“引力的量子性质”
高级研讨班

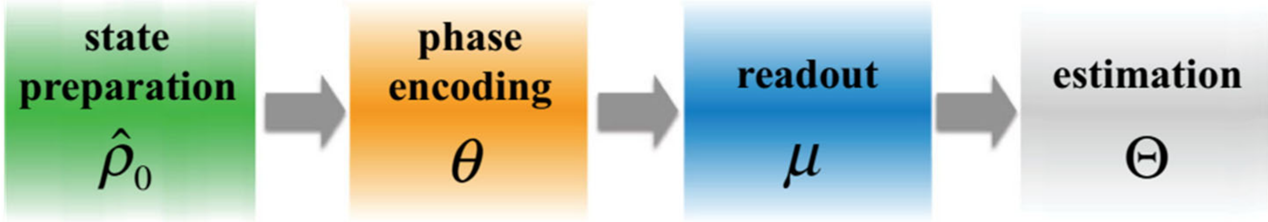
From gravitational wave detection to beating Heisenberg limit

Geng Chen (陈 耕)
Key Laboratory of Quantum Information, CAS
USTC

Outlines

1. Background and paradigms of optical metrology
2. Suppress technical noises to achieve **standard quantum limit**
3. Achieve **Heisenberg limit** and provide precision advantage
4. **Super-Heisenberg limited metrology**
5. Quantum **nonlinear** enhancement of precision
6. Discussions and perspective

Background and paradigms



(i) preparation of the probe state $\hat{\rho}_0$

(ii) encoding of a phase shift θ that depends on the physical quantity of interest

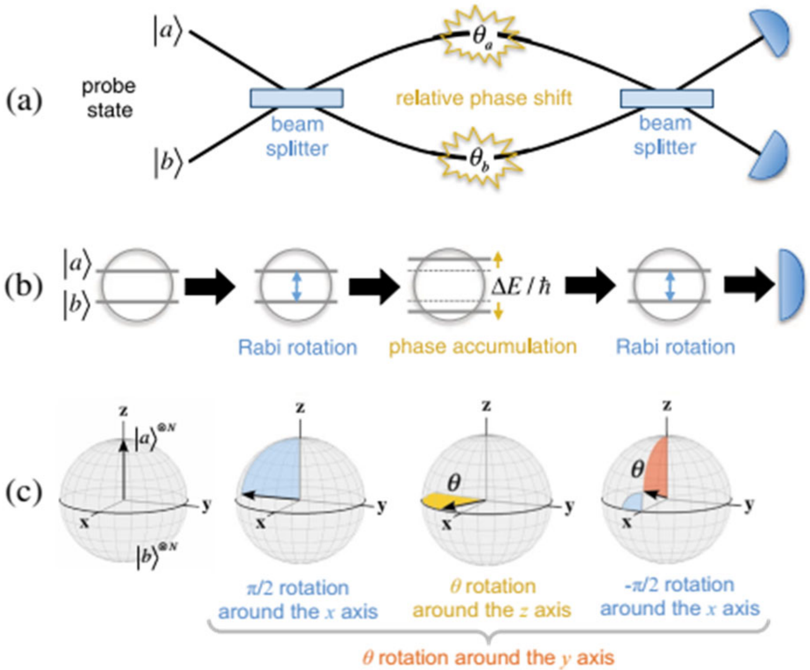
(iii) readout, where μ indicates ageneric measurement result

(iv) estimation, where the estimator $\Theta(\mu)$ is a function of the measurement result(s)

The uncertainty Δθ of the estimation depends crucially on all of these operations

Luca Pezze *et al.*, REVIEWS OF MODERN PHYSICS **90**, 035005 (2018)

Background and paradigms



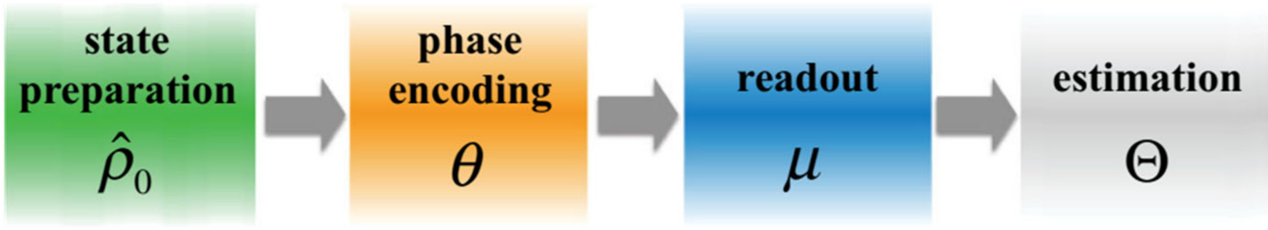
(a) **Mach-Zehnder interferometer**
relative phase between spatial modes

(b) **Ramsey interferometer on internal energy level**
relative phase depending on energy difference

(c) **Spin interferometer**
relative phase depending on spin rotation

Luca Pezze *et al.*, REVIEWS OF MODERN PHYSICS **90**, 035005 (2018)

Background and paradigms



Fisher information

$$F(\theta) = \sum_{\mu} \frac{1}{P(\mu|\theta)} \left(\frac{\partial P(\mu|\theta)}{\partial \theta} \right)^2$$

Cramér-Rao bound reads

$$\Delta\theta \geq \Delta\theta_{\mathrm{CR}} = \frac{1}{\sqrt{\nu F(\theta)}},$$

Maximizing over all possible generalized measurements in quantum mechanics

$$F_{\mathrm{Q}}[\hat{\rho}_\theta] = \max_{\{\hat{E}\}} F(\theta) \qquad F(\theta) \leq F_{\mathrm{Q}}[\hat{\rho}_\theta]$$

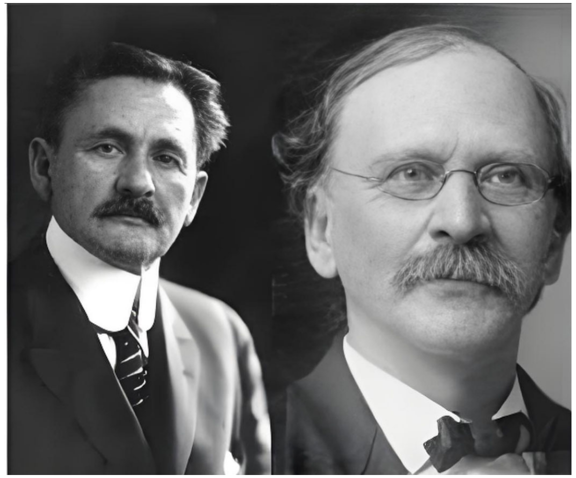
quantum Cramér-Rao bound

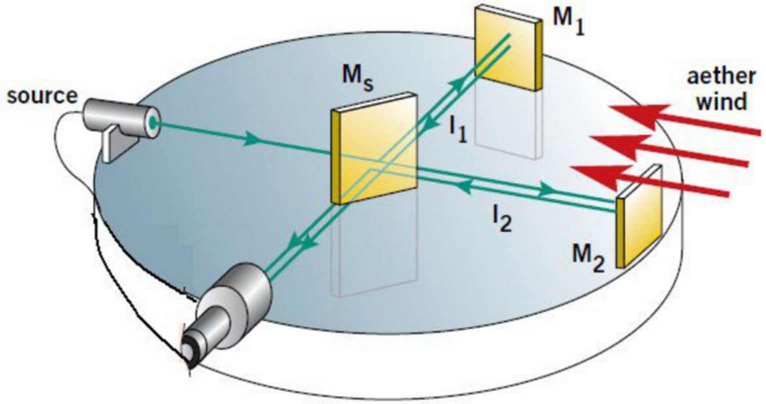
$$\Delta\theta_{\mathrm{CR}} \geq \Delta\theta_{\mathrm{QCR}} = \frac{1}{\sqrt{\nu F_{\mathrm{Q}}[\hat{\rho}_\theta]}}$$

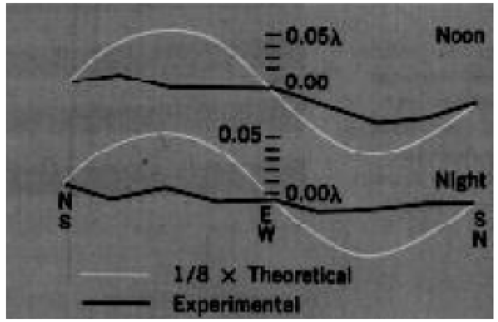
Luca Pezze *et al.*, REVIEWS OF MODERN PHYSICS **90**, 035005 (2018)

Background and paradigms

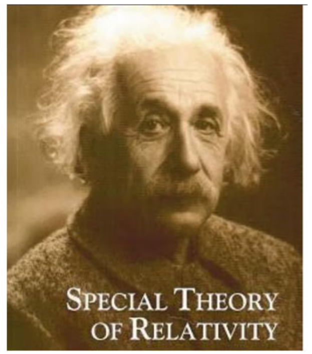
Michelson-Morley Experiment Detecting the aether wind







One black cloud leading to the development of Special Theory of Relativity



Background and paradigms

Gravitational wave detection

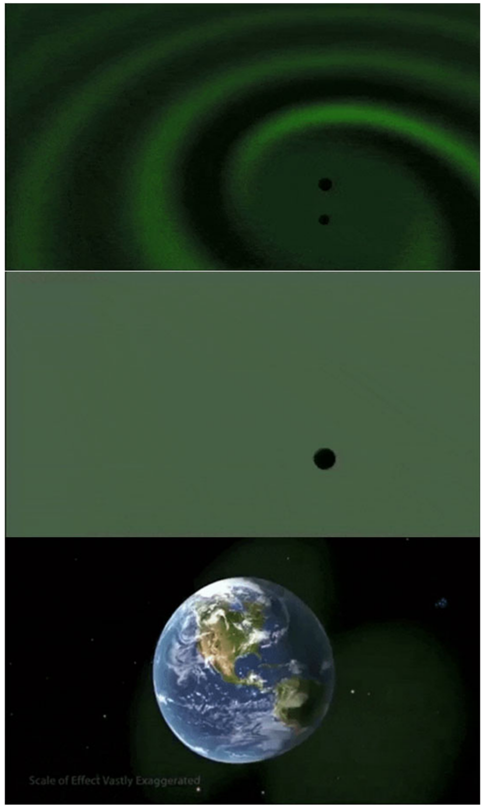
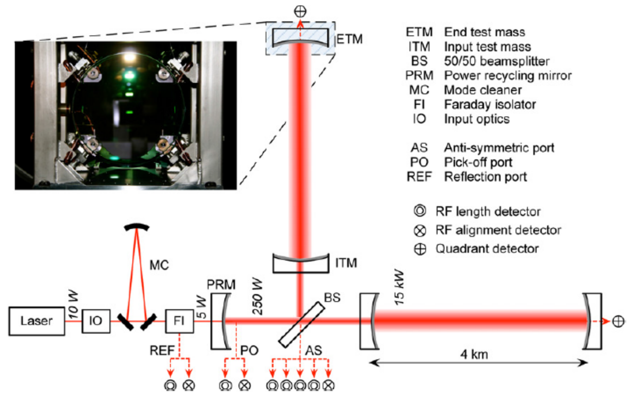
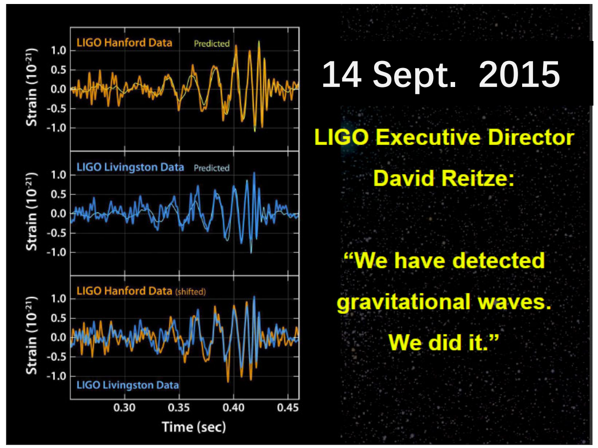
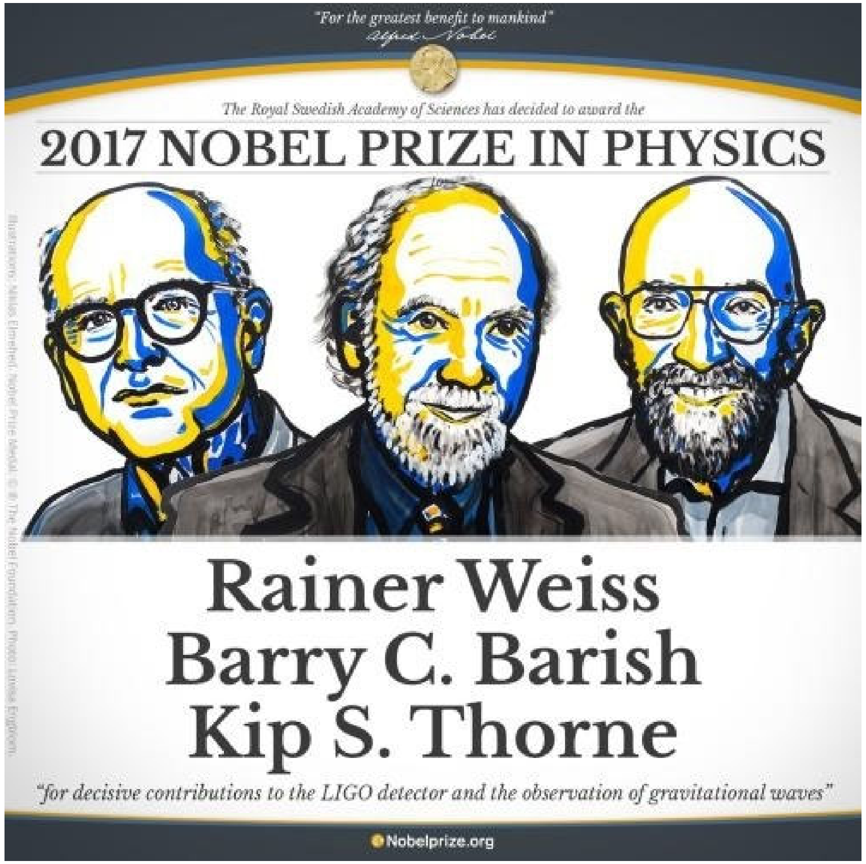


Figure 2. Aerial photograph of the LIGO observatories at Hanford, Washington (top) and Livingston, Louisiana (bottom). The lasers and optics are contained in the large corner buildings. From each corner building, evacuated beam tubes extend at right angles for 4 km in each direction (the full length of only one of the arms is seen in each photo); the tubes are covered by the arched, concrete enclosures seen here.





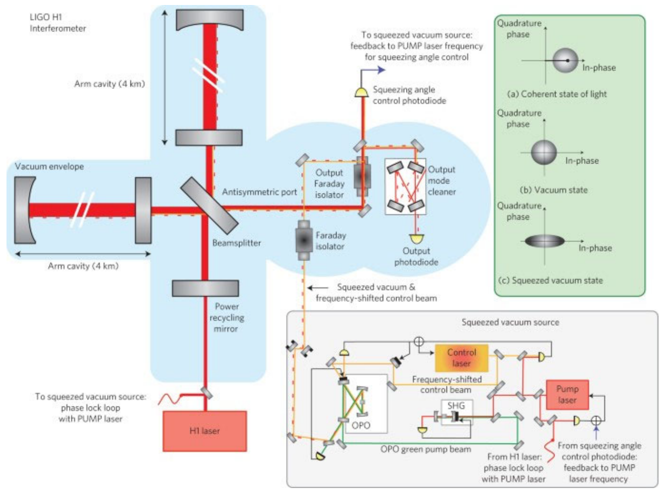
Background and paradigms



Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

J. Aasi, J. Abadie, [...] J. Zweizig

Nature Photonics **7**, 613–619 (2013) | Cite this article



Background and paradigms

Classical methods

Coherent state

Standard quantum limit (SQL)

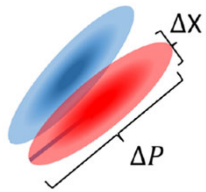
$$\Delta \sim \frac{1}{\sqrt{N}}$$

PRL **125**, 080501 (2020)

Light: Sci. & Appl. **10**, 103 (2021)

Quantum probes enhancing

Squeezed state



Surpassing SQL

$$\frac{1}{N} < \Delta < \frac{1}{\sqrt{N}}$$

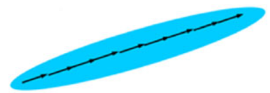
Beat SQL by utmost ～15dB

PRL **123**, 231108 (2019)

Nature **594**, 201 (2021)

Nature **581**, 159 (2020)

Entangled state



Heisenberg limit

$$\Delta \sim \frac{1}{N}$$

Low scalability

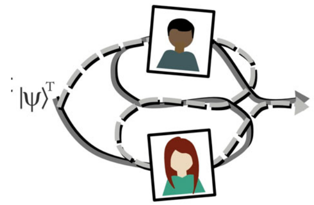
Science **316**, 726 (2007)

Nat. Photon. **11**, 700 (2017)

Nature **608**, 677 (2022)

Quantum evolution

Indefinite causal order



Super-Heisenberg limit

$$\Delta \sim \frac{1}{N^2}$$

Physical consistence?
Practical advantage?

PRL, **124** 090503 (2020)

Nat. Phys. **19**, 1122 (2023)

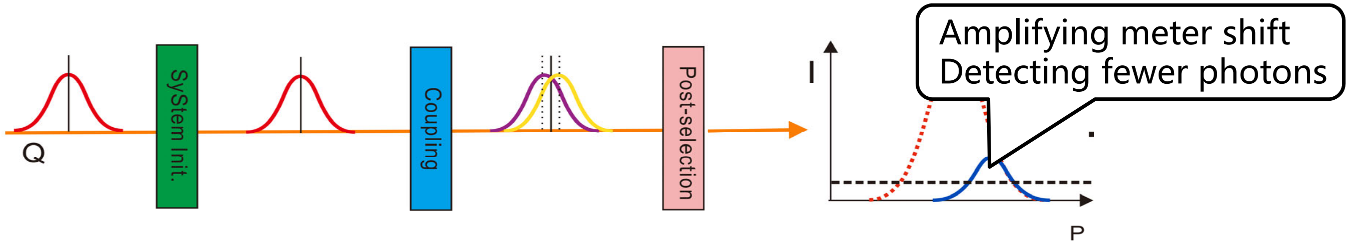
Suppress technical noises to achieve SQL

AAV amplification effect (1988)


Y. Aharonov


D. Albert


L. Vaidman



Weak value (amplify factor):

$$A_w \equiv \frac{\langle \phi | A | \psi \rangle}{\langle \phi | \psi \rangle}$$

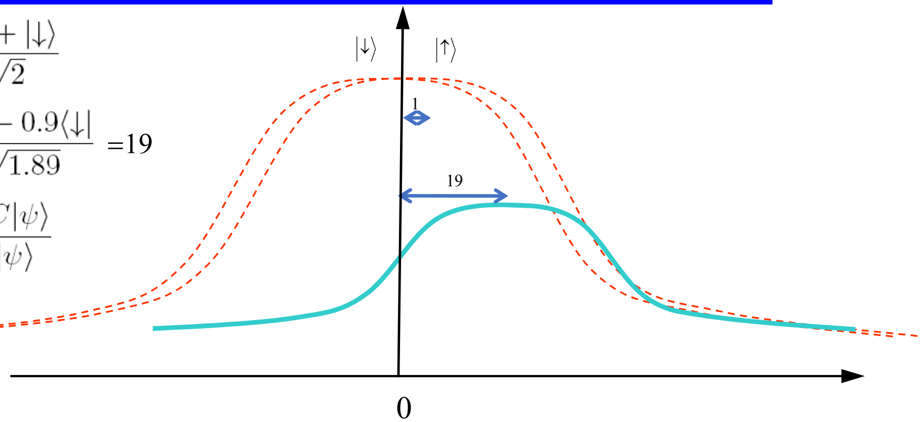
Post- & Pre-selection state

Amplification of the meter shift

$$|\psi\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\langle \phi | = \frac{\langle \uparrow | - 0.9 \langle \downarrow |}{\sqrt{1.89}} \quad =19$$

$$C_w = \frac{\langle \phi | C | \psi \rangle}{\langle \phi | \psi \rangle}$$



$$\delta Q = gReA_w$$

$$\delta P = 2g\mathrm{Var}(P) ImA_w$$

Phys. Rev. Lett. **60**, 1351–1354 (1988)

Suppress technical noises to achieve SQL

Science

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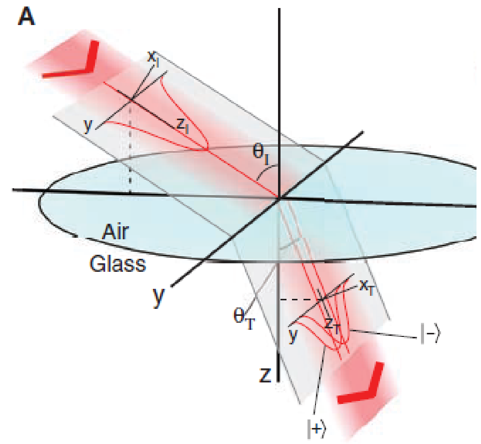
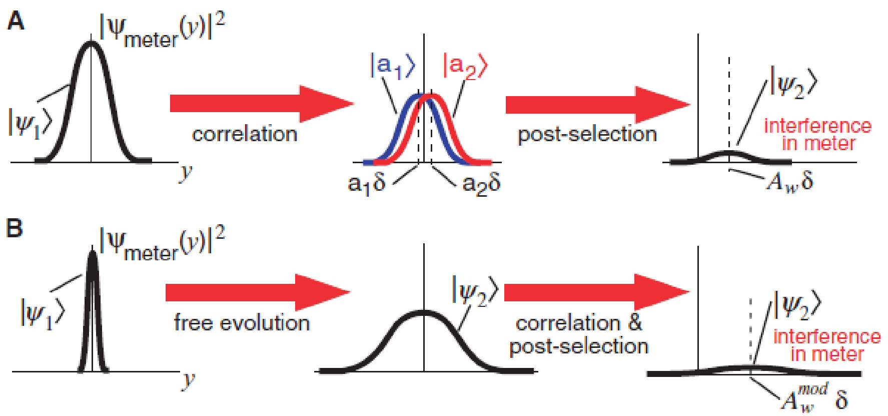
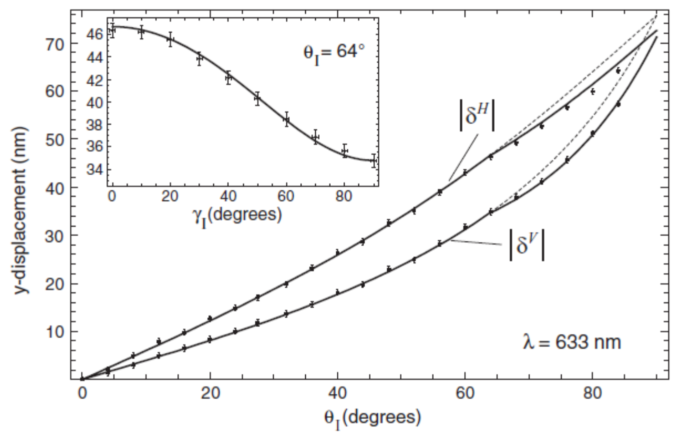
HOME > SCIENCE > VOL. 319, NO. 5864 > OBSERVATION OF THE SPIN HALL EFFECT OF LIGHT VIA WEAK MEASUREMENTS

REPORTS

Observation of the Spin Hall Effect of Light via Weak Measurements







Science **319**, 787 (2008)

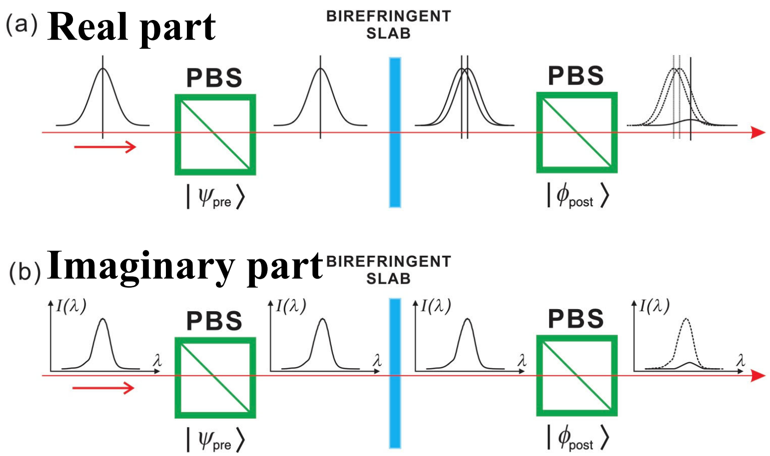
Suppress technical noises to achieve SQL

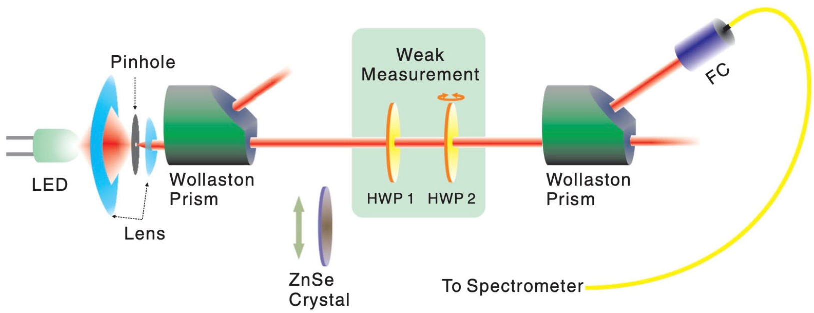
PRL **111,** 033604 (2013) PHYSICAL REVIEW LETTERS week ending 19 JULY 2013

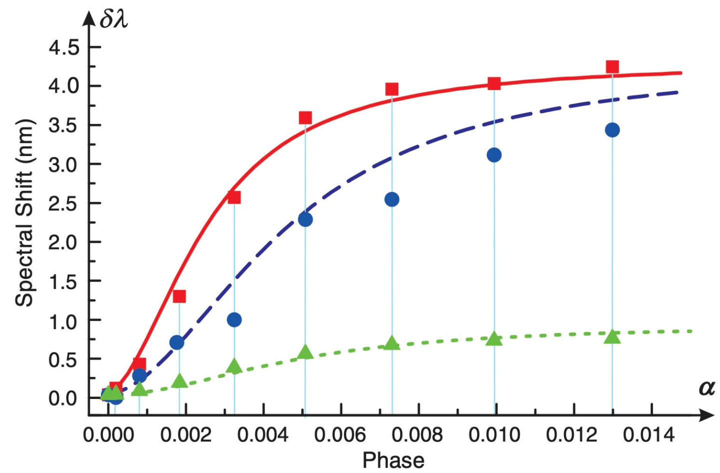
Phase Estimation with Weak Measurement Using a White Light Source

Xiao-Ye Xu,[1] Yaron Kedem,[2] Kai Sun,[1] Lev Vaidman,[2] Chuan-Feng Li,[1,*] and Guang-Can Guo[1]
[1]*Key Laboratory of Quantum Information, University of Science and Technology of China, CAS, Hefei 230026, People's Republic of China*
[2]*Raymond and Beverly Sackler School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel*
(Received 13 November 2012; published 17 July 2013)

We report results of a high precision phase estimation based on a weak measurement scheme using a commercial light-emitting diode. The method is based on a measurement of the imaginary part of the weak value of a polarization operator. The imaginary part of the weak value appeared due to the measurement interaction itself. The sensitivity of our method is equivalent to resolving light pulses of the order of a attosecond and it is robust against chromatic dispersion.







Phys. Rev. Lett. **111,** 033604 (2013)

Suppress technical noises to achieve SQL

Intense debates surrounding the weak value amplification

PRL **112,** 040406 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Weak Value Amplification is Suboptimal for Estimation and Detection

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 25 July 2013; revised manuscript received 21 November 2013; published 31 January 2014)

We show by using statistically rigorous arguments that the technique of weak value amplification does not perform better than standard statistical techniques for the tasks of single parameter estimation and signal detection. Specifically, we prove that postselection, a necessary ingredient for weak value amplification, decreases estimation accuracy and, moreover, arranging for anomalously large weak values is a suboptimal strategy. In doing so, we explicitly provide the optimal estimator, which in turn allows us to identify the optimal experimental arrangement to be the one in which all outcomes have equal weak values (all as small as possible) and the initial state of the meter is the maximal eigenvalue of the square of the system observable. Finally, we give precise quantitative conditions for when weak measurement (measurements without postselection or anomalously large weak values) can mitigate the effect of uncharacterized technical noise in estimation.

PRL **114,** 210801 (2015) PHYSICAL REVIEW LETTERS week ending 29 MAY 2015

Precision Metrology Using Weak Measurements

Lijian Zhang,[1,2,3,*] Animesh Datta,[4,5] and Ian A. Walmsley[5]
[1]*National Laboratory of Solid State Microstructures and College of Engineering and Applied Sciences, Nanjing University, Nanjing 210093, China*
[2]*Collaborative Innovation Center of Advanced Microstructures, Nanjing University, Nanjing 210093, China*
[3]*Max Planck Institute for Structure and Dynamics of Material, Hamburg 22761, Germany*
[4]*Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom*
[5]*Clarendon Laboratory, Department of Physics, University of Oxford, Oxford OX1 3PU, United Kingdom*
(Received 17 October 2013; revised manuscript received 16 June 2014; published 27 May 2015)

Weak values and measurements have been proposed as a means to achieve dramatic enhancements in metrology based on the greatly increased range of possible measurement outcomes. Unfortunately, the very large values of measurement outcomes occur with highly suppressed probabilities. This raises three vital questions in weak-measurement-based metrology. Namely, (Q1) Does postselection enhance the measurement precision? (Q2) Does weak measurement offer better precision than strong measurement? (Q3) Is it possible to beat the standard quantum limit or to achieve the Heisenberg limit with weak measurement using only classical resources? We analyze these questions for two prototypical, and generic, measurement protocols and show that while the answers to the first two questions are negative for both protocols, the answer to the last is affirmative for measurements with phase-space interactions, and negative for configuration space interactions. Our results, particularly the ability of weak measurements to perform at par with strong measurements in some cases, are instructive for the design of weak-measurement-based protocols for quantum metrology.

Phys. Rev. Lett. **111,** 033604 (2013)

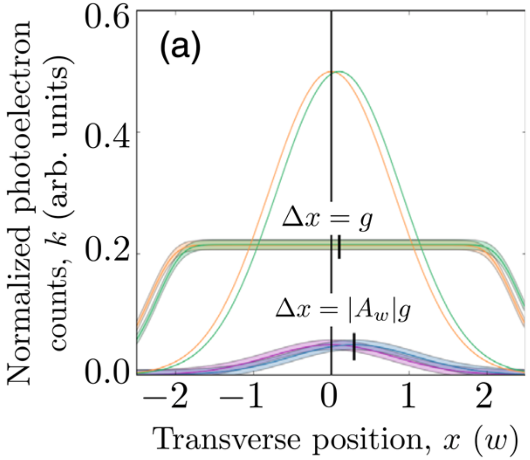
Suppress technical noises to achieve SQL

Intense debates surrounding the weak value amplification

PRL **118,** 070802 (2017) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2017

Weak Value Amplification Can Outperform Conventional Measurement in the Presence of Detector Saturation

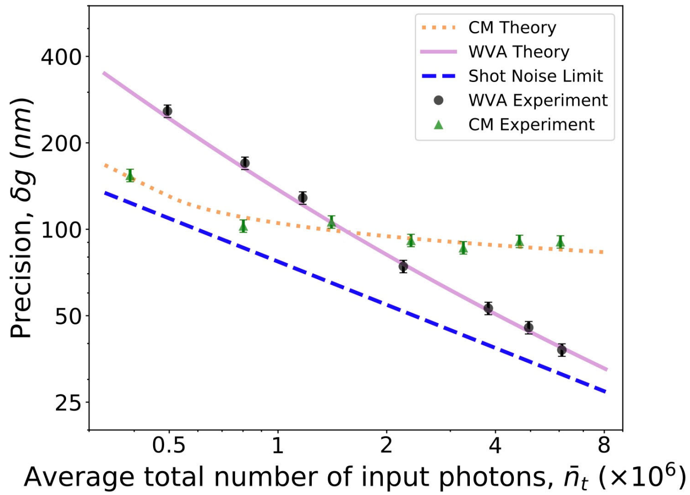
Jérémie Harris,[1,*] Robert W. Boyd,[1,2] and Jeff S. Lundeen[1]
[1]*Max Planck Centre for Extreme and Quantum Photonics, Department of Physics, University of Ottawa, 25 Templeton Street, Ottawa, Ontario K1N 6N5, Canada*
[2]*Institute of Optics, University of Rochester, Rochester, New York 14627, USA*
(Received 7 July 2016; published 15 February 2017)



PHYSICAL REVIEW LETTERS **125,** 080501 (2020)

Approaching Quantum-Limited Metrology with Imperfect Detectors by Using Weak-Value Amplification

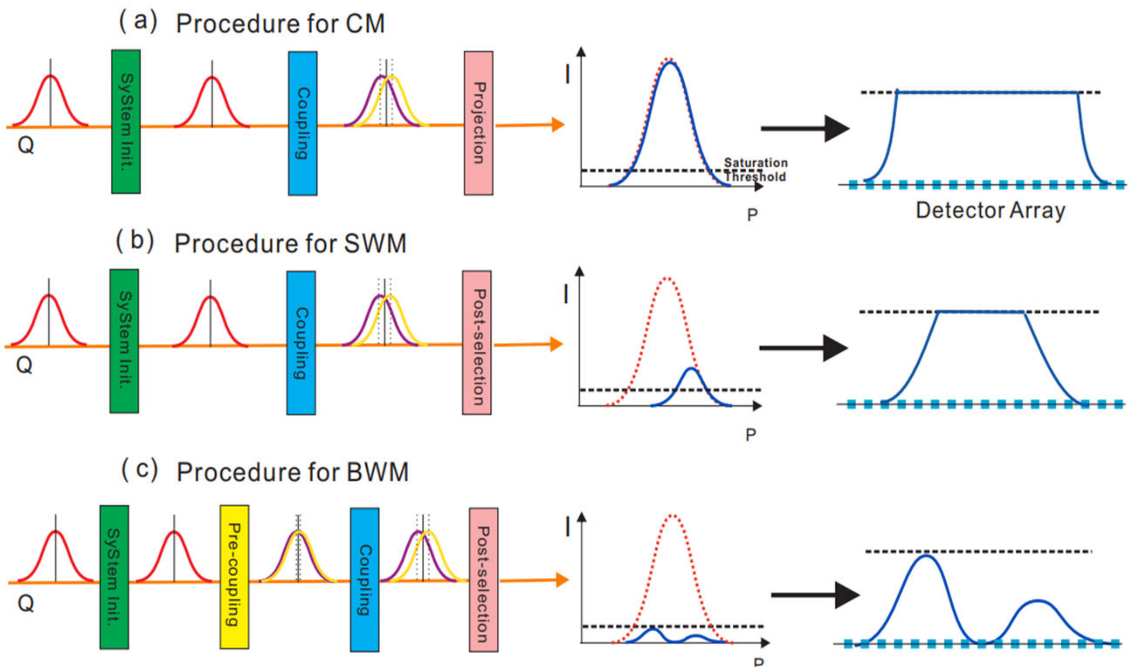
Liang Xu,[1] Zexuan Liu,[1] Animesh Datta,[2] George C. Knee,[2] Jeff S. Lundeen,[3] Yan-qing Lu,[1,*] and Lijian Zhang[1,†]



Phys. Rev. Lett. **111,** 033604 (2013)

Suppress technical noises to achieve SQL

Three Protocols



Meter shift

$$\delta p_{CM} = \frac{2k(\Delta p)^2 cos(2kp_0)}{sin(2kp_0) + e^{2k^2\Delta p^2}} \simeq 2k(\Delta p)^2.$$

$$\delta p_{SWM} = 2k(\Delta p)^2 \cot \epsilon \simeq \frac{2k(\Delta p)^2}{\epsilon}$$

$$\delta p_{BWM} \simeq \frac{2k(p_0)^2}{\epsilon}$$

Z.H. Zhang, **G. Chen*** …, C. F. Li*, *et al*., Phys. Rev. A **94**, 053843 (2016)

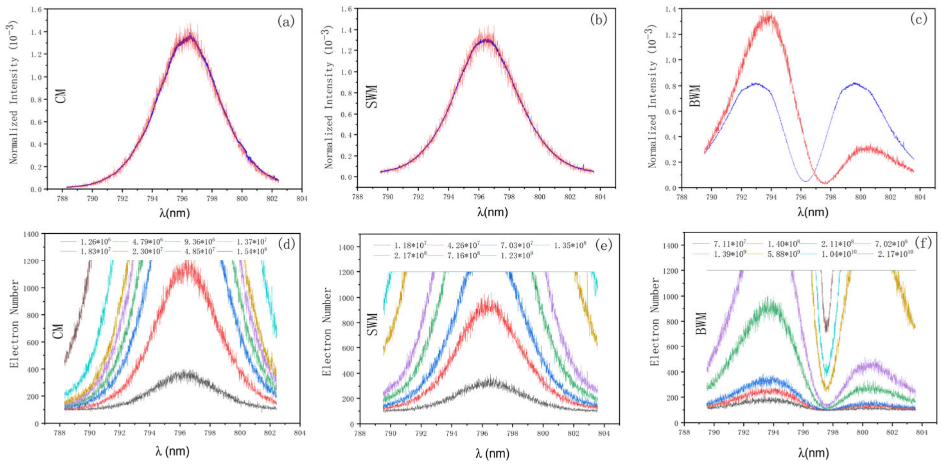
Suppress technical noises to achieve SQL

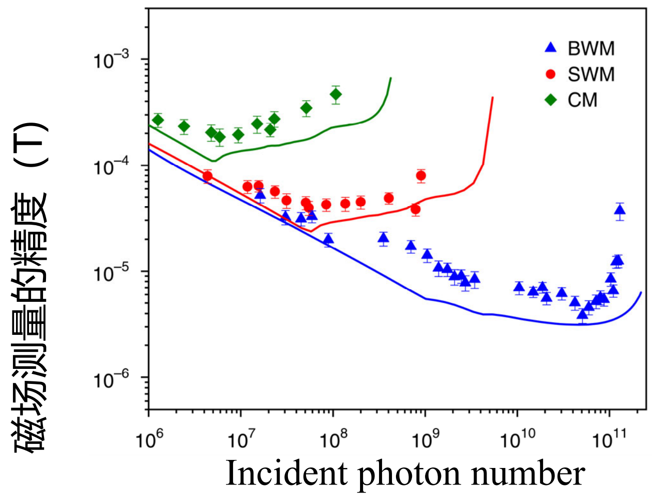
Yin et al. *Light: Science & Applications* (2021)10:103
https://doi.org/10.1038/s41377-021-00543-4

Official journal of the CIOMP 2047-7538
www.nature.com/lsa

ARTICLE Open Access

Improving the precision of optical metrology by detecting fewer photons with biased weak measurement





- Biased weak measurement outperforms conventional method and standard weak measurement by at least one order of magnitude.
- Biased weak measurement is more impervious to detector saturation and allows the usage of more photons

P. Yin, ..., **G. Chen***, C. F. Li*, *et al.*, Light: Science and Applications **10**, 103 (2021)

Suppress technical noises to achieve SQL

Conclusion on weak measurement

	Conventional measurement:	Weak measurement:
Postselection Probability:	1	ε^2
Sensitivity (Signal):	$s=\frac{\partial\langle Q\rangle}{\partial g}=c_i t$	$s=\frac{\partial\langle Q\rangle}{\partial g}=\frac{1}{\varepsilon}c_i t$
Uncertainty (Shot Noise):	$\triangle Q=\frac{\sigma}{\sqrt{N}}$	$\triangle Q=\frac{\sigma}{\sqrt{\varepsilon^2 N}}$ or σ
Precision:	$\Delta g=\frac{\triangle Q}{s}=\frac{\sigma}{c_i t\sqrt{N}}$	$\Delta g=\frac{\triangle Q}{s}=\frac{\sigma}{c_i t\sqrt{N}}$ or $\frac{\varepsilon\sigma}{c_i t}$

Weak measurement can suppress technical noises and achieve but not beat SQL by detecting fewer photons

Suppress technical noises to achieve SQL

Brief Summaries:

- ✓ Weak measurement (WM) cannot outperform conventional methods (CM) if the shot noise dominates; but when detector saturation occurs, WM can surpass CM by detecting fewer photons.

- ✓ Weak measurement can suppress technical noise due to the weak value amplification, and thus approaches the SQL and maintains the amplified meter shift.

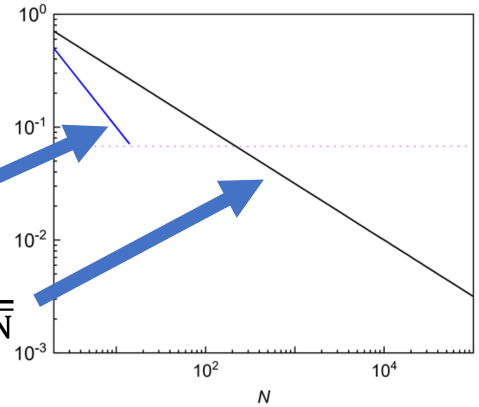
- ✓ Weak measurement cannot beat SQL since no quantum resources are exploited.

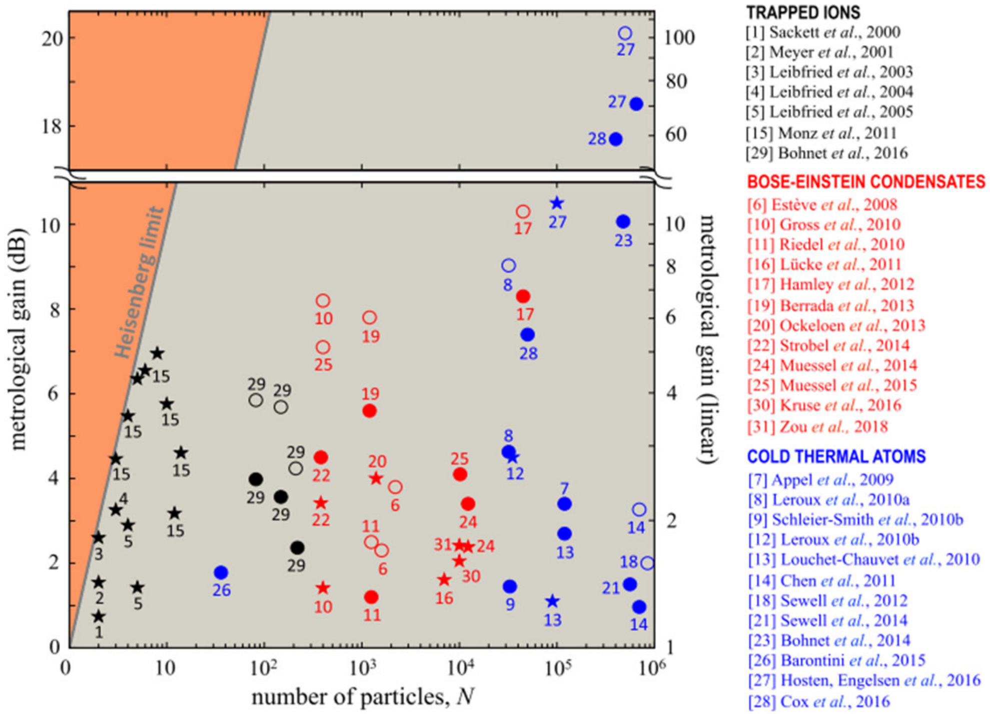
Achieve **Heisenberg limit** and provide precision advantage

Scaling advantage does not necessarily lead to precision advantage due to the low scalability and quality of the quantum probe

Heisenberg limit $\frac{1}{N}$

Standard quantum limit $\frac{1}{\sqrt{N}}$

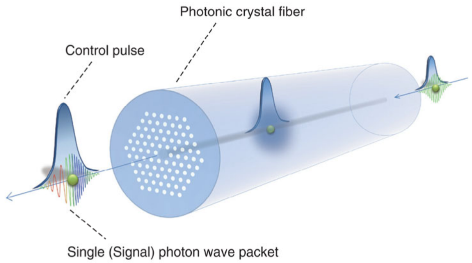




REVIEWS OF MODERN PHYSICS **90**, 035005 (2018)

Achieve **Heisenberg limit** and provide precision advantage

Approaching Heisenberg-limit with mixed state probe



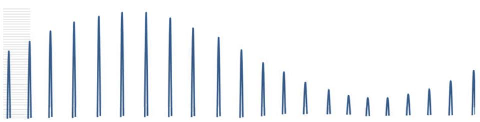
→ Quantum Logic Gate

→ Quantum non-demolition

Precision of WM: $\delta g \sim \frac{1}{|A_w|\Delta}$ —Coherent state→ $\Delta \sim \sqrt{n} \rightarrow \delta g \propto \frac{1}{\sqrt{n}}$

(A_w: weak value, Δ: SD of the probe) (n: mean photon number)

Modulate the coherent state and increase the variance up to $\Delta \sim n$

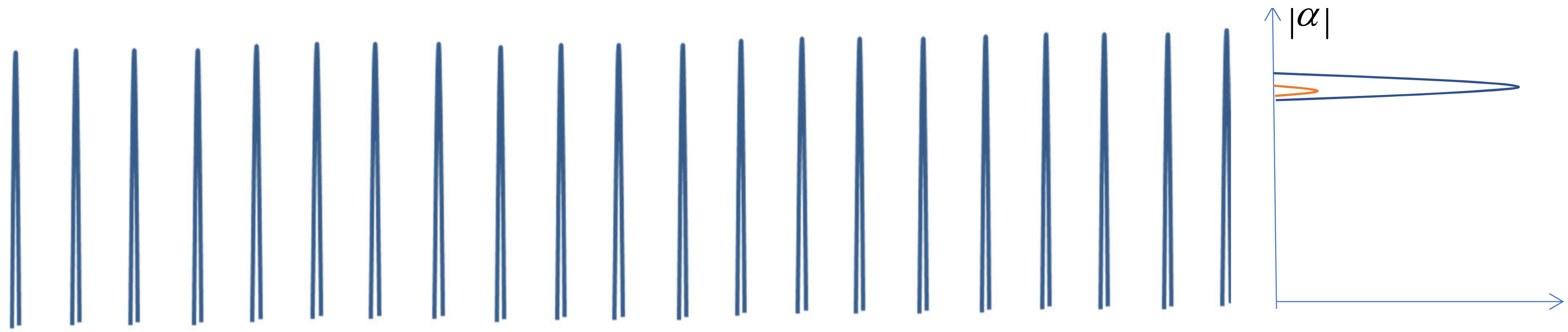


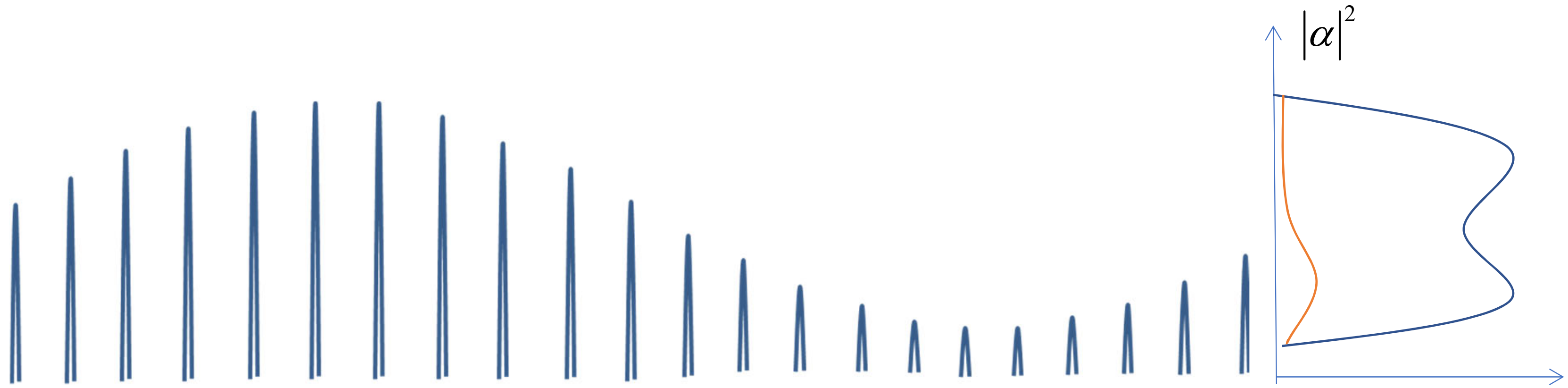
Intensity modulation → $\Delta \sim n \rightarrow \delta g \propto \frac{1}{n}$

G. Chen *et. al*., Nat. Commun. **9** , 93 (2018)

Achieve **Heisenberg limit** and provide precision advantage

Approaching Heisenberg-limit with mixed state probe

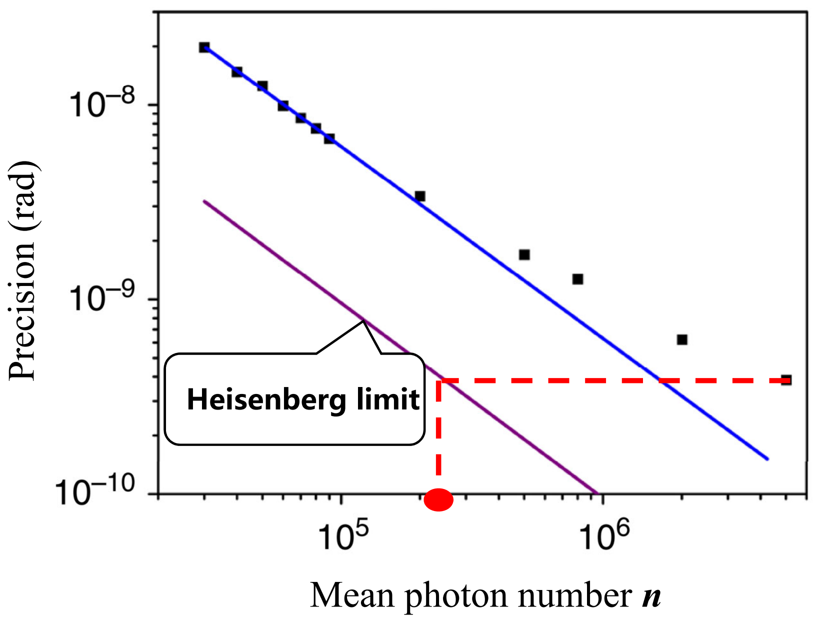




G. Chen *et. al*., Nat. Commun. **9** , 93 (2018)

Achieve **Heisenberg limit** and provide precision advantage

Approaching Heisenberg-limit with mixed state probe



- Merely takes use of single qubit superposition and achieves Heisenberg scaling of 6.3/n;

- The ultimate precision is approximtely 10^{-9}rad, which is equivalent to the precision using 100,000 maximally entangled photons, and outperforms former classical method by one order of magnitude (*Nat. Photon.* **3**, 95 (2008))。

G. Chen *et. al*., Nat. Commun. **9** , 93 (2018)

Achieve **Heisenberg limit** and provide precision advantage

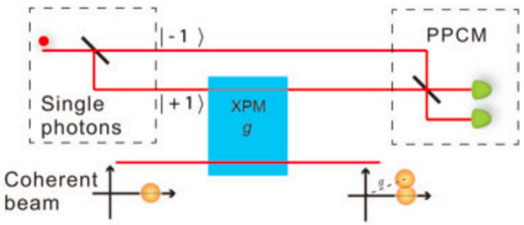
PHYSICAL REVIEW LETTERS **121,** 060506 (2018)

Achieving Heisenberg-Scaling Precision with Projective Measurement on Single Photons

Geng Chen,[1,2] Lijian Zhang,[3,4,*] Wen-Hao Zhang,[1,2] Xing-Xiang Peng,[1,2] Liang Xu,[3,4] Zhao-Di Liu,[1,2] Xiao-Ye Xu,[1,2] Jian-Shun Tang,[1,2] Yong-Nan Sun,[1,2] De-Yong He,[1,2] Jin-Shi Xu,[1,2] Zong-Quan Zhou,[1,2] Chuan-Feng Li,[1,2,†] and Guang-Can Guo[1,2]

[1]*CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, China*
[2]*Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[3]*National Laboratory of Solid State Microstructures and College of Engineering and Applied Sciences, Nanjing University, Nanjing, China*
[4]*Collaborative Innovation Center of Advanced Microstructures, Nanjing University, Nanjing 210093, China*

(a)



Fisher Info. Analysis

FI in projective probability: $F_p = n^2$

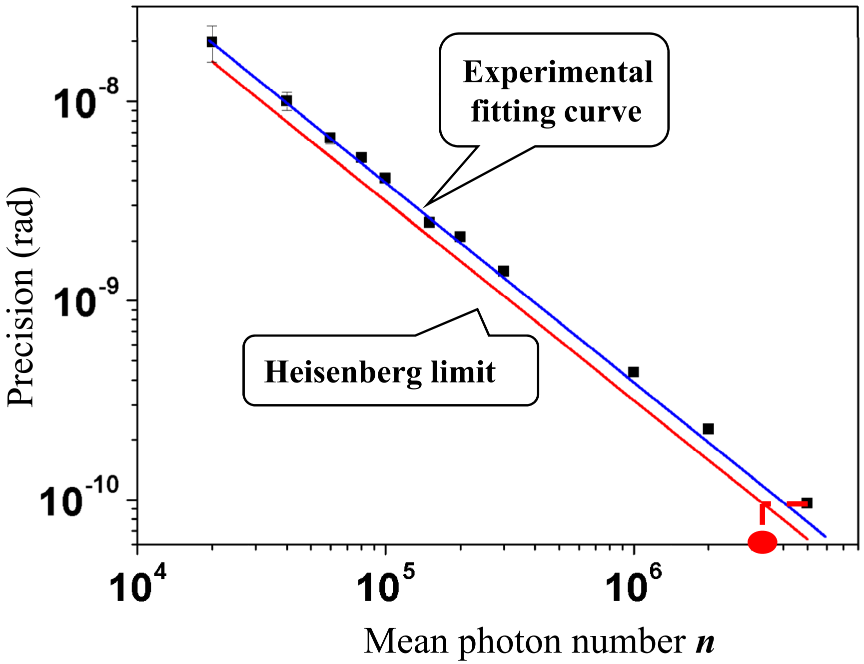
FI in post-selected state: $P_d Q_d = (1 - \varepsilon^2/4)n$

FI in discarded state: $P_r Q_r = \varepsilon^2 n/4$

L. J. Zhang *et al.*, Phys. Rev. Lett. **114**, 210801 (2015)

Achieve **Heisenberg limit** and provide precision advantage

Approaching HL by projective measurement on single photons



- Merely takes use of single qubit superposition and achieves Heisenberg scaling of 1.2/n;
- The ultimate precision is 10^{-10}rad, which is equivalent to the precision using 1,000,000 maximally entangled photons, and outperforms former classical method by two orders of magnitude (*Nat. Photon.* **3**, 95 (2008))。

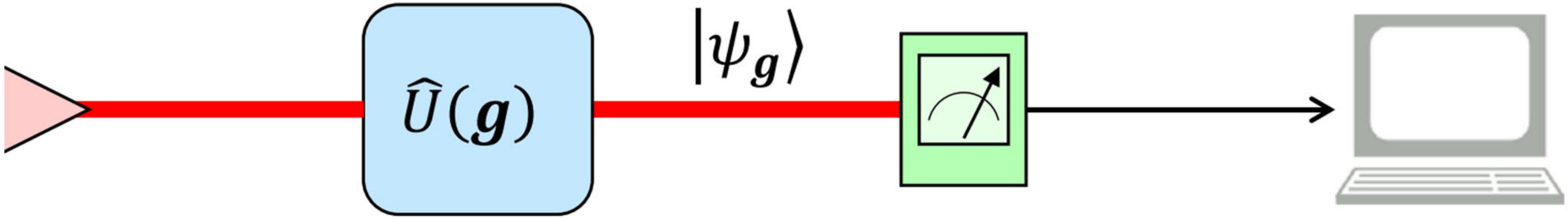
G. Chen *et al*., Phys. Rev. Lett. **121**, 060506 (2018)

Achieve **Heisenberg limit** and provide precision advantage

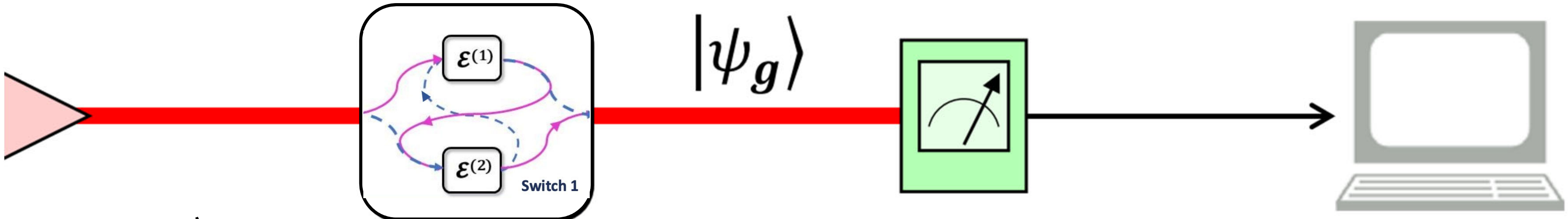
Brief Summaries:

- ✓ It is currently insurmountable to meanwhile achieve both scaling and precision advantages.
- ✓ In some specific scenarios, single qubits could render a scalable Heisenberg scaling and eventual achieve a better precision.
- ✓ Fisher information analysis is a useful to guide the experimental engineering.

Super-**Heisenberg limit** metrology

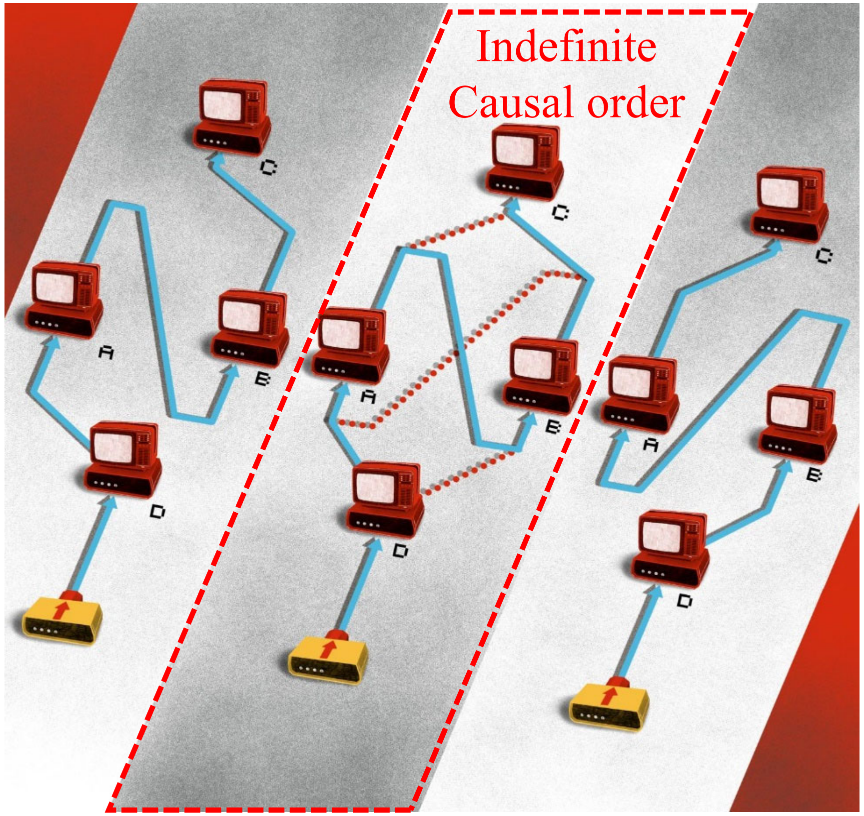


Quantum probe $|\varphi_0\rangle$ Evolution to encode g Readout Estimation



Classical probe $|\alpha\rangle$ Quantized Evolution to encode g Readout Estimation

Super-**Heisenberg limit** metrology



Definite causal order

The causal order is definite for two related events in classical world

Left： A occurs before B， A causes B；
Right： B occurs before A， B causes A

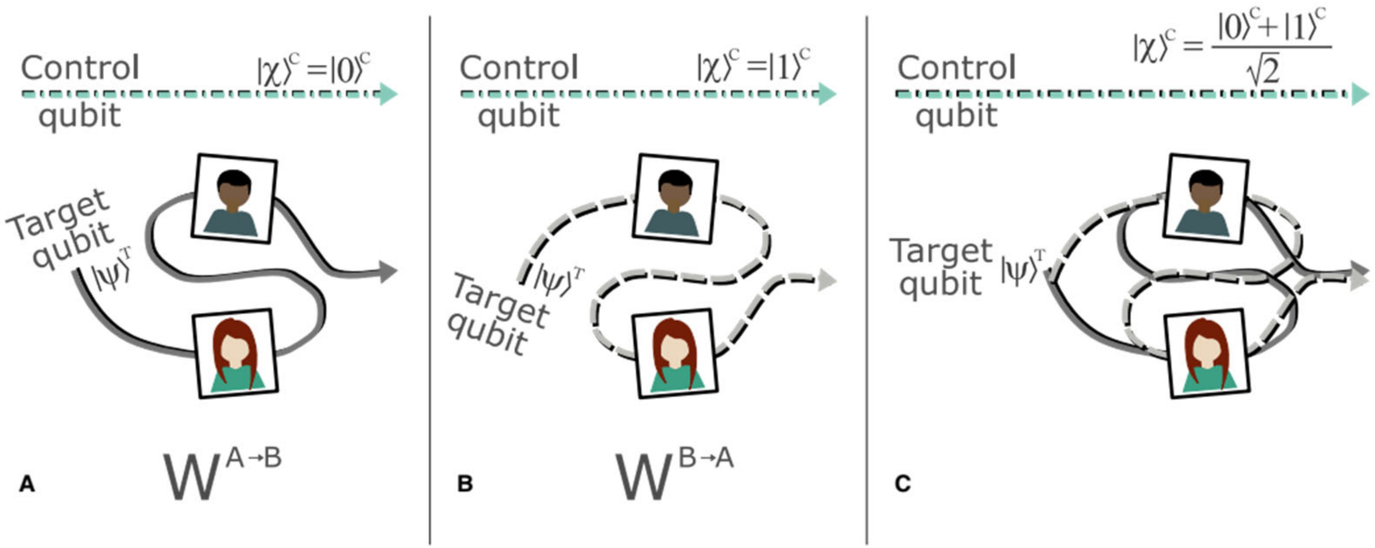
Indefinite causal order

Quantum mechanics allows the superposition of eigenstates, and also the **superposition of orders**

Middle： superposition of two alternative orders between A and B, forms a supermap of evolution

Super-**Heisenberg limit** metrology

To build this quantum supermap, a control qubit has to be introduced to control the evolution order, and the whole structure is named as **Quantum SWITCH**.

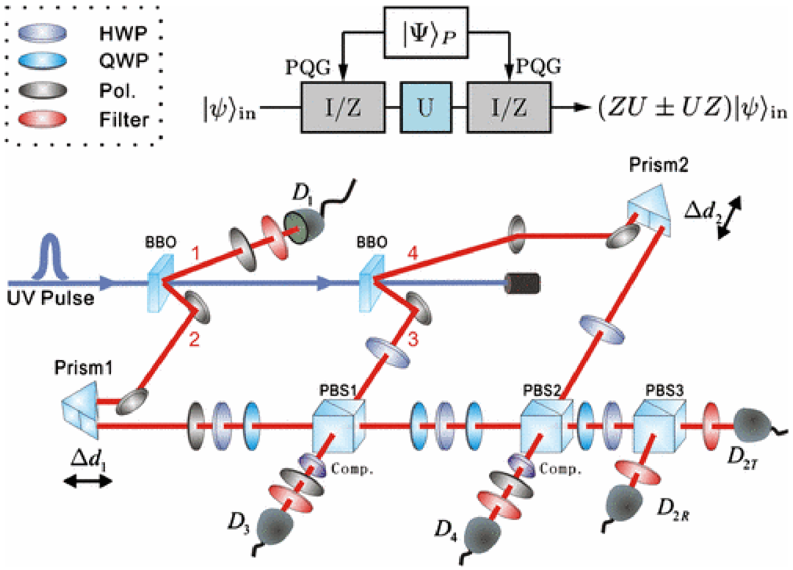


Sci. Adv. **3**: e1602589 (2017)

Super-**Heisenberg limit** metrology

Discriminating two unitary operations commute or anti-commute

$[U_A, U_B]|\psi> = 0 \; or \; \{U_A, U_B\}|\psi> = 0$



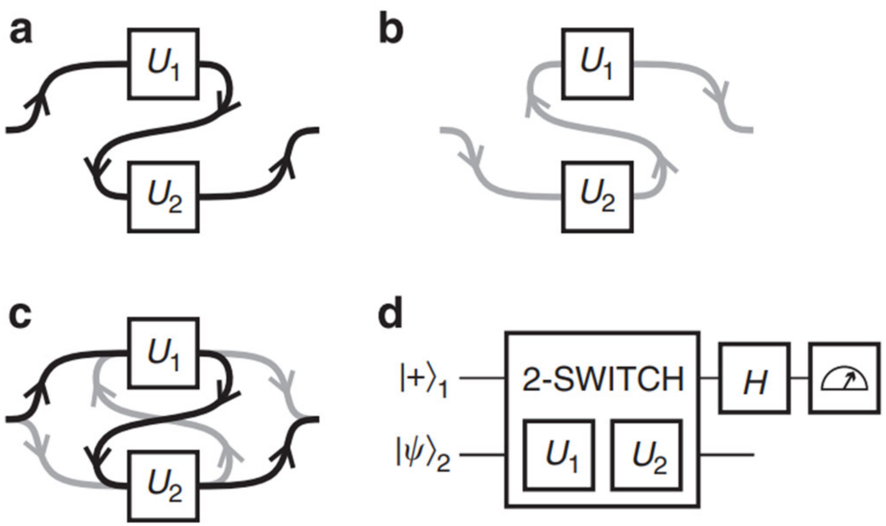
Definite casual order method: one of the gates has to used at least twice

Phys. Rev. Lett. **105**, 120402(2010)

Super-**Heisenberg limit** metrology

Discriminating two unitary operations commute or anti-commute

$$[U_A, U_B]|\psi> = 0 \; or \; \{U_A, U_B\}|\psi> = 0$$



I. Preparing control Qubit:

$$|+\rangle_c = 1/\sqrt{2}(|0\rangle_c + |1\rangle_c)$$

II. Imposing quantum switch on the target qubit:

$$\frac{1}{\sqrt{2}}(|0\rangle_c \otimes U_B U_A|\psi\rangle_t + |1\rangle_c \otimes U_A U_B|\psi\rangle_t)$$

III. Hadamard gate on the control qubit:

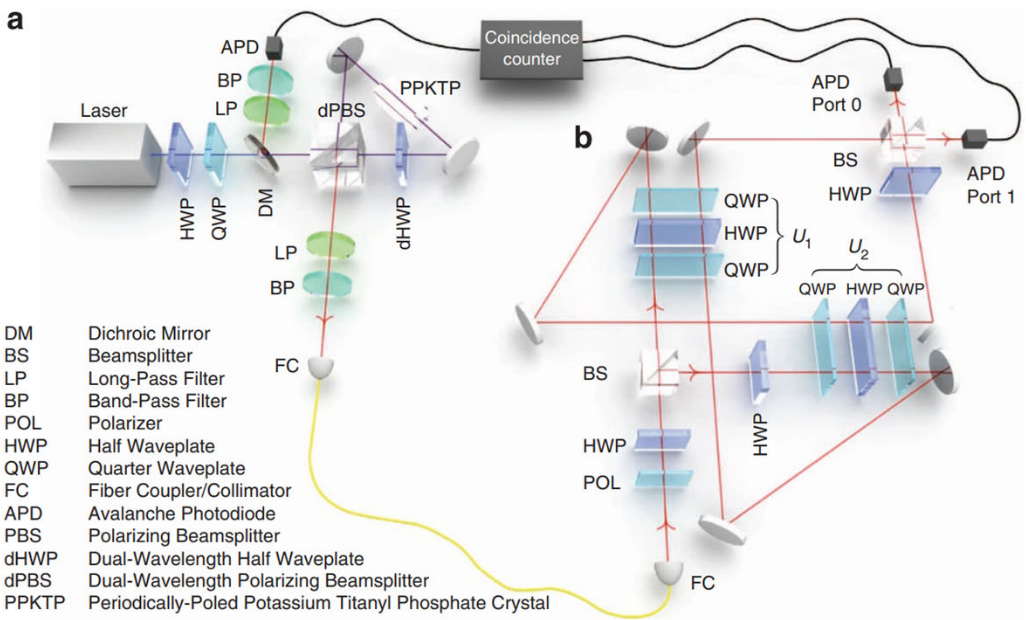
$$\frac{1}{2}(|0\rangle_c \otimes \{U_A, U_B\}|\psi\rangle_t - |1\rangle_c \otimes [U_A, U_B]|\psi\rangle_t)$$

Only one query of each black box

Phys. Rev. A **86**, 040301(R) (2012)

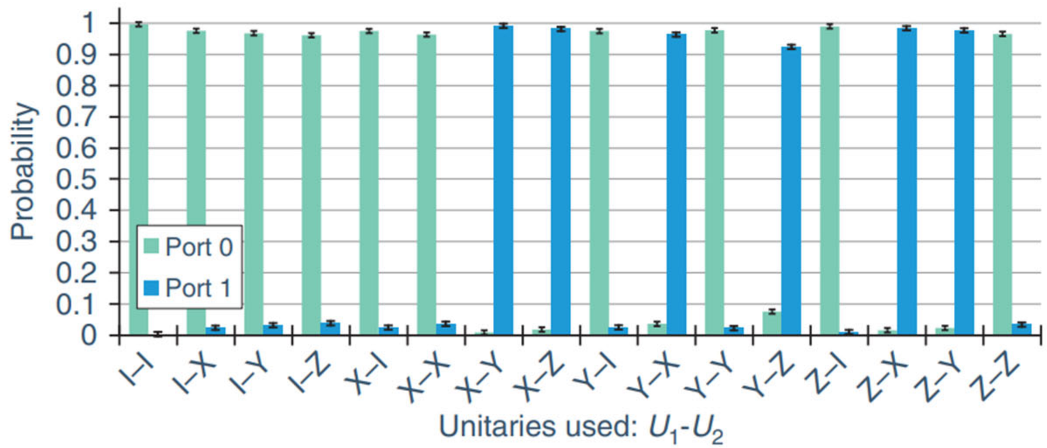
Super-**Heisenberg limit** metrology

Discriminating two unitary operations commute or anti-commute



Control qubit: paths of photons

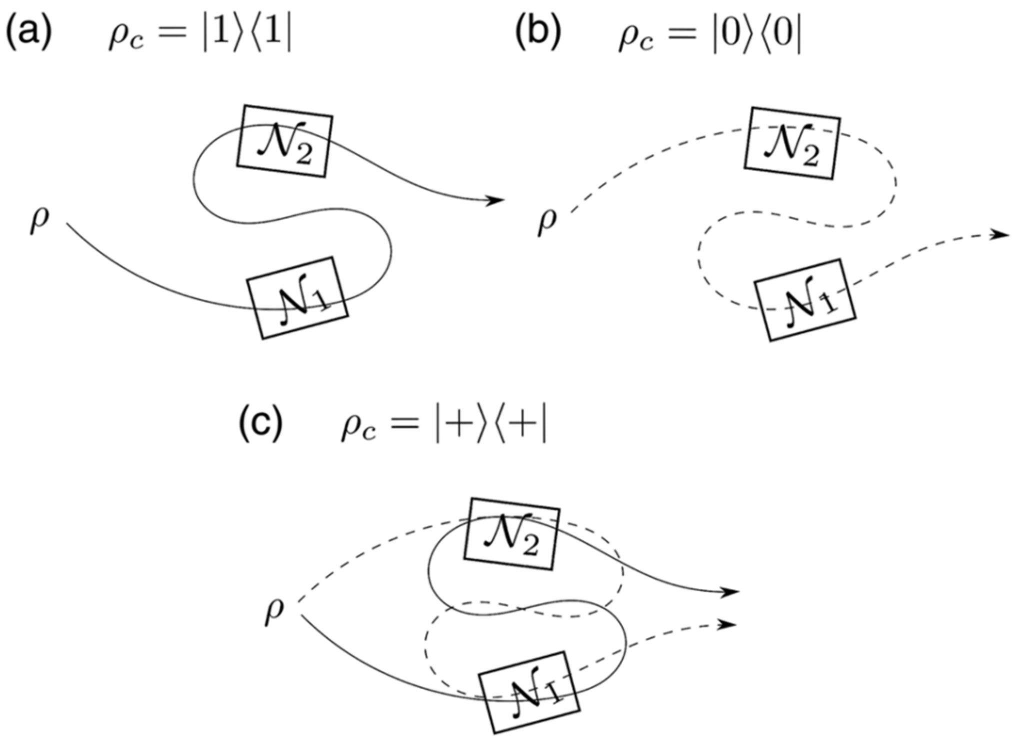
Target qubit: polarization of photons



Nat. Commun. **6**, 7913 (2015)

Super-**Heisenberg limit** metrology

To overcome noisy channels



completely depolarizing channel

$$\mathcal{N}^D(\rho) = \frac{1}{d^2}\sum_{i=1}^{d^2} U_i \rho U_i^\dagger = \mathrm{Tr}[\rho]\frac{I}{d}$$

Quantum switch channel and its outcome

$$W_{ij} = \frac{1}{d^2}(U_i U_j \otimes |0\rangle\langle 0|_c + U_j U_i \otimes |1\rangle\langle 1|_c)$$

$$\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)(\rho \otimes \rho_c) = \sum_{i,j} W_{ij}(\rho \otimes \rho_c) W_{ij}^\dagger.$$

Outcome of Hadamard measurement

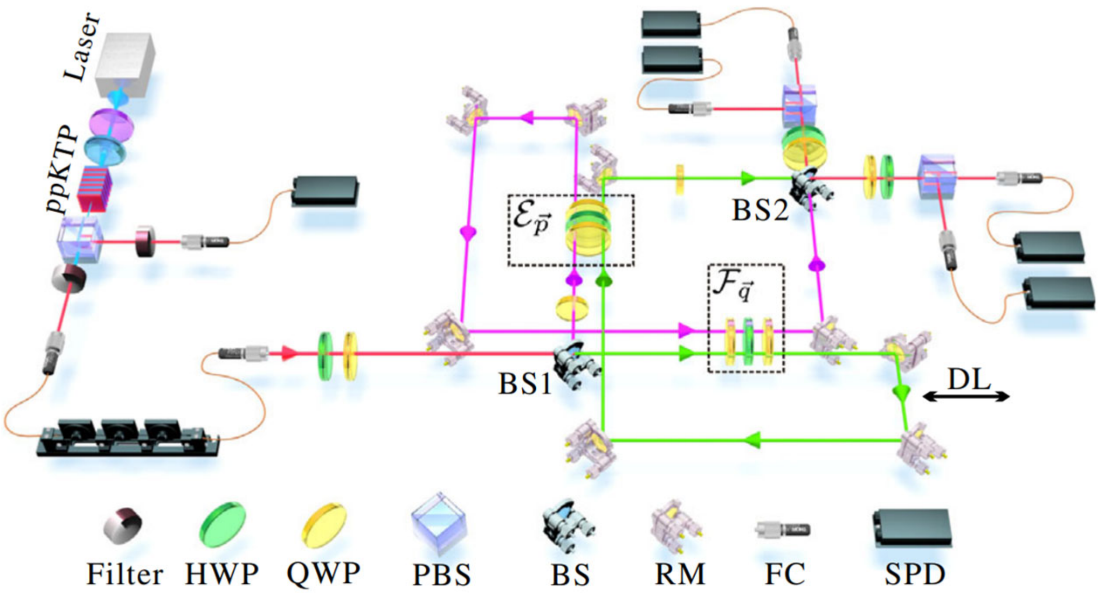
$$\langle \pm | \mathcal{S}(\mathcal{N}^D, \mathcal{N}^D)(\rho \otimes \rho_c) | \pm \rangle = \frac{I}{2d} \pm \sqrt{p(1-p)}\frac{\rho}{d^2}$$

$$|\psi_c\rangle := \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle$$

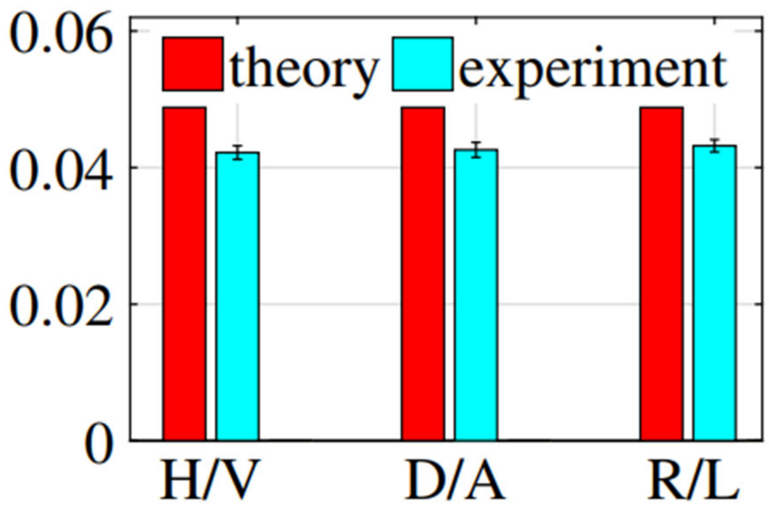
Phys. Rev. Lett. **120**, 120502(2018)

Super-**Heisenberg limit** metrology

To overcome noisy channels



Mutual information for $\mathcal{S}(\mathcal{D},\mathcal{D})$.



completely depolarizing channel

$$\mathcal{D}(\rho) = 1/4 \sum_{i=0}^{3} \sigma_i \rho \sigma_i$$

Phys. Rev. Lett. **124**, 030502 (2020)

Super-**Heisenberg limit** metrology

N position displacements
$D_{x_j} = e^{-i x_j P}$

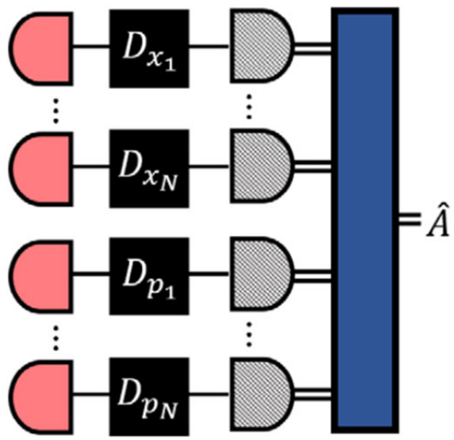
N momentum displacements
$D_{p_k} = e^{i p_k X}$

estimate

$$A := \bar{x} \cdot \bar{p}$$

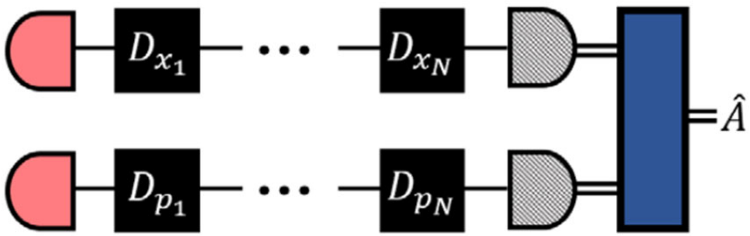
$$\bar{x} := \sum_{j=1}^{N} x_j / N \quad \bar{p} := \sum_{j=1}^{N} p_j / N$$

Parallel scheme



Standard quantum limit $1/\sqrt{N}$

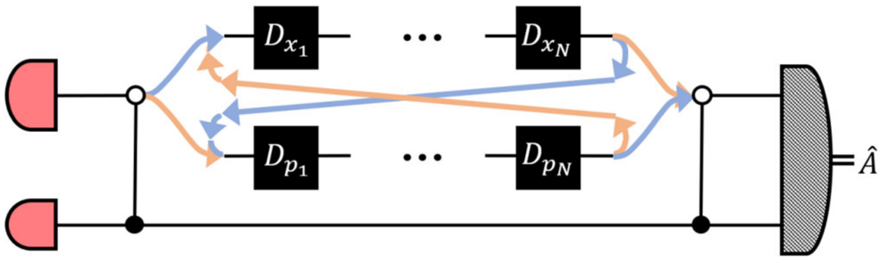
Sequential scheme



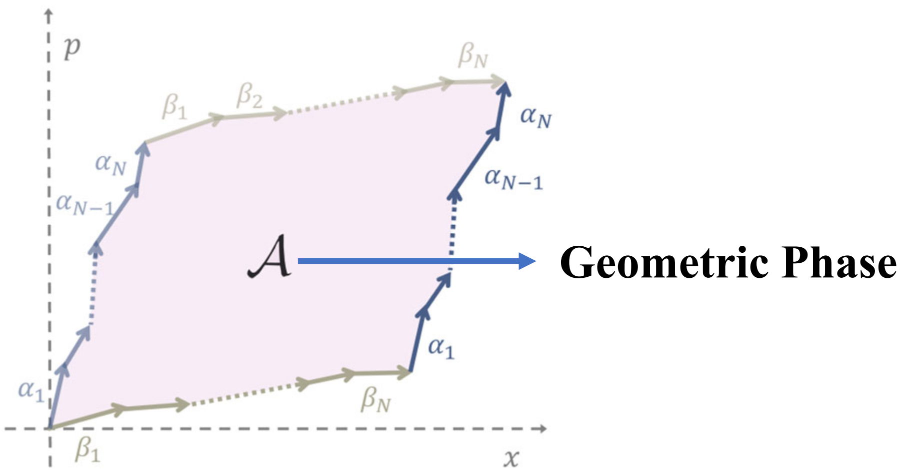
Heisenberg scaling $1/N$

Phys. Rev. Lett. **124**, 190503 (2020)

Super-**Heisenberg limit** metrology



Super − Heisenberg scaling $\propto \frac{1}{N^2}$



$$W = |0\rangle\langle 0| \otimes \prod_{j=1}^{N} D_{p_j} \prod_{j=1}^{N} D_{x_j} + |1\rangle\langle 1| \otimes \prod_{j=1}^{N} D_{x_j} \prod_{j=1}^{N} D_{p_j}.$$

$$W = (|0\rangle\langle 0| + e^{iN^2 A}|1\rangle\langle 1|) \otimes \left(\prod_{j=1}^{N} D_{p_j} \prod_{j=1}^{N} D_{x_j}\right)$$

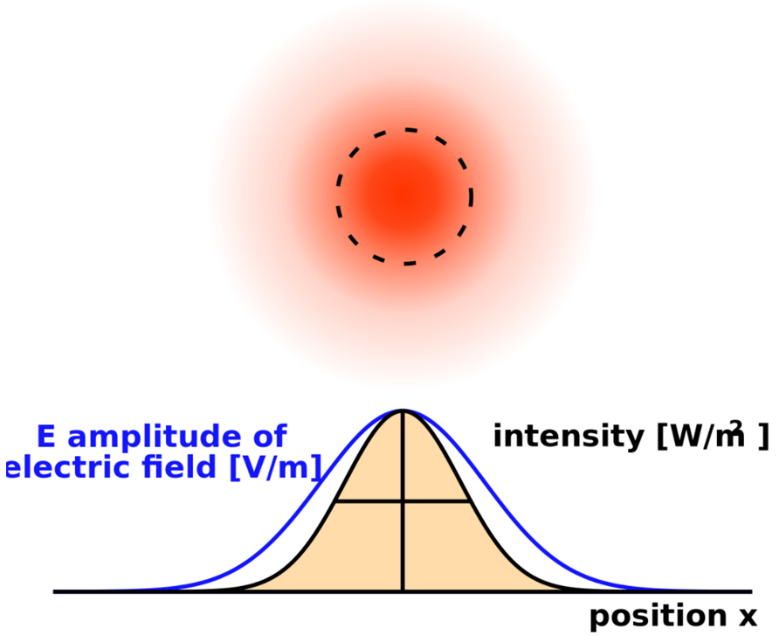
Control qubit

Fisher Info. $$F_A := \sum_{m\in\{+,-\}} p(m|A)\left[\frac{\partial \ln p(m|A)}{\partial A}\right]^2 = N^4.$$

Enhanced scaling $$\Delta A_{\mathrm{SWITCH}} = \frac{1}{\sqrt{\nu} N^2}$$

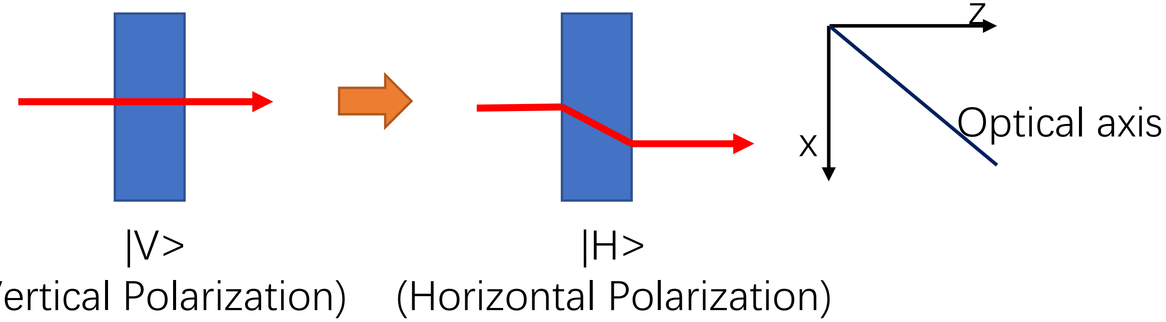
Phys. Rev. Lett. **124**, 190503 (2020)

Super-**Heisenberg limit** metrology



Transverse mode of photons——
quantum switch on continuous variables

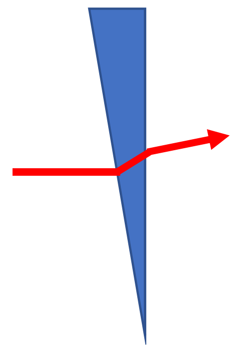
X-displacement $18.6\ \mu m$



P-displacement $\theta^{\mathrm{eff}} \approx 2.8 \times 10^{-4}\ \mathrm{rad}(0.016°)$

$$e^{ipX} = \int dx\ e^{ipx} |x >< x|$$

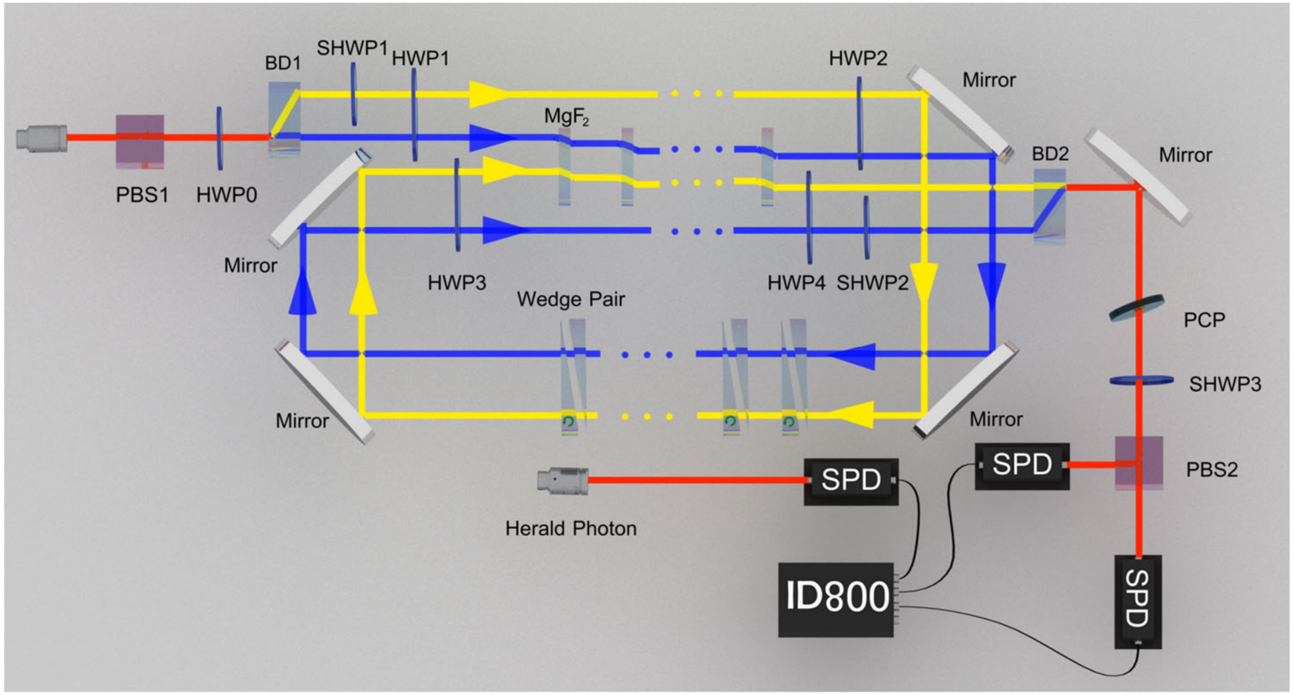
$$e^{ipx} * \ e^{ikz} = e^{i(px+kz)}$$



P. Yin et al. Nat. Phys. **19**, 1122-1127 (2023).

Super-**Heisenberg limit** metrology

Experimental Setup



Control system: photon polarization (finite dimensional)

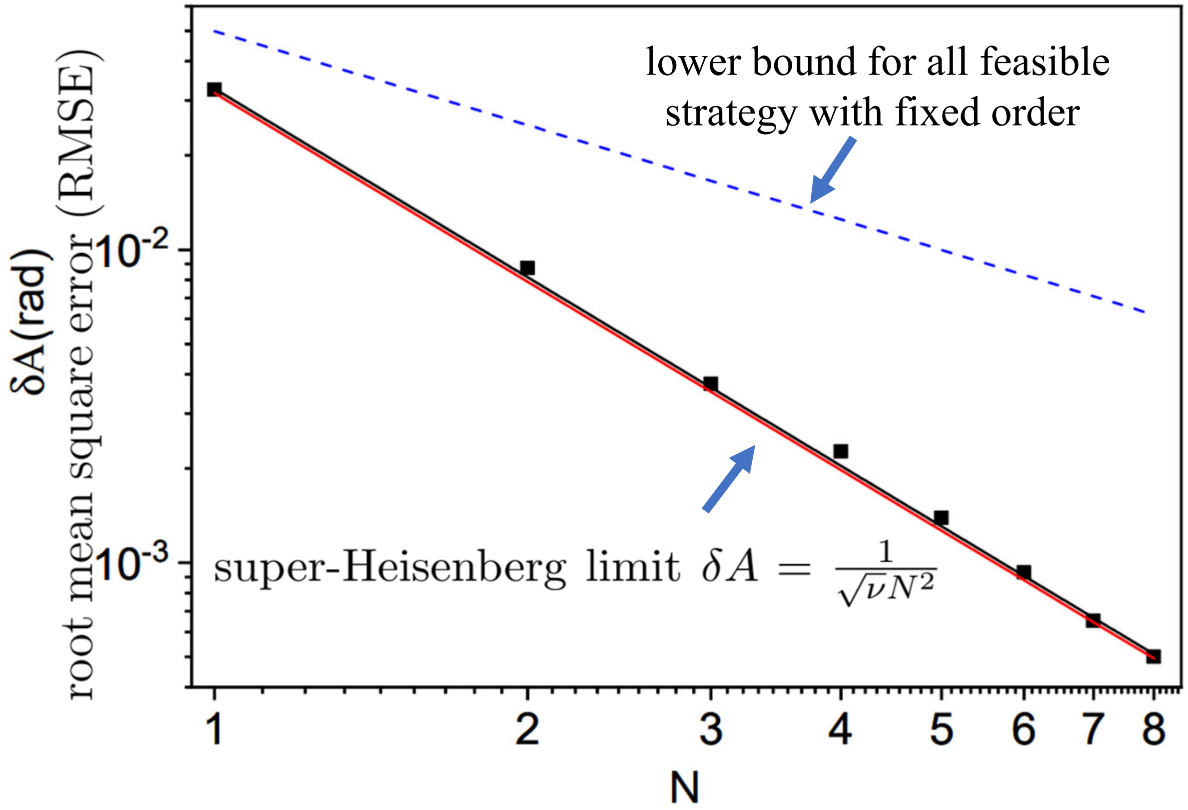
Control

Target system: transverse mode (infinite dimensional)

Initialization: eliminate the stochastic phase introduced by the optical elements: Cancel x-displacement by changing the polarization

P. Yin et al. Nat. Phys. **19**, 1122-1127 (2023).

Super-**Heisenberg limit** metrology



Fitting curve of experimental points

$$\frac{1}{cN^2} \qquad c \approx 30.65$$

Super-Heisenberg limit

$$\delta A = \frac{1}{\sqrt{\nu}N^2} \qquad \nu = 1000$$

P. Yin et al. Nat. Phys. **19**, 1122-1127 (2023).

Super-**Heisenberg limit** metrology

Brief Summaries:

- ✓ Any setup using a superposition of alternative orders and a finite amount of energy in the probes is necessarily bound to the N^(-2) scaling, two orders are optimal.
- ✓ Irrespective of the definition of Heisenberg limit, our scheme realizes a realistic advantage compared to definite causal order schemes, not merely a better scaling.
- ✓ Our experiment implements ICO on a hybrid system for the first time, which utilizes discrete control qubit to decide the alternative order of two sets of evolutions.
- ✓ The resource count in terms of the energy does not exceed the energy of a single photon, and no nonlinear effect occurs.

Quantum **nonlinear** enhancement of precision

Discussions

Classcial Nonlinear schemes are widely debated

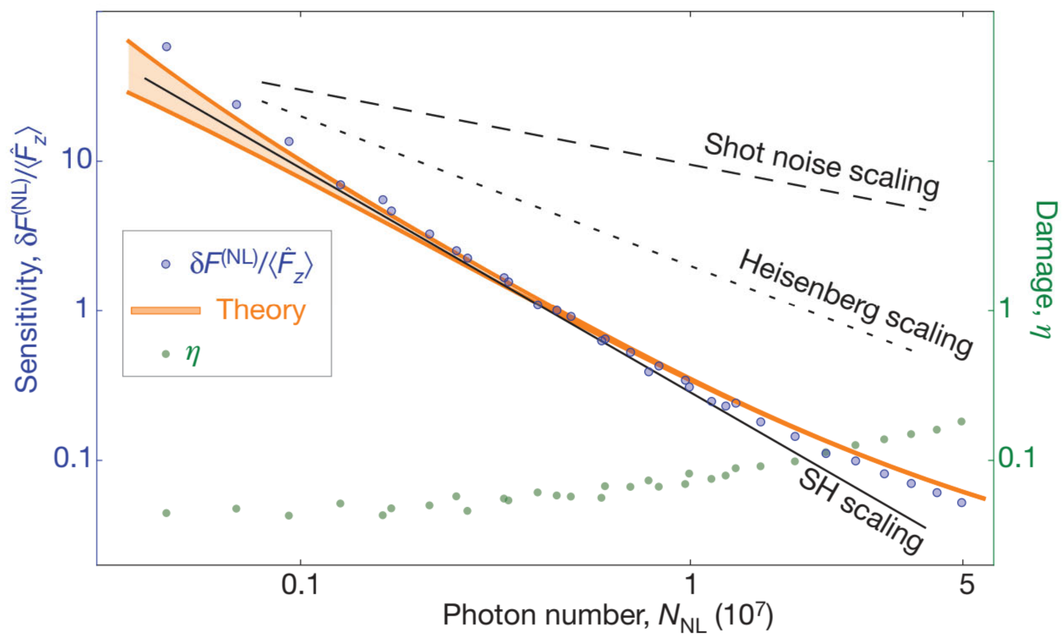
Interaction-based quantum metrology showing scaling beyond the Heisenberg limit

M. Napolitano, M. Koschorreck, B. Dubost, N. Behbood, R. J. Sewell & M. W. Mitchell

Nature **471**, 486–489 (2011) | Cite this article

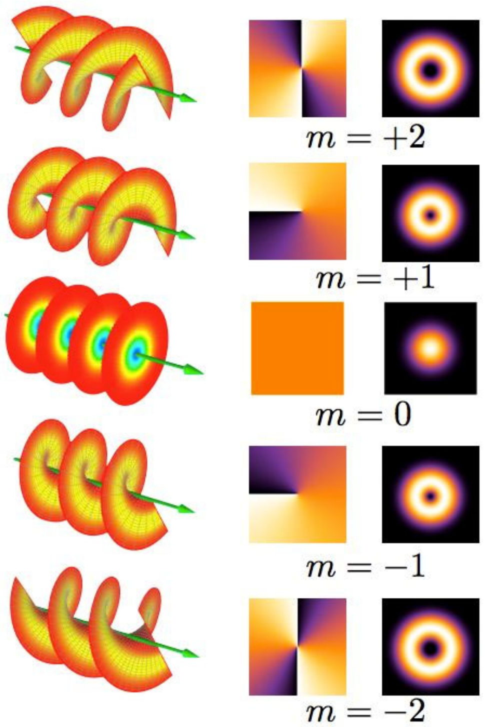
Abstract

Quantum metrology aims to use entanglement and other quantum resources to improve precision measurement[1]. An interferometer using N independent particles to measure a parameter χ can achieve at best the standard quantum limit of sensitivity, $\delta_{\chi} \propto N^{-1/2}$. However, using N entangled particles and exotic states[2], such an interferometer[3] can in principle achieve the Heisenberg limit, $\delta\chi \propto N^{-1}$. Recent theoretical work[4,5,6] has argued that interactions among particles may be a valuable resource for quantum metrology, allowing scaling beyond the Heisenberg limit. Specifically, a k-particle interaction will produce sensitivity $\delta\chi \propto N^{-k}$ with appropriate entangled states and $\delta\chi \propto N^{-(k-1/2)}$ even without entanglement[7]. Here we demonstrate 'super-Heisenberg' scaling of $\delta\chi \propto N^{-3/2}$ in a nonlinear, non-destructive[8,9] measurement of the magnetization[10,11] of an atomic ensemble[12]. We use



Quantum **nonlinear** enhancement of precision

Measure the rotating angle with vortex beam



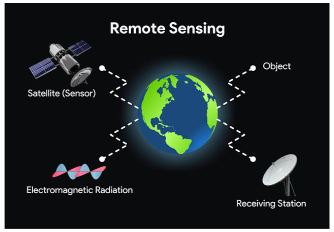
Twisting of light around rotating black holes

Fabrizio Tamburini, Bo Thidé, Gabriel Molina-Terriza & Gabriele Anzolin

Nature Physics 7, 195–197 (2011) | Cite this article

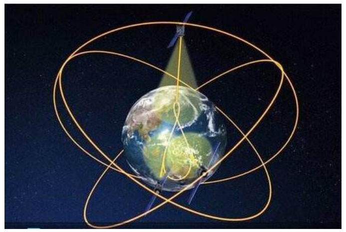
Abstract

Kerr black holes are among the most intriguing predictions of Einstein's general relativity theory[1,2]. These rotating massive astrophysical objects drag and intermix their surrounding space and time, deflecting and phase-modifying light emitted near them. We have found that this leads to a new relativistic effect that imprints orbital angular momentum on such light. Numerical experiments, based on the integration of the null geodesic equations of light from orbiting point-like sources in the Kerr black hole equatorial plane to an asymptotic observer[3], indeed identify the phase change and wavefront warping and predict the associated light-beam orbital angular momentum spectra[4]. Setting up the best existing telescopes properly, it should be possible to detect and measure this twisted light, thus allowing a direct observational demonstration of the existence of rotating black holes. As non-rotating objects are more an exception than a rule in the Universe, our findings are of fundamental importance.

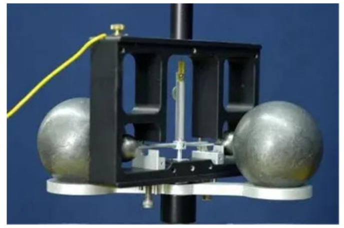


Remote Sensing

Asteroid Defense

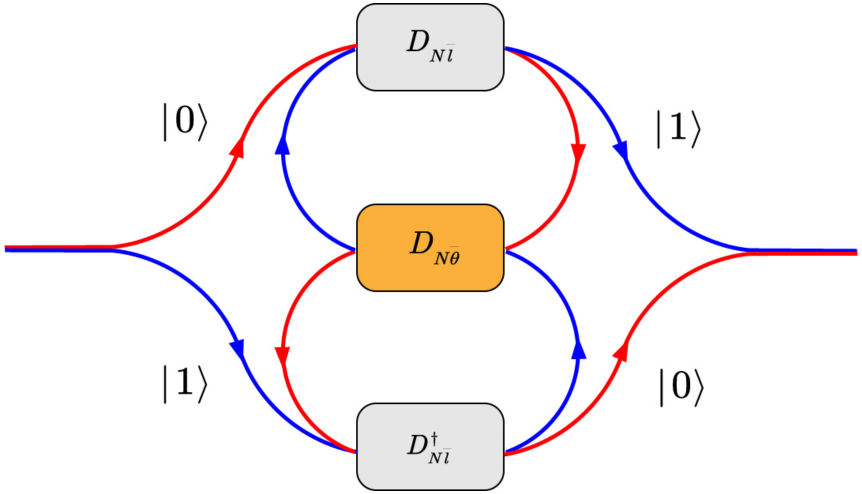


PNT



Gravitation constant

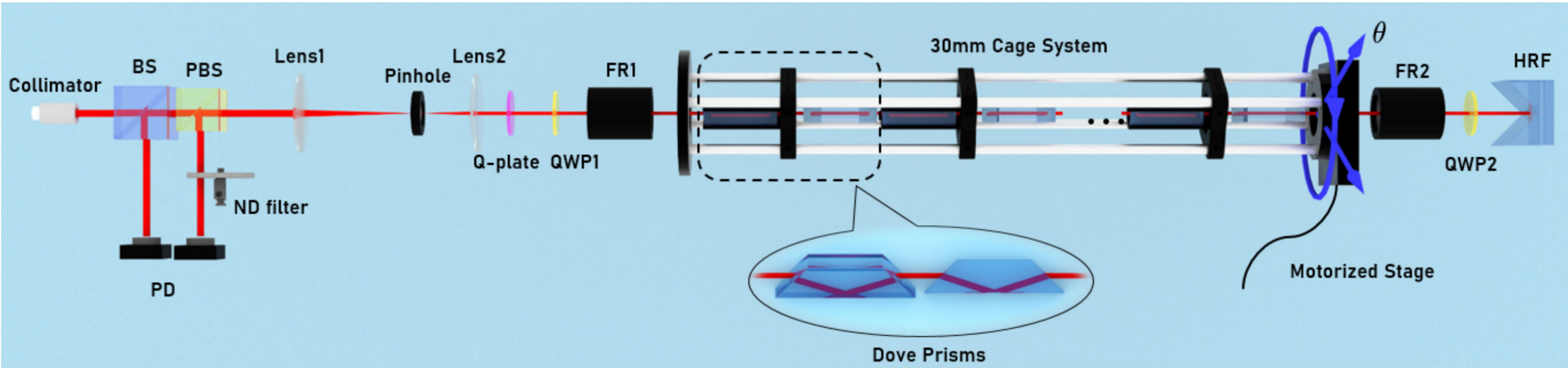
Quantum **nonlinear** enhancement of precision



Quantum SWITCH operator:

$$W = D^{\dagger}_{l\hbar} D_{2m\theta} D_{l\hbar} \otimes |0\rangle\langle 0| + D_{l\hbar} D_{2m\theta} D^{\dagger}_{l\hbar} \otimes |1\rangle\langle 1|$$

$$W_{QS} = D_{2m\theta} D_{2l\hbar} \otimes |0\rangle\langle 0| + D_{2l\hbar} D_{2m\theta} \otimes |1\rangle\langle 1|$$



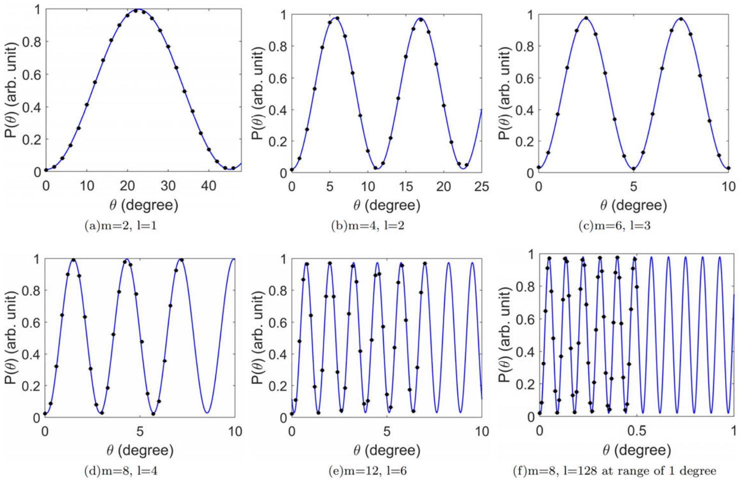
Paper in preparation

Quantum **nonlinear** enhancement of precision

Projection probability of control qubit:

$$P(\theta) = \frac{1}{2}\left[1 - \cos(4ml\theta + \phi_0)\right]$$

Interference fringe for increasing m and l



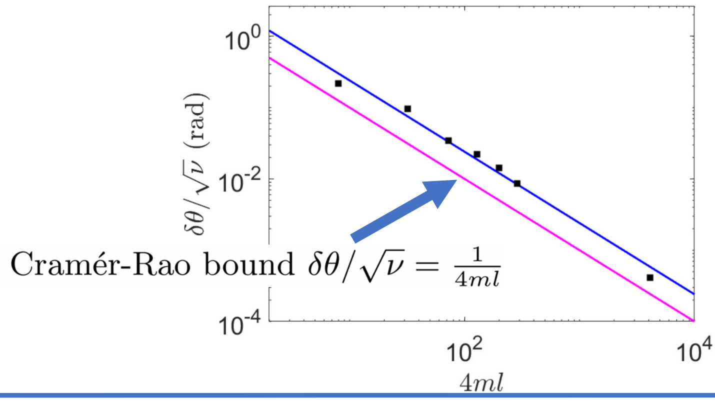
Fisher info. analysis:

$$F_\theta^p = \frac{1}{P_+(\theta)}\left[\frac{\partial P_+(\theta)}{\partial\theta}\right]^2 + \frac{1}{P_-(\theta)}\left[\frac{\partial P_-(\theta)}{\partial\theta}\right]^2 = 16m^2l^2$$

From the Cramer-Rao bound, the RMSE to estimate θ satisfies

$$\delta\theta \geq \frac{1}{\sqrt{\nu F_\theta}} = \frac{1}{4\sqrt{\nu}ml}$$

Nonlinear enhancement of normalized precision



Paper in preparation

Quantum **nonlinear** enhancement of precision

Measure the rotating angle with vortex beam

nature COMMUNICATIONS

ARTICLE

Received 4 Jul 2013 | Accepted 12 Aug 2013 | Published 18 Sep 2013 DOI: 10.1038/ncomms3432 OPEN

Photonic polarization gears for ultra-sensitive angular measurements

Vincenzo D'Ambrosio[1], Nicolò Spagnolo[1], Lorenzo Del Re[1], Sergei Slussarenko[2], Ying Li[3], Leong Chuan Kwek[3,4,5], Lorenzo Marrucci[2,6], Stephen P. Walborn[7], Leandro Aolita[8,9] & Fabio Sciarrino[1,10]

Quantum metrology bears a great promise in enhancing measurement precision, but is unlikely to become practical in the near future. Its concepts can nevertheless inspire classical or hybrid methods of immediate value. Here we demonstrate NOON-like photonic states of m quanta of angular momentum up to $m = 100$, in a setup that acts as a 'photonic gear', converting, for each photon, a mechanical rotation of an angle θ into an amplified rotation of the optical polarization by $m\theta$, corresponding to a 'super-resolving' Malus' law. We show that this effect leads to single-photon angular measurements with the same precision of polarization-only quantum strategies with m photons, but robust to photon losses. Moreover, we combine the gear effect with the quantum enhancement due to entanglement, thus exploiting the advantages of both approaches. The high 'gear ratio' m boosts the current state of the art of optical non-contact angular measurements by almost two orders of magnitude.

In summary, we have reported a photonic scheme to measure rotation angles with greatly enhanced precision. In the regime of single-photon probes, a precision of $\sim 55\sqrt{\nu N}$ has been

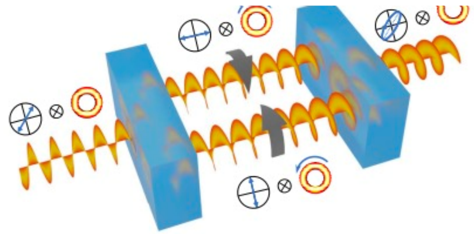
SCIENCE ADVANCES | RESEARCH ARTICLE

OPTICS

Nanoradian-scale precision in light rotation measurement via indefinite quantum dynamics

Binke Xia[1]†, Jingzheng Huang[1,2,3]*†, Hongjing Li[1,2,3], Zhongyuan Luo[1], Guihua Zeng[1,2,3]*

The manipulation and metrology of light beams are pivotal for optical science and applications. In particular, achieving ultrahigh precision in the measurement of light beam rotations has been a long-standing challenge. Instead of using quantum probes like entangled photons, we address this challenge by incorporating a quantum strategy called "indefinite time direction" into the parameterizing process of quantum parameter estimation. Leveraging this quantum property of the parameterizing dynamics allows us to maximize the utilization of orbital angular momentum resources for measuring ultrasmall angular rotations of beam profile. Notably, a nanoradian-scale precision of light rotation measurement is lastly achieved in the experiment, which is the highest precision by far to our best knowledge. Furthermore, this scheme holds promise in various optical applications due to the diverse range of manipulable resources offered by photons.



OAM resources. Consequently, a remarkable precision of 12.9 nrad on light rotation measurement has been achieved with assistance of 150-order LG beam. Fur-

Quantum **nonlinear** enhancement of precision

Brief Summaries:

- ✓ When one of the processes in quantum SWITCH can be precisely applied, it can be used as the leverage to boost the Fisher information and creates quantum nonlinear enhancement.

- ✓ Other scheme may also render approximate amount of quantum Fisher information, but with ICO we can conveniently approach the quantum Cramer-Rao bound merely with projective measurement on control qubit.

Discussion and perspective

Heisenberg uncertainty principle (HUP)

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

The measurement uncertainties between **two inherent observables** of a particle

Heisenberg limit

$$\Delta \sim \frac{1}{N}$$

The scaling of the precision with N to estimate the coupling strength g in evolution operator $\hat{U}(g) = gA \otimes \hat{P}$ but **not the inherent property** of a particle.

Quantum Cramer-Rao (QCR) bound for single parameter estimation

$$(\Delta g)^2 \geq \frac{1}{\nu Q}$$

The QCR bound can only be attained of with an optimal measurement

Relate HUP with quantum Fisher info. for multi-parameter estimation

$$\langle \Delta \hat{H}_i^2 \rangle \langle \Delta \hat{H}_j^2 \rangle \geq \frac{1}{4} |\langle [\hat{H}_i, \hat{H}_j] \rangle|^2 \qquad Q_{ii} = 4 \langle \Delta \hat{H}_i^2 \rangle$$

From the Fisher information view, HUP decides the information loss in one measurement of two parameters

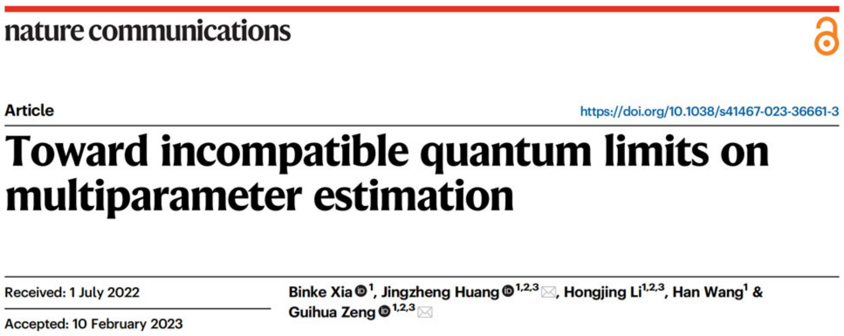
Discussion and perspective

$H_1 = \hat{x}, and\ H_2 = \hat{p},$ $\langle\Delta\hat{H}_i^2\rangle\langle\Delta\hat{H}_j^2\rangle \geq \frac{1}{4}|\langle[\hat{H}_i,\hat{H}_j]\rangle|^2$ → $\Delta x\ \Delta p \geq \frac{\hbar}{2}$

$H_1 \neq \hat{x}, and\ H_2 \neq \hat{p},$ $\langle\Delta\hat{H}_i^2\rangle\langle\Delta\hat{H}_j^2\rangle \geq \frac{1}{4}|\langle[\hat{H}_i,\hat{H}_j]\rangle|^2$ **Optimize probe and measurement** → $(\Delta g_i)^2 \to \frac{1}{\nu Q_{ii}}$, $\&(\Delta g_j)^2 \to \frac{1}{\nu Q_{jj}}$

$\hat{H}_I = \left(g_1\hat{P}\otimes\hat{A} + g_2\hat{X}\otimes\hat{A}\right)\delta(t-t_0)$

Both $\langle\Delta\hat{H}_i^2\rangle$ and $\langle\Delta\hat{H}_j^2\rangle$ can be infinitely enhanced, and the two parameter simultaneously saturate the QCR bound without any information loss.


nature communications

Article

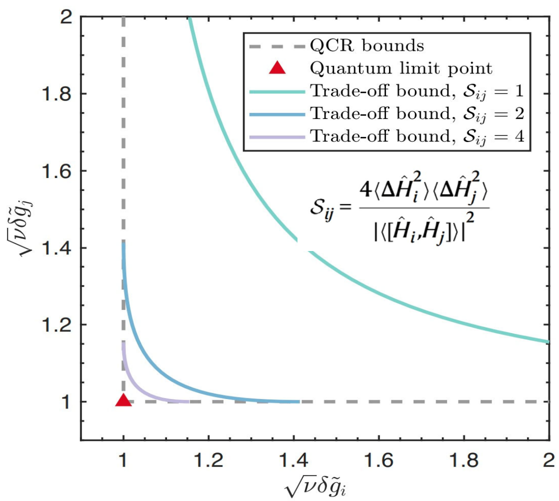
https://doi.org/10.1038/s41467-023-36661-3

Toward incompatible quantum limits on multiparameter estimation

Received: 1 July 2022

Accepted: 10 February 2023

Binke Xia[1], Jingzheng Huang[1,2,3], Hongjing Li[1,2,3], Han Wang[1] & Guihua Zeng[1,2,3]



Discussion and perspective

Rediscover of Heisenberg uncertainty principle (HUP)

I. HUP does not put constraint on the scaling of precision in single parameter estimation, and **super-Heisenberg limit is allowed** and consistent with current physical framework.

II. HUP relates to the incompatibility of two parameters and causes information loss, but with specific probe and measurement engineering, the incompatibility can be minimized and uncertainties of the two parameters can **simultaneously approach the QCR bound**.

Discussion and perspective

Two further questions:

I. In the sense that the HUP does not restrict the scaling in single parameter estimation, is it possible to achieve higher limit than $1/N^2$?

II. Is it possible to meanwhile approach the QCR bound and achieve dual super-Heisenberg limit in bi-parameter estimation?

感谢各位老师和同学，
欢迎提问！