3d N=2 from M-theory on CY4 and IIB brane box

2312.17082 w/ Marwan Najjar and Jiahua Tian

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Conformal Field Theories

• Classification of CFTs is an interesting but hard question

(1) 2d CFT: Virasoro algebra provides strong constraints, rational CFT

(2) For higher dimensional CFTs (e. g. $d \geq 3$), the full operator spectrum, OPEs ... are not known

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• In the SCFT cases, partial classification comes from geometric constructions

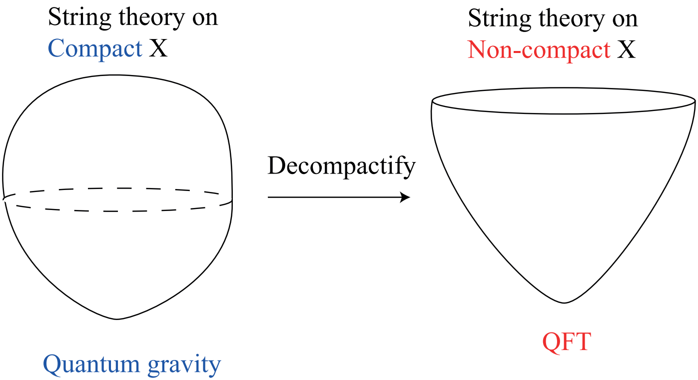
(1) Superstring/M/F-theory on a non-compact space

(2) Dimensional reduction of 6d SCFTs on a compact space

(3) Worldvolume theory of brane objects in superstring/M/F-theory (AdS/CFT)

Geometric Engineering

- Superstring/M/F-theory on a non-compact space, decouple gravity



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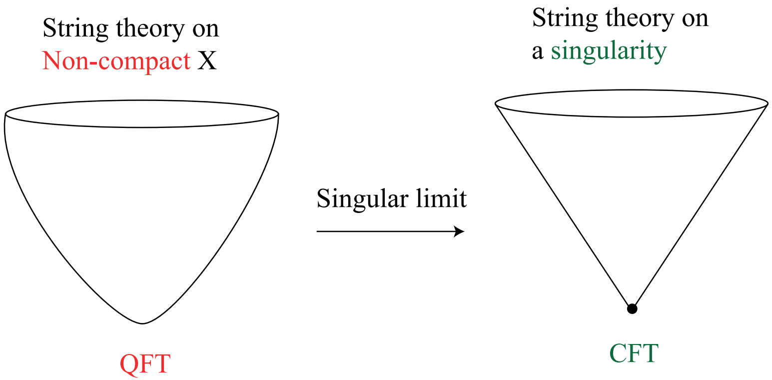
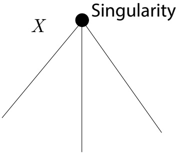


- The CFT degree of freedoms are localized around the origin

5d SCFTs

(1) 11d M-theory on canonical threefold singularity



(Xie, Yau 15')(Apruzzi, Bhardwaj, Closset, Collinucci, De Marco, Del Zotto, Eckhard, Giacomelli, Heckman, Hubner, Jefferson, Katz, Kim, Lawrie, Lin, Morrison, Mu, Sangiovanni, Saxena, Schafer-Nameki, Tarazi, Tian, Vafa, Valandro, YNW, Zafrir, Zhang. . .).

(2) Brane web constructions in IIB superstring

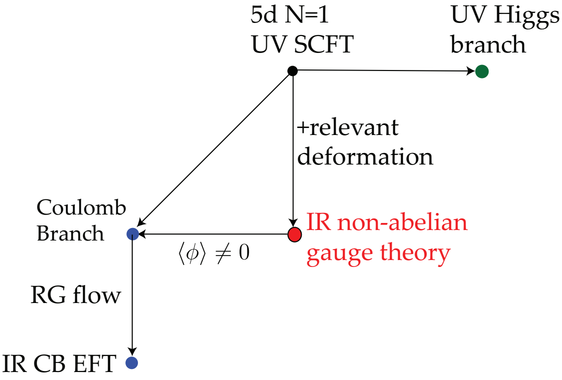
(Akhond, van Beest, Bergman, Bourget, Cabrera, Carta, Dwivedi, Eckhard, Ferlito, Giacomelli, Grimminger, Hanany, Hayashi, He, Kalveks, Kim, Kim, Kim, Lee, Mekareeya, Ohmori, Schafer-Nameki, Shimizu, Sperling, Tachikawa, Taki, Uhlemann, Yagi, Zafrir, Zajac, Zoccarato, Zhong . . .).

Deformations of SCFTs

- Directly study the operator spectrum/ OPE etc. Hard!

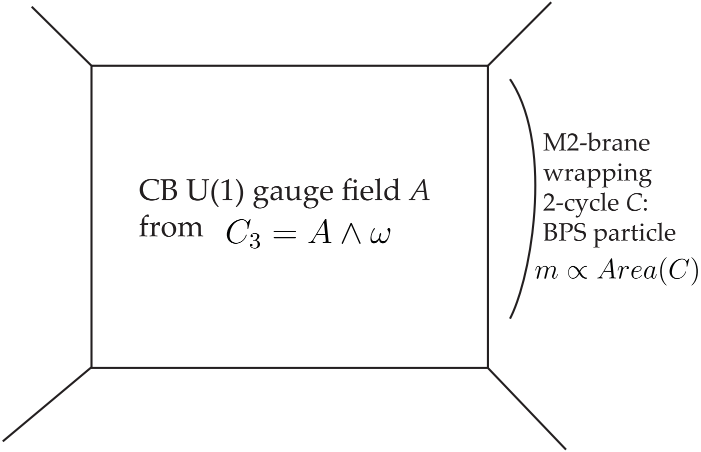
(1) Coulomb branch: scalars ϕ^i in the vector multiplets have non-zero vev.

(2) Higgs branch: scalars in the hypermultiplets have non-zero vev.



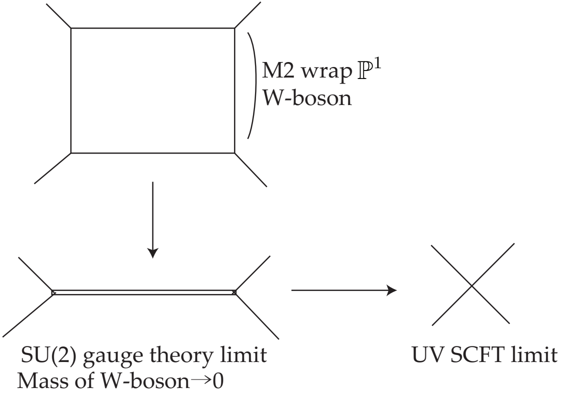
5d CB and M-theory on resolved CY3

- M-theory on a resolved CY3 $\to$ CB physics, $U(1)^r +$ massive charged matter



Non-abelian and SCFT limit

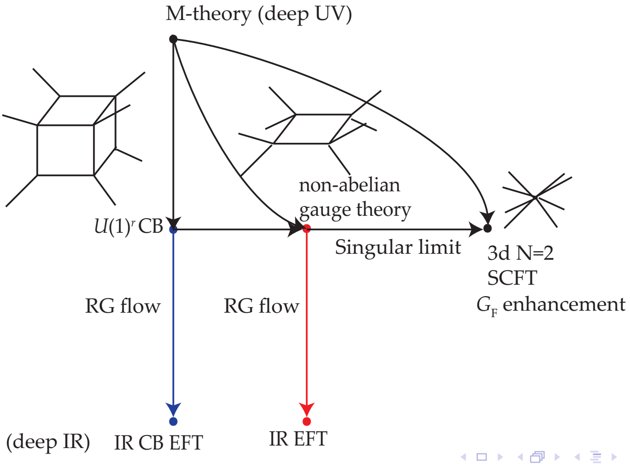
- Non-abelian gauge theory description exists when the CY3 has a $\mathbb{P}^1$-fibration structure, e. g. the local $\mathbb{P}^1 \times \mathbb{P}^1$ gives 5d $SU(2)_0$ theory in the non-abelian limit.



- Similar picture in the IIB (p, q) 5-brane web constructions!

What about 3d $\mathcal{N} = 2$?

- Naturally, M-theory on local CY4 singularity $\rightarrow$ 3d $\mathcal{N} = 2$ SCFT, because of the absence of geometric scale
- Build up geometric dictionary, investigate 3d $\mathcal{N} = 2$ physics from M-theory on CY4 (Najjar, Tian, YNW 23').



3d $\mathcal{N} = 2$ basics

- Vector multiplet: A_μ, λ, $\tilde{\lambda}$, real scalar σ
- Chiral multiplet: ϕ, ψ (same d.o.f. as 4d $\mathcal{N} = 1$ chiral multiplet)
- Anti-chiral multiplet: $\tilde{\phi}$, $\tilde{\psi}$, in the conjugate rep. of chiral multiplet

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- Integrate out "chiral" fermions $\rightarrow$ IR effective Chern-Simons terms

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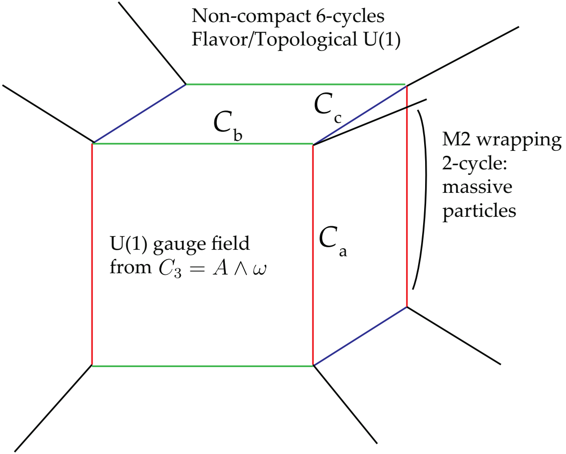
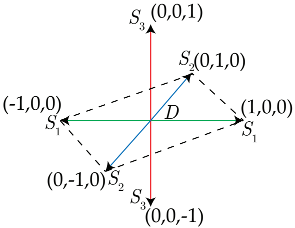
- Lots of IR dualities (Aharony, Hanany, Intriligator, Seiberg, Strassler 97')....

Resolved CY4 (CB)

- M-theory on resolved local CY4 X_4, e. g. local $D = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \rightarrow$ 3d $\mathcal{N} = 2$ U(1) gauge theory+ massive matter fields





Uncharged sector

$$C_3 = \sum_{i=1}^{r} A_i \wedge \omega_i^{(1,1)} + \sum_{\alpha=1}^{f} B_\alpha \wedge \omega_\alpha^{(1,1),F} \tag{2}$$

(1) Dynamical gauge fields A_i

- Gauge rank $r = b_6(X_4)$
- $\omega_i^{(1,1)}$ Poincaré dual to compact divisor (6-cycle) D_i

(2) Background gauge fields B_α for geometric flavor symmetries

- Flavor rank $f = b_2(X_4) - b_6(X_4)$
- $\omega_\alpha^{(1,1),F}$ Poincaré dual to non-compact divisor (6-cycle) S_α

Kähler form and CB parameters

- To compute volume of various cycles in X_4, we need the Kähler (1,1)-form

$$J(X_4) = \sum_{i=1}^{r} a_i \omega_i^{(1,1)} + \sum_{\alpha=1}^{f} b_\alpha \omega_\alpha^{(1,1),F} \,. \tag{3}$$

- It is Poincaré dual to

$$J^c(X_4) = \sum_{i=1}^{r} a_i D_i + \sum_{\alpha=1}^{f} b_\alpha S_\alpha \,. \tag{4}$$

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(1) $a_i = \langle \sigma_i \rangle$: Coulomb branch parameters
(2) $b_\alpha = \langle \xi_\alpha \rangle$: vev for the real scalar in the background gauge field vector multiplet; real mass for flavor symmetry

- Volume of 2-cycles C, 4-cycles $\mathcal{S}$ and 6-cycles D are computed as

$$V_C = \int_C J \ , \ V_{\mathcal{S}} = \frac{1}{2}\int_{\mathcal{S}} J \wedge J \ , \ V_D = \frac{1}{6}\int_D J \wedge J \wedge J \, . \tag{5}$$

Gauge coupling

- $U(1)$ Gauge coupling $1/g^2$ given by what?
- Reduce the kinetic term in 11D SUGRA action on X_4 (leading term)

$$\begin{aligned} \frac{1}{2}\int_{\mathbb{R}^{1,2}\times X_4} G_4\wedge\star G_4 &= \frac{1}{2}\int_{\mathbb{R}^{1,2}} F\wedge\star F\int_{X_4}\omega^{(1,1)}\wedge\star\omega^{(1,1)}+(\dots) \\ &= \frac{1}{2g^2}\int_{\mathbb{R}^{1,2}} F\wedge\star F+(\dots) \end{aligned} \tag{6}$$

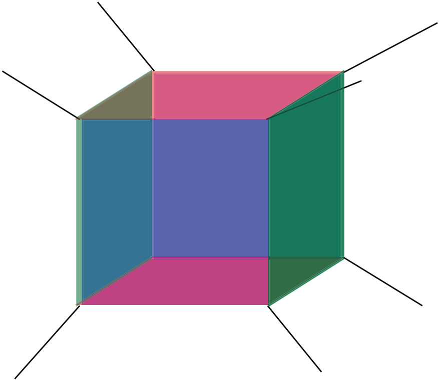
where

$$\begin{aligned} \frac{1}{g^2} &= \int_{X_4}\omega^{(1,1)}\wedge\star\omega^{(1,1)} \\ &= -\frac{1}{2}\int_{X_4}\omega^{(1,1)}\wedge\omega^{(1,1)}\wedge J\wedge J \\ &= -\frac{1}{2}\int_{D\cdot D} J\wedge J \\ &= \mathrm{Vol}(-K_D)\,. \end{aligned} \tag{7}$$

- Volume of the anti-canonical divisor of D!

Gauge coupling

- In the local $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ case, the compact divisor $D = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ is toric, $\frac{1}{g^2}$ given by the sum of the volumes of all 4-cycles (walls)!



M2-brane wrapping modes

- BPS states from M2-brane wrapping $\mathbb{P}^1$ curves C. Hint from 4d/3d F/M-duality (Beasley, Heckman, Vafa 08')(Intriligator, Jockers, Katz, Morrison, Plesser 12')(Jockers, Katz, Morrison, Plesser 16'). We first assume no G_4 flux

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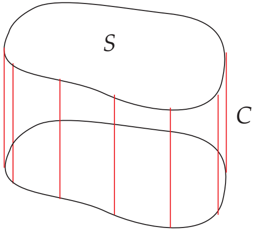
(1) $N_{C|X_4} = \mathcal{O} \oplus \mathcal{O} \oplus \mathcal{O}(-2)$, C is locally a $\mathbb{P}^1$ fiber, moduli space is a 4-cycle $\mathcal{S}$.



- Adiabatically, the zero modes on C is the twisted reduction of 7d $\mathcal{N} = 1$ vector multiplet on $\mathcal{S}$
- 1 vector multiplet + $(h^{0,1} + h^{0,2})$ vector-like pairs of chiral multiplets

M2-brane wrapping modes

(2) $N_{C|X_4} = \mathcal{O} \oplus \mathcal{O}(-1) \oplus \mathcal{O}(-1)$, C is locally a $\mathbb{P}^1$ fiber, moduli space is a Riemann surface Σ.

• The zero modes on C is the twisted reduction of 5d $\mathcal{N} = 1$ vector multiplet on Σ

• BPS states come from zero modes of Dirac operators on $\Sigma \rightarrow$ vector-like pairs chiral multiplets.

• In particular, when $\Sigma = \mathbb{P}^1$, there is no zero mode and thus no BPS particles.

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• In general: mass of the BPS particle $m \propto Area(C)$

• Charge under Cartan: $q = C \cdot D$

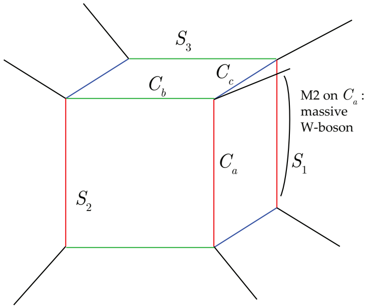
• Charge under flavor Cartan $q_i^F = C \cdot F_i$

Local $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

- Denote the non-compact divisors to be S_1, S_2, S_3, compact divisor is D

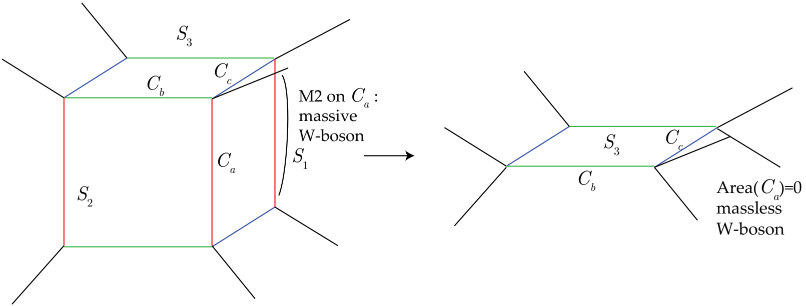
$$C_a = D \cdot S_2 \cdot S_3 \ , \ \ C_b = D \cdot S_1 \cdot S_3 \ , \ \ C_c = D \cdot S_1 \cdot S_2 \tag{8}$$

- C_a, C_b, C_c all have normal bundle $\mathcal{O} \oplus \mathcal{O} \oplus \mathcal{O}(-2)$, moduli space $\mathcal{S} = \mathbb{P}^1 \times \mathbb{P}^1$
- M2-brane wrapping mode: 1 vector multiplet
- $U(1)$ gauge charge $C_a \cdot D = C_b \cdot D = C_c \cdot D = -2$, hence one can choose C_a, C_b or C_c as gauge W-boson.



SU(2) limit

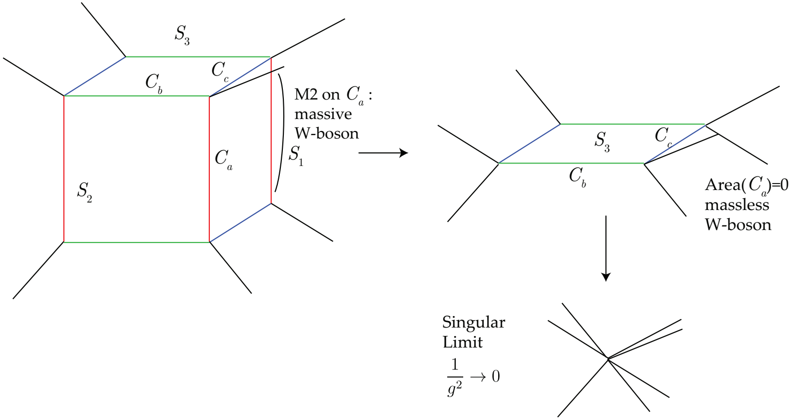
- In the limit of e. g. $Area(C_a) \to 0$, M2-brane wrapping C_a becomes massless W-boson.



- $1/g^2 \sim$Vol(S)
- $SU(2)$ gauge theory+massive charged particle from M2-brane wrapping C_b and C_c
- Interpreted as disorder operators! (Dyonic instanton in 5d $SU(2)_0$ theory on S^2)

SCFT limit

- Singular limit: all compact cycles shrink to zero volume, $1/g^2 \to 0$
- Absence of scale parameter $\to$ SCFT! $W = 0$



Shrinkability condition

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(1) At the singular limit when all compact cycles of X_4 shrinks to a point, still have $1/g_\alpha^2 \to \infty$ for all non-compact divisors S_α ($U(1)^f$ flavor symmetry still persists).
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(2) Exists strongly coupled limit $1/g_i^2 \to 0$ for all $U(1)_i$ gauge groups only when the 4-cycles on all compact divisors D_i in X_4 shrink to zero volume.
(Counter example: 3d $\mathcal{N} = 4$ models such as local T^2)

Flavor symmetry enhancement

- In the singular limit of X_4, 3d $\mathcal{N} = 2$ SCFT with non-abelian flavor symmetry enhancement G_F
- Read off from the CB picture from M-theory on resolved CY4

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• Identify flavor W-bosons as M2 wrapping C_i.

(1) Vector multiplet: $N_{C_i|X_4} = \mathcal{O}\oplus\mathcal{O}\oplus\mathcal{O}(-2)$

(2) Charge under $U(1)^f$ forming the Cartan matrix of G_F

(3) Neutral under $U(1)^r$ gauge symmetry

Flavor symmetry enhancement

- In the example of local $(\mathbb{P}^1)^3$, flavor Cartans

$$F_1 = S_1 - S_2 \ , \ F_2 = S_2 - S_3 \, . \tag{9}$$

- Flavor W-bosons

$$C_1 = D \cdot (S_1 - S_2) \cdot S_3 \ , \ C_2 = D \cdot (S_2 - S_3) \cdot S_1 \, . \tag{10}$$

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$$\begin{array}{c|cc} & F_1 & F_2 \\ \hline C_1 & -2 & 1 \\ C_2 & 1 & -2 \end{array} \quad , \quad \text{Exactly the Cartan matrix for } SU(3)! \tag{11}$$

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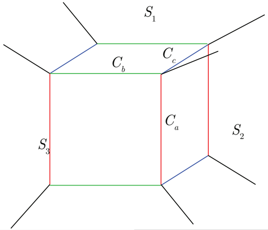
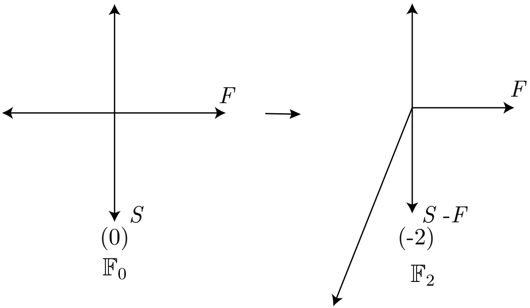
- C_a, C_b and C_c form the **3** rep. of $SU(3)$!



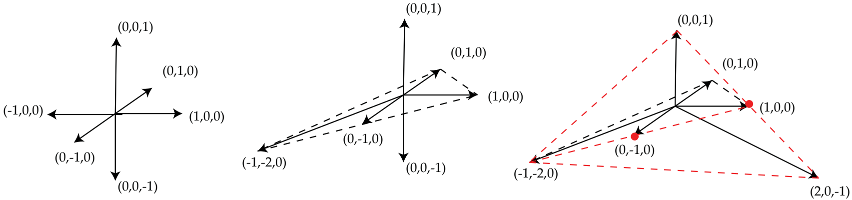
Flavor symmetry enhancement

- Flavor W-boson being non-effective?
- Similar to the 5d case, local $\mathbb{F}_0 \approx$ local $\mathbb{F}_2$ (Seiberg rank-1 E_1 theory with $G_F = SU(2)$)
- Deformation $\mathbb{F}_0 \to \mathbb{F}_2$ gives the same SCFT!
- Flavor W-boson is only an effective curve on $\mathbb{F}_2$



Flavor symmetry enhancement

- CY4 case, toric diagram from local $(\mathbb{P}^1)^3$:

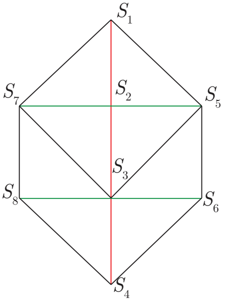


- After the deformation, see the $SU(3)$ flavor symmetry explicitly

Flavor symmetry duality

• Sometimes, one can assign different sets of flavor W-bosons $\rightarrow$ Different non-abelian flavor symmetry enhancements

• Consider a 2d facet of a 3d toric diagram, with two $\mathbb{P}^1$-fibration structures



• $SU(3) \leftrightarrow SU(2)^2$ flavor symmetry duality! Phenomenon is not present in 5d

1-form symmetry

- 1-form global symmetry symmetry acting on Wilson loops of gauge theory (Gaiotto, Kapustin, Seiberg, Willett 14')
- Pure d-dim. $U(1)$ Maxwell theory has a $U(1)$ 1-form symmetry

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- Pure d-dim. $U(1)$ Maxwell theory has a $U(1)$ 1-form symmetry
- $U(1)^r$ gauge theory w/ charged matter ϕ_i with charge q_{ij} under $U(1)_j$, matter breaks the $U(1)^r$ 1-form symmetry to a subgroup Γ
- Compute Smith Normal Form D

$$D = \begin{pmatrix} l_1 & 0 & \dots & 0 \\ 0 & l_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & l_r \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} = A\{q_{ij}\}B \tag{12}$$

$$\Gamma = \bigoplus_{i=1}^{r} (\mathbb{Z}/l_i\mathbb{Z}) \tag{13}$$

1-form symmetry

- On the resolved X_4 CB,

(1) $U(1)^r$ gauge fields from compact divisors D_j

(2) Charged particles from M2-brane wrapping 2-cycles C_i

- $q_{ij} = C_i \cdot D_j$, compute SNF, get 1-form symmetry

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1-form symmetry

- On the resolved X_4 CB,

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- In the toric CY4 case, equivalent computation using SNF of list of toric rays (Morrison, Schafer-Nameki, Willett 19')

G_4 flux

- For M-theory/F-theory on CY4, (free) G_4 flux is usually a crucial ingredient

$$G_4 + \frac{1}{2}c_2(X_4) \in H^4(X_4, \mathbb{Z}) \tag{14}$$

- In the non-compact CY4 case, G_4 should have compact support (dual to a compact 4-cycle)(Gukov, Vafa, Witten, 99')

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(1) Induce non-zero chirality of matter fields

- Integrating out chiral matter $\rightarrow$ deep IR Chern-Simons term

$$S_{CS} = \frac{1}{4\pi} \int \sum_{i,j=1}^{r} k_{ij} A_i \wedge F_j \,. \tag{15}$$

$$\begin{aligned} k_{ij} &= \int_{X_4} G_4 \wedge \omega_i^{(1,1)} \wedge \omega_j^{(1,1)} \\ &= G_4^c \cdot D_i \cdot D_j \,. \end{aligned} \tag{16}$$

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(2) GVW superpotential, D-term superpotential

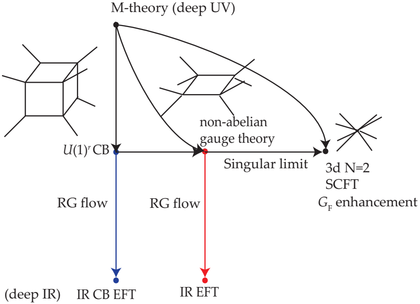
$$W_{GVW} = \int_{X_4} G_4 \wedge \Omega_4 \;,\; W_D = \int_{X_4} G_4 \wedge J \wedge J \,. \tag{17}$$

G_4 flux

- On the resolved X_4 (CB)

(1) $G_4 = 0 \rightarrow$ no CS term in the deep IR

(2) $G_4 \neq 0 \rightarrow$ CS term in the deep IR



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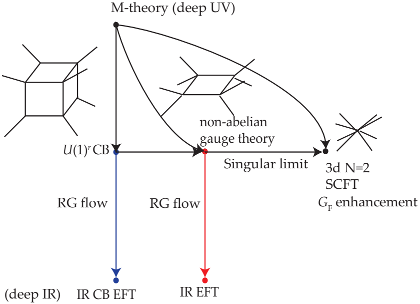

- Adding G_4 obstructs the singular limit of X_4! Because G_4 cannot pass through shrinking 4-cycles
- To have the SCFT description at singular limit, $G_4 = 0$

Superpotential

• A detailed computation of superpotential W is still unknown, several sources

(1) Euclidean M5 brane wrapping compact 6-cycle D with $h^{0,1}(D) = h^{0,2}(D) = h^{0,3}(D) = 0$ (Witten, 96')

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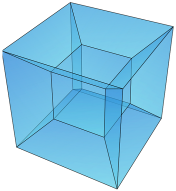
(2) Euclean M2 brane wrapping rigid 3-cycles, absent in toric CY4.

(3) GVW superpotential w/ G_4 flux

- A detailed calculation of W in the future?

Brane web picture in IIB

- In the case of toric CY4, a dual brane web description in IIB! (Leung, Vafa 97')



- First consider M-theory on T^3 $(8, 9, 10)$ directions
- The toric CY4 is equivalent to the system of $(6+1)$-dim. KK7M monopoles

	0	1	2	3	4	5	6	7	S^1_8	S^1_9	S^1_{10}
KK7M$^{(10)}$	✓	✓	✓	•	•	•	✓	✓	✓	✓	TN
KK7M$^{(9)}$	✓	✓	✓	•	•	✓	•	✓	✓	TN	✓
KK7M$^{(8)}$	✓	✓	✓	•	•	✓	✓	•	TN	✓	✓

Brane web picture in IIB

- M-theory on $S^1_{10} \to$ IIA

	0	1	2	3	4	5	6	7	S^1_8	S^1_9
$D_6^{(1,0,0)}$	✓	✓	✓	•	•	•	✓	✓	✓	✓
$KK6A^{(0,1,0)}$	✓	✓	✓	•	•	✓	•	✓	✓	TN
$KK6A^{(0,0,1)}$	✓	✓	✓	•	•	✓	✓	•	TN	✓

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- T-duality along $S^1_9 \to$ IIB on $\widetilde{S}^1_9$

	0	1	2	3	4	5	6	7	S^1_8	$\widetilde{S}^1_9$
$D_5^{(1,0,0)}$	✓	✓	✓	•	•	•	✓	✓	✓	•
$NS_5^{(0,1,0)}$	✓	✓	✓	•	•	✓	•	✓	✓	•
$KK6B^{(0,0,1)}$	✓	✓	✓	•	•	✓	✓	•	TN	✓

Brane web picture in IIB

- M2-brane wrapping $\mathbb{P}^1$: C_a, C_b, C_c:

	0	1	2	3	4	5	6	7	S^1_8	S^1_9	S^1_{10}
$M2^{(1,0,0)}$	✓	•	•	•	•	✓	•	•	•	•	✓
$M2^{(0,1,0)}$	✓	•	•	•	•	•	✓	•	•	✓	•
$M2^{(0,0,1)}$	✓	•	•	•	•	•	•	✓	✓	•	•

- In IIB description

	0	1	2	3	4	5	6	7	S^1_8	$\widetilde{S}^1_9$
$F_1^{(1,0,0)}$	✓	•	•	•	•	✓	•	•	•	•
$D_1^{(0,1,0)}$	✓	•	•	•	•	•	✓	•	•	•
$D_3^{(0,0,1)}$	✓	•	•	•	•	•	•	✓	✓	✓

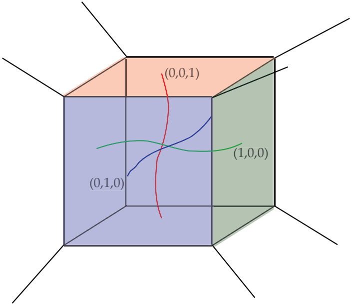
Brane web picture in IIB

- Can be viewed as a web of (p,q,r) 4-branes in 8d SUGRA (remove 8, 9 directions)! (Leung, Vafa 97')(Lu, Roy 98')
- (p,q,r) transforms under $SL(3,\mathbb{Z})$ (part of 8d U-duality)

	0	1	2	3	4	5	6	7
(1,0,0) 4-brane	✓	✓	✓	•	•	•	✓	✓
(0,1,0) 4-brane	✓	✓	✓	•	•	✓	•	✓
(0,0,1) 4-brane	✓	✓	✓	•	•	✓	✓	•
(1,0,0)-string	✓	•	•	•	•	✓	•	•
(0,1,0)-string	✓	•	•	•	•	•	✓	•
(0,0,1)-string	✓	•	•	•	•	•	•	✓

- M2-brane wrapping $\mathbb{P}^1$: C_a, C_b, C_c: $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ strings!
- M2-brane wrapping 2-cycle in M-theory $\leftrightarrow$ open string modes on 4-string junction!

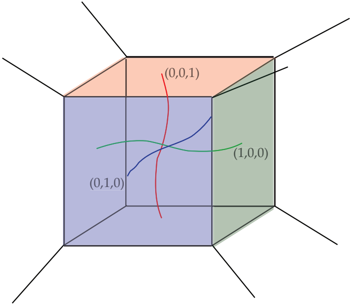
Brane web picture in IIB



- Generally for (p, q, r)-string, mass of BPS open strings states

$$m \sim \text{length} \times T_{(p,q,r)} \tag{19}$$

Brane web picture in IIB



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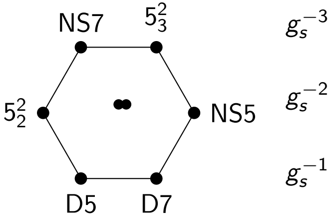
- $U(1)$ gauge field given by a linear combination of $U(1)$s of all finite 4-branes
- Electric charge of a string state $Q_e = -N_b$, total # of end points of the string

Flavor branes giving flavor symmetry

- Flavor branes in IIB: classified by exotic branes
- Flavor branes in 8D SUGRA: 5-brane objects

	0	1	2	3	4	5	6	7
$\widetilde{S}^1_9 \times S^1_8$-wrapped (D7,NS7)	✓	✓	✓	✓	✓	•	•	✓
(NS5,5^2_2)	✓	✓	✓	✓	✓	✓	•	•
(D5,5^2_3)	✓	✓	✓	✓	✓	•	✓	•



What's next?

- Superpotential from geometry? Hard even for 3d $\mathcal{N}=2$ $SU(2)+N_f\mathbf{F}$!
- Higher derivative/quantum correction to the 11D SUGRA action, more precise formula for $1/g^2$
- Realize known 3d $\mathcal{N}=2$ dualities, e. g. SQED-XYZ duality
- Relations to other 3d $\mathcal{N}=2$ constructions, e. g. 6d (2,0) on 3-manifolds?
- 4d $\mathcal{N}=1$ uplift in the elliptic cases
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Thank you for your attention!