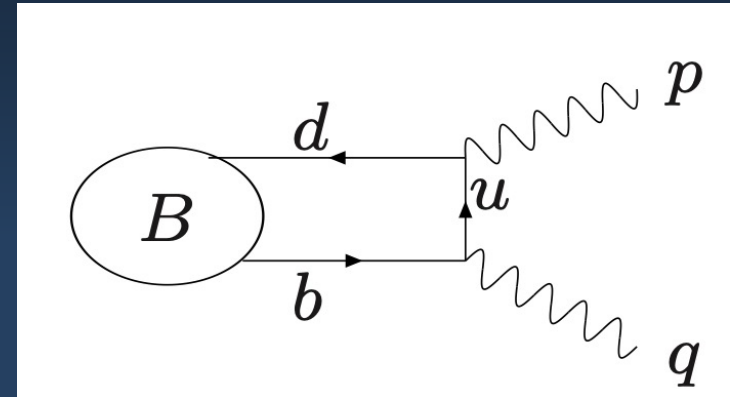
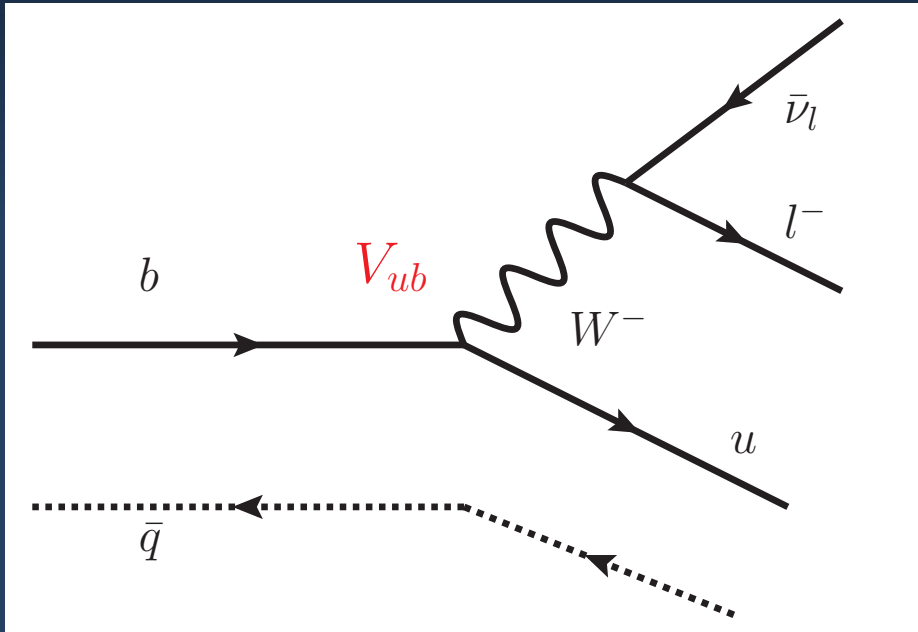
$B \rightarrow \pi$ form factor

Hard kernel

B-meson LCDA

QCD Factorization: BBNS, PRL 83, 1914 (1999)

For PQCD, See: Keum, Li, Sanda PRD 63, 054008 (2001)



B meson LCSR:

De Fazio, Feldmann, Hurth, NPB 733, 1 (2006)
 Khodjamirian, Mannel, Offen, PLB620,52 (2005)

$$\begin{aligned} \Xi_{\perp}(\tau, n \cdot p) = & -\frac{\alpha_s C_F}{2\pi} \frac{U_2(\mu_{h2}, \mu)}{f_{V,\perp}(\nu)} \frac{\tilde{f}_B(\mu)}{m_b} \frac{m_B}{m_b} [(1-\tau)\theta(\tau)\theta(1-\tau)] \\ & \times \int_0^{\omega_s} d\omega' \exp\left[-\frac{n \cdot p \omega' - m_V^2}{n \cdot p \omega_M}\right] \int_{\omega'}^{\infty} d\omega \frac{\phi_B^+(\omega, \mu)}{\omega} + \mathcal{O}(\alpha_s^2) \end{aligned}$$

Gao, et.al, PRD101, 074035(2020)

What do we know about HM LCDA?

- Equation of motion: *[Kawamura, Kodaira, Qiao, Tanaka, PLB523, 111 (2001)]*
- Evolution equations: *[Lange, Neubert, 2003; Bell, Feldmann, 2008]*

$$\frac{d}{d \ln \mu} \phi_B^+(\omega, \mu) = -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\omega' \gamma_+^{(1)}(\omega, \omega', \mu) \phi_B^+(\omega', \mu) + \mathcal{O}(\alpha_s^2)$$

$$\gamma_+^{(1)}(\omega, \omega', \mu) = \left(\Gamma_{\text{cusp}}^{(1)} \ln \frac{\mu}{\omega} - 2 \right) \delta(\omega - \omega') - \Gamma_{\text{cusp}}^{(1)} \omega \left[\frac{\theta(\omega' - \omega)}{\omega'(\omega' - \omega)} + \frac{\theta(\omega - \omega')}{\omega(\omega - \omega')} \right]_+$$

- Solution of evolution equations. *[Bell, Feldmann, Wang, Yip, 2013; Braun, Manashov, 2014]*
- RG equations of $\phi_B^+(\omega, \mu)$ at two-loops. *[Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]*
- RG equations of the higher-twist B-meson distribution amplitudes. *[Braun, Ji, Manashov, 2017]*
- Perturbative constraint for large ω *[Lee, Neubert, PRD72 (2005) 094028]*

$$\phi_+(\omega, \mu) = \frac{C_F \alpha_s}{\pi \omega} \left[\left(\frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}}{3\omega} \left(2 - \ln \frac{\omega}{\mu} \right) \right]$$

Models for heavy meson LCDAs

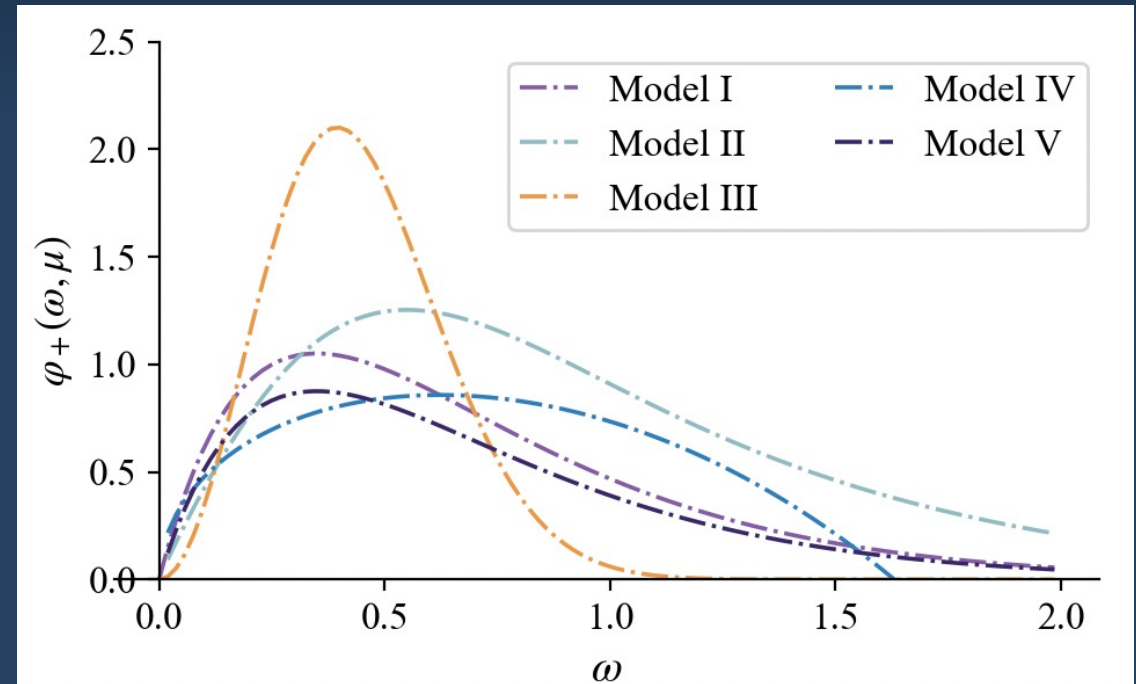
$$\varphi_{\text{I}}^+(\omega, \mu_0) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0},$$

$$\varphi_{\text{II}}^+(\omega, \mu_0) = \frac{4}{\pi\omega_0} \frac{k}{k^2 + 1} \left[\frac{1}{k^2 + 1} - \frac{2(\sigma_B^{(1)} - 1)}{\pi^2} \ln k \right]$$

$$\varphi_{\text{III}}^+(\omega, \mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2} e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\text{IV}}^+(\omega, \mu_0) = \frac{\omega}{\omega_0\omega_2} \frac{\omega_2 - \omega}{\sqrt{\omega(2\omega_2 - \omega)}} \theta(\omega_2 - \omega),$$

$$\varphi_{\text{V}}^+(\omega, \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta - \alpha, 3 - \alpha, \omega/\omega_0),$$

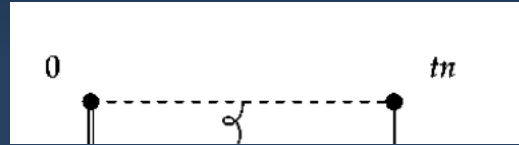


$$f_{B \rightarrow \pi}^+(0) = 0.122 \times \left[1 \pm 0.07 \Big|_{S_0^\pi} \pm 0.11 \Big|_{\bar{\Lambda}_q} \pm 0.02 \Big|_{\lambda_E^2/\lambda_H^2} \begin{matrix} +0.05 \\ -0.06 \end{matrix} \Big|_{M^2} \pm 0.05 \Big|_{2\lambda_E^2 + \lambda_H^2} \right. \\ \left. \begin{matrix} +0.06 \\ -0.10 \end{matrix} \Big|_{\mu_h} \pm 0.04 \Big|_{\mu} \begin{matrix} +1.36 \\ -0.56 \end{matrix} \Big|_{\lambda_{B_q}} \begin{matrix} +0.25 \\ -0.43 \end{matrix} \Big|_{\hat{\sigma}_1, \hat{\sigma}_2} \right]$$

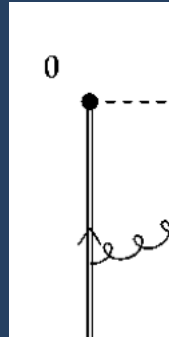
Cui, et.al, JHEP 03 (2023) 140

$$\langle H(p_H) | \bar{h}_v(0) \not{n}_+ \gamma_5 [0, tn_+] q_s(tn_+) | 0 \rangle = -i \tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega e^{i\omega t n_+ \cdot v} \varphi_+(\omega; \mu)$$

Lightcone:

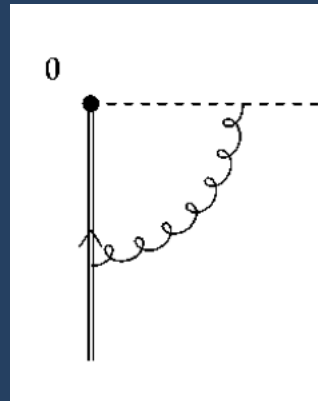


Heavy quark Field:



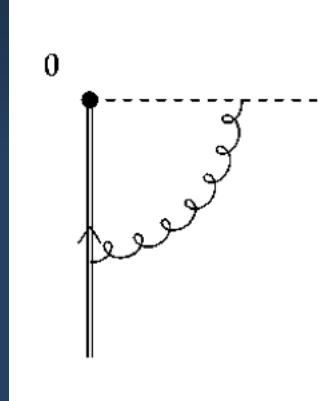
$$\frac{1}{i v \cdot k}$$

Cusp divergence:



$$O_+^{\text{ren}}(t, \mu) = O_+^{\text{bare}}(t) + \frac{\alpha_s C_F}{4\pi} \left\{ \left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}} \ln(it\mu) \right) O_+^{\text{bare}}(t) - \frac{4}{\hat{\epsilon}} \int_0^1 du \frac{u}{1-u} [O_+^{\text{bare}}(ut) - O_+^{\text{bare}}(t)] \right\}$$

Cusp divergence:



$$\cos \theta = \frac{n \cdot v}{\sqrt{n^2} \sqrt{v^2}}$$

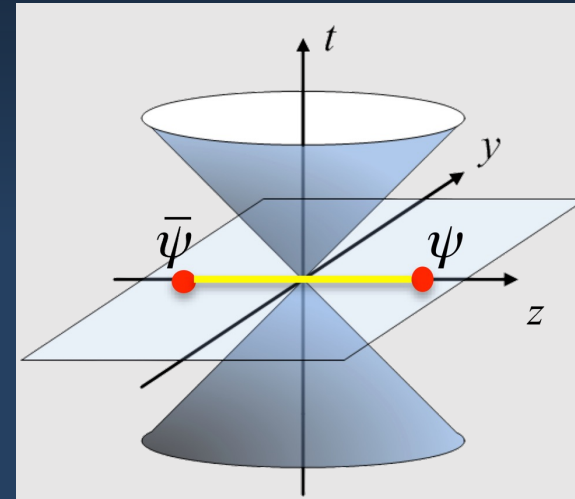
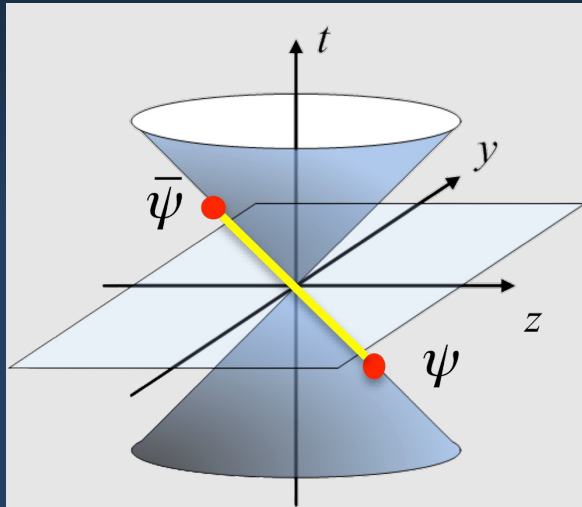
- ✓ $n^2 \neq 0$, still heavy quark field h_v

WW, Wang, Xu, Zhao, PRD 102, 011502 (2020); Xu, Zhang, Zhao, PRD106, L011503 (2022)

- ✓ No h_v : QCD heavy quark

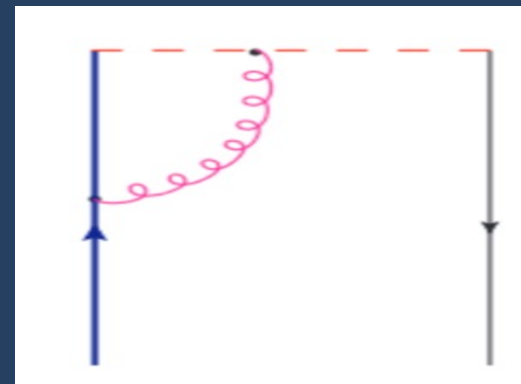
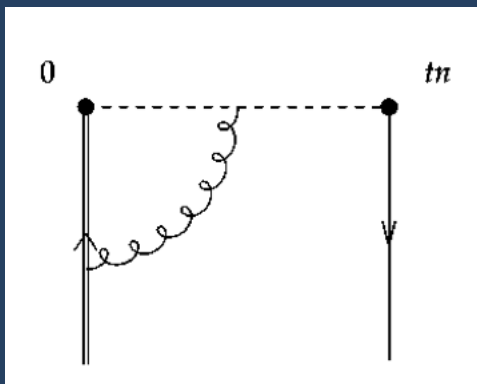
Han, Wang, Zhang, et.al,2403.17492;

Han, Wang, Zhang, Zhang, 2408.13486; Deng, Wang, Wei, Zeng, 2409.00632

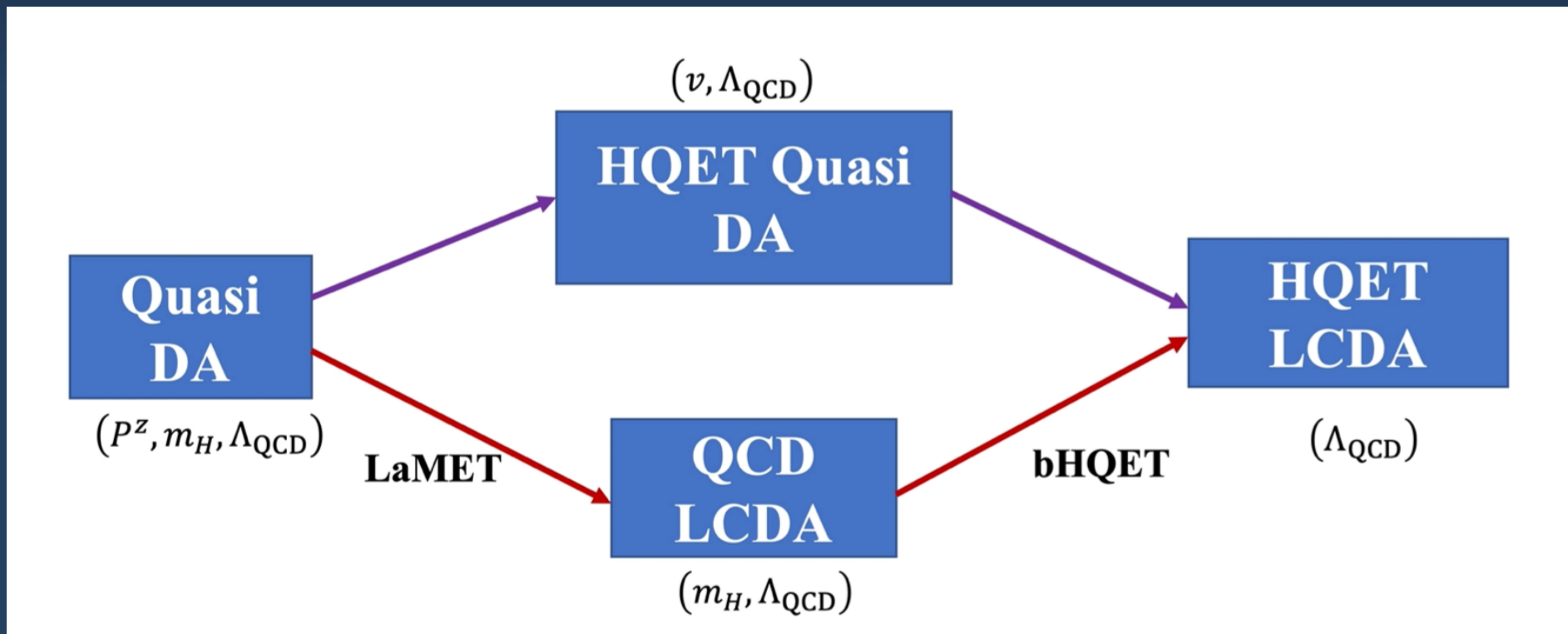


LaMET[*Ji, PRL 110, 262002 2013*]:

lightcone can be accessed by simulating correlation functions with a large but finite P^z



HQET fields can be accessed by simulating correlation functions with a large but finite m_Q



A two-step matching method

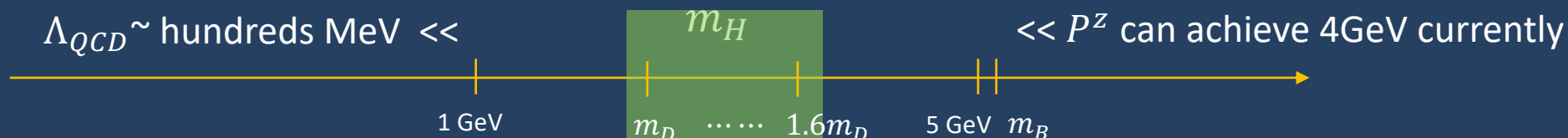
➤ Start from Quasi DA, calculable from LQCD



• A multi-scale processes:

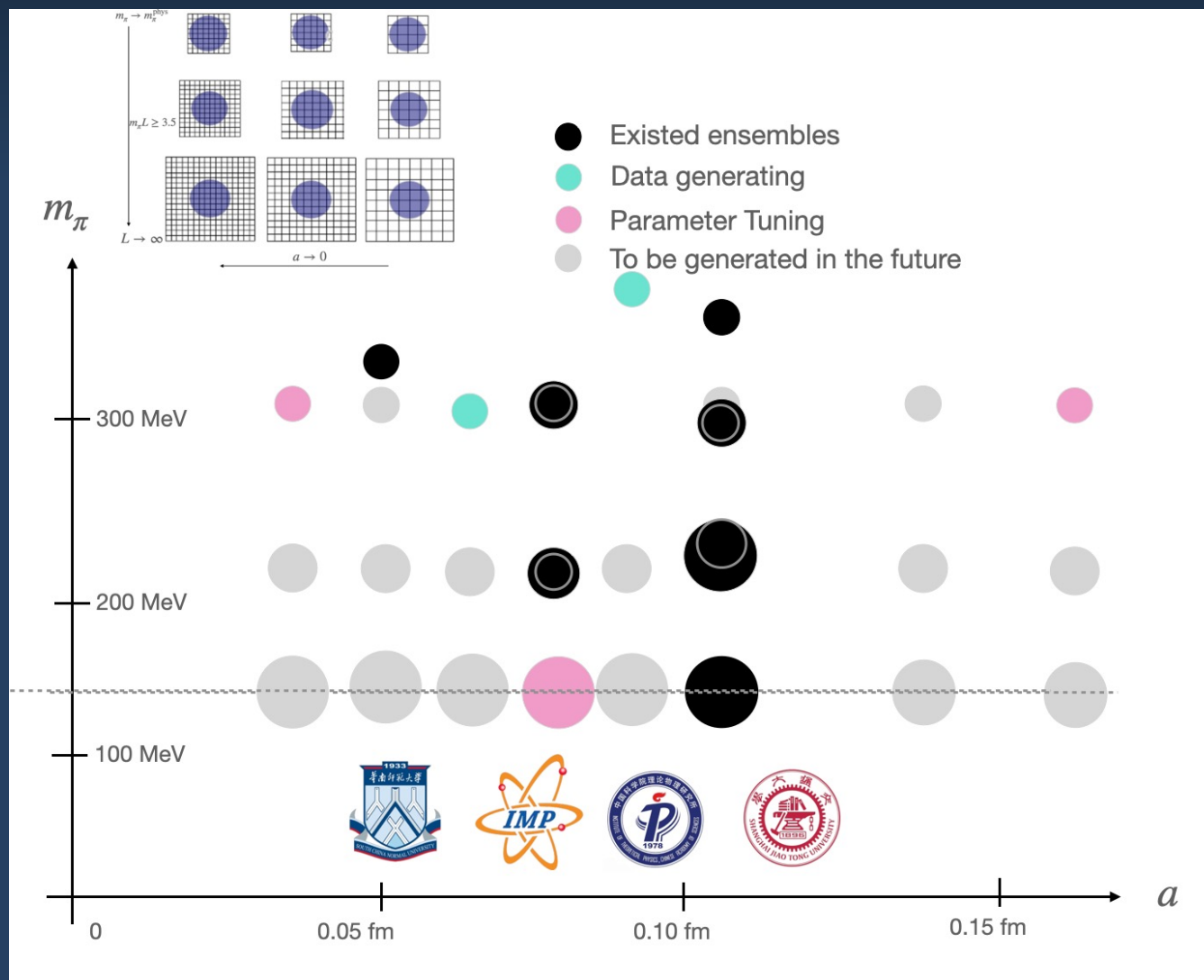
1. LaMET requires $\Lambda_{\text{QCD}}, m_H \ll P^z$ and finally integrate out P^z ;
2. bHQET requires $\Lambda_{\text{QCD}} \ll m_H$ and integrate out m_H ;

⇒ **Hierarchy $\Lambda_{\text{QCD}} \ll m_H \ll P^z$** : A big challenge for lattice simulation but **still calculable on the lattice**

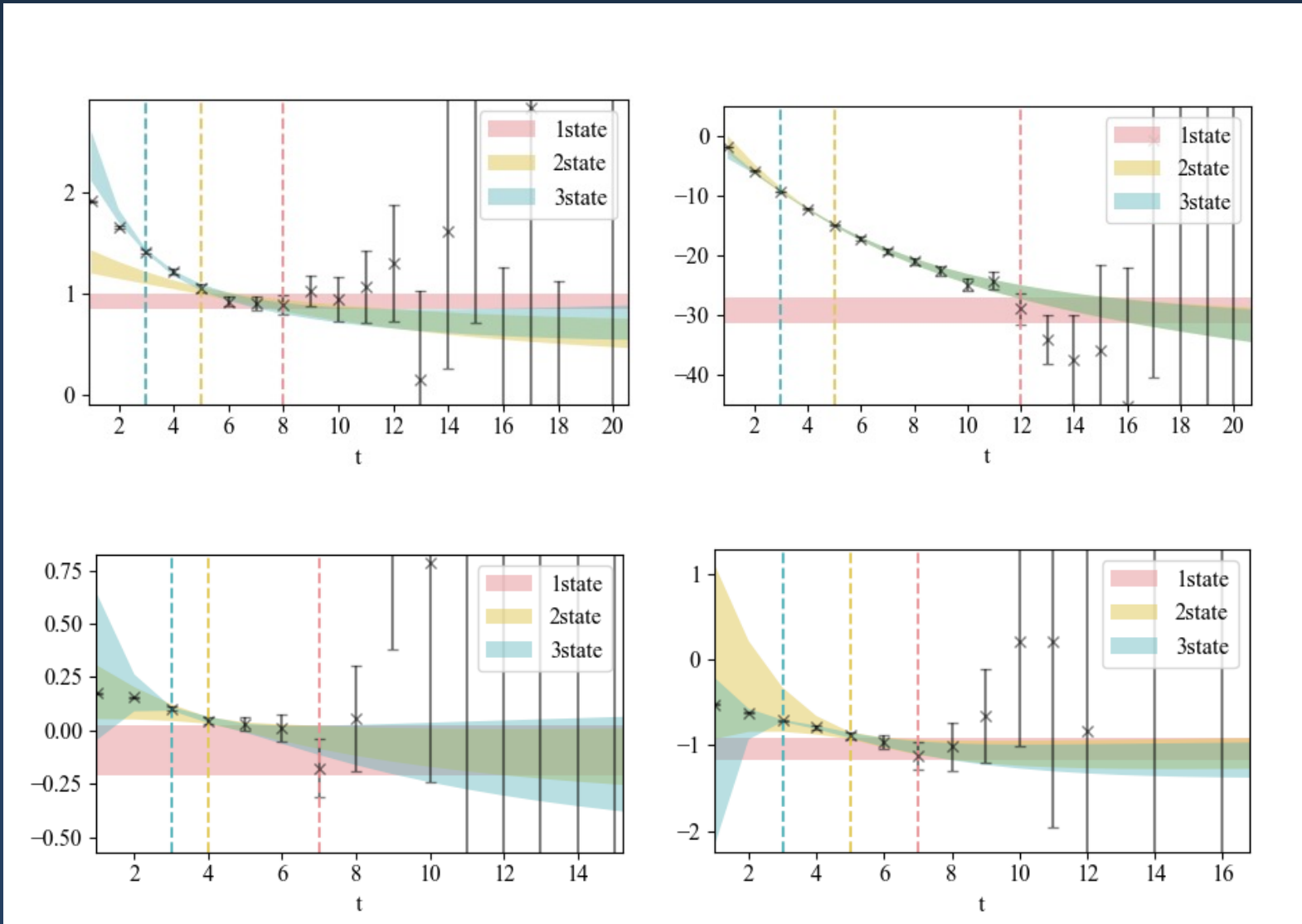


The image features two 3D lattice QCD diagrams. Each diagram consists of a 3x3x3 grid of white lines representing lattice sites. Purple spheres are placed at various lattice sites, representing quarks. Yellow and orange lines connect the sites, representing gluons. The left diagram shows a quark-antiquark pair with a large, bright yellow and orange glow between them, indicating a strong interaction. The right diagram shows a quark-antiquark pair with a similar glow, but the interaction appears slightly different. The background is a dark blue space with stars and a blue surface with ripples.

Lattice QCD verification



- H48P32, $n_s^3 \times n_t = 48^3 \times 144$, $a = 0.05187\text{fm}$;
- $m_\pi \simeq 317\text{MeV}$, $m_{\eta_s} = 700\text{MeV}$;
- Determine the charm quark mass by tuning $m_{J/\psi}$ to its physical value, then $m_D \simeq 1.90\text{GeV}$;
- Coulomb gauge fixed grid source with grid = $1 \times 1 \times n_s$; 549 configurations
 $\times 8$ measurements.

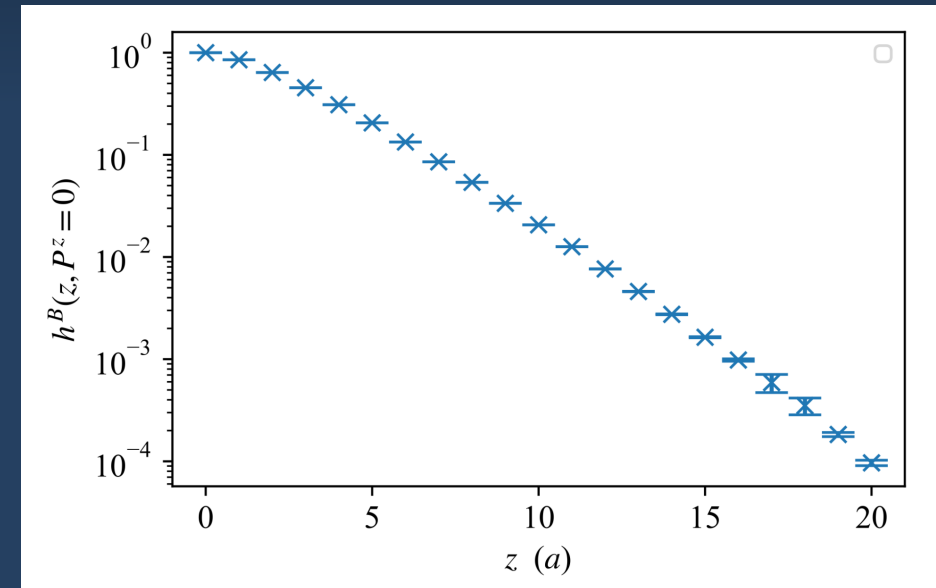


- We compare the 1, 2, 3-state fits. All the fit results are consistent with each other.
- Different fit strategy valid at different t-range.
- To balance the signal of data and reliability of the multi-state fits, we prefer the 1-state fit, and select the 2-state fit when the former one is inadequate to describe the data. Result from 3-state fit is only used as a reliability check for the first two strategy.

Step 1: Renormalization in the hybrid-ratio scheme

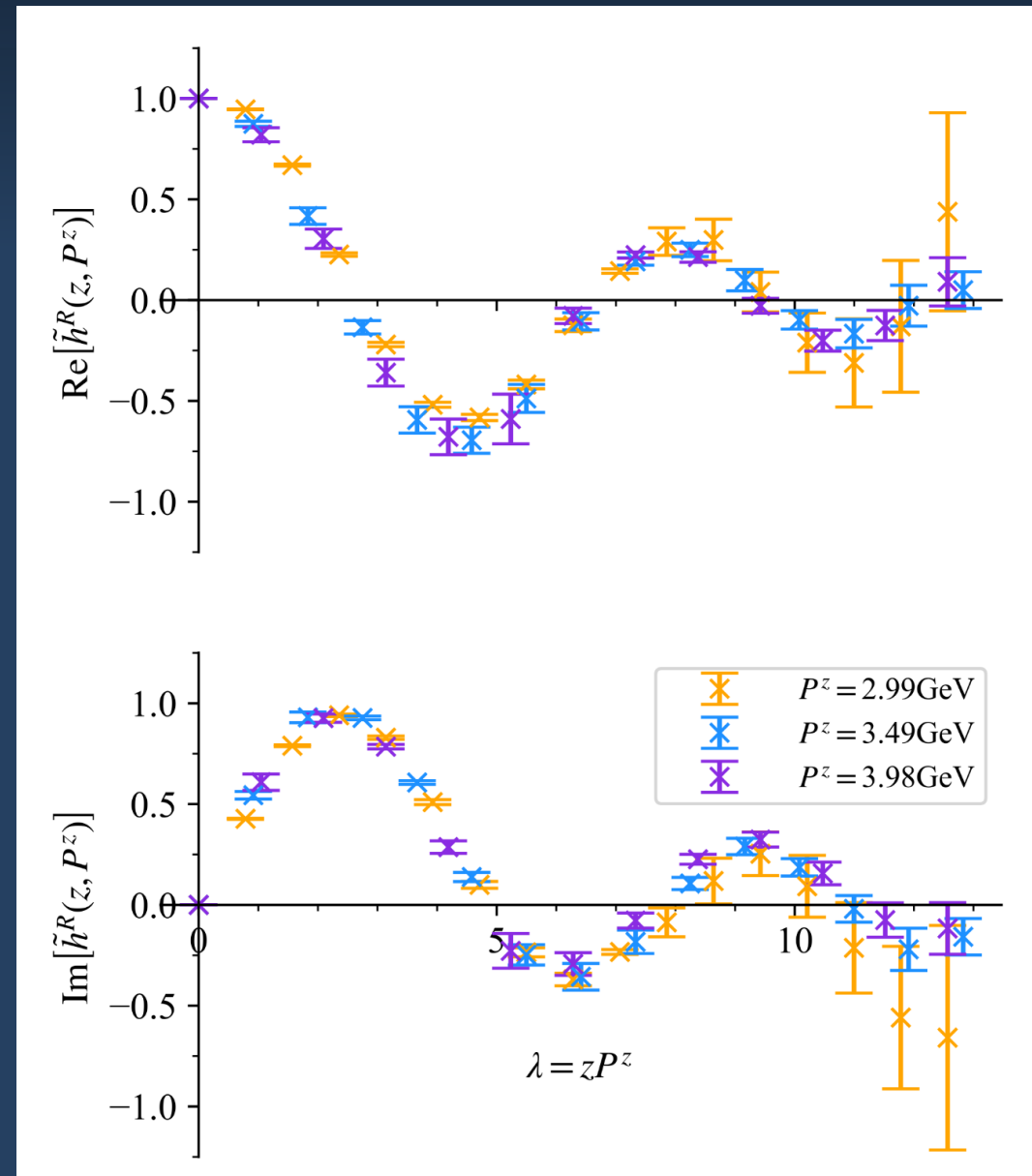
$$\tilde{h}^R(z, P^z) = \begin{cases} \frac{\tilde{h}^B(z, P^z)}{\tilde{h}^B(z, P^z=0)} & |z| < z_s \\ e^{(\delta m + m_0)(z - z_s)} \frac{\tilde{h}^B(z, P^z)}{\tilde{h}^B(z_s, P^z=0)} & |z| \geq z_s \end{cases},$$

- We use the zero momentum matrix element $\tilde{h}^B(z, P^z = 0)$ to renormalize the bare ones.
- The Dirac structure of zero momentum matrix element is $\gamma^t \gamma_5$, it contains same UV divergence as the one with $\gamma^z \gamma_5$.



Ji, Liu, Schäfer, Wang, Yang, Zhang, Zhao, NPB 964, 115311 (2021)

- The renormalized matrix elements at different momenta are basically consistent with each other.

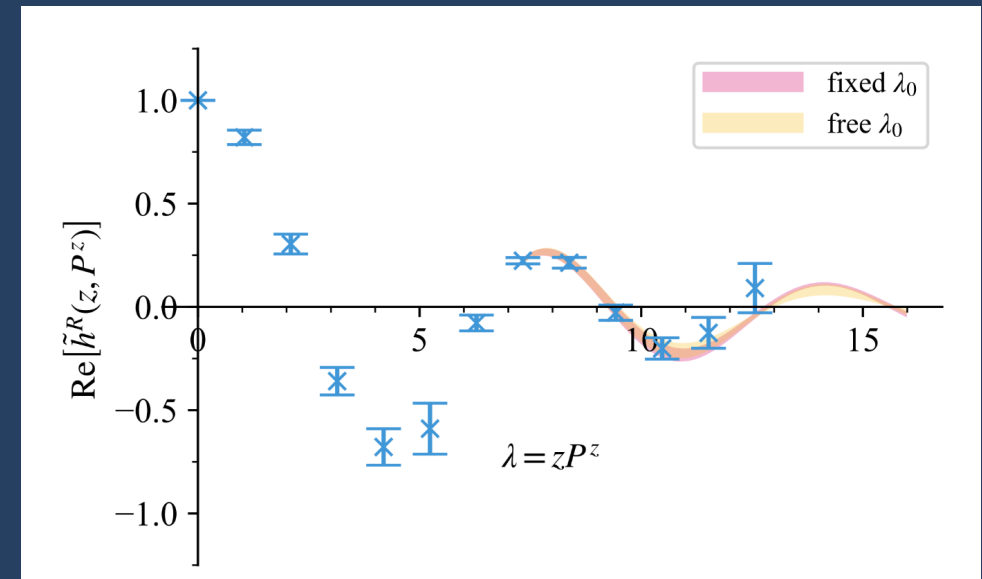


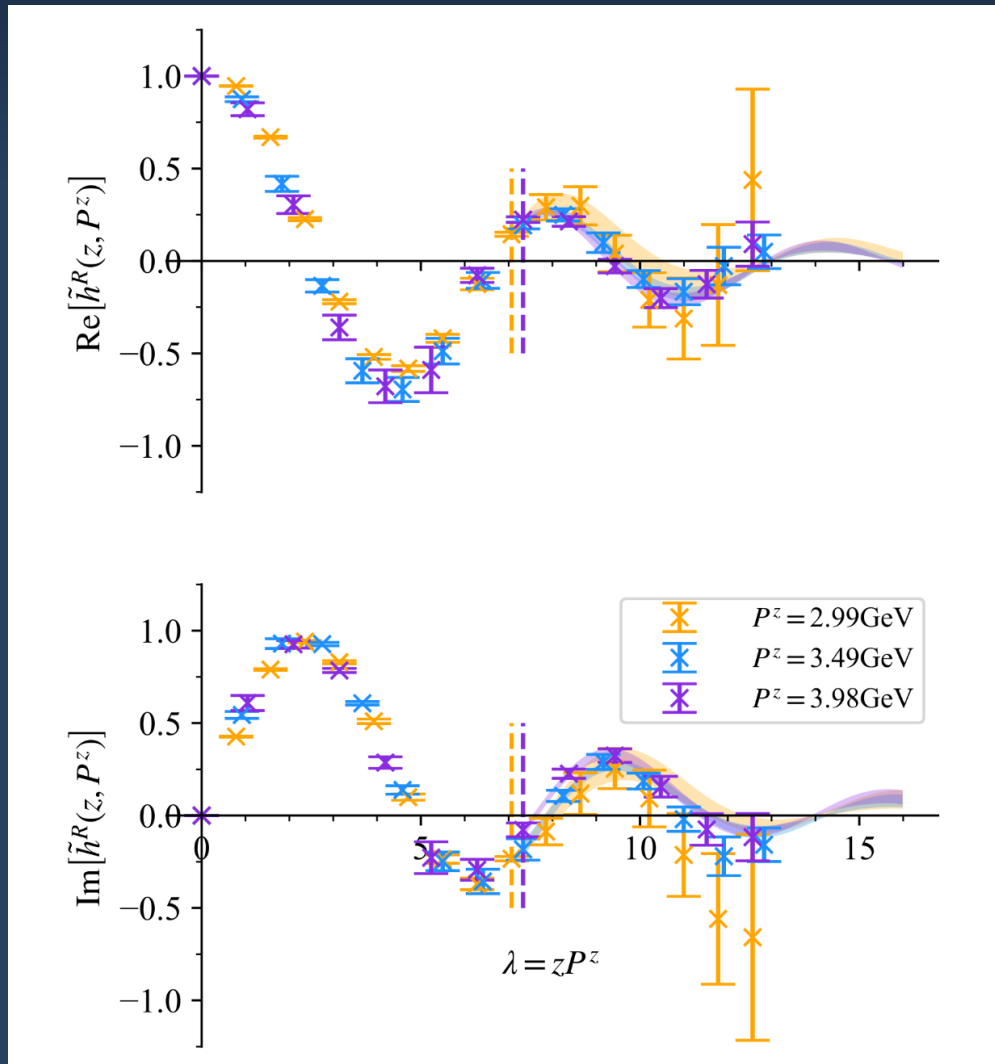
Step 1: λ -extrapolation

We extrapolate the renormalized matrix elements to infinity based on the data at large λ :

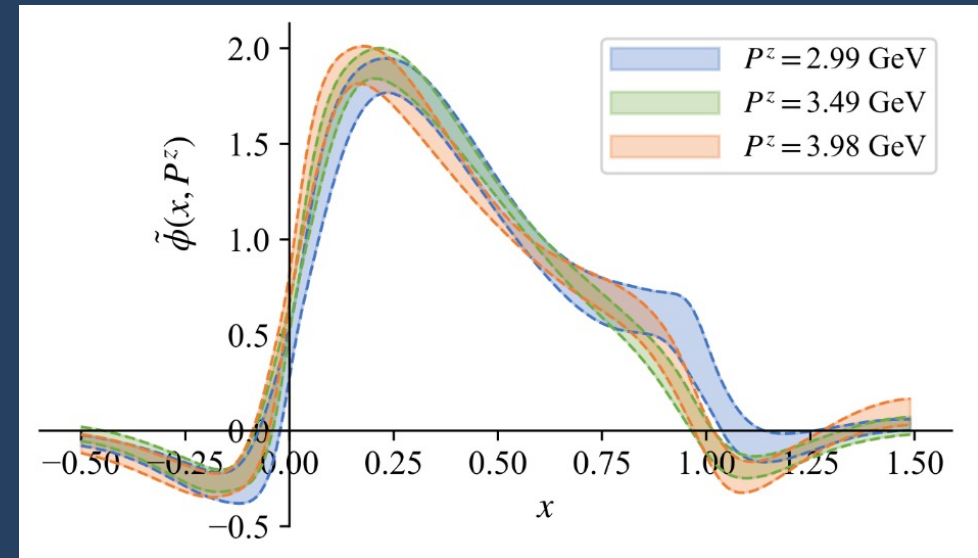
- The parameterization inside the square brackets account for the algebraic behavior and motivated by the Regge behavior of the light-cone distributions at endpoint regions.
- The exponential decay behavior is governed by the decaying $\propto e^{-\delta m z}$ at long-tail region. Based on the definition of hybrid ratio scheme, the renormalized matrix elements decaying with $e^{m_0(z-z_s)}$, which related to the finite correlation length $\lambda_0 \sim -P^z/m_0$.
- We compare the extrapolation from “fixed λ_0 ” and “free λ_0 ”. The results from two strategies are consistent with each other.

$$\tilde{h}^R(\lambda) = \left[\frac{c_1}{(-i\lambda_1)^{d_1}} + e^{i\lambda} \frac{c_2}{(i\lambda_2)^{d_2}} \right] e^{-\lambda/\lambda_0},$$





- We extrapolate the renormalized matrix elements to infinity, and then Fourier transform them to momentum space to obtain the quasi DA.
- We use the “free λ_0 ” strategy for conservative and adopt $\lambda_L = \{7.07, 7.34, 7.32\}$ for $P^z = \{2.99, 3.49, 3.98\} \text{ GeV}$.



- The matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int dy C(x, y, P^z) \phi(y),$$

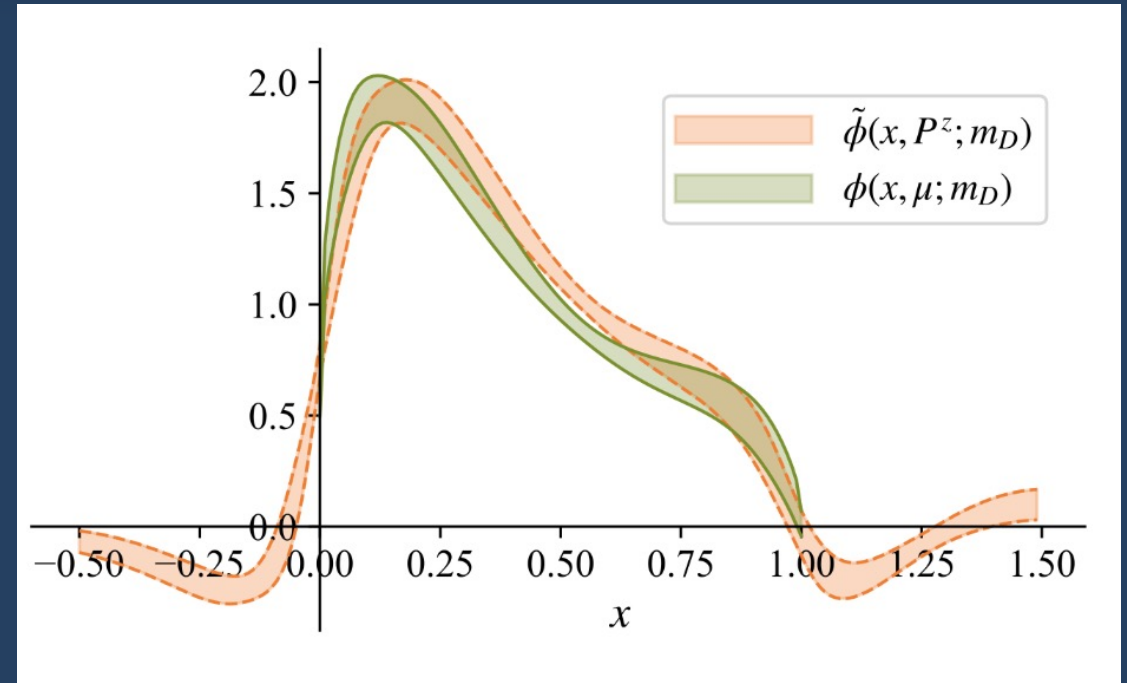
the perturbative matching kernel up to NLO at leading power:

$$C_B^{(1)}\left(x, y, \frac{P^z}{\mu}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} [H_1(x, y)]_{+(y)} & x < 0 < y < 1 \\ [H_2\left(x, y, \frac{P^z}{\mu}\right)]_{+(y)} & 0 < x < y < 1 \\ [H_2\left(1-x, 1-y, \frac{P^z}{\mu}\right)]_{+(y)} & 0 < y < x < 1 \\ [H_1(1-x, 1-y)]_{+(y)} & 0 < y < 1 < x \end{cases}$$

$$C^{(0)}(x, y) = \delta(x - y),$$

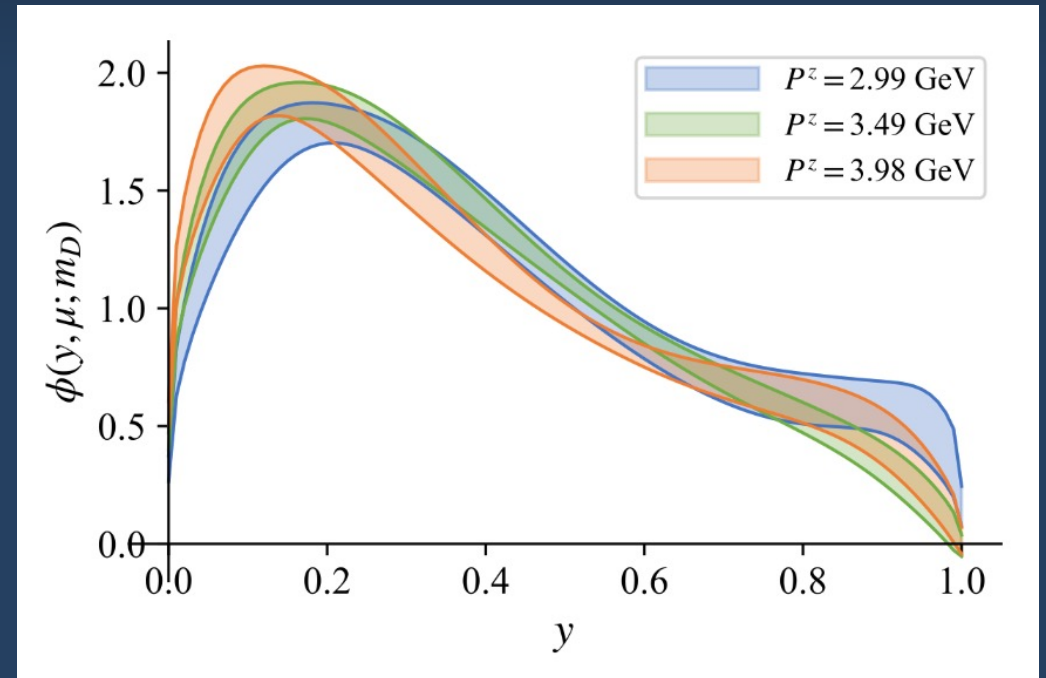
$$C^{(1)}(x, y) = C_B^{(1)} - C_{CT}^{(1)}.$$

$P^z = 3.98\text{GeV}, \mu = 2\text{GeV}$



The results with different momenta are consistent within $1\text{-}\sigma$.

The matching kernel without renormalon resummation still contains some large $\log P^Z$ terms, these terms will give the more major contribution than the polynomial P^Z terms at the limit of $P^Z \rightarrow \infty$.

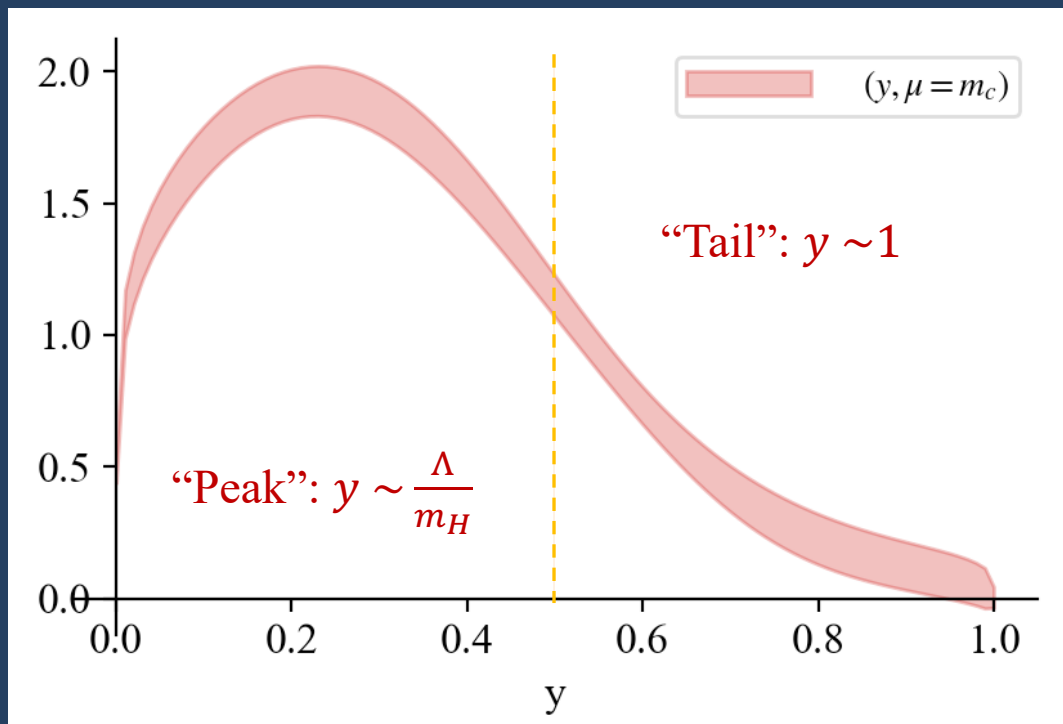


Step 3: LCDAs in HQET

➤ The LCDAs in QCD defined as:

$$\begin{aligned} & \langle 0 | \bar{q}(tn_+) \not{n}_+ \gamma_5 W_c(tn_+, 0) Q(0) | H(P_H) \rangle \\ & = i f_H n_+ \cdot P_H \int_0^1 dy e^{iy P_H \cdot tn_+} \phi(y, \mu), \end{aligned}$$

can be divided into 2 parts based on the hierarchy of y :



- For very large scale $\mu \gg m_Q$, $\phi(y, \mu)$ will tend to asymptotic form;

- For the scale $\mu \lesssim m_Q$,

⇒ Light quark carries small momentum fraction $y \sim \Lambda/m_H$
 ⇒ peak region, related to the HQET LCDA;

⇒ $y \sim O(1)$ region be suppressed in LCDA:

P_q is soft-collinear, $\ll P_Q$, only contribute through power corrections;

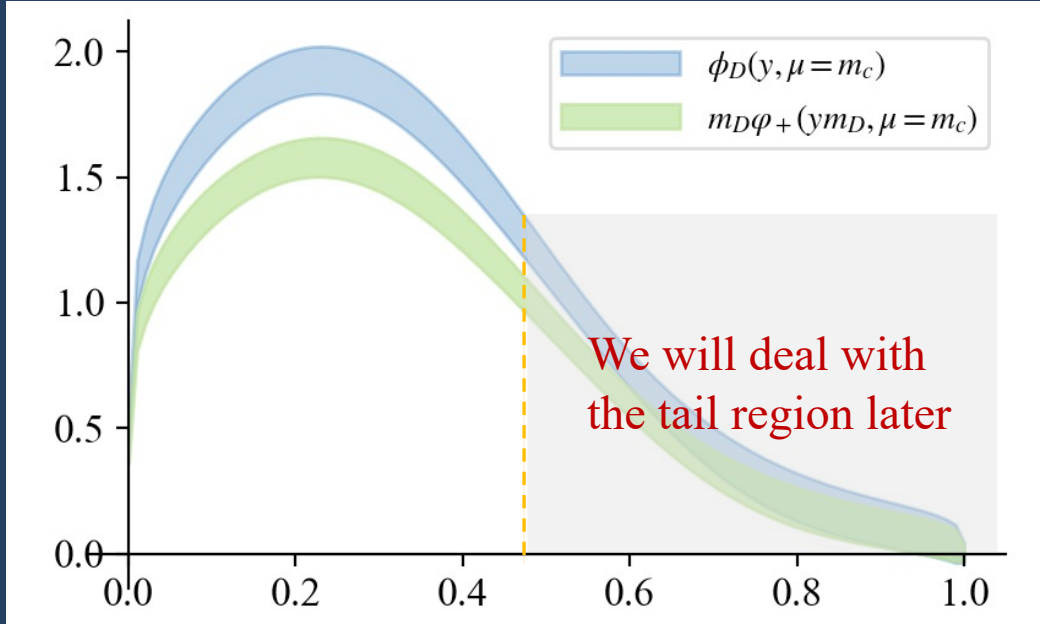
SCET renormalized matrix element in this region contain only hard-collinear physics, and starts at the one-loop level.

➤ The leading twist heavy meson LCDA in HQET:

$$\begin{aligned} & \langle 0 | \bar{q}(tn_+) \not{n}_+ \gamma_5 W_c(tn_+, 0) h_v(0) | H(v) \rangle \\ & = iF_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega t n_+ \cdot v} \varphi_+(\omega, \mu), \end{aligned}$$

is connected with the QCD LCDA through a multiplicative factorization in the peak region:

$$\begin{aligned} \phi(u, m_H) &= \frac{\tilde{f}_H}{f_H} J_{\text{peak}} m_H \varphi_+(\omega = u m_H), \\ J_{\text{peak}} &= 1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{2} \ln^2 \frac{\mu^2}{m_H^2} + \frac{1}{2} \ln \frac{\mu^2}{m_H^2} + \frac{\pi^2}{12} + 2 \right) + O(\alpha_s^2), \\ F_H &= \tilde{F}_H(\mu) \left[1 - \frac{\alpha_s C_F}{4\pi} \left(\frac{3}{2} \ln \frac{\mu^2}{m_H^2} + 2 \right) + O(\alpha_s^2) \right], \end{aligned}$$



[Beneke, Finauri, Keri Vos, Wei, JHEP 09, 066 (2023)]

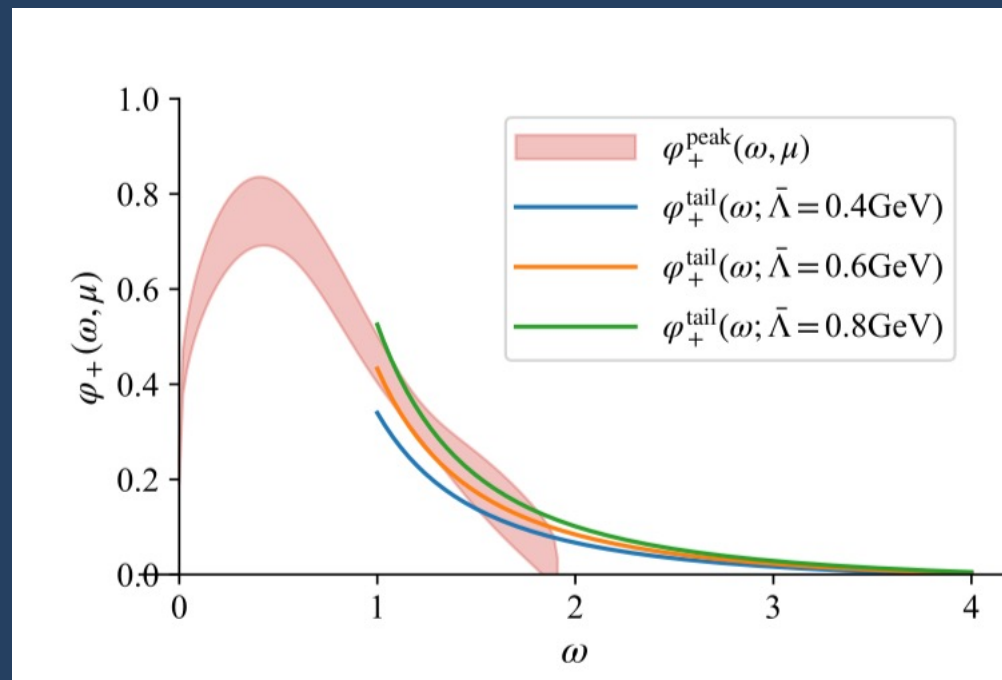
Step 3: Tail of HQET LCDA

- The tail region of HQET LCDA is perturbative and its 1-loop result: *[Lee, Neubert, PRD72 (2005) 094028]*

$$\phi_+(\omega, \mu) = \frac{C_F \alpha_s}{\pi \omega} \left[\left(\frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}}{3\omega} \left(2 - \ln \frac{\omega}{\mu} \right) \right]$$

where $\bar{\Lambda} \equiv m_H - m_Q^{\text{pole}}$ reflect the power correction, and usually be chosen as 400~800MeV.

- $\bar{\Lambda} = 0$: neglect the power correction;
- We use the difference between the lines to estimate the power correction.



The final results of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).

➤ Models for HQET LCDAs

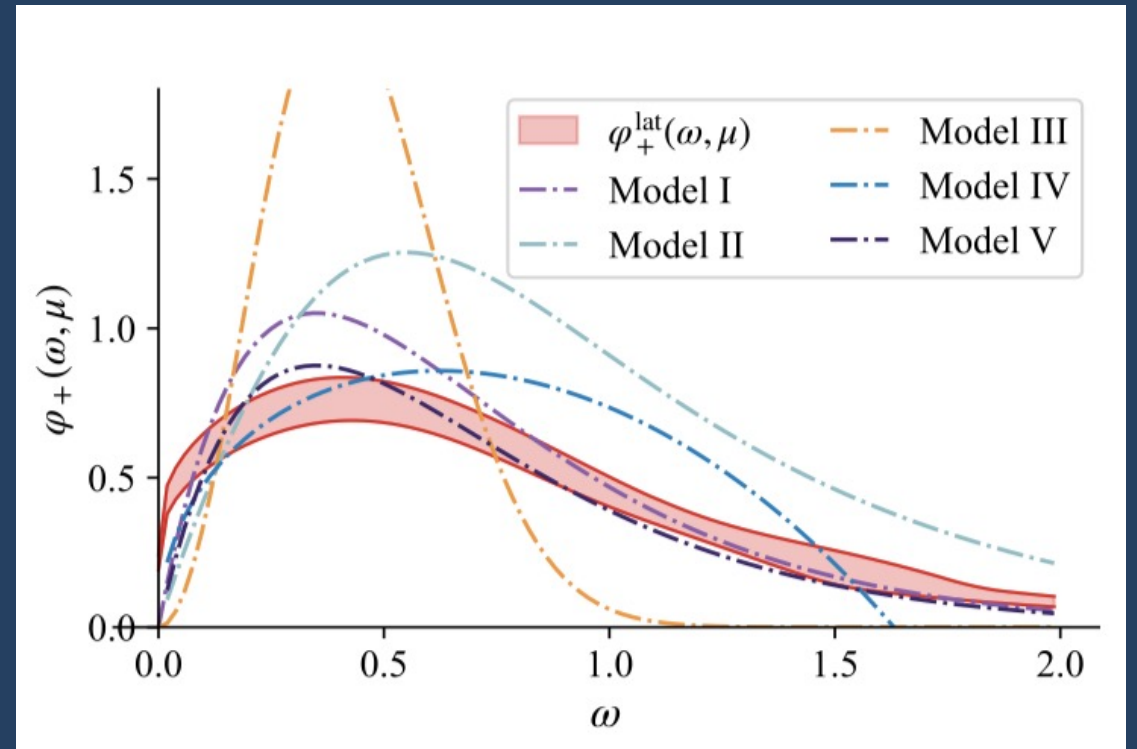
$$\varphi_{\text{I}}^+(\omega, \mu_0) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0},$$

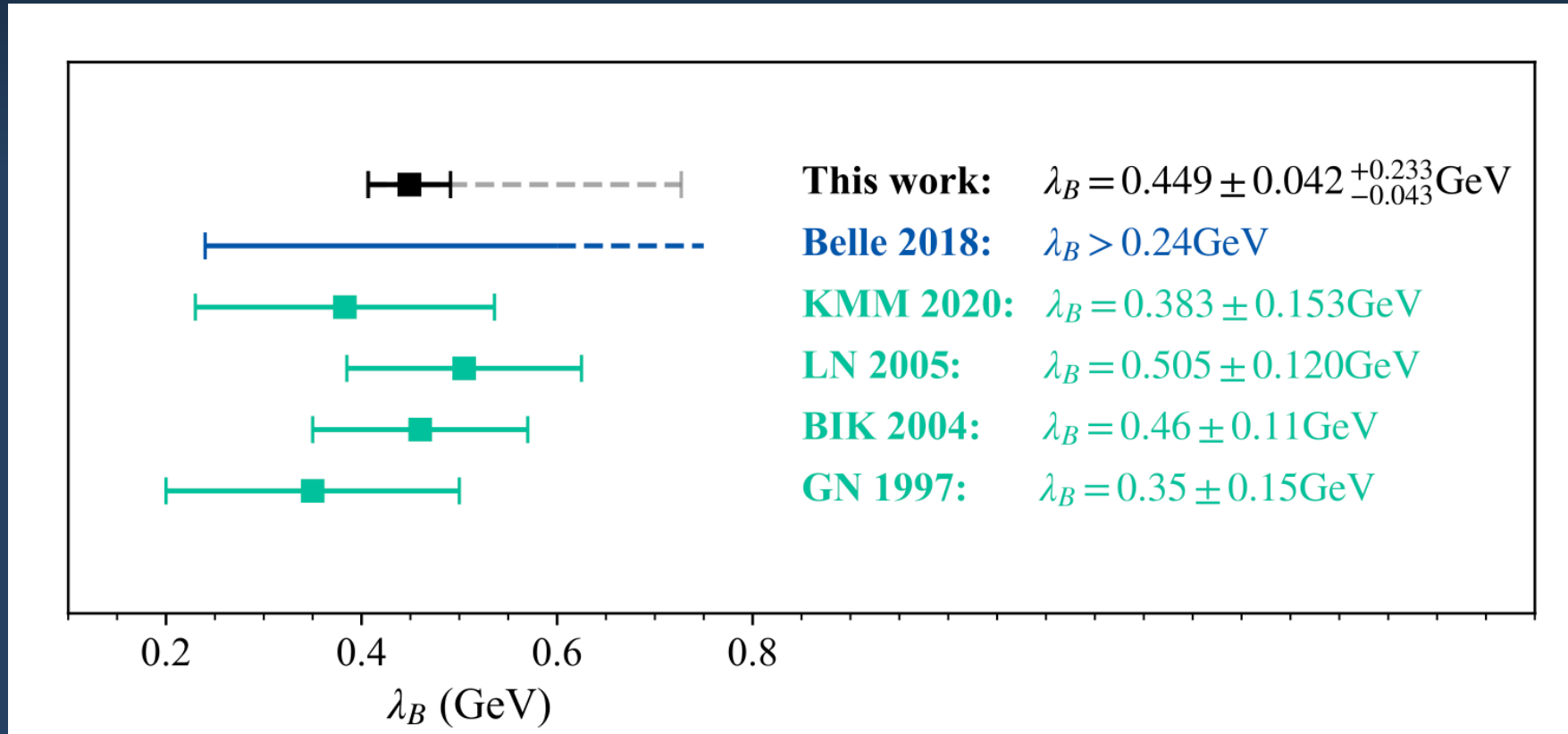
$$\varphi_{\text{II}}^+(\omega, \mu_0) = \frac{4}{\pi\omega_0} \frac{k}{k^2 + 1} \left[\frac{1}{k^2 + 1} - \frac{2(\sigma_B^{(1)} - 1)}{\pi^2} \ln k \right]$$

$$\varphi_{\text{III}}^+(\omega, \mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2} e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\text{IV}}^+(\omega, \mu_0) = \frac{\omega}{\omega_0\omega_2} \frac{\omega_2 - \omega}{\sqrt{\omega(2\omega_2 - \omega)}} \theta(\omega_2 - \omega),$$

$$\varphi_{\text{V}}^+(\omega, \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta - \alpha, 3 - \alpha, \omega/\omega_0),$$





KMM 2020: Khodjamirian, Mandal, Mannel, JHEP 10, 043 (2020)

LN 2005: Lee, Neubert, PRD 72, 094028 (2005)

BIK2004: Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

GN 1997: Grozin, Neubert, Phys. Rev. D 55, 272- 290 (1997)

- ✓ Generally, weak decays of bottom hadrons offer an ideal platform to explore new physics (NP), but accurate hadronic inputs are essential for making reliable predictions.
- ✓ A method to determine heavy meson LCDA from Lattice QCD:
 - ✓ Two-step effective field theories
 - ✓ CLQCD ensemble (0.05fm) to simulate Heavy (D) meson quasi Das
 - ✓ Hybrid renormalization on lattice and λ -extrapolation scheme
 - ✓ The (preliminary) results for LCDAs are consistent with models

The first step towards heavy meson LCDAs

Theory

- Heavy quark spin symmetry
- $1/P^z$ corrections
- $1/m_Q$ corrections
- m_Q dependence
- ...

Lattice

- Finer Lattices
- Renormalization
- Different sources
- ...

Precise results on heavy meson LCDAs

Thank you for your attention!