Exact Holographic Euclidean correlators

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Based on arXiv: JHEP 06 (2023) 116, 2308.13518[hep-th], 2311.09636[hep-th]

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Outline

- **Motivations**
- **Holographic prescriptions**

1. **Torus correlators of stress tensor in AdS3/CFT2**
2. **Finite temperature correlators dual to AdS-Planar BH**

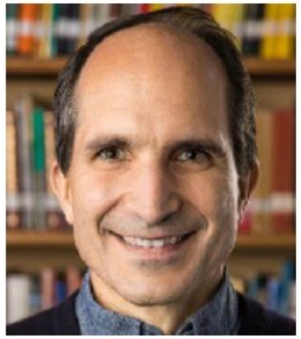
- **Summary and perspectives**

Motivations

To understand nature of QG
proposed by t' Hooft and Susskind

AdS/CFT correspondence, Maldacena 1997

$$AdS_5 \times S^5 \Longleftrightarrow N = 4 \text{ SYM theory}$$



Maldacena

I. AdS5/CFT4
II. AdS4/CFT3 (ABJM)
III. AdS3/CFT2
IV. nAdS2/nCFT1, nAdS2/SYK4
V. Non-AdS/CFT (Celestial Holograph)
VI. DS/CFT
VII....

Symmetry, field contents

Partion function

Lower point correlation function, & application

Higher point correlation function…

AdS/CFT correspondence, Maldacena 1997

Dictionary: GKPW

S. S. Gubser, I. R. Klebanov and A. M. Polyakov，9802109
E. Witten，9802150

$$Z_{\mathrm{CFT}}[g_{ij}, J] = \int_{G_{\mu\nu}|_{\mathrm{bdy}}=g_{ij}, \Phi|_{\mathrm{bdy}}=J} [dG_{\mu\nu}][d\Phi] e^{-S_{\mathrm{grav}}[G_{\mu\nu}, \Phi]}$$

To check ("prove") the AdS3/CFT2 correspondence:

$$\langle O \rangle = -i \frac{\delta Z[\phi_0]}{\delta \phi_0} = \frac{\delta S[\phi_0]}{\delta \phi_0}$$

Partition functions, generic correlation functions, etc.

$$\langle O(x_1) \dots O(x_n) \rangle_{CFT} \sim \frac{\delta^n I_{grav}}{\delta \psi_0(x_1) \dots \delta \psi_0(x_n)}$$

Recent Progress on Holographic correlators

- **Most previous research focuses on holographic correlators in pure AdS**
- **Holographic correlators from Minkowski AdS planar blackhole**

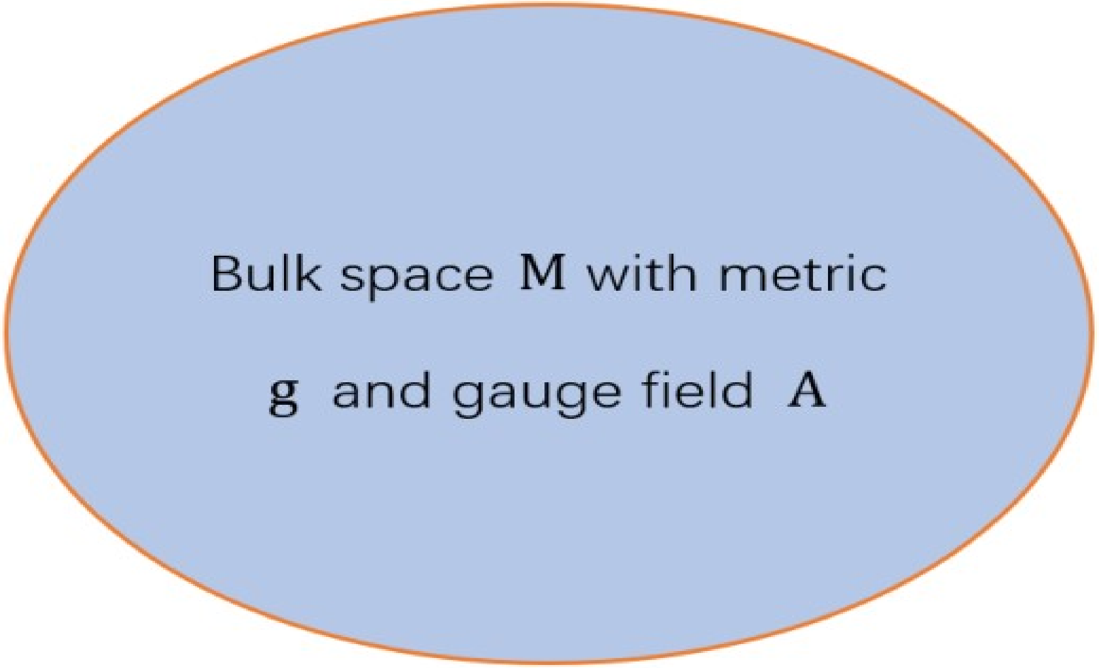
  holographic transport coefficients (specify B.C. on the horizon, ingoing)
- **Holographic correlators from Euclidean AdS planar blackhole**

  Scalar operator correlators worked out (arXiv 2206.07720),
  Only near-boundary analysis for stress tensor correlators (JHEP 09 (2022), 234)

We focus on correlators in the Euclidean spacetime with nontrivial topology.

$$\langle T_{i_1 j_1}(x_1) \dots T_{i_n j_n}(x_n) \rangle_{CFT} \sim \frac{\delta^n I_{grav}}{\delta \gamma^{i_1 j_1}(x_1) \dots \delta \gamma^{i_n j_n}(x_n)}$$

Boundary Value Problem



Conformal boundary ∂M

with boundary metric γ

and gauge field $\mathcal{A}$

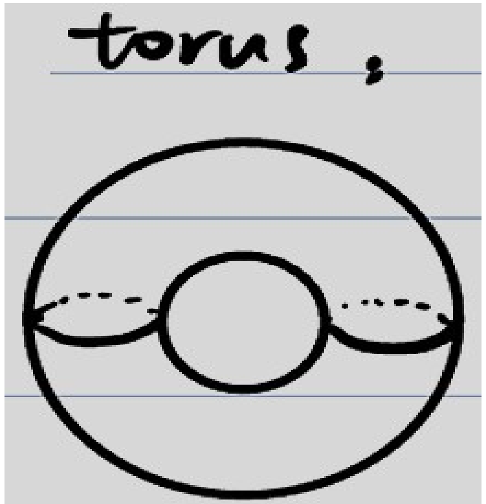
$$\gamma = r^2 g|_{r=0}, \mathcal{A} = A|_{r=0}$$

- **In general, for the given conformal boundary e.g., torus, we need to consider all gravity saddles with different bulk topology and metric.**

- **Near boundary geometry is well-understood** **[Charles Fefferman, C. Robin Graham, arXiv: 0710.0919, Commun. Math. Phys. 217 (2001) 595-622)]**

- **The global boundary value problem is much more difficult.**

Lower Dimensional: AdS3/CFT2



AdS3/CFT2

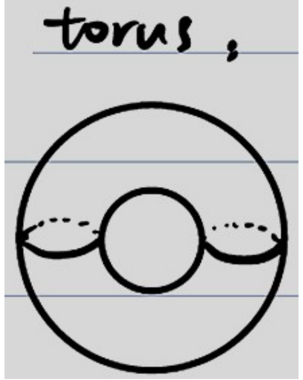
In AdS3/CFT2, the partition function Alexander Maloney, Edward Witten, 0712.0155

$$\sum_{\alpha} e^{-I^{(\alpha)}_{on-shell}} = Z_{CFT}$$

α labels the saddle points.

- **Thermal AdS3** for low temperature
- ↕ S transformation of moduli parameter
- **BTZ black hole** for high temperature

$$S_E = -\frac{1}{16\pi G}\int\sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G}\int\sqrt{\gamma}K + \frac{1}{8\pi Gl}\int\sqrt{\gamma}.$$



Holographic stress tensor correlators:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\left[dzd\bar{z} - r^2\pi^2(dz^2 + d\bar{z}^2) + r^4\pi^4 dzd\bar{z}\right]$$

Variation of boundary metric

$$\delta\gamma_{ij}dx^i dx^j = \epsilon f_{ij}(z,\bar{z})dx^i dx^j.$$

Variation of bulk metric

Ensures solution is well-behaved

$$\langle T_{i_1 j_1}(z_1)\ldots T_{i_n j_n}(z_n)\rangle$$

$$= -\frac{(-2)^n \delta^n I[\gamma]}{\sqrt{\det(\gamma(z_1))}\ldots\sqrt{\det(\gamma(z_n))}\,\delta\gamma^{i_1 j_1}(z_1)\ldots\delta\gamma^{i_n j_n}(z_n)}$$

Holographic prescriptions:

1. **Top-down: Regularity Boundary Conditions**

2. **Bottom-up: Modular Variation"-" Boundary Coordinate Transformation =Metric variation**

AdS3 gravity

$$S_E=-\frac{1}{16\pi G}\int\sqrt{g}(R-2\Lambda)-\frac{1}{8\pi G}\int\sqrt{\gamma}K+\frac{1}{8\pi Gl}\int\sqrt{\gamma}.$$

Fefferman-Graham series truncates as Banados space-time

$$ds^2=\frac{dr^2}{r^2}+\frac{1}{r^2}g_{ij}(x,r)dx^idx^j.$$

truncates

$$g_{ij}(x,r)=g^{(0)}_{ij}(x)+g^{(2)}_{ij}(x)r^2+g^{(4)}_{ij}(x)r^4.$$

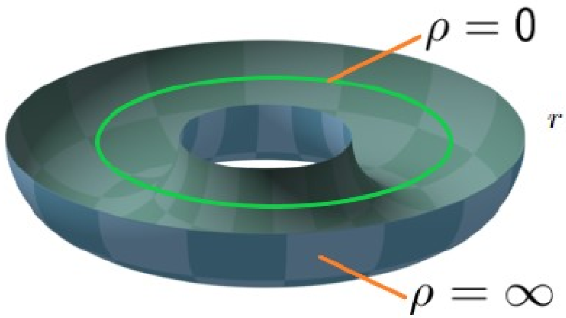
$$\langle T_{ij}\rangle=\frac{1}{8\pi G}\left(g^{(2)}_{ij}-g^{(0)kl}g^{(2)}_{kl}g^{(0)}_{ij}\right)$$

$$g^{(4)}_{ij}=\frac{1}{4}g^{(2)}_{ik}g^{(0)kl}g^{(2)}_{lj}.$$

Thermal AdS3：

(z, zbar) ~ (z + 1, zbar + 1) ~ (z + τ, zbar + τbar)

$$ds^2=d\rho^2+\cosh^2\rho dt^2+\sinh^2\rho d\phi^2$$



$$r=\frac{1}{\pi e^{\rho}},\ z=\frac{\phi+it}{2\pi},\ \bar{z}=\frac{\phi-it}{2\pi}.$$

$$ds^2=\frac{dr^2}{r^2}+\frac{1}{r^2}\left[dzd\bar{z}-r^2\pi^2(dz^2+d\bar{z}^2)+r^4\pi^4dzd\bar{z}\right]$$

conformal boundary at rho = ∞ or r = 0

Top-down approach

To the first order $ds^2 = (1+\epsilon\mathcal{L}_{V^{[1]}})(d\rho^2 + \cosh^2\rho dt^2 + \sinh^2\rho d\phi^2) + \epsilon g_{ij}^{FG[1]} dx^i dx^j.$

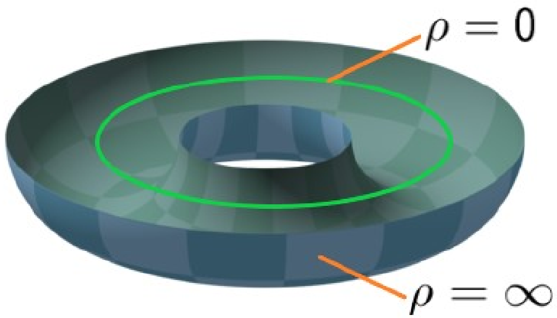
C. Fefferman and C. R. Graham, Ann. Math. Stud. 178, 1 (2011), arXiv:0710.0919 [math.DG]

boundary preserving diffeomorphism

$$V = \sum_{n=1}^{\infty} \epsilon^n V^{[n]} \qquad \langle T_{ij}^{\epsilon} \rangle = \sum_{n=1}^{\infty} \epsilon^n T_{ij}^{[n]}.$$

ensures solution is well-behaved

Thermal AdS3：



Regularity boundary conditions: bulk metric at rho=0 be regular.

$$\int_{T^2} d^2z\, g_{0t\phi}^{FG[1]} = 0. \qquad \int_{T^2} d^2z\, g_{2\phi\phi}^{FG[1]} = 0.$$

Two-point function:

SH, Yi Li, Yun-Ze Li, Yunda Zhang, JHEP 06 (2023) 116

$$T^{[1]}_{zz}(z) = \int d^2w\mathcal{G}(z-w)(2T^{[0]}_{zz}\partial_w - \frac{1}{16\pi G}\partial_w^3)f_{\bar{z}\bar{z}}(w) + \frac{1}{16\pi G}(-\partial_z\partial_{\bar{z}}f_{zz} + 2\partial_z^2 f_{z\bar{z}})(z) + C^{[1]}(z),$$

$$\frac{1}{\pi}\partial_{\bar{z}}G_\tau(z,w) = \delta(z,w) - \frac{1}{\mathrm{Im}\tau}$$

$$\frac{1}{\pi}\partial_z\partial_{\bar{z}}\tilde{G}_\tau(z,w) = \delta(z,w) - \frac{1}{\mathrm{Im}\tau},$$

Regularity at rho=0

$$C^{[1]} = \frac{\pi}{4G\mathrm{Im}\tau}\int_{T^2} d^2z f_{\bar{z}\bar{z}}$$

vary $T^{[1]}_{zz}$ with respect to $f_{\bar{z}\bar{z}}$

$$\langle T(z)T(w)\rangle = \frac{c}{12}[\wp''_\tau(z-w) + 4\pi^2\wp_\tau(z-w) + 8\pi^2\zeta_\tau(\frac{1}{2})]$$

Consistent with:
T. Eguchi and H. Ooguri, Nucl. Phys. B 282, 308 (1987)
SH and Y. Sun，arXiv:2004.07486

Hard to obtain the higher point correlation function by using top-down approach !!!

Bottom-up approach

For bounday torus: $ds^2 = dzd\bar{z}$ $\quad (z,\bar{z}) \sim (z+1, \bar{z}+1) \sim (z+\tau, \bar{z}+\bar{\tau})$

variation $\delta\gamma_{\bar{z}\bar{z}}(z) = \alpha$ and $\delta\gamma_{zz}(z) = \bar{\alpha}$,

$$\begin{aligned} ds^2 &= dzd\bar{z} + \bar{\alpha}dz^2 + \alpha d\bar{z}^2 \\ &= (1+\alpha+\bar{\alpha})d(z+\alpha(\bar{z}-z))d(\bar{z}+\bar{\alpha}(z-\bar{z})) + o(\alpha^2) \end{aligned}$$

Wely transformation $1-\alpha-\bar{\alpha}$ $\quad \phi : z' = z + \alpha(\bar{z}-z), \bar{z}' = \bar{z} + \bar{\alpha}(z-\bar{z})$

$$ds^2 = dz'd\bar{z}' \quad \tau' = \tau + \alpha(\bar{\tau} - \tau)$$

Variations of moduli on the torus "minus" boundary local coordinate transformations are equivalent to variations of the torus metric

Bottom-up approach

SH, Yi Li, Yun-Ze Li, Yunda Zhang, JHEP 06 (2023) 116

1. Global Variation
   $\bar{\alpha} dz^2 + \alpha d\bar{z}^2$
2. Wely Transformation
   $(1 - \alpha - \bar{\alpha})$
3. Diffeomorphism
   $z + \alpha(\bar{z} - z)$

$\simeq$

- Modular Parameter Variation
  $\Delta\tau = \alpha(\bar{\tau} - \tau)$

$$\int d^2z\left(\frac{\delta}{\delta\gamma_{\bar{z}\bar{z}}(z)} - \frac{\delta}{\delta\gamma_{z\bar{z}}(z)}\right) + \mathcal{L}_{(\bar{z}-z)\partial_z} = (\bar{\tau} - \tau)\frac{\partial}{\partial\tau},$$
$$\int d^2z\left(\frac{\delta}{\delta\gamma_{zz}(z)} - \frac{\delta}{\delta\gamma_{z\bar{z}}(z)}\right) + \mathcal{L}_{(z-\bar{z})\partial_{\bar{z}}} = (\tau - \bar{\tau})\frac{\partial}{\partial\bar{\tau}}.$$

Key ingredient to obtain the higher point correlation function !!!

Acting on lower-point functional

SH, Yi Li, Yun-Ze Li, Yunda Zhang, JHEP 06 (2023) 116

$$(\bar{\tau}-\tau)\partial_\tau\langle O\rangle = \mathcal{L}_{(z-\bar{z})\partial_z}\langle O\rangle + \int_{T^2} d^2z\left(\frac{\delta\langle O\rangle}{\delta\gamma_{\bar{z}\bar{z}}(z)} - \frac{\delta\langle O\rangle}{\delta\gamma_{z\bar{z}}(z)}\right)$$

$$T^{[1]}_{zz}(z) = \int d^2w\,\mathcal{G}(z-w)\left(2T^{[0]}_{zz}\partial_w - \frac{1}{16\pi G}\partial_w^3\right)f_{\bar{z}\bar{z}}(w) + \frac{1}{16\pi G}\left(-\partial_z\partial_{\bar{z}}f_{zz} + 2\partial_z^2 f_{z\bar{z}}\right)(z) + C^{[1]}(z),$$

Then the integral constant is determined as

$$\frac{\delta C^{[1]}}{\delta f_{\bar{z}\bar{z}}(z)} = -\frac{2}{\mathrm{Im}\tau}T^{[0]}_{zz} - 2i\frac{\partial}{\partial\tau}T^{[0]}_{zz}$$

Consistent with top-down approach!

Higher point correlations are consistent with field theory.

SH and Y. Sun, arXiv:2004.07486

Holographic torus correlators from thermal AdS_3 saddle

- Two point correlator

$$\langle T_{zz}(z_1)T_{zz}(z_2)\rangle = \frac{1}{32\pi^2 G}\left[\wp''_\tau(z_1-z_2)+4\pi^2\wp_\tau(z_1-z_2)+8\pi^2\zeta_\tau(\frac{1}{2})\right]$$

T. Eguchi and H. Ooguri, Nucl. Phys. B 282, 308 (1987)

- Three-point correlator

$$\langle T_{zz}(z_1)T_{zz}(z_2)T_{zz}(z_3)\rangle = -\frac{1}{64\pi^3 G}\Big[12\wp_\tau(z_1-z_2)\wp_\tau(z_2-z_3)\wp_\tau(z_3-z_1)$$
$$+4\pi^2\Big(\wp_\tau(z_1-z_2)\wp_\tau(z_2-z_3)+\wp_\tau(z_2-z_3)\wp_\tau(z_3-z_1)+\wp_\tau(z_3-z_1)\wp_\tau(z_1-z_2)\Big)$$
$$+(16\pi^2\zeta_\tau(\frac{1}{2})-g_{2,\tau})\Big(\wp_\tau(z_1-z_2)+\wp_\tau(z_2-z_3)+\wp_\tau(z_3-z_1)\Big)\Big]+C_{TTT,\tau}$$

with

$$C_{TTT,\tau}=\frac{1}{320\pi^3 G}\left[-4\left(g_{2,\tau}+60\pi^2\zeta_\tau(\frac{1}{2})\right)\zeta_\tau(\frac{1}{2})-i\pi\partial_\tau g_{2,\tau}+18g_{3,\tau}\right],$$

$$g_{2,\tau}=60\sum_{(m,n)\neq(0,0)}\frac{1}{(m+n\tau)^4},$$

$$g_{3,\tau}=140\sum_{(m,n)\neq(0,0)}\frac{1}{(m+n\tau)^6}$$

SH and Y. Sun, arXiv:2004.07486

The recurrence relation

- Holographic Virasoro Ward identity for $\gamma_{\bar{z}\bar{z}} = F$

$$\partial_{\bar{z}}\langle T_{zz}\rangle - 2\partial_z F\langle T_{zz}\rangle - F\partial_z\langle T_{zz}\rangle + \frac{1}{16\pi G}\partial_z^3 F = 0.$$

- We obtain a differential equation relating higher and lower point correlators by taking the functional derivative with respect to F, we solve the differential equation with integration constants to obtain the recurrence relation

$$\begin{aligned}
&\langle T_{zz}(z)T_{zz}(z_1)\dots T_{zz}(z_n)\rangle = -i\partial_\tau\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle + \frac{1}{32\pi^2 G}\delta_{n,1}\wp''_\tau(z-z_1)\\
&-\frac{1}{2\pi}\sum_{i=1}^{n}\Big[2(\wp_\tau(z-z_i)+2\zeta_\tau(\frac{1}{2}))\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle\\
&+(\zeta_\tau(z-z_i)-2\zeta_\tau(\frac{1}{2})(z-z_i))\partial_{z_i}\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle\Big]
\end{aligned}$$

SH and Y. Sun，arXiv:2004.07486

Higher Dimensional Correlators:

Euclidean AdS5 Planar Black Hole

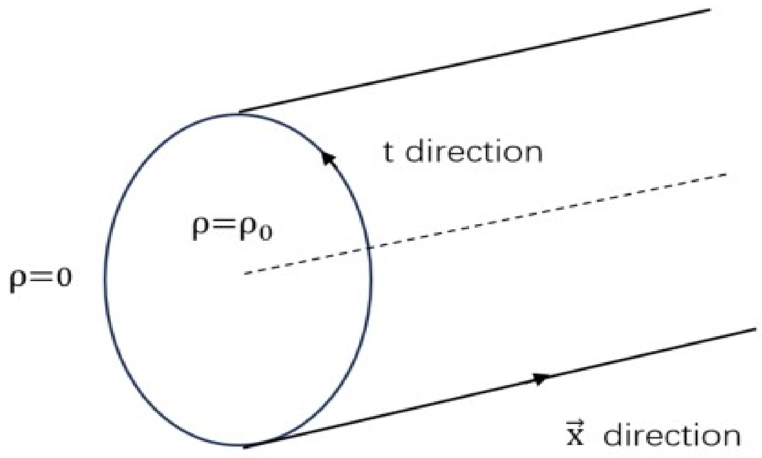
Euclidean AdS5 planar black hole

Thermal states of CFT_4 holographically described by AdS_5 planar black hole

The black hole is a solid cylinder $\mathbb{B}^2 \times \mathbb{R}^3$ with the metric

$$ds^2 = \frac{1}{\rho^2}[(1 - \frac{\rho^4}{\rho_0^4})^{-1} d\rho^2 + (1 - \frac{\rho^4}{\rho_0^4}) dt^2 + d\vec{x}^2]$$

The conformal boundary is at $\rho = 0$, and the horizon is at $\rho = \rho_0$



Set $\rho_0 = 1$

SH, Yi Li, 2308.13518[hep-th]

$$d \star F = 0$$

Gauge Fixing and Boundary Conditions

Gauge fixing: set $A_\rho = 0$ in the region $0 \le \rho < 1$ (excluding the horizon) by a $U(1)$ gauge transformation

$$A = \mathbf{A}_i dx^i \qquad\qquad A + d\Lambda$$

Boundary condition at the horizon: the solution has a regular limit as $\rho \to 1$ after a gauge transformation parametrized by Λ

$$ds^2 \sim d\mathfrak{s}^2 + \mathfrak{s}^2 d(2t)^2 + d\vec{x}^2$$

"Cartesian coordinates"

"cylindrical radial coordinate" $\mathfrak{s} = \frac{1}{2}\cosh^{-1}\frac{1}{\rho^2}$

$$X = \mathfrak{s}\cos 2t$$

$$Y = \mathfrak{s}\sin 2t$$

$$\vec{x} = \vec{x}$$

Gauge Fixing and Boundary Conditions

Components in the "Cartesian coordinates" are regular

$$\lim_{\mathfrak{s}\to 0} A + d\Lambda = A^*_X(\vec{x})dX + A^*_Y(\vec{x})dY + A^*_a(\vec{x})dx^a$$

⇓

$$\lim_{\mathfrak{s}\to 0} \partial_{\mathfrak{s}}\Lambda = A^*_X(\vec{x})\cos 2t + A^*_Y(\vec{x})\sin 2t$$

$$\lim_{\mathfrak{s}\to 0} \frac{\mathbf{A}_t + \partial_t\Lambda}{\mathfrak{s}} = -2A^*_X(\vec{x})\sin 2t + 2A^*_Y(\vec{x})\cos 2t$$

$$\lim_{\mathfrak{s}\to 0} \mathbf{A}_a + \partial_a\Lambda = A^*_a(\vec{x})$$

⇓

$\mathbf{A}_a$ regular as $\rho \to 1$

$$\int_0^\pi dt\mathbf{A}_t|_{\rho=1} = 0$$

Equations of Motion

The Maxwell equation

$$d \star F = 0$$

Work with Fourier modes $\tilde{\mathbf{A}}_i$ with Matsubara frequency $\omega = 2m$, $m \in \mathbb{Z}$ and spatial momentum $\vec{p}$ rotated to the x^1 direction for simplicity. Also use the substitution $z = \rho^2$

Transverse channel

$$(\partial_z^2 - \frac{2z}{1-z^2}\partial_z - \frac{\omega^2 + p^2(1-z^2)}{4z(1-z^2)^2})\tilde{\mathbf{A}}_2 = 0$$

By the substitution $\tilde{\mathbf{A}}_2(z) = (1-z^2)^{-\frac{1}{2}} w(z)$, we get a Heun equation

$$\left(\partial_z^2 + \frac{\frac{1}{4}-(\frac{1}{2})^2}{z^2} + \frac{\frac{1}{4}-(\frac{m}{2})^2}{(z-1)^2} + \frac{\frac{1}{4}-(\frac{m}{2}i)^2}{(z+1)^2} + \frac{p^2+4m^2-2}{8z(z-1)} - \frac{p^2+4m^2+2}{8z(z+1)}\right) w(z) = 0,$$

$$t = -1,\ a_0 = \frac{1}{2},\ a_1 = \frac{|m|}{2},\ a_t = \frac{m}{2}i,\ a_\infty = \frac{1}{2},\ u = -\frac{p^2+4m^2+2}{8}$$

By the boundary condition $\mathbf{A}_2$ regular as $z \to 1$, we must have

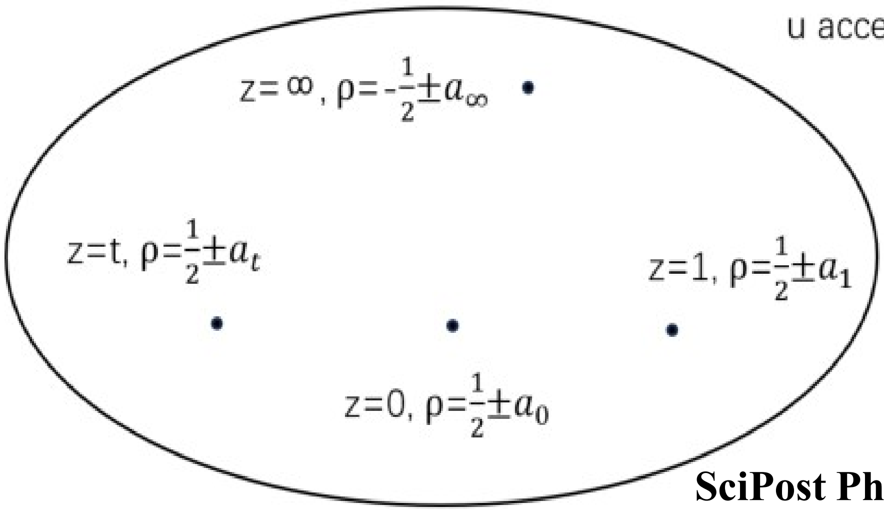
$$\tilde{\mathbf{A}}_2(z) \sim (1-z^2)^{-\frac{1}{2}} w_+^{(1)}(z)$$

How to connect with UV (z=0) and IR (z=1) ???

Heun equation ↔ semiclassical Liouville CFT ↔ SUSY gauge theory

$$\Big(\partial_z^2 + \frac{\frac{1}{4} - a_0^2}{z^2} + \frac{\frac{1}{4} - a_1^2}{(z-1)^2} + \frac{\frac{1}{4} - a_t^2}{(z-t)^2} - \frac{\frac{1}{2} - a_1^2 - a_t^2 - a_0^2 + a_\infty^2 + u}{z(z-1)} + \frac{u}{z(z-t)}\Big) w(z) = 0$$

F: Nekrasov-Shatashvili function



u accessory parameter

Commun. Math. Phys. 397 (2023) 635-727

$$w_\theta^{(1)}(z) = \sum_{\theta'=\pm} \mathcal{M}_{\theta\theta'}(a_1, a_0; a) e^{(\frac{\theta}{2}\partial_{a_1} - \frac{\theta'}{2}\partial_{a_0}) F\left(\begin{smallmatrix} a_t & & a_1 \\ a_\infty & a & a_0 \end{smallmatrix}; \frac{1}{t}\right)} w_{\theta'}^{(0)}(z),$$

$$\mathcal{M}_{\theta\theta'}(a_1, a_0; a) = \frac{\Gamma(-2\theta' a_0)\Gamma(1 + 2\theta a_1)}{\Gamma(\frac{1}{2} + \theta a_1 - \theta' a_0 + a)\Gamma(\frac{1}{2} + \theta a_1 - \theta' a_0 - a)}$$

and a is to be implicitly determined from the relation

SciPost Phys. 14, 116 (2023)

$$u = -\frac{1}{4} - a^2 + a_t^2 + a_0^2 + t\partial_t F$$

Connect UV and IR

Transverse channel

$$(\partial_z^2 - \frac{2z}{1-z^2}\partial_z - \frac{\omega^2 + p^2(1-z^2)}{4z(1-z^2)^2})\tilde{\mathbf{A}}_2 = 0$$

$$\tilde{\mathbf{A}}_2(\omega = 2m, p, z) = \tilde{\mathcal{A}}_2(\omega, p)(1-z^2)^{-\frac{1}{2}}\Big[w_-^{(0)} + \frac{p^2 + 4m^2}{4}(-2\psi(1) - 1 + \frac{1}{2}\sum_{\theta,\sigma=\pm}\psi(\theta\frac{m}{2} + \sigma a) - \frac{1}{2}\partial_{a_0}^2 F - \frac{2}{p^2+4m^2}(1 + 2\partial_t\partial_{a_0}F))w_+^{(0)}\Big]$$

Resulting two point function

$$\langle \tilde{J}_t(\omega, p)\tilde{J}_t(-\omega, -p)\rangle = \frac{p^2}{2}\mathcal{C}_2(\omega, p)$$

$$\mathcal{C}_2(\omega = \frac{2m}{\rho_0}, p) = (2\psi(1) + 1 - \frac{1}{2}\sum_{\theta,\sigma=\pm}\psi(\frac{1}{2} + \theta\frac{m}{2} + \sigma a) + \frac{1}{2}\partial_{a_0}^2 F)\Big|_{t=-1, a_0=0, a_1=\frac{|m|}{2}, a_t=\frac{m}{2}i, a_\infty=1, u=-\frac{\rho_0^2 p^2 + 4m^2 + 6}{8}}$$

JJ & TT refer to

SH, Yi Li, 2308.13518[hep-th]

How about energy momentum tensor?

The linearized Einstein equation

$$\frac{1}{2}(\nabla^\lambda\nabla_\mu\delta g_{\lambda\nu}+\nabla^\lambda\nabla_\nu\delta g_{\lambda\mu}-\nabla^\lambda\nabla_\lambda\delta g_{\mu\nu}-\nabla_\mu\nabla_\nu\delta g^\lambda_\lambda)+4\delta g_{\mu\nu}=0$$

IR boundary condition? $\mathcal{L}_V(ds^2)+\delta ds^2$

Gauge fixing: make the solid cylinder coordinates $\rho, t, \vec{x}$ the Fefferman-Graham coordinates of the perturbed bulk metric in the region $0 \le \rho < 1$ by a diffeomorphism

$$\delta ds^2 = \delta \mathbf{g}_{ij} dx^i dx^j$$

$$\delta \mathbf{g}_{ab} \text{ regular as } \rho \to 1$$

IR B.C.:

$$\int_0^\pi dt \delta \mathbf{g}_{ta}|_{\rho=1} = 0$$

In the scalar and shear channel we find

$$\langle \tilde{T}_{23}(\omega = \frac{2m}{\rho_0}, p)\tilde{T}_{23}(-\omega, -p)\rangle = \frac{1}{4\pi G}\frac{(p^2+\omega^2)^2}{32}\mathcal{C}_3(\omega = \frac{2m}{\rho_0}, p)$$

$$\langle \tilde{T}_{t2}(\omega = \frac{2m}{\rho_0}, p)\tilde{T}_{t2}(-\omega, -p)\rangle = \frac{1}{4\pi G}\frac{p^2+\omega^2}{32}p^2\mathcal{C}_4(\omega = \frac{2m}{\rho_0}, p)$$

$$\langle \tilde{T}_{t2}(\omega = \frac{2m}{\rho_0}, p)\tilde{T}_{12}(-\omega, -p)\rangle = -\frac{1}{4\pi G}\frac{p^2+\omega^2}{32}\omega p\mathcal{C}_4(\omega = \frac{2m}{\rho_0}, p)$$

$$\langle \tilde{T}_{12}(\omega = \frac{2m}{\rho_0}, p)\tilde{T}_{12}(-\omega, -p)\rangle = \frac{1}{4\pi G}\frac{p^2+\omega^2}{32}\omega^2\mathcal{C}_4(\omega = \frac{2m}{\rho_0}, p),$$

$$\mathcal{C}_3(\omega = \frac{2m}{\rho_0}, p) = \Big[2\psi(1) + \frac{5}{2} - \frac{1}{2}\sum_{\theta,\sigma=\pm}\psi(-\frac{1}{2} + \theta\frac{m}{2} + \sigma a)$$

$$+ \frac{1}{2}\partial_{a_0}^2 F - \frac{16}{(\rho_0^2 p^2 + 4m^2)^2}(4a^2 - 2a^2 m^2 + \frac{1}{4}m^4 + 4(\partial_t F)^2 + (-8a^2 + 2m^2)\partial_t F$$

$$- 4\partial_t F\partial_t\partial_{a_0}F + (-2 + 4a^2 - m^2)\partial_t\partial_{a_0}F)\Big]\Big|_{t=-1, a_0=1, a_1=\frac{|m|}{2}, a_t=\frac{m}{2}i, a_\infty=0, u=-\frac{\rho_0^2 p^2+4m^2-2}{8}}$$

$$\mathcal{C}_4(\omega = \frac{2m}{\rho_0}, p) = \Big(2\psi(1) + 1 - \frac{1}{2}\sum_{\theta,\sigma=\pm}\psi(\theta\frac{m}{2} + \sigma a)$$

$$+ \frac{1}{2}\partial_{a_0}^2 F + \frac{2}{\rho_0^2 p^2 + 4m^2}(1 + 2\partial_t\partial_{a_0}F)\Big)\Big|_{t=-1, a_0=\frac{1}{2}, a_1=\frac{|m|}{2}, a_t=\frac{m}{2}i, a_\infty=\frac{3}{2}, u=-\frac{\rho_0^2 p^2+4m^2+10}{8}}$$

We can reduce the sound channel to first-order equations of variables $\tilde{\mathbf{h}}_{tt}, \tilde{\mathbf{h}}_{11}, \frac{\tilde{\mathbf{h}}_{22}+\tilde{\mathbf{h}}_{33}}{2}, \tilde{\mathbf{h}}_{t1}, \partial_z \tilde{\mathbf{h}}_{t1}$, and by the substitution

$$\begin{pmatrix} \tilde{\mathbf{h}}_{tt} \\ \tilde{\mathbf{h}}_{11} \\ \frac{\tilde{\mathbf{h}}_{22}+\tilde{\mathbf{h}}_{33}}{2} \\ \tilde{\mathbf{h}}_{t1} \\ \partial_z \tilde{\mathbf{h}}_{t1} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{3}(1-z^2)^2 & \frac{2}{3}z(1-z^2) & 0 & 0 \\ -z^2 & 1-z^2 & \frac{2}{3}z & 0 & 0 \\ \frac{1}{2}z^2 & 0 & -\frac{1}{3}z & 0 & 0 \\ 0 & 0 & 0 & 1-z^2 & 0 \\ 0 & 0 & 0 & 0 & z \end{pmatrix} H$$

We don't know connection relation of local solutions of this Fuchsian system.

$\mathbf{h}_{ij} = \rho^2 \delta \mathbf{g}_{ij}$

$$\partial_z H = \left(\frac{M_0}{z} + \frac{M_1}{z-1} + \frac{M_{-1}}{z+1}\right) H$$

Unsolved sound channel in a analytical way

Summary

- □Proposed prescription to study Holographic torus stress tensor correlator， which are consistent with CFTs data.
- □Offer a precise a check AdS3/CFT2.
- □JJ and TT in Thermal CFT4 by holographical approach.
- □Other topologies (higher genus, cross cap), Mixing operators, etc.

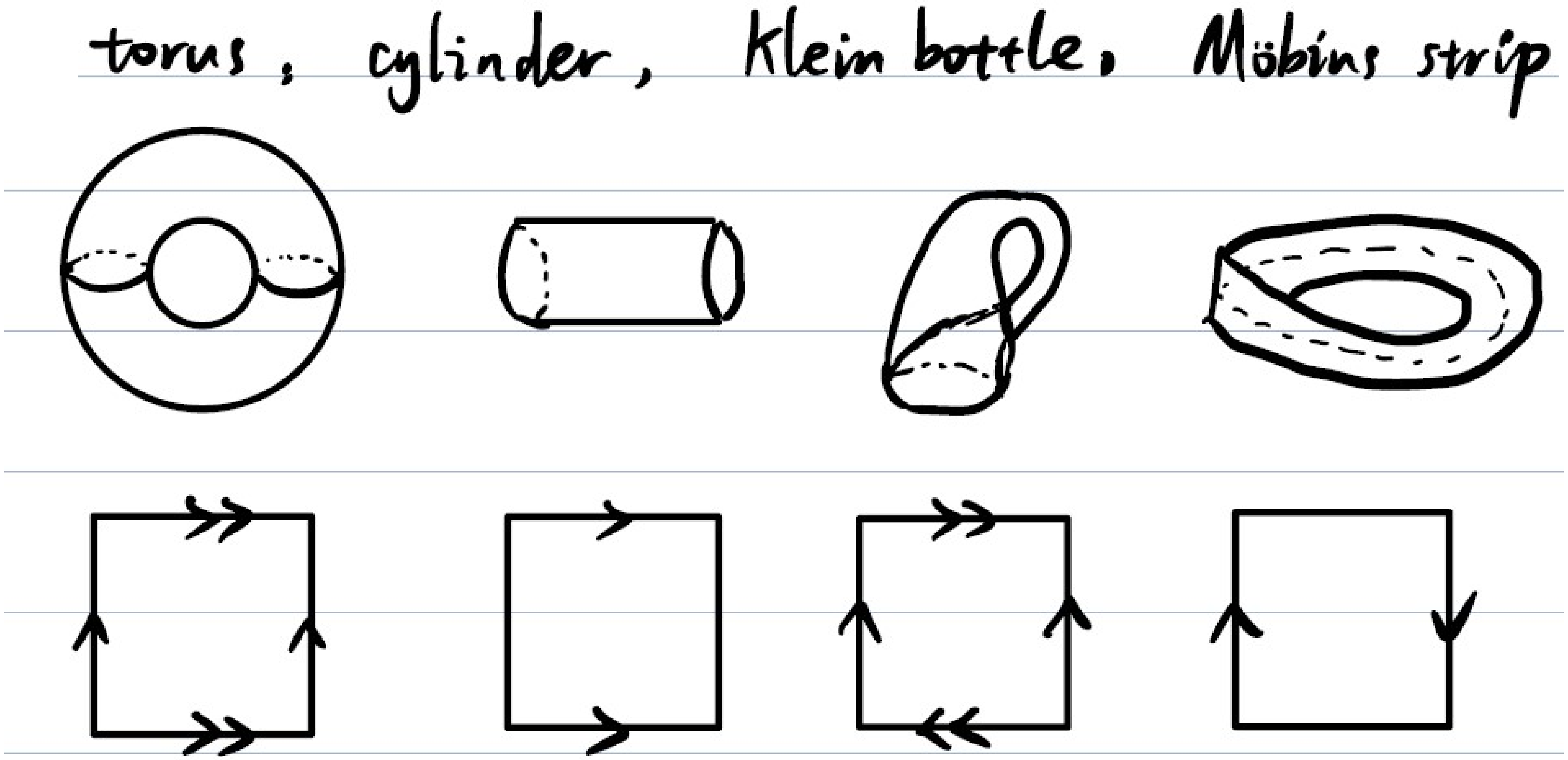
AdS3/CFT2

$$S_E = -\frac{1}{16\pi G}\int \sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G}\int \sqrt{\gamma}K + \frac{1}{8\pi G l}\int \sqrt{\gamma}.$$

Holography for generic 2D Riemann surface

Euclidean AdS3 by discrete identifications

Kirill Krasnov, 0005106



Thermal AdS3

Partition functions, correlation functions, etc.

Thanks for your attention