Gravitational radiation pulses from EMRI system with a supermassive boson star

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[1] arXiv: 2108.13170, **Y.P. Zhang**, Y.B. Zeng, Q.Y. Wang, S.W. Wei, P. Amaro Seoane, YXL

[2] arXiv: 2107.04848, **Y.P. Zhang**, Y.B. Zeng, Q.Y. Wang, S.W. Wei, YXL

[3] arXiv: 1811.08795, **Q.Y. Wang**, YXL, S.W. Wei

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1. Introduction

- Classical scalar fields can be proposed as possible candidates for the dark matter component of the universe. [1]
- Boson stars as the black hole mimickers had been proposed in the 1970s. [2]
- Supermassive black holes locate at the center of most galaxies [3].
- Extreme-Mass-Ratio-Inspiral (EMRI) systems described by the stellar-mass compact objects orbiting around central supermassive black holes are the promising gravitational wave sources of the future space-borne gravitational wave detectors.[4]

[1] Rev. Mod. Phys. 75, 559 (2003); PRD 62, 103517 (2000); Astrophys. J. Lett. 534, L127 (2000).
[2] D. A. Feinblum and W. A. McKinley, Phys. Rev. 168, 1445 (1968); David J. Kaup, Phys. Rev. 172, 1331 (1968); R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969).
[3] Astrophys. J. 875 (2019) no.1, L1-L6.
[4] P. Amaro-Seoane, Living Rev. Rel. 21, 4 (2018).

1. Introduction

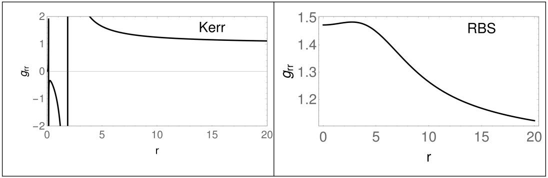


Figure 1: Plots of the metric function g_{rr} of a Kerr black hole and a rotating boson star.

- Boson stars are the horizonless compact object. There might be novel orbits in boson star background.
- It has been shown that a supermassive boson star can be in a galactic center. [5]
- The GWs from the EMRI system with a central supermassive spherical boson star have been obtained. [6]
- Are there novel GWs when the EMRI system has a central supermassive rotating boson stars (RBS)?

[5] Diego F. Torres et al., PRD, 62, 104012 (2000); Y.-F. Yuan et al., Astrophys. J. 606, 1112 (2004).
[6] M. Kesden, J. Gair, and M. Kamionkowski, PRD 71, 044015 (2005).

2. Model of rotating boson star (action of system)

Rotating boson stars are equilibrium strong gravity systems.

action (mini boson star [a]) ($G = c = \hbar = 1$)

[a]Schunck and Mielke, CQG 20, R301 (2003).

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \nabla_\mu\Phi\nabla^\mu\Phi^* - \frac{\mu^2}{\hbar^2}\Phi\Phi^*\right]. \tag{1}$$

metric [a]

[a]Herdeiro and Radu, PRL 112, 221101 (2014)

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + 2g_{t\varphi}dtd\varphi + g_{\varphi\varphi}d\varphi^2 \\ &= -e^{2F_0}dt^2 + e^{2F_1}\left(dr^2 + r^2d\theta^2\right) \\ &\quad + e^{2F_2}r^2\sin^2\theta(d\varphi - Wdt)^2. \end{aligned} \tag{2}$$

metric functions: F_i, W

2. Model of rotating boson star (symmetry and boundary)

Ansatz for the complex scalar field Φ

$$\Phi = \phi(r,\theta)\exp\left[i(\omega t - k\varphi)\right], \quad (3)$$

where ω is frequency of the scalar field and k is angular number.

reflection symmetry (at $\theta = \pi/2$)

$$\partial_\theta F_i(r,\theta)|_{\frac{\pi}{2}} = \partial_\theta W(r,\theta)|_{\frac{\pi}{2}} = 0 \quad (4)$$

$$\begin{cases} \partial_\theta \phi(r,\theta)|_{\frac{\pi}{2}} = 0, & \text{even parity,} \\ \phi(r,\theta)|_{\frac{\pi}{2}} = 0, & \text{odd parity.} \end{cases} \quad (5)$$

at $\theta = (0,\pi)$:

$$\partial_\theta F_i(r,\theta)|_{0,\pi} = \partial_\theta W(r,\theta)|_{0,\pi} = 0 \quad (6)$$

$$\phi(r,\theta)|_{0,\pi} = 0, \quad (7)$$

Asymptotic flat for a rotating boson star demands:

$$\lim_{r\to\infty} F_i = \lim_{r\to\infty} W = \lim_{r\to\infty} \phi = 0. \quad (8)$$

2. **Model of rotating boson star** (properties of boson stars)

We have obtained the numerical solutions[7] of the rotating boson star.

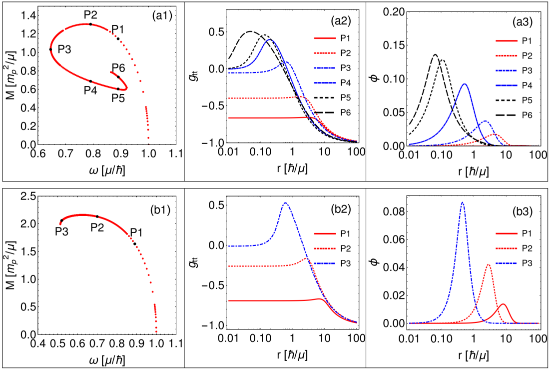


Figure 2: Plots of the ADM mass M, metric function $g_{tt}(r, \pi/2)$, scalar field modulus $|\phi(r, \pi/2)|$ of the boson star. We set $k = 1, 2$ for subfigures (a1-a3), (b1-b3), respectively.

[7] Y.-Q. Wang, Y.-X. Liu, and S.-W. Wei, PRD 99, 064036, (2019); KADATH, J. Comp. Physics 229, 3334 (2010).

2. Model of rotating boson star (properties of boson stars)

ergoregion and profile of scalar field

A highly relativistic rapidly rotating boson star will have an ergoregion [a] when

$$g_{tt} = r^2 \sin^2\theta \, W^2(r,\theta) e^{2F_2(r,\theta)} - e^{2F_0(r,\theta)} > 0. \quad (9)$$

The corresponding profiles of the metric component $g_{tt}(r,\pi/2)$ and scalar field $\phi(r,\pi/2)$ for each point are given in Fig. 2.

[a] Philippe, Claire, and Eric, PRD 90, 024068, (2014).

gravitational mass scale of boson stars

$$\bar{\lambda} = \frac{\hbar}{\mu c}. \quad (\mu \text{ is the scalar field mass parameter}) \quad (10)$$

The gravitational mass is [a]

$$M_{\text{bos}} \sim \frac{c^2 \, \bar{\lambda}}{G} = \frac{m_p^2}{\mu}. \quad (\text{Planck mass } m_p = \sqrt{\hbar c/G} \approx 2.18 \times 10^{-8} \text{ kg}) \quad (11)$$

[a] S. L. Liebling and C. Palenzuela, Living Rev. Relativ. 15, 6 (2012).

3. Geodesics in rotating boson star background (four-velocity)

conserved quantities of a test particle
(timelike killing vector $\xi^\mu = (\partial_t)^\mu$ and a spacelike killing vector $\eta^\mu = (\partial_\varphi)^\mu$)

$$\bar{E} = -(\partial_t)^\mu u_\mu = -g_{tt}u^t - g_{t\varphi}u^\varphi, \tag{12}$$

$$\bar{J} = (\partial_\varphi)^\mu u_\mu = g_{t\varphi}u^t + g_{\varphi\varphi}u^\varphi, \tag{13}$$

where $\bar{E}$ and $\bar{J}$ are the energy and angular momentum of the particle per unit mass.

four-velocity of the test particle (only consider the equatorial orbits)

$$u^\mu = \left(\frac{dt}{d\tau}, \frac{dr}{d\tau}, \frac{d\theta}{d\tau}, \frac{d\varphi}{d\tau}\right) = \left(u^t, u^r, 0, u^\varphi\right), \tag{14}$$

$$u^\mu u_\mu = -1, \tag{15}$$

$$u^t = \frac{\bar{E}g_{\varphi\varphi} + \bar{J}g_{t\varphi}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}, \quad u^\varphi = \frac{\bar{E}g_{t\varphi} + \bar{J}g_{tt}}{g_{tt}g_{\varphi\varphi} - g_{t\varphi}^2} \propto (\bar{E} - V_\varphi), \tag{16}$$

$$(u^r)^2 = -\frac{1 + g_{\varphi\varphi}u^\varphi u^\varphi + 2g_{t\varphi}u^t u^\phi + g_{tt}u^t u^t}{g_{rr}} \propto \left(\bar{E} - V_{\text{eff}}^+\right)\left(\bar{E} - V_{\text{eff}}^-\right). \tag{17}$$

3. Geodesics in rotating boson star background

radial effective potential

$$(u^r)^2 \propto \left(\bar{E} - V_{\text{eff}}\right)\left(\bar{E} - V'_{\text{eff}}\right) \qquad (18)$$

angular effective potential

$$u^\varphi \propto (\bar{E} - V_\varphi), \quad V_\varphi = -\frac{\bar{J} g_{tt}}{g_{t\varphi}} \qquad (19)$$

Here, we consider a rotating boson star with $k = 1$ and $\omega = 0.79\mu$ as an example.

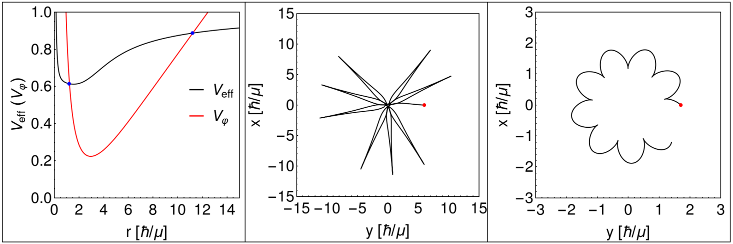


Figure 3: Orbits of a test particle with angular momentum $\bar{J} = -0.248$ in the rotating boson star with $k = 1$, $\omega = 0.79\mu$. The velocity satisfies $u^r = 0$ and $u^\varphi = 0$ in the peaks of both the two orbits.

3. Geodesics in rotating boson star background

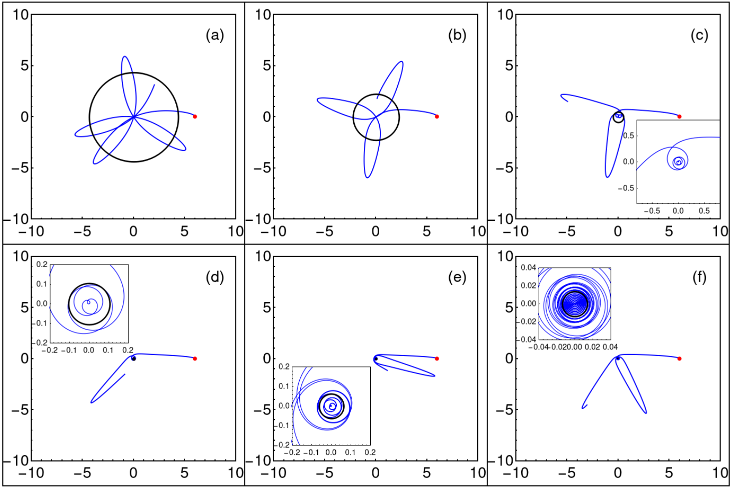


Figure 4: Zero-angular-momentum orbits of a test particle in the rotating boson star with $k = 1$, where the orbits in subfigures (a), (b), (c), (d), (e), and (f) are in the background of the boson stars described respectively by the points P_1, P_2, P_3, P_4, P_5, and P_6 in subfigure (a1) of Fig. 2.

3. Geodesics in rotating boson star background

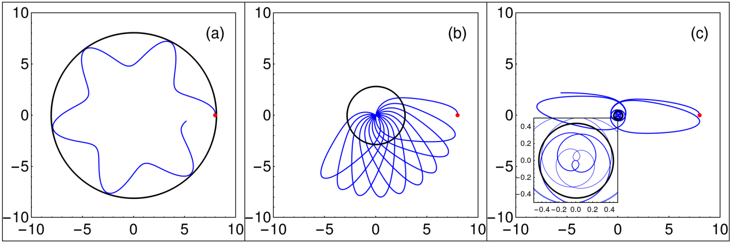


Figure 5: Zero-angular-momentum orbits of a test particle in the rotating boson star with $k = 2$. The orbits in subfigures (a), (b), and (c) are related with the points P_1, P_2, and P_3 in subfigure (b1) of Fig. 2.

These novel orbits do not appear in the Kerr black hole case.

3. Geodesics in rotating boson star background

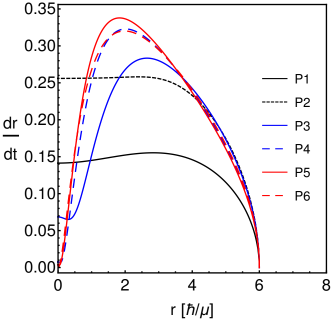


Figure 6: Plot of $\frac{dr}{dt}$ as a function of r for the orbits in Fig. 4.

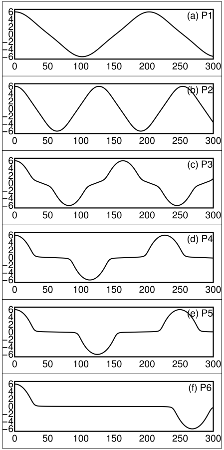


Figure 7: Relations between the radius and coordinate time for the orbits in Fig. 4, where the horizontal axis is the coordinate time and the vertical axis is the radius.

3. Geodesics in rotating boson star background

- a test particle with zero angular momentum can pass through the center of rotating boson star;

- a boson star will transit from the low rotating state to the highly relativistic rapidly rotating state with the increase of the compactness of background scalar field;

- for a given boson star with fixed frequency ω, the compactness of the background scalar field decreases with the angular momentum k while the frame-dragging effect increases with k;

- the time of a test particle stays in the central region of the rotating boson star increases with the increase of the compactness of the boson star.

3. Geodesics in rotating boson star background

Comparison of Kerr black hole and rotating boson star

$$M = 1.3124,\ J = 1.3776, \text{ and } a = J/M^2 = 0.7998.$$

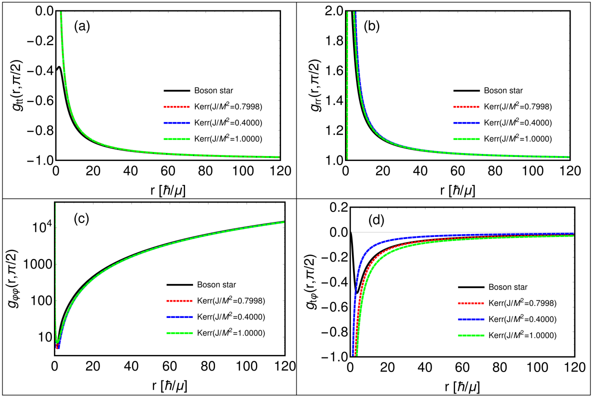


Figure 8: Relations of the metric functions between the rotating boson star with $(k = 1, \omega = 0.79)$ and Kerr black hole with same mass but different spins.

3. Geodesics in rotating boson star background

comparison the orbits in the backgrounds of rotating boson star and Kerr black hole with the same mass and spin angular momentum

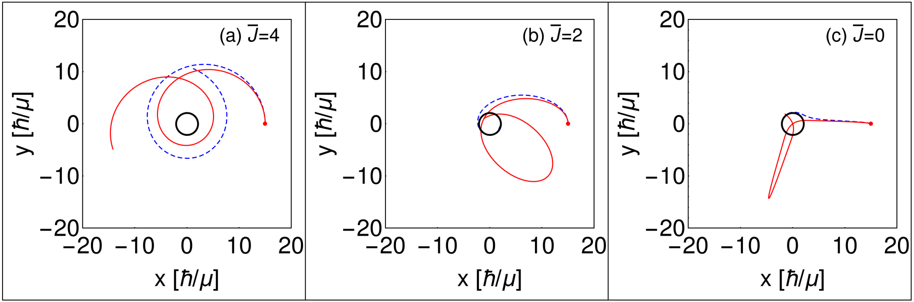


Figure 9: Orbits with the same orbital angular momenta and same initial positions $(r = 15\hbar/\mu)$ in the backgrounds of rotating boson star $(k = 1, \omega = 0.79)$ and Kerr black hole, where the masses and spins of the rotating boson star and Kerr black hole are the same, the black circle is the event horizon of the black hole.

4. Stability of rotating boson star

Stability of rotating boson star

- Rotating mini boson stars are not stable.
- It can collapse to a black hole and emit gravitational waves.
- Numerical relativity are needed for studying the nonlinear stability and the final state of rotating boson star.

Method for the nonlinear evolution of rotating boson star

- Constructing the configurations of the rotating boson stars;
- Constructing the initial data of rotating boson star for the Einstein Toolkit (reading the configurations of rotating boson stars);
- Evolving the spacetime and scalar field by using the Einstein Toolkit with our own codes (MAYA codes developed at Georgia Institute of Technology).

4. Stability of rotating boson star

Nonlinear evolution of rotating boson star

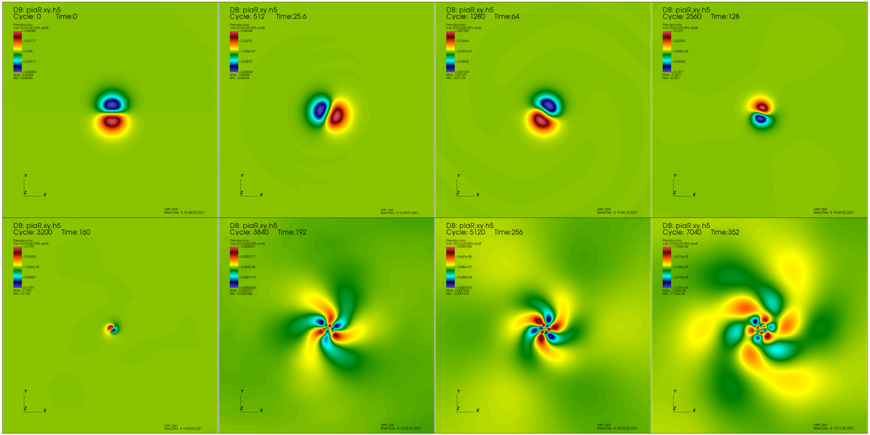


Figure 10: Nonlinear evolution of rotating boson star. This figure describes the configuration of the real part of the scalar field in the equatorial plane at different time ($\omega = 0.79, k = 1$).

4. Stability of rotating boson star

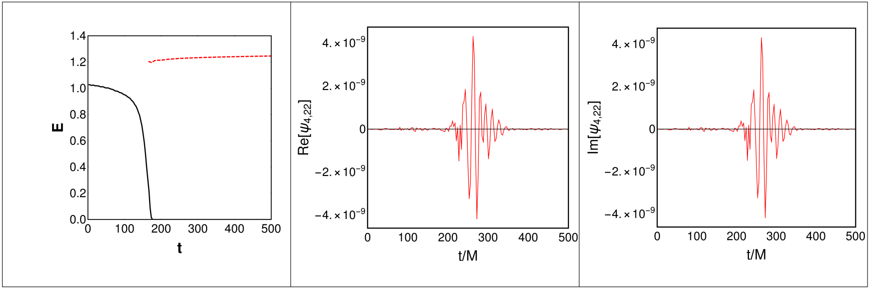


Figure 11: Information of rotating boson star of a rotating boson star collapse.

- Energy of scalar field (black solid line) and the irreducible mass (red dashed line) of black hole from the collapse of a rotating boson star (left channel);
- middle and right channels are the corresponding gravitational waves emitted from the rotating boson star when it collapses to a black hole.

4. Stability of rotating boson star

- Current observations indicate that the supermassive objects in the center of the galaxy are stable or have extremely long life spans.
- the central supermassive galactic RBS should be stable or extremely long lived.

nonlinear interactions can stabilize boson stars [a] ($G = c = \hbar = 1$)

[a] N. Siemonsen and W. E. East, PRD 103, 044022 (2021)

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \nabla_\mu\Phi\nabla^\mu\Phi^* - \frac{\mu^2}{\hbar^2}\Phi\Phi^* - \lambda|\Phi|^4\right]. \tag{20}$$

stable rotating boson stars [a] ($G = c = \hbar = 1$)

[a] N. Siemonsen and W. E. East, PRD 103, 044022 (2021)

- As a comparison, we consider a stable boson star and an unstable boson star with the same $k = 1$ and $\lambda = 500$, their frequencies are $\omega = (0.78, 0.90)$.
- The one with $\omega = 0.78$ is unstable and the one with $\omega = 0.90$ is stable.

5. Gravitational radiation pulses (numerical kludge (NK) method)

GWs from EMRI in terms of numerical kludge (NK) method [a]

[a] S. Babak, H. Fang, J. R. Gair, K. Glampedakis, PRD 75, 024005 (2007); Erratum PRD 77, 049902 (2008).

- Using the solutions of the linearized gravitational perturbation;
- Integrating the geodesics of the test particle in the rotating boson star background.

Constructing the effective trajectories by projecting the Boyer-Lindquist coordinates into a pseudo-flat space as follows

$$x = r\sin\theta\cos\varphi,\ y = r\sin\theta\sin\varphi,\ z = r\cos\theta. \tag{21}$$

$$I^{jk}(t') = \int x'^j x'^k T^{00}(t', x')d^3x'.\ \text{(quadrupole of the test particle)} \tag{22}$$

Gravitational waves (adopting the transverse-traceless projection [a])

[a] M. Kesden, J. Gair, and M. Kamionkowski, PRD 71, 044015 (2005).

$$\bar{h}^{ij} = \frac{2G}{d}\ddot{I}^{jk}|_{t'=t-d},\ (d\ \text{is the distance from source to observation point}) \tag{23}$$

5. Gravitational radiation pulses (novel orbits)

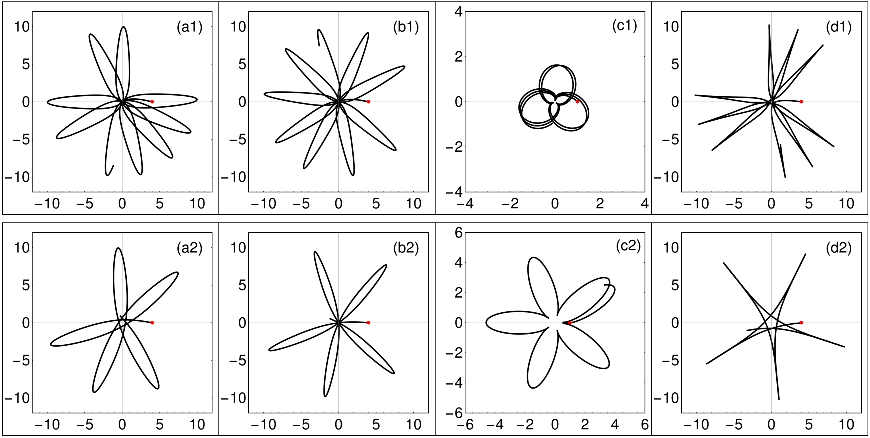


Figure 12: Four types of special orbits.

5. Gravitational radiation pulses (details of orbits)

We give the angular momentum $\bar{J}$, apastron r_a, periastron r_p, and energy $\bar{E}$ of these orbits in Table 1. With the help of the definition of the orbital eccentricity

$$e = \frac{r_a - r_p}{r_a + r_p}, \tag{24}$$

we can prove that the orbital eccentricity e for a test particle in a rotating boson star could be 1, this result is unacceptable for a black hole.

Table 1: We consider four types of special orbits in a rotating boson star.

orbit	$\bar{J}_1$	$\bar{E}_1$	e_1	$\bar{J}_2$	$\bar{E}_2$	e_2
a	0.1500	0.8165	0.8121	0.1500	0.9470	0.9215
b	0.0000	0.8161	1.0000	0.0000	0.9470	1.0000
c	-0.0050	0.5392	0.8282	-0.0050	0.7521	0.7521
d	-0.4302	0.8156	0.9584	-0.2800	0.8625	0.8626

5. Gravitational radiation pulses (GWs)

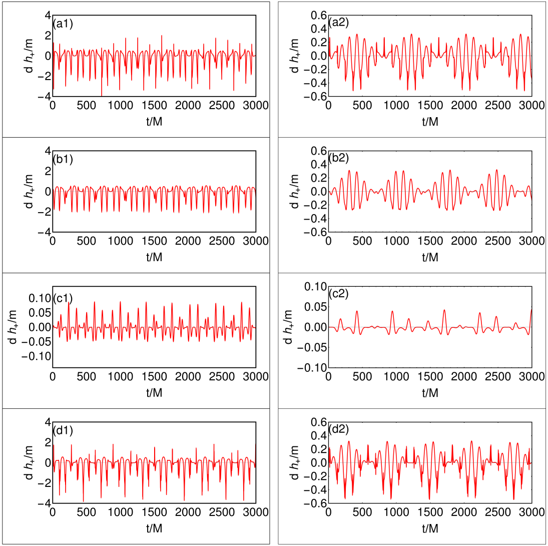
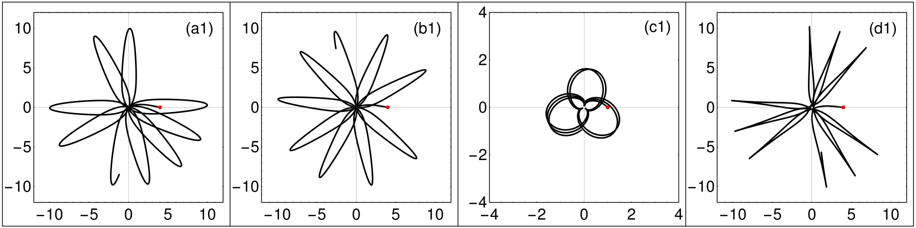


Figure 13: Amplitudes of the plus polarization for the GWs generated by the orbits in Fig. 12.

5. Gravitational radiation pulses (relations between GWs and orbits)



- Orbits (c) and (d) have peaks.
- All orbits can generate pulses.

- GWs depend on the orbit location, velocity, and acceleration.
- The orbital eccentricities of the orbits (a) and (d) are close to but not equal to 1, the corresponding velocity (u^r, u^φ) will change greatly in a very short time when the test particle is passing through the periastron r_p.

5. Gravitational radiation pulses (relations between GWs and orbits)

velocity (u^r, u^φ) and acceleration $(du^r/d\tau, du^\varphi/d\tau)$ of the orbits in Fig. 12.

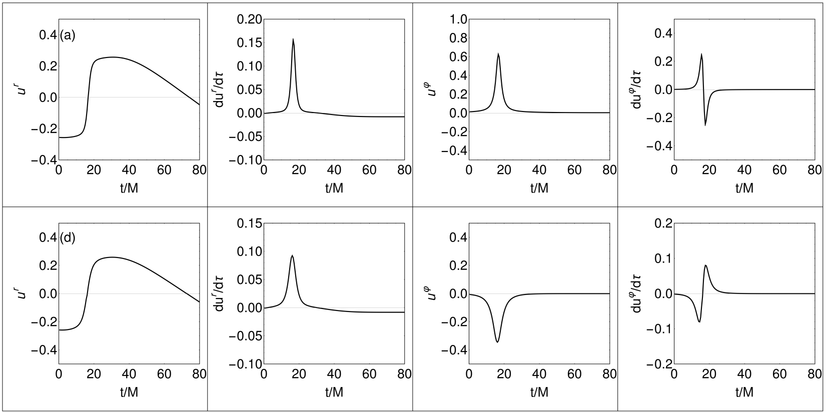


Figure 14: The results in subfigures (a1) and (d1).

5. Gravitational radiation pulses (detection of these pulses)

supermassive rotating boson star

- Reference [JCAP **11**, 002 (2010)] exploits the orbits of the S-stars at the Galactic Center and showed that the mass parameter of the scalar field is of about

$$\mu \in [300\,\text{eV},\ 2 \times 10^4\,\text{eV}] \tag{25}$$

  for the boson star described by action (20).
- With this constraint, the masses of the RBSs that we considered are in the order of $M \sim 10^6 M_\odot$ [Phys. Rev. D, **90**, 024068, (2014)].

mass of the small compact object

We set the mass of the small star to be

$$m = 10M_\odot. \tag{26}$$

distance from the detector to the source

The distance from the detector to the source is set up to 100Mpc.

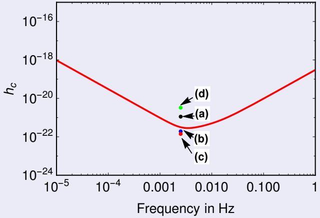
5. Gravitational radiation pulses (detection of these pulses)

amplitudes and frequencies of gravitational radiation pulses

- The frequencies of the pulse part are in the magnitude of $f_{\text{pulses}} \sim 10^{-1}$Hz.
- The frequencies of the whole GW signal are still in the milli-Hz.

maximal amplitude of GWs given in Fig. 13 and LISA's sensitivity window [a]

[a] C. J. Moore, R. H. Cole, and C. P. L. Berry, CQG, 32, 015014 (2015).



These novel pulses could be detected by the LISA.

6. Summary and outlook

Summary

- The horizonless property of the rotating boson star leads to the stellar-mass compact object could pass through the center of the rotating boson star and the orbital eccentricity could be $e = 1$.
- We found the novel gravitational radiation pulses from the EMRI system with a central supermassive rotating boson star.
- These novel GWs are quite different from the EMRI system with a central supermassive Kerr black hole, and can therefore be envisaged as a new way to discover supermassive rotating boson stars.

Outlook

- Study how the GWs depends on the configurations of rotating boson stars.
- Get the GWs of an inspiral into a supermassive rotating boson star.
- Study the GWs of the extended small body inspiral into a supermassive rotating boson star.
- Study the GWs of the collapse of a rotating boson star.
- $\cdots$

Thanks for your listening!