微(弱)引力量子系统

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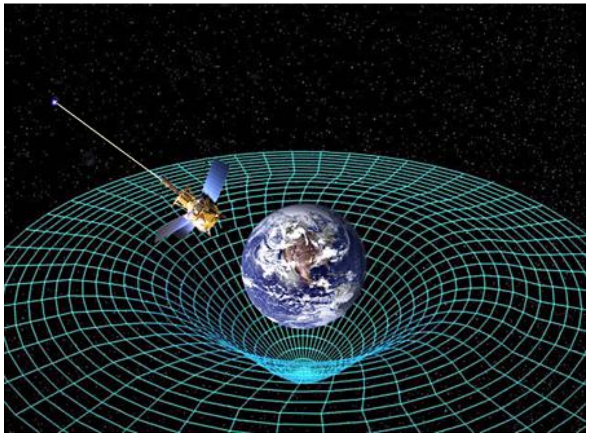
OUTLINE

- I: Relativistic Quantum Information
- II: RQI in strong gravitational quantum systems
- III: RQI in Micro/weak gravitational systems
- IV: QCS and QI in micro gravitational systems

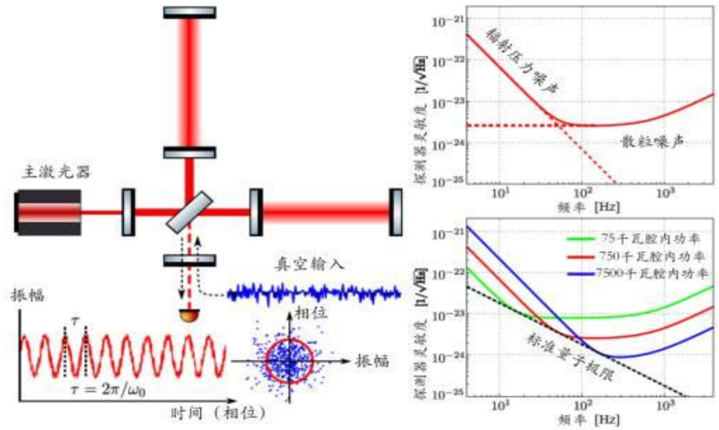
I: Relativistic Quantum Information

Relativistic Quantum Information

Quantum information tasks go relativistic



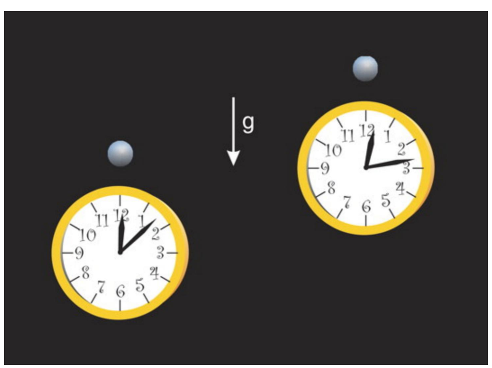
Space-based QC

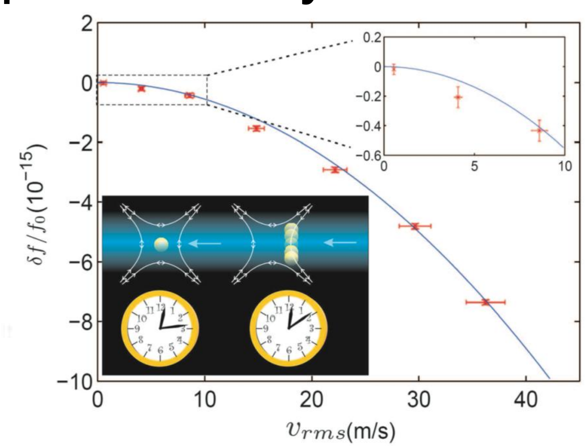


GW detection

Y. Ma et.al. Nat. Phys. 13, 776 (2017)

Observable relativistic effects in quantum systems

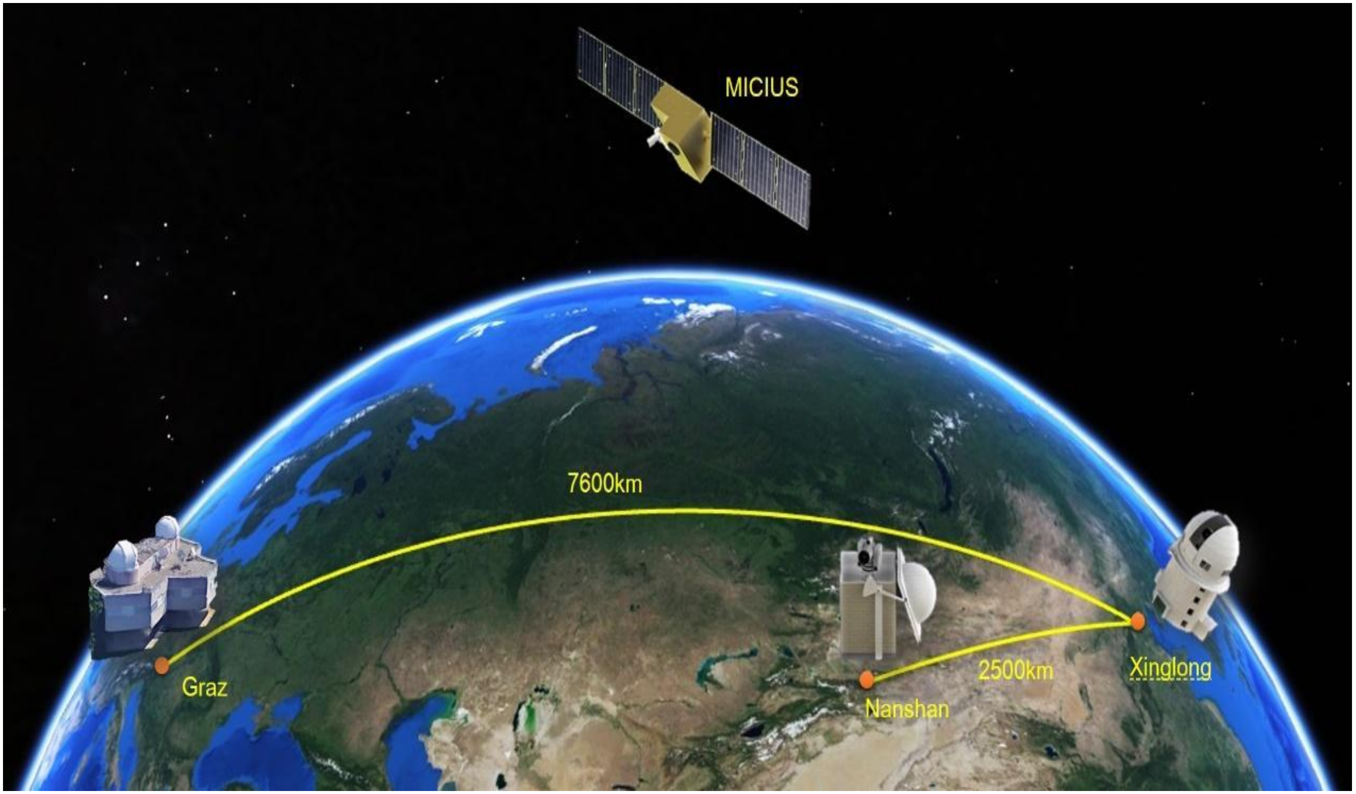




Wineland et.al, Optical Clocks and Relativity, *Science* 329, 1630 (2010).

1. Practical aspects

LEO (low Earth orbit) satellite (500-1000km), distance 10^6, relative speed $10^{-5}c$

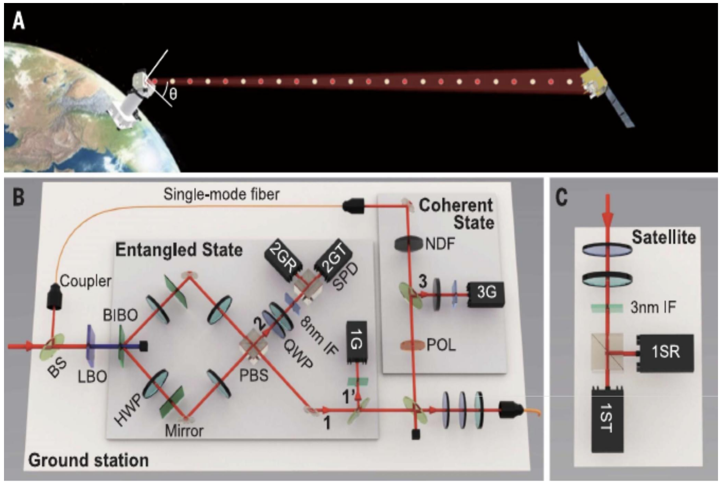


Satellite-based etanglement distribution

Quantum teleportation

Y. Jin *et.al.*, Science 356, 1140 (2017);
J. Ren *et.al.,* Nature 549, 70 (2017).

2. Fundamental aspects:



Xu *et.al*, Science, 366, 132(2019)

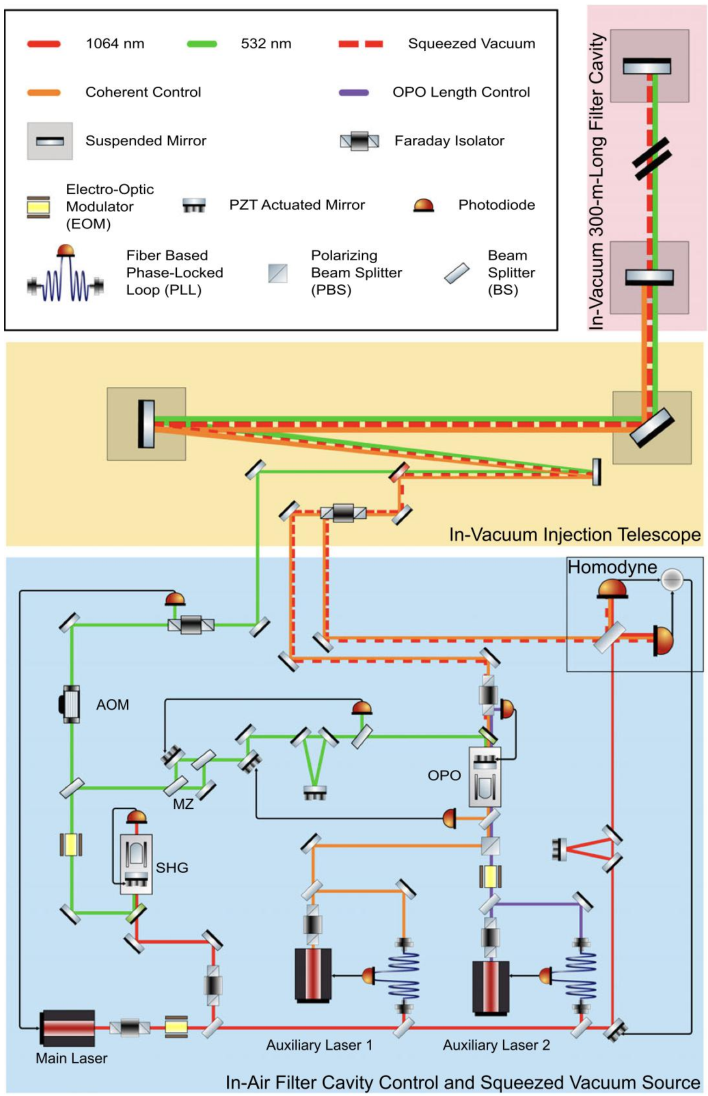
Test of GR in lab.

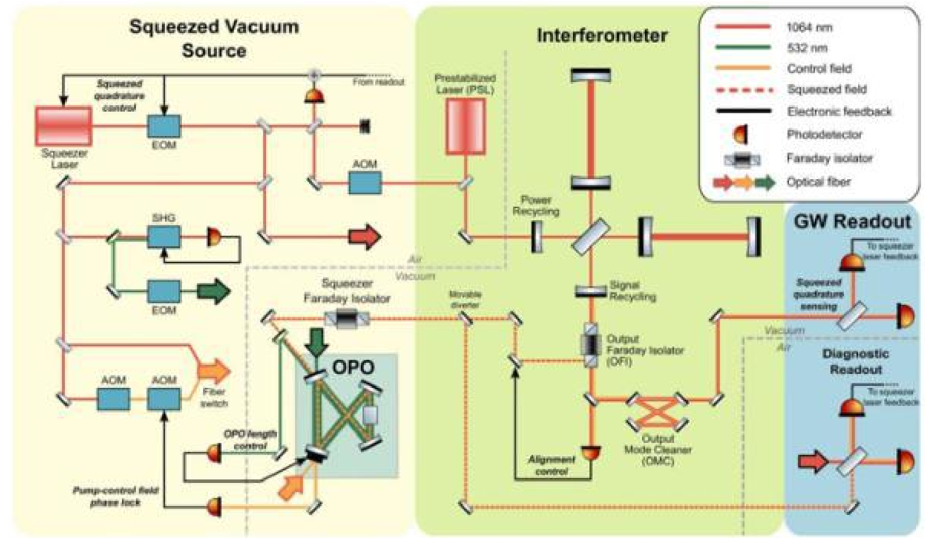
Understanding of nonlocality and causality.

Entropy, thermodynamics and information paradox of black holes!

At these regimes relativistic quantum information kicks in!

Application: Quantum enhanced GW detectors



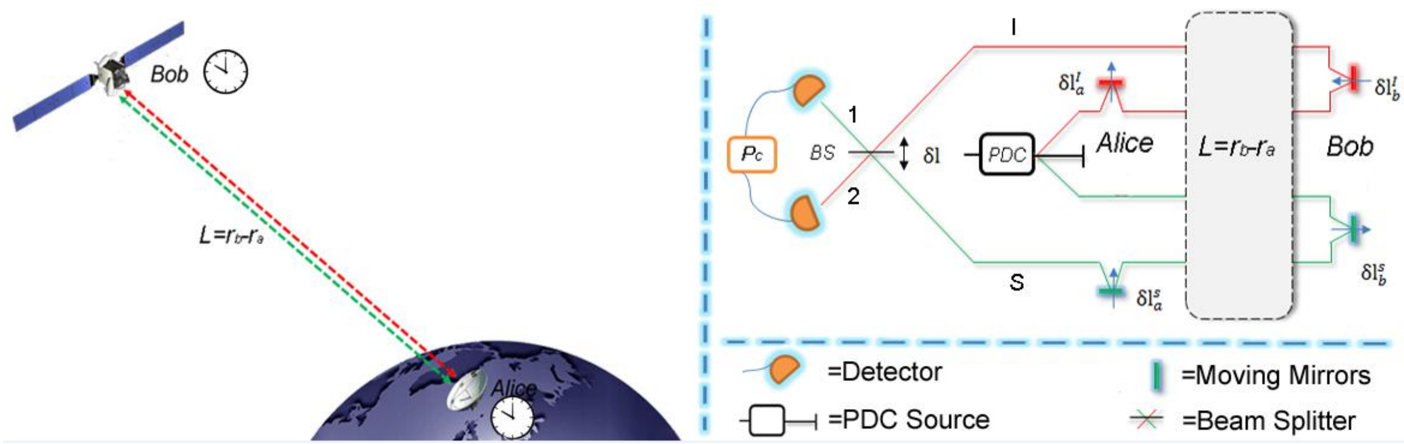


M. Tse et al., PRL 123, 231107 (2019)

C. M. Caves, PRD 23, 1693 (1981);
F. Acernese *et.al.*, PRL 123, 231108 (2019);
Y. Zhao *et.al.*, PRL 124, 171101 (2020);
L. McCuller *et.al.*, PRL 124, 171102 (2020).

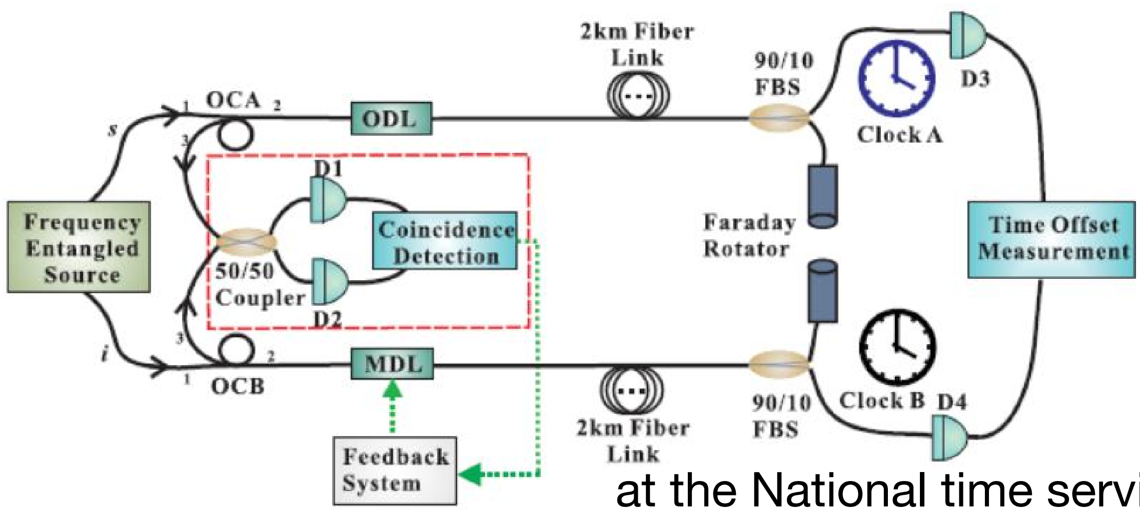
Application: space-based QCS

Theoretical proposal: Wang, Tian, Jing, Fan, Phys. Rev. D 93, 065008 (2016), arXiv:1501.01478



Effects of gravity on space-based quantum clock synchronization

Experimental studies: R. Quan et.al, Sci. Rep., 6: 30453 (2016), arXiv:1602.06371



at the National time service center of China

II. RQI in strong gravitational quantum systems

RQI in strong gravitational quantum systems

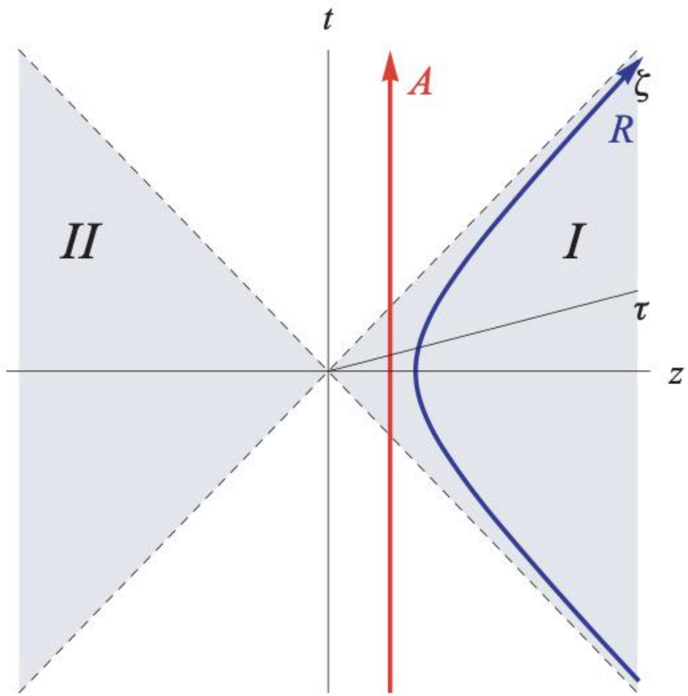
QFT in curved spacetime

- Classical spacetime+ quantum fields
- Incorporates Lorentz invariance
- Combines quantum mechanics with relativity at scales reachable by near-future experiments

RQI in strong gravitational quantum systems

- Hawking radiation (Unruh, Faccio, Koenig, Steinhauer)
- Unruh effect
- Dynamical Casimir effect (Delsing)
- Expanding Universe (Westbrook)

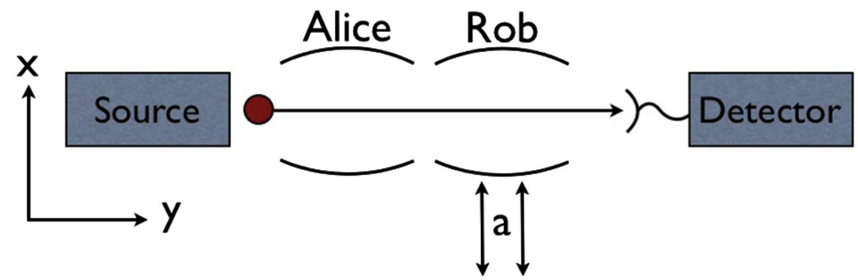
Free modes and moving cavities in noninertial frames



$$|0_k\rangle^{\mathcal{M}} \sim \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r \; |n_k\rangle_I \, |n_k\rangle_{II} \,,$$

$$\cosh r = (1 - e^{-2\pi\Omega})^{-1/2}, \quad \Omega = |k|c/a.$$

Fuentes and Mann, PRL 95, 120404 (2005);
Wang and Jing, PRA 82 032324 (2010) .



Downes et.al., PRL 106 210502 (2011)

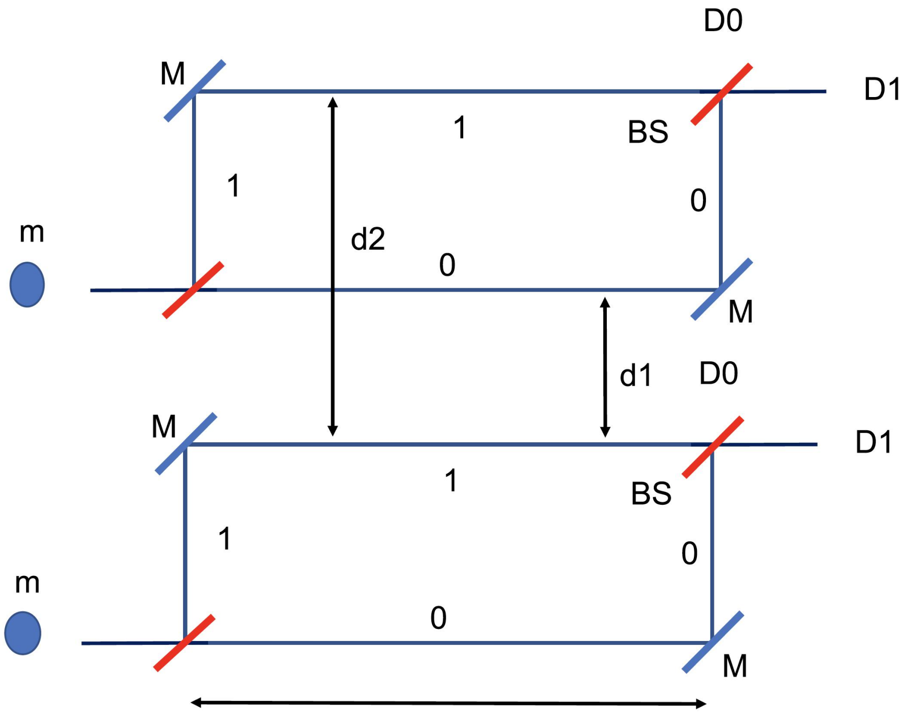
symplectic formalism

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \cdots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \cdots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathcal{M}_{mn} = \begin{pmatrix} \Re(\alpha_{mn} - \beta_{mn}) & \Im(\alpha_{mn} + \beta_{mn}) \\ -\Im(\alpha_{mn} - \beta_{mn}) & \Re(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

III. RQI in micro/weak gravitational quantum systems

Type I: Massive particles



Each mass is put in the state:

$$1/\sqrt{2}(|0\rangle + |1\rangle)$$

Since the masses on different paths interact via the gravitational field, the state of the composite system becomes entangled

$$\frac{1}{2}|0\rangle[|0\rangle + \exp(i\phi_1)|1\rangle] + \frac{1}{2}\exp(i\phi_1)|1\rangle[|0\rangle + \exp(i(\Delta\phi))|1\rangle],$$

$$\phi_i = (m^2 G/\hbar d_i)\Delta t$$

In each of the interferometers, the probabilities p_α for the mass to emerge on path $\alpha = 0, 1$ are

$$p_0 = \frac{1}{2}\left(\cos^2\frac{\phi_1}{2} + \cos^2\frac{\Delta\phi}{2}\right),$$

$$p_1 = \frac{1}{2}\left(\sin^2\frac{\phi_1}{2} + \sin^2\frac{\Delta\phi}{2}\right). \quad (2)$$

two massive molecules,
two split Bose condensates ,
two nanomechanical oscillators.

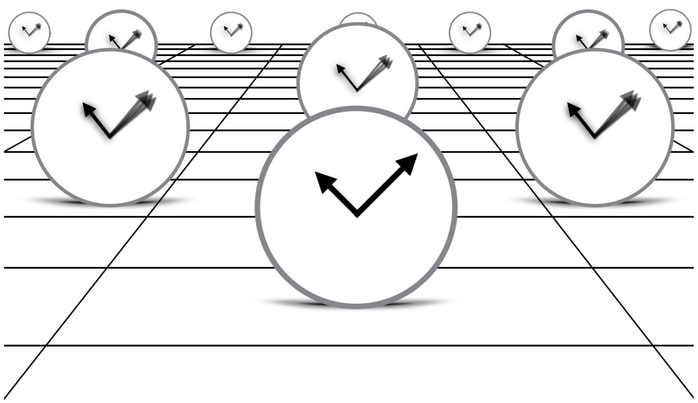
10^{-12} kg $\quad d \approx 10^{-6}$ m $\quad \Delta t = 10^{-6}$ s

S. Bose et.al., Phys. Rev. Lett. 119, 240401 (20
V.Vedral et.al., Phys. Rev. Lett. 119, 240402 (2(

Type II: Quantum clocks and oscillators

Two assumptions of GQC:

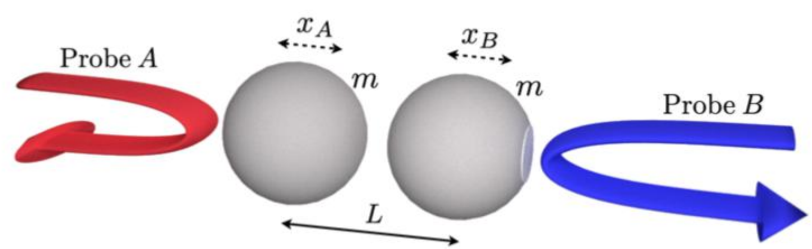
- mass-energy equivalence
- superposition of energy eigenstates



$$\hat{H}_{free} = \Delta E(j\mathbb{1} - \hat{Z}), \text{ where } \hat{Z} = \textstyle\sum_{-j}^{j} m\,|m\rangle\langle m|.$$

$$\hat{H} = \sum_{a=0}^{N} \hat{H}_a - \frac{G}{c^4 x} \sum_{a<b} \hat{H}_a \hat{H}_b,$$

Ruiz *et. al.*, PNAS 114, 2303 (2017)



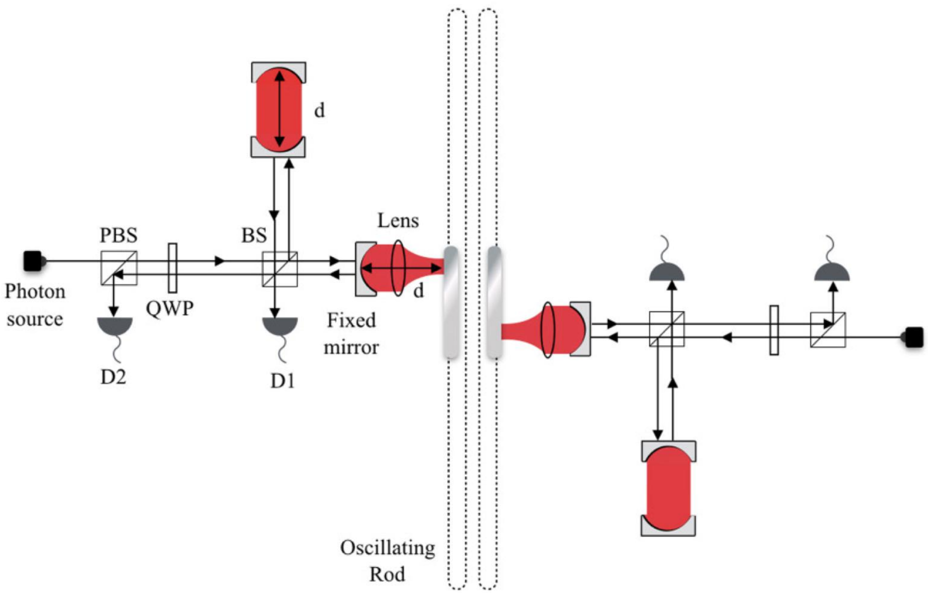
$$H_0 = \frac{p_A^2}{2m} + \frac{1}{2}m\omega^2 x_A^2 + \frac{p_B^2}{2m} + \frac{1}{2}m\omega^2 x_B^2$$

$$H_{\rm g} = -\frac{Gm^2}{L}\left(1 + \frac{(x_A - x_B)}{L} + \frac{(x_A - x_B)^2}{L^2} + \cdots\right)$$

Krisnanda *et. al.*, npj Quantum Information 6, 12 (2020)

Type III: Gravitational optomechanical systems

Quantum Cavendish experiment:



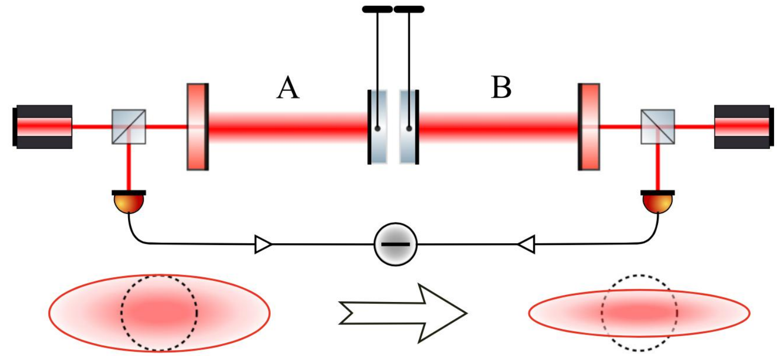
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle_c + |1,0\rangle_c)\,|\beta_m\rangle$$

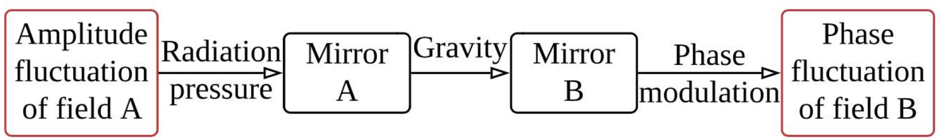
$$\times \otimes \frac{1}{\sqrt{2}}(|0,1\rangle_d + |1,0\rangle_d)\,|\beta_M\rangle$$

$$H_1 = \hbar\omega_c(c_1^\dagger c_1 + c_2^\dagger c_2) + \hbar\Omega_a a^\dagger a - \Lambda_m \hbar\Omega_a c_1^\dagger c_1(a^\dagger + a)$$
$$+ \hbar\omega_d(d_1^\dagger d_1 + d_2^\dagger d_2) + \hbar\Omega_b b^\dagger b - \Lambda_M \hbar\Omega_b d_1^\dagger d_1(b^\dagger + b)$$

Balushi *et. al.*, PRA 98, 043811 (2018)

Optomechanical pendulum:

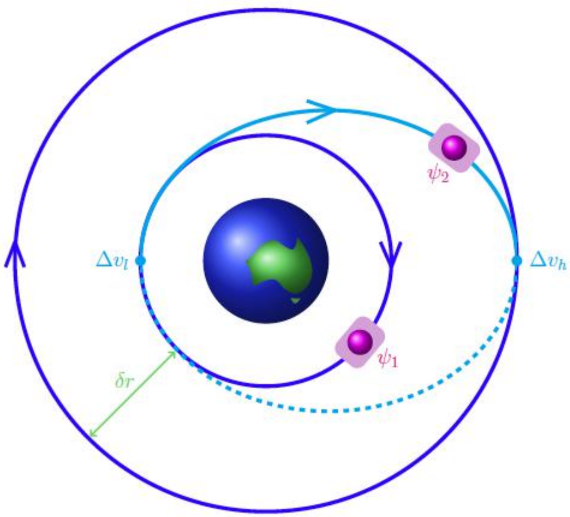




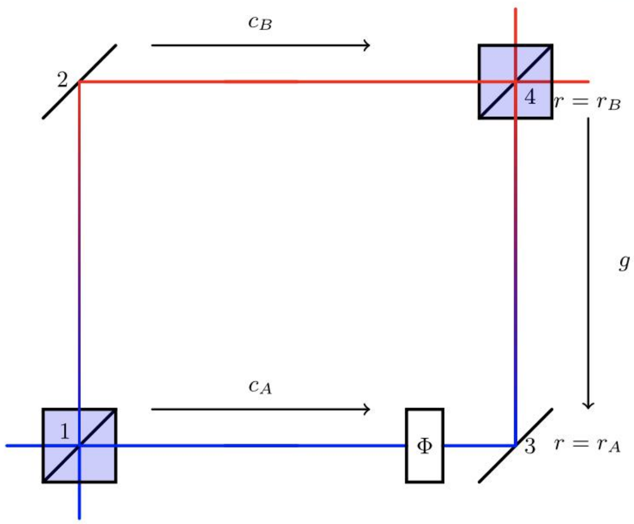
$$\begin{bmatrix} \hat{X}_A^{\rm out} \\ \hat{Y}_A^{\rm out} \\ \hat{X}_B^{\rm out} \\ \hat{Y}_B^{\rm out} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathcal{K} & 1 & \mathcal{G} & 0 \\ 0 & 0 & 1 & 0 \\ \mathcal{G} & 0 & \mathcal{K} & 1 \end{bmatrix} \begin{bmatrix} \hat{X}_A^{\rm in} \\ \hat{Y}_A^{\rm in} \\ \hat{X}_B^{\rm in} \\ \hat{Y}_B^{\rm in} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \alpha & \beta \\ 0 & 0 \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} \hat{Q}_A^{\rm th} \\ \hat{Q}_B^{\rm th} \end{bmatrix}$$

Miao *et. al.*, PRA 101, 063804 (2020)

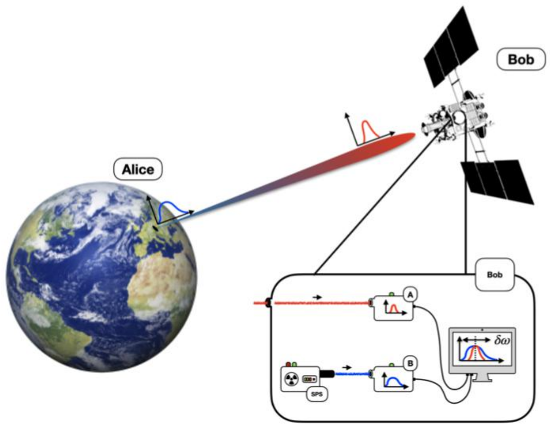
Type IV: Space-based QIP in weak gravity



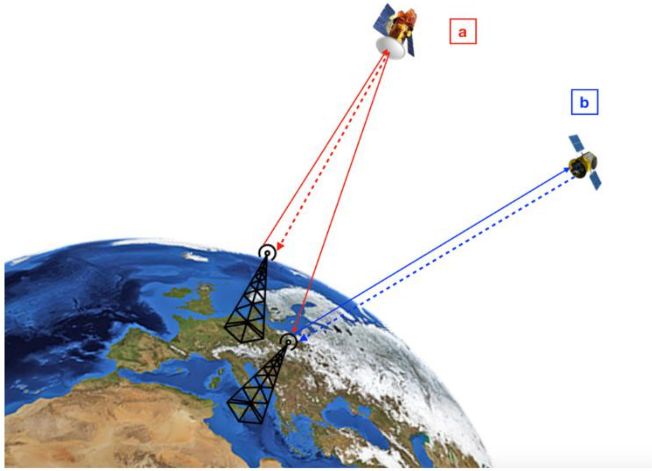
Bruschi *et.al.*, Phys. Rev. D 90, 045041 (2014)



Kish *et.al.*, Phys. Rev. D 99, 124015 (20



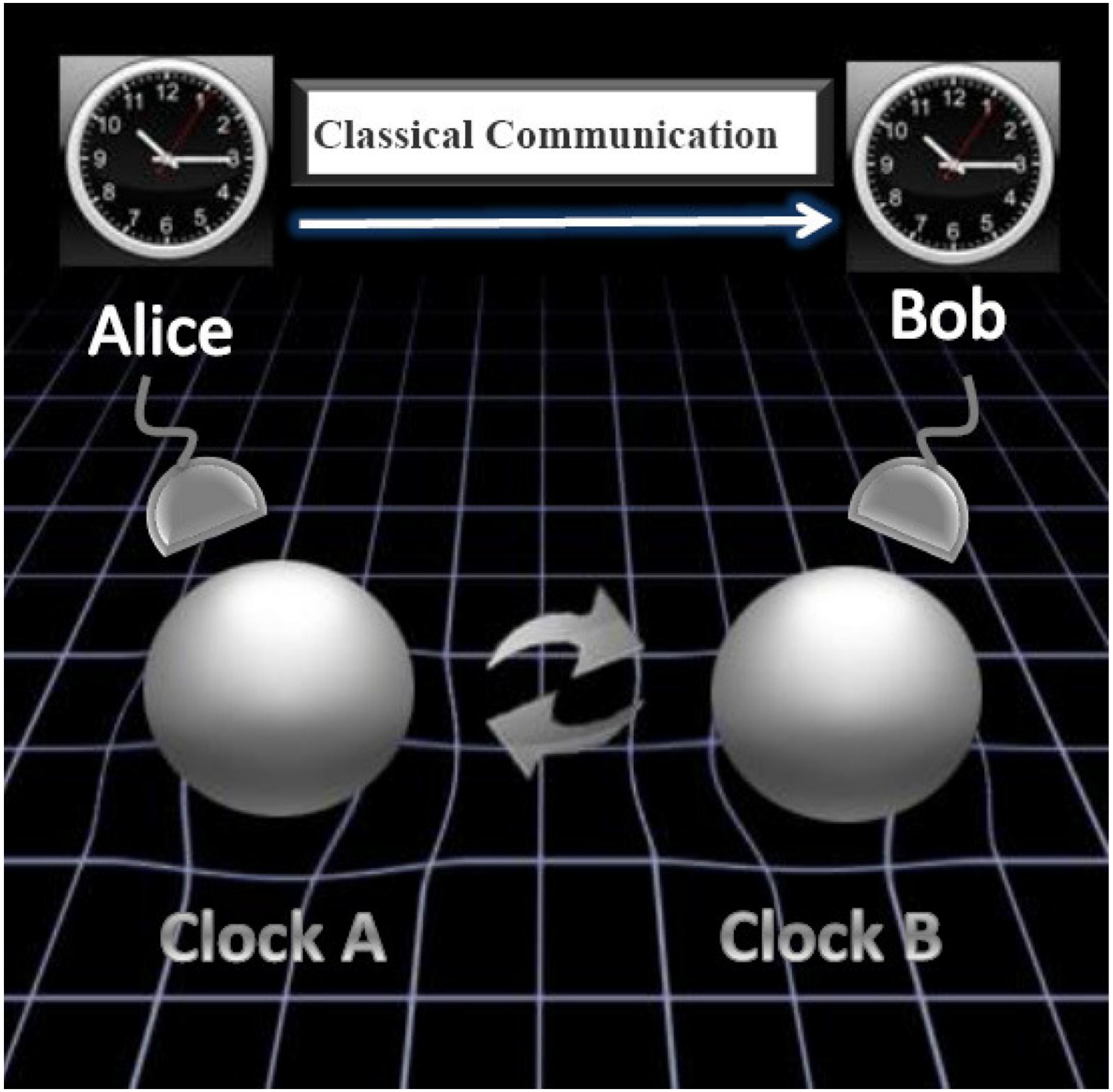
Bruschi *et.al.,*Phys. Rev. D 104, 085015 (2021)



Jan *et.al.*, Phys. Rev. A 99, 032350 (2019)

IV. QCS of gravity-induced time difference and gravity-enhanced quantum illumination

QCS of gravity-induced time difference



Q-clocks: two-level particles in the super-position of energy eigenstates

Gravitational effects only originate from the clock themselves

Wang, Liu, Jing, Adv. Quant. Tech. 2, 1900003 (2019)

The model of quantum clocks

We implement the mass energy equivalence by considering the composite particles are emerged from the interaction of two scalar fields φ_A and φ_B. The Lagrangian density of the scalar field is

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\left(\sum_A g^{\mu\nu}\left(\partial_\mu\varphi_A\right)\left(\partial_\nu\varphi_A\right) + \sum_{AB} M_{AB}^2\varphi_A\varphi_B\right), \tag{1}$$

where g is the determinant of the spacetime metric $g_{\mu\nu}$, and M_{AB} is the symmetric matrix in which the fields φ_1 and φ_2 are coupled.

In the weak field limit, the non-fixed metric background of the particle reads

$$\mathrm{d}s^2 = -(1+2\Phi(\mathbf{x}))\mathrm{d}t^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}, \tag{2}$$

where Φ is the gravitational potential. Then the Hamiltonian is obtained via the Legendre transformation, which is

$$H = \frac{1}{2}\int \mathrm{d}^3\mathbf{x}\left(\sum_A\left(\pi_A^2 + (\nabla\varphi_A)^2\right) + \sum_{AB} M_{AB}^2\varphi_A\varphi_B\right)\mathbf{\Phi}, \tag{3}$$

Evolution of the quantum clocks under gravitational interaction

To obtain the evolution of the two-particle state in the Fock space, we calculate the matrix element $\langle \xi^{(1)}, \eta^{(1)} \,|\, H \,|\xi^{(2)}, \eta^{(2)}\rangle$, where

$$\left|\xi^{(i)}, \eta^{(i)}\right\rangle = 2^{-1/2} \sum_{AB} \int \mathrm{d}^3\mathbf{x} d^3\mathbf{x}' \xi_A(\mathbf{x}) \eta_B(\mathbf{x}') \phi_A^\dagger(\mathbf{x}) \phi_B^\dagger(\mathbf{x}') \left|0\right\rangle \tag{9}$$

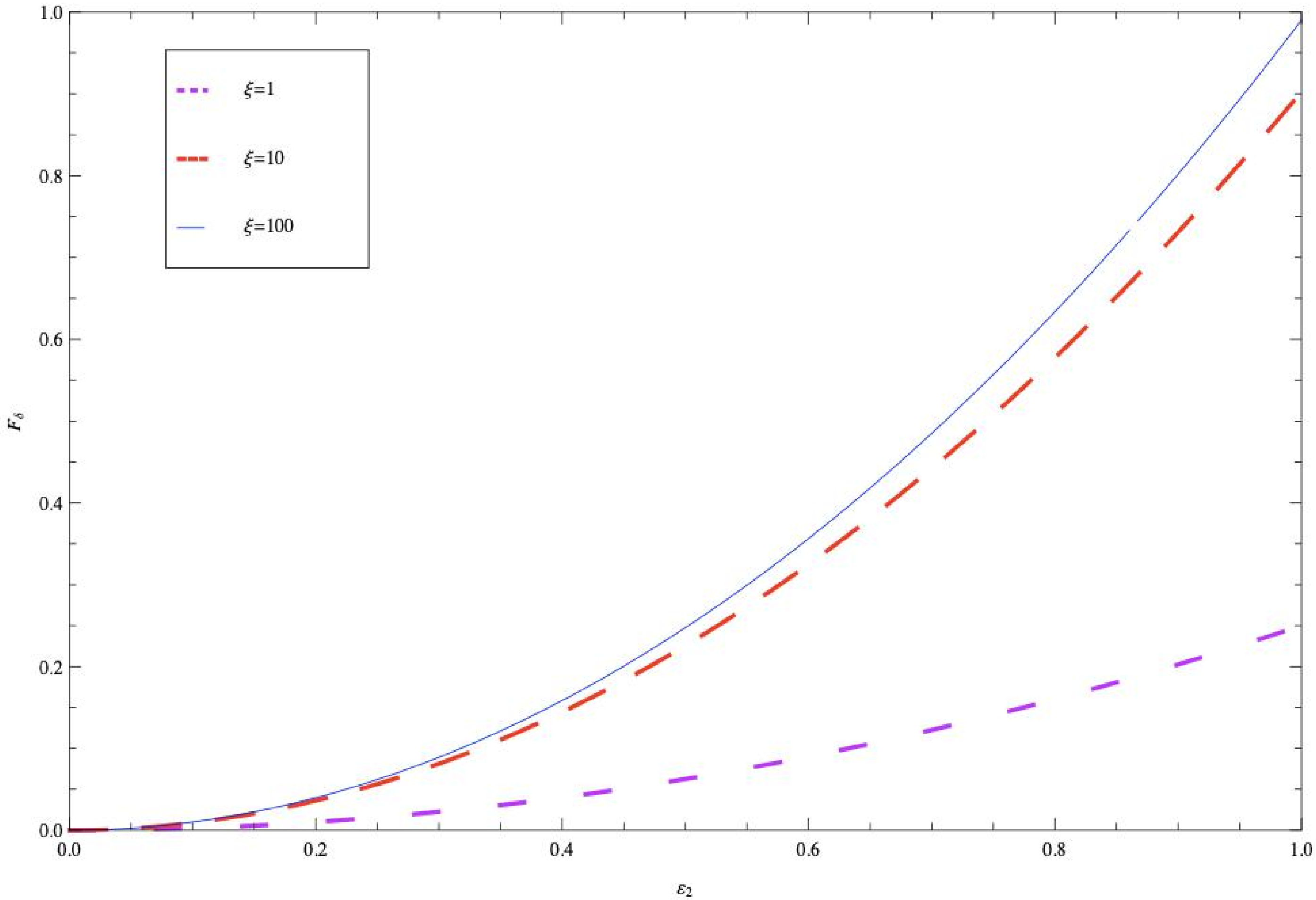
is a two-particle state for $i = 1, 2$, and $|0\rangle$ denotes the vacuo of the field. Then the two particle Hamiltonian is found to be

$$\begin{aligned} \hat{H} \;=\; & \hat{M}_r \otimes I + I \otimes \hat{M}_{ren} + \frac{1}{2}\hat{M}^{-1}\hat{\mathbf{p}}^2 \otimes I \\ & + \frac{1}{2}\hat{M}^{-1} I \otimes \hat{\mathbf{p}}^2 - G\frac{\hat{M} \otimes \hat{M}}{|\hat{\mathbf{x}} \otimes I - I \otimes \hat{\mathbf{x}}|}, \end{aligned} \tag{10}$$

where $\hat{M}_r = \hat{M} - (\pi\delta^2)^{-1/2} G \hat{M}^2$ is the renormalized mass matrix and $\langle \xi \,|\, \hat{M} \,|\eta\rangle = \sum_{AB} \int \mathrm{d}^3\mathbf{x} \bar{\xi}_A(\mathbf{x}) M_{AB} \eta_B(\mathbf{x})$. By projecting it in the corresponding subspace, one can obtain the Hamiltonian for an arbitrary number of particles.

Wang, Liu, Jing, Adv. Quant. Tech. 2, 1900003 (2019)

The estimation of time difference



Wang, Liu, Jing, Adv. Quant. Tech. 2, 1900003 (2019)

Quantum coherence for accelerated detectors

The initial state of the detectors is

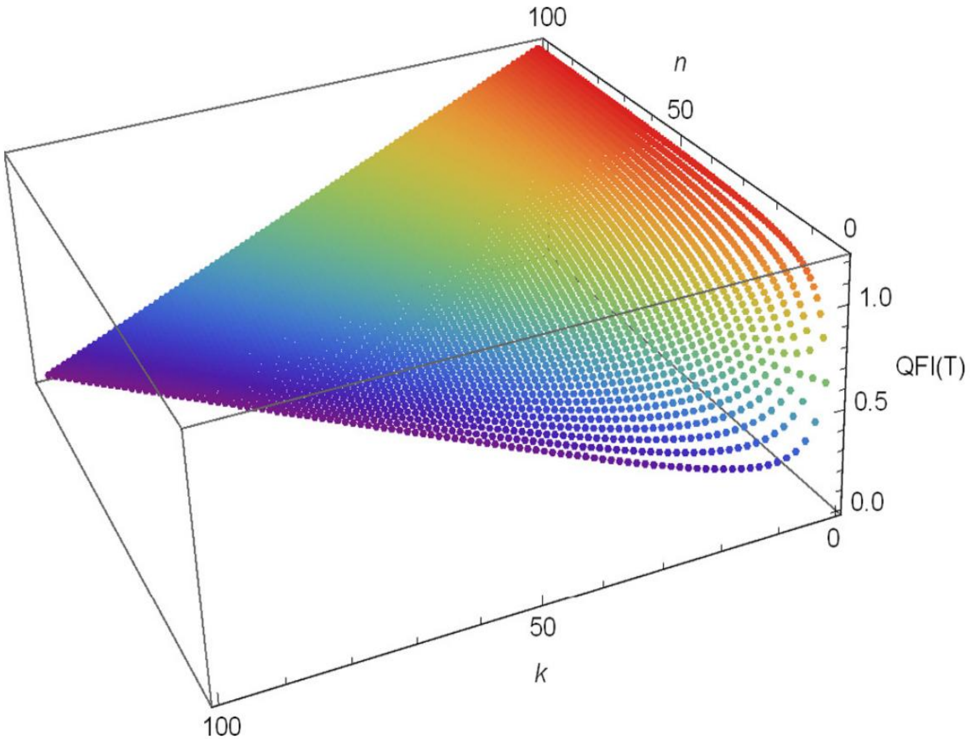
$$|\psi_{t_0}\rangle = \sqrt{\frac{(n-k)!k!}{n!}}(|111...000\rangle + |11...01...00\rangle + ... + |000...111\rangle),$$

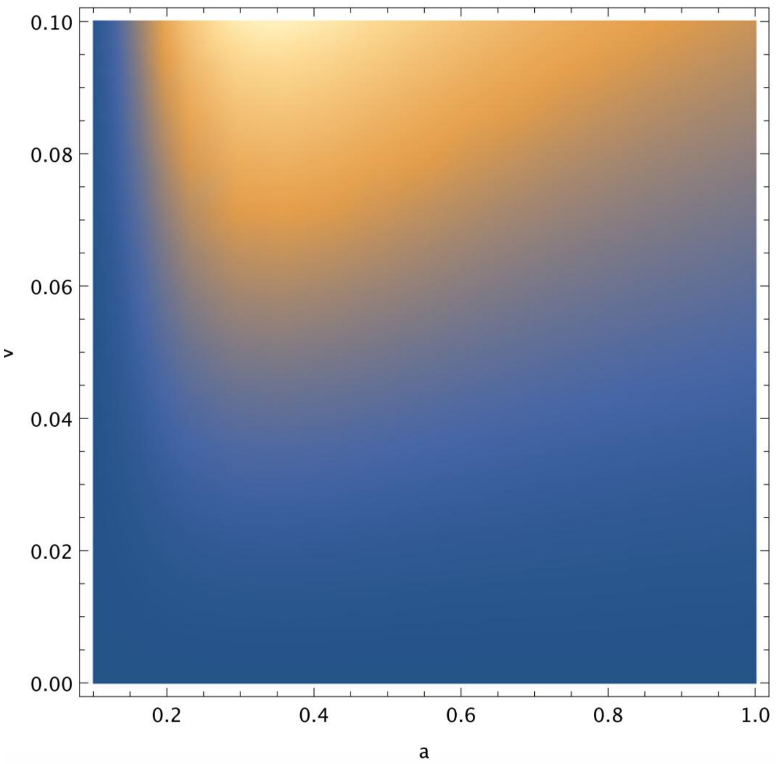
QFI for the final state is found to be

$$\mathcal{F}_Q(T) = \frac{\Omega^2 q v^2 [n^2 - (1-q)k^2 v^2 + (1-q)kn(v^2-1)]}{(1-q)T^4[(1-q)kv^2 + n(1-q+qv^2)]^2}.$$

Wang, Zhang, Chen, Jing, Phy. Lett. B 802, 135239(2020)

Estimating the Unruh effect via entangled probes





The quantum Fisher information for estimating the Unruh temperature T

Wang, Zhang, Chen, Jing, Phy. Lett. B 802, 135239(2020)

Conclusions：

1. 微引力系统的之间的纯引力相互作用可以产生量子纠缠和时间差.

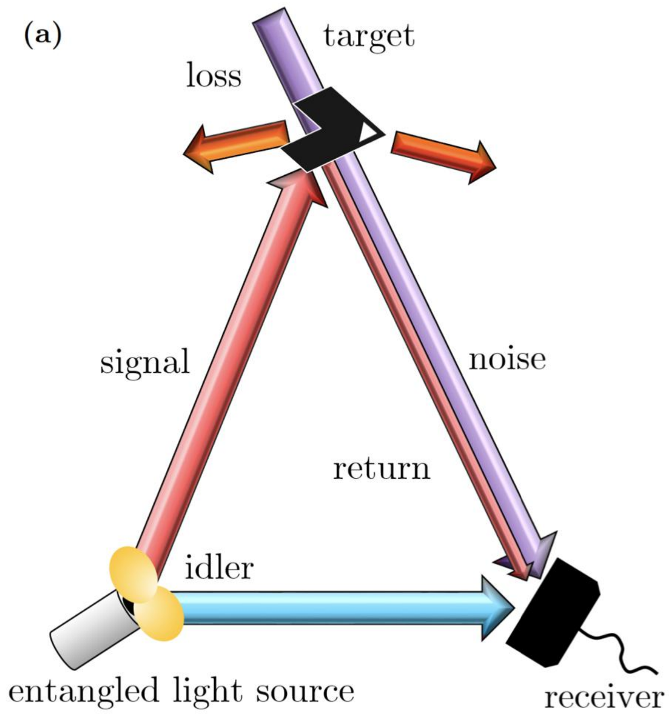
2. 时间差的量子精密测量参数与引力场的强度密切相关，且背景量子钟的能级差越大，量子精密测量的误差越小.

3. 量子钟之间因为纯引力相互作用产生的纠缠和通过测量消除的时间差是引力量子性的直接证据.

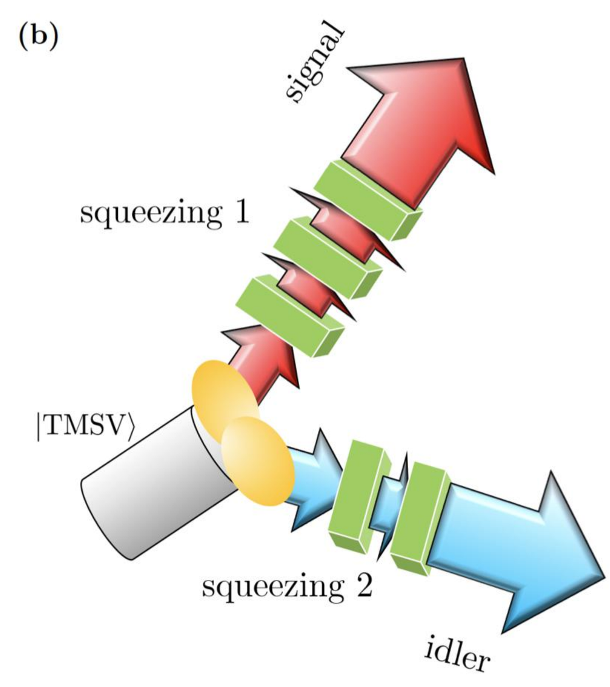
Gaussian quantum illumination

[1] S. Lloyd, Science 321, 1463 (2008);

[2] S. H. Tan et al.,Phys. Rev. Lett. 101, 253601(2008).



An object located in a noisy environment.



An object is not present.

Yonggi Jo et al., arXiv:2103.17006

The near-earth spacetime background

The reduced metric for Kerr metric in Boyer-Lindquist coordinates (t, r, ϕ) reads

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{\Delta}dr^2$$

$$+\left(r^2 + a^2 + \frac{2Ma^2}{r}\right)d\phi^2 - \frac{4Ma}{r}dt\, d\phi,$$

$$\Delta = 1 - \frac{2M}{r} + \frac{a^2}{r^2},$$

where M, r, J, $a = \frac{J}{M}$ are the mass, radius, angular momentum and Kerr parameter of the Earth, respectively. For the sake of simplicity, our work will be constrained to the equatorial plane $\theta = \frac{\pi}{2}$.

Liu, Wang and Jing, Adv. Quant. Tech. 1, 1800072 (2018)

How gravitational effects influence a photon?

A photon can be properly modeled by a wave packet of electromagnetic fields with a distribution $F^{(K)}_{\omega_{K,0}}$

$$a_{\omega_{K,0}}(t_K) = \int_0^{+\infty} d\omega\Omega_K \, e^{-i\omega_K t_K} F^{(K)}_{\omega_{K,0}}(\omega_K) \, a_{\omega_K} ,$$

where $K = A, B$ labels either Alice or Bob, and ω_K is the physical frequencies as measured in their labs.

The relation between the frequency distributions $F^{(K)}_{\Omega_{K,0}}$ of the photons before and after the propagation is

$$F^{(B)}_{\Omega_{B,0}}(\Omega_B) = \sqrt[4]{\frac{f(r_B)}{f(r_A)}} F^{(A)}_{\Omega_{A,0}} \left(\sqrt{\frac{f(r_B)}{f(r_A)}} \Omega_B \right) .$$

Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

The fidelity of the gravity-induced noise channel

Indeed, such a nonlinear gravitational effect is found to influence the fidelity of the quantum channel

$$\bar{a}' = \Theta a' + \sqrt{1-\Theta^2}\, a'_{\perp},$$

where Θ is the wave packet overlap between the distributions $F^{(B)}_{\Omega_{B,0}}(\Omega_B)$ and $F^{(A)}_{\Omega_{A,0}}(\Omega_B)$,

$$\Theta := \int_0^{+\infty} d\Omega_B\, F^{(B)\star}_{\Omega_{B,0}}(\Omega_B) F^{(A)}_{\Omega_{A,0}}(\Omega_B),$$

and we have $\Theta = 1$ for a perfect channel.

Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

How gravitational effects influence the final state?

We assume that Alice employs a real normalized Gaussian wave packet

$$F_{\Omega_0}(\Omega) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{-\frac{(\Omega-\Omega_0)^2}{4\sigma^2}},$$

with wave packet width σ. In this case the overlap Θ is given by

$$\Theta = \sqrt{\frac{2}{1+(1+\delta)^2} \frac{1}{1+\delta}} e^{-\frac{\delta^2 \Omega_{B,0}^2}{4(1+(1+\delta)^2)\sigma^2}},$$

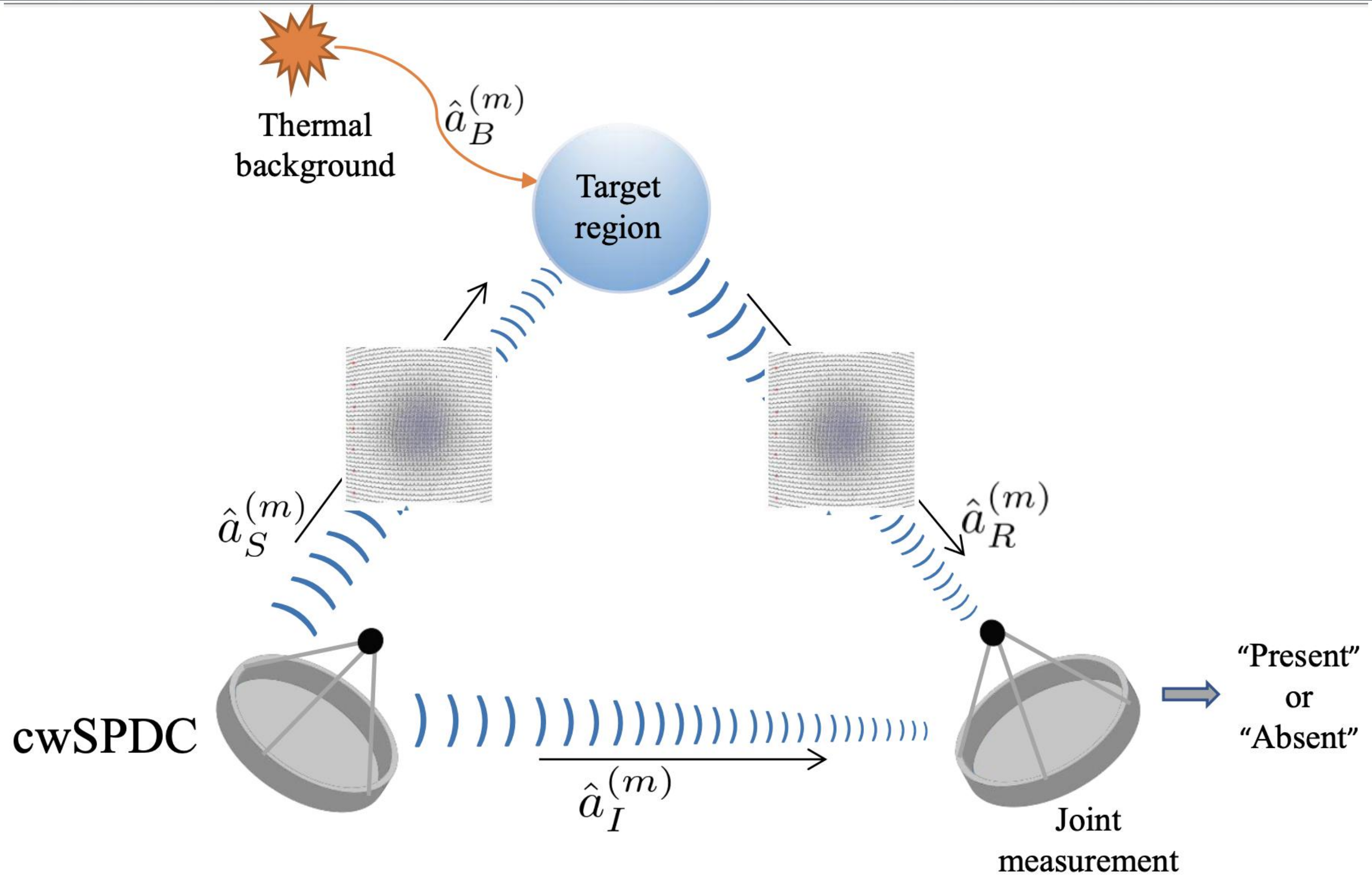
with δ being

$$= \frac{1}{8} \frac{r_S}{r_A} \left(\frac{1 - 2\frac{h}{r_A}}{1 + \frac{h}{r_A}} \right) - \frac{(r_A \omega)^2}{4} - \frac{(r_A \omega)^2}{4} \left(\frac{3}{4} \frac{r_S}{r_A} - \frac{4Ma}{\omega r_A^3} \right),$$

where $h = r_B - r_A$.

Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

Gravity enhanced quantum spatial target detection



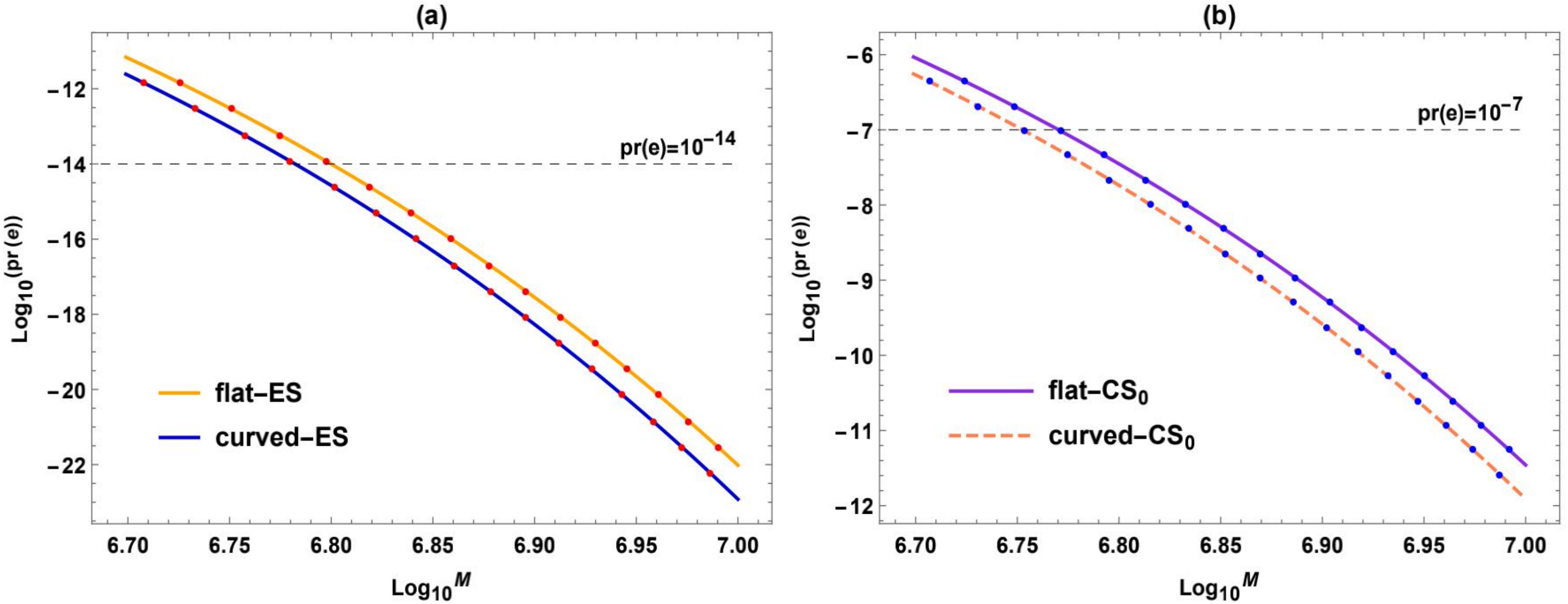
纠缠光源：$$|\phi\rangle_{IS} = \sum_{n=0}^{\infty} \sqrt{\frac{N_S^n}{(N_S+1)^{n+1}}} |n\rangle_I |n\rangle_S,$$

Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

Gravity enhanced quantum spatial target detection

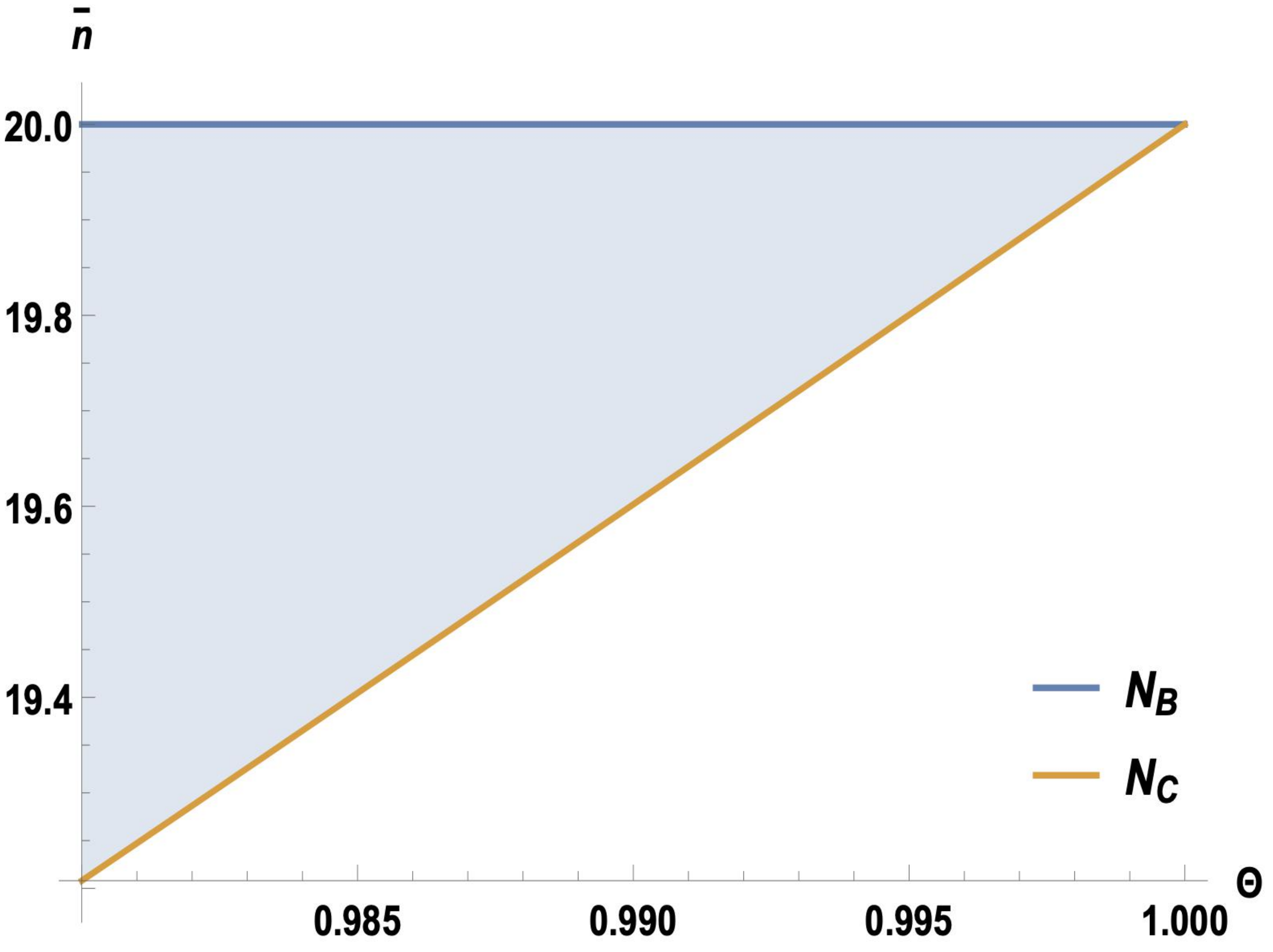
$$\Pr(e)_{\mathrm{CS}_0} \geq \frac{1}{2}\left(1-\sqrt{1-\exp[-2M\kappa\,\Theta^2 N_S(\sqrt{N_B+1/\Theta^2}-\sqrt{N_B})^2]}\right)$$

$$\approx \frac{1}{4}\exp[-M\kappa\,N_S/2\Theta^2 N_B], \qquad \text{when } N_B \gg 1 \text{ and } M\kappa\,\Theta^2 N_S/2N_B \gg 1.$$



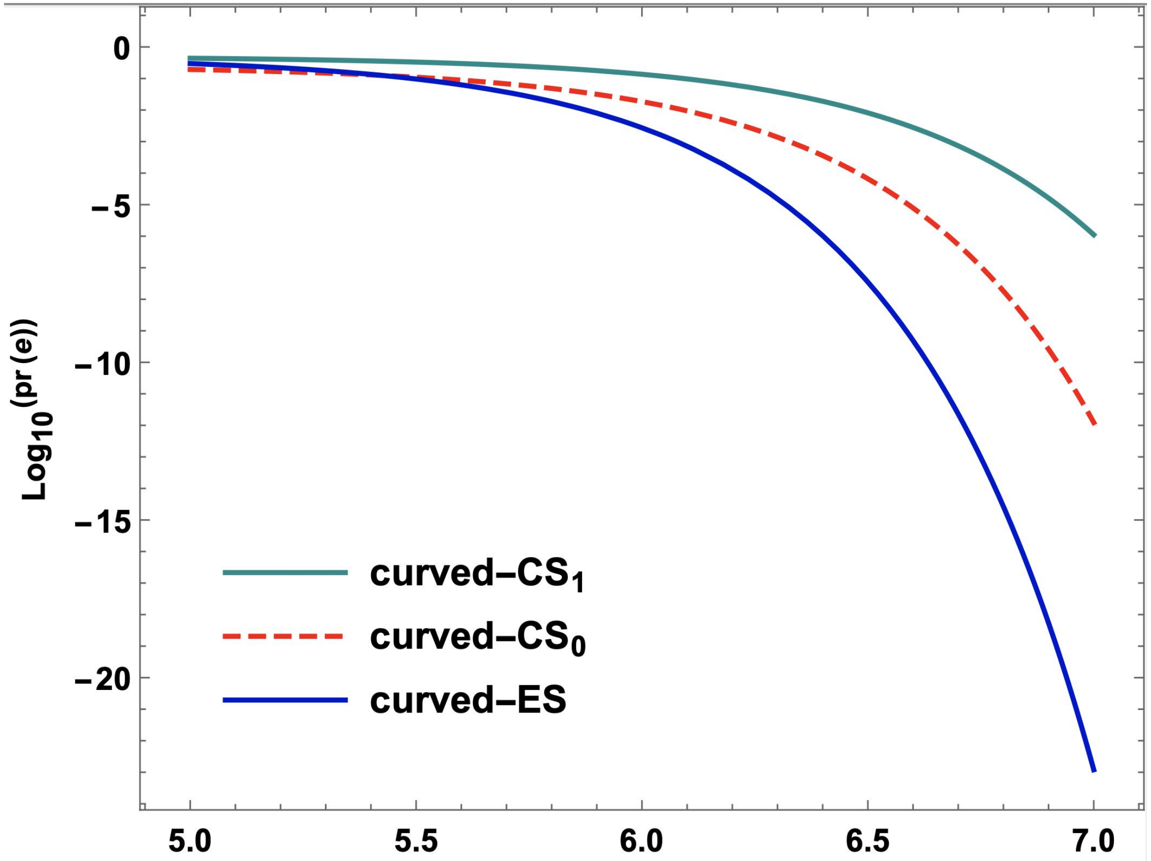
Liu, Wen, Tian, Jing, Wang, arXiv: 2104.02314 (2021)

The average particle number of reflected thermal signal



Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

The bounds on the target-detection error-probability



The bounds on the target-detection error-probability as a function of the copies of transmitted modes M

Liu,Wen, Tian, Jing,Wang, arXiv: 2104.02314 (2021)

结论:

1. 在地球弱引力背景下，纠缠态资源量子照明和相干态资源量子照明的误差概率都会降低.

2. 对于相同的误差，弯曲时空下的量子照明方案所需要的量子资源更少.

3. 引力场的存在可以减少热光子的反射，从而可以实现更低误差率的空间目标探测.

Thanks!