Enhancing gravitationally induced entanglement

吴普训

湖南师范大学

Collaborators: 王锦成, 余洪伟, in preparation

“引力的量子性质”高级研讨班—彭桓武高能基础理论研究中心

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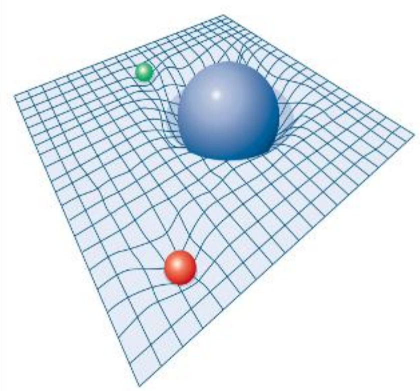
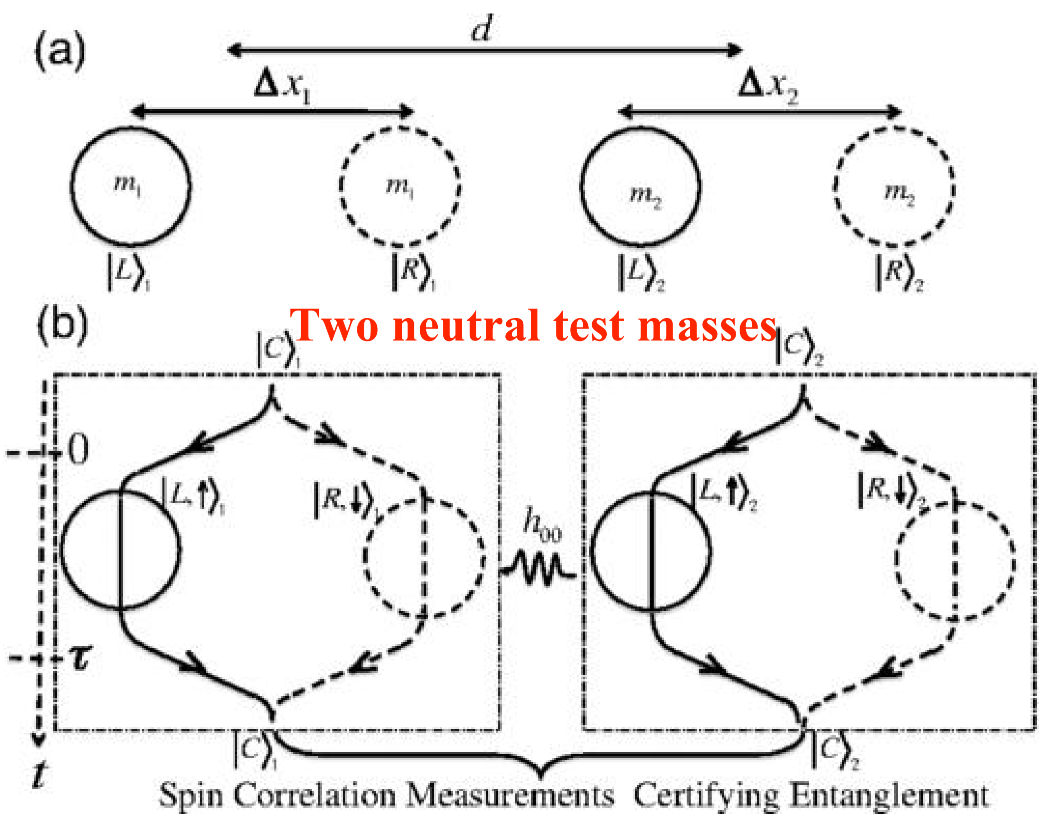
Motivation

引力主宰宇宙的演化，决定时空的性质

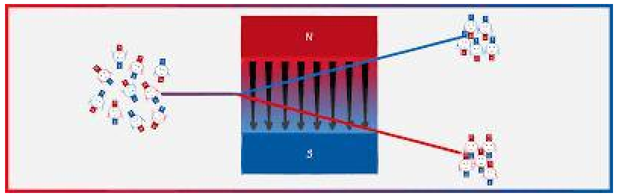


缺乏公认的量子引力理论

引力是否具有量子性质？



Two neutral test masses



Stern–Gerlach setting

$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$

$$\to |\Psi(t=\tau)\rangle_{12} = \frac{e^{i\phi}}{\sqrt{2}}\Big\{|L\rangle_1 \frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2) + |R\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)\Big\},$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi$, $\Delta\phi_{LR} = \phi_{LR} - \phi$, and

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \qquad \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},$$

$$\phi \sim \frac{Gm_1m_2\tau}{\hbar d}.$$

LOCC: entanglement between two systems cannot be created by local operations and classical communication.

Bose, at al., Phys. Rev. Lett. 119, 240401 (2017); Marletto and Vedral, Phys. Rev. Lett. 119, 240402 (2017)

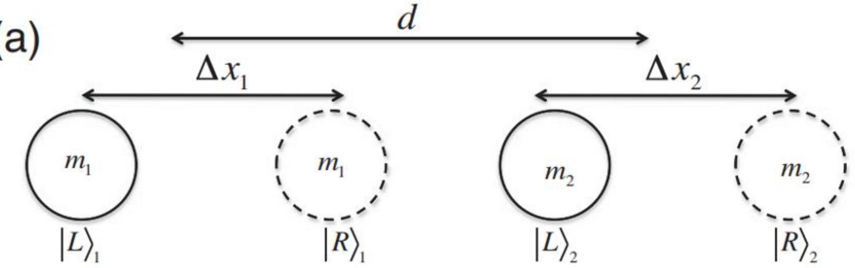
Motivation

密度矩阵：

$$\rho_{tot}(\tau) = |\Psi(\tau)\rangle\langle\Psi(\tau)|$$

$$= \frac{1}{4}\begin{pmatrix} 1 & e^{i(\phi_{RL}-\phi)\tau} & e^{i(\phi_{LR}-\phi)\tau} & 1 \\ e^{-i(\phi_{RL}-\phi)\tau} & 1 & e^{i(\phi_{LR}-\phi_{RL})\tau} & e^{-i(\phi_{RL}-\phi)\tau} \\ e^{-i(\phi_{LR}-\phi)\tau} & e^{-i(\phi_{RL}-\phi_{LR})\tau} & 1 & e^{-i(\phi_{LR}-\phi)\tau} \\ 1 & e^{i(\phi_{RL}-\phi)\tau} & e^{i(\phi_{LR}-\phi)\tau} & 1 \end{pmatrix}$$

(a)



$$d - \Delta x \ll d$$

约化密度矩阵：

$$\rho_2 = \mathrm{Tr}_1[\rho_{tot}] = \frac{1}{2}\begin{pmatrix} 1 & \frac{e^{i(\phi_{RL}-\phi)\tau}+e^{-i(\phi_{LR}-\phi)\tau}}{2} \\ \frac{e^{i(\phi_{LR}-\phi)\tau}+e^{-i(\phi_{RL}-\phi)\tau}}{2} & 1 \end{pmatrix}$$

$$\mathcal{C} = \left|\sin\left(\frac{\phi_{RL}+\phi_{RL}-2\phi}{2}\tau\right)\right|$$

$$\simeq |\sin\phi_{RL}| = \left|\sin\left(\frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}\right)\right|$$

纠缠（以子系统的纯度作度量）： $\mathcal{C} \equiv \sqrt{2[1-\mathrm{Tr}(\rho_2^2)]}$

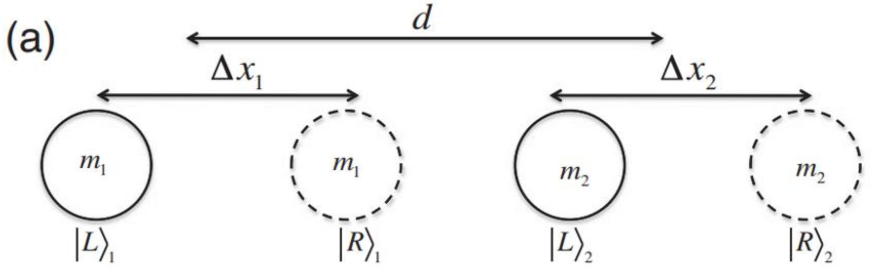
Motivation

$$\phi_{RL} = \frac{G m_1 m_2 \tau}{\hbar (d - \Delta x)} \simeq 1$$

$$\tau \simeq 1 \text{ s}$$

$$d - \Delta x \simeq 200 \ \mu\text{m}$$

$$\Longrightarrow \quad m \approx 10^{-14}\text{kg}$$



$$d - \Delta x \ll d$$

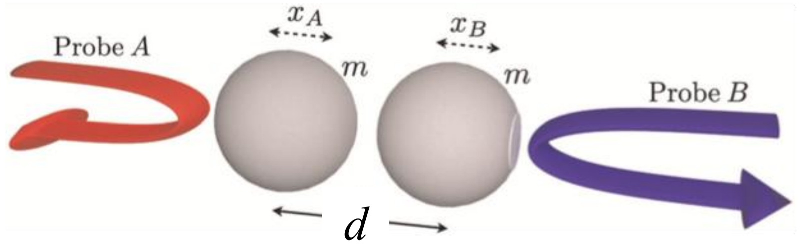
目前实验:

- **10^{-17}~ 10^{-18} kg**
- Moderate sized (~ **10 nm - 1μm**) superpositions
- Still **several order of magnitude less** than what we need

Credit: Fuwen Shu

Motivation

考虑两个有质量物体囚禁在一维谐振腔，两个粒子可看作理想的谐振子



$$F_N = \frac{Gm^2}{r^2}$$

系统哈密顿量为：$H = H_0 + H_g$

$$H_0 = \frac{p_A^2}{2m} + \frac{1}{2}m\omega^2 x_A^2 + \frac{p_B^2}{2m} + \frac{1}{2}m\omega^2 x_B^2$$

$$H_g = -\frac{Gm^2}{d + x_B - x_A}, \qquad x_A - x_B \ll d$$

引力相互作用按小量展开：

$$H_g \approx -\frac{Gm^2}{d}\left(1 + \frac{(x_A - x_B)}{d} + \frac{(x_A - x_B)^2}{d^2} + \ldots\right)$$

相互作用

引入：

$$X_j = \sqrt{m\omega/\hbar}\, x_j, P_j = p_j/\sqrt{\hbar m\omega},$$

$$\nu = Gm^2/\sqrt{\hbar m\omega d^4}$$

$$\eta = 2Gm/\omega^2 d^3 \ll 1$$

哈密顿量可表示为：

$$H = \frac{\hbar\omega}{2}\left((1-\eta)X_A^2 + P_A^2 + (1-\eta)X_B^2 + P_B^2\right) - \nu(X_A - X_B) + \eta\hbar\omega X_A X_B$$

T. Krisnanda, G. Yao Tham, M. Paternostro & T. Paterek, npj Quantum Information, 6, 12 (2020)

Motivation

由 $$H=\frac{\hbar\omega}{2}\left((1-\eta)X_A^2+P_A^2+(1-\eta)X_B^2+P_B^2\right)-\nu(X_A-X_B)+\eta\hbar\omega X_A X_B$$

和 $$\dot{P}_j=-\frac{\partial H}{\partial X_j},\ \dot{X}_j=\frac{\partial H}{\partial P_j}$$

可得系统满足的方程

$$\dot{X}_j=\omega P_j$$
$$\dot{P}_A=-\omega(1-\eta)X_A-\omega\eta X_B+\nu$$
$$\dot{P}_B=-\omega(1-\eta)X_B-\omega\eta X_A-\nu$$

引入矩阵

$$u(t)=\begin{pmatrix}X_A(t) & P_A(t) & X_B(t) & P_B(t)\end{pmatrix}^T$$

则方程可表示为：

$$\dot{u}(t)=Ku(t)+\kappa$$

其中

$$K=\begin{pmatrix}0 & \omega & 0 & 0\\ -\omega(1-\eta) & 0 & -\omega\eta & 0\\ 0 & 0 & 0 & \omega\\ -\omega\eta & 0 & -\omega(1-\eta) & 0\end{pmatrix}\qquad \kappa=\nu\begin{pmatrix}0 & 1 & 0 & -1\end{pmatrix}^T$$

方程的解为：

$$u(t)=e^{Kt}u(0)+e^{Kt}\int_0^t dt' e^{-Kt'}\kappa$$

Motivation

系统的协方差矩阵可以表示为：

$$V_{ij} = \frac{1}{2}\langle\{\Delta u_i, \Delta u_j\}\rangle \qquad V(t) = \begin{pmatrix} I_A & L \\ L^T & I_B \end{pmatrix}$$

系统的纠缠（用对数负度作度量）：

$$LN \equiv \log_2 ||\rho_{AB}^{T_B}||$$

迹范数： $\|\rho\|_1 = \mathrm{Tr}\sqrt{\rho\rho^\dagger}$.

$$= max\left\{0, \frac{1}{2}\left|\log_2 2\left(\Sigma - \sqrt{\Sigma^2 - 4\det(V)}\right)\right|\right\}$$

其中：

$$\Sigma = \det\ I_A + \det\ I_B - 2\det\ L$$

对于高斯态（包含基态，热态，压缩态）

以初始基态为例： $|\psi_i\rangle = |0_A 0_B\rangle$

基态满足最小不确定性：

$$(\Delta X_j)^2 = (\Delta P_j)^2 = \frac{1}{2} \qquad V(0) = \frac{1}{2} I_4$$

四维单位矩阵

可得：

$$\Sigma(t) \approx \frac{1}{2} + \eta^2 \sin^2[\omega t(1-\eta)]$$

其中 $\eta = 2Gm/\omega^2 d^3 \ll 1$

$$LN \approx -\frac{1}{2}\log_2\left(1 - 2\eta|\sin[\omega t(1-\eta)]|\right)$$

纠缠呈周期性变化

$$t_{max} = \frac{\pi}{2\omega(1-\eta)}, \qquad LN_{max} \approx \frac{\eta}{\ln 2}$$

Motivation

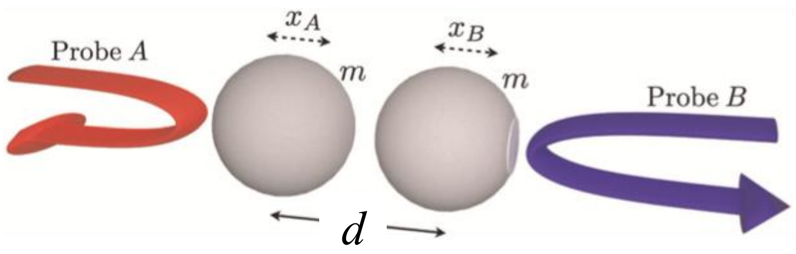
$$t_{max} = \frac{\pi}{2\omega(1-\eta)}, \quad LN_{max} \approx \frac{\eta}{\ln 2}$$

其中 $\eta = 2Gm/\omega^2 d^3 \ll 1$

取一组可制备的谐振子数据作参考：

$$m = 100\mu\text{g}, \ \omega = 100\text{kHz}, \ d = 0.1\text{mm}$$

$\eta \sim 10^{-12}$



可得： $t_{max} \sim 10^{-5}\text{s}, \ LN_{max} \sim 10^{-12}$

所需时间较短，但纠缠太弱！

对于初始压缩态： $|\psi_i\rangle = \hat{S}_A(r_A)\hat{S}_B(r_B)|0_A 0_B\rangle$

纠缠依旧呈周期性振荡

$$t_{max} = \frac{\pi}{2\eta\omega}, \ LN_{max} \approx \frac{r_A + r_B}{\ln 2} \qquad (\eta \ll r_A, r_B)$$

可以产生显著纠缠，但所需时间较长（大约 10^7s = 110 天）！

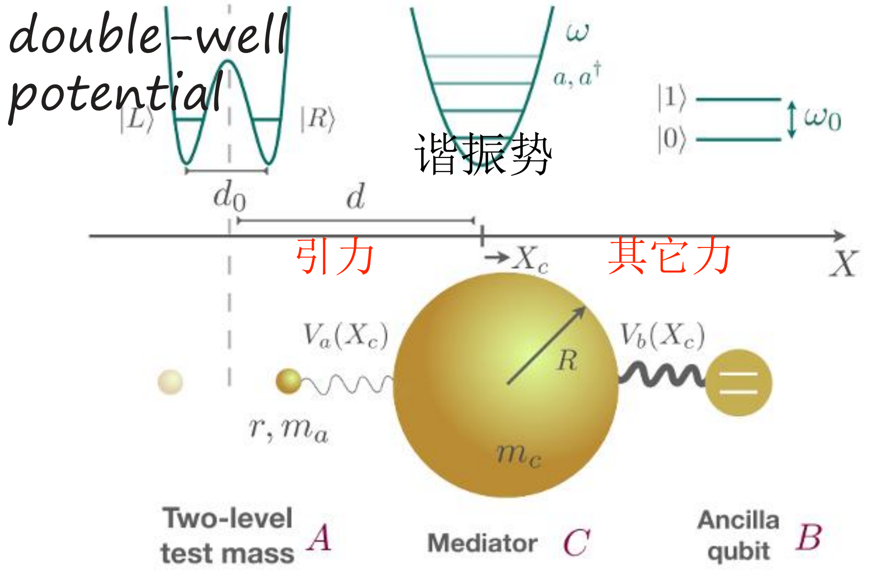
Motivation

系统哈密顿量为（忽略了A与B之间的相互作用）：

$$\hat{H} = \hbar\omega_0\hat{\sigma}_b^z + \frac{1}{2m_c}\hat{P}^2 + \frac{1}{2}m_c\omega^2\hat{X}^2 + \sum_{\alpha=L,R}\hat{V}_{a,\alpha}|\alpha\rangle\langle\alpha| + \sum_{\alpha=0,1}\hat{V}_{b,\alpha}|\alpha\rangle\langle\alpha|$$

A与C引力相互作用　　B与C相互作用

$$\hat{V}_{b,\alpha} \gg \hat{V}_{a,\alpha}$$



将引力势展开到二阶

$$\hat{V}_{a,\pm}(\hat{X}) = -\frac{Gm_am_c}{|d \mp \frac{d_0}{2} + \hat{X}|}$$

$$= -\frac{Gm_am_c}{d}\left(1 + \frac{d_0^2}{4d^2} \pm \frac{d_0}{2d} - \left(1 \pm \frac{d_0}{d}\right)\frac{\hat{X}}{d} + \frac{\hat{X}^2}{d^2}\right)$$

$$\hat{V}_{a,+} \to \hat{V}_{a,R}$$

$$\hat{V}_{a,-} \to \hat{V}_{a,L}$$

哈密顿量可表示为

$$\hat{H} = \hbar\omega_a\hat{\sigma}_a^z + \hbar\omega_b\hat{\sigma}_b^z + \hbar\tilde{\omega}\hat{a}^\dagger\hat{a} + \hbar(g_a\hat{\sigma}_a^z + g_b\hat{\sigma}_b^z)(\hat{a} + \hat{a}^\dagger)$$

$$\tilde{\omega}^2 = \omega^2 - (2Gm_a/d^3) + (2/m_c)V_b^{(2)}$$

$$\omega_a = Gm_am_cd_0/(2\hbar d^2)$$

$$g_a = -(Gm_ad_0/d^3)\sqrt{m_c/(2\tilde{\omega}\hbar)}$$

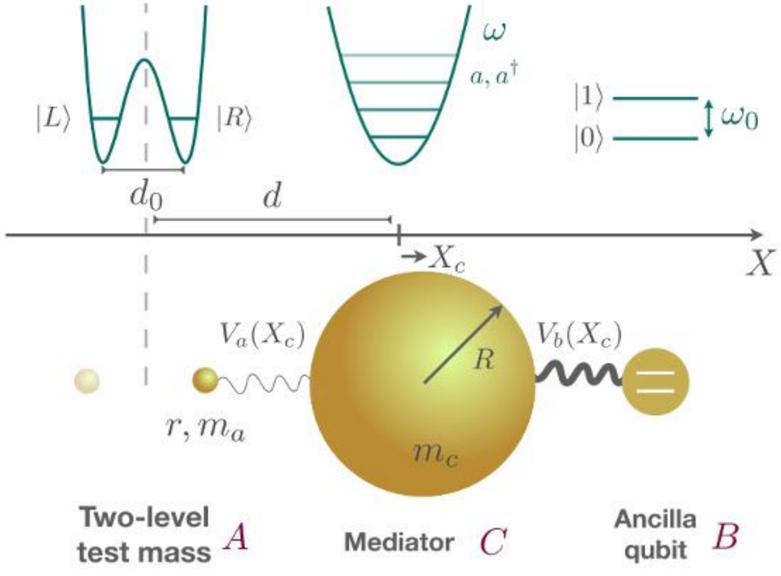
$$\hat{\sigma}_z^a = |L\rangle\langle L| - |R\rangle\langle R|$$

J. S. Pedernales, K. Streltsov, and M. B. Plenio Phys. Rev. Lett. 128, 110401 (2022)

Motivation

在相互作用绘景下计算

$$\hat{H} = \underbrace{\hbar\omega_a\hat{\sigma}_a^z + \hbar\omega_b\hat{\sigma}_b^z + \hbar\tilde{\omega}\hat{a}^\dagger\hat{a}}_{H_0} + \underbrace{\hbar(g_a\hat{\sigma}_a^z + g_b\hat{\sigma}_b^z)(\hat{a} + \hat{a}^\dagger)}_{H_I}$$



相互作用哈密顿量的演化：

$$\hat{H}_I(t) = e^{i\hat{H}_0 t}\hat{H}_I(0)e^{-i\hat{H}_0 t} = \hbar(g_a\hat{\sigma}_a^z + g_b\hat{\sigma}_b^z)(\hat{a}e^{-i\tilde{\omega}t} + \hat{a}^\dagger e^{i\tilde{\omega}t})$$

对应的时间演化算符：

$$\hat{U}_I(t) = \exp\left[\hat{\Omega}^{(1)}(t) + \hat{\Omega}^{(2)}(t) + \ldots\right]$$

其中：

$$\hat{\Omega}^{(1)}(t) = -i\int_0^t dt'\hat{H}_I(t') = -i\left(g_a\hat{\sigma}_a^z + g_b\hat{\sigma}_b^z\right)\left(\hat{a}\frac{e^{-i\tilde{\omega}t} - 1}{-i\tilde{\omega}} + \hat{a}^\dagger\frac{e^{i\tilde{\omega}t} - 1}{i\tilde{\omega}}\right)$$

$$\hat{\Omega}^{(2)}(t) = -\frac{1}{2}\int_0^t dt'\int_0^{t'} dt''\left[\hat{H}_I(t'), \hat{H}_I(t'')\right] = i\left(g_a\hat{\sigma}_a^z + g_b\hat{\sigma}_b^z\right)^2\left(\frac{t}{\tilde{\omega}} - \frac{\sin(\tilde{\omega})}{\tilde{\omega}^2}\right)$$

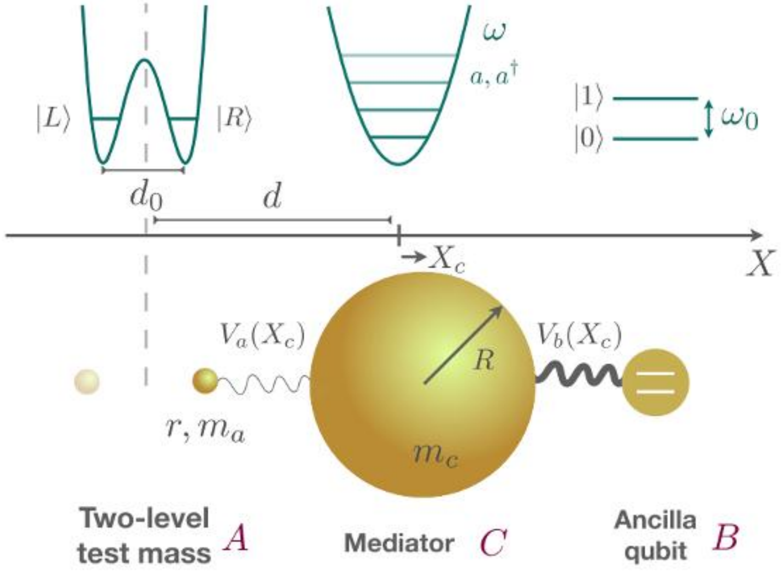
A、B之间的二阶相互作用

Motivation

时间演化算符保留前两阶：

$$\hat{U}_I(t) = \exp\left\{\left(g_a\hat{\sigma}_z^a + g_b\hat{\sigma}_z^b\right)\left(-\hat{a}\alpha_t + \hat{a}^\dagger\alpha_t^*\right)\right\}$$
$$\times \exp\left\{-i\frac{2g_ag_b}{\tilde{\omega}}\hat{\sigma}_z^a\hat{\sigma}_z^b\left(t - \frac{\sin\tilde{\omega}t}{\tilde{\omega}}\right)\right\}$$

其中： $\alpha_t = [(e^{-i\tilde{\omega}t} - 1)/\tilde{\omega}]$



显然当： $t_n = 2n\pi/\tilde{\omega}$ $\alpha_{t_n} = 0$ $\sin\tilde{\omega}t_n = 0$

时间演化算符可以约化为：

$$\hat{U}_I(t_n) = \exp\left\{-i\frac{2g_ag_b}{\tilde{\omega}}\hat{\sigma}_z^a\hat{\sigma}_z^b t_n\right\}$$

在该特定时间，AC与BC间没有相互作用，但AB间存在间接相互作用

Motivation

考虑系统初态为：

$$|\psi_i\rangle = \frac{1}{2}(|L\rangle + |R\rangle) \otimes (|1\rangle + |2\rangle)$$

密度矩阵：

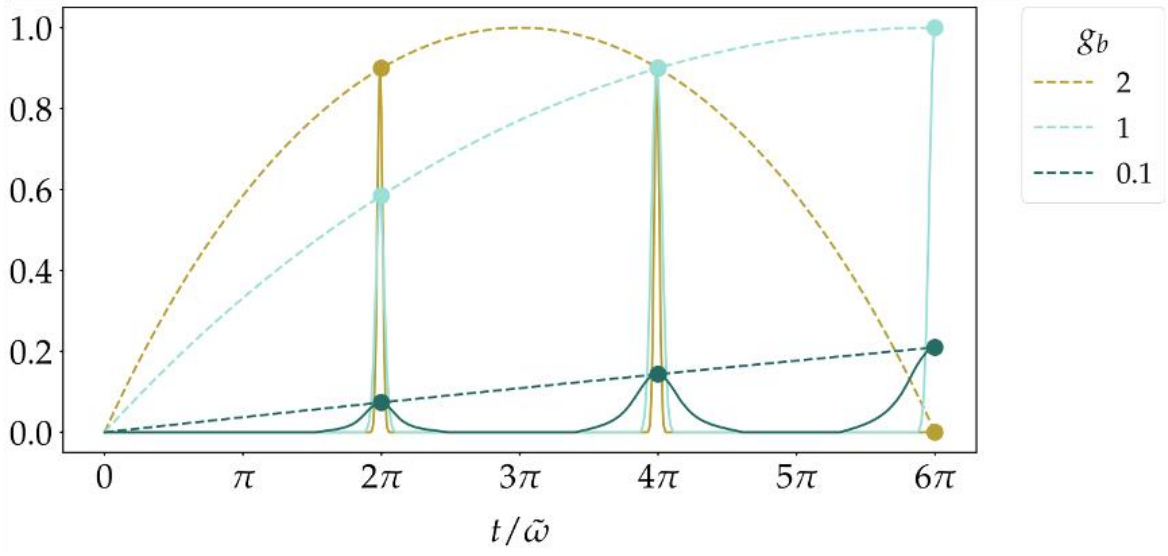
$$\hat{\rho}(t_n) = \frac{1}{4}\begin{pmatrix} 1 & e^{-2i\tilde{\Omega}t_n} & e^{-2i\tilde{\Omega}t_n} & 1 \\ e^{2i\tilde{\Omega}t_n} & 1 & 1 & e^{2i\tilde{\Omega}t_n} \\ e^{2i\tilde{\Omega}t_n} & 1 & 1 & e^{2i\tilde{\Omega}t_n} \\ 1 & e^{-2i\tilde{\Omega}t_n} & e^{-2i\tilde{\Omega}t_n} & 1 \end{pmatrix}$$

其中：
$$\tilde{\Omega} = 2g_a g_b/\tilde{\omega}$$

$$g_a = -(Gm_a d_0/d^3)\sqrt{m_c/(2\tilde{\omega}\hbar)}$$

可得纠缠（用对数负度作度量）：

$$LN(t_n) = \log_2(1 + |\sin(2\tilde{\Omega}t_n)|)$$



实线：the *entanglement between the TLTM and the AQ for different values of the coupling*

虚线：*the evolution of entanglement between two generic qubits governed by* $\hat{H} = (2gagb/\tilde{\omega})\hat{\sigma}_z\hat{\sigma}_z$.

纠缠相位相比无媒介直接相互作用放大了约 $\frac{g_b}{\tilde{\omega}}$ 倍

通过增大 $\frac{g_b}{\tilde{\omega}}$ 减少达到最大纠缠所需时间

Motivation

由 $\Delta_x \ll d$ 可导出 $\frac{g_b}{\tilde{\omega}} \ll \frac{1}{2}\sqrt{\frac{m_c\tilde{\omega}d^2}{\hbar}}$

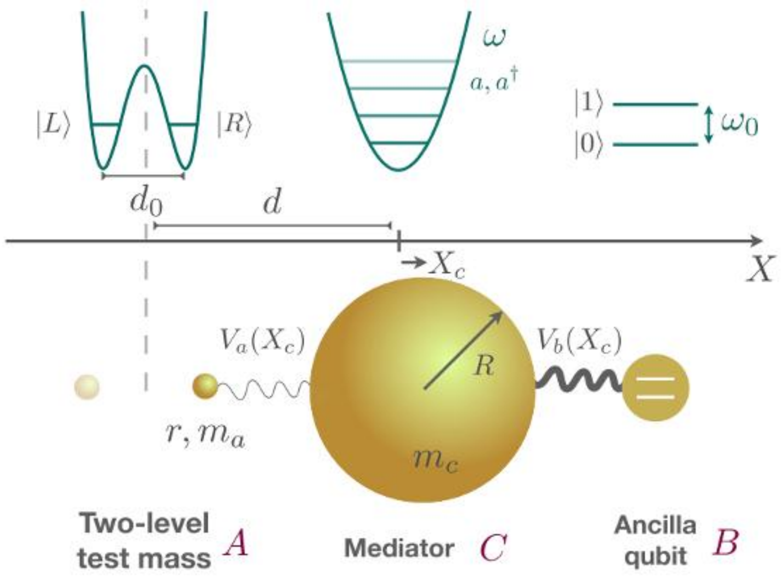
取一组可行的数据（硅）作参考：

$$r = 70\text{nm},\ R = 7\mu\text{m},\ \rho \sim 2400\text{kg/m}^3,\ d = 166\mu\text{m},\ d_0 = 500\text{nm}$$

$$\tilde{\omega} = 100\ \text{Hz}$$

可得：

$$\frac{g_b}{\tilde{\omega}} \ll 10^8$$



达到最大纠缠所需时间可减少不超过10^8倍

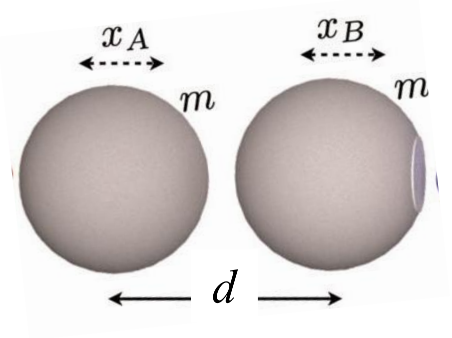
Our work

两个引力相互作用谐振子： $H_0=\frac{p_A^2}{2m}+\frac{1}{2}m\omega^2x_A^2+\frac{p_B^2}{2m}+\frac{1}{2}m\omega^2x_B^2$ $H_g=-\frac{Gm^2}{d+x_B-x_A}$, $x_A-x_B\ll d$

定义修正频率： $\tilde{\omega}^2=\omega^2(1-\eta)$

和前面类似，哈密顿量可表示为：

$$H\approx\frac{\tilde{\omega}}{2}(P_A^2+X_A^2+P_B^2+X_B^2)+\eta\tilde{\omega}X_AX_B$$



量子化： $X_j=\sqrt{m\omega/\hbar}x_j, P_j=p_j/\sqrt{\hbar m\omega}$,

$$\hat{X}_j=\frac{1}{\sqrt{2}}\left(\hat{a}_j+\hat{a}_j^\dagger\right),\ \hat{P}_j=\frac{1}{\sqrt{2}}\left(\hat{a}_j-\hat{a}_j^\dagger\right)$$

可得：

$$H=\underbrace{\tilde{\omega}\left(\hat{a}_A^\dagger\hat{a}_A+\hat{a}_B^\dagger\hat{a}_B\right)}_{H_0}+\underbrace{\frac{\eta\tilde{\omega}}{2}\left(\hat{a}_A^\dagger\hat{a}_B+\hat{a}_A\hat{a}_B^\dagger+\hat{a}_A\hat{a}_B+\hat{a}_A^\dagger\hat{a}_B^\dagger\right)}_{H_I}$$

Our work

相互作用哈密顿量的演化为：

$$\hat{H}_I(t) = e^{i\hat{H}_0 t}\hat{H}_I(0)e^{-i\hat{H}_0 t} = \frac{\eta\tilde{\omega}}{2}\left(\hat{a}_A^\dagger\hat{a}_B + \hat{a}_A\hat{a}_B^\dagger + \hat{a}_A\hat{a}_B e^{-2i\tilde{\omega}t} + \hat{a}_A^\dagger\hat{a}_B^\dagger e^{2i\tilde{\omega}t}\right)$$

对应的时间演化算符：

$$\hat{U}_I(t) = \exp\left[\hat{\Omega}^{(1)}(t) + \hat{\Omega}^{(2)}(t) + \ldots\right]$$

对 η 作小量展开，保留到一阶：

$$\hat{U}_I(t) \approx 1 - \frac{i\eta\tilde{\omega}t}{2}\left(\hat{a}_A^\dagger\hat{a}_B + \hat{a}_A\hat{a}_B^\dagger\right) - \frac{i\eta\sin(\tilde{\omega}t)}{2}\left(\hat{a}_A\hat{a}_B e^{-i\tilde{\omega}t} + \hat{a}_A^\dagger\hat{a}_B^\dagger e^{i\tilde{\omega}t}\right) \qquad (\eta\tilde{\omega}t \ll 1)$$

随时间增长

周期性变化

Our work

对于初始基态： $|\psi_i\rangle = |0_A 0_B\rangle$

$$\hat{U}_I(t) \approx 1 - \frac{i\eta\tilde{\omega}t}{2}\left(\hat{a}_A^\dagger \hat{a}_B + \hat{a}_A \hat{a}_B^\dagger\right) - \frac{i\eta\sin(\tilde{\omega}t)}{2}\left(\hat{a}_A\hat{a}_B e^{-i\tilde{\omega}t} + \hat{a}_A^\dagger \hat{a}_B^\dagger e^{i\tilde{\omega}t}\right)$$

只有该部分起作用

密度矩阵：

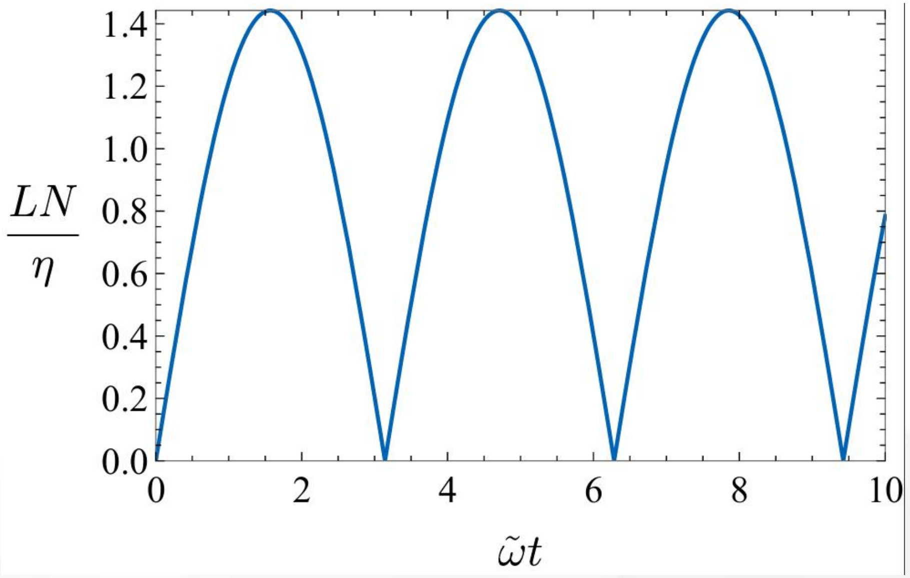
$$\rho_{AB}(t) \approx |0_A 0_B\rangle\langle 0_A 0_B| + \frac{i\eta}{2} e^{-i\tilde{\omega}t}\sin(\tilde{\omega}t)\left(|0_A 0_B\rangle\langle 1_A 1_B| - |1_A 1_B\rangle\langle 0_A 0_B|\right)$$

纠缠：

$$LN \equiv \max\left(0, \log_2 ||\rho_{AB}^{T_B}||\right)$$
$$= \log_2\left(1 + |\eta\sin(\tilde{\omega}t)|\right)$$

$$t_{max} = \frac{\pi}{2\tilde{\omega}}, LN_{max} \approx \frac{\eta}{\ln 2}$$

表达式与已有结果一致



Our work

对于初始激发态：以 $|\psi_i\rangle = |0_A 1_B\rangle$ 为例

$$\hat{U}_I(t) \approx 1 - i\eta\tilde{\omega}t\left(\hat{a}_A^\dagger \hat{a}_B + \hat{a}_A \hat{a}_B^\dagger\right) - i\eta \sin(\tilde{\omega}t)\left(\hat{a}_A \hat{a}_B e^{-i\tilde{\omega}t} + \hat{a}_A^\dagger \hat{a}_B^\dagger e^{i\tilde{\omega}t}\right)$$

有贡献

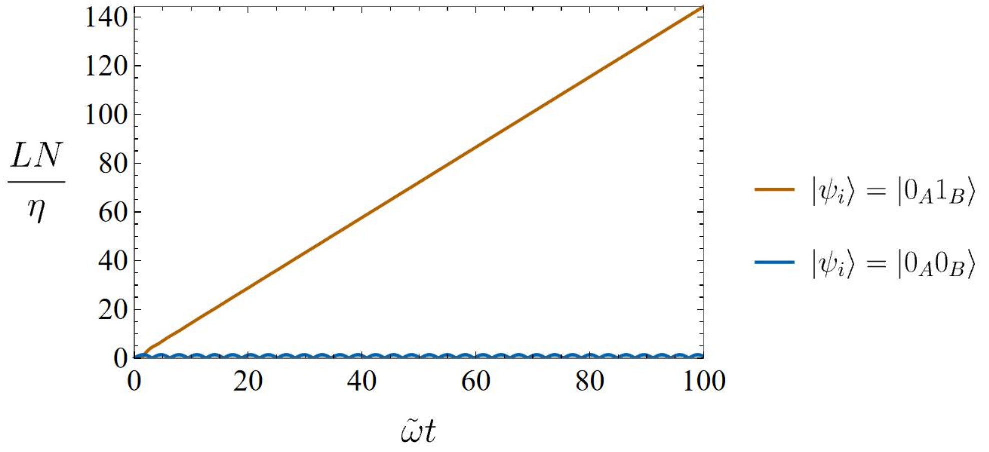
密度矩阵：

$$\rho_{AB}(t) \approx |0_A 1_B\rangle\langle 0_A 1_B| + \frac{i\eta\tilde{\omega}t}{2}\left(|0_A 1_B\rangle\langle 1_A 0_B| - |1_A 0_B\rangle\langle 0_A 1_B|\right) + \frac{i\eta}{2}\sqrt{2}\sin(\tilde{\omega}t)\left(e^{-i\tilde{\omega}t}|0_A 1_B\rangle\langle 1_A 2_B| - e^{i\tilde{\omega}t}|1_A 2_B\rangle\langle 0_A 1_B|\right).$$

纠缠：

$$LN = \log_2\left(1 + \eta|2\sin^2(\tilde{\omega}t) - (\tilde{\omega}t)^2|^{\frac{1}{2}}\right)$$

随时间增长



Our work

对于一般的初始激发态： $|\psi_i\rangle = |n_A n_B\rangle$

纠缠：

$$LN \approx \log_2\left(1 + \eta\tilde{\omega}t(\sqrt{(1+n_A)n_B} + \sqrt{(1+n_B)n_A})\right) \qquad (\eta\tilde{\omega}t \ll 1)$$

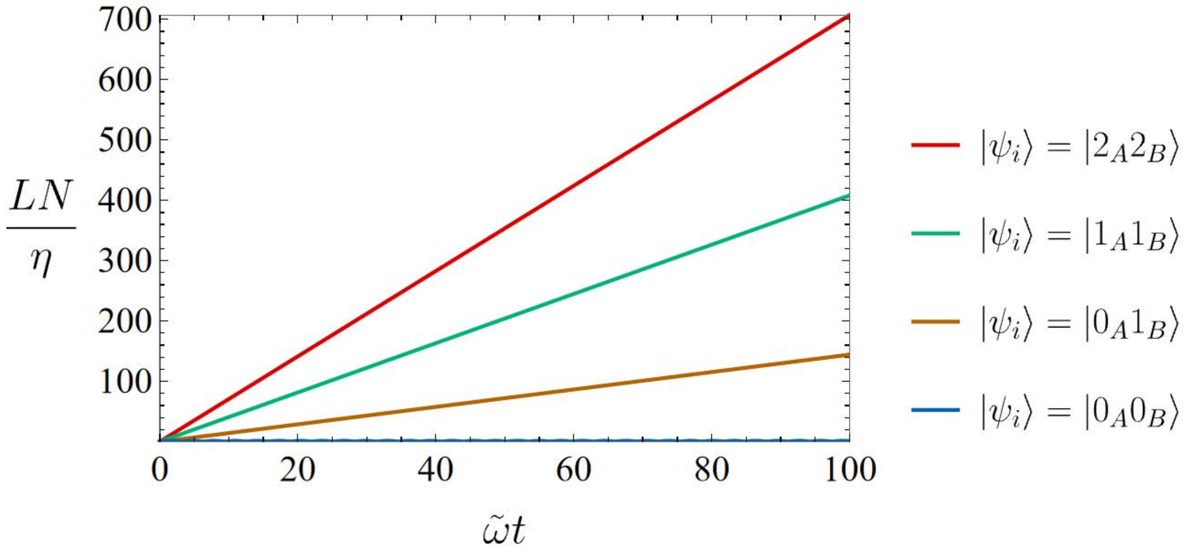
纠缠与时间以及初态的能级呈正相关

相比初始基态： $LN_{max} \approx \frac{\eta}{\ln 2}$

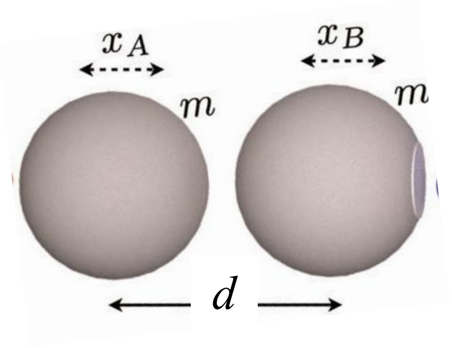
增大约： $\tilde{\omega}t(\sqrt{(1+n_A)n_B} + \sqrt{(1+n_B)n_A})$ 倍

例如： $n_A = 1,\ n_B = 1,\ \tilde{\omega} = 10^5\ \text{Hz}$

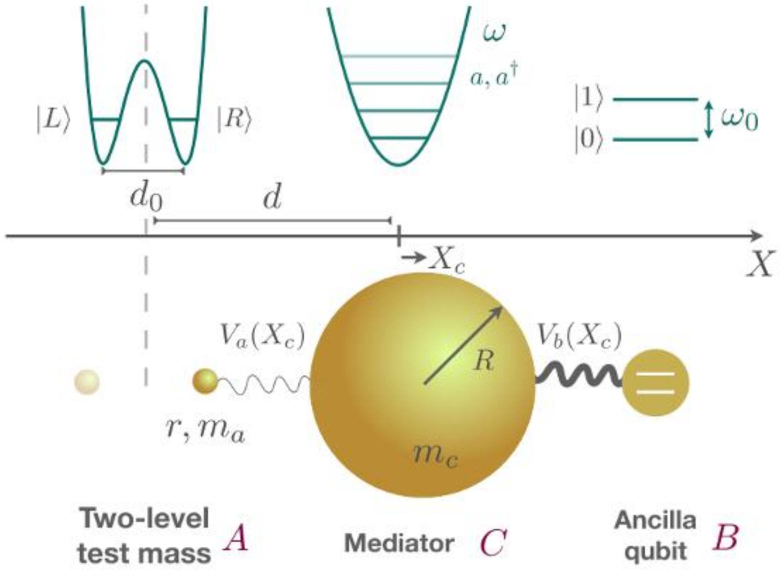
纠缠可在秒量级内增大约10^5倍



讨 论



制备、操控



$$\tilde{\omega}t(\sqrt{(1+n_A)n_B}+\sqrt{(1+n_B)n_A})$$

$$\frac{g_b}{\tilde{\omega}}$$

谢谢大家

Motivation

设谐振子振幅为A，有 $\Delta_x^2 = \frac{A^2}{2}$ $\langle X^2 \rangle = \frac{A^2}{2}$

最大位置不确定度大小（平方平均值减平均值平方开根号）

谐振子的平均动能可表示为：$\frac{1}{2} m_c \tilde{\omega}^2 \langle X^2 \rangle = \frac{1}{2}(\langle n(t) \rangle + 1/2)\hbar\tilde{\omega}$

平均声子占居数

$$\Rightarrow m_c \tilde{\omega}^2 \Delta_x^2 \simeq \langle n(t) \rangle \hbar \tilde{\omega} \Rightarrow \Delta_x \sim \sqrt{\hbar/(m_c \tilde{\omega})} \sqrt{\langle n(t) \rangle}$$

$$\begin{aligned} \langle n(t) \rangle &= \langle \hat{U}_I \hat{a}^\dagger \hat{a} \hat{U}_I^{-1} \rangle_t \\ &= \bar{n}_0 + |\alpha_t|^2 \left\langle \left(g_a \hat{\sigma}_z^a + g_b \hat{\sigma}_z^b \right)^2 \right\rangle \\ &= \bar{n}_0 + 4 \frac{g_a^2 + g_b^2}{\tilde{\omega}^2} \sin^2 \frac{\tilde{\omega} t}{2} \end{aligned}$$

其中：$\bar{n}_0 = \langle \hat{a}^\dagger \hat{a} \rangle$

取近似：$\langle n(t) \rangle_{\max} \approx \bar{n}_0 + 4 \frac{g_b^2}{\tilde{\omega}^2}$

由 $\Delta_x \ll d$

可得：$\frac{g_b}{\tilde{\omega}} \ll \frac{1}{2} \sqrt{\frac{m_c \tilde{\omega} d^2}{\hbar} - \bar{n}_0}$

END

THANK YOU FOR LISTENING

Results

	bin 1	bin 2	bin 3	bin 4	bin 5
z_i	0.017	0.049	0.144	0.296	0.544
Redshift range	$0.010 < z \leq 0.027$	$0.027 < z \leq 0.087$	$0.087 < z \leq 0.237$	$0.237 < z \leq 0.370$	$0.370 < z \leq 0.799$
N_{SN}	312	312	312	312	312
m_i	1[illegible]15	17.338 ± 0.009	19.800 ± 0.010	21.563 ± 0.008	23.088 ± 0.011
m'_i	12[illegible]571	45.46 [illegible] 1	16.530 ± 0.223	8.543 ± 0.202	4.704 ± 0.076
Δm_i	0.024 ± 0.016	0.006 ± 0.011	-0.003 ± 0.013	0.006 ± 0.014	-0.017 ± 0.020
$\Delta m'_i$	-6.894 ± 2.571	-0.463 ± 0.512	0.084 ± 0.224	0.146 ± 0.203	-0.072 ± 0.079

$> 1\sigma$

2.7σ

[a] The mean values [illegible]ertainty are shown.

Consistent with the prediction of ΛCDM model

[b] $\Delta m_i(\Delta m'_i)$ denot[illegible]nces between the constraints on $m_i(m'_i)$ and the fiducial model: ΛCDM model with $\Omega_{m0} = 0.333 \pm 0.018$ and $\mathcal{M} = 25 + 5\log_{10}\left(\frac{c}{H_0}\right) + M = 23.808 \pm 0.007$.

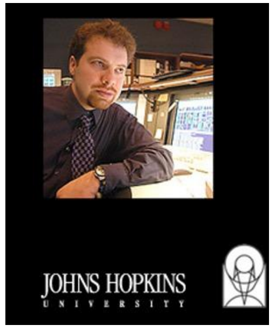
A variation of the absolute magnitude M in the very low redshift region

Motivation

宇宙加速膨胀

Perlmutter

Schmidt

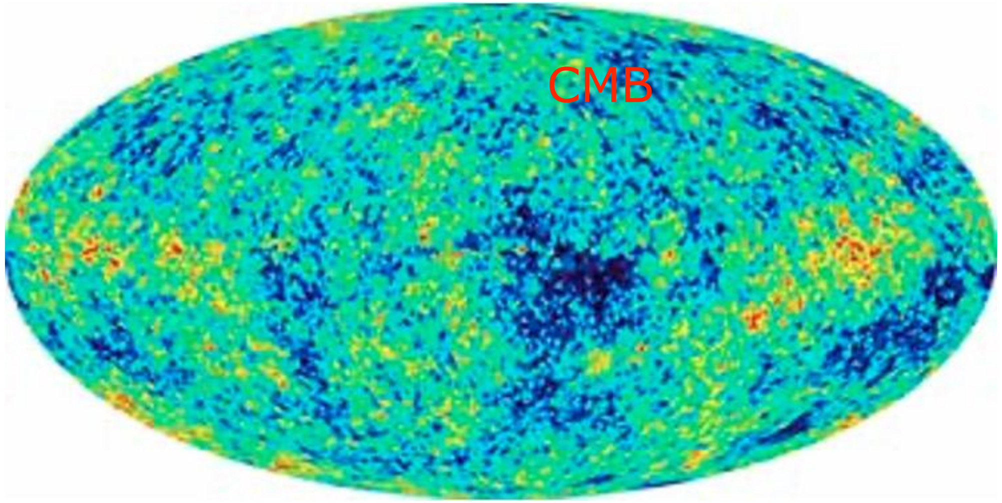


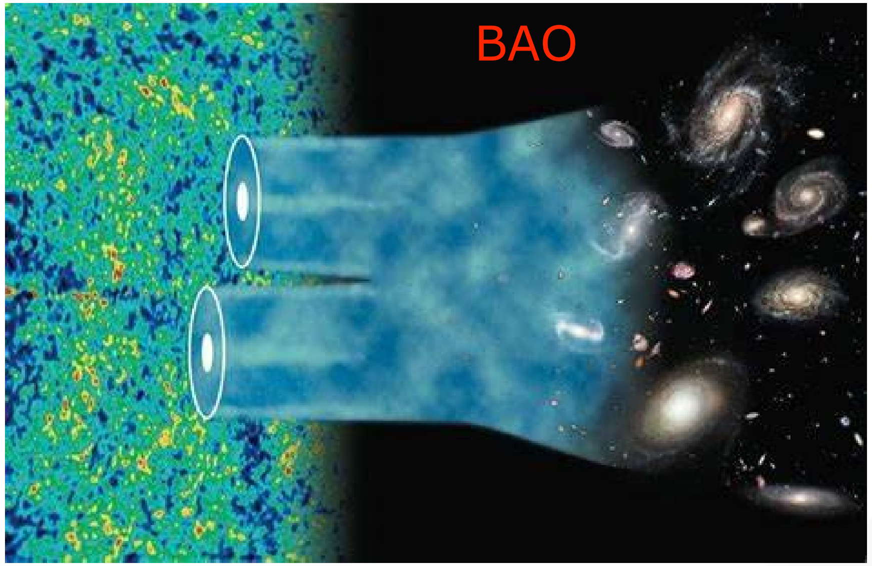
Riess

获2011年Nobel物理学奖

$$G^{\mu\nu} = 8\pi G T^{\mu\nu}$$

修改引力　　　　暗能量





Motivation

$$\mathcal{E} = \log_2 ||\rho_{AB}^{T_B}|| \\ = max\left\{0, \frac{1}{2}\left|\log_2 2\left(\Sigma - \sqrt{\Sigma^2 - 4\det(V)}\right)\right|\right\}$$

$$\Sigma = \det\ I_A + \det\ I_B - 2\det\ L$$

$$(\Delta X_j)^2 = (\Delta P_j)^2 = \frac{1}{2} \qquad V(0) = \frac{1}{2} I_4$$

$$\Sigma(t) \approx \frac{1}{2} + \eta^2 \sin^2[\omega t(1-\eta)] \qquad \eta = 2Gm/\omega^2 d^3 \ll 1$$

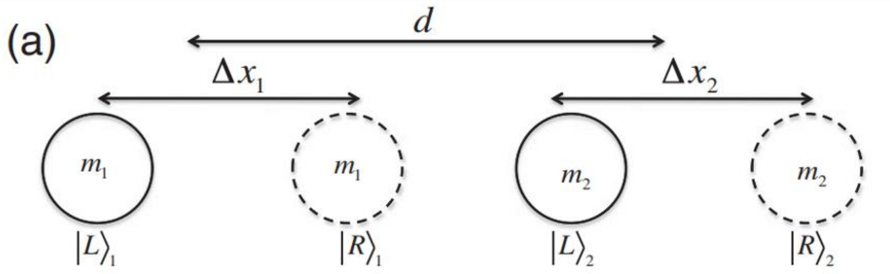
$$\mathcal{E} \approx -\frac{1}{2}\log_2\left(1 - 2\eta\,|\sin[\omega t(1-\eta)]|\right)$$

$$\omega \sim 0.1\text{Hz},\ t_{max} \sim 15\text{s} \qquad R = 7\mu\text{m},\ \rho \approx 2400\,\text{Kg/m}^3,\ d = 166\,\mu\text{m},\ \eta \sim 10^{-8}$$

The time becomes shorter, while the entanglement is

Motivation

$$\mathcal{C} = \left|\sin\left(\frac{\phi_{RL} + \phi_{RL} - 2\phi}{2}\tau\right)\right| \approx \sin\left(\frac{Gm_1m_2\tau}{\hbar d}\left(\frac{\Delta x}{d}\right)^2\right)$$



$$r = 70\,\mathrm{nm},\ \rho \approx 2400\,\mathrm{Kg/m^3},\ \Delta x = 500\mathrm{nm},\ d = 166\,\mu\mathrm{m}$$

$$\frac{Gm_1m_2\tau}{\hbar d}\left(\frac{\Delta x}{d}\right)^2 \sim \frac{\pi}{2} \Rightarrow \tau \sim 3.8 \times 10^{12}\mathrm{s} \sim 120000\mathrm{year}$$

If we can reduce the time required to respond to entanglement?