Quantum effects of gravity induced by fluctuations and superposition of spacetime

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Outline

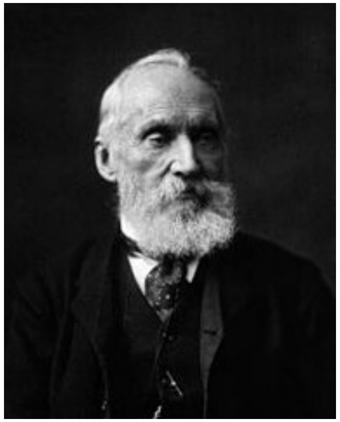
- Motivation
- Quantum gravitational effects induced by fluctuations of spacetime
- Quantum gravitational effects induced by quantum superposition of spacetime
- Summary

Background and motivation

"The beauty and clearness of the dynamical theory, which asserts heat and light to be modes of motion, is at present obscured by two clouds...."

Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light

(27th April 1900, *Lord Kelvin*)



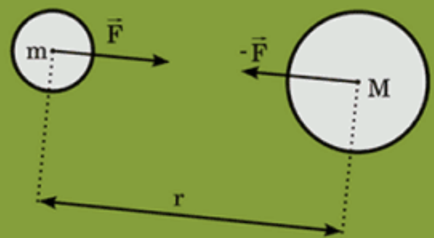
The question of the existence of an electromagnetic medium—aether.

The failure of classical physics to explain blackbody radiation.

Theory of relativity ⟹ Quantum gravity? ⟸ Quantum theory

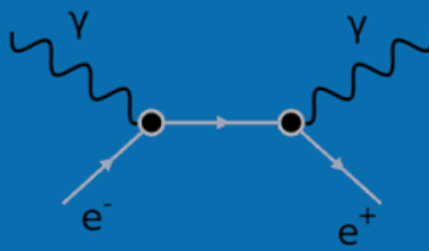
- **A full theory of quantum gravity is still elusive**

$$G_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$



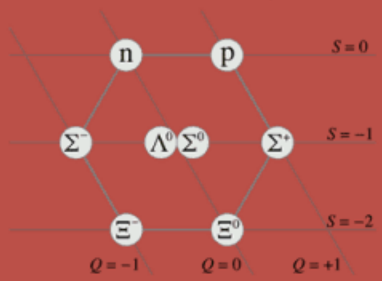
Gravity Graviton?

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



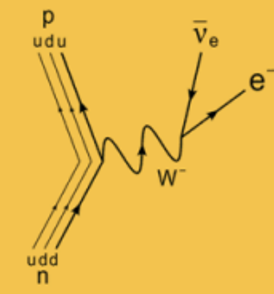
Electromagnetism Photon

$$\mathcal{L} = \bar{\psi}_i\left(i\gamma^\mu\left(D_\mu\right)_{ij} - m\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$



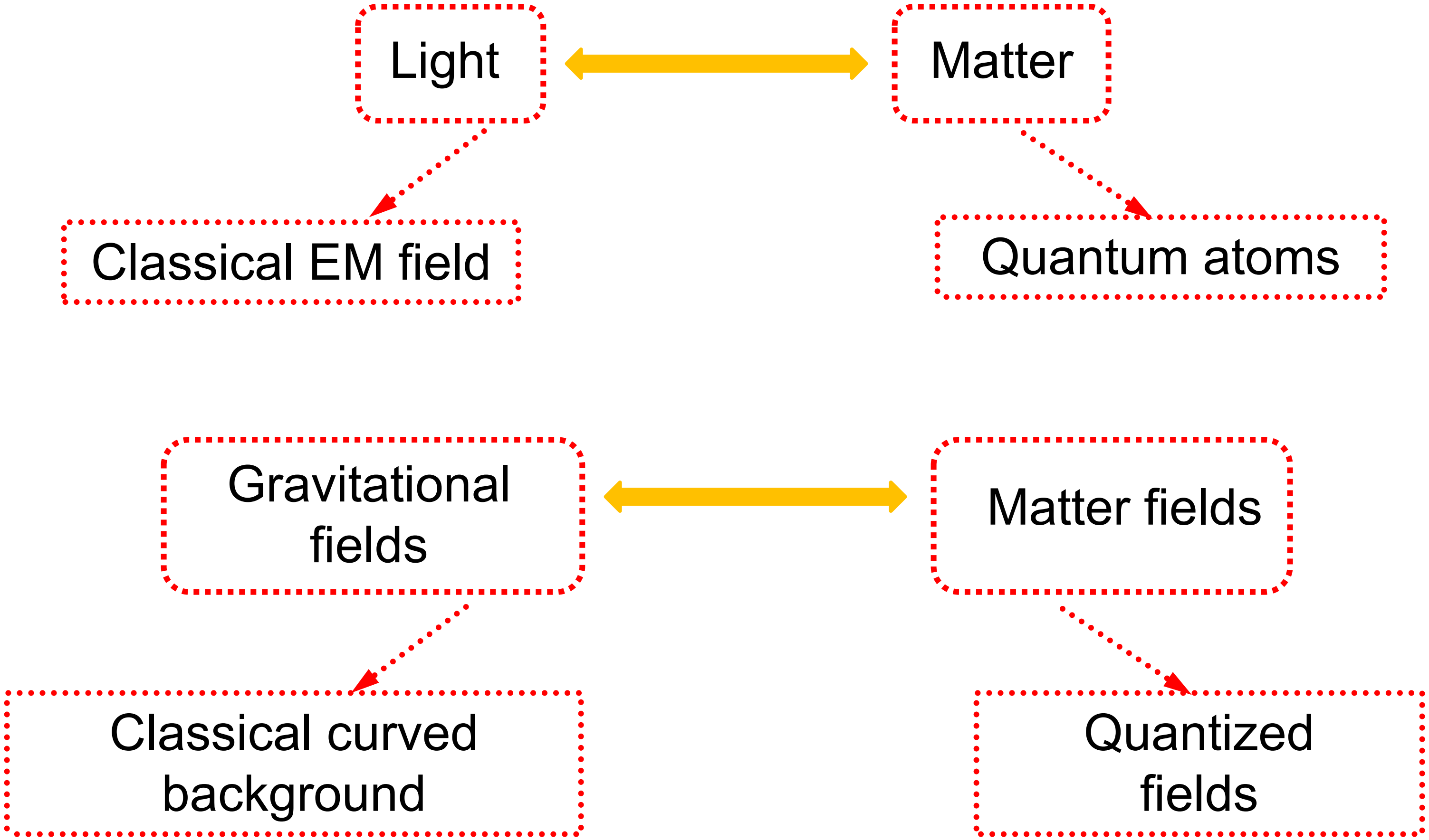
Strong Gluon

$$\mathcal{L} = g\left(\bar{\nu}_{eL}, \bar{e}\right)\gamma^\mu\left\{\begin{pmatrix} -\sqrt{1+\xi^2}Z_\mu & 0 \\ 0 & \frac{\xi A_\mu}{\sqrt{1+\xi^2}} - \frac{\xi^2}{\sqrt{1+\xi^2}}Z_\mu \end{pmatrix} + \frac{1-\gamma^5}{4}\begin{pmatrix} -\sqrt{1+\xi^2}Z_\mu & -\sqrt{2}W^+_\mu \\ -\sqrt{2}W_\mu & \sqrt{1+\xi^2}Z_\mu \end{pmatrix}\right\}\begin{pmatrix} \nu_{eL} \\ e \end{pmatrix}$$



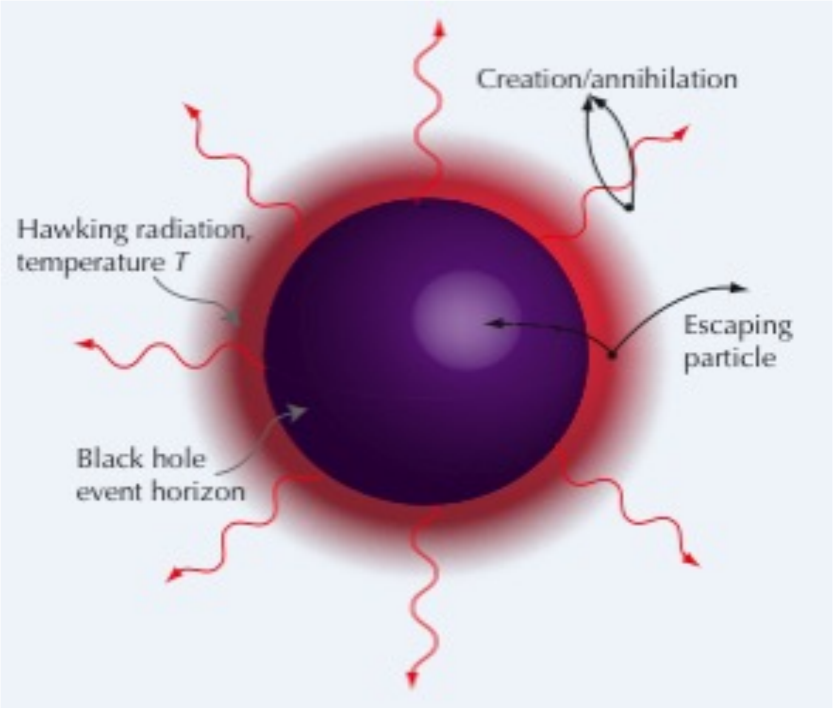
Weak W^+, W^-, Z^0

Can we investigate quantum effects of gravity?



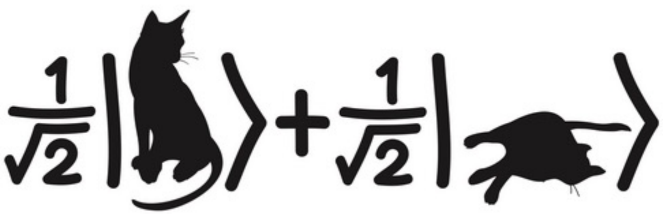
The basic idea of QFT in curved spacetime

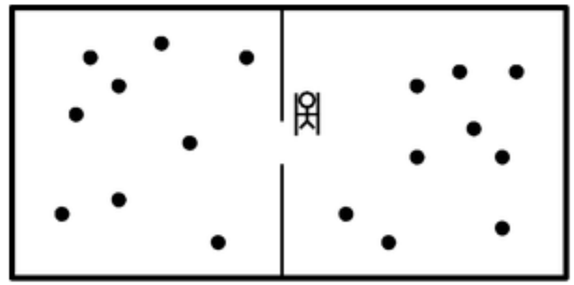
- Hawking radiation (1974)

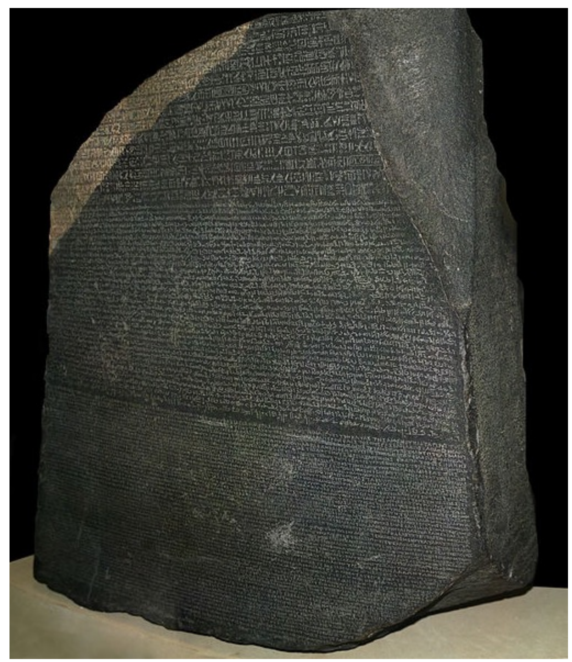


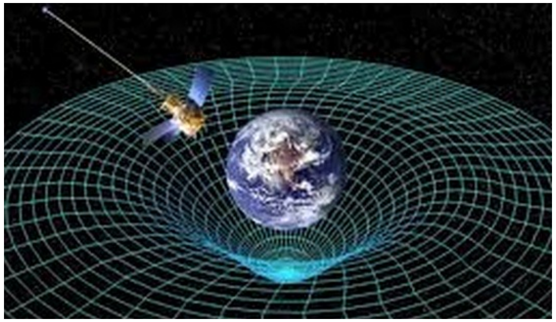
S. Hawking, Nature 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975).

- Rosetta Stone



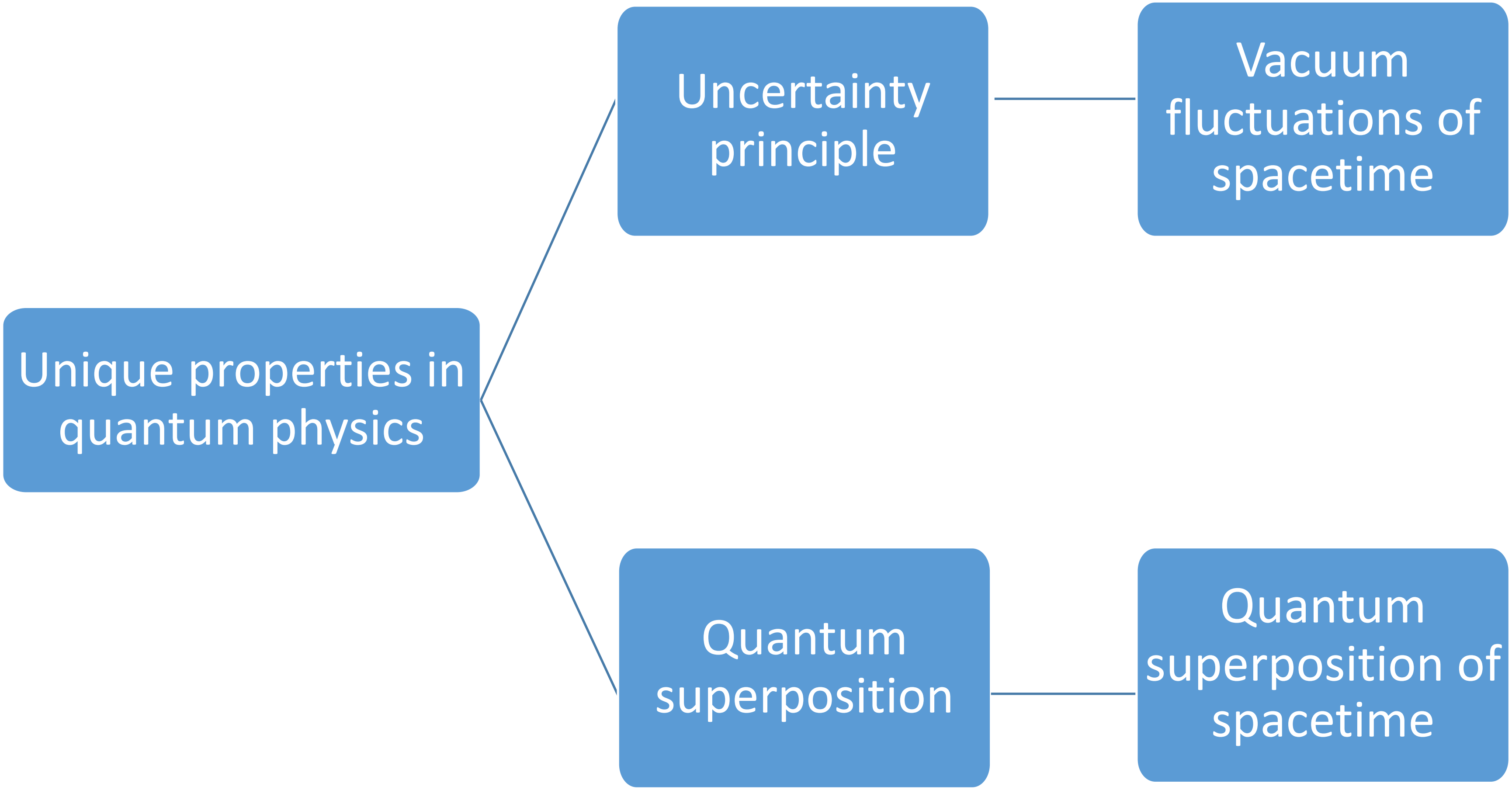






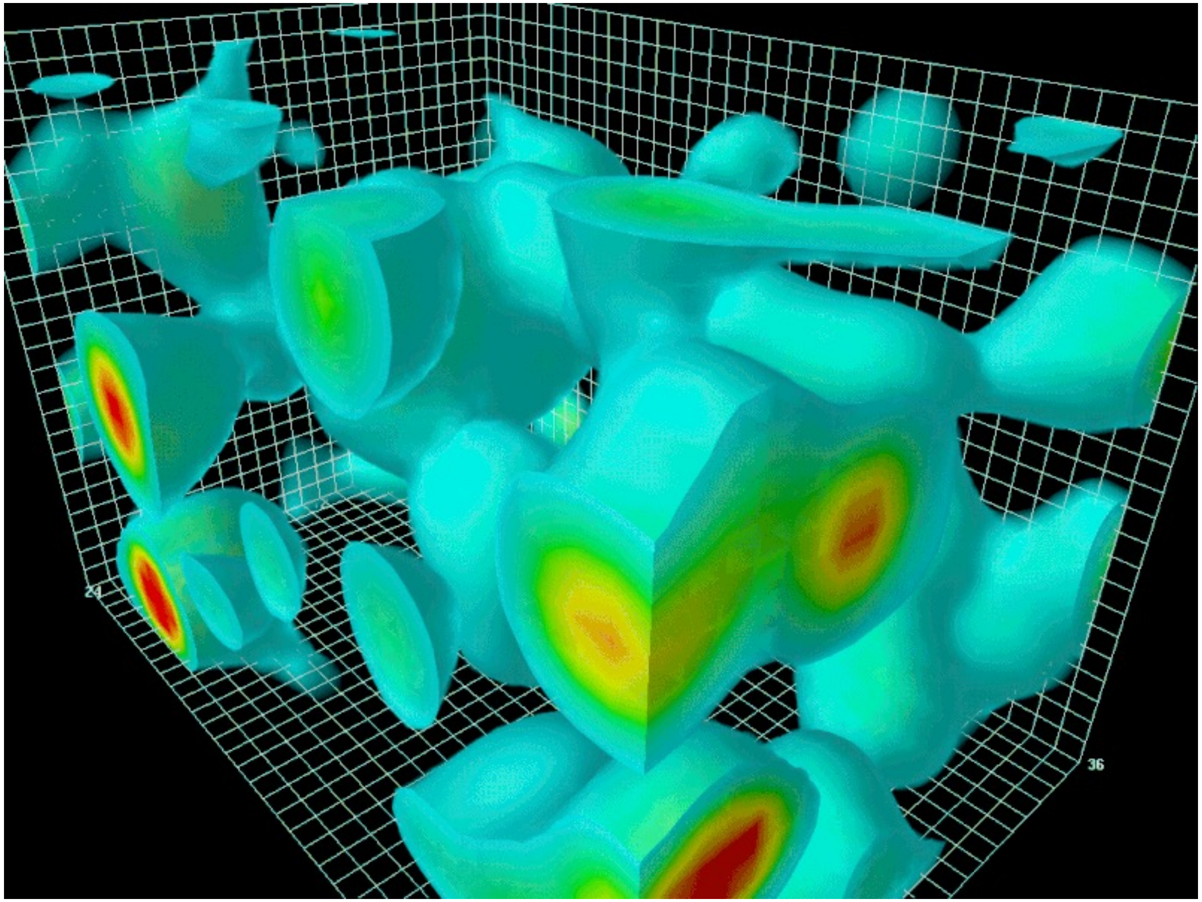
- Quantum effects unique to curved spacetime
    - Hawking radiation
    - Unruh effect
    - Cosmological particle creation
    - Gibbons-Hawking effect
    - …
- **However, the spacetime itself is still classical.**

How about the spacetime itself is quantum?

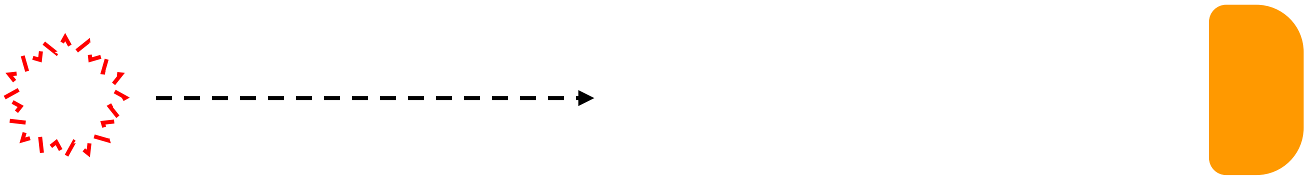


Vacuum fluctuations of spacetime

- Quantum fluctuations of spacetime as necessitated by the uncertainty principle



- **Lightcone fluctuations**

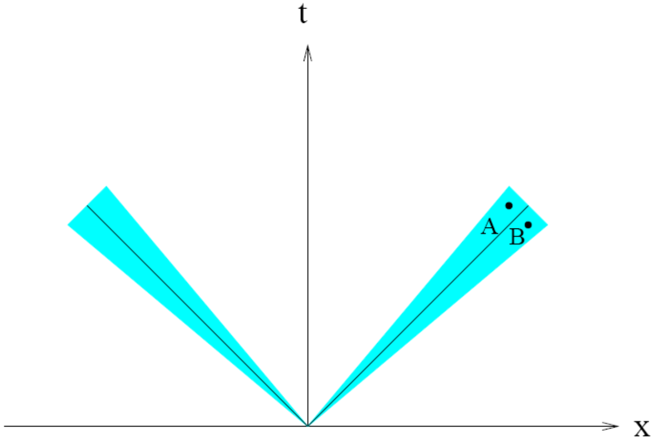


Classical physics: a fixed propagating time

Quantum gravity: a fluctuating propagating time

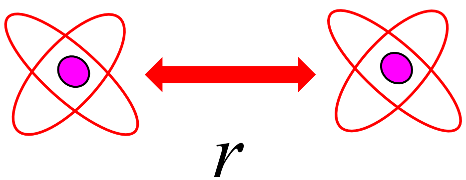
$$\Delta t = \frac{\sqrt{\langle \sigma_1^2 \rangle}}{r}$$

$$\langle \sigma_1^2 \rangle = \frac{1}{4}(\Delta r)^2 \int_{r_0}^{r_1} dr \int_{r_0}^{r_1} dr' \, n^\mu n^\nu n^\rho n^\sigma \times \langle h_{\mu\nu}(x) h_{\rho\sigma}(x') \rangle_R \, .$$



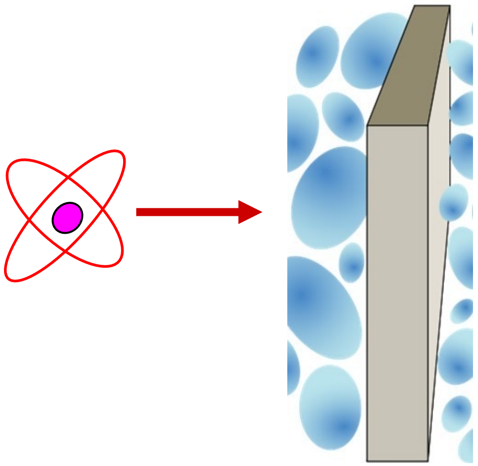
L.H. Ford, PRD 51, 1692 (1995); H. Yu and L.H. Ford, PRD 60, 084023 (1999)

- **Classical interaction**



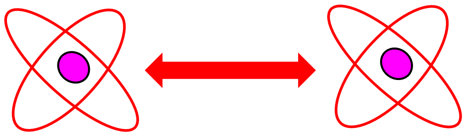
- The Newtonian/coulomb interaction:

$$V(r) \propto -\frac{1}{r}$$

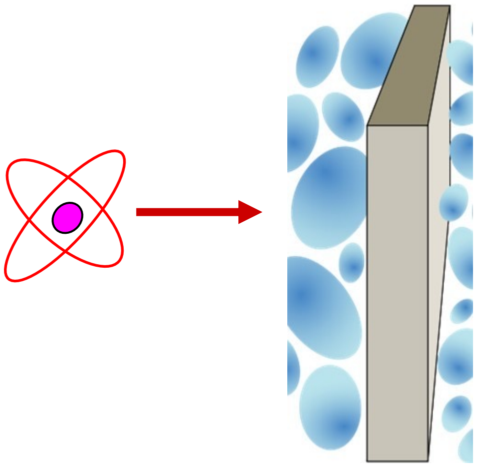


- Nothing but the universal gravitation

- **In the presence of electromagnetic vacuum fluctuations**



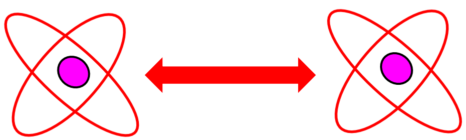
- For two polarizable atoms, the interaction is proportional to r^{-6} in the near region, and is proportional to r^{-7} in the far region



- For polarizable object and a boundary, the interaction is proportional to r^{-3} in the near region, and is proportional to r^{-4} in the far region

H. B. G. Casimir, D. Polder, Phys. Rev. 73, 360 (1948).

- **In the presence of gravitational vacuum fluctuations**



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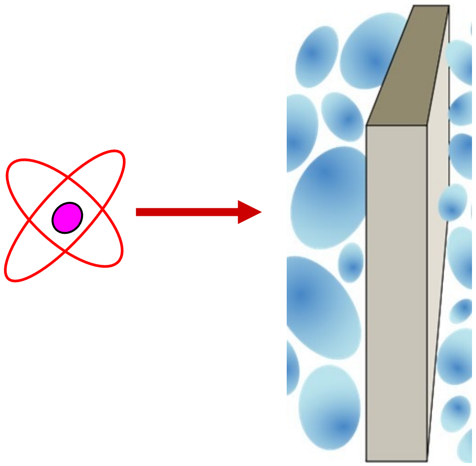

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H. B. G. Casimir, D. Polder, Phys. Rev. 73, 360 (1948).

QGI between a nonpointlike object and a boundary

- **Model:**

  A nonpointlike object modeled as a two-level quantum system at a distance *z* from a reflecting boundary

- **The Hamiltonian**

$$H = H_A + H_B + H_I$$

$$H_A = \hbar\omega_0 S_z$$

$$H_I = -\frac{1}{2}\sum_{ij} Q_{ij}E_{ij}$$

$$S_z = \tfrac{1}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

$$E_{ij} = -\nabla_i\nabla_j\phi$$

$$= -c^2 C_{0i0j} = \tfrac{1}{2}\ddot{h}_{ij}$$

$$B_{ij} = \tfrac{1}{2}c^2\epsilon_{i0\alpha\beta}C^{\alpha\beta}{}_{j0}$$

QGI between a nonpointlike object and a boundary

- **Model:**

  A nonpointlike object modeled as a two-level quantum system at a distance *z* from a reflecting boundary

- **The Hamiltonian**

$$
\begin{aligned}
E_{ik,k} &= \kappa\rho_i^E \ , \\
B_{ik,k} &= \kappa\rho_i^M \ , \\
\epsilon_{jkl}B_{il,k} - E_{ij,0} &= \kappa J_{ij}^E \ , \\
-\epsilon_{jkl}E_{il,k} - B_{ij,0} &= \kappa J_{ij}^M \ ,
\end{aligned}
$$

$$S_z = \tfrac{1}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

$$
\begin{aligned}
E_{ij} &= -\nabla_i\nabla_j\phi \\
&= -c^2C_{0i0j} = \tfrac{1}{2}\ddot{h}_{ij}
\end{aligned}
$$

$$B_{ij} = \tfrac{1}{2}c^2\epsilon_{i0\alpha\beta}C^{\alpha\beta}{}_{j0}$$

- **How – Linearized quantum gravity**

For a flat background spacetime with a linearized perturbation $h_{\mu\nu}$ propagating upon it, the metric can be expanded as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

In the transverse tracefree (TT) gauge, the gravitational field can be quantized as

$$h_{ij} = \sum_{\mathbf{k},\lambda} [a_{\mathbf{k},\lambda} e_{ij}(\mathbf{k},\lambda) f_{\mathbf{k}} + \mathrm{H.c.}]$$

with

$$f_{\mathbf{k}} = \frac{1}{\sqrt{2\omega(2\pi)^3}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

The vacuum state of gravitational fields is defined as

$$a_{\mathbf{k},\lambda}|0\rangle = 0$$

- ## How – Theory of open quantum systems

The von Neumann equation for the whole system

$$\frac{d\rho_{\text{tot}}(\tau)}{d\tau} = -i\left[H_I(\tau), \rho_{\text{tot}}(\tau)\right]$$

The evolution of the reduced system

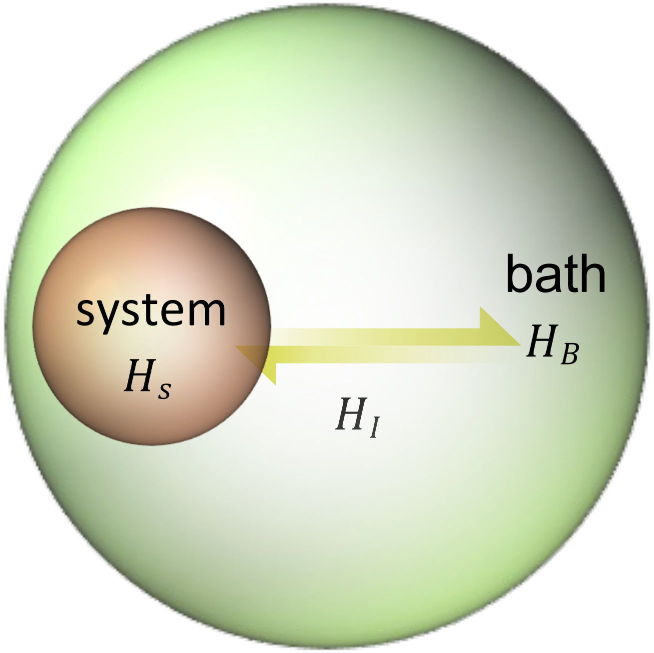
$$\frac{d}{d\tau}\rho(\tau) = -\frac{i}{\hbar}[H_{LS}, \rho(\tau)] + \mathcal{D}(\rho(\tau))$$

$$H_{LS} = \hbar \sum_{\omega}\sum_{ijkl} S_{ijkl}(\omega) A^{\dagger}_{ij}(\omega) A_{kl}(\omega)$$

$$S_{ijkl}(\omega) = \frac{i}{2}\mathcal{G}_{ijkl}(\omega) - i\Gamma_{ijkl}(\omega)$$

$$\mathcal{G}_{ijkl}(\omega) = \frac{1}{\hbar^2}\int_{-\infty}^{\infty} ds e^{i\omega s}\langle E_{ij}(s) E_{kl}(0)\rangle$$

$$\Gamma_{ijkl}(\omega) = \frac{1}{\hbar^2}\int_{0}^{\infty} ds e^{i\omega s}\langle E_{ij}(s) E_{kl}(0)\rangle$$



$$A_{ij}(\omega) = -\tfrac{1}{2}\sum_{\nu'-\nu=\omega}\Pi(\nu) Q_{ij}\Pi(\nu'),$$

$$S_{ijkl}(\omega) = -\frac{\mathrm{P}}{2\pi}\int_{-\infty}^{\infty}\frac{\mathcal{G}_{ijkl}(\lambda)}{\lambda-\omega}d\lambda$$

The trajectory of the object

$$t(\tau) = \tau, \; x(\tau) = y(\tau) = 0, \; z(\tau) = z$$

The Wightman function for gravitons in the TT gauge

$$\langle h_{ij}(x)\, h_{kl}(x') \rangle = \frac{32\pi G\hbar^2}{c^4}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + D_{ijkl})\langle 0|\phi(x)\phi(x')|0\rangle$$

$$D_{ijkl} = \left(\frac{\partial_i \partial'_j}{\nabla^2}\delta_{kl} + \frac{\partial_k \partial'_l}{\nabla^2}\delta_{ij} - \frac{\partial_i \partial'_k}{\nabla^2}\delta_{jl} - \frac{\partial_i \partial'_l}{\nabla^2}\delta_{jk} - \frac{\partial_j \partial'_l}{\nabla^2}\delta_{ik} - \frac{\partial_j \partial'_k}{\nabla^2}\delta_{il} + \frac{\partial_i \partial'_j \partial_k \partial'_l}{\nabla^4} \right)$$

$$\begin{aligned} \langle 0|\phi(x)\phi(x')|0\rangle = &- \frac{c}{4\pi^2\hbar}\frac{1}{(ct - ct' - i\epsilon)^2 - (x - x')^2 - (y - y')^2 - (z - z')^2} \\ &+ \frac{c}{4\pi^2\hbar}\frac{1}{(ct - ct' - i\epsilon)^2 - (x - x')^2 - (y - y')^2 - (z + z')^2} \end{aligned}$$

$$H_{LS} = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(\omega_0)|+\rangle\langle+|Q_{ij}|-\rangle\langle-|Q_{kl}|+\rangle\langle+|$$

$$+\frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(-\omega_0)|-\rangle\langle-|Q_{ij}|+\rangle\langle+|Q_{kl}|-\rangle\langle-|$$

$$\delta\mathcal{E}_- = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(-\omega_0)\langle-|Q_{ij}|+\rangle\langle+|Q_{kl}|-\rangle \ ,$$

$$\delta\mathcal{E}_+ = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(\omega_0)\langle+|Q_{ij}|-\rangle\langle-|Q_{kl}|+\rangle \ ,$$

$$\delta\mathcal{E}_- = \frac{G}{z^5} \sum_{ijkl} Q_{ij} Q^*_{kl} \, f_{ijkl}(\omega_0, z)$$

$$Q_{ij} = \langle-|Q_{ij}|+\rangle$$

$$Q^*_{ij} = \langle+|Q_{ij}|-\rangle$$

$$f_{1111}(\omega_0, z) = \frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{16u^4 + 16u^3 + 20u^2 + 18u + 9}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

$$f_{3333}(\omega_0, z) = \frac{\omega_0 z}{8\pi c} \int_0^\infty du \frac{4u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

$$f_{1122}(\omega_0, z) = -\frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{(2u+1)(8u^3 + 4u^2 - 3)}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

$$f_{1133}(\omega_0, z) = -\frac{\omega_0 z}{16\pi c} \int_0^\infty du \frac{4u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

$$f_{1212}(\omega_0, z) = \frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{16u^4 + 16u^3 + 12u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

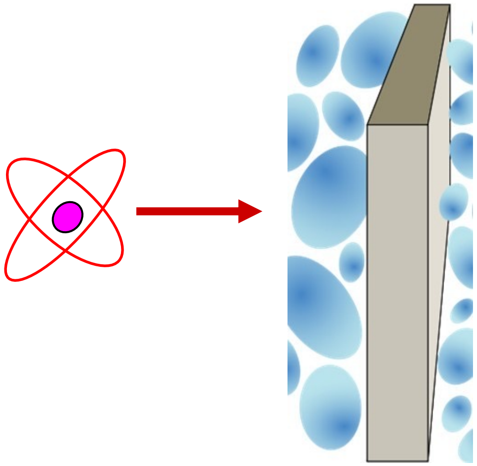
$$f_{1313}(\omega_0, z) = -\frac{\omega_0 z}{16\pi c} \int_0^\infty du \frac{4u^3 + 6u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u} \ ,$$

$$f_{1111}(\omega_0, z) = f_{2222}(\omega_0, z) \ , \quad f_{1122}(\omega_0, z) = f_{2211}(\omega_0, z) \ , \quad f_{1212}(\omega_0, z) = f_{2121}(\omega_0, z) \ ,$$

$$f_{1133}(\omega_0, z) = f_{3311}(\omega_0, z) = f_{2233}(\omega_0, z) = f_{3322}(\omega_0, z)$$

$$f_{1313}(\omega_0, z) = f_{3131}(\omega_0, z) = f_{2323}(\omega_0, z) = f_{3232}(\omega_0, z) \ ,$$

- **Quantum gravitational interaction between an object and a boundary**



Near regime, $\omega_0 z/c \ll 1$

$$\delta\mathcal{E}_- = \frac{3\hbar\omega_0 G}{128 z^5}(2\alpha_{11} + 2\alpha_{22} + 17\alpha_{33} + 2\alpha_{12} - 8\alpha_{13} - 8\alpha_{23})$$

Far regime, $\omega_0 z/c \gg 1$

$$\delta\mathcal{E}_- = \frac{3\hbar G c}{4\pi z^6}(\alpha_{11} + \alpha_{22} + \alpha_{33} + \alpha_{12} - \alpha_{13} - \alpha_{23})$$

$$\alpha_{ij} \equiv |Q_{ij}|^2/\hbar\omega_0$$

Is it observable?

- **A BEC in a harmonic trap**

The polarizability of a BEC $\alpha \sim \frac{MR^2}{\omega_0^2}$

M=*N***m*: mass of the BEC $N = 10^6$

R: radius of the BEC $z \simeq R \sim 1\ \mu\text{m}$

ω_0: center-of-mass oscillating frequency of the BEC $\omega_0 \sim 10^2\ \text{Hz}$

The interaction due to the surface V_{surf} causes a relative shift to ω_0

$$\gamma \equiv \frac{\omega_0 - \omega}{\omega_0} \simeq -\frac{1}{2M\omega_0^2}\frac{\partial^2}{\partial z^2}V_{\text{surf}}$$

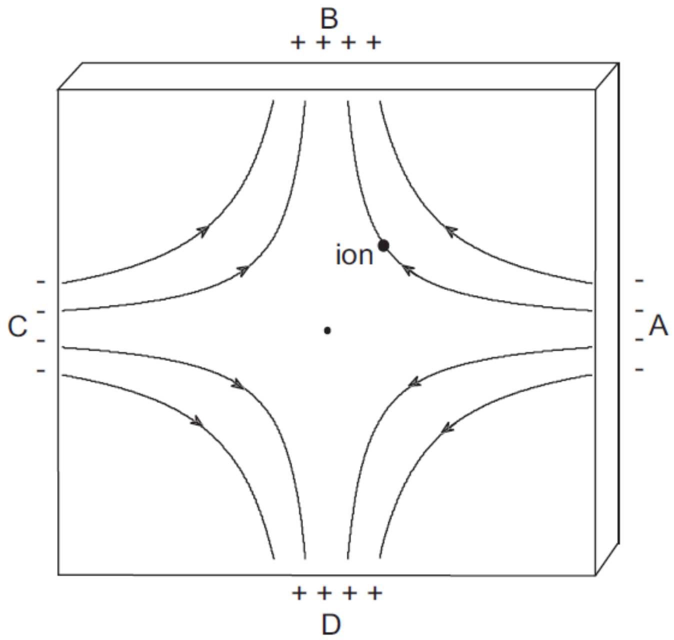
If the frequency of a harmonic oscillator is suddenly changed from ω_0 to ω, its amplitude will be changed from *R* to *R'*

$$\frac{R' - R}{R} \simeq \frac{1}{2}\gamma \qquad \gamma \sim \frac{\hbar G R^2}{z^7 \omega_0^3} \sim 10^{-21}$$

JH and H. Yu, Phys. Lett. B 767 (2017) 16-19.

- **A mirror for gravitational waves ?**

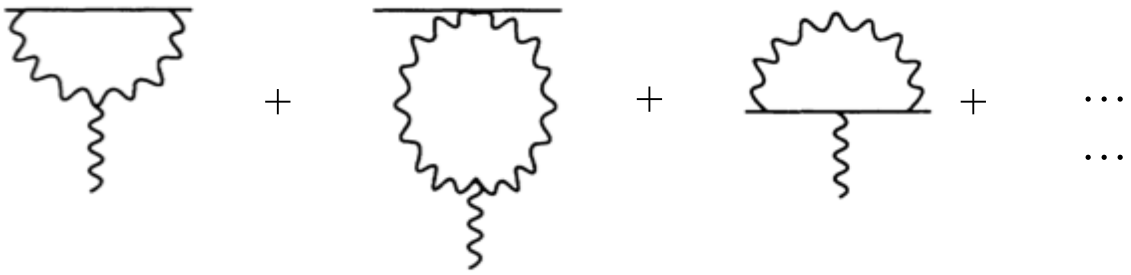
- In ordinary metal plates, the ions and normal electrons locally co-move together along the same geodesics in the presence of a GW.
- In superconducting plates, the quantum-mechanical nonlocalizability of the negatively charged Cooper pair undergoes non-geodesic motion, whereas the positive charged ions of the lattice remain on the geodesic path.
- The Cooper pairs and ions are oppositely charged, a strong Coulomb force will resist this separation of charge caused by the GW, resulting in its reflection.



S. J. Minter, K. Wegter-McNelly, R. Y. Chiao, Physica E 42 (2010) 234–255

QGI between two nonpointlike objects

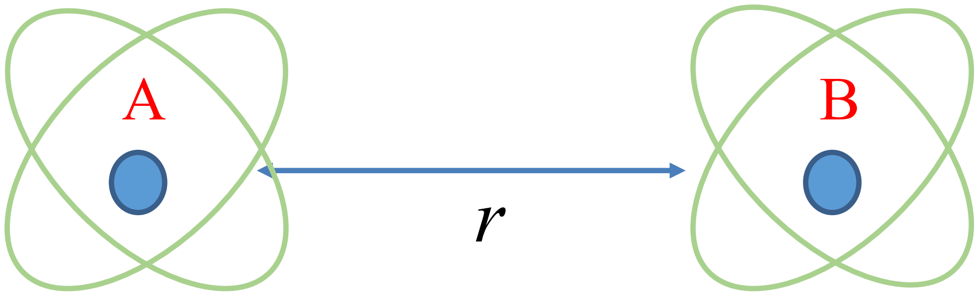
- At low energy: GR as an effective field theory (Donoghue, PRL 72, 2996 (1994); Bjerrum-Bohr, et al., PRD 71, 069903 (2005))



$$V(r) \propto -\frac{41}{10\pi r^3}$$ → Monopole--Monopole

Quadrupole-quadrupole?

- **QGI between two objects - vacuum**

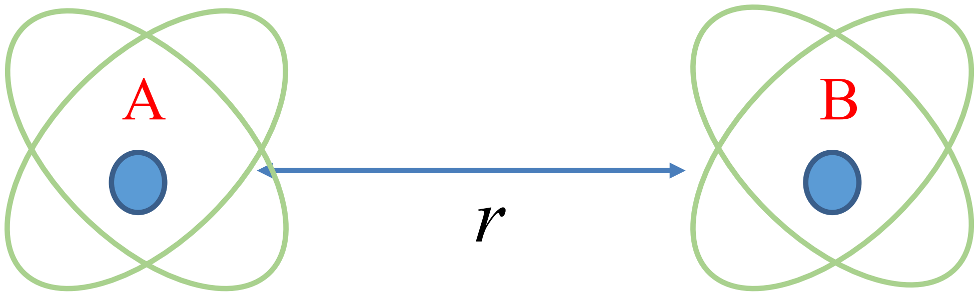


$$Q_{A,lm}(\omega) = \alpha_A(\omega) E^{\mathbf{k}}_{lm}(\mathbf{r}_A) \qquad Q_{B,lm}(\omega) = \alpha_B(\omega) E^{\mathbf{k}}_{lm}(\mathbf{r}_B)$$

$$E^{\mathbf{k}}_{ij}(A \to B) = \mathrm{Re}\left\{ \frac{G e^{ir\omega}}{r^5} \Lambda^{lm}_{ij}(\omega r, \hat{\mathbf{n}})\, Q_{A,lm}(\omega) \right\} \qquad E^{\mathbf{k}}_{ij}(B \to A) = \mathrm{Re}\left\{ \frac{G e^{ir\omega}}{r^5} \Lambda^{lm}_{ij}(\omega r, \hat{\mathbf{n}})\, Q_{B,lm}(\omega) \right\}$$

$$\begin{aligned}
\Lambda^{lm}_{ij}(\gamma, \hat{\mathbf{n}}) \;=\; & \frac{1}{2}[(6 - 6\gamma^2 + 2\gamma^4 - 6i\gamma + 4i\gamma^3)\delta^l_i \delta^m_j \\
& +(-15 + 9\gamma^2 - \gamma^4 + 15i\gamma - 4i\gamma^3)(n^i n^l \delta^m_j + n^j n^l \delta^m_i + n^i n^m \delta^l_j + n^j n^m \delta^l_i) \\
& +(-15 + 3\gamma^2 + \gamma^4 + 15i\gamma + 2i\gamma^3) n^l n^m \delta_{ij} \\
& +(105 - 45\gamma^2 + \gamma^4 - 105i\gamma + 10i\gamma^3) n^i n^j n^l n^m] \;. \qquad (15)
\end{aligned}$$

- **QGI between two objects - vacuum**



$$Q_{A,lm}(\omega) = \alpha_A(\omega) E^{\mathbf{k}}_{lm}(\mathbf{r}_A) \qquad Q_{B,lm}(\omega) = \alpha_B(\omega) E^{\mathbf{k}}_{lm}(\mathbf{r}_B)$$

$$E^{\mathbf{k}}_{ij}(A \to B) = \mathrm{Re}\left\{ \frac{Ge^{ir\omega}}{r^5} \Lambda^{lm}_{ij}(\omega r, \hat{\mathbf{n}})\, Q_{A,lm}(\omega) \right\} \qquad E^{\mathbf{k}}_{ij}(B \to A) = \mathrm{Re}\left\{ \frac{Ge^{ir\omega}}{r^5} \Lambda^{lm}_{ij}(\omega r, \hat{\mathbf{n}})\, Q_{B,lm}(\omega) \right\}$$

$$V(r) = -\frac{1}{2} \sum_{\mathbf{k}} \langle \{0\} | Q_{B,ij}(\omega) E^{\mathbf{k}}_{ij}(A \to B) | \{0\} \rangle = -\frac{1}{2} \sum_{\mathbf{k}} \langle \{0\} | Q_{A,ij}(\omega) E^{\mathbf{k}}_{ij}(B \to A) | \{0\} \rangle$$

$$\langle \{0\} | E^{\mathbf{k}}_{A,ij}(\mathbf{r}_A) E^{\mathbf{k}}_{B,lm}(\mathbf{r}_B) | \{0\} \rangle \ = \ ?$$

The two-point function:

$$\begin{aligned}
&\langle\{0\}|E^{\mathbf{k}}_{A,ij}(\mathbf{r}_A)E^{\mathbf{k}}_{B,lm}(\mathbf{r}_B)|\{0\}\rangle \\
&\quad= \frac{\omega^4}{4}\langle\{0\}|h^{\mathbf{k}}_{ij}(\mathbf{r}_A)h^{\mathbf{k}}_{lm}(\mathbf{r}_B)|\{0\}\rangle \\
&\quad= \frac{G\omega^3}{(2\pi)^2}\sum_{\lambda} e_{ij}(\mathbf{k},\lambda)e_{lm}(\mathbf{k},\lambda)e^{i\mathbf{k}\cdot(\mathbf{r}_A-\mathbf{r}_B)} \\
&\quad= \frac{G\omega^3}{(2\pi)^2} g_{ijlm}(\hat{\mathbf{k}})e^{i\mathbf{k}\cdot\mathbf{r}},
\end{aligned}$$

$$\begin{aligned}
g_{ijlm}(\hat{\mathbf{k}}) = {} & \delta_{il}\delta_{jm}+\delta_{im}\delta_{jl}-\delta_{ij}\delta_{lm}+\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m+\hat{k}_i\hat{k}_j\delta_{lm} \\
& +\hat{k}_l\hat{k}_m\delta_{ij}-\hat{k}_i\hat{k}_m\delta_{jl}-\hat{k}_i\hat{k}_l\delta_{jm}-\hat{k}_j\hat{k}_m\delta_{il}-\hat{k}_j\hat{k}_l\delta_{im}
\end{aligned}$$

The quantum potential:

$$V(r) = -\frac{G^2}{8\pi^2}\mathrm{Re}\left\{\sum_{\mathbf{k}}\alpha_A(\omega)\alpha_B(\omega)\frac{\omega^3 e^{i(\mathbf{k}\cdot\mathbf{r}+\omega r)}}{r^5}\Lambda^{lm}_{ij}(\omega r,\hat{\mathbf{n}})g_{ijlm}(\hat{\mathbf{k}})\right\}$$

In the far region:

$$V_{\text{far}}(r) = -\frac{3987\hbar c G^2}{4\pi r^{11}} \alpha_{1S}\alpha_{2S}$$

In the near region:

$$V_{\text{near}}(r) = -\frac{315\hbar G^2}{\pi r^{10}} \int_0^\infty d\omega \alpha_1(i\omega)\alpha_2(i\omega)$$

The result is in agreement with Ford et al., PRL 116, 151301 (2016).

- **QGI between two objects - a thermal bath of gravitons**

In the high-temperature limit, $\beta \ll r$

$$V_T(r) = -\frac{315G^2}{\beta r^{10}}\alpha_A(0)\alpha_B(0)$$

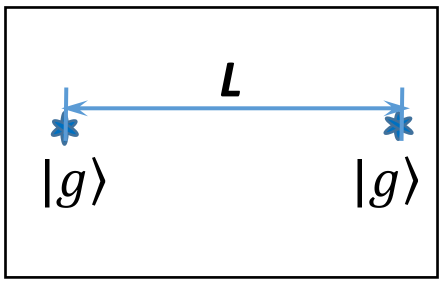
In the low-temperature limit, $\beta \gg r$

$$V_T(r) = -\frac{83456\pi^9 G^2}{10395}\frac{1}{r\beta^{10}}\alpha_A(0)\alpha_B(0)$$

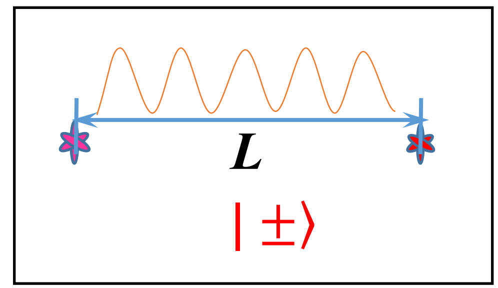
P. Wu, **JH** and H. Yu, Phys. Rev. D 95, 104057 (2017)

- **What happens if an entangled state is involved?**

Ground state



Entangled state



$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|g_A e_B\rangle \pm |e_A g_B\rangle)$$

$$\delta E(L) \sim \begin{cases} L^{-6}, & L \ll \lambda \\ L^{-7}, & L \gg \lambda \end{cases}$$

$$\delta E(L) \sim \begin{cases} L^{-3}, & L \ll \lambda \\ L^{-1}, & L \gg \lambda \end{cases}$$

Is it possible to amplify quantum gravitational effects?

- **QGI between two objects - entangled states**

- **The initial state of the objects** $|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|g_A e_B\rangle \pm |e_A g_B\rangle)$

- In the near regime $r \ll \omega_0^{-1}$,

$$\delta E_{AB} \simeq \pm \frac{21 G\hbar\omega_0\alpha}{r^5}.$$

- In the far regime, i.e., $r \gg \omega_0^{-1}$,

$$\delta E_{AB} \simeq \mp \frac{G\hbar\omega_0^5\alpha}{rc^4}\cos\left(\frac{\omega_0 r}{c} - \phi\right).$$

- The objects are assumed to be isotropically polarizable, i.e. $\alpha^{ijkl} = \alpha$,

- **Compared with the monopole-monopole QGI in the far regime**

For an with radius *R*, mass *M* and frequency ω_0, the gravitational quadrupole polarizability $\alpha \sim MR^2\omega_0^2$. Then,

$$\frac{\delta E_{AB}}{V_m} \sim \frac{R^2\omega_0^3 r^2}{GMc} = \frac{R^2\omega_0 c}{GM}\left(\frac{r}{\lambda}\right)^2, \qquad V_m = -\frac{41}{10\pi r^3}$$

For a gravitationally bound system, the orbital frequency $\Omega = \sqrt{GM/R^3}$, which gives a lower bound on ω_0 for any physical system. Thus,

$$\frac{\delta E_{AB}}{V_m} \gtrsim \frac{R^2\Omega c}{\sqrt{2}GM}\left(\frac{r}{\lambda}\right)^2 = \left(\frac{R}{R_S}\right)^{\frac{1}{2}}\left(\frac{r}{\lambda}\right)^2,$$

$$R_S = 2GM/c^2$$

$$r \gg c/\omega_0 = \lambda$$

Thus, **in the far regime**, the resonance quantum gravitational interaction can give the **dominating** quantum correction to the Newtonian potential.

- **Compared with the classical Newtonian potential**

$$\frac{\delta E_{AB}}{V_N} \simeq \frac{\hbar R^2 \omega_0^3}{Mc^4} = \frac{\hbar\omega_0}{Mc^2}\left(\frac{R}{\lambda}\right)^2 .$$

$$V_N = GM^2/r$$

It is obvious that $\hbar\omega_0 \ll Mc^2$ For electrically bound objects, e.g. hydrogen atoms, the ratio above can be estimated as

$$\frac{\delta E_{AB}}{V_N} \sim \left(\frac{1\ \mathrm{eV}}{1\ \mathrm{GeV}}\right)\left(\frac{10^{-11}\ \mathrm{m}}{10^{-7}\ \mathrm{m}}\right)^2 \sim 10^{-17}.$$

If the objects are bound gravitationally, we have

$$\Omega = \sqrt{GM/R^3}$$

$$\frac{\delta E_{AB}}{V_N} \sim \left(\frac{l_P}{R}\right)^2 \left(\frac{R_S}{R}\right)^{1/2}$$

$$l_P = \sqrt{\hbar G/c^3}$$

Therefore, the resonance quantum gravitational interaction is **small** compared with the classical Newtonian potential.

Quantum Gravitomagnetic Interaction

The action for a quantum field reads $S = \int d^4x \sqrt{-g}\mathcal{L},$

$$S = \int d^4x \left\{ \left(\sqrt{-g}\mathcal{L}\right)_{h=0} + \left[\frac{\delta}{\delta g^{\alpha\beta}}\left(\sqrt{-g}\mathcal{L}\right)\right]_{h=0} h^{\alpha\beta} \right\} + O(h^2),$$

The stress-energy tensor $T_{\alpha\beta} = -\dfrac{2}{\sqrt{-g}}\dfrac{\delta}{\delta g^{\alpha\beta}}\left(\sqrt{-g}\mathcal{L}\right),$

We may rewrite $S = \int d^4x \left(\mathcal{L}^{(0)} - \frac{1}{2}T^{(0)}_{\alpha\beta}h^{\alpha\beta}\right),$ and then $\mathcal{L}_g = -\frac{1}{2}T^{(0)}_{\alpha\beta}h^{\alpha\beta}$

$$\mathcal{H} = -\mathcal{L} = -\frac{1}{2}h_{\mu\nu}T^{\mu\nu}$$

$$T^{\mu\nu} = \begin{pmatrix} \rho_m & j^1 & j^2 & j^3 \\ j^1 & T^{11} & T^{12} & T^{13} \\ j^2 & T^{21} & T^{22} & T^{23} \\ j^3 & T^{31} & T^{32} & T^{33} \end{pmatrix},$$

$$T^{00} = \rho_m\ ,\ T^{0i} = j^i(x) = \rho_m(x)v^i$$

$$\mathcal{H} = -\frac{1}{2}h_{\mu\nu}T^{\mu\nu} = -\frac{1}{2}h_{00}T^{00} - \boxed{\frac{1}{2}h_{0i}T^{0i}}$$

$$= -\frac{1}{2}h_{00}\rho_m(x) - \frac{1}{2}h_{0i}\boxed{\left(\rho_m(x)v^i\right)}$$

The localized mass-current density

$$\mathcal{H} = -\frac{1}{2}h_{00}\rho_m(x) - \frac{1}{2}h_{0i}(\rho_m(x)v^i)$$

In the **Fermi normal coordinate**, we have

$$h_{00} = -R_{0j0k}x^j x^k \ ,$$
$$h_{0i} = -\frac{2}{3}R_{0jik}x^j x^k \ .$$

$$\mathcal{H} = \frac{1}{2}(\rho_m(x)x^j x^k)R_{0j0k} + \frac{1}{3}R_{0jik}x^j x^k(\rho_m(x)v^i)$$

$$E_{ij} = -R_{0i0j},$$
$$B_{ij} = \frac{1}{2}\epsilon_{ikl}R_{kl0j}.$$

$$\mathcal{H} = -\frac{1}{2}(\rho_m(x)x^j x^k)E_{jk} - \frac{1}{3}\rho_m(x)\,(\mathbf{x}\times\mathbf{v})^{(l}x^{j)}B_{lj}$$

According to the **Weyl gravito-electromagnetism**, we have

$$H = \int d^3x\,\mathcal{H} = -\frac{1}{2}\int d^3x\,\rho_m(x)\left(x^j x^k - \frac{1}{3}\delta^{jk}r^2\right)E_{jk} - \frac{1}{3}\int d^3x\,\rho_m(x)\frac{1}{2}[(\mathbf{x}\times\mathbf{v})^l x^j + (\mathbf{x}\times\mathbf{v})^j x^l]B_{lj}$$

$$= -\frac{1}{2}Q^{jk}E_{jk} - \frac{1}{3}S^{lj}B_{lj}$$

The mass quadrupole moment: $Q^{jk} = \int d^3x\,\rho_m(x)\left(x^j x^k - \frac{1}{3}\delta^{jk}r^2\right)$

The mass-current quadrupole moment: $S^{lj} = \int d^3x\,\rho_m(x)\frac{1}{2}[(\mathbf{x}\times\mathbf{v})^l x^j + (\mathbf{x}\times\mathbf{v})^j x^l]$

- **The ground-state interaction energy**

$$\Delta E_{AB}^{GM}(r) = -\frac{16G^2}{81\pi r^{10}}\int_0^{+\infty} du\,\chi_A(iu)\chi_B(iu)T(ur)e^{-2ur},$$

where $T(x) = 315 + 630x + 585x^2 + 330x^3 + 129x^4 + 42x^5 + 14x^6 + 4x^7 + x^8.$

- **Asymptotic behaviors**

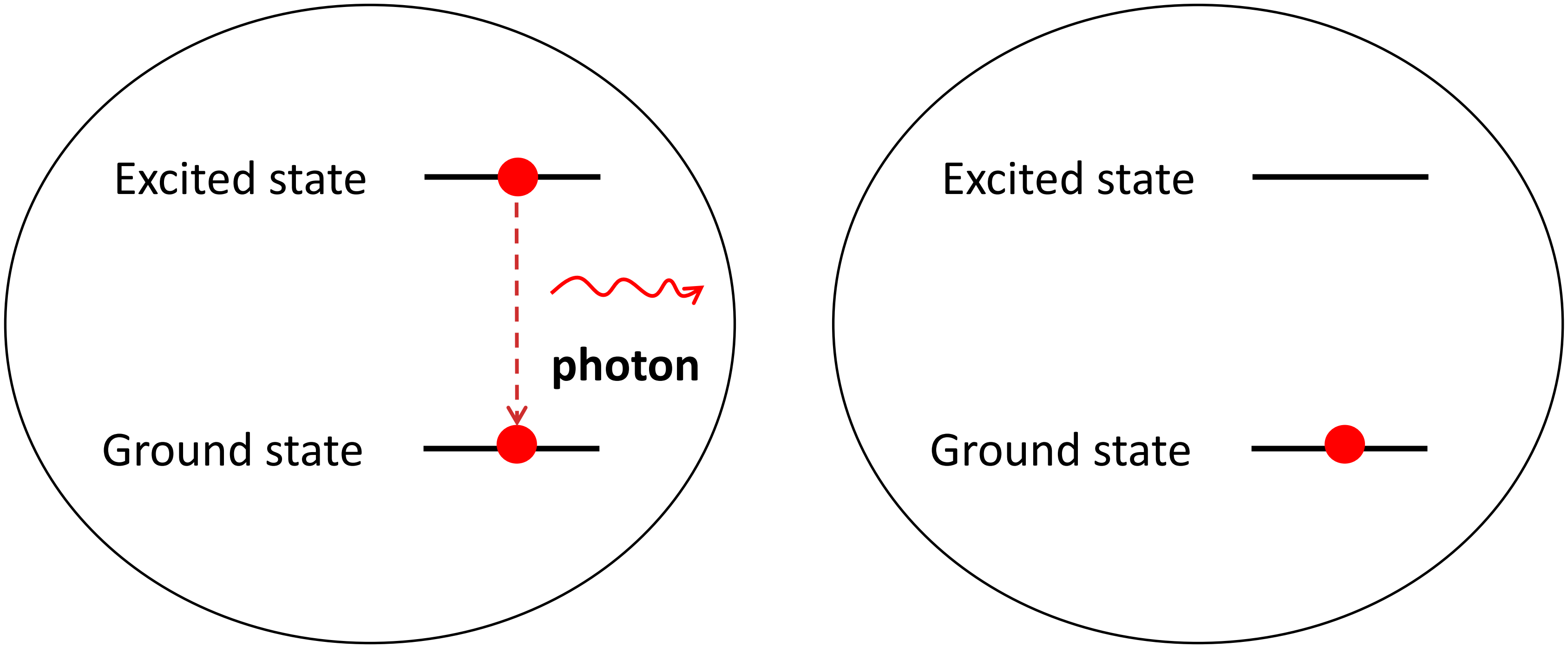
① In the near regime $r \ll \omega_{A(B)}^{-1}$,

$$\Delta E_{AB}^{\text{near}}(r) = -\frac{560\hbar G^2}{9\pi r^{10}}\int_0^{+\infty} du\,\chi_A(iu)\chi_B(iu), \quad \text{(SI)}$$

② In the far regime $r \gg \omega_{A(B)}^{-1}$,

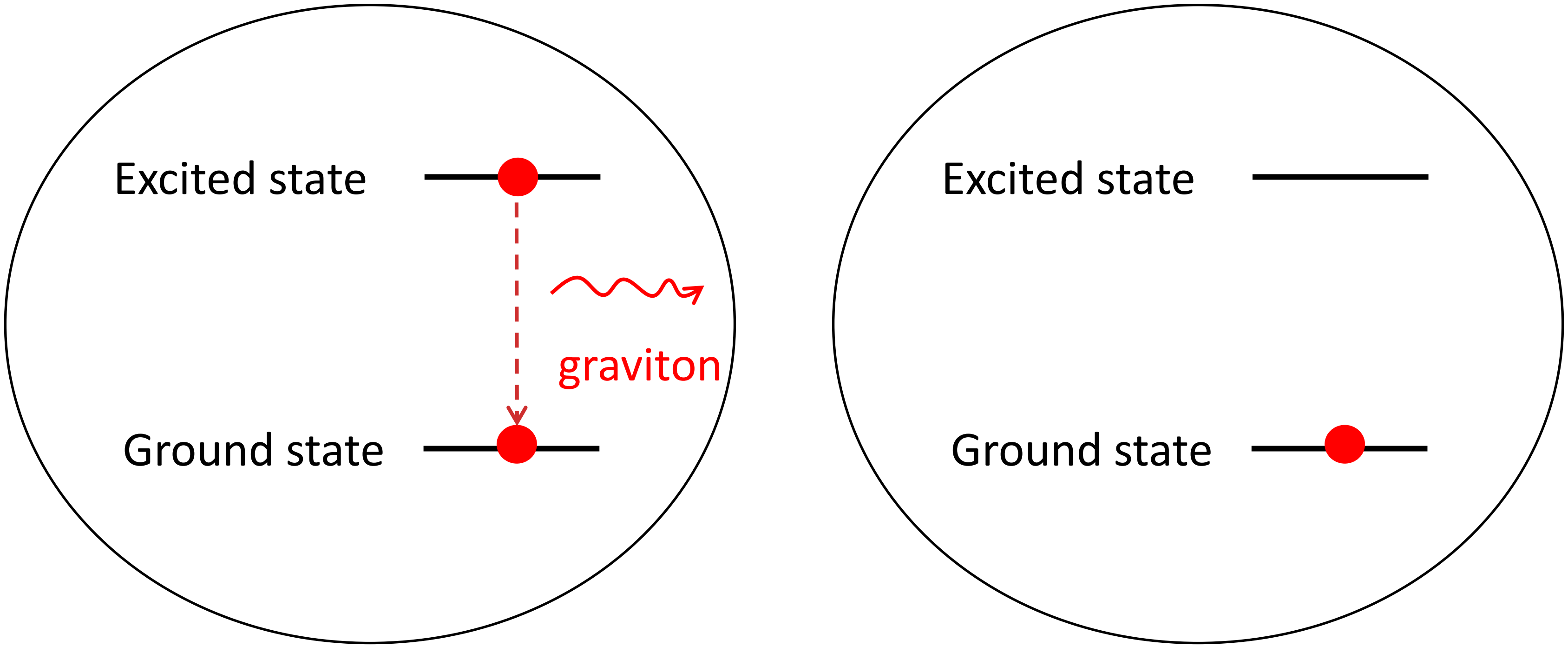
$$\Delta E_{AB}^{GM,\,far}(r) = -\frac{1772\hbar c G^2}{9\pi r^{11}}\chi_A(0)\chi_B(0)\ , \quad \text{(SI)}$$

- **Spontaneous emission of an atom**



electromagnetic vacuum fluctuations

- **Spontaneous emission of an atom**



gravitational vacuum fluctuations

- **The model:**

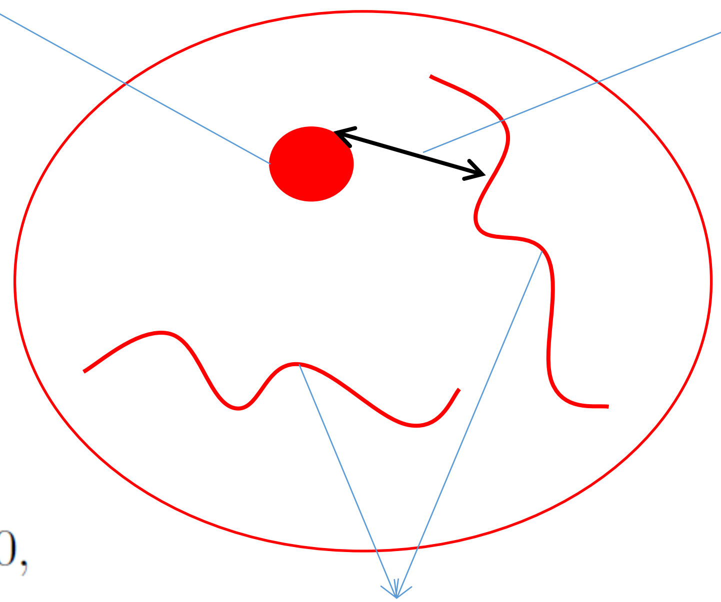
A multi-level atom in vacuum

$$H_A(\tau) = \sum_n \omega_n \sigma_{nn}(\tau)$$

Trajectory of the atom

$$t(\tau) = \tau,$$

$$x(\tau) = y(\tau) = z(\tau) = 0,$$



The fluctuating gravitational field

$$H_F(\tau) = \sum_k \omega_{\vec{k}} a^\dagger_{\vec{k}} a_{\vec{k}} \frac{dt}{d\tau}$$

The quadrupolar interaction

$$H_I(\tau) = -\frac{1}{2} Q_{ij}(\tau) E_{ij}(x(\tau))$$

$$E_{ij} = C_{i0j0}$$

$$B_{ij} = -\frac{1}{2}\epsilon_{imn} C^{mn}{}_{0j}$$

$$E_{ik,k} = \kappa \rho_i^E \ ,$$

$$B_{ik,k} = \kappa \rho_i^M \ ,$$

$$\epsilon_{jkl} B_{il,k} - E_{ij,0} = \kappa J_{ij}^E \ ,$$

$$-\epsilon_{jkl} E_{il,k} - B_{ij,0} = \kappa J_{ij}^M \ ,$$

The mean rate of change of atomic energy

$$\left\langle \frac{d}{d\tau} H_A(\tau) \right\rangle = -\frac{2\hbar G}{15c^5} \sum_{\omega_{bd}>0} \omega_{bd}^7 \big(2\alpha_{1111} + 2\alpha_{2222} + 2\alpha_{3333} - \alpha_{1122} - \alpha_{2211} - \alpha_{1133} - \alpha_{3311} - \alpha_{2233} - \alpha_{3322} + 6\alpha_{1212} + 6\alpha_{1313} + 6\alpha_{2323}\big).$$

where we have defined the gravitational polarizability as

$$\alpha_{ijkl} = \langle b|Q_{ij}^F(0)|d\rangle\langle d|Q_{kl}^F(0)|b\rangle/\hbar\omega_{bd}$$

The transition rate

$$\Gamma = \frac{\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle}{\hbar\omega}$$

- **For hydrogen atoms**

$$\Psi_{1s} = \frac{1}{\sqrt{\pi}a^{3/2}} e^{-r/a}, \qquad \Psi_{3d} = \frac{1}{162\sqrt{\pi}a^{3/2}} \left(\frac{r^2}{a^2}\right) e^{-r/3a} \sin^2\theta \, e^{2i\phi},$$

Direct calculations show that

$$\alpha_{1111} = \alpha_{2222} = \alpha_{1122} = \alpha_{2211} = \frac{243}{8192} \frac{m_e a^2}{\omega_{31}^2},$$

$$\alpha_{3333} = \frac{243}{2048} \frac{m_e a^2}{\omega_{31}^2},$$

$$\alpha_{1133} = \alpha_{3311} = \alpha_{2233} = \alpha_{3322} = -\frac{243}{4096} \frac{m_e a^2}{\omega_{31}^2},$$

$$\alpha_{1212} = \alpha_{1313} = \alpha_{2323} = 0,$$

and $$\left\langle \frac{d}{d\tau} H_A(\tau) \right\rangle = -\frac{3^8 G m_e^2 a^4 \omega_{bd}^6}{5 \times 2^{13} c^5},$$

$$\Gamma = \frac{\frac{dH}{d\tau}}{\hbar\omega} \sim 5.7 * 10^{-40}\ s^{-1}$$

Lifetime: $\sim 10^{31}$ years

Age of the Universe: $\sim 10^{10}$ years

Weinberg 1972, Kiefer 2004, Boughn and Rothman 2006

A possible source of high frequency background GWs?

The gravitational energy density emitted by hydrogen atoms from the time τ to $\tau + d\tau$ can be expressed as

$$d\rho = -\left(\frac{R(\tau)}{R_0}\right)^4 N(\tau)\left\langle\frac{dH_A}{d\tau}\right\rangle d\tau,$$

where *R* is the scale factor, and *N* is the number density of hydrogen atoms in an excited state.

We assume that all atoms (ordinary matter) today are hydrogen atoms, and the number is conserved, so the number density of hydrogen atoms in the 3d state at time *t* can be estimated as

$$N(\tau) \approx \frac{\rho_c P_B P_{3d}(\tau)}{m_H c^2}\left(\frac{R_0}{R(\tau)}\right)^3,$$

$$P_{3d}(\tau) \approx \frac{n_{3d}(\tau)}{\sum_{n=1}^{4}\sum_{l=0}^{n-1} n_{ns}(\tau)}.$$

$$n_{nl}(\tau) = (2l+1)n_{1s}(\tau)e^{-\frac{B_1-B_n}{k_B T(\tau)}},$$

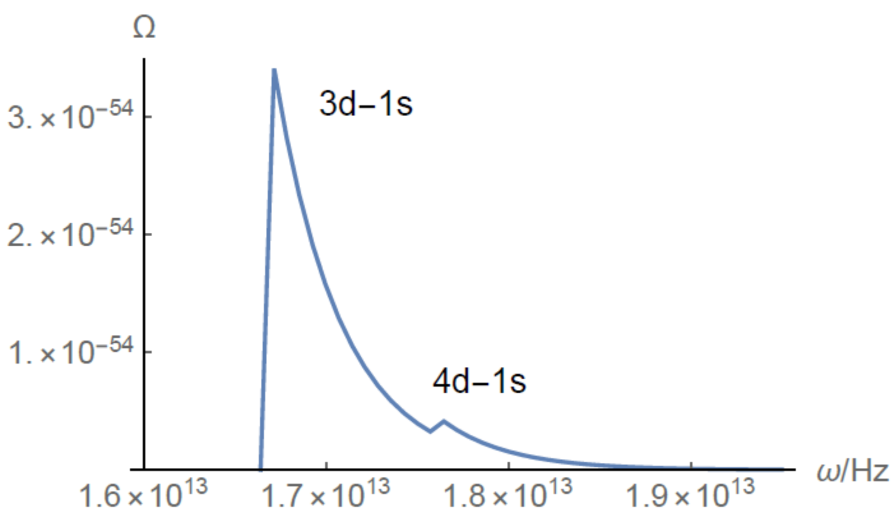
The energy density normalized with respect to the critical energy density

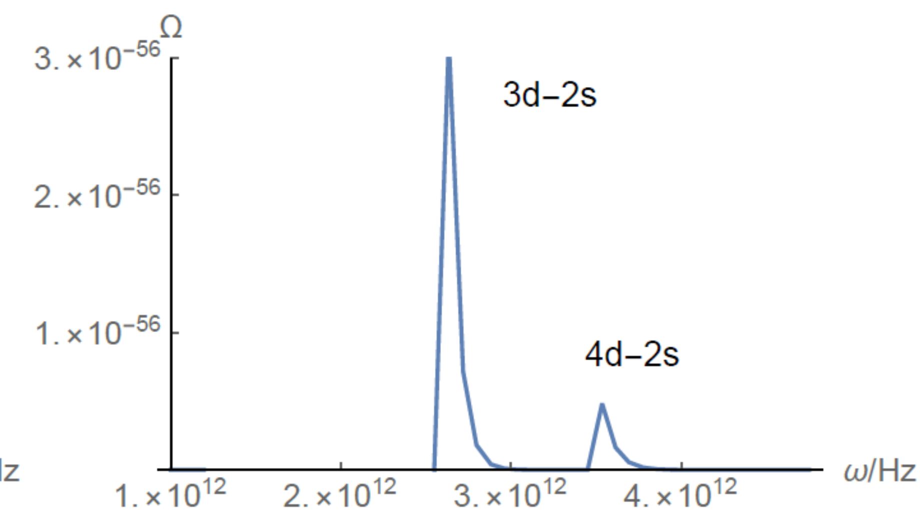
$$\Omega = \frac{1}{\rho_c}\frac{d\rho}{d\ln\omega}.$$

can be calculated as

$$\Omega = \frac{3^8\, G m_e^2 a^4 \omega_{31}^5 P_B P_{3d}\,\omega}{5\times 2^{13}\, m_H c^7 H_0},$$

The spectrum



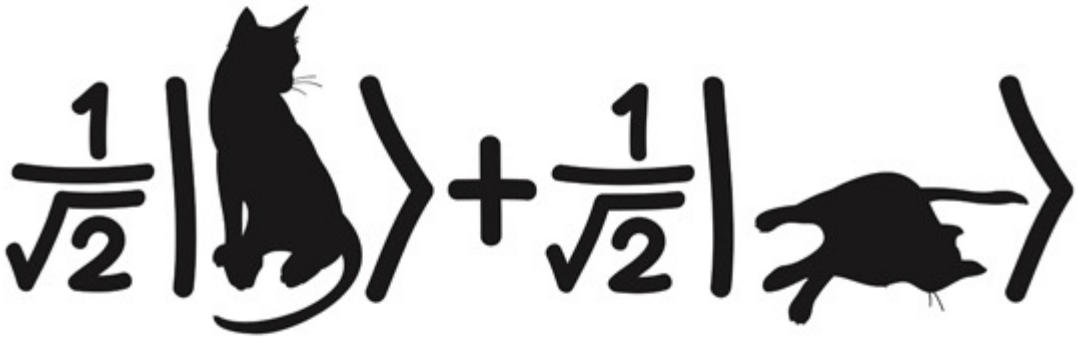


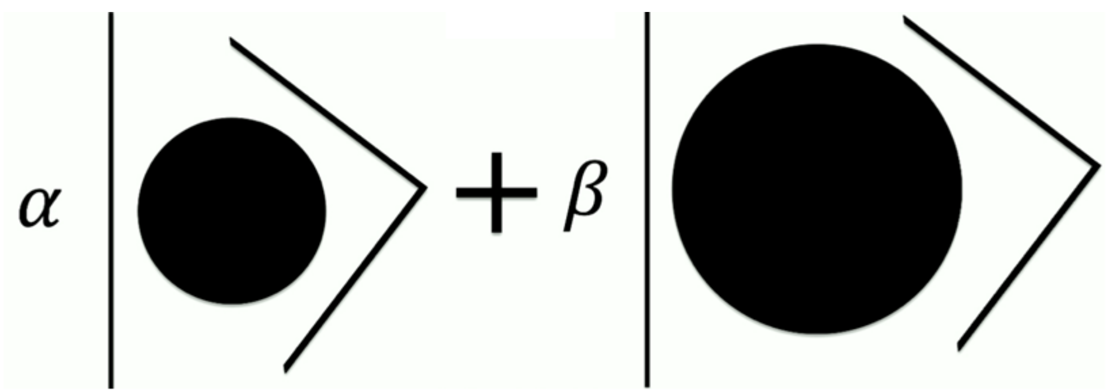
- **Contributions from helium**

- Helium is the second most abundant element in the Universe, constituting $\sim$ 24% of the baryonic matter.
- The binding energy of helium is larger than that of hydrogen, so the recombination comes earlier. The recombination of He^+ takes place around redshift $z \approx 6000$, and the recombination of neutral helium takes place around redshift $z \approx 2000$.
- The dominant contribution comes from the 3d − 1s transition of He^+ at the redshift $z \sim 6000$, which gives a peak in frequency at $\omega = 1.22 \times 10^{13}$ Hz, and the relative energy density is $\sim 10^{-48}$, which is 6 orders of magnitude larger than that from hydrogen atoms.
- This significant difference mainly comes from a much larger population of higher-lying excited states since the recombination of He^+ is much earlier and therefore the Universe is much hotter.

JH and H. Yu, Eur. Phys. J. C 81, 470 (2021).

Quantum superposition of spacetime





Quantum superposition of spacetime

- **Superposition of accelerations**
- J. Foo, S. Onoe and M. Zych, Unruh-deWitt detectors in quantum superpositions of trajectories, Phys. Rev. D 102 085013 (2020).
- L. C. Barbado, E. Castro-Ruiz, L. Apadula and C. Brukner, Unruh effect for detectors in superposition of accelerations, Phys. Rev. D 102 045002 (2020).
- **Superposition of spacetimes**
- J. Foo, R. B. Mann, and M. Zych, Schrödinger's cat for de Sitter spacetime, Class. Quantum Grav. 38, 115010 (2021).
- J. Foo, C. S. Arabaci, M. Zych, and R. B. Mann, Quantum Signatures of Black Hole Mass Superpositions, Phys. Rev. Lett. 129, 181301 (2022).
- J. Foo, C. S. Arabaci, M. Zych, and R. B. Mann, Quantum superpositions of Minkowski spacetime, Phys. Rev. D 107, 045014 (2023).
- ……

The basic formalism

To initialize the detector in a trajectory superposition, we introduce a control degree of freedom, c_i, whose states $|c_i\rangle$ designate the individual paths which the detector takes.

The total Hilbert space of the relevant systems is

$$\mathcal{H} = \mathcal{H}_{\mathrm{UdW}} \otimes \mathcal{H}_F \otimes \mathcal{H}_C$$

where H_{UdW}, H_F and H_C are associated are respectively associated with the internal states of the detector, the field degrees of freedom, and a control.

The state of the system can be expressed as

$$|\Psi\rangle_{\mathrm{S}} = |c\rangle|0_M\rangle|g\rangle \quad \text{where}|c\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|c_i\rangle,$$

The coupling of the detector to the field is described by

$$\hat{H}_I(\tau) = \sum_{i=1}^{N} \hat{\mathcal{H}}_i \otimes |c_i\rangle\langle c_i|$$

where $\hat{\mathcal{H}}_i(\tau) = \lambda\eta(\tau)\sigma(\tau)\hat{\Phi}(\mathbf{x}_i(\tau))$ $\quad \sigma(\tau) = (|e\rangle\langle g|e^{i\Omega\tau} + |g\rangle\langle e|e^{-i\Omega\tau})$

The evolution of the initial system state can be obtained by perturbatively expanding the time evolution operator using the Dyson series,

$$\hat{U} = \sum_{i=1}^{N} \hat{U}_i |c_i\rangle\langle c_i|,$$

$$\hat{U}_i = 1 - i\lambda \int_{\tau_0}^{\tau} \mathrm{d}\tau' \hat{\mathcal{H}}_i(\tau') - \lambda^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \int_{\tau_0}^{\tau'} \mathrm{d}\tau'' \hat{\mathcal{H}}_i(\tau')\hat{\mathcal{H}}_i(\tau'') + \mathcal{O}(\lambda^3)$$

The time evolution is thus given by

$$\hat{U}|\Psi\rangle_{\mathrm{S}} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{U}_i |c_i\rangle|0_M\rangle|g\rangle$$

We consider the conditional transition probability of the detector given that the control is measured in a superposition state, which for simplicity we take to be $|c\rangle$. The final state of the detector-field system is given by

$$\langle c|\hat{U}|\Psi\rangle_{\mathrm{S}} = |\Psi\rangle_{\mathrm{FD}} = \frac{1}{N}\sum_{i=1}^{N}\hat{U}_i|0_M\rangle|g\rangle.$$

The density matrix of the detector-field system is

$$\hat{\rho}_{\mathrm{FD}} = \frac{1}{N^2}\sum_{i,j=1}^{N}\underbrace{\hat{U}_i|0_M\rangle|g\rangle\langle g\langle 0_M|\hat{U}_j^\dagger}_{\hat{\rho}_{ij,\mathrm{FD}}}.$$

The final density matrix of the detector is,

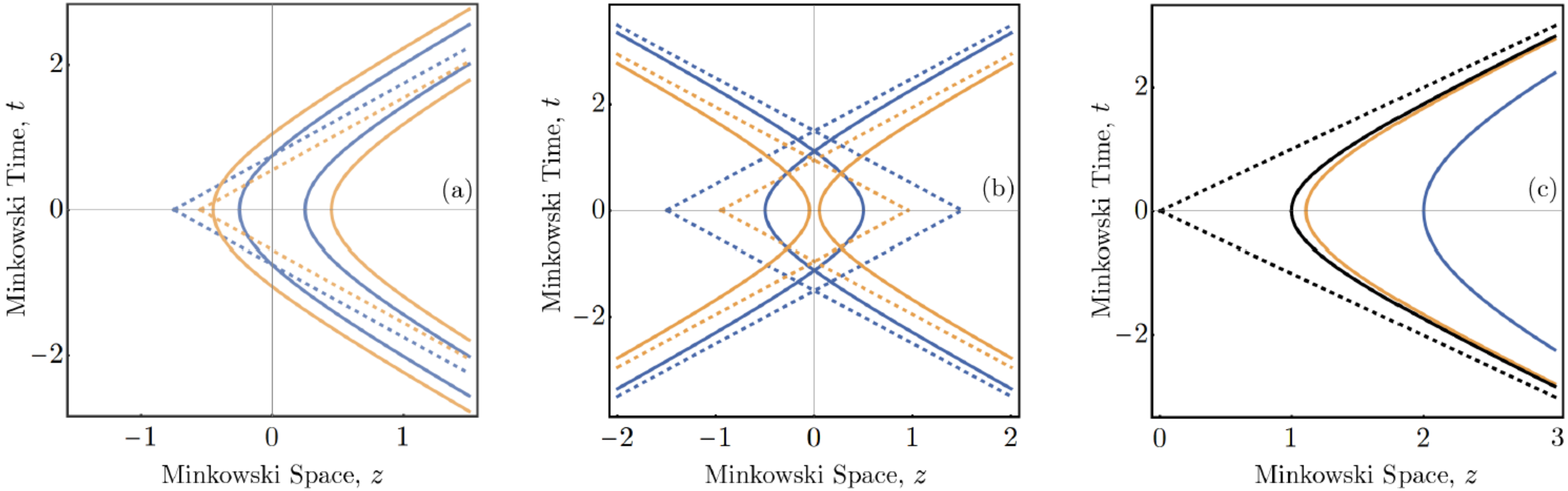
$$\hat{\rho}_{\mathrm{D}} = \begin{pmatrix} 1-\mathcal{P}_{\mathrm{D}} & 0 \\ 0 & \mathcal{P}_{\mathrm{D}} \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\mathcal{P}_{\mathrm{D}} = \frac{\lambda^2}{N^2}\sum_{i,j=1}^{N}\int \mathrm{d}\tau \int \mathrm{d}\tau' \chi_i(\tau)\bar{\chi}_j(\tau')\mathcal{W}^{ji}\left(x_i, x_j'\right)$$

$i = j$, local

$i \neq j$, nonlocal

Superposition of accelerated trajectories



parallel accelerations

antiparallel accelerations

differing accelerations

$$z_1 = \kappa^{-1}(\cosh(\kappa\tau) - 1) + \mathcal{L}/2,$$
$$z_2 = \pm\kappa^{-1}(\cosh(\kappa\tau) - 1) - \mathcal{L}/2,$$
$$t_1 = t_2 = \kappa^{-1}\sinh(\kappa\tau),$$

$$z_i^{\text{diff.}} = \kappa_i^{-1}\cosh(\kappa_i\tau),$$

$$t_i^{\text{diff.}} = \kappa_i^{-1}\sinh(\kappa_i\tau),$$

Detector thermalization

For a uniformly accelerated detector, the transition rate must satisfy the detailed balance form of the Kubo-Martin-Schwinger (KMS) condition

$$\frac{\dot{\mathcal{P}}_E(\Omega)}{\dot{\mathcal{P}}_E(-\Omega)} = \exp(-2\pi\Omega/\kappa).$$

Recalling that L defines the distance of closest approach between the two trajectories, for both the parallel and anti-parallel configurations, the KMS condition is only satisfied in the limit $L \to \infty$.

Infinitely separated trajectories possessing different proper accelerations do not produce a thermal response either.

- **Superposition of spacetimes**

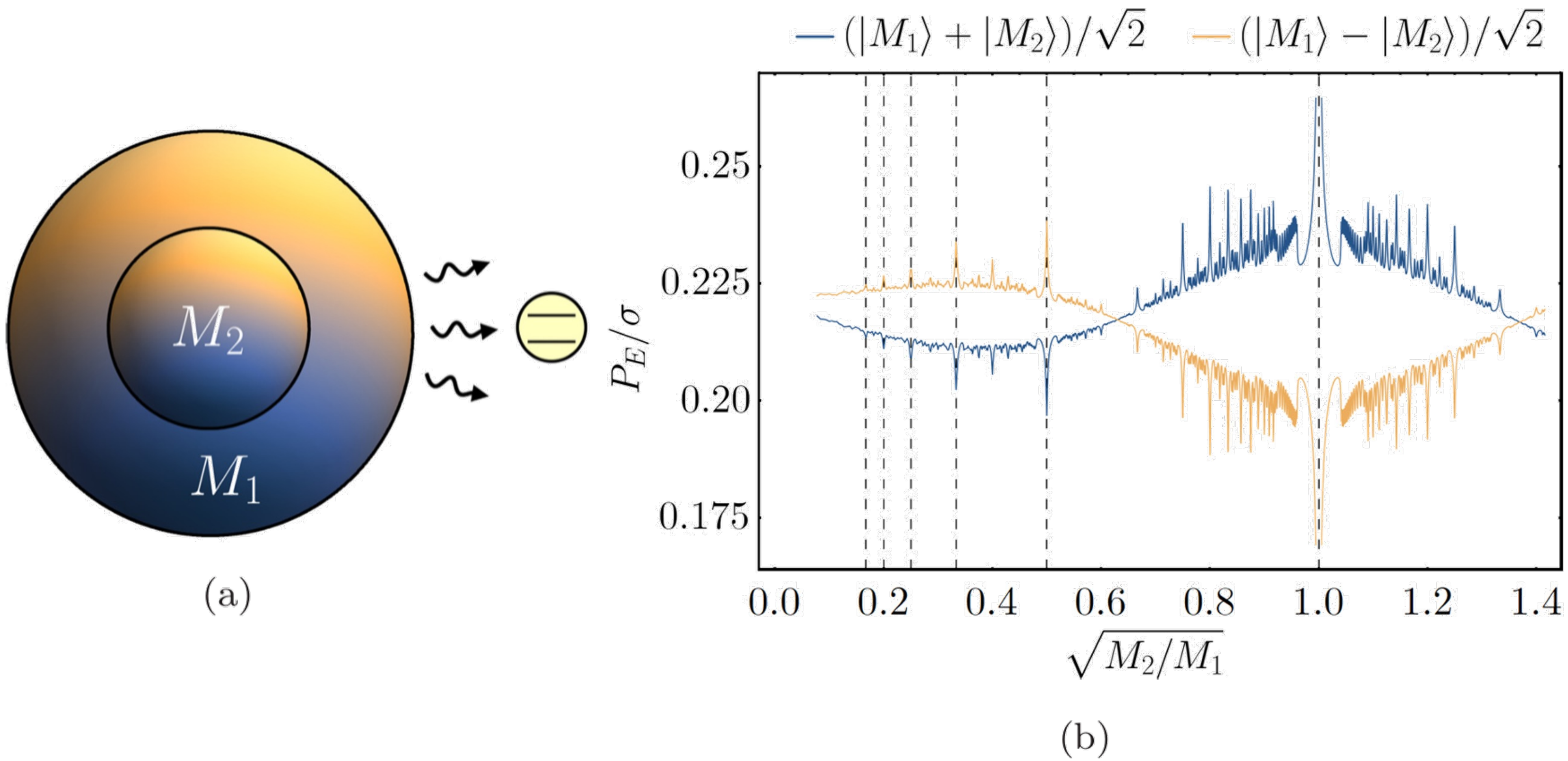
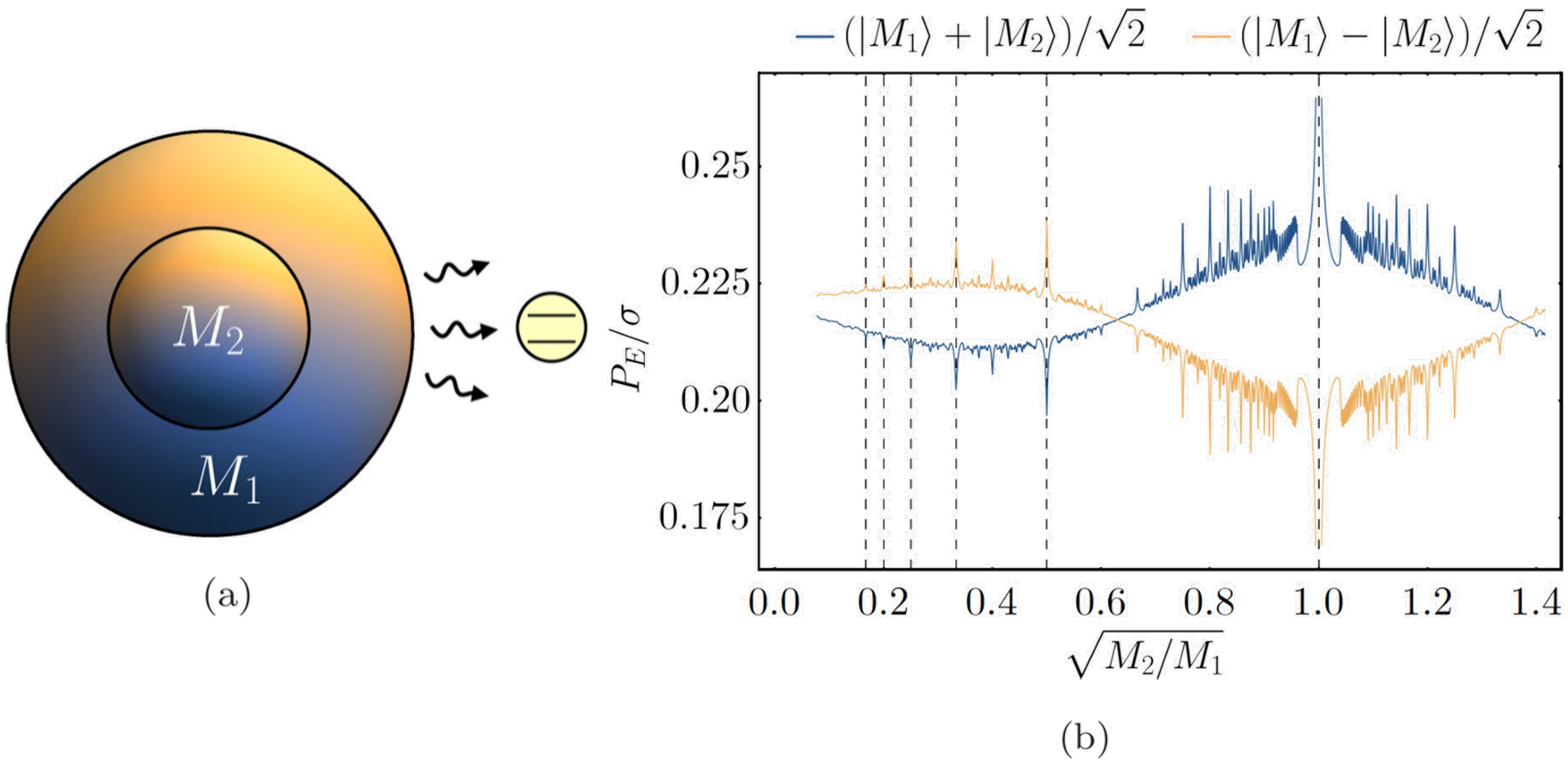


Fig. 3. (a) Schematic diagram of the black hole mass superposition, with a detector situated at a fixed radial distance from the origin of coordinates. (b) Normalized probability P_E/σ of the detector as a function of $\sqrt{M_2/M_1}$ (σ is the timescale of the interaction). The measurement basis of the control is indicated in the legend. The dashed lines correspond to $\sqrt{M_2/M_1} = 1/n$ where $n = \{1, \ldots, 6\}$.

- **Superposition of spacetimes**



A novel, independent signature that supports and extends Bekenstein's conjecture regarding the discrete mass eigenspectrum of quantum black holes.

- **Deducing the spacetime metric from the correlator**

Example: 4-dimensional Minkowski spacetime

The Wightman function

$$G^+(x,x') = -\frac{1}{4\pi^2}\frac{1}{(t-t'-i\epsilon)^2-(x-x')^2-(y-y')^2-(z-z')^2}.$$

It is straightforward to check that,

$$\eta_{ij} = -\frac{1}{2}\left[\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right]^{\frac{2}{D-2}}\frac{\partial}{\partial x^i}\frac{\partial}{\partial y^j}\left(G^+(x,y)^{\frac{2}{2-D}}\right).$$

M. Saravani, S. Aslanbeigi and A. Kempf, Phys. Rev. D 93, 045026 (2016).

- **Deducing the spacetime metric from the correlator**

Utilizing the fact that the strength of correlations in quantum field theory is an operational measure of spacetime distance, we can write a "conditional quantum metric" (i.e. conditioned on the measurement of the control system)

$$g_{\mu\nu} \propto \lim_{x \to x'} \frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x'^{\nu}} \sum_{i,j} \underbrace{\left[{}_F \langle \varphi | \hat{\Phi}(x_i) \hat{\Phi}(x_j) | \varphi \rangle_F \right]}_{\text{two}-\text{point correlator}}^{\frac{2}{d-2}}$$

M. Saravani, S. Aslanbeigi and A. Kempf, Phys. Rev. D 93, 045026 (2016).

Summary

- Although a full theory of quantum gravity is absent, one can study quantum gravitational effects by assuming that basic principles of quantum mechanics, such as Heisenberg's uncertainty principle and the quantum superposition principle, still apply in quantum gravity.

- We have studied the quantum gravitational interaction between two objects, as well as between an object and a gravitational boundary, in several cases.

- We have introduced some recent works on the response of an Unruh-DeWitt detector in superposed trajectories and superposed spacetimes.

Thanks!