Decoherence and Bell test of cosmological perturbations: boundary terms and the WKB wave functional

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引力的量子性质, PCFT

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JHEP 04 (2023) 092 (arXiv:2207.04435) with **Duc Huy Tran** & **Yi Wang**

Outline

Introductory part

The **two-mode squeezed state** of cosmological perturbations

→

Proposals to probe their quantum nature

- **Cosmological Bell test**
- Quantum noise of gravitons in detectors

→

Decoherence of cosmological perturbations

- Quantum-to-classical transition as **a fundamental constraint of the proposals**

Technical part

Boundary terms (total time derivative) of cosmological perturbations

- **slow-roll unsuppressed** terms in the action of single-field inflation

Non-Gaussian phase from the **WKB approximation of Wheeler-DeWitt equation**

→

Improved estimation of cosmic decoherence

→

Cosmological Bell test with decoherence

- **Quantify the constraint** in the simplest single-field inflation
- Subtlety of defining squeezing with **boundary terms**

The two-mode squeezed state of cosmological perturbations

Inflation: primordial universe experienced exponential expansion

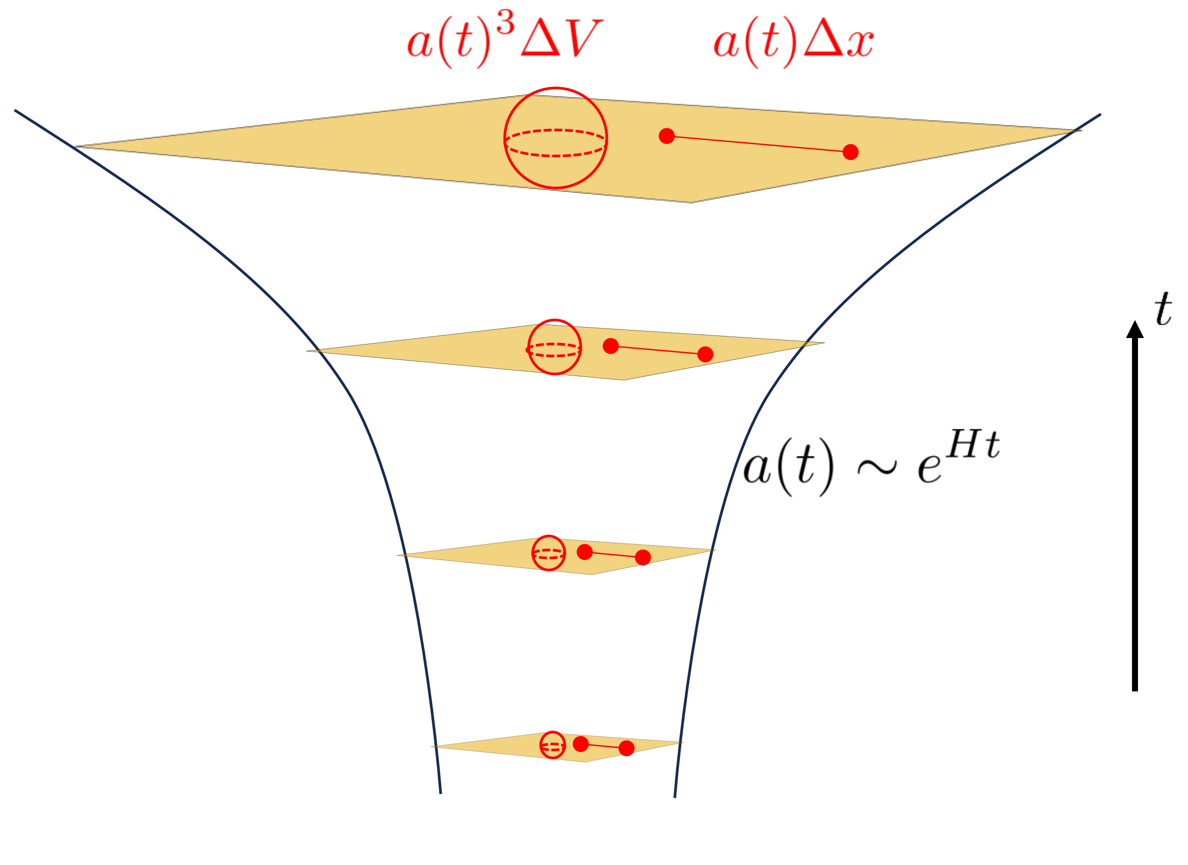
The geometry of the expanding universe

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a^2 \left(-d\tau^2 + d\mathbf{x}^2\right)$$

conformal time

- Nearly flat, homogenous and isotropic on large scale
- $a(t) \sim e^{Ht}$ as the **scale factor**, with a nearly constant **Hubble parameter** (how fast the expansion)

$$H = \frac{\dot{a}}{a}$$



The dynamics of slow-roll inflation

The simplest inflation model is driven by a real scalar field, **inflaton** $\phi(\mathbf{x}, t)$

- The Hubble parameter changes slowly

$$H^2 = \frac{1}{3M_p^2}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

Planck mass

energy density of inflaton homogenous background $\phi(t)$

  - *Not strictly constant as the equation of motion (EOM)*

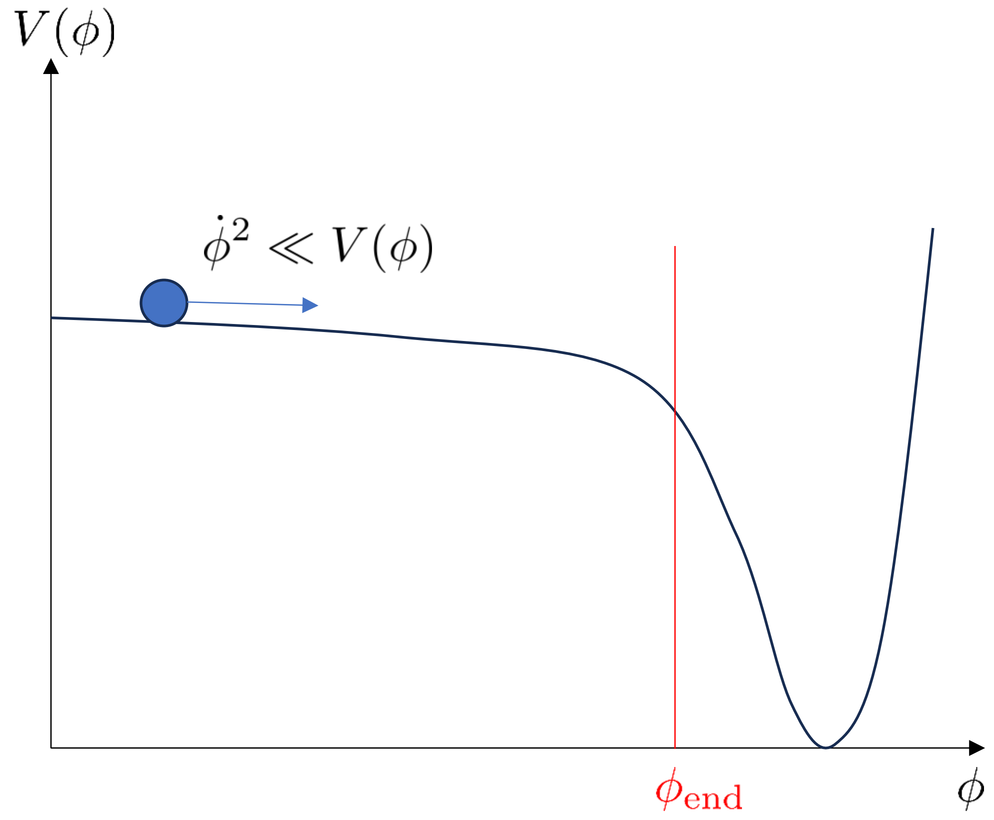
$$\ddot{\phi} + \partial_\phi V(\phi) = -3H\dot{\phi}$$

- "Slow" described by the **slow-roll parameters**

$$\epsilon = -\frac{\dot{H}}{H^2} < \mathcal{O}(10^{-3}) \quad \eta = \frac{\dot{\epsilon}}{H\epsilon} \approx 0.03$$

Observational constraints

  - As small parameters
  - Identify **the dominated order** of decoherence effect

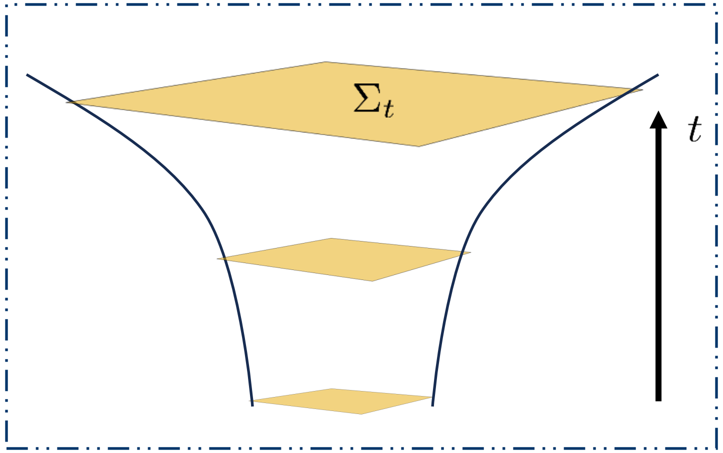


Cosmological perturbations from quantum fluctuation

Spatial metric of 3d hypersurface at time t during inflation

$$h_{ij}(\mathbf{x},t) = a(t)^2 e^{2\zeta(\mathbf{x},t)} \left(e^{\gamma(\mathbf{x},t)}\right)_{ij}$$

Maldacena's convention



in the ADM (3+1) decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Lapse and shift by solving constraint equations

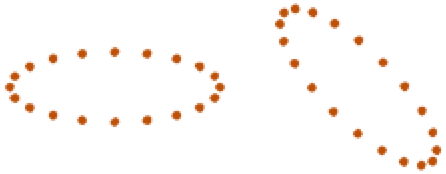
$$N = 1 + \frac{\dot{\zeta}}{H}\,,\ N^i = \frac{1}{a^2}\left(-\frac{\partial_i \zeta}{H} + a^2 \epsilon \partial_i \partial^{-2} \dot{\zeta}\right)$$

$^{(3)}R$

Perturb 3d Ricci scalar

- **Scalar curvature perturbation** ζ by inflaton's **quantum fluctuation**

$$\delta\phi = \phi(\mathbf{x},t) - \phi(t)$$

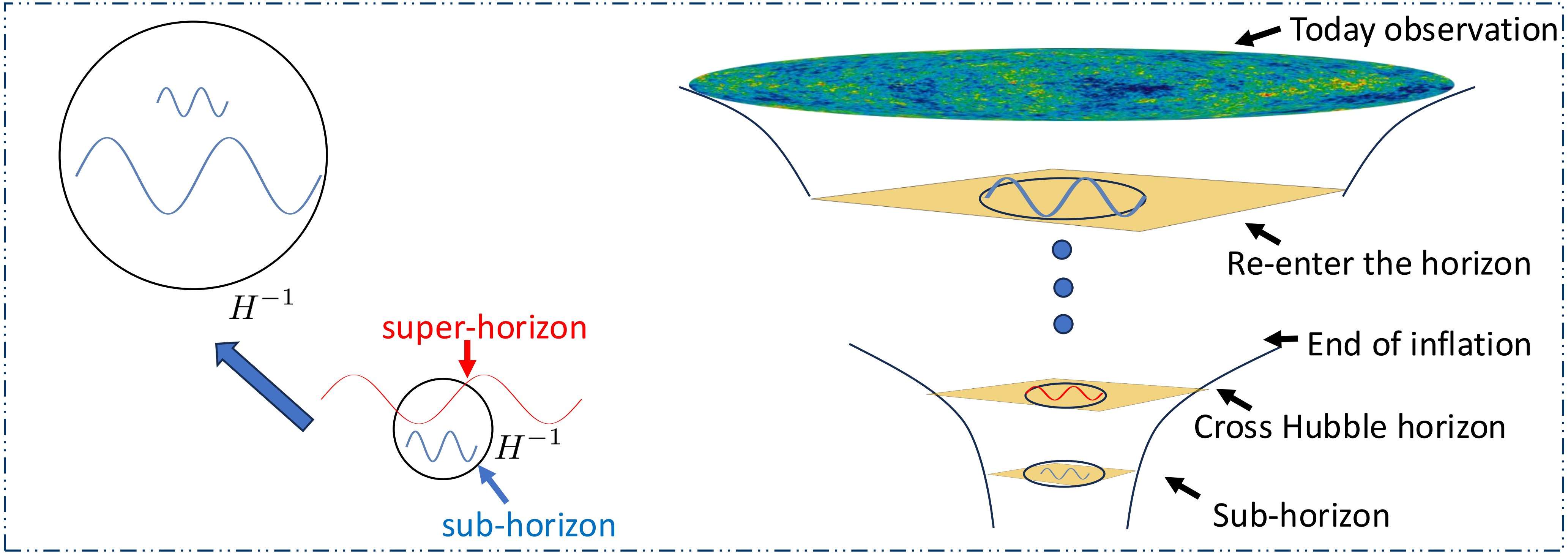


- **Tensor perturbation** γ_{ij} is related to primordial gravitational wave
  - **Primordial gravitons** as the quantum fluctuation

The primordial perturbations are related to today's observations

The statistics of perturbations fitting observation

$$\langle\zeta\zeta\rangle \propto \Delta_\zeta^2 \approx 2\times 10^{-9}$$



- The perturbations **stop evolving** when the wavelengths larger than Hubble horizon H^{-1}

The two-mode squeezed state of cosmological perturbations

In Heisenberg picture, the canonical quantization of cosmological perturbations

$$\zeta_{\mathbf{k}} = u_k a_{\mathbf{k}} + u_k^* a_{-\mathbf{k}}^\dagger \ , \ \gamma_{ij}(\mathbf{k}) = \sum_{s=\pm} (v_k b_{\mathbf{k}}^s + v_k^* b_{-\mathbf{k}}^{s\dagger}) e_{ij}^s(\mathbf{k})$$

comoving wavenumber

mode function

annihilation/creation operators

In **Schrödinger picture**, the BD vacuum becomes **squeezed** during inflation, e.g. for scalar perturbation

- (Grishchuk & Sidorov, PRD 42, 3413, 1990) (Albrecht, Ferreira, Joyce & Prokopec, astro-ph/9303001)

$$\mathcal{H}_{\mathbf{k}} = \underbrace{\frac{k}{2}(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + a_{-\mathbf{k}}^\dagger a_{-\mathbf{k}} + 1)}_{\text{Harmonic oscillator}} - \frac{i}{2}\frac{(a\sqrt{\epsilon})'}{a\sqrt{\epsilon}} \underbrace{(a_{\mathbf{k}} a_{-\mathbf{k}} - a_{-\mathbf{k}}^\dagger a_{\mathbf{k}}^\dagger)}_{\text{squeezing}}$$

It evolves into a **two-mode squeezed state**, and particles with opposite directions are **entangled**

squeezing angle

particle number eigenstate

$$|\Psi_{\mathbf{k},-\mathbf{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{\infty} e^{-2in\varphi_k} \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

squeezing parameter

The wave functional of cosmological perturbations (free theory)

The wave functional in field basis has **Gaussian form**, but **the entanglement is not manifest**

$$\langle \zeta,\gamma|\Psi_G\rangle = \Psi_G(\zeta,\gamma) \propto \exp\left[-\frac{1}{2}\left(\int \frac{d^3k}{(2\pi)^3} A_k^{(\zeta)}(t)\underbrace{\zeta_{\mathbf{k}}\zeta_{-\mathbf{k}}} + \sum_{s=\pm} A_k^{(\gamma)}(t)\underbrace{\gamma^s_{\mathbf{k}}\gamma^s_{-\mathbf{k}}}\right)\right] \stackrel{?}{=} \prod_{\mathbf{k}} \Psi_G^{(\zeta)}(\zeta_{\mathbf{k}})\Psi_G^{(\gamma)}(\gamma_{\mathbf{k}})$$

mixing of modes with opposite direction

- Basis by the eigenstates of field operators $\hat{\zeta}|\zeta\rangle = \zeta|\zeta\rangle \quad \hat{\gamma}_{ij}|\gamma_{ij}\rangle = \gamma_{ij}|\gamma_{ij}\rangle$
- The mixing of modes with opposite direction $\pm\mathbf{k}$ is resolved by **changing the basis**

Schrödinger picture

$$\hat{\zeta}^S_{\mathbf{k}} \propto a_{\mathbf{k}} + a^\dagger_{-\mathbf{k}} \quad \xrightarrow{-\mathbf{k}\to\mathbf{k}} \quad \hat{x}^S_{\mathbf{k}} \propto a_{\mathbf{k}} + a^\dagger_{\mathbf{k}} \propto \zeta_{\mathbf{k}} + \zeta_{-\mathbf{k}} + \frac{i}{k}(\zeta'_{\mathbf{k}} - \zeta'_{-\mathbf{k}})$$

new field: "position" operator

conformal time derivative $\frac{d}{d\tau} = \frac{1}{a(t)}\frac{d}{dt}$

- The wave functional in this basis is **not separable**, thus entangling

$$\langle x_{\mathbf{k}}, x_{-\mathbf{k}}|\Psi\rangle = \Psi(x_{\mathbf{k}}, x_{-\mathbf{k}}) = \frac{1}{\sqrt{\pi}} e^{-\frac{\cosh(2r_k)}{2}(x^2_{\mathbf{k}}+x^2_{-\mathbf{k}})+\sinh(2r_k)x_{\mathbf{k}}x_{-\mathbf{k}}} \neq \Psi(x_{\mathbf{k}})\Psi(x_{-\mathbf{k}})$$

Probe the quantum nature with the two-mode squeezed state

Facts about the two-mode squeezed state

The **squeezing parameter** is proportional to the **e-folds after crossing the Hubble horizon**	For inflation which can **last 60 e-folds**	squeezed states produced in lab (Hosten et al, Nature 529, 7587, 2016):
$r_k \sim \log(aH/k) = N_{\rm cross}$	$r_k \sim 60$ **in decibel** $-10\log_{10}\left(e^{-r_k}\right)\,\mathrm{dB} \approx 260\ \mathrm{dB}$	$\sim 20\ \mathrm{dB}$

Means universe expands for $e^{N_{\rm cross}}$ times

utilize this **squeezing** to construct **tests of quantum nature**

Cosmological Bell test: with the **entanglement** between $\pm\mathbf{k}$

Quantum noise of gravitons: noise in GW detectors **enhanced by squeezing**

Proposals to probe their quantum nature

Cosmological Bell test in the literature

<table>
<tr><th>References</th><th>Bell violation</th><th>Limitations</th></tr>
<tr><td>• Campo & Parentani, astro-ph/0510445</td><td>CH inequality with <u>joint probabilities</u> on two-mode coherent-state projectors
$|v,\mathbf{k}\rangle\langle v,\mathbf{k}| \otimes |w,-\mathbf{k}\rangle\langle w,-\mathbf{k}|$
$P(v,w)+P(v,w')+P(v',w)-P(v',w') > P(v)+P(w)$</td><td rowspan="2">• Practically difficult to measure <u>field's conjugate momentum</u>
depends on $\dot{\zeta}$
• Affected by minimal <u>decoherence from gravitational non-linearity</u>
we focus on this: theoretical constraint</td></tr>
<tr><td>• Martin & Vennin, 1706.05001
• Kanno & Soda, 1705.06199
• (See also for pseudo spin: Revzen, Mello, Mann & Johansen, quant-ph/0405100)</td><td>CHSH inequality with the <u>pseudo spin operators</u> constructed in field's phase space
(discussed in details in the following slides)</td></tr>
</table>

See also (Maldacena, 1508.01082) for constructing a "cosmological Bell violation" **independent to conjugate momentum**

- **BUT**, it is **not** measuring the entanglement of cosmological perturbations, the **isospin of a massive field** instead
- Require additional massive field, axion and a **quite contrived mass term** coupling between them and inflaton

$$m^2(\phi)h^\dagger h + \lambda(\phi)h^\dagger(\sigma_x \cos n\theta + \sigma_y \sin n\theta)h$$

Cosmological Bell test with the pseudo spin

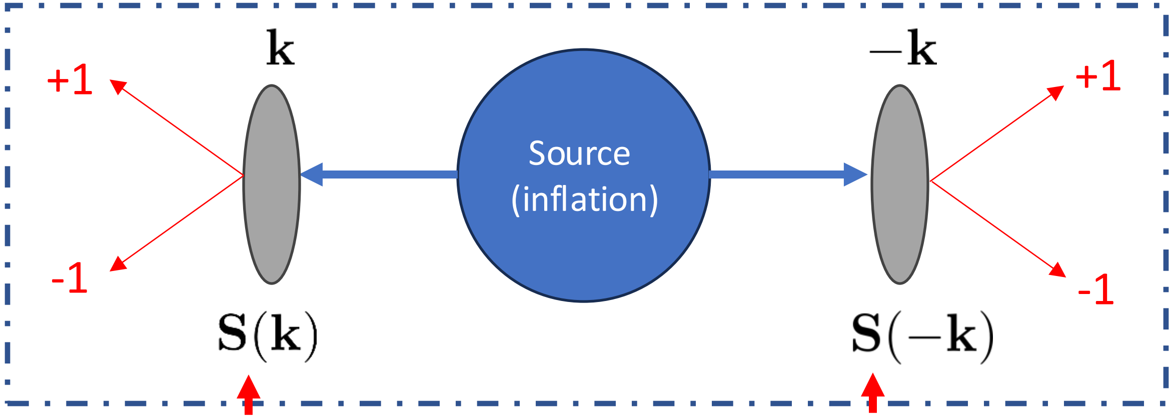
The <u>**cosmological Bell test**</u> relies on the <u>**pseudo spins**</u> of the entangled $(\mathbf{k}, -\mathbf{k})$ modes

- (Martin & Vennin, 1706.05001) (see also Revzen, Mello, Mann & Johansen, quant-ph/0405100)
- Constructed with the eigenstates of $\hat{x}_{\pm\mathbf{k}}$

$$S_x(\mathbf{k}) = \int_0^{+\infty} dx_{\mathbf{k}} (|x_{\mathbf{k}}\rangle\langle x_{\mathbf{k}}| - |-x_{\mathbf{k}}\rangle\langle -x_{\mathbf{k}}|)$$

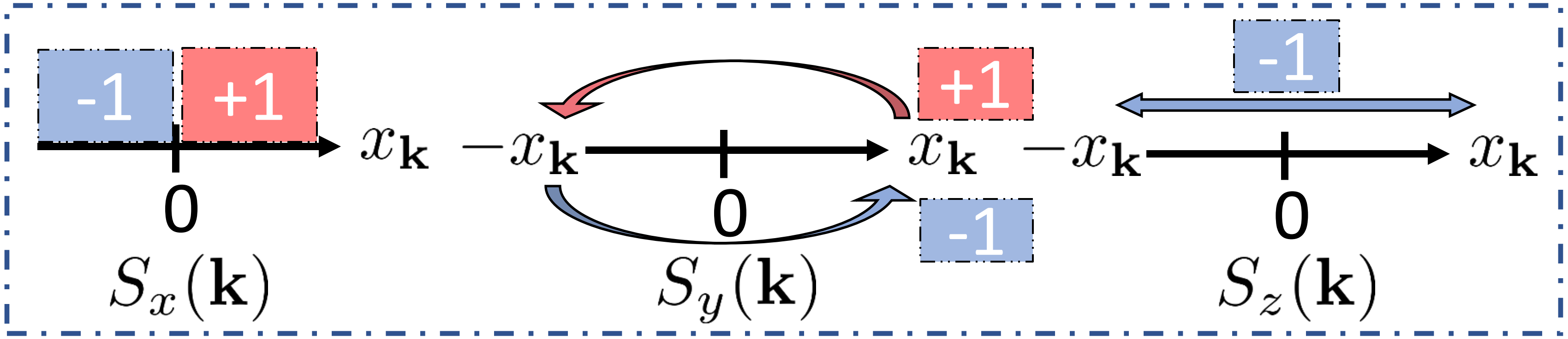
$$S_y(\mathbf{k}) = -\int_0^{+\infty} dx_{\mathbf{k}} (|x_{\mathbf{k}}\rangle\langle -x_{\mathbf{k}}| - |-x_{\mathbf{k}}\rangle\langle x_{\mathbf{k}}|)$$

$$S_z(\mathbf{k}) = -\int_{-\infty}^{+\infty} dx_{\mathbf{k}} |x_{\mathbf{k}}\rangle\langle -x_{\mathbf{k}}|$$



key point: operators to test the entanglement

- <u>**Discrete**</u> operators with eigenvalues ± 1 constructed from the <u>**continuous phase space**</u>:



Cosmological Bell test with the pseudo spin (Cont.)

Construct **Bell inequality** to test the quantum correlation between $\pm\mathbf{k}$ modes, which **violates for the two-mode squeezed state**

- The standard CHSH setup with four unit vectors $\hat{\mathbf{n}},\ \hat{\mathbf{n}}',\ \hat{\mathbf{m}},\ \hat{\mathbf{m}}'$

$$\langle\Psi|\mathcal{B}(\mathbf{k},-\mathbf{k})|\Psi\rangle = E(\theta_n,\theta_m) + E(\theta_n,\theta_{m'}) + E(\theta_{n'},\theta_m) - E(\theta_{n'},\theta_{m'})$$

polar angles of unit vectors

where the correlation function $E(\hat{\mathbf{n}},\hat{\mathbf{m}}) = \langle\Psi|\hat{\mathbf{n}}\cdot\mathbf{S}(\mathbf{k})\otimes\hat{\mathbf{m}}\cdot\mathbf{S}(-\mathbf{k})|\Psi\rangle$

- Optimize the violation $\theta_n = 0,\ \theta_{n'} = \pi/2,\ \theta_{m'} = -\theta_m$

$$\langle\Psi|\mathcal{B}(\mathbf{k},-\mathbf{k})|\Psi\rangle \overset{\text{optimize}}{=} 2\sqrt{\langle S_x(\mathbf{k})S_x(-\mathbf{k})\rangle^2 + \langle S_z(\mathbf{k})S_z(-\mathbf{k})\rangle^2}$$

$$= 2\sqrt{1+\tanh(2r_k)^2\cos(2\varphi_k)^2} > 2$$

$r_k \to +\infty$ $\quad$ $\varphi_k \to -\pi/2$

Quantum noise of Gravitational wave detectors

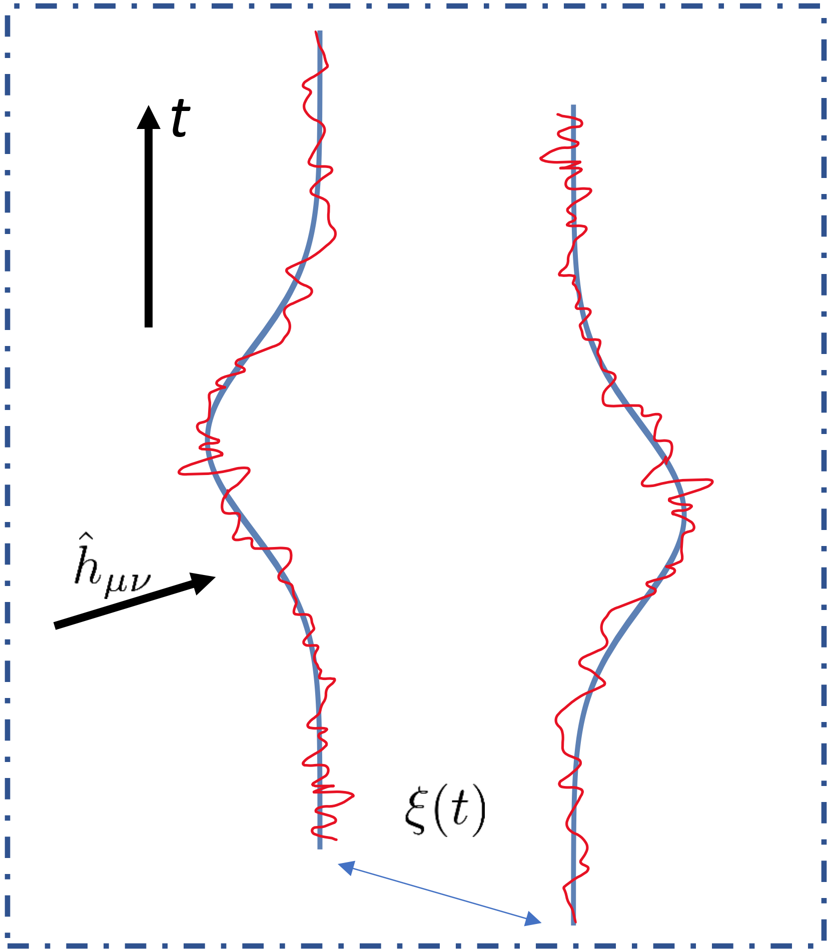
Parikh, Wilczek and Zahariade proposed that **gravitons can produce noise term to the detector's classical EOM** (2005.07211, 2010.08208 & 2010.08205)

- The EOM of GW detector's arm $\xi(t)$

$$\ddot{\xi}(t) - \frac{1}{2}\left[\ddot{h}(t) - \frac{m_0 G}{c^5}\frac{d^5\xi^2(t)}{dt^5} + \ddot{N}(t)\right]\xi(t) = 0$$

classical GW effect — quantum noise

- The quantum noise depends on **the quantum state of the incoming GW**
- This can verify **the quantum nature of gravity**



Quantum noise depends on the state of gravitons

Transition probability for the detector from initial state $|A\rangle$ to the final state $|B\rangle$ is by summing over graviton's final state $|f\rangle$ (not measured)

$$P_\psi(A \to B) = \sum_{|f\rangle} \left| \langle f, B | \hat{U}(T) | \psi, A \rangle \right|^2$$

$$\propto \int \mathcal{D}\xi \mathcal{D}\xi' e^{i \int dt \frac{m_0}{2} (\dot{\xi}^2 - \dot{\xi}'^2)} F_\psi(\xi, \xi')$$

Path integral (2 copies since it is probability)

Action difference of detector's paths

Feynman-Vernon influence functional

Noise from the trick

Function of *ξ* and *ξ'*

$$F_{\psi_{\mathbf{k}}}(\xi, \xi') \propto \langle \psi_{\mathbf{k}} | e^{-W^* a^\dagger_{\mathbf{k}}} e^{W a_{\mathbf{k}}} | \psi_{\mathbf{k}} \rangle$$

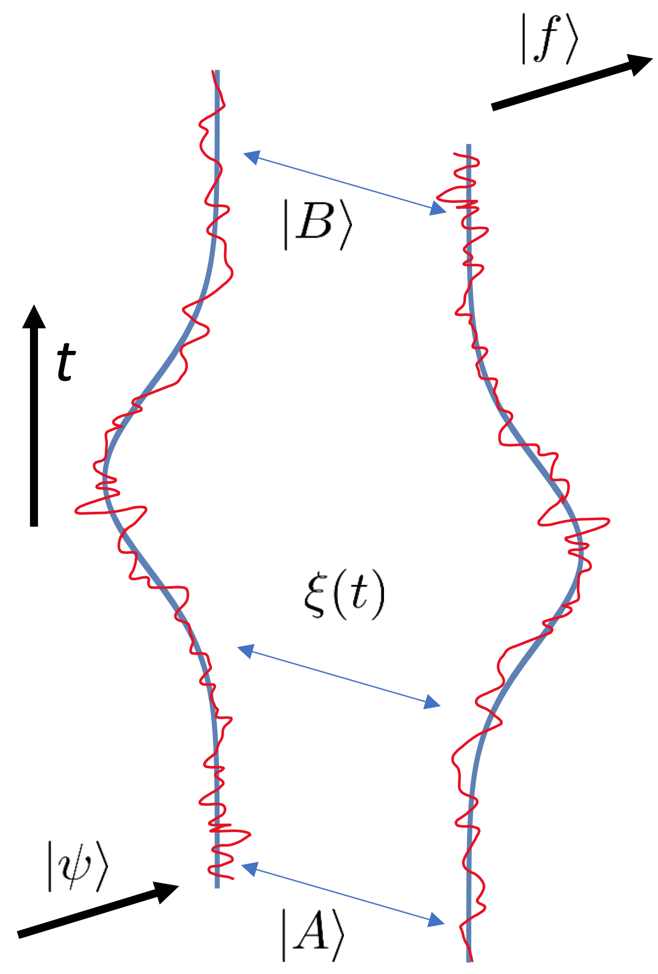
Enhanced exponentially by the squeezing parameter *r*

$$\supset \int \mathcal{D}N \exp \left[-\frac{1}{2} \int dt dt' A^{-1}_\psi(t, t') N(t) N(t') + i \int dt \frac{m_0}{4} N(t) (\ddot{\xi}^2 - \ddot{\xi}'^2) \right]$$

Auxiliary field makes the exponent into the form *S(ξ)-S(ξ')*

Auto-correlation of the noise, FT gives power spectrum

noise term in EOM of detector's arm



Detectability of the squeezed-state quantum noise

Power spectra of noise by **vacuum and squeezed gravitons** (Parikh, Wilczek and Zahariade, 2010.08208)

require precision of detector to Planck's length (Dyson, Int. J. Mod. Phys. A 28 (2013) 1330041)

$$S_{\text{vacuum}} = 4G\omega \longrightarrow \sqrt{S} \propto \sqrt{G} = l_p \sim 10^{-35}\text{m}$$

$$S_{\text{squeezed}} = 4G\omega\sqrt{\cosh(2r_k)}$$

← accompanied by a **non-stationary** part with auto-correlation $A(t,t') = A(t+t')$

exponentially enhanced by squeezing

The effective strain by the noise (Kanno, Soda & Tokuda, 2007.09838)

- Reaching the **sensitivity** of the next-generation space-based detector, e.g. DECIGO, at 0.1 Hz requires:

$$r_k \sim 42 \quad \text{← possible from inflation} \qquad h_{\text{eff}}(f) \sim 10^{-24}\text{Hz}^{-\frac{1}{2}}$$

However, the current proposals **DO NOT consider decoherence effect during inflation**, will it change the results/interpretations of quantum noise?

Motivation of studying cosmic decoherence during inflation

Not only **explaining** the quantum-to-classical transition, but also **constraining** the probe of quantum origin

Quantumness vs Decoherence:
Quantitative results are important

Decoherence of cosmological perturbations

Decoherence by tracing out unobserved modes

In general, the **pure** full quantum state of system $\{\xi_{\mathbf{q}}\}$ and environment $\{\mathcal{E}_{\mathbf{k}}\}$ can be described by the **density matrix**

$$\rho(\{\xi,\mathcal{E}\},\{\tilde{\xi},\tilde{\mathcal{E}}\}) = \Psi(\xi,\mathcal{E})\Psi^*(\tilde{\xi},\tilde{\mathcal{E}}) \longleftarrow \Psi(\xi,\mathcal{E}) = \sqrt{\mathcal{P}(\xi,\mathcal{E})}e^{iS(\xi,\mathcal{E})}$$

$$\hat{\rho} = \begin{pmatrix} |\Psi(\xi_1,\mathcal{E}_1)|^2 & \Psi(\xi_1,\mathcal{E}_1)\Psi^*(\xi_2,\mathcal{E}_2) & \cdots \\ \Psi(\xi_2,\mathcal{E}_2)\Psi^*(\xi_1,\mathcal{E}_1) & |\Psi(\xi_2,\mathcal{E}_2)|^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \mathcal{P}(\xi_1,\mathcal{E}_1) & \sqrt{\mathcal{P}(\xi_1,\mathcal{E}_1)\mathcal{P}(\xi_2,\mathcal{E}_2)}e^{i(S(\xi_1,\mathcal{E}_1)-S(\xi_2,\mathcal{E}_2))} & \cdots \\ \sqrt{\mathcal{P}(\xi_2,\mathcal{E}_2)\mathcal{P}(\xi_1,\mathcal{E}_1)}e^{i(S(\xi_2,\mathcal{E}_2)-S(\xi_1,\mathcal{E}_1))} & \mathcal{P}(\xi_2,\mathcal{E}_2) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- Diagonal elements act like **classical probability distribution** $\mathcal{P}(\xi,\mathcal{E})$
- Off-diagonal involve **quantum interference** with phase difference $e^{i(S(\xi,\mathcal{E})-S(\tilde{\xi},\tilde{\mathcal{E}}))}$
- Decoherence is characterized by the **decaying of off-diagonal elements** when the environment is **traced out**, i.e. **loss of quantum interference** ⟶ **classical statistics**

Frameworks to calculate cosmic decoherence

The **reduced density matrix**, describing the open system's quantum state, is usually **not analytically solvable**. In the literature, there are 2 popular types of approximations

Perturbative method

- **Feynman-Vernon influence functional** by the Schwinger-Keldysh path integral in open system (e.g. Hu et al., gr-qc/9512049)
- Or approximate the **Schrödinger wave functional**, then tracing out the environment (e.g. Nelson, 1601.03734)

  - Used in our works
  - **Intuitively** explain the decoherence from cosmological perturbation theory

Markovian approximation

- For the relaxation time of environment is much shorter than the time scale of system's evolution
- Directly calculate the **evolution of reduced density matrix,** the **master equation** (e.g. Burgess et al., 2211.11046)

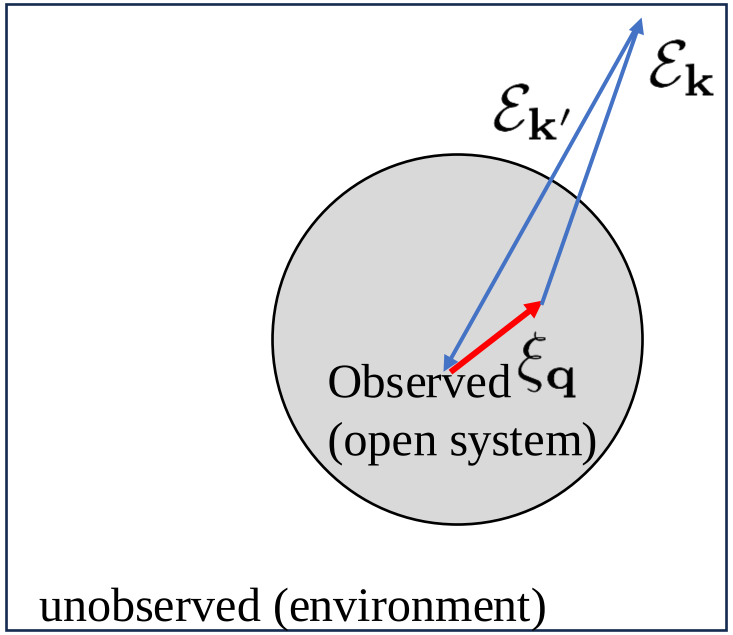
  - Applied to handle **IR (late-time) divergence** in recently years
  - For estimating decoherence, both of the methods give **comparable results**

Decoherence starts from cubic (non-Gaussianity)

Interaction makes **observable modes** (system) coupling with **unobserved modes** (environment)

- e.g. the observable range for Cosmic Microwave Background (Planck, 2018)

$$10^{-4} < q < 10^{-1}\mathrm{Mpc}^{-1} \Rightarrow \zeta = \int_{sys.} \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \xi_{\mathbf{q}} + \int_{env.} \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \mathcal{E}_{\mathbf{k}}$$



- The perturbative wave functional has the **non-Gaussian** form, e.g.

$$\Psi(\zeta) \propto \exp\left(\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\mathbf{p}_1,\ldots,\mathbf{p}_n} A^{(n)}_{\mathbf{p}_1,\ldots,\mathbf{p}_n} \zeta_{\mathbf{p}_1} \cdots \zeta_{\mathbf{p}_n}\right)$$

$$\int \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_n}{(2\pi)^3} (2\pi)^3 \delta^3\left(\sum \mathbf{p}_i\right)$$

- System-environment coupling **starts from cubic interaction**

$$\int d^3x\, \zeta(\mathbf{x},t)^2 = \int \frac{d^3p}{(2\pi)^3} \zeta_{\mathbf{p}} \zeta_{-\mathbf{p}} \qquad \int d^3x\, \zeta(\mathbf{x},t)^3 = \int_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3} \zeta_{\mathbf{p}_1} \zeta_{\mathbf{p}_2} \zeta_{\mathbf{p}_3}$$

quadratic term only couples modes with opposite directions

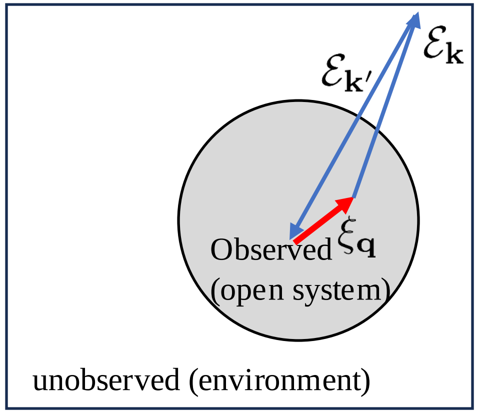
Decoherence by tracing out unobserved modes

Through the cubic interaction, wave functional has cubic term (Nelson, 1601.03734)

$$\Psi(\xi,\mathcal{E}) \propto \exp\left(\int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}}\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}'}\xi_{\mathbf{q}}\right)\Psi_G(\mathcal{E},\xi)$$

← Gaussian part

key point: cubic coefficient of wave functional



- Property which **turns out to be general**: non-Gaussian phase **dominates**

$$\mathrm{Re}\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \to \mathcal{O}(a^0) \quad \mathrm{Im}\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \to \mathcal{O}(a^n)$$

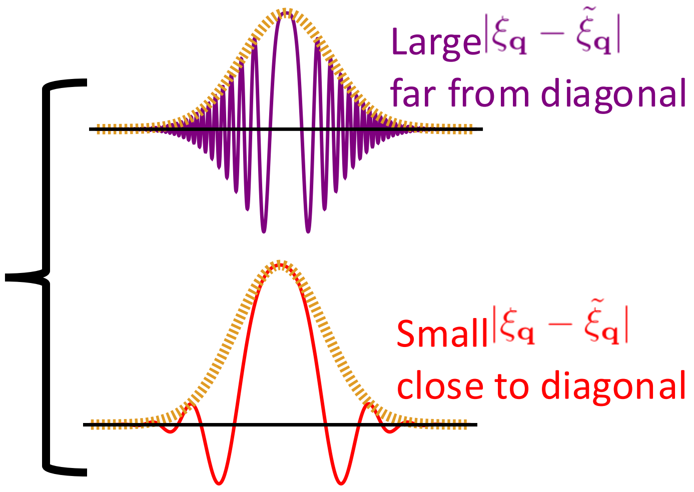
Make sense as it is related to 3-pt function $\langle\zeta^3\rangle$

Loss of coherence when environment are traced out (taking average)

Reduced density matrix:

$$\rho_R(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}}) = \langle\xi_{\mathbf{q}}|\mathrm{Tr}_{\mathcal{E}}\left(|\Psi\rangle\langle\Psi|\right)|\tilde{\xi}_{\mathbf{q}}\rangle$$

$$= \langle\Psi(\xi_{\mathbf{q}},\mathcal{E})\Psi^*(\tilde{\xi}_{\mathbf{q}},\mathcal{E})\rangle_{\mathcal{E}} = \int D\mathcal{E}$$



$$\propto e^{-(\mathrm{Im}\mathcal{F})^2|\xi_{\mathbf{q}}-\tilde{\xi}_{\mathbf{q}}|^2\times\ldots}$$

$$\sim e^{-\Gamma_{\mathrm{deco}}}$$

Decoherence exponent

Non-Gaussian phase is important!

Example: scalar decoherence by bulk interaction (Nelson, 1601.03734)

Ansatz of the perturbative wave functional

$$\Psi(\xi,\mathcal{E}) \propto \exp\left(\int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}}\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}'}\xi_{\mathbf{q}}\right)\Psi_G(\mathcal{E},\xi)$$

Cubic interaction Hamiltonian

$$H_{\rm int}(\tau) \supset \int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \tilde{H}^{\rm int}_{\mathbf{k},\mathbf{k}',\mathbf{q}}(\tau)\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}'}\xi_{\mathbf{q}}$$

Match the Schrödinger equation up to $\mathcal{O}(\zeta^3)$

$$i\partial_t\Psi(\xi,\mathcal{E}) = H(t)\psi(\xi,\mathcal{E})$$

Solve the cubic coefficient at the leading order as a **time integral**

$$\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \approx i\int_{\tau_i}^{\tau}\frac{d\tau'}{H\tau'}\tilde{H}^{\rm int}_{\mathbf{k},\mathbf{k}',\mathbf{q}}(\tau')\frac{u_k(\tau')u_{k'}(\tau')u_q(\tau')}{u_k(\tau)u_{k'}(\tau)u_q(\tau)} + \mathcal{O}\left((\tilde{H}^{\rm int})^2\right)$$

with the mode function $u_k^{(\zeta)}(\tau) = \dfrac{H}{2M_p\sqrt{\epsilon k^3}}(1-ik\tau)\,e^{ik\tau}$

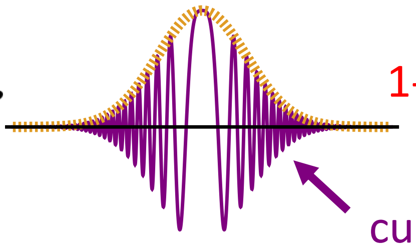
Example: scalar decoherence by bulk interaction (Cont.)

For single-field inflation, the **leading bulk interaction** (from gravitational non-linearity) causing decoherence is

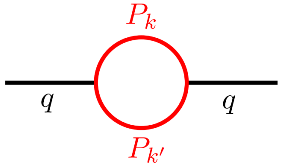
$$\mathcal{L}_{\mathrm{bulk},\zeta} = -\frac{M_p^2}{2}\epsilon(\epsilon+\eta)a\zeta^2\partial_i^2\zeta$$

The **ratio of off-diagonal to diagonal** elements of reduced density matrix (Nelson, 1601.03734)

$$D(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}}) = \left|\frac{\rho_R(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}})}{\sqrt{\rho_R(\xi_{\mathbf{q}},\xi_{\mathbf{q}})\rho_R(\tilde{\xi}_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}})}}\right| = \int D\mathcal{E}$$



1-loop $\approx \exp\Big(-$



$\Big) \sim e^{-\Gamma_{\mathrm{deco}}}$

cubic phase with Gaussian envelope

slow-roll suppressed

power spectrum

IR cut off

$$\Gamma_{\mathrm{deco}} = \frac{\pi}{2}\left(\frac{\epsilon+\eta}{12}\right)^2\Delta_\zeta^2\left(\frac{aH}{q}\right)^3 + \frac{(\epsilon+\eta)^2\Delta_\zeta^2}{18}\left(\frac{aH}{q}\right)^2\left[\log\frac{q}{k_{\min}} - \frac{19}{48}\right] \gtrsim \mathcal{O}(1)$$

sub-horizon environment

super-horizon environment

decoherence happens or by purity

$$\mathrm{Tr}\rho_R^2 \approx \frac{2}{\Gamma_{\mathrm{deco}}} \to 0$$

Summary of the estimation of decoherence in the literature

system	environment	bulk interaction	**e-folds after crossing horizon** for decoherence	references
ζ_{observed}	ζ_{sub} $\zeta_{\text{unobserved, super}}$	$-\frac{M_p^2}{2}\epsilon(\epsilon+\eta)a\zeta^2\partial_i^2\zeta$	10 - 15 ← Uncertainty by slow-roll parameters	Nelson, 1601.03734 (perturbative wave functional)
ζ_{observed}	ζ_{sub} $\gamma_{ij,\text{ sub}}$	$M_p^2\epsilon^2\zeta\partial_i\zeta\partial_i\zeta$ $\frac{M_p^2}{8}\epsilon\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}$	13	Burgess et al., 2211.11046 (Markovian approximation)
γ_{ij} ← Maldacena's convention	ζ_{sub}	$M_p^2\epsilon a\gamma_{ij}\partial_i\zeta\partial_j\zeta$	10	Burgess et al., 2211.11046
h_{ij}^{TT} ← SVT decomposition	$h_{ij,\text{ sub}}^{\text{TT}}$	All three-tensor interactions of h_{ij}^{TT}	5 - 10 ← Uncertainty by IR cutoff	Gong & Seo, 1903.12295

Not the end of the story, still has boundary terms (total time derivative)!

So far we have seen:

the cosmic decoherence as a **fundamental constraint** of testing the quantumness of cosmological perturbations, besides the practical difficulty of measurement

However, neglecting boundary terms, which exist by **well-defined variational principle in GR**, cause the following problems:

1. wrong estimation of cosmic decoherence, as **boundary term terms dominate the effect**
2. wrong estimation of the two-mode squeezing, as it depends on **canonical transformation** (Grain & Vennin, 1910.01916)

Summary the flow of the technical part

Boundary term

↓

Non-Gaussian phase

↓

Improved decoherence of ζ and γ_{ij}

↓

Cosmological Bell test with decoherence

Boundary terms (total time derivative, independent to $\dot{\zeta}$, $\dot{\gamma}_{ij}$), usually neglected in the non-Gaussianity literature, e.g.

$$\mathcal{L}_{\mathrm{bd},\zeta} = M_p^2 \frac{d}{dt}(-2Ha^3 e^{3\zeta}) \quad \mathcal{L}_{\mathrm{bd},\zeta-\gamma} = M_p^2 \frac{d}{dt}\left[-\frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H} - \frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H} \right]$$

- From the standard integration by parts (IBP), the boundary terms cause **slow-roll unsuppressed** NG phase to the wave functional $\Psi(\zeta, \gamma_{ij})$
- **Independent to the IBP**, seen from the WKB approximation of the Wheeler-DeWitt equation

Lead to **great improvement** of estimating the cosmic decoherence, by **several slow-roll order**

- **Squeezing and thus Bell violation** are affected by the boundary terms
- Estimate the **possible window** of having Bell violation for scalar curvature perturbation

Boundary terms in cosmological perturbation theory

Deriving the cubic interactions from the gravitational action is **complicated**

Follow the famous paper of primordial non-Gaussianity (Maldacena, astro-ph/0210603)

- Consider the simplest single-field inflation

$$= -M_p^2 \int_{\partial\mathcal{M}} d^3y\sqrt{h}K$$

$$S = \int d^4x \mathcal{L} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_{\text{GHY}} \qquad \textbf{(1)}$$

Gibbons-Hawking-York boundary term

Adding it = Ignoring covariance derivative (as usual) in

$R = {}^{(3)}R - K^2 + K^\mu_\nu K^\nu_\mu - 2\nabla_\mu\left(-Kn^\mu + n^\nu\nabla_\nu n^\mu\right)$

- For fixed induced metric h^{ij} on boundary (hypersurface at equal time), the GHY term is the **only option** to make the **variation well-defined**, so boundary terms **unique** (Chakraborty, 1607.05986)

The correct non-Gaussian correlators like $\langle\zeta\zeta\zeta\rangle$ **is NOT simply from expanding (1)**

- **Integration by parts** and **rearrange with EOM** to select bulk terms with **correct slow-roll orders**:

$$\mathcal{L}^{(3)} = \underbrace{\mathcal{L}_{\zeta\zeta\zeta} + \mathcal{L}_{\zeta\zeta\gamma} + \mathcal{L}_{\zeta\gamma\gamma} + \mathcal{L}_{\gamma\gamma\gamma}}_{\text{Bulk interaction}} + \underbrace{f(\zeta,\gamma)\frac{\delta L_2}{\delta\zeta} + f_{ij}(\zeta,\gamma)\frac{\delta L_2}{\delta\gamma_{ij}}}_{\text{Equation of motion (EOM)}} + \underbrace{\mathcal{L}_{\text{bd},\zeta\zeta\zeta} + \mathcal{L}_{\text{bd},\zeta\zeta\gamma} + \mathcal{L}_{\text{bd},\zeta\gamma\gamma}}_{\text{Boundary interaction}}$$

Deriving the cubic interactions from the gravitational action is complicated

Done with the Mathematica package *MathGR* (Ning, **Sou** & Wang, 2305.08071)

- Bulk terms

$$\mathcal{L}_{\zeta\zeta\zeta} = M_p^2\left[a^3\epsilon(\epsilon-\eta)\zeta\dot{\zeta}^2 + a\epsilon(\epsilon+\eta)\zeta(\partial_i\zeta)^2 + \left(\frac{\epsilon}{2}-2\right)\frac{\partial^2\chi}{a}\partial_i\chi\partial_i\zeta + \frac{\epsilon}{4a}\partial^2\zeta\left(\partial_i\chi\right)^2\right]$$

$$\mathcal{L}_{\zeta\zeta\gamma} = M_p^2\left[-\frac{1}{2}a\epsilon\chi\partial_i\partial_j\zeta\dot{\gamma}_{ij} + \frac{\partial_i\chi\partial_j\chi\partial^2\gamma_{ij}}{4a} + a\epsilon\partial_i\zeta\partial_j\zeta\gamma_{ij}\right]$$

slow-roll suppressed

$$\mathcal{L}_{\zeta\gamma\gamma} = M_p^2\left[\frac{1}{8}a^3\epsilon\zeta\dot{\gamma_{ij}}^2 - \frac{1}{4}a\partial_l\chi\dot{\gamma}_{ij}\partial_l\gamma_{ij} + \frac{1}{8}a\epsilon\zeta\left(\partial_l\gamma_{ij}\right)^2\right]$$

$$\mathcal{L}_{\gamma\gamma\gamma} = M_p^2\left[\frac{1}{4}a\partial_m\gamma_{il}\partial_l\gamma_{jm}\gamma_{ij} + \frac{1}{8}a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}\right],$$

slow-roll unsuppressed

- EOM terms

$$f(\zeta,\gamma) = -\frac{\dot{\zeta}\zeta}{H} + \frac{1}{4a^2H^2}\left[(\partial_i\zeta)^2 - \partial^{-2}\partial_i\partial_j\left(\partial_i\zeta\partial_j\zeta\right)\right] - \frac{1}{2a^2H}\left[\partial_i\zeta\partial_i\chi - \partial^{-2}\partial_i\partial_j\left(\partial_i\zeta\partial_j\chi\right)\right] + \frac{\partial_i\partial_j\zeta\gamma_{ij}}{4H}\partial^{-2}$$

$$f_{ij}(\zeta,\gamma) = -\frac{\zeta\dot{\gamma}_{ij}}{H} + \frac{\partial_i\zeta\partial_j\zeta}{a^2H^2} + \frac{2\chi\partial_i\partial_j\zeta}{a^2H}$$

$$\frac{\delta L_2}{\delta\zeta} = 2M_p^2\left[-\frac{d}{dt}\left(\epsilon a^3\dot{\zeta}\right) + \epsilon a\partial^2\zeta\right]$$

$$\frac{\delta L_2}{\delta\gamma_{ij}} = \frac{M_p^2}{4}\left[-\frac{d}{dt}\left(a^3\dot{\gamma}_{ij}\right) + a\partial^2\gamma_{ij}\right],$$

EOM terms are zero at the leading order

- **Boundary terms**

We focus on these

$$\mathcal{L}_{\mathrm{bd},\zeta\zeta\zeta} = M_p^2\frac{d}{dt}\left\{-9a^3H\zeta^3 + \frac{a}{H}(1-\epsilon)\zeta\left(\partial_i\zeta\right)^2 - \frac{1}{4aH^3}\left(\partial_i\zeta\right)^2\partial^2\zeta - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 - \frac{\zeta}{2aH}\left[\left(\partial_i\partial_j\chi\right)^2 - \left(\partial^2\chi\right)^2\right] + \frac{\zeta}{2aH^2}\left(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi\right)\right\}$$

$$\mathcal{L}_{\mathrm{bd},\zeta\zeta\gamma} = M_p^2\frac{d}{dt}\left(-\frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H} + \frac{a\partial_i\zeta\partial_j\zeta\dot{\gamma}_{ij}}{4H^2} + \frac{a\chi\partial_i\partial_j\zeta\dot{\gamma}_{ij}}{2H}\right)$$

$$\mathcal{L}_{\mathrm{bd},\zeta\gamma\gamma} = M_p^2\frac{d}{dt}\left[-\frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H} - \frac{a^3\zeta\dot{\gamma_{ij}}^2}{8H}\right].$$

$$\chi = a^2\epsilon\partial^{-2}\dot{\zeta}$$

Depends on $\dot{\zeta}$ or $\dot{\gamma}_{ij}$

red boxes equivalent to non-linear field redefinitions (Burrage, Ribeiro & Serry, 1103.4126) (Arroja & Tanaka, 1103.1102)

Field redefinition can only remove boundary terms with $\dot{\zeta}$, $\dot{\gamma}_{ij}$

For the field redefinition (Burrage, Ribeiro & Serry, 1103.4126) (Arroja & Tanaka, 1103.1102)

$$\zeta \to \zeta_n - f(\zeta_n, \tilde{\gamma}) \, , \, \gamma_{ij} \to \tilde{\gamma}_{ij} - f_{ij}(\zeta_n, \tilde{\gamma})$$

the quadratic actions change as

$$S_2^{(\zeta)}(\zeta) \to S_2^{(\zeta)}(\zeta_n) - M_p^2 \int_{\partial\mathcal{M}} d^3x \, 2\epsilon a^3 \dot{\zeta}_n f - \int_{\mathcal{M}} d^4x \, f \frac{\delta L_2}{\delta\zeta} + \dots$$

$$S_2^{(\gamma)}(\gamma) \to S_2^{(\gamma)}(\tilde{\gamma}) - M_p^2 \int_{\partial\mathcal{M}} d^3x \, \frac{a^3}{4} \dot{\tilde{\gamma}}_{ij} f_{ij} - \int_{\mathcal{M}} d^4x \, f_{ij} \frac{\delta L_2}{\delta\gamma_{ij}} + \dots$$

so **this type of boundary terms contribute to correlators** $\langle\zeta^n\rangle$, $\langle\gamma^n\rangle$

- This agrees with the interaction picture calculation (in-in formalism)

$$\langle 0|\bar{T}e^{i\int_{-\infty}^{t}\partial_{t'}K(\zeta_I,t')dt'}\zeta_I^n(t)Te^{-i\int_{-\infty}^{t}\partial_{t'}K(\zeta_I,t')dt'}|0\rangle = \langle 0|e^{iK(\zeta_I,t)}\zeta_I^n(t)e^{-iK(\zeta_I,t)}|0\rangle$$

- But for the boundary terms **independent** to $\dot{\zeta}$, $\dot{\gamma}_{ij}$, they are neglected in the literature because of **no contribution** to usual correlators

We will see that they **contribute to decoherence**

Slow-roll order estimation of cubic interaction terms

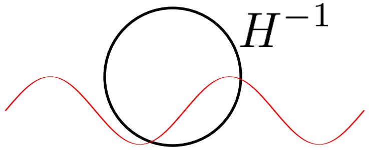
Bulk/Boundary	Type	Leading interaction of each type	Slow-roll order
Bulk	$\zeta\zeta\zeta$	$\epsilon(\epsilon+\eta)a(\partial_i\zeta)^2\zeta$	$\epsilon(\epsilon+\eta)\zeta^3$
Bulk	$\zeta\zeta\gamma$	$\epsilon a\partial_i\zeta\partial_j\zeta\gamma_{ij}$	$\epsilon^{\frac{3}{2}}\zeta^3$
Bulk	$\zeta\gamma\gamma$	$\epsilon a\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}$	$\epsilon^2\zeta^3$
Bulk	$\gamma\gamma\gamma$	$a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}$	$\epsilon^{\frac{3}{2}}\zeta^3$
Boundary	$\zeta\zeta\zeta$	$\partial_t\left(a^3\zeta^3\right)$	ζ^3
Boundary	$\zeta\zeta\gamma$	$\partial_t\left(a\partial_i\zeta\partial_j\zeta\gamma_{ij}\right)$	$\epsilon^{\frac{1}{2}}\zeta^3$
Boundary	$\zeta\gamma\gamma$	$\partial_t\left(a\zeta\partial_l\gamma_{ij}\partial_l\gamma_{ij}\right)$	$\epsilon\zeta^3$

- The **slow-roll order** is estimated with $\Delta_\gamma^2 \sim \mathcal{O}(\epsilon)\Delta_\zeta^2 \implies \gamma \sim \mathcal{O}\left(\sqrt{\epsilon}\right)\zeta$
- Boundary terms are **less slow-roll suppressed**

Revisit the boundary term of ζ: contribute a phase

The dynamics **is dominated by a boundary term** in long wavelength limit (Maldacena, astro-ph/0210603)

$$\mathcal{L} \to M_p^2 \frac{d}{dt}(-2Ha^3 e^{3\zeta})$$



Check with **the in-in formalism**, it **does not** contribute to correlators

$$\langle \zeta^n \rangle = \int D\zeta |\Psi(\zeta)|^2 \zeta^n$$

It **contributes** to conjugate momentum $\Pi_\zeta = -i\frac{\delta}{\delta\zeta}$

$$\langle \Pi_\zeta^n \rangle = \int D\zeta \Psi^*(\zeta) \left(-i\frac{\delta}{\delta\zeta}\right)^n \Psi(\zeta)$$

$$\Psi(\zeta) = \sqrt{\mathcal{P}(\zeta)} e^{iS(\zeta)}$$

Pure phase

Probability distribution

the boundary term contributes a pure phase

The non-Gaussian phase from the WKB approximation of Wheeler-DeWitt

Wave functional with the boundary term

Several ways to see that the boundary terms contribute a **non-Gaussian phase** to the wave functional (**Sou**, Tran & Wang, 2207.04435) (Ning, **Sou** & Wang, 2305.08071)

$$\mathcal{L} = \mathcal{L}_2 - \partial_t \mathcal{K} = f_{aa}(t)\dot{\alpha}^a\dot{\alpha}^a + j_{aa}(t)\alpha^a\alpha^a - \partial_t\left(F_{abc}(t)\alpha^a\alpha^b\alpha^c\right)$$

Include spatial-derivative terms

1. Calculate evolution operator at the cubic order $|\Psi(t)\rangle = U(t,t_i)|\Psi(t_i)\rangle$

$$H_{\mathrm{bd}}(\zeta,\gamma,t) = \int \partial_t \mathcal{K}(\zeta,\gamma,t) \quad U(t,t_i) = \exp\left(-i\int \mathcal{K}\right)U_{\mathrm{free}}(t,t_i) \quad \langle\zeta,\gamma|\Psi(t)\rangle = \exp\left(-i\int \mathcal{K}(\zeta,\gamma,t)\right)\Psi_G(\zeta,\gamma,t)$$

Spatial integral

2. Canonical quantization in the Schrödinger picture

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{\alpha}^a} = 2f_{bb}\delta^b_a\dot{\alpha}^b - \left(F_{dbc}+F_{bdc}+F_{bcd}\right)\delta^d_a\alpha^b\alpha^c \quad \Psi(\vec{\alpha}) = e^{-i\int F_{abc}\alpha^a\alpha^b\alpha^c}\Psi_{\mathrm{free}}(\vec{\alpha})$$

3. The WKB limit of the Wheeler-DeWitt equation

Systematic way to find out the slow-roll unsuppressed phase?

So far our analysis is based on integration by parts (IBP) and rearrangement with EOM terms (Maldacena, astro-ph/0210603)

- Question: there are **(infinitely) many ways to do IBP to the action**, how to **ensure the correct phase factor** in the wave functional?

Goal: finding a method independent to integration by parts

The form of wave functional with long wavelength

At the long wavelength limit $a(t) \to +\infty$, the wave functional looks like (Pimentel, 1309.1793):

$$\Psi(h_{ij}, \phi) = e^{iW(h_{ij},\phi)} Z(h_{ij}, \phi)$$

Real, local, grows as $\mathcal{O}(a^n)$

Non-local, converges at large $a(t)$

- Only $Z(h_{ij}, \phi)$ contributes to usual cosmological correlators

$$\langle O(h_{ij}) \rangle = \int Dh_{ij} \left|\Psi(h_{ij}, \phi)\right|^2 O(h_{ij}) = \int Dh_{ij} \left|Z(h_{ij}, \phi)\right|^2 O(h_{ij})$$

- e.g. the free wave functional of scalar curvature perturbation

$$\Psi(\zeta) \propto \exp\left[-\epsilon \frac{M_p^2}{H^2} \int_{\mathbf{k}} \left(k^3 + ik^2 Ha\right) \zeta_{\mathbf{k}} \zeta_{-\mathbf{k}}\right] \Longrightarrow \langle \zeta_{\mathbf{k}} \zeta_{-\mathbf{k}} \rangle \propto \frac{H^2}{4\epsilon M_p^2 k^3}$$

$\subset Z(h_{ij}, \phi)$ $\quad \propto {}^{(3)}R \subset W(h_{ij}, \phi)$

Some reasoning

Phase of the wave functional

→ In what limit?

Phase dominates in the long wavelength limit

→ How does the quantum state behave?

$\Psi \sim e^{iS}$ characterizes the WKB limit (close to semi-classical limit)

↓ What is the corresponding classical theory?

Classical General Relativity (GR)

← What does the phase S mean in GR?

The Hamilton-Jacobi functional of GR

← Which quantum theory is approximated?

WKB approximation of the Wheeler-DeWitt equation

The WKB approximation of Wheeler-DeWitt equation

To obtain the phase dominated at long wavelength, apply the **WKB approximation** to the Wheeler-DeWitt equation

$$\mathcal{H}\left(\phi, h_{ab}, \frac{\delta}{\delta\phi}, \frac{\delta}{\delta h_{ab}}\right)\Psi(h_{ij},\phi)=0 \qquad \Psi(h_{ij},\phi)\sim e^{i\frac{W(h_{ij},\phi)}{\hbar}}$$

Hamiltonian constraint

- the leading order $\mathcal{O}(\hbar^0)$ is **the solution of the Hamilton-Jacobi equation** (Salopek & Stewart, Class. Quantum Grav., 9 1943, 1992)

$$W(h_{ij},\phi)\approx M_p^2\int_\Sigma d^3x\sqrt{h}\left(U(\phi)+M(\phi)h^{ij}\partial_i\phi\partial_j\phi+\Phi(\phi)^{(3)}R\right)+\mathcal{O}\left(a^0\right)$$

Only include ζ

Include terms with γ_{ij}

$$\approx M_p^2\int_\Sigma d^3x a^3 e^{3\zeta}\left(-2H+\frac{1}{2H}{}^{(3)}R\right)+\mathcal{O}(\epsilon,\eta)$$

$$\supset M_p^2\int_\Sigma d^3x\left[-9a^3H\zeta^3+\frac{a\zeta\left(\partial_i\zeta\right)^2}{H}-\frac{a\zeta\left(\partial_l\gamma_{ij}\right)^2}{8H}-\frac{a\partial_i\zeta\partial_j\zeta\gamma_{ij}}{H}+\frac{a\partial_m\gamma_{il}\partial_l\gamma_{jm}\gamma_{ij}}{4H}+\frac{a\partial_i\gamma_{lm}\partial_j\gamma_{lm}\gamma_{ij}}{8H}\right]$$

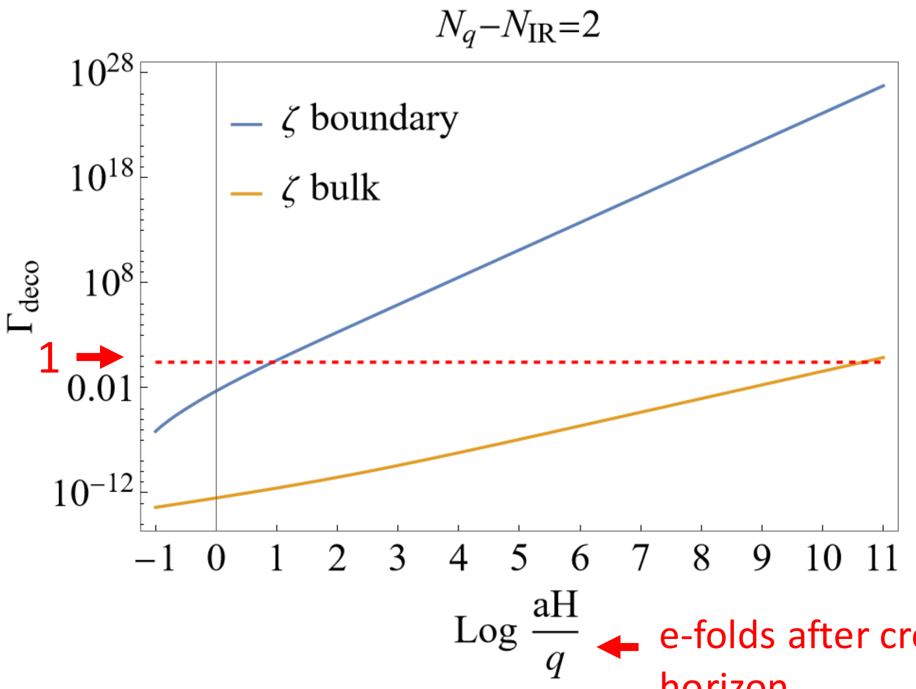
Slow-roll unsuppressed boundary terms

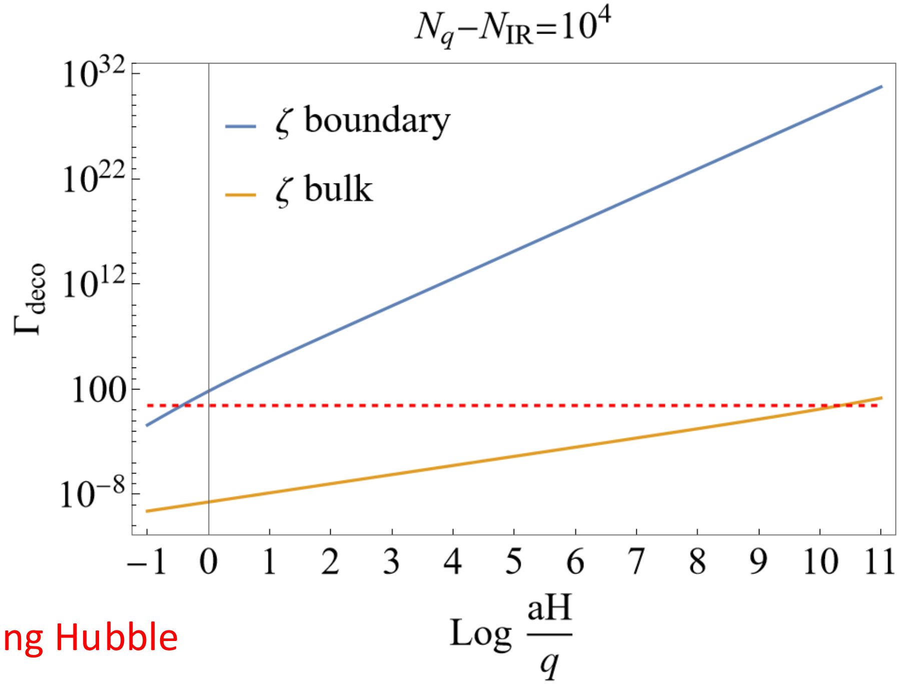
Phase from the slow-roll unsuppressed bulk interaction $\mathcal{L}_{\gamma\gamma\gamma}$

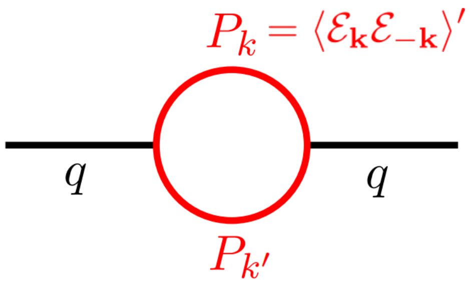
Independent to integration by parts (not needed)! (Ning, **Sou** & Wang, 2305.08071)

Improved estimation of cosmic decoherence

Compare the decoherence exponent for scalar curvature perturbation ζ







Our result with the dominated three-scalar boundary term (**Sou**, Tran & Wang, 2207.04435):

$$\Gamma_{\mathrm{bd},\zeta} \approx \frac{729}{4\epsilon^2} \Delta_\zeta^2 (N_q - N_{\rm IR} - 1) \left(\frac{aH}{q}\right)^6$$

IR cutoff

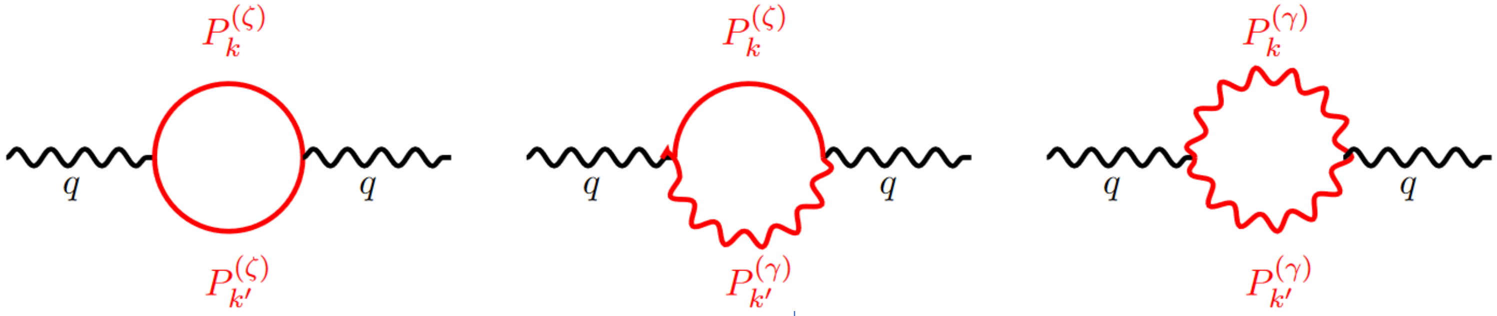
The previous result with bulk term (Nelson, 1601.03734):

$$\Gamma_{\mathrm{bulk},\zeta} \approx \boxed{\frac{\pi}{2}\left(\frac{\epsilon+\eta}{12}\right)^2} \Delta_\zeta^2 \left(\frac{aH}{q}\right)^3 + \frac{(\epsilon+\eta)^2 \Delta_\zeta^2}{18} \left(\frac{aH}{q}\right)^2 \left[N_q - N_{\rm IR} - \frac{19}{48}\right]$$

Slow-roll suppressed

Decoherence of gravitons γ_{ij}

Solid: scalar curvature perturbation ζ, wavy: primordial graviton γ_{ij}



3 decoherence exponents

(Ning, **Sou** & Wang, 2305.08071)

$$\Gamma^{\rm bd}_{\zeta\zeta\gamma} \approx \frac{\pi\Delta_\zeta^2}{15\epsilon}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bd}_{\zeta\gamma\gamma} \approx \frac{\Delta_\zeta^2}{120}\left[60(N_q - N_{\rm IR}) - 79\right]\left(\frac{aH}{q}\right)^2$$

$$\Gamma^{\rm bulk}_{\gamma\gamma\gamma} \approx \boxed{\epsilon\Delta_\zeta^2}\left[\frac{\pi}{4}\left(\frac{aH}{q}\right)^3 + 2(N_q - N_{\rm IR})\left(\frac{aH}{q}\right)^2\right]$$

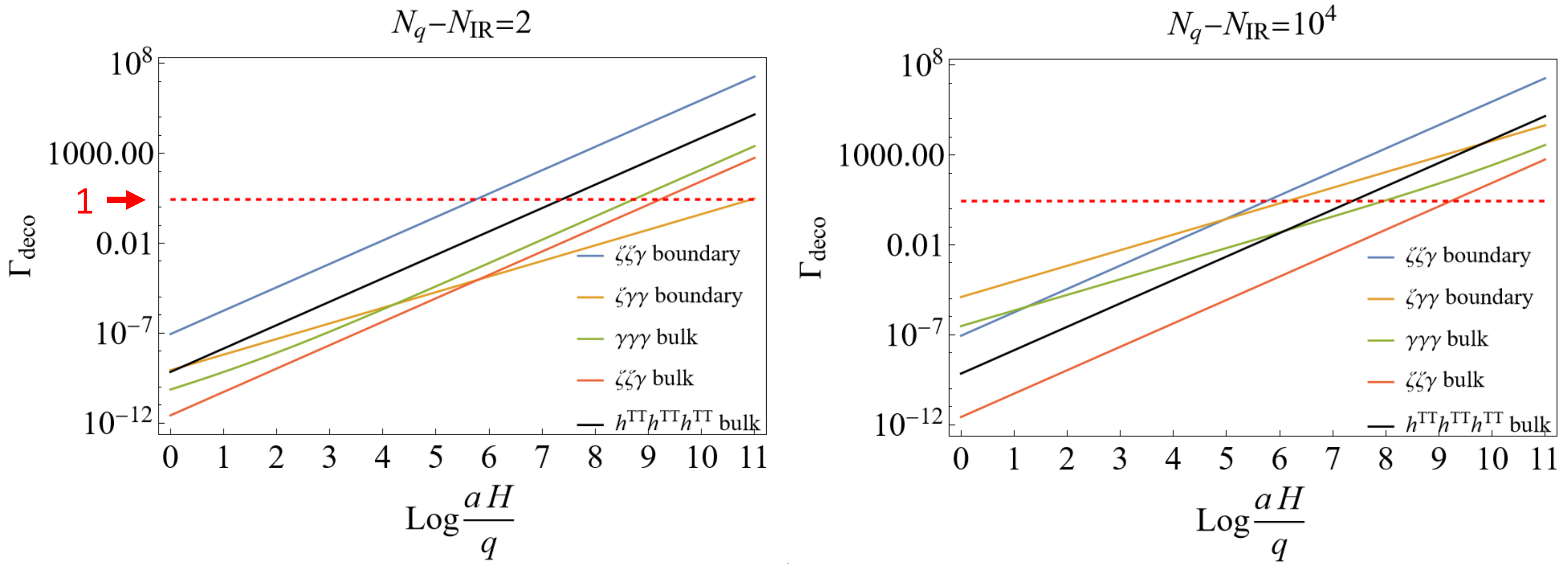
Previous results with bulk interactions (Gong & Seo, 1903.12295) (Burgess et al., 2211.11046)

$$\Gamma^{\rm bulk}_{hhh} \approx \boxed{\frac{2048\epsilon\Delta_\zeta^2}{45}}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bulk}_{\zeta\zeta\gamma} \approx \boxed{\frac{\pi\epsilon\Delta_\zeta^2}{18}}\left(\frac{aH}{q}\right)^3$$

Bulk terms give slow-roll suppressed

Decoherence of primordial gravitons γ_{ij} by different interactions



$$\Gamma^{\rm bd}_{\zeta\zeta\gamma} \approx \frac{\pi\Delta_\zeta^2}{15\epsilon}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bd}_{\zeta\gamma\gamma} \approx \frac{\Delta_\zeta^2}{120}\left[60(N_q - N_{\rm IR}) - 79\right]\left(\frac{aH}{q}\right)^2$$

$$\Gamma^{\rm bulk}_{\gamma\gamma\gamma} \approx \boxed{\epsilon\Delta_\zeta^2}\left[\frac{\pi}{4}\left(\frac{aH}{q}\right)^3 + 2(N_q - N_{\rm IR})\left(\frac{aH}{q}\right)^2\right]$$

$$\Gamma^{\rm bulk}_{hhh} \approx \boxed{\frac{2048\epsilon\Delta_\zeta^2}{45}}\left(\frac{aH}{q}\right)^3$$

$$\Gamma^{\rm bulk}_{\zeta\zeta\gamma} \approx \boxed{\frac{\pi\epsilon\Delta_\zeta^2}{18}}\left(\frac{aH}{q}\right)^3$$

Bulk terms give slow-roll suppressed

Cosmological Bell test with decoherence

The effect of boundary term on squeezing

There is an **inconsistency** in the values of squeezing parameter in the literature, e.g.

$$\sinh r_k = \left|\frac{1}{2k\tau}\right| \propto e^{N_{\rm cross}} \quad (1)$$

in (Polarski & Starobinsky, gr-qc/9504030)

$$\sinh r_k = \left|\frac{1}{2k^2\tau^2}\right| \propto e^{2N_{\rm cross}} \quad (2)$$

in (Kanno & Soda, 1705.06199)

This is related to the **boundary term** while redefining scalar perturbation to the Mukhanov-Sasaki variable $y = aM_p\sqrt{2\epsilon}\zeta = z\zeta$

commonly used in calculating squeezing

$$S_2^{(\zeta)} = \int d\tau d^3x \frac{z^2}{2}\left[\zeta'^2 - (\partial_i\zeta)^2\right] = \frac{1}{2}\int d\tau d^3x \left[\underbrace{y'^2 - (\partial_i y)^2 + \frac{z''}{z}y^2}_{\text{canonically normalized}} - \partial_\tau\left(\frac{z'}{z}y^2\right)\right]$$

- Including the boundary term corresponds to different state: $S \to S + S_{\rm bd}$
- If the boundary term is included, then (1), otherwise (2) $\Psi(\zeta) \to e^{iS_{\rm bd}(\zeta)}\Psi(\zeta)$

The effect of boundary term on squeezing (Cont.)

With the dominated boundary term

$$\mathcal{L}_{\mathrm{bd},\zeta} \supset -M_p^2 \partial_t(2a^3 H e^{3\zeta}) \supset -M_p^2 \partial_t(9a^3 H \zeta^2)$$

the quadratic action with the Mukhanov-Sasaki variable is

$$S_2^{(\zeta)} = \frac{1}{2}\int d\tau d^3x \left[y'^2 - (\partial_i y)^2 + \frac{z''}{z} y^2 - \partial_\tau\left(\frac{z'}{z}y^2\right) + \partial_\tau\left(\frac{9}{\epsilon\tau}y^2\right)\right]$$

- lead to change of **conjugate momentum** $p_y = y' - \frac{z'}{z}y + \frac{9}{\epsilon\tau}y$

Thus the squeezing parameter is affected (**Sou**, Wang & Wang, 2405.07141)

$$\sinh r_k = \left|\sqrt{\frac{k}{2}} f_k - \sqrt{\frac{1}{2k}} g_k\right| = \sqrt{\frac{81 + (k\tau)^2(\epsilon+9)^2}{4\epsilon^2 (k\tau)^4}}$$

field's mode function (f_k) conjugate-momentum mode function (g_k)

- the **squeezing is enhanced** by the factor $\propto 1/\epsilon$
- For primordial gravitons, there is **no such an enhancement**, as there is no such a boundary term at quadratic order

Possible window of Bell violation for scalar perturbation

- With the **quadratic** boundary term:

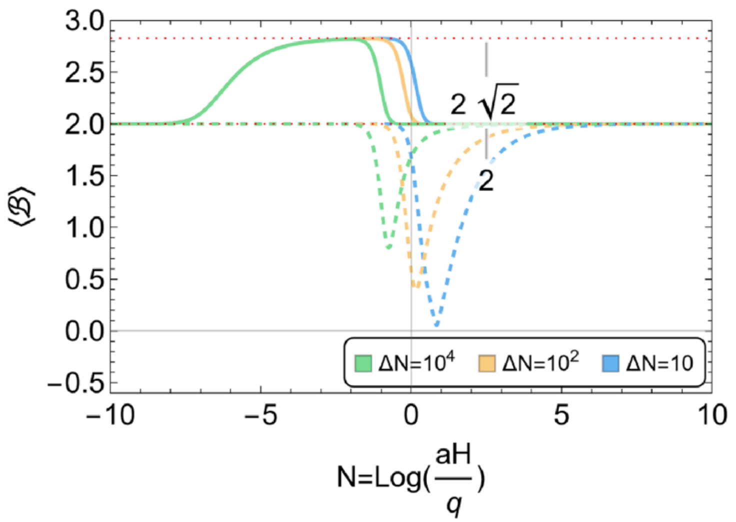
Matrix elements are **unchanged**
$\langle\tilde{\zeta}|S_{x/z}(\mathbf{k})S_{x/z}(-\mathbf{k})|\zeta\rangle$

The state is **changed** (squeezing enhanced)
$\langle\zeta|\Psi\rangle \rightarrow e^{iS_{\mathrm{bd}}}\langle\zeta|\Psi\rangle$

check with $\langle x_{\mathbf{k}}, x_{-\mathbf{k}}|\zeta_{\mathbf{k}}, \zeta_{-\mathbf{k}}\rangle$ after adding the quadratic bd term

Expectation value (Bell violation) is **changed**
$\langle\mathcal{B}\rangle$

- With the decoherence by gravitational non-linearity:



Quadratic Boundary

No Quadratic Boundary

There is a window of around **5 e-folds** with Bell violation (**Sou**, Wang & Wang, 2405.07141)

Conclusion

- **Quantifying** cosmic decoherence is **essential** for testing the quantum nature of cosmological perturbations, as it is a **fundamental constraint**

- The **boundary terms**, naturally exist in the action of cosmological perturbations, can **contribute faster decoherence effect** by trancing out unobserved modes
    - ➢ Improve the decoherence calculations for both scalar curvature perturbation and primordial gravitons

- The non-Gaussian phase can be analyzed systematically with **the WKB approximation of the Wheeler-DeWitt equation**, a way **independent** to the tedious IBP

- Revisit the cosmological Bell test with **decoherence** and **squeezing** by the boundary terms
    - There is a window of **5 e-folds** having the cosmological Bell violation