NuclearScience
Computing CenteratCCNU

Mass and Topology in QCD: A Lattice Perspective

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Bollweg, HTD, 高翔, 罗冉, Mukherjee, PRL 137 (2026) 6, 061901

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

Workshop on Non-Perturbative Methods and Their Applications in High-Energy Physics

Nov. 7-9, 2026 @Peng Huanwu Center for Fundamental Theory (Hefei)

Two quantum anomalies in QCD

Trace anomaly

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{a}_{\mu\nu} F^{a\mu\nu} + \sum_{f} (1 + \gamma_m) m_f \bar{\psi}_f \psi_f$$

Scale transformations

Running coupling and the mass scale

Axial anomaly

$$\partial_\mu J^{\mu}_{5} = 2i \sum_{f} m_f \bar{\psi}_f \gamma_5 \psi_f + 2N_f \, q(x)$$

$$q(x) = \frac{g^2}{32\pi^2} F^{a}_{\mu\nu} \widetilde{F}^{a\mu\nu}, \qquad Q = \int d^4x \, q(x)$$

Singlet axial rotations

Topology and quark-mass dependence

Two anomalous identities, two complementary lattice probes

The questions behind the two anomalies

Mass decomposition

How much of a hadron mass comes from quark and gluon operators?

Can the energy and trace sum rules be tested independently?

η_c and J/ψ in the vacuum

Bollweg, HTD, 高翔, 罗冉, Swagato Mukherjee, PRL 137 (2026) 6, 061901

QCD topology

How do topological fluctuations respond to the light-quark mass?

Does the local mass dependence change across the thermal crossover?

χ_{top} and its chiral response at finite T

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$\langle H(p)|T^{\mu\nu}|H(p)\rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^\alpha_\alpha\right) + \frac{1}{4}g^{\mu\nu}T^\alpha_\alpha$$

Traceless Trace

For QCD: $T^{\mu\nu}_{\rm QCD} = T^{\mu\nu}_q + T^{\mu\nu}_g =$ Quark part $M_E + M_m$ + Gluon part $M_g + M_a$

M_E: quark kinetic energy

M_m: explicit quark mass

M_g: gluon energy

M_a: anomaly

Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

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$$T^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^\alpha_\alpha\right) + \frac{1}{4}g^{\mu\nu}T^\alpha_\alpha$$

Traceless Trace

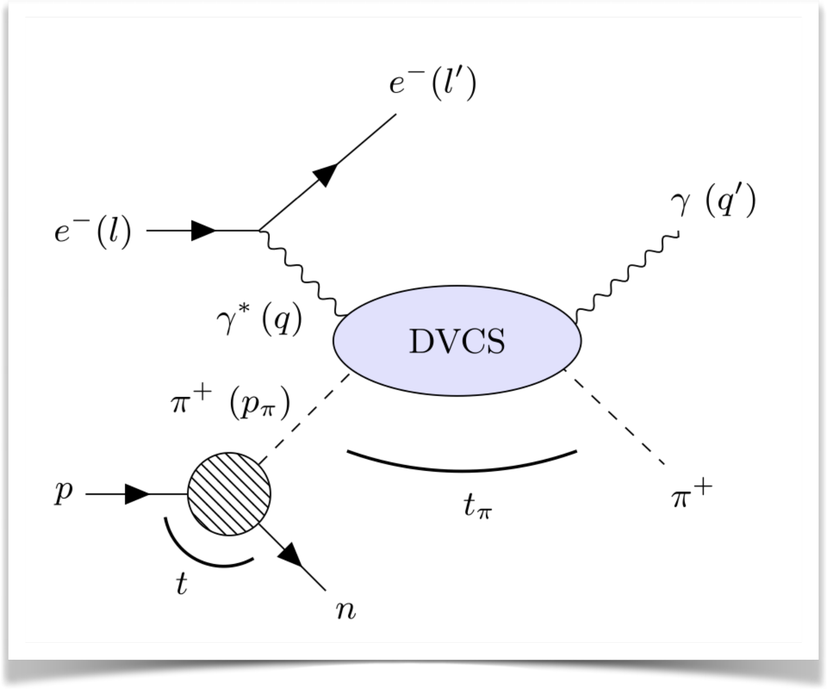
same EMT information⟶ different organizations ⟶ different hadron-mass sum rules

Ji | HRT | Lorcé | MPR

Ji, PRL 74 (1995) 1071 ;Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

Current status: partial EMT access from experiment and lattice QCD

Deeply virtual exclusive processes
⇒ partial GFF / traceless EMT access

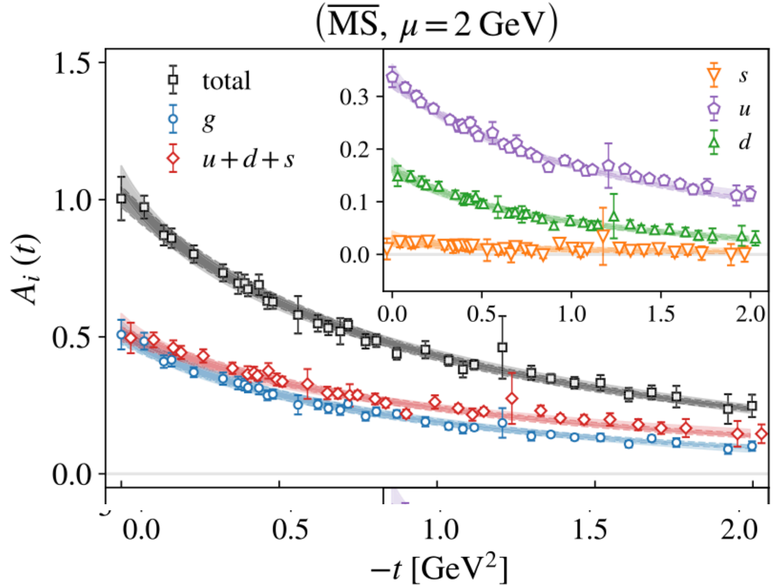


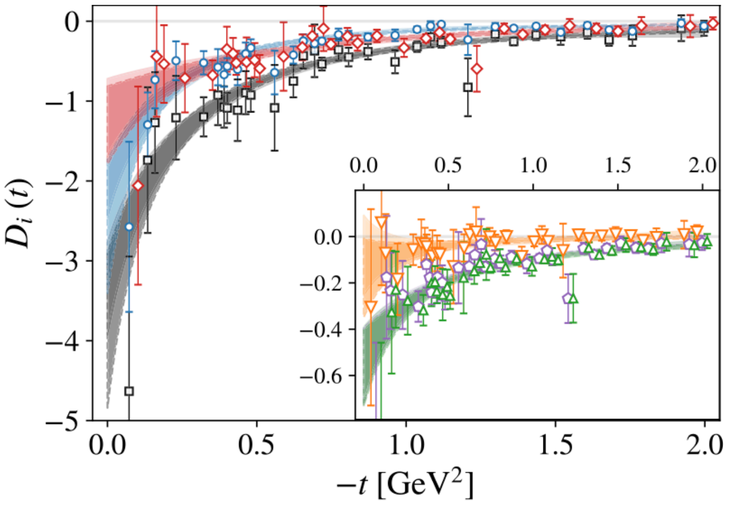
See EXP extracted pressure distribution in e.g.
Burkert, Elouadrhiri & Girod, Nature 557 (2018)396

Recent lattice QCD:
GFFs and spatial structure are accessible but the trace sector remained difficult

Traceless Trace

$$\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)\frac{iP^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2m} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m} + \bar{C}(t)mg^{\mu\nu}\right]u(p)$$





Hackett et al., PRL 132 (2024) 251904

Current status: partial EMT access from experiment and lattice QCD

Earlier lattice QCD

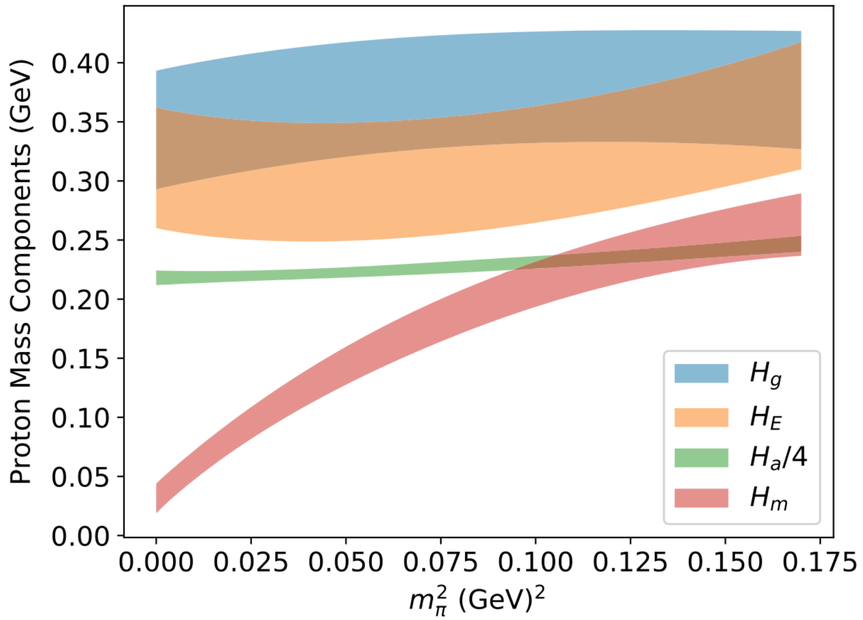
Hadron mass decompositions relied on partial EMT input or indirect closure

$$T^{\mu}_{\mu} = \underbrace{\sum_f m_f \bar{\psi}_f \psi_f}_{\text{explicit quark-mass term}} + \underbrace{\left[\frac{\beta(g)}{2g} F^2 + \gamma_m \sum_f m_f \bar{\psi}_f \psi_f\right]}_{\text{quantum trace anomaly}}.$$

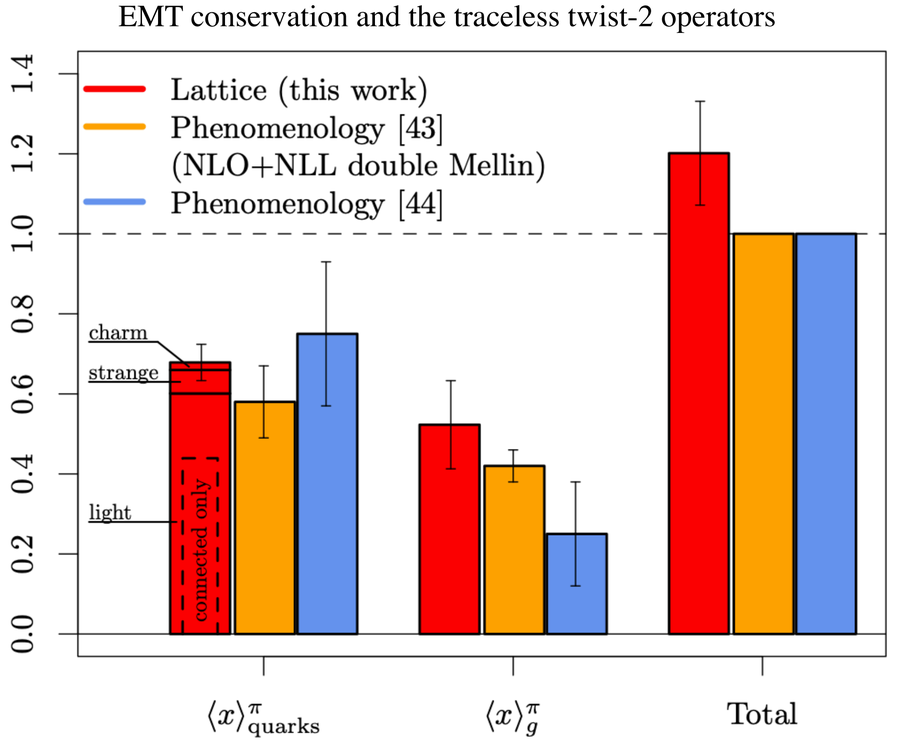
Trace

$$\langle \pi(\mathbf{p}) | \bar{T}^X_{\mu\nu} | \pi(\mathbf{p}) \rangle = 2\langle x \rangle^X \left(p_\mu p_\nu - \delta_{\mu\nu} \frac{p^2}{4} \right)$$

Traceless



Yang, et al.,[χQCD], PRL 121 (2018) 212001



Alexandrou, et al.,[ETMC], PRL 127 (2021) 25, 252001

Current status: partial EMT access from experiment and lattice QCD

Earlier lattice QCD

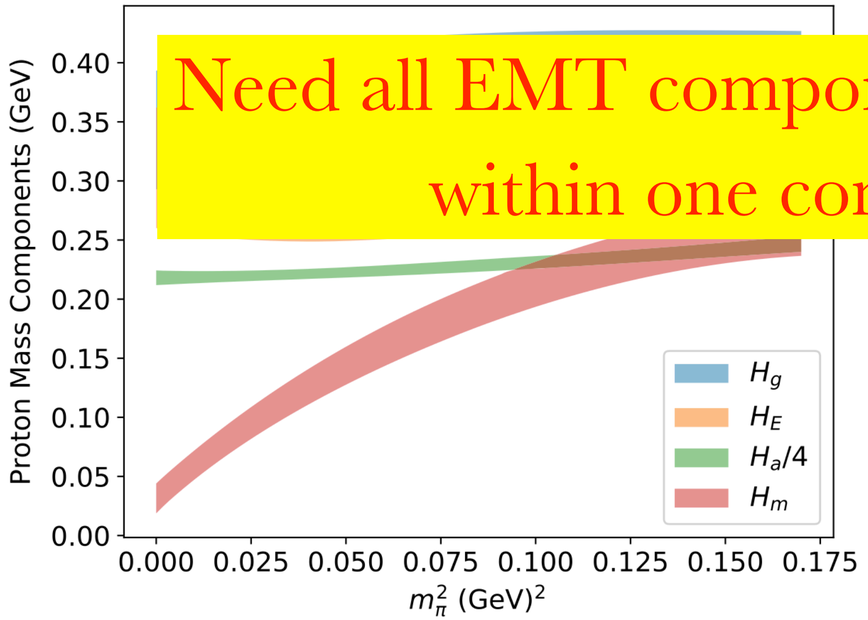
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Trace

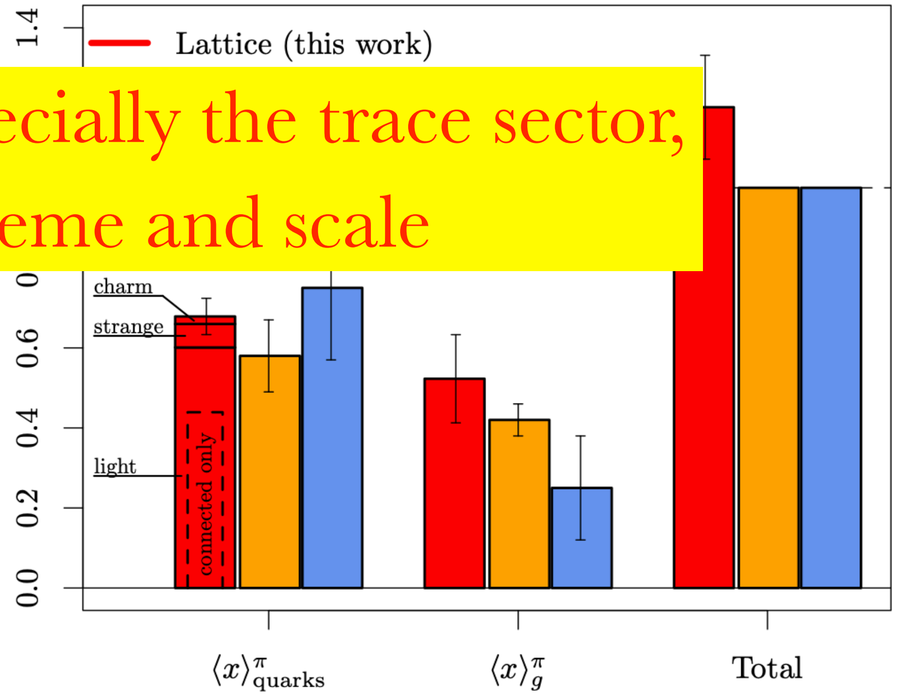
$$\langle \pi(\mathbf{p}) | \bar{T}^X_{\mu\nu} | \pi(\mathbf{p}) \rangle = 2\langle x \rangle^X \left(p_\mu p_\nu - \delta_{\mu\nu} \frac{p^2}{4} \right)$$

Traceless



Yang, et al.,[χQCD], PRL 121 (2018) 212001

EMT conservation and the traceless twist-2 operators



Alexandrou, et al.,[ETMC], PRL 127 (2021) 25, 252001

Need all EMT components, especially the trace sector, within one common scheme and scale

What was difficult before

$$T_{\rho\sigma}(x) = \mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4}\mathcal{O}_{2,\rho\sigma}(x) + \frac{1}{4}\mathcal{O}_{3,\rho\sigma}(x) - \frac{1}{2}\mathcal{O}_{4,\rho\sigma}(x) - \mathcal{O}_{5,\rho\sigma}(x)$$

Gluon part: $T_{g,\rho\sigma}$ Quark part: $T_{q,\rho\sigma}$

$$\mathcal{O}_{1,\rho\sigma} = \frac{2}{g_0^2}\mathrm{Tr}^c\left[F_{\rho\omega}F_{\sigma\omega}\right], \quad \mathcal{O}_{2,\rho\sigma} = \frac{2}{g_0^2}\delta_{\rho\sigma}\mathrm{Tr}^c\left[F_{\omega\xi}F_{\omega\xi}\right]$$

$$\mathcal{O}_{3,\rho\sigma} = \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f\left(\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho\right)\psi_f$$

$$\mathcal{O}_{4,\rho\sigma} = \delta_{\rho\sigma}\sum_f \bar{\psi}_f \overleftrightarrow{\not{D}}\psi_f, \quad \mathcal{O}_{5,\rho\sigma} = \delta_{\rho\sigma}\sum_f m_f\bar{\psi}_f\psi_f$$

Tensor operators: $\mathcal{O}_1, \mathcal{O}_3$ Pure-trace operators: $\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5$

- EMT trace lies in the scalar channel under lattice hypercubic symmetry ⇒ mixing with lower-dimensional operators
- power-divergent subtractions are required
- conventional RI/MOM-type renormalization: gauge dependent/IR sensitive/ noisy

Belinfante, Phys. Rev. 128 (1962) 2832; Belitsky & Radyushkin, Phys. Rept. 418 (2005) 1; Freese, Phys. Rev. D 106 (2022) 125012,…

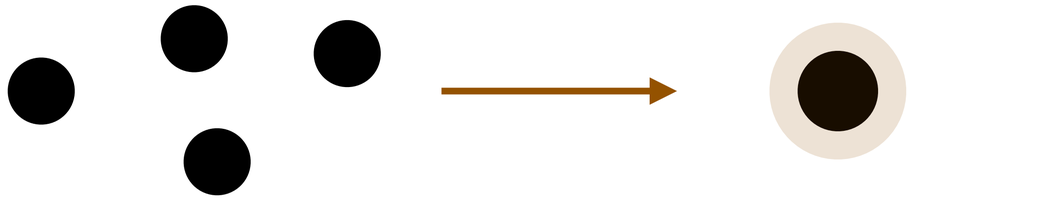
Our strategy: from gradient flow to a common $\overline{\text{MS}}$ EMT scheme

Gradient Flow (GF): Lüscher et al., 10', 11', 14'

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(0,x) = A_\mu(x)$$

Flow time t_f smooth UV fluctuations over a radius of $\sqrt{8t_f}$

$$B_\mu(t,p) \simeq e^{-tp^2} A_\mu(p)$$



rough field — flowed/smeared field

Small-flow-time EMT reconstruction

$$T^{\overline{\text{MS}}}_{\rho\sigma}(x,\mu) = \lim_{t_f \to 0} \Sigma_i c_i(t_f,\mu) \bar{\mathcal{O}}_{i,\rho\sigma}(t_f,x)$$

flowed operators are matched to the physical EMT; the coefficients c_i are known perturbatively

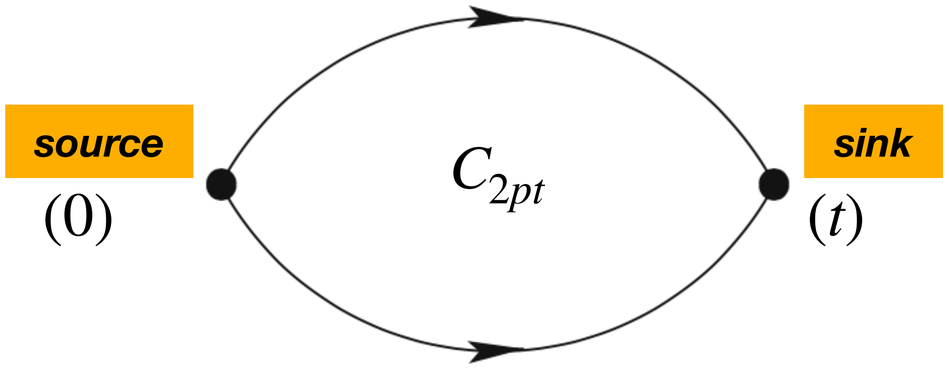
- simplifies renormalization and avoids power-divergent subtractions
- puts traceless and trace sectors into one common framework

Operationally in this work

Flowed EMT at fixed t_f → Take $a \to 0$ at each fixed t_f → 2-loop matching GF → MSbar → Zero-Flow-time extrapolation $t_f \to 0$ → All EMT ingredients in one common scheme

common scheme-and-scale determination of traceless and trace contributions

Hadron ground states from 2-pt correlators



$$C_{\Gamma}^{2\mathrm{pt}}(t) \equiv \sum_{x} \left\langle \bar{\psi}(x)\Gamma\psi(x)\ \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle$$

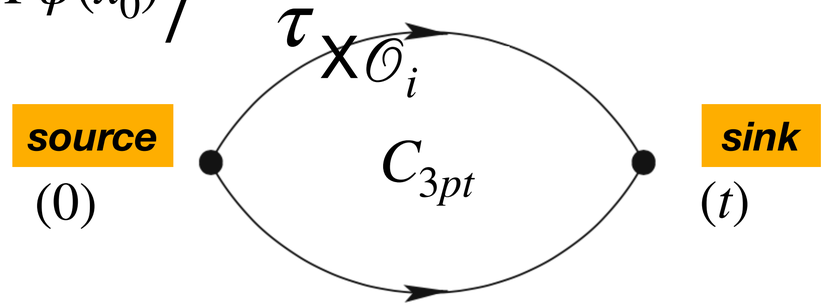
- Determine energy of states from the energy decomposition:

$$C^{2\mathrm{pt}}(t) = \sum_{n=0}^{n_{\max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n (N_t a - t)} \right]$$

EMT matrix elements from the ratio of 3pt to 2pt correlators

- 3-pt correlation function $C^{\rho\sigma}_{q,\Gamma}(t,\tau) \equiv \sum_x \left\langle \bar{\psi}(x)\Gamma\psi(x)\, \tilde{\mathcal{O}}_{3,\rho\sigma}(\tau)\, \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle$

Spectral decomposition $C^{\rho\sigma}_{X,\Gamma}(t,\tau) = \sum_{m,n} Z^*_m Z_n \frac{\langle m|\tilde{\mathcal{O}}_{i,\rho\sigma}|n\rangle}{2M} e^{-\tau E_n} e^{-(t-\tau)E_m}$



- 2-pt correlation function $C^{\rm 2pt}(t) = \sum_{n=0}^{n_{\rm max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$

Ratio of 3pt- to 2pt- correlator

$$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z^*_1 Z_0 \langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E\tau} + e^{-\Delta E(t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$

Wanted matrix element

asymptotic limit
$t \gg \tau \gg 0$

3-point correlation function: Demonstration for the gluonic part

- 3-pt corr: $C^{\rho\sigma}_{g,\Gamma}(t,\tau) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \mathcal{O}_\Gamma(\boldsymbol{x},t)\widetilde{\mathcal{O}}_{1,\rho\sigma}(\boldsymbol{y},\tau)\mathcal{O}^\dagger_\Gamma(\boldsymbol{0},0)\rangle - \langle C^{\mathrm{2pt}}_\Gamma(t)\rangle \sum_{\boldsymbol{y}} \langle \widetilde{\mathcal{O}}_{1,\rho\sigma}(\boldsymbol{y},\tau)\rangle$
- Gluon operator $\widetilde{\mathcal{O}}_{1,\rho\sigma}(t_\mathrm{f},x) \equiv 2\mathrm{Tr}^\mathrm{c}\left[F_{\rho\omega}(t_\mathrm{f},x)F_{\sigma\omega}(t_\mathrm{f},x)\right]$

  Clover-type discretization: $F_{\rho\sigma}(x) = -i\left[F'_{\rho\sigma}(x) - \frac{\mathrm{Tr}^\mathrm{c}F'_{\rho\sigma}(x)}{N_\mathrm{c}}I\right]$

  $F'_{\rho\sigma}(x) = \frac{1}{8}\left[P_{\rho\sigma}(x) - P^\dagger_{\rho\sigma}(x)\right],\ P_{\rho\sigma}(x) = U_{\rho\sigma}(x) + U_{\sigma,-\rho}(x) + U_{-\rho,-\sigma}(x) + U_{-\sigma,\rho}(x)$

  $U_{\rho\sigma}(x) = U_\rho(x)U_\sigma(x+\hat{\rho})U^\dagger_\rho(x+\hat{\sigma})U^\dagger_\sigma(x)$
- Gauge links $U_\rho(x)$ are replaced by the flowed counterparts

  $\partial_{t_\mathrm{f}} B_\mu(t_\mathrm{f},x) = D_\nu F_{\nu\mu}(t_\mathrm{f},x), \qquad B_\mu(0,x) = A_\mu(x)$

3-point correlation function: Demonstration for the quark part

- 3-pt corr: $C^{\rho\sigma}_{q,\Gamma}(t,\tau) \equiv \sum_x \left\langle \bar{\psi}(x)\Gamma\psi(x)\, \tilde{\mathcal{O}}_{3,\rho\sigma}(\tau)\, \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle$, $\quad \tilde{\mathcal{O}}_{3,\rho\sigma}(\tau) = \sum_{\boldsymbol{y}} \tilde{\mathcal{O}}_{3,\rho\sigma}(y)$

- Quark operator $\tilde{\mathcal{O}}_{3,\rho\sigma}(t_{\rm f},x) \equiv \sum_f \mathring{\bar{\chi}}_f(t_{\rm f},x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right) \mathring{\chi}_f(t_{\rm f},x)$

$$\partial_{t_{\rm f}}\chi(t_{\rm f},x) = \Delta\chi(t_{\rm f},x)$$
$$\chi(0,x) = \psi(x)$$

Ringed fermion fields: cancel the wave function renormalization by construction

$$\mathring{\bar{\chi}}_f(t_{\rm f},x) \equiv \sqrt{\frac{-6}{(4\pi t_{\rm f})^2 \langle \bar{\chi}_f(t_{\rm f},x) \overleftrightarrow{\not{D}} \chi_f(t_{\rm f},x)\rangle}}\, \bar{\chi}_f(t_{\rm f},x) \qquad \mathring{\chi}_f(t_{\rm f},x) \equiv \sqrt{\frac{-6}{(4\pi t_{\rm f})^2 \langle \bar{\chi}_f(t_{\rm f},x) \overleftrightarrow{\not{D}} \chi_f(t_{\rm f},x)\rangle}}\, \chi_f(t_{\rm f},x)$$

- Contractions among flowed and un-flowed fermion fields: No additional Dirac inversions are needed

$$\left\langle \bar{\chi}(t_f,y)\,\psi(x)\right\rangle = \sum_z K(t_f,y;0,z)\, S_\psi(z\,|\,x)\,, \qquad \left\langle \psi(x)\,\bar{\chi}(t_f,y)\right\rangle = \sum_z S_\psi(x\,|\,z)\, K^\dagger(t_f,y;0,z), \qquad \lim_{t_f\to 0} K(t_f,y;0,z) = \delta_{yz}\,.$$

Is J/ψ simply two charm quarks?



$$M_{J/\psi} = 3.097 \text{ GeV}$$

Current quark mass: $m_c^{\overline{MS}}(m_c) \approx 1.27$ GeV

Pole/Constituent-like mass:

$$m_c^{\text{pole}} = m_c^{\overline{\text{MS}}}(m_c)\left(1 + \frac{4}{3}\frac{\alpha_s}{\pi} + \cdots\right) \approx 1.5 \text{ GeV}$$

Much less challenging computationally compared with light mesons or proton

Lattice setup and extracted matrix elements

Ensembles

- 2+1 flavor HISQ, HotQCD ensembles
- $a \approx 0.06, 0.08, 0.12$ fm
- $64 \times 48^3, 64^4, 64^4$
- physical strange, near-physical light quarks
- physical charmonium: η_c & J/ψ

Measured quantities

- flowed EMT operators $\mathcal{O}_1, \cdots \mathcal{O}_5$
- flowed quark and gluon matrix elements
- both traceless & trace sectors

Extraction

- 2-point & 3-point correlators $\rightarrow < H | \mathcal{O}_i | H >$
- $a \rightarrow 0$ at fixed t_f
- GF $\rightarrow \overline{\text{MS}}$ matching
- $t_f \rightarrow 0$

Dennis Bollweg, HTD, Xiang Gao, Ran Luo, Swagato Mukherjee, PRL 137 (2026) 6, 061901

Measurements done in

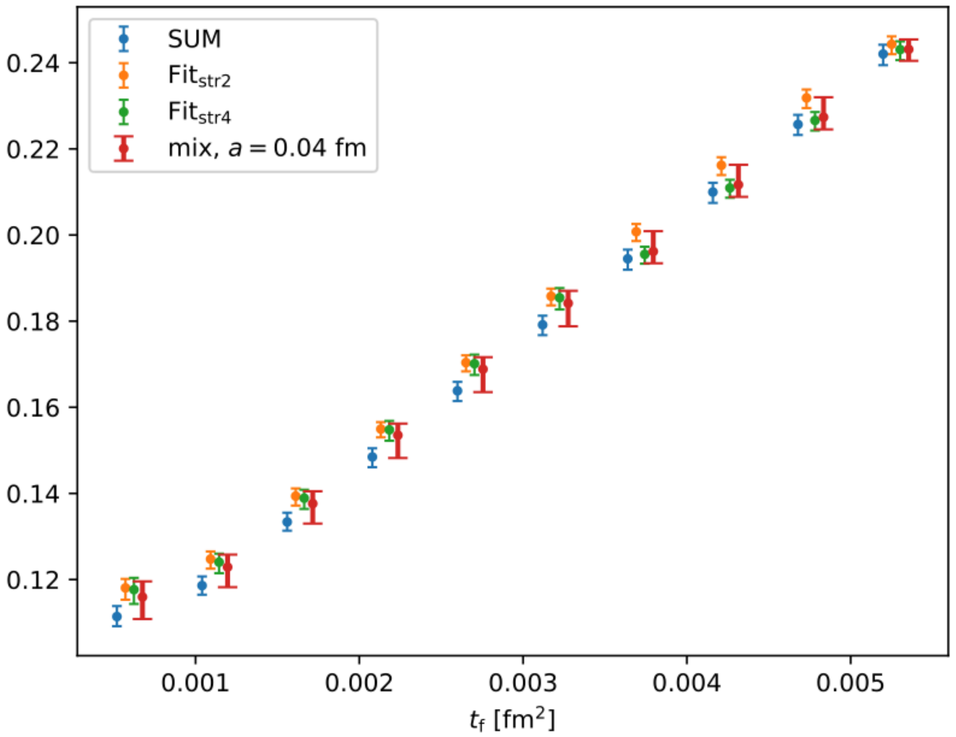


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Stable extraction of flowed EMT matrix elements

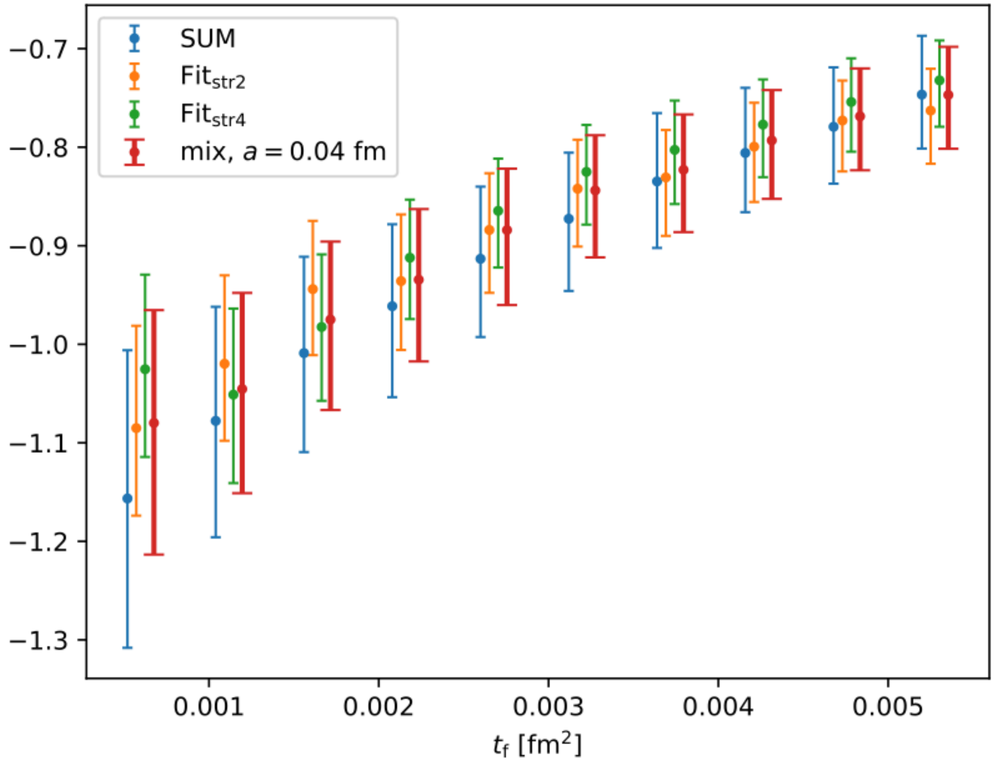
quark matrix element

$$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{3,kk}/3|\eta_c\rangle$$



gluon matrix element

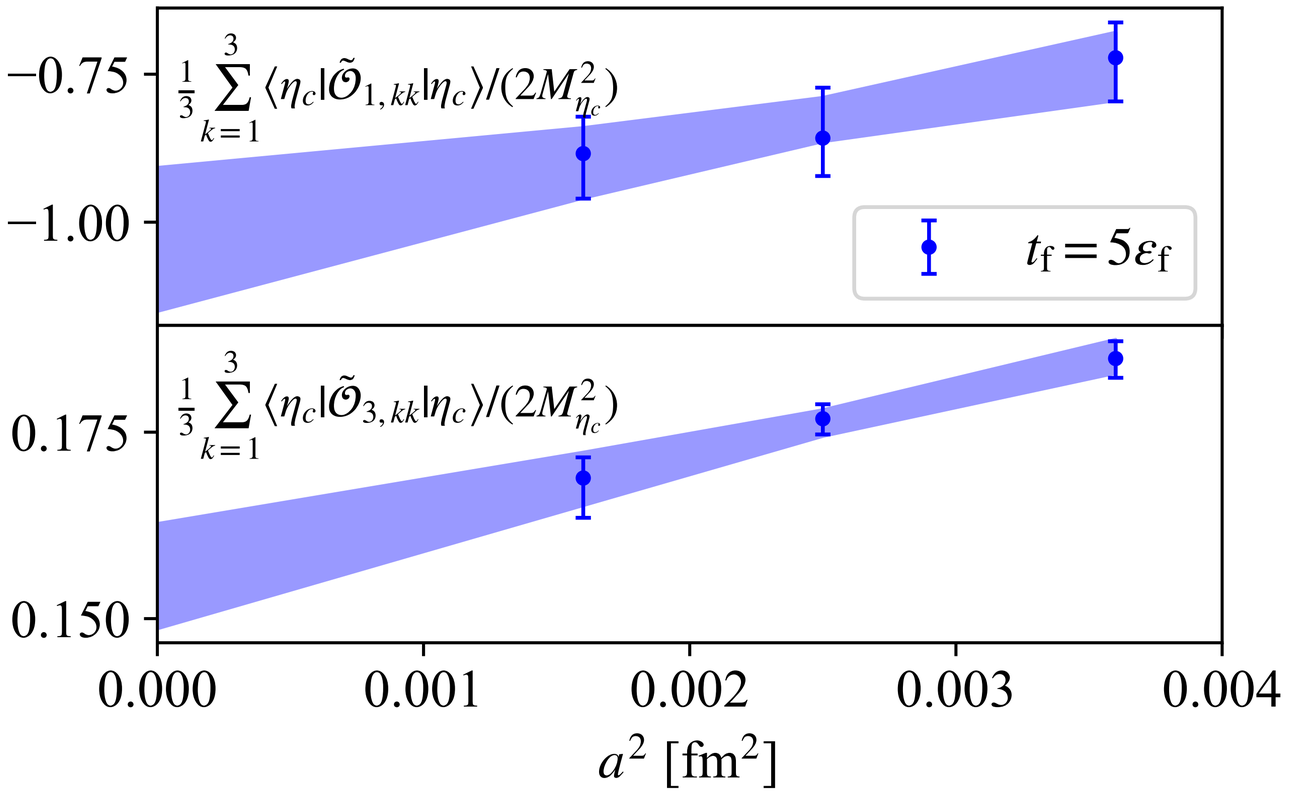
$$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{1,kk}/3|\eta_c\rangle$$



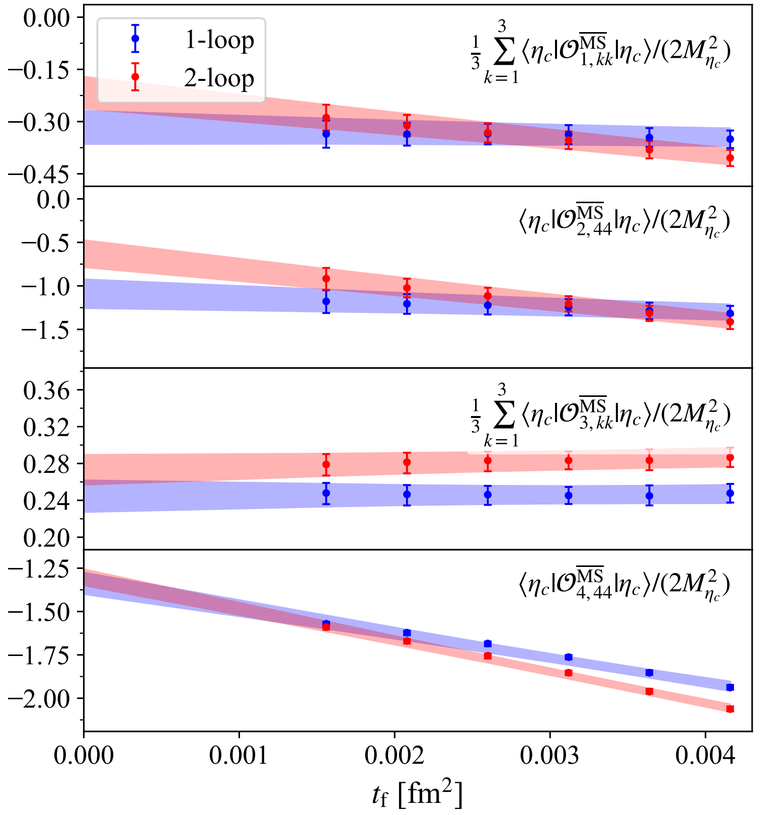
Flowed quark and gluon EMT matrix elements are extracted stably from standard 3pt/2pt ratios

From flowed matrix elements to the physical EMT

Continuum extrapolation at fixed flow time t_f



GF → $\overline{\text{MS}}$ matching and $t_f \to 0$ extrapolation



$\sim \sigma_f$

Pert. matching from: Suzuki, 14'15'; Harlander et al., 18'

Smooth continuum, matching and $t_f \to 0$ behavior enables a common determination of all EMT components

Forward-limit sum rules for GFF

$A(0)$: momentum fraction type term; $\bar{C}(0)$: trace-related GFF ingredient; $\bar{f}(0)$: extra spin-1 structure for J/ψ

- Spin-0 hadron

$$\langle \eta_c | T^{\overline{\rm MS}}_{X,00} | \eta_c \rangle = 2M^2_{\eta_c}[A_X(0) + \bar{C}_X(0)]$$

$$\sum_{k=1}^{3} \langle \eta_c | T^{\overline{\rm MS}}_{X,kk} | \eta_c \rangle = -6M^2_{\eta_c}\bar{C}_X(0)$$

Poincaré symmetry $\sum_X A_X(0) = 1$

Conservation of the total EMT $\sum_X \bar{C}_X(0) = 0$

X=c,g

- Spin-1 hadron

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,11} | J/\psi \rangle = 2M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) + \frac{3}{4}\bar{f}_X(0)\right]$$

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,00} | J/\psi \rangle = 2M^2_{J/\psi}[A_X(0) - \frac{1}{2}\bar{C}_X(0) + \frac{1}{4}\bar{f}_X(0)]$$

$$\sum_{k=2,3} \langle J/\psi | T^{\overline{\rm MS}}_{X,kk} | J/\psi \rangle = 4M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) - \frac{1}{4}\bar{f}_X(0)\right]$$

Rotational symmetry & polarization structure

$$\sum_X \bar{f}_X(0) = 0$$

Ji, PRL 78 (1997) 610, Hackett et al., PRD108 (2023) 114504, Pefkou et al., 105 (2022) 054509, Polyakov & Sun, PRD 100 (2019) 036003

Forward-limit GFF ingredients at rest for η_c and J/ψ

$A(0)$: momentum fraction type term; $\bar{C}(0)$: trace-related GFF ingredient; $\bar{f}(0)$: extra spin-1 structure for J/ψ

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
gluon(g)	0.084(45)	0.058(18)	0.077(35)	-0.146(29)	-0.012(32)
charm(c)	0.918(17)	-0.068(4)	0.926(34)	0.176(87)	0.023(5)
total	1.000(46)	−0.010(18)	0.997(43)	0.030(30)	0.010(32)

$$\langle \eta_c | T^{\overline{\rm MS}}_{X,00} | \eta_c \rangle = 2M^2_{\eta_c}[A_X(0) + \bar{C}_X(0)]$$

$$\sum_{k=1}^{3} \langle \eta_c | T^{\overline{\rm MS}}_{X,kk} | \eta_c \rangle = -6M^2_{\eta_c}\bar{C}_X(0)$$

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,11} | J/\psi \rangle = 2M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) + \frac{3}{4}\bar{f}_X(0)\right]$$

$$\langle J/\psi | T^{\overline{\rm MS}}_{X,00} | J/\psi \rangle = 2M^2_{J/\psi}[A_X(0) - \frac{1}{2}\bar{C}_X(0) + \frac{1}{4}\bar{f}_X(0)]$$

$$\sum_{k=2,3} \langle J/\psi | T^{\overline{\rm MS}}_{X,kk} | J/\psi \rangle = 4M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) - \frac{1}{4}\bar{f}_X(0)\right]$$

- $\bar{C}(0)$ determined directly for η_c and J/ψ
- $A_g(0) + A_c(0) = 1$ for both states
- $\bar{C}_g(0) + \bar{C}_c(0) = 0$ within uncertainties
- $\bar{f}_g(0) + \bar{f}_c(0) = 0$ within uncertainties for J/ψ

Consistent with the forward-limit GFF sum rules

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , PRL 137 (2026) 6, 061901

Four sum rules of hadron mass decomposition

$$M_H = \sum_{X=q,g} M_X^{\rm HRT}, \quad \text{with} \quad M_X^{\rm HRT} = \frac{\langle H| \left(T_X^{\overline{\rm MS}}\right)_\rho^\rho |H\rangle}{2M_H}.$$

Hatta, Rajan, & Tanaka, JHEP 12 (2018) 008

$$M_H = \sum_{X=q,g,m} M_X^{\rm MPR}, \quad \text{with} \quad M_m^{\rm MPR} = \sum_f \sigma_f,$$
$$M_q^{\rm MPR} = M_q^{\rm Lor} - \sum_f \sigma_f, \quad M_g^{\rm MPR} = M_g^{\rm Lor}.$$

Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

$$M_H = \sum_{X=q,g} M_X^{\rm Lor}, \quad \text{with} \quad M_X^{\rm Lor} = \frac{\langle H|T_{X,00}^{\overline{\rm MS}}|H\rangle}{2M_H}.$$

Lorcé, EPJC 78 (2018)120

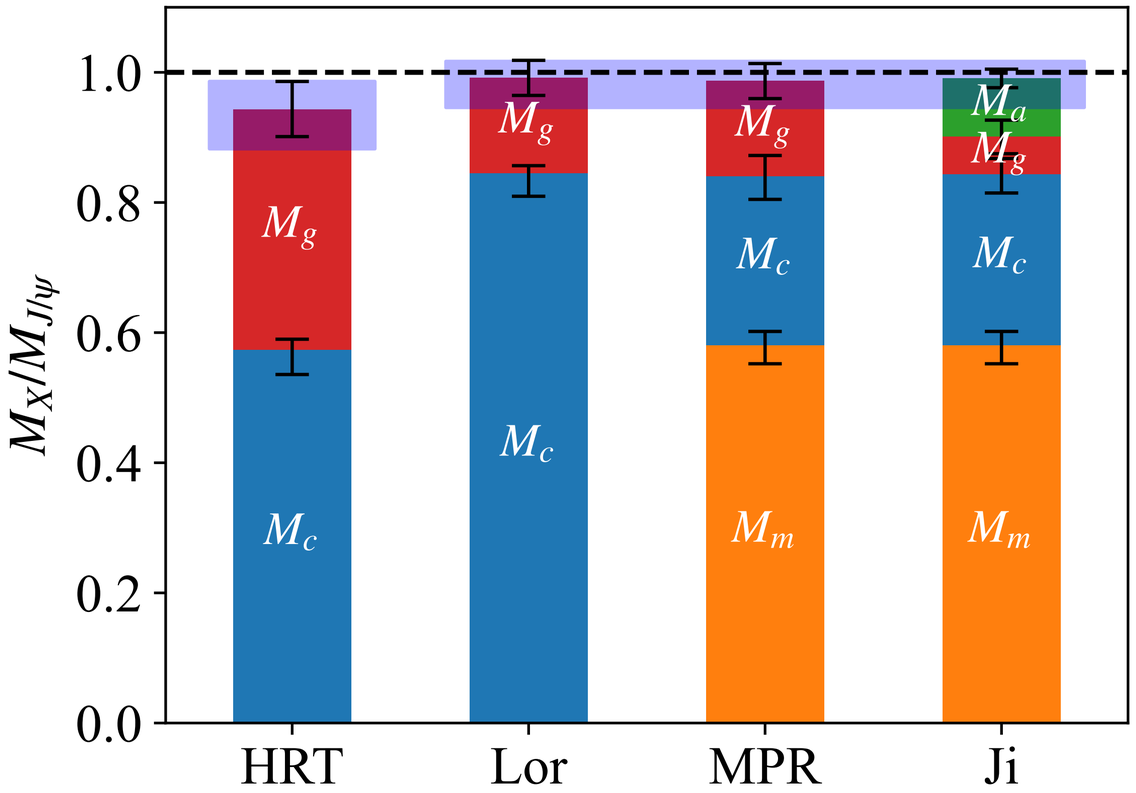
$$M_H = \sum_{X=q,g,m,a} M_X^{\rm Ji}, \quad \text{with} \quad M_m^{\rm Ji} = M_m^{\rm MPR},$$
$$M_q^{\rm Ji} = M_q^{\rm Lor} - \frac{1}{4} M_q^{\rm HRT} - \frac{3}{4} M_m^{\rm MPR},$$
$$M_g^{\rm Ji} = M_g^{\rm Lor} - \frac{1}{4} M_g^{\rm HRT},$$
$$M_a^{\rm Ji} = \frac{1}{4}\left[M_q^{\rm HRT} + M_g^{\rm HRT} - M_m^{\rm MPR}\right].$$

Ji, PRL 74 (1995) 1071

Lattice QCD closes the J/ψ & η_c mass budget across independent sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split

J/ψ mass budget



Gluon energy is sizable

About 15% in Lorce/MPR,
About 30% in HRT

The Ji anomaly term is visible

277 MeV in J/ψ , close to 9%

Forward-limit GFF tests close

$A_c + A_g = 1, \bar{C}_c + \bar{C}_g = 0$

First direct results for $\bar{C}$

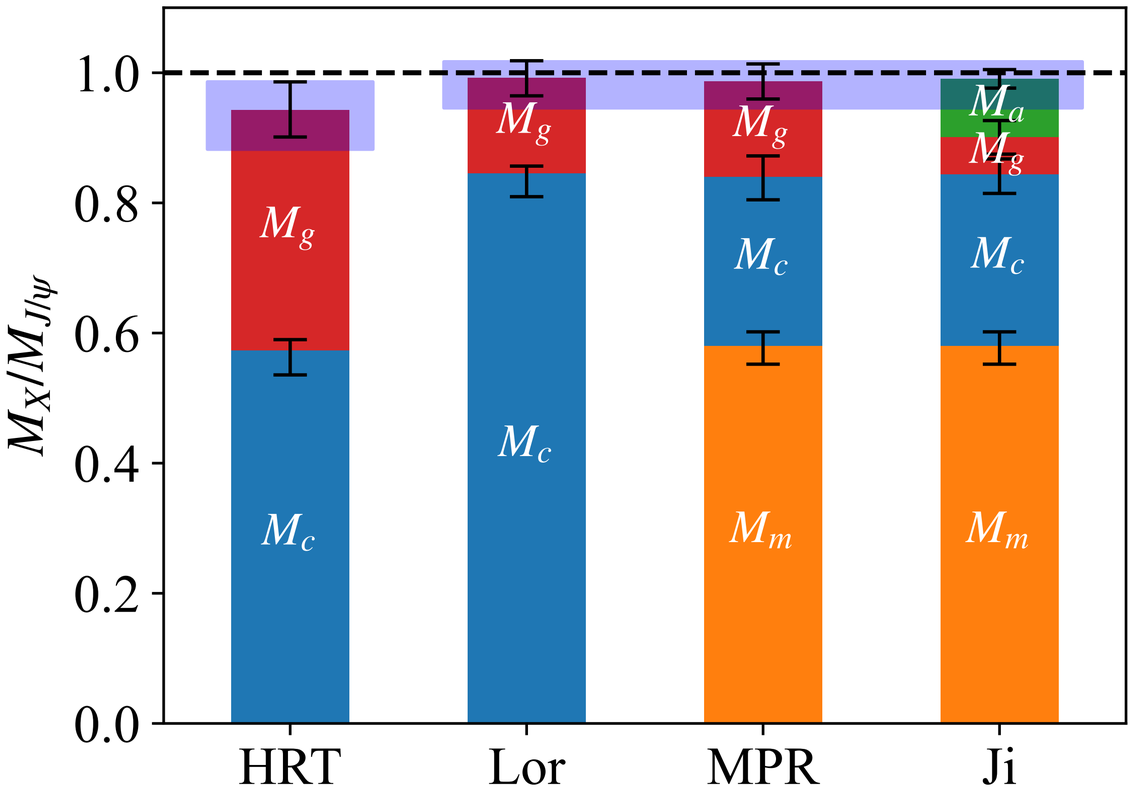
M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , PRL 137 (2026) 6, 061901

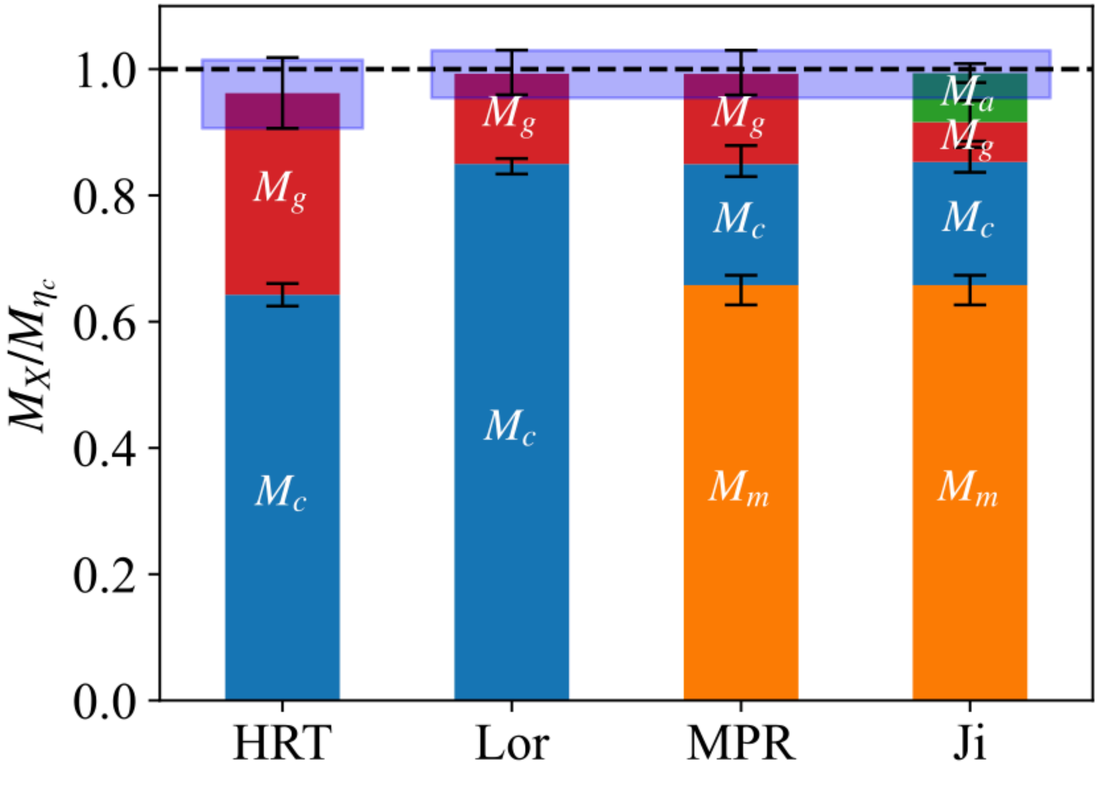
Lattice QCD closes the J/ψ & η_c mass budget across independent sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split

J/ψ mass budget



η_c mass budget

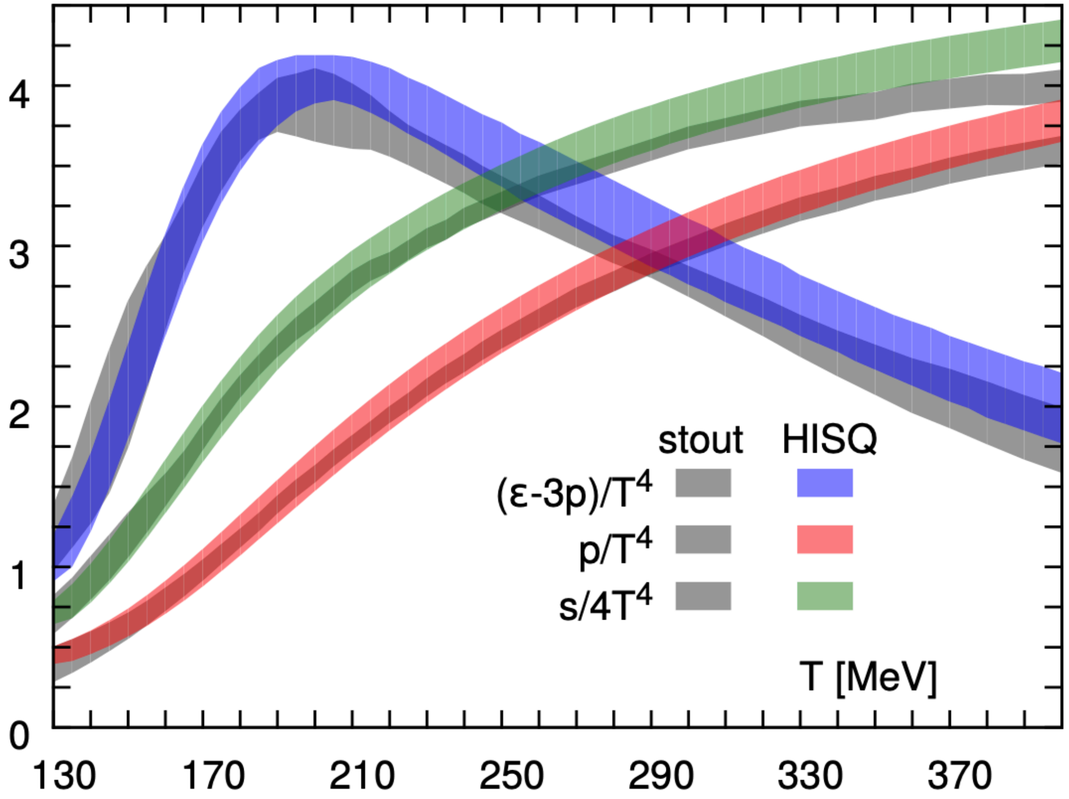


M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , PRL 137 (2026) 6, 061901

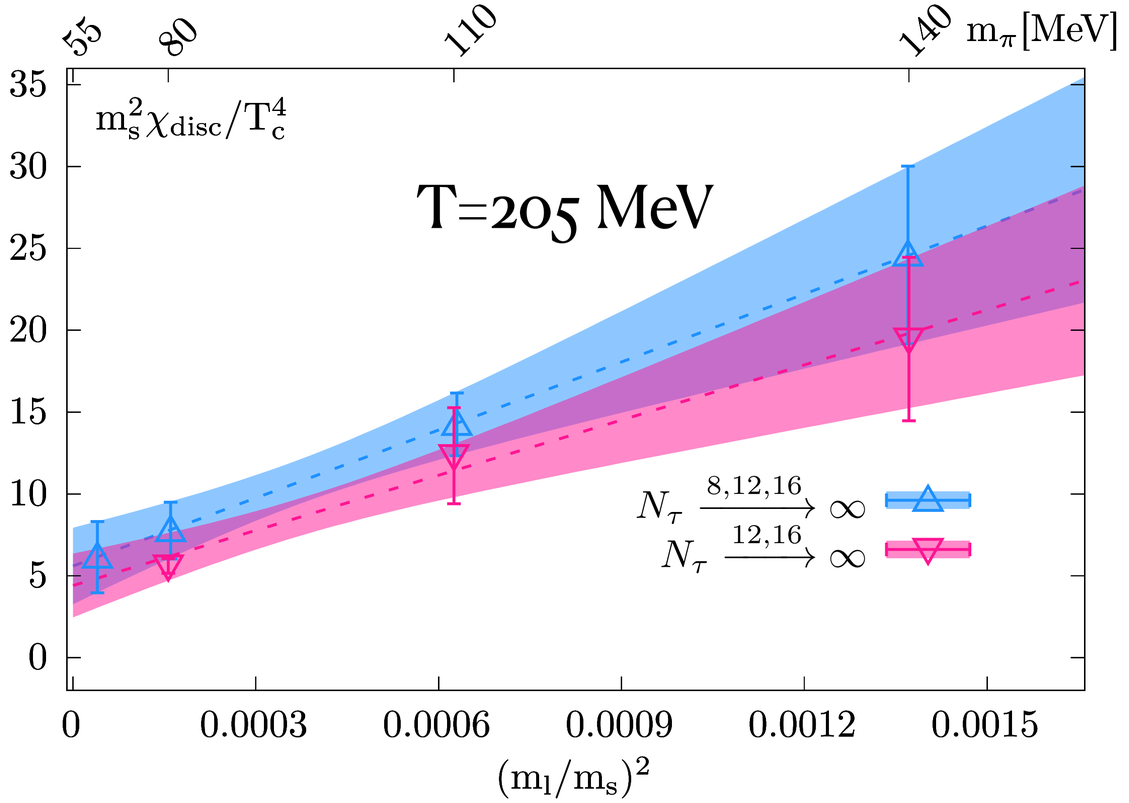
Anomalies at nonzero temperature

Trace anomaly



HotQCD, PRD 2014, arXiv:1407.6387

Axial U1 anomaly

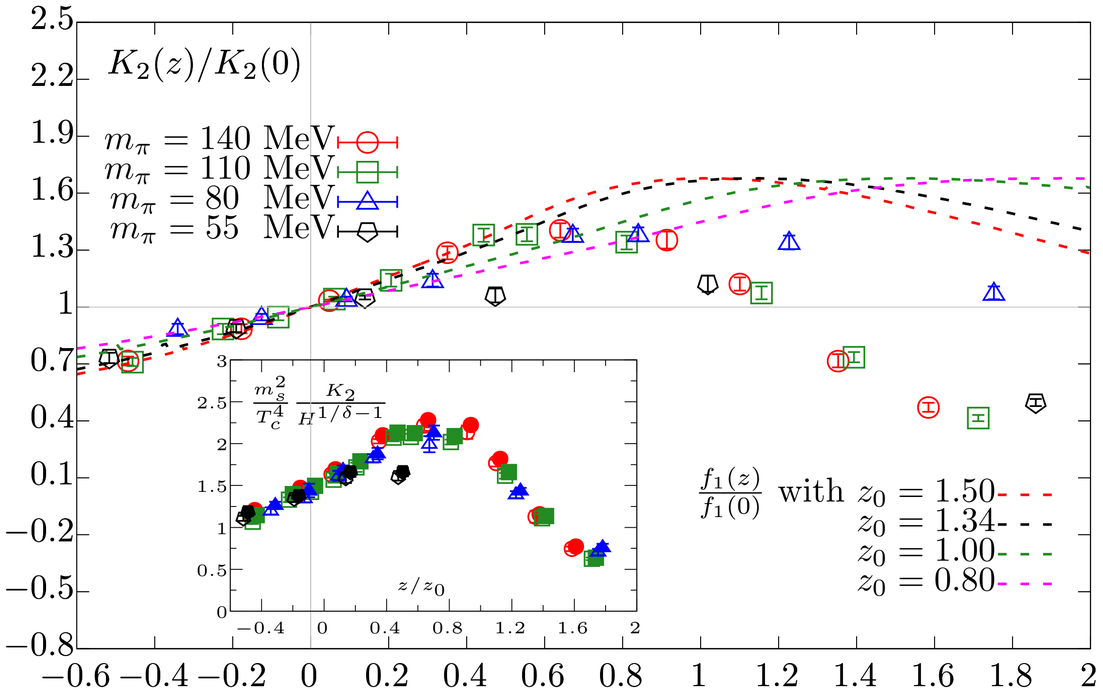


HTD et al., PRL 126 (2021) 082001

Nature of the QCD transition

O(N) scaling towards chiral limit from cumulants of chiral condensates

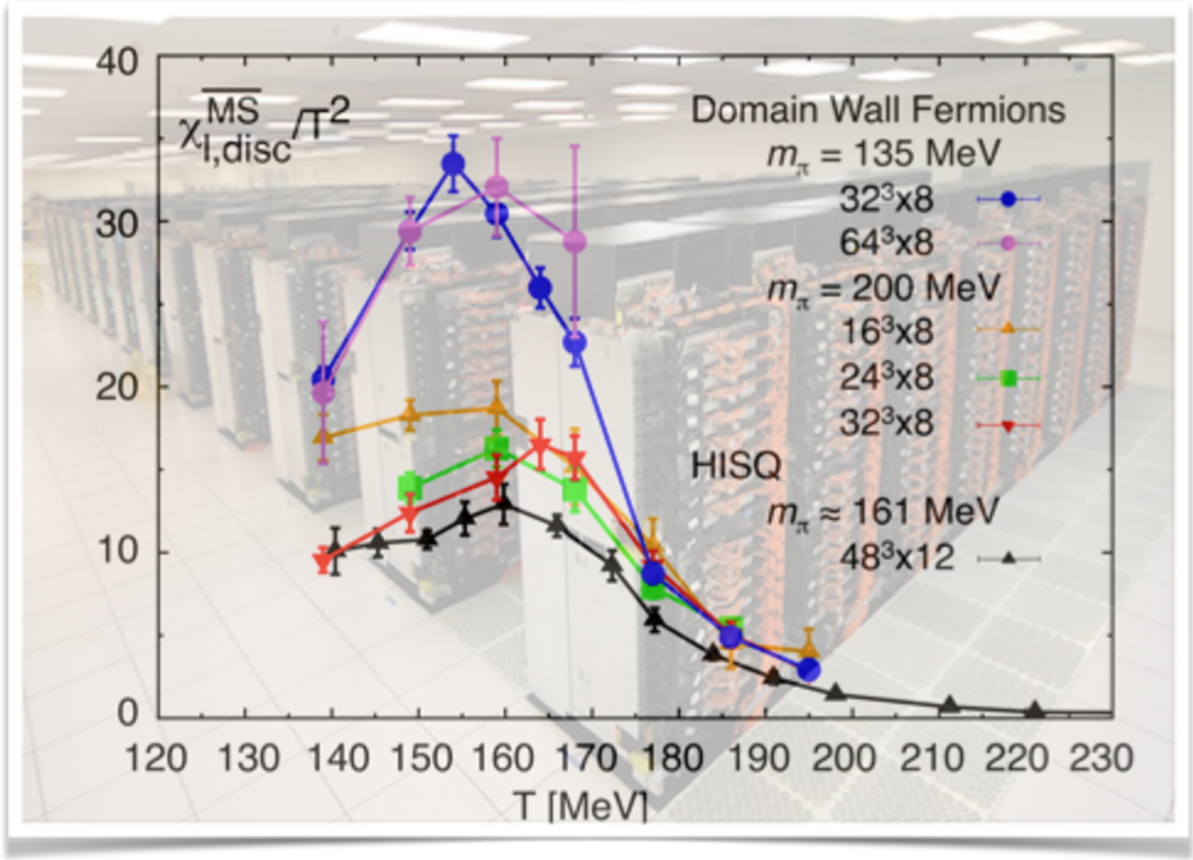
$$\mathbb{K}_2(z) \sim m_l^{1/\delta-1} f_2(z), \quad \mathbb{K}_2(z)/\mathbb{K}_2(z=0) \sim f_2(z)/f_2(0)$$



$$z/z_0 = H^{-1/\beta\delta}(T - T_c)/T_c$$

rapid chiral crossover at the physical point

chiral susceptibility K2

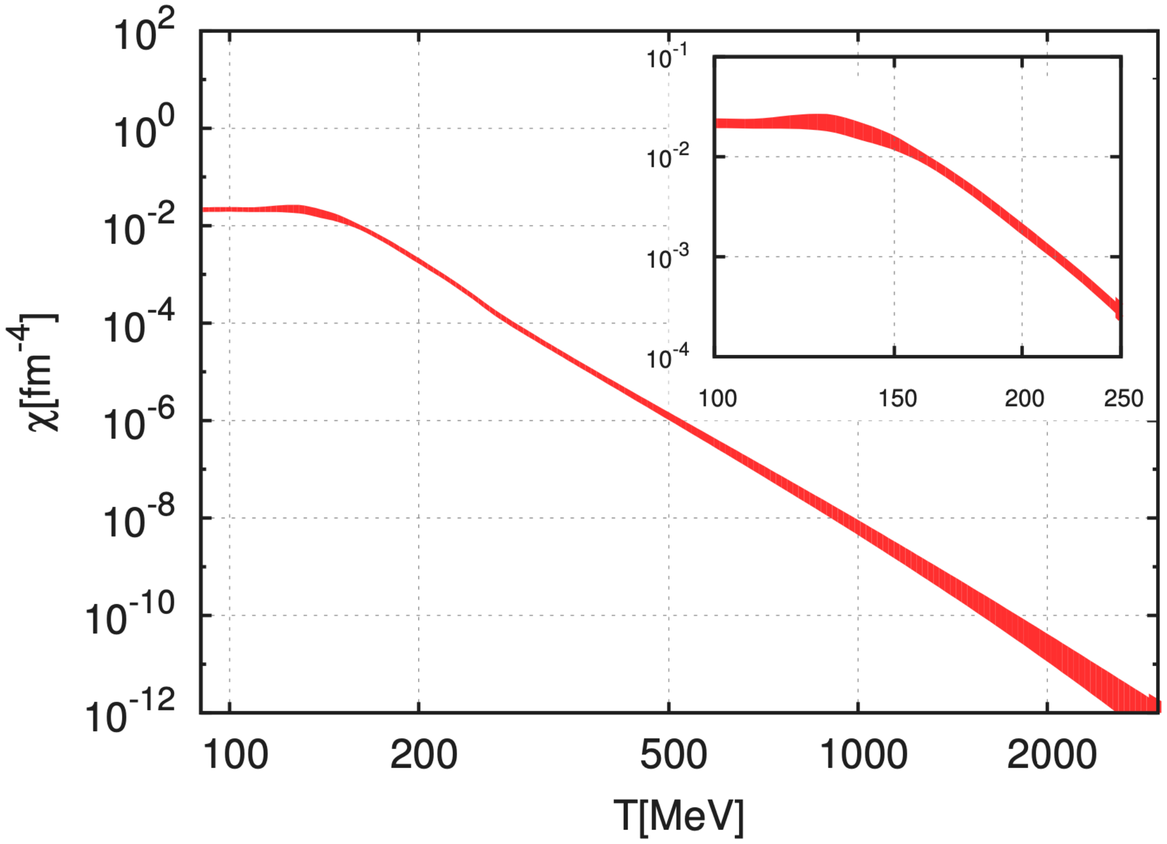


HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, PRL 131(2023)161903

HotQCD, Phys.Rev.Lett. 113 (2014) 8, 082001

How does the topology feel about the QCD transition

topological susceptibility



[Wuppertal-Budapest], arXiv:1606.07494, Nature 2016

Topological charge:

$$Q[U] = \int d^4x\, q(x), \qquad q(x) \propto F^a_{\mu\nu}\, \widetilde{F}^a_{\mu\nu}\,.$$

Topological susceptibility:

$$\chi_{\text{top}}(T) = \left.\frac{\partial^2 f}{\partial\theta^2}\right|_{\theta=0} = \frac{\langle Q^2\rangle}{V_4}$$

Topological fluctuations and quark masses

- Index theorem: $Q[U] = \text{index}(\mathscr{D})$

- Q is gluonic, but the fermion determinant controls its equilibrium weight

$$\chi_{top} = < Q^2 > = \int DU\ P[U]\ Q^2, \qquad P[U] \propto e^{-S_g[U]} \det(\mathscr{D} + m_\ell)^2 \det(\mathscr{D} + m_s)\,.$$

However, thermal suppression of χ_{top} does not determine its mass dependence

$$q_u \mapsto e^{i\alpha\gamma_5} q_u, \qquad \theta \mapsto \theta - 2\alpha$$

In the chiral limit: $\quad m_u = 0: \quad f(T,\theta) = f(T,0), \qquad \chi_{\text{top}} = \dfrac{\partial^2 f}{\partial \theta^2} = 0$

The chiral response measures a local mass slope

- Low temperature:
  chiral perturbation theory

$$\chi_{\rm top}^{\rm LO} = \frac{\Sigma}{2/m_\ell + 1/m_s}, \qquad C_{\rm top}^{\rm LO} = \frac{2m_s}{2m_s + m_\ell}$$

Grilli di Cortona et al., JHEP 01(2016) 034 ,Leutwyler & Smilga,, PRD46 (1992) 5607.

- High temperature:
  leading quadratic dependence

$$\chi_{\rm top}(T) \propto m_\ell^2 \quad (m_s \text{ fixed}), \qquad C_{\rm top} \simeq 2$$

Gross, Pisarski & Yaffe 1981, Schaffer & Shuryak 1998, Kanazawa & Yamamoto 2015



Chiral response
of QCD topology

$$C_{\rm top}(T) = \left.\frac{\partial \ln \chi_{\rm top}}{\partial \ln m_\ell}\right|_{T,V_4,m_s} \qquad \chi_{\rm top} \propto m_\ell^p \quad \Longrightarrow \quad C_{\rm top} = p$$



Mass derivative of O: $\dfrac{\partial O}{\partial x} = \lim_{\epsilon \to 0} \dfrac{O(x+\epsilon) - O(x)}{\epsilon}$

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

A single-ensemble estimator for the mass derivative

- QCD partition function: $Z = \int \mathcal{D}U\, e^{-S_g} \det(D + m_\ell)^2 \det(D + m_s)$

- Derivative of χ_{top} without additional situations on shifted quark masses

$$\frac{\partial \chi_{\text{top}}}{\partial m_\ell} = \frac{\langle Q^2 X_1 \rangle - \langle Q^2 \rangle \langle X_1 \rangle}{V_4} \qquad X_1 \equiv \partial_{m_\ell} \ln \det(D + m_\ell)^2 = 2\,\text{Tr}(D + m_\ell)^{-1}$$

- The covariance supplies the response directly

$$C_{\text{top}} = m_\ell \frac{\langle Q^2 X_1 \rangle_c}{\langle Q^2 \rangle}$$

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

Thermal scan and a continuum check

2+1-flavor HISQ, $m_\pi \approx 220$ MeV

$m_\ell = m_s/10$, physical m_s

$a \approx 0.117$ fm, spatial volume 32^3

N_τ	T [MeV]
24, 18, 16, 14	70.2, 93.6, 105.3, 120.4
12, 10, 8, 6	140.4, 168.5, 210.8, 280.9

Continuum check at T ≈ 211 MeV

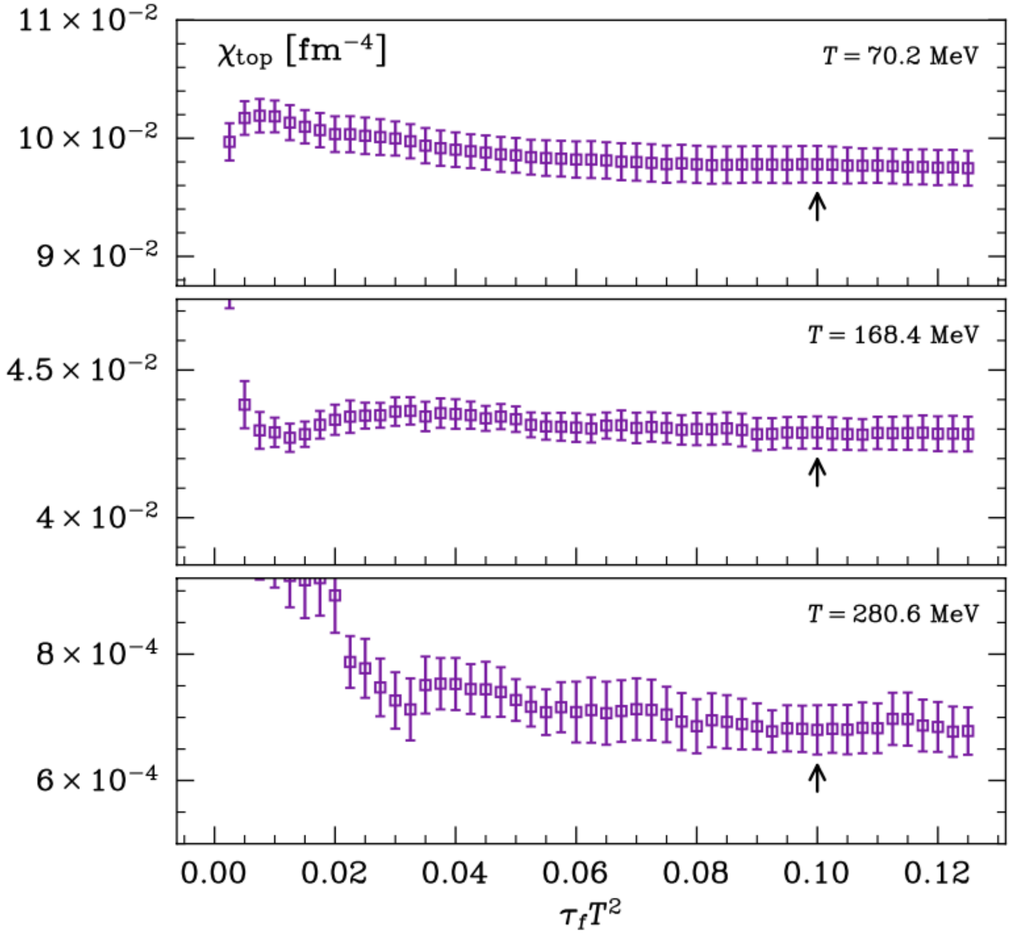
$32^3 \times 8$ $40^3 \times 10$ $48^3 \times 12$

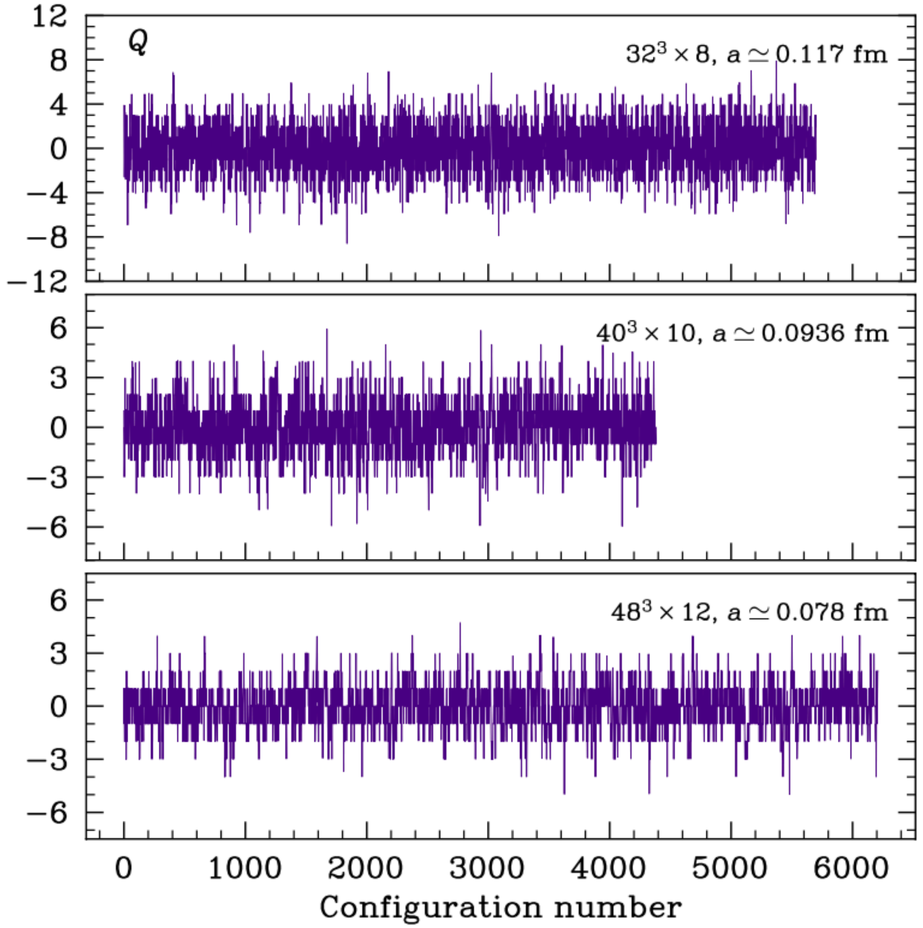
Matched masses and physical volume

Pseudocritical region: $T_{pc} \approx 170$ MeV

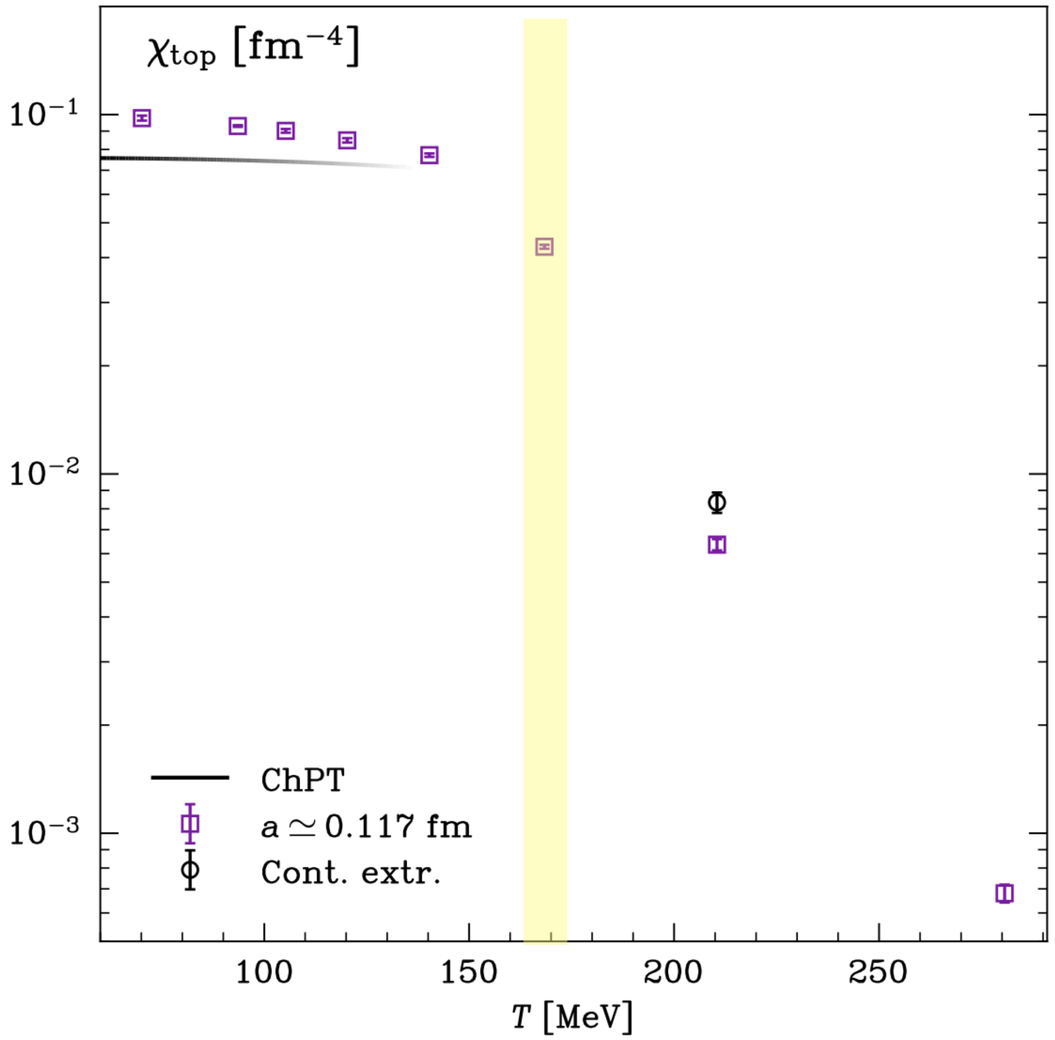
NSCC Nuclear Science Computing Center at CCNU

Flow time dependence and time history of Q





Topology is strongly suppressed by temperature



Fixed masses $m_\pi \simeq 220$ MeV

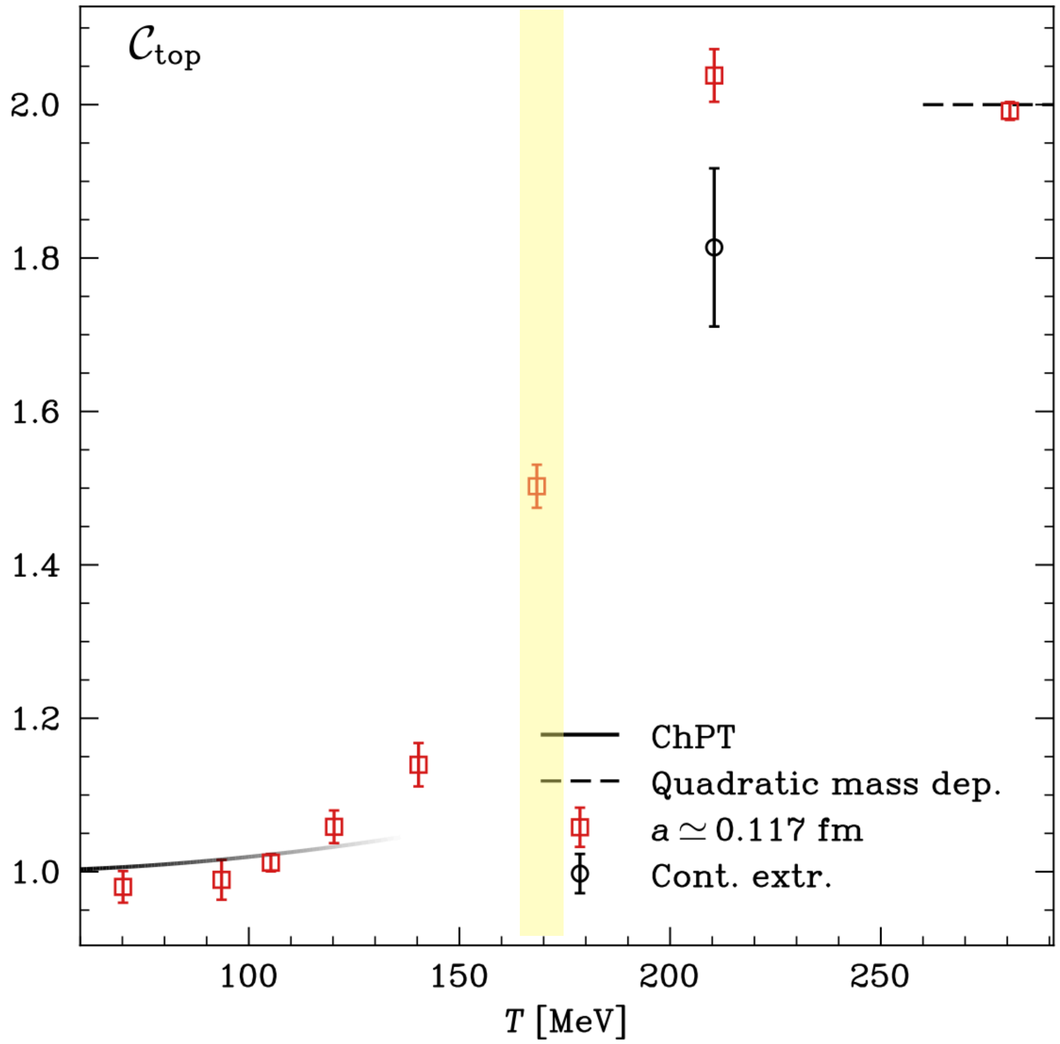
T=70–281 MeV

About two orders of magnitude of thermal suppression

What changes in the mass response?

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

The response rises through the crossover



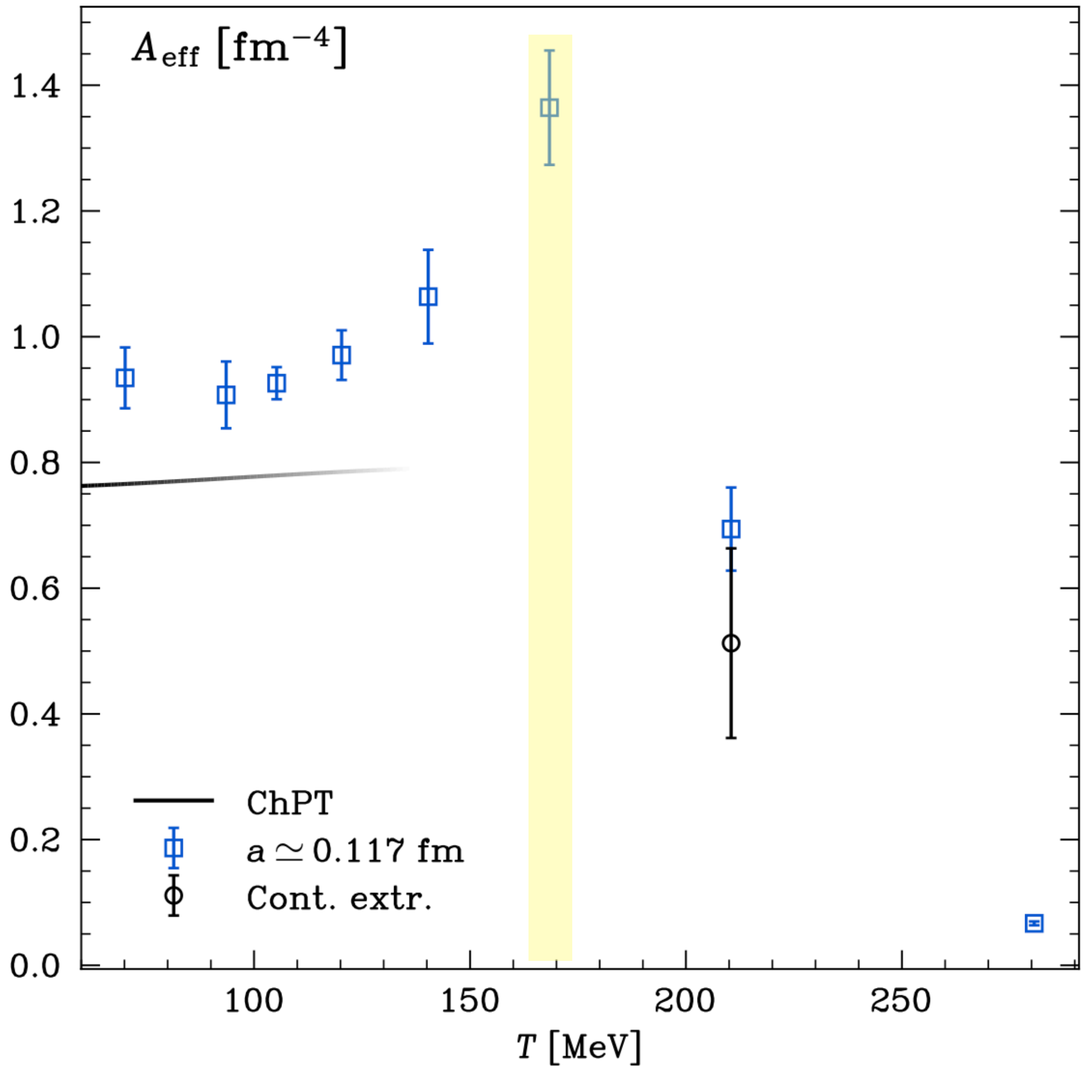
Low T: $C_{\text{top}} \approx 1$

Near $T_{pc} \simeq 170$ MeV: $C_{\text{top}} \approx 1.5$

Above T_{pc}: C_{top} approaches 2

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

The local amplitude peaks near the crossover



$$H \equiv m_\ell / m_s, \qquad A_{\rm eff}(T) = \chi_{\rm top}(T)\, H^{-C_{\rm top}(T)}$$

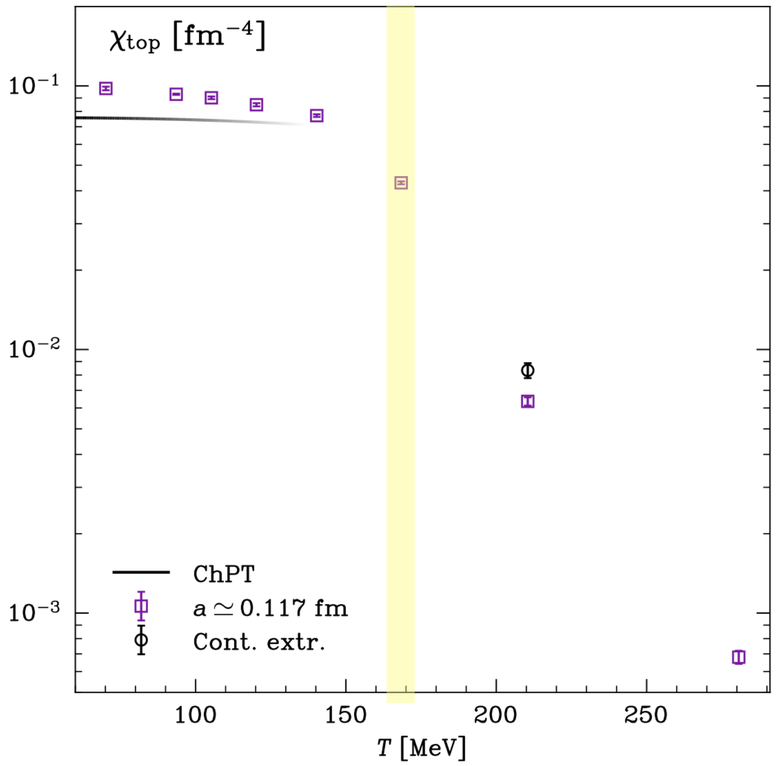
Largest sampled value near $T_{pc} \simeq 170$ MeV

Competition between decreasing $\chi_{\rm top}$ and increasing local exponent

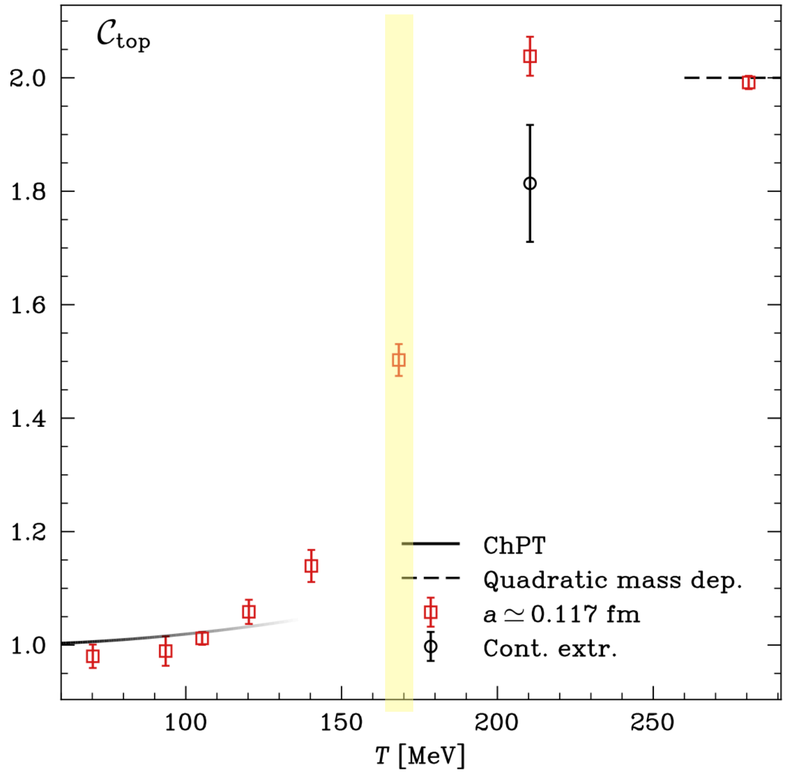
HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

The crossover appears in the mass response of topology

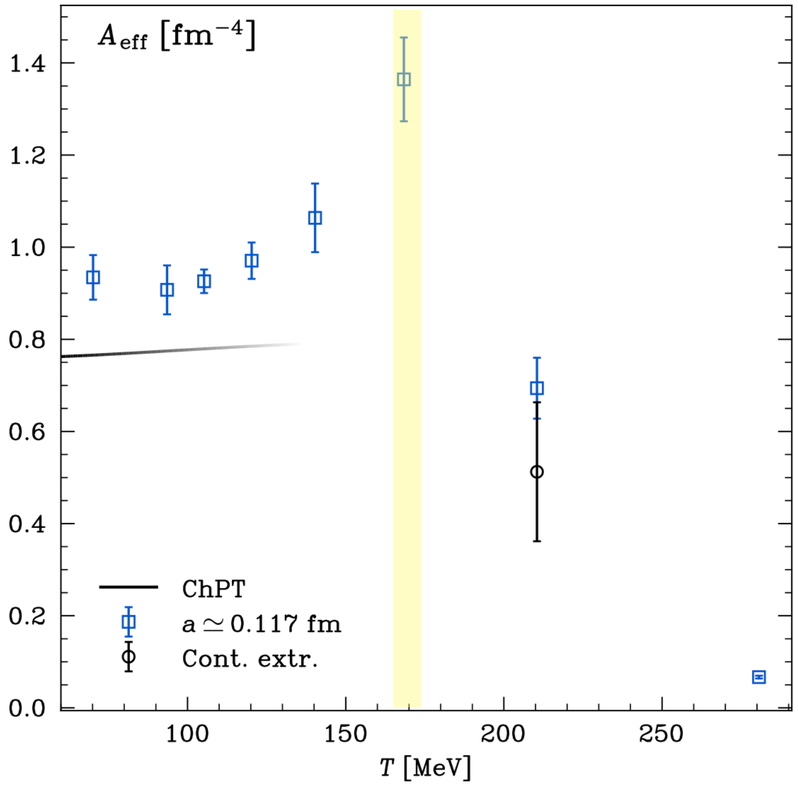
χ_{top}: suppression



C_{top}: increasing response



A_{eff}: a crossover peak

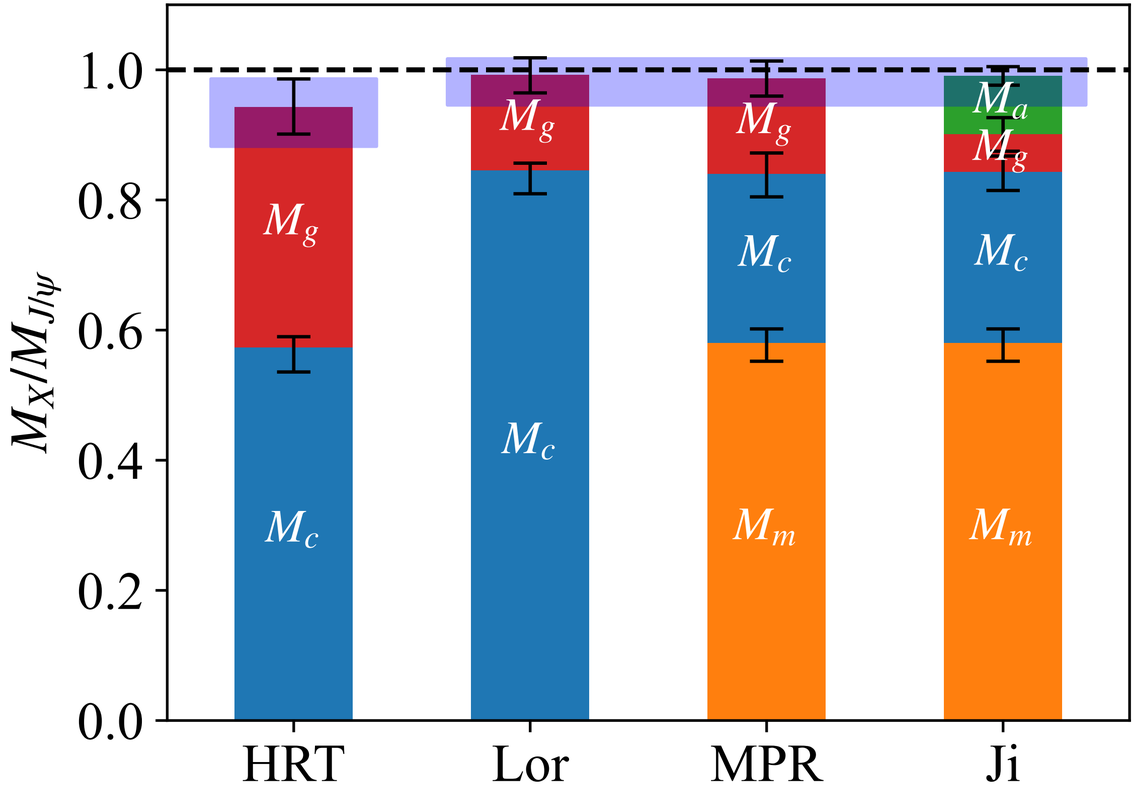


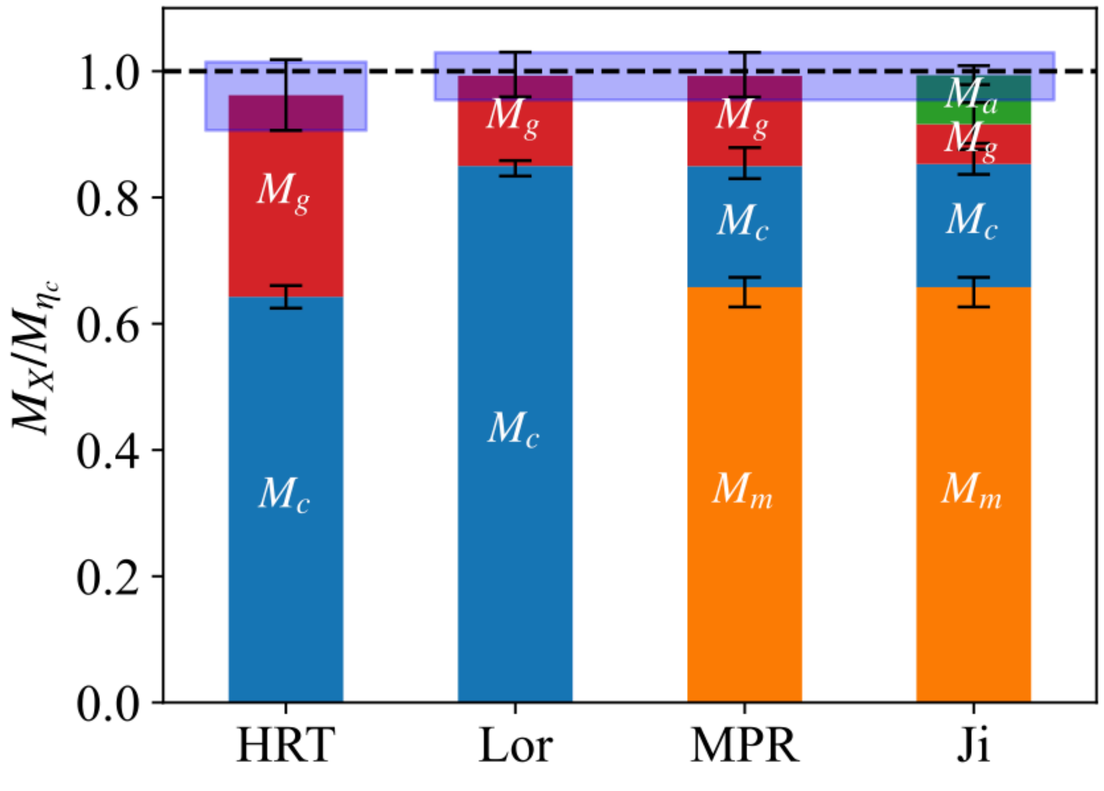
$T_{pc} \approx 170$ MeV for this mass and lattice setup

HTD, 顾锦彪, Hernández Hernández, 李胜泰 & 倪佳, arXiv:2611.xxxx

Summary

- Constructed the full QCD EMT, including the trace sector, in a common $\overline{\text{MS}}$ framework
- Provides a general first-principles framework for studying hadron mass, spin, and GFFs

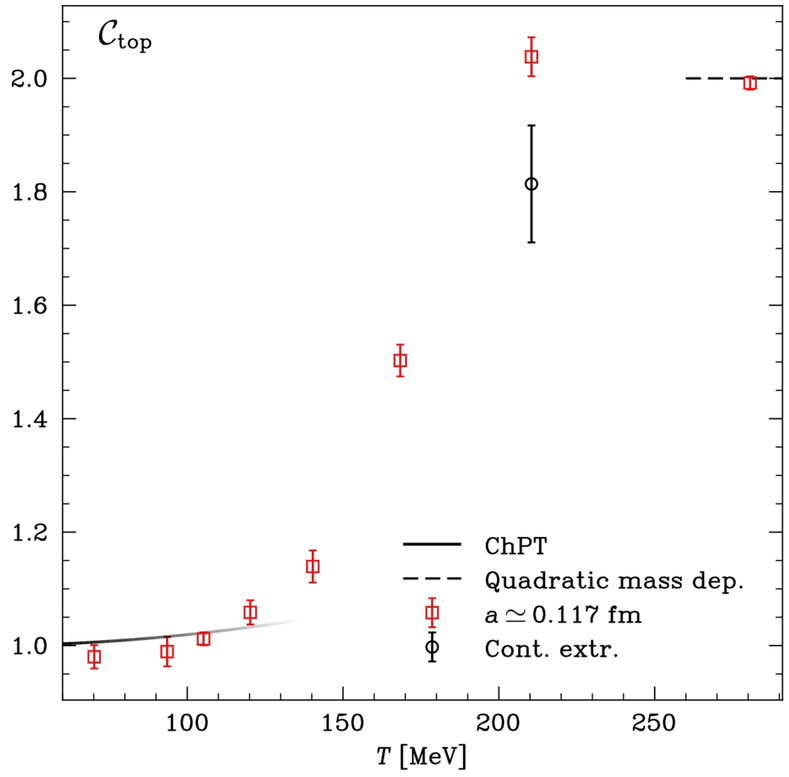


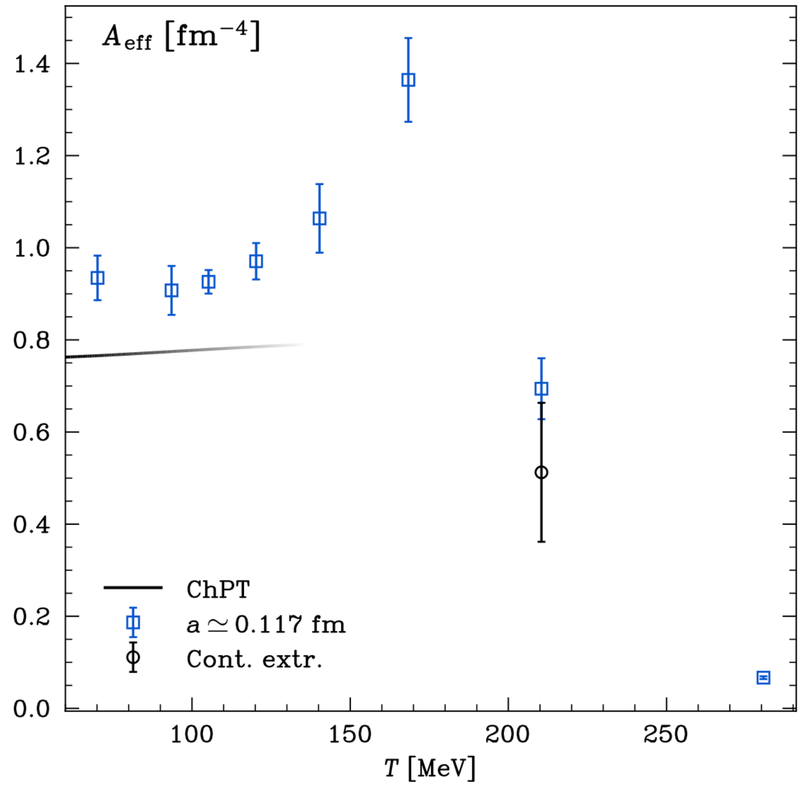


Same EMT input, different organizations - one consistent first-principles picture

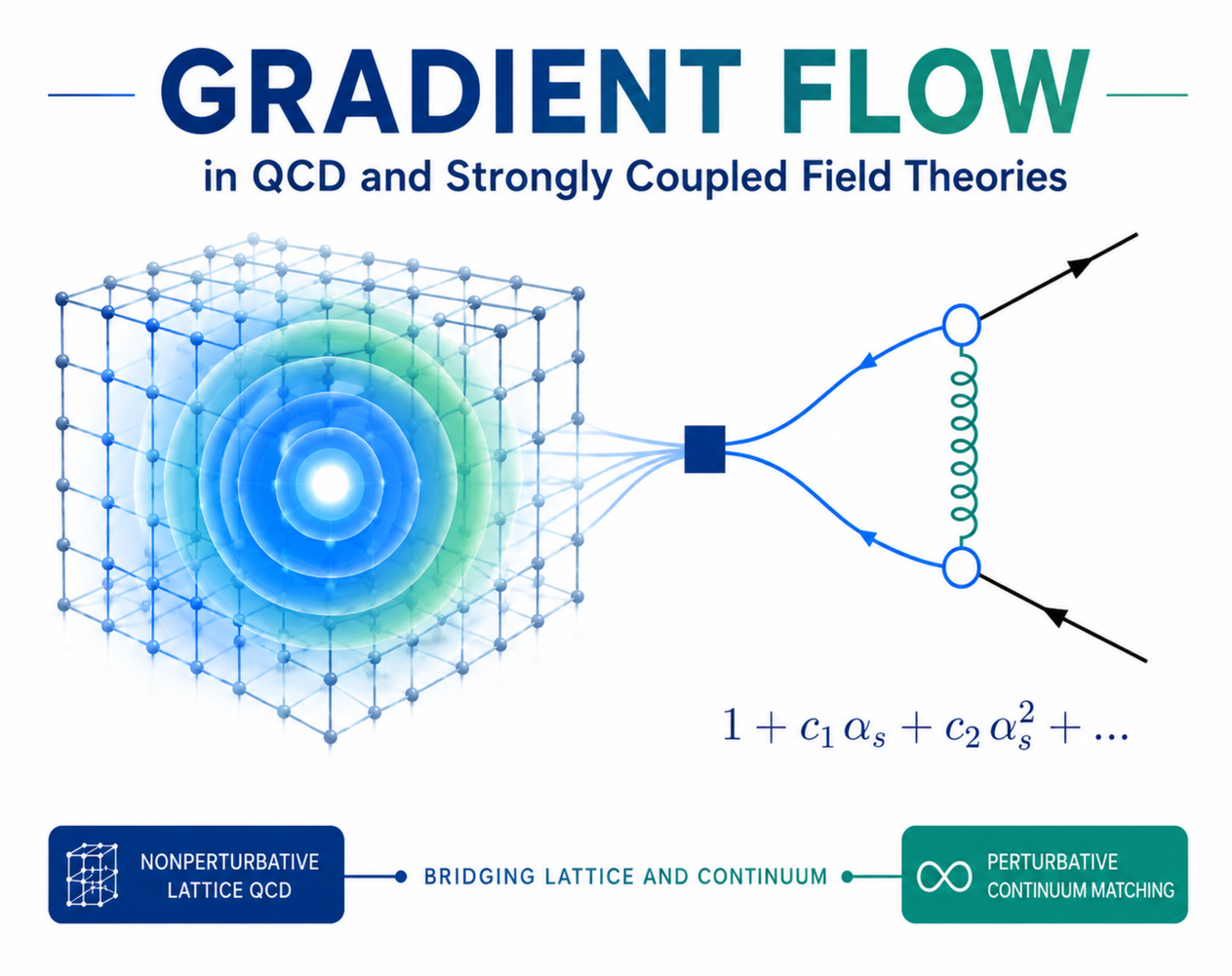
Summary

- A novel methodology to compute $\partial\chi_{top}/\partial m_\ell$ directly without the simulations at shifted masses
- Chiral response increases through chiral crossover while effective amplitude peaks near crossover





Methodology can be applied to other discretization and other physics problems



梯度流应用 国际研讨会

2027年3月2-5日,华中核理论中心, 武汉

Central China
Center for Nuclear Theory
华 中 核 理 论 中 心

https://indico.ihep.ac.cn/event/30090/

Local organizers:

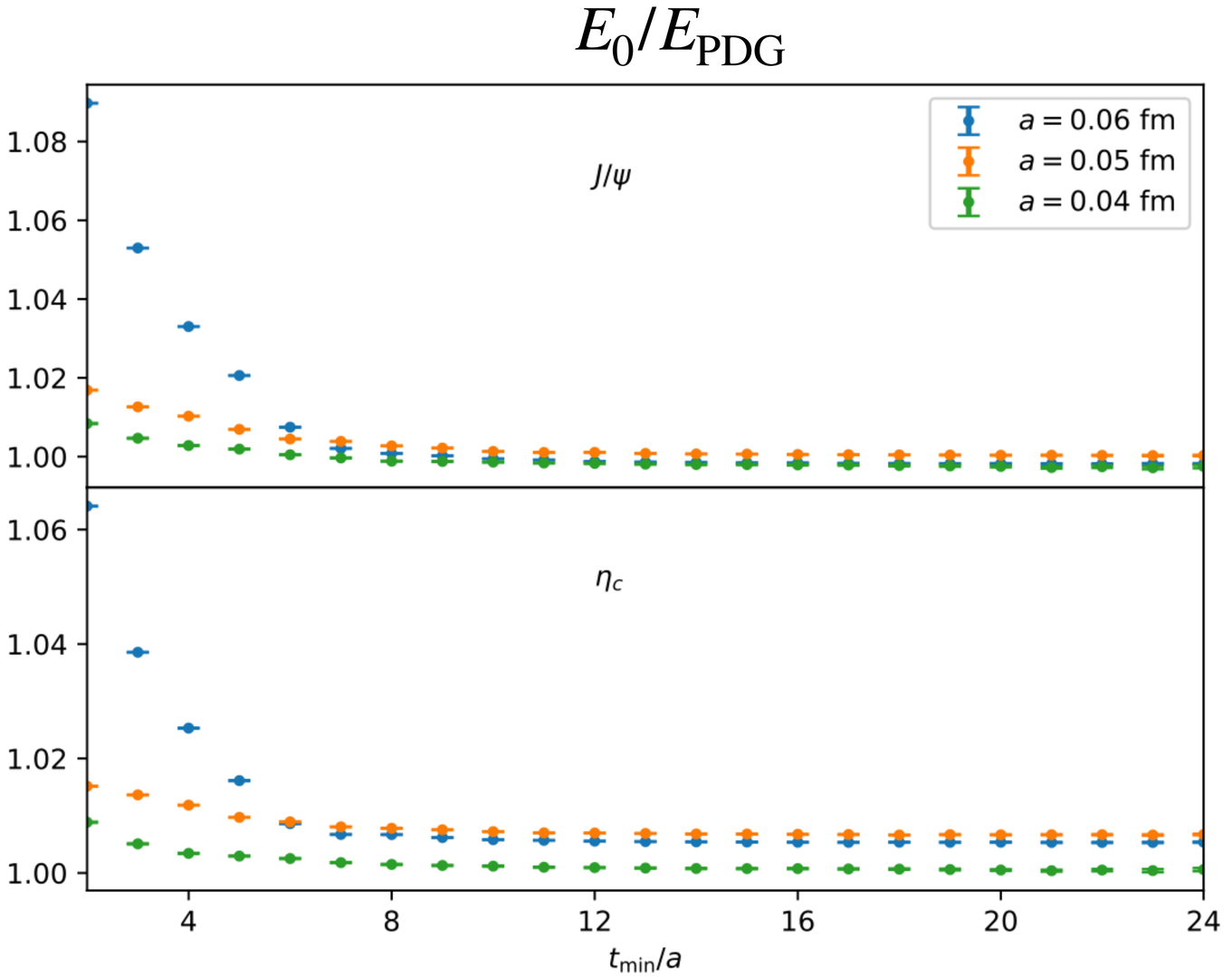
王翔鹏 (Central China Normal University)

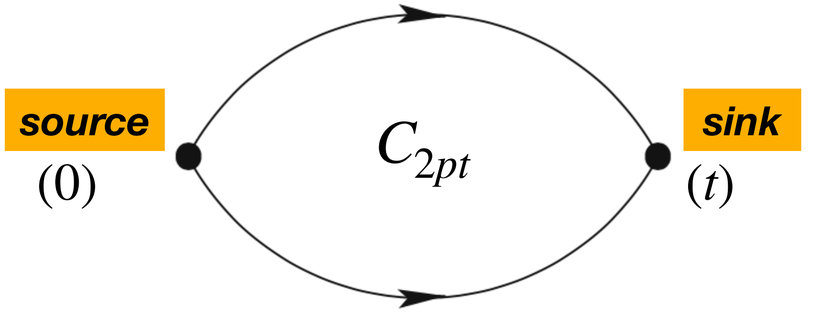
舒海涛(Central China Normal University)

丁亨通 (Central China Normal University)

Backup

Construction of quarkonia states via 2-pt correlation function





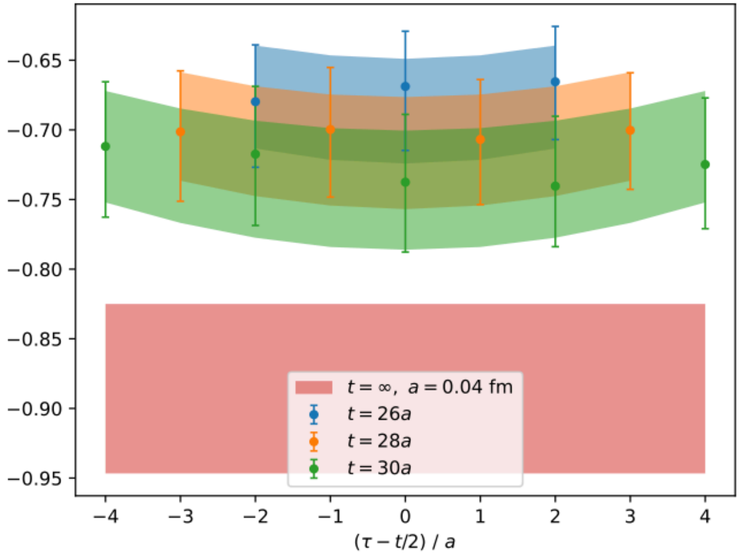
$$C_{\Gamma}^{2\mathrm{pt}}(t) \equiv \sum_{\boldsymbol{x}} \langle \bar{c}(x)\Gamma c(x)\bar{c}(x_0)\Gamma c(x_0)\rangle$$

$$C^{2\mathrm{pt}}(t) = \sum_{n=0}^{n_{\mathrm{max}}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$$

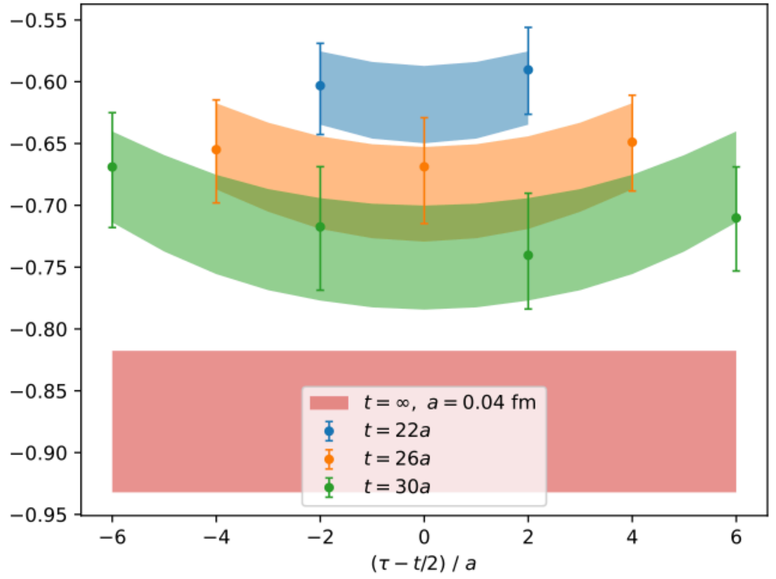
- Ratios are remarkably close to unity
- Bare charm-quark mass is tuned to reproduce the physical charmonium states

Demonstration: Extraction of gluon matrix elements $< \eta_c | \tilde{\mathcal{O}}_{1,kk} | \eta_c >$ at a= 0.04 fm & $t_f = 5\epsilon_f$

2-state fits: $$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0 \langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E\tau} + e^{-\Delta E(t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$



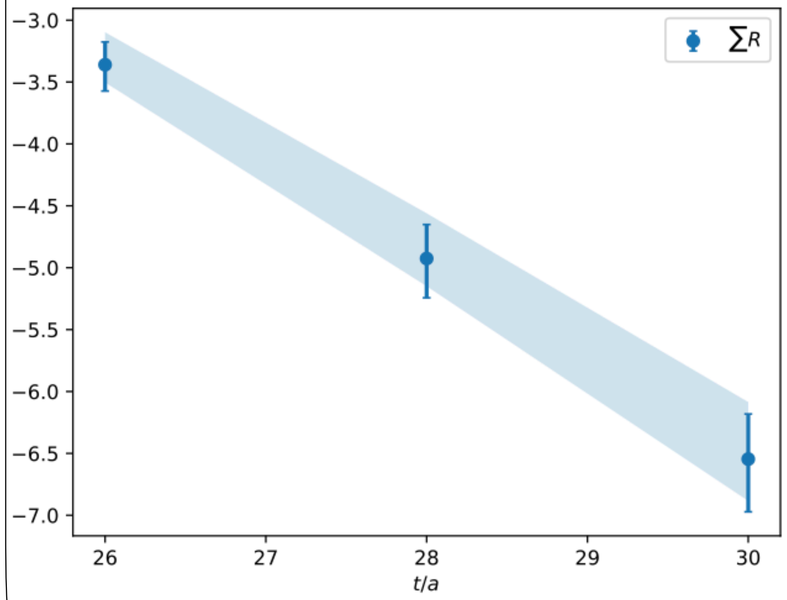
2-stride fit ($N_{\mathrm{stride}} = 2$)



4-stride fit ($N_{\mathrm{stride}} = 4$)

$\tau = N_{\mathrm{skip}}, N_{\mathrm{skip}} + N_{\mathrm{stride}}, \cdots, \tau \in [N_{\mathrm{skip}}, t - N_{\mathrm{skip}}]$

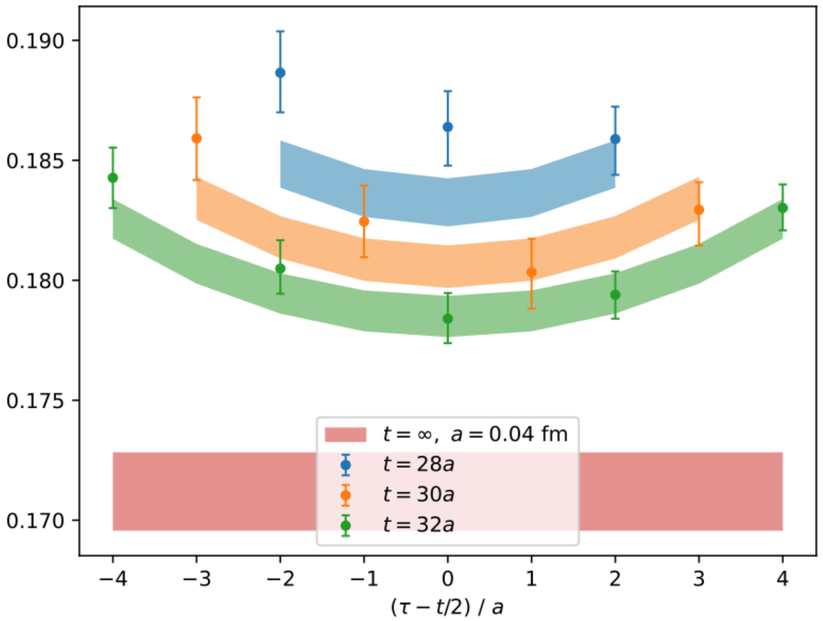
$$\sum_{\tau=aN_{\mathrm{skip}}}^{t-aN_{\mathrm{skip}}} R^{\rho\sigma}_{X,\Gamma}(t,\tau) \underset{t\to\infty}{\longrightarrow} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle}{2M} t + B\,.$$



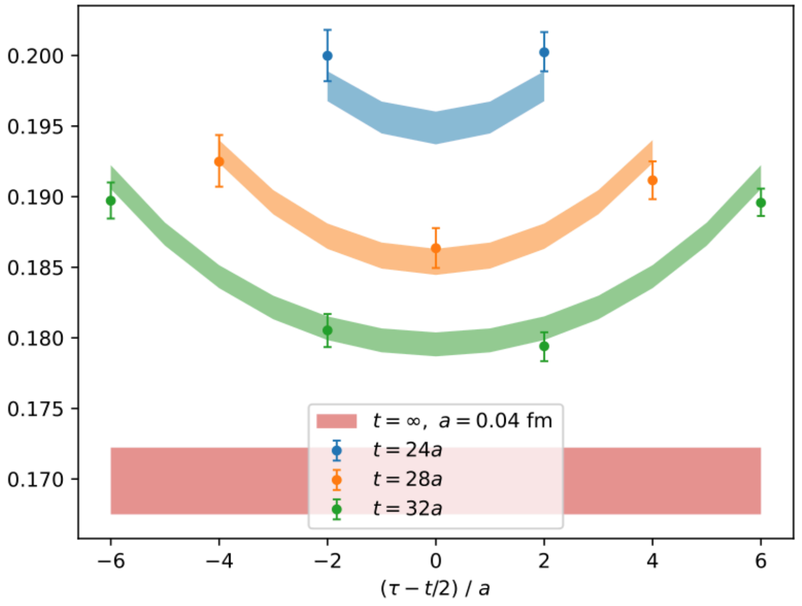
Summation fit

Demonstration: Extraction of quark matrix elements $< \eta_c | \tilde{\mathcal{O}}_{3,kk} | \eta_c >$ at a= 0.04 fm & $t_f = 5\epsilon_f$

2-state fits: $$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0 \langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2} \left[e^{-\Delta E \tau} + e^{-\Delta E (t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2) e^{-\Delta E t}}$$



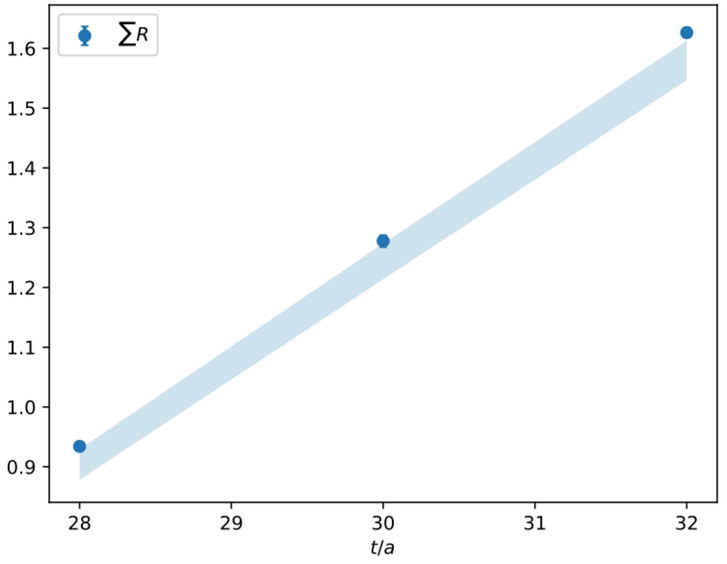
2-stride fit ($N_{\text{stride}} = 2$)



4-stride fit ($N_{\text{stride}} = 4$)

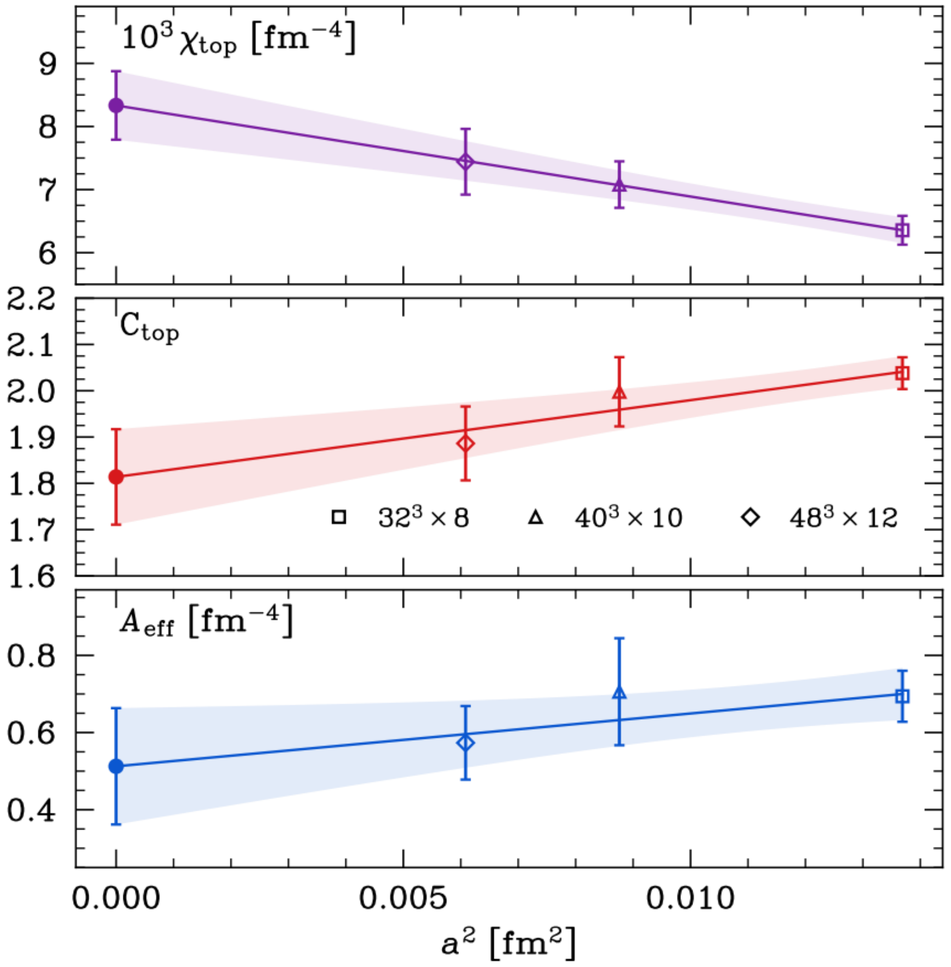
$$\tau = N_{\text{skip}}, N_{\text{skip}} + N_{\text{stride}}, \cdots, \tau \in [N_{\text{skip}}, t - N_{\text{skip}}]$$

$$\sum_{\tau=aN_{\text{skip}}}^{t-aN_{\text{skip}}} R^{\rho\sigma}_{X,\Gamma}(t,\tau) \underset{t\to\infty}{\longrightarrow} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle}{2M} t + B\,.$$



Summation fit

Continuum check at T ≈ 211 MeV



Matched masses and volume

Nτ = 8, 10, 12

Ctop remains above the low-T benchmark

Full continuum scan and lighter masses remain to be studied