Coupled-Channel Effects on Near-Threshold Exotic States

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Based on Phys.Rev.Lett. 128,112001(2022); JHEP01(2023)058; arXiv:2510.01564; arXiv:2610.29392

In collaboration with Guang-Juan Wang, Jia-Jun Wu, Makoto Oka, Shi-Lin Zhu

2026 非微扰方法及其在高能物理中的应用

2026. 11. 7, 中国科学技术大学，合肥

Contents

- Near $DK(D^*K)$ thresholds states: $D_{s1}(2460/2536)$

  Phys.Rev.Lett. 128,112001(2022); JHEP01(2023)058

- From mass spectra to decay properties

  arXiv:2510.01564

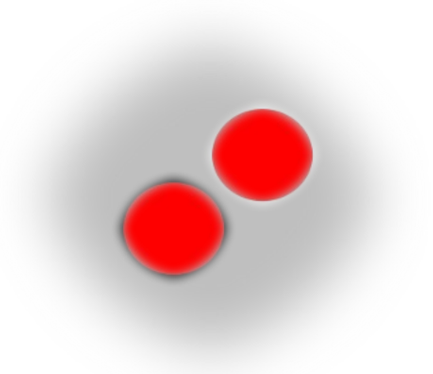
- Near $DD^*(D\bar{D}^*)$ thresholds states: T_{cc} and $X(3872)$

  Sci.Bull. 69,3036(2024)

- From mass spectra to production properties

  arXiv:2610.29392

Hadrons



Meson



Baryon

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest



8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING II *)

G. Zweig
CERN---Geneva

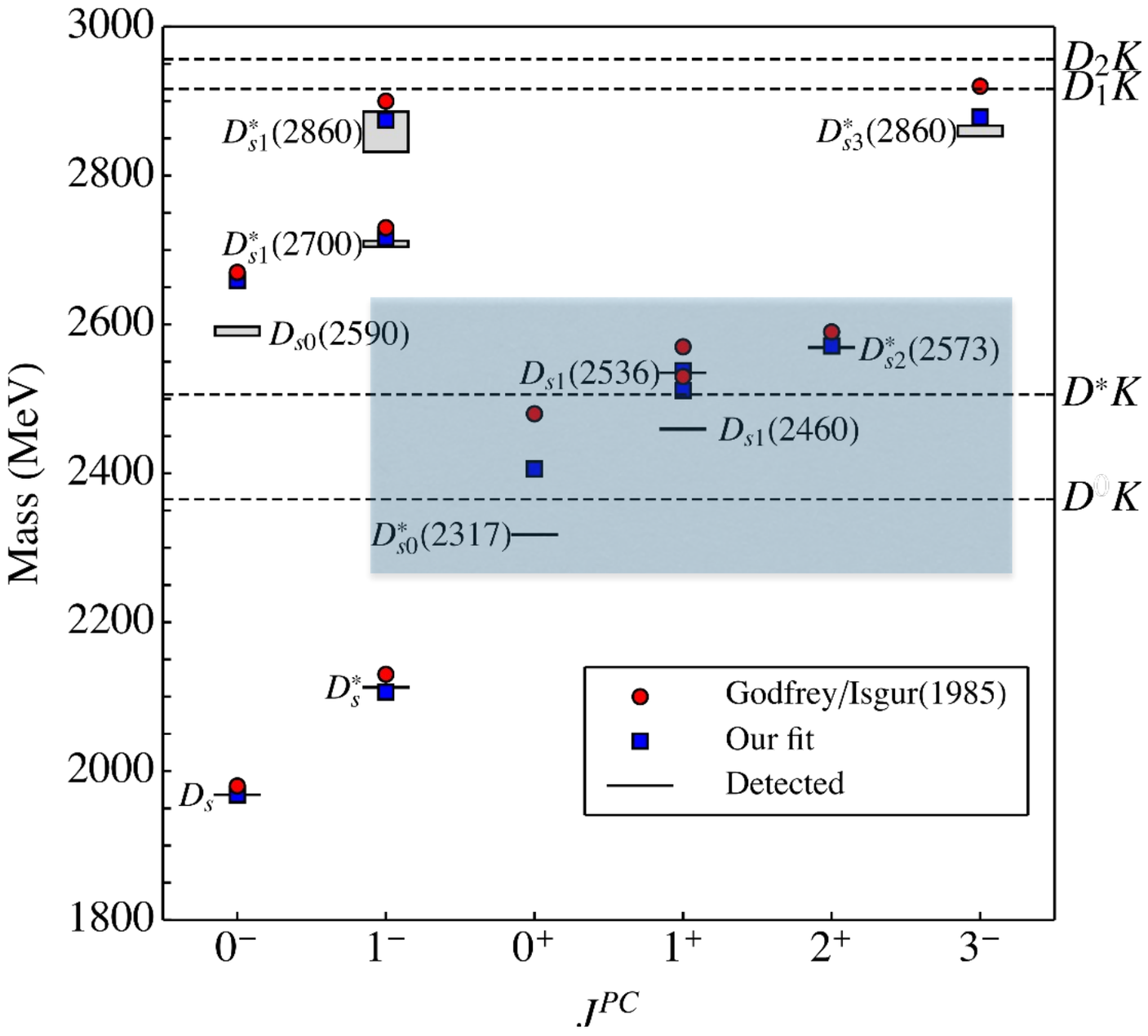
*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".



Four near-threshold D_s states in quark model



- Fit the updated masses of the low-lying states away from thresholds
- Our fit is more consistent with observation.

Four near-threshold D_s states in quark model

Transformation from physical bases to the heavy quark limit bases.

	$c\bar{s}$ cores			channel
	$B(\lvert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D^*_{s0}(2317)$	$\lvert^3P_0\rangle$	2405.9	DK	S
$D^*_{s1}(2460)$	$0.68\lvert^1P_1\rangle - 0.74\lvert^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D^*_{s1}(2536)$	$-0.74\lvert^1P_1\rangle - 0.68\lvert^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D^*_{s2}(2573)$	$\lvert^3P_2\rangle$	2571.2	DK, D^*K	D

GI results

$J^P = 1^+$	$B(\lvert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D^*_{s1}(2460)$	$-0.97\lvert^1P_1\rangle + 0.24\lvert^3P_1\rangle$ $= 0.76\phi_s - 0.65\phi_d$	2549.7	D^*K	S, D
$D^*_{s1}(2536)$	$-0.24\lvert^1P_1\rangle - 0.97\lvert^3P_1\rangle$ $= -0.65\phi_s - 0.76\phi_d$	2559.46	D^*K	S, D

$$\phi_s = |\tfrac{1}{2}_l \otimes \tfrac{1}{2}_h\rangle$$
$$\phi_d = |\tfrac{3}{2}_l \otimes \tfrac{1}{2}_h\rangle$$

- Ds(2317) and Ds(2460) are much heavier than predicted.
- The bare 1^+ states are almost purely given by the states with heavy-quark spin bases.

Coupled-channel framework

The Hamiltonian reads

$$H = H_0 + H_I,$$

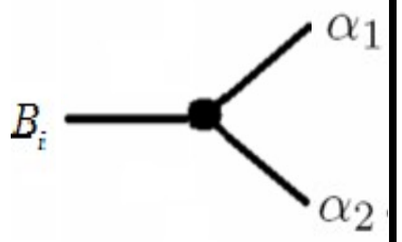
where the non-interacting one is

$$H_0 = \sum_B |B\rangle m_B \langle B| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

And the interacting one includes two parts

$$H_I = g + v$$

bare state core -> channel：



$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

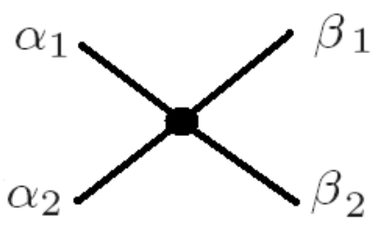
Quark pair creation model (QPC):

$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

truncate the hard vertices given by usual QPC

P. G. Ortega, et al, **Phys. Rev. D 94, 074037 (2016)**

channel -> channel：



$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Effective Lagrangian: (exchanging ρ/ω)

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{PPV} + \mathcal{L}_{VVV} \\ &= ig_v \mathrm{Tr}(\partial^\mu P[P, V_\mu]) + ig_v \mathrm{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu])\end{aligned}$$

Form factor: $\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2 \left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$

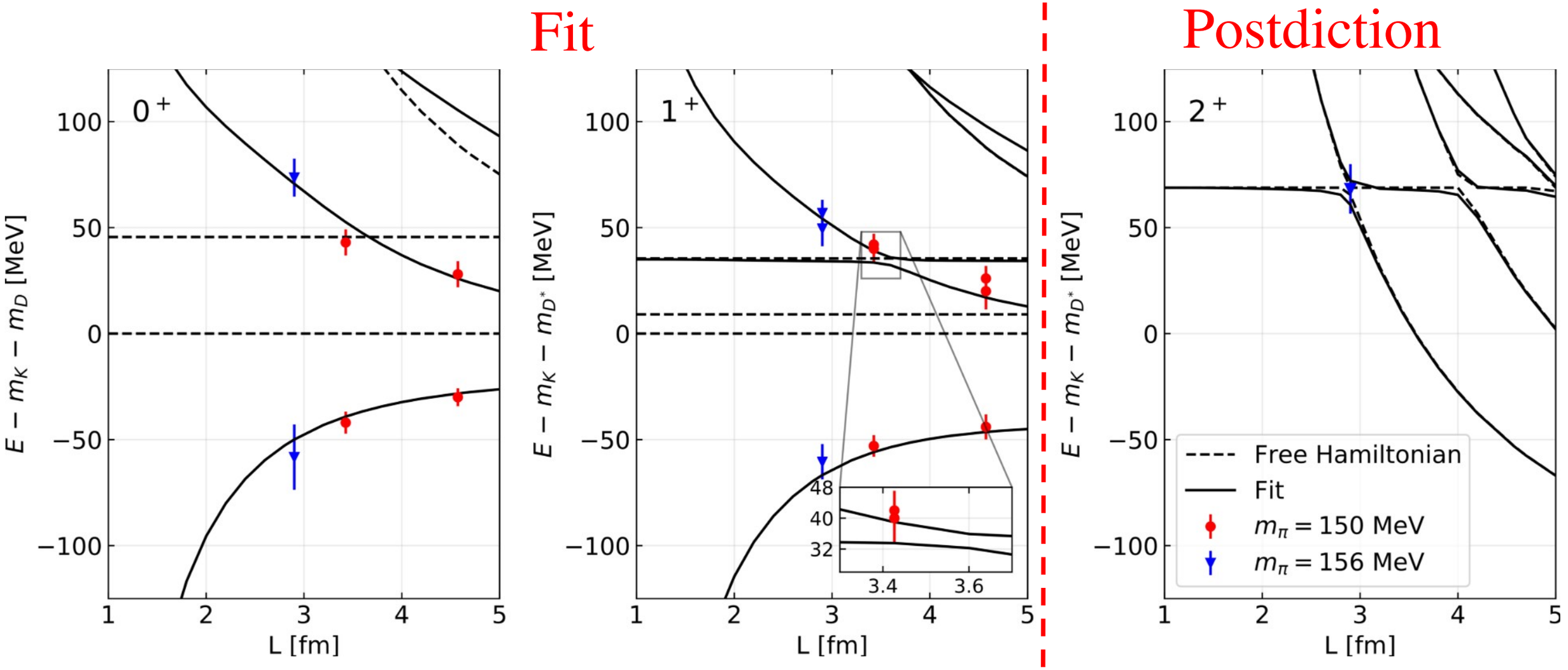
Fit the lattice data: $D_s(2317,2460,2536)$

- With fixed $\Lambda = 1.0$ GeV, $\chi^2/\mathrm{dof} = 0.95$

$$g_c = 4.2^{+2.2}_{-3.1}\,, \Lambda' = 0.323^{+0.033}_{-0.031}\ \mathrm{GeV},$$

$$\gamma = 10.3^{+1.1}_{-1.0}$$

Lattice data from: C. B. Lang et al., **Phys. Rev. D 90, 034510 (2014);**
G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**



Component and pole mass

state | L=4.57 fm | Pole mass at $L \to \infty$

G. S. Bali et al., **Phys. Rev. D 96, 074501 (2017)**

	$P(c\bar{s})[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

Lattice
2348(4)(+6)

$D_{s0}(2317), D_{s1}(2460)$

- Bare $c\bar{s}$ has strong coupling to S-wave $D^{(*)}K$ channels, and significant mass shift.
- Both the bare $c\bar{s}$ core and molecular components are significant and essential.

$D_{s1}(2536), D_{s2}(2573)$

- Coupling to D-wave $D^{(*)}K$ channels can be neglected.
- Mainly pure $c\bar{s}$.

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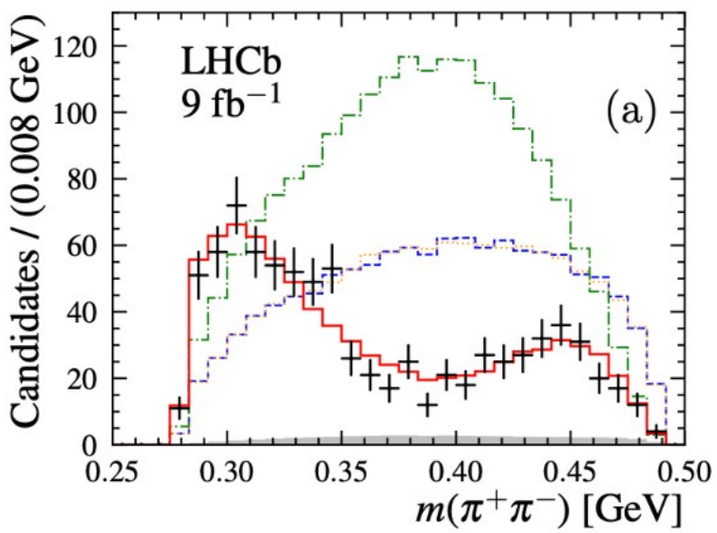
Exotic structure in $B \to D^{(*)} D_s^+ \pi^+ \pi^-$ decays

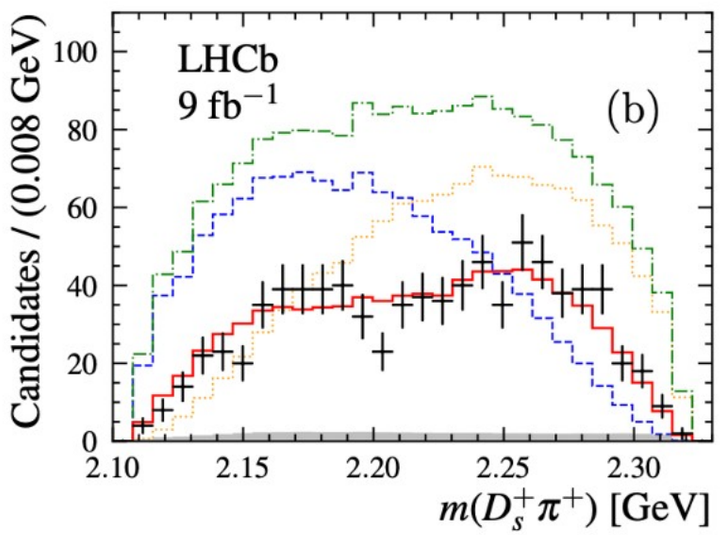
$$B \to D^{(*)} D_{s1}(2460)$$

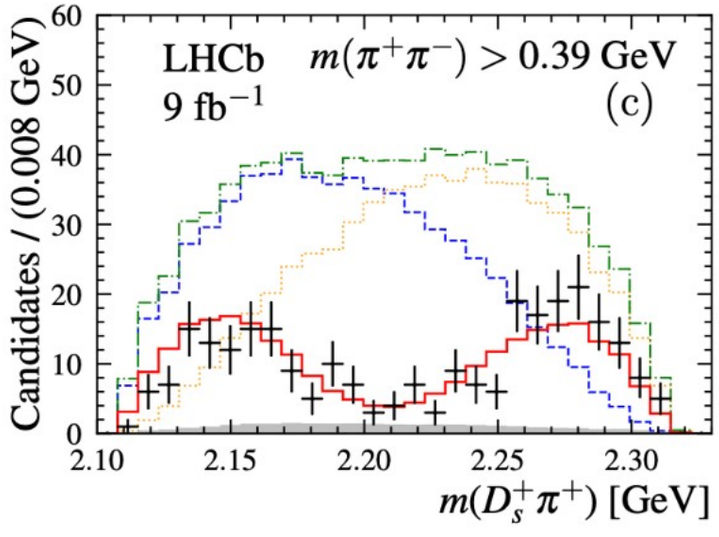
$$\downarrow$$

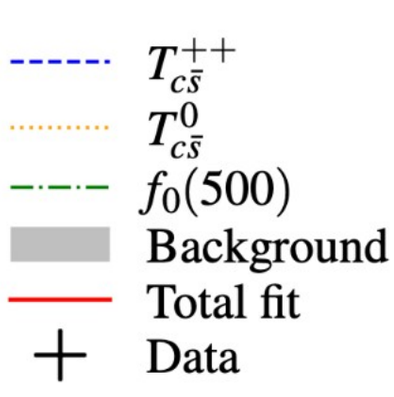
$$D_s^+ \pi^+ \pi^-$$

LHCb, arXiv:2411.03399







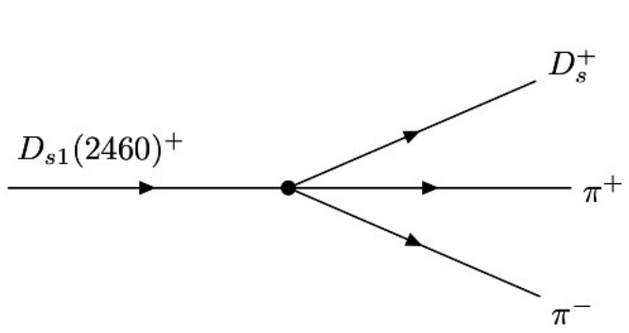


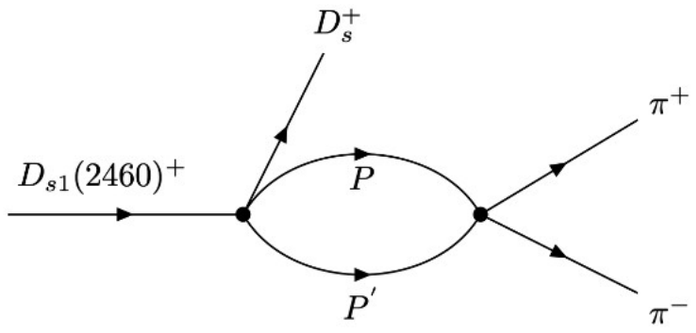
Model	Resonance	Mass (MeV)	Width (MeV)
$f_0(500) + \text{RBW } T_{c\bar{s}}(0^+)$	$f_0(500)$	$464 \pm 23 \pm 14$	$214 \pm 28 \pm 8$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2312 \pm 27 \pm 11$	$264 \pm 46 \pm 21$
$f_0(500) + \text{K-matrix } T_{c\bar{s}}(0^+)$	$f_0(500)$	$472 \pm 32 \pm 19$	$226 \pm 24 \pm 18$
	$T_{c\bar{s}}^{++}/T_{c\bar{s}}^{0}$	$2328 \pm 12 \pm 12$	$96 \pm 16 \pm 23$

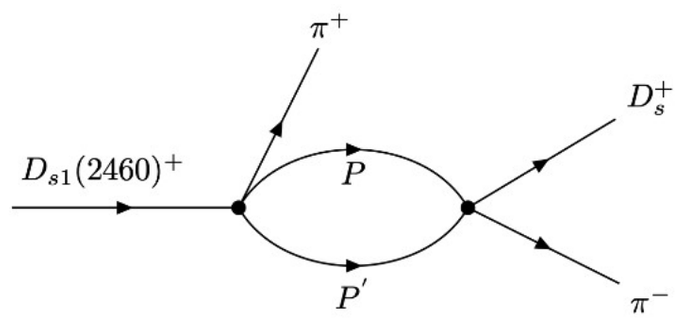
Theoretical interpretation for $T_{c\bar{s}}$

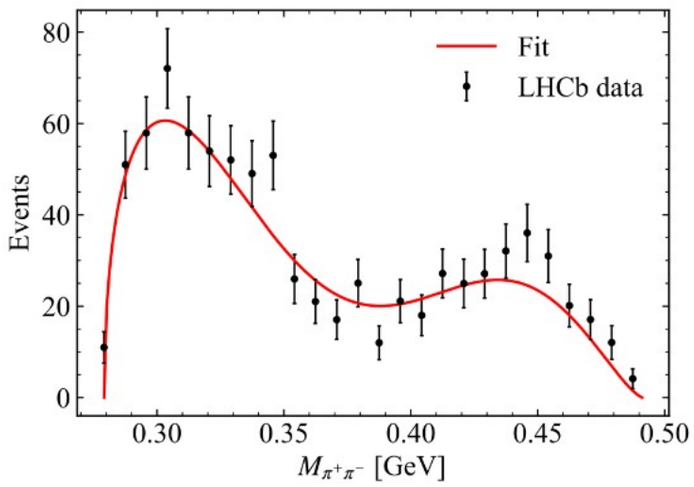
- $D_s\pi - DK$ coupled channel scattering: $T_{c\bar{s}}$ resonances is dynamically generated from the pseudoscalar-pseudoscalar meson interaction within the chiral unitary approach.

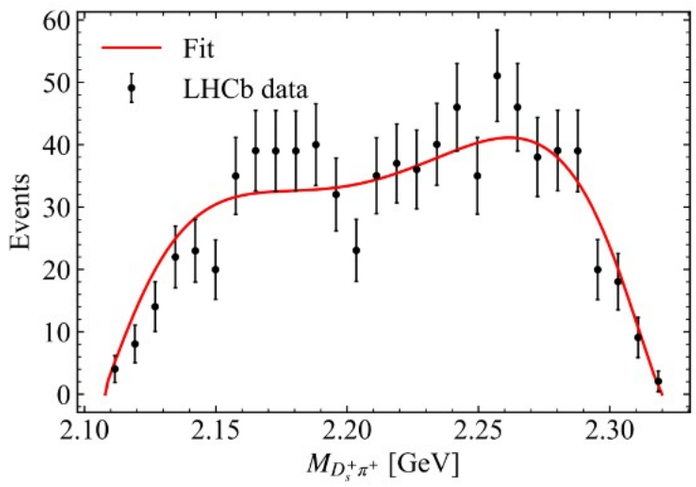
ZY Wang, YS Li and SQ Luo, Phys.Rev.D 111 (2025) 7, 076009

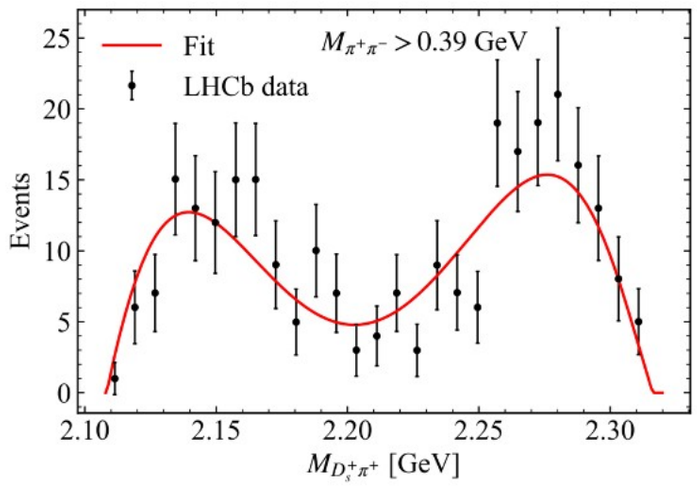










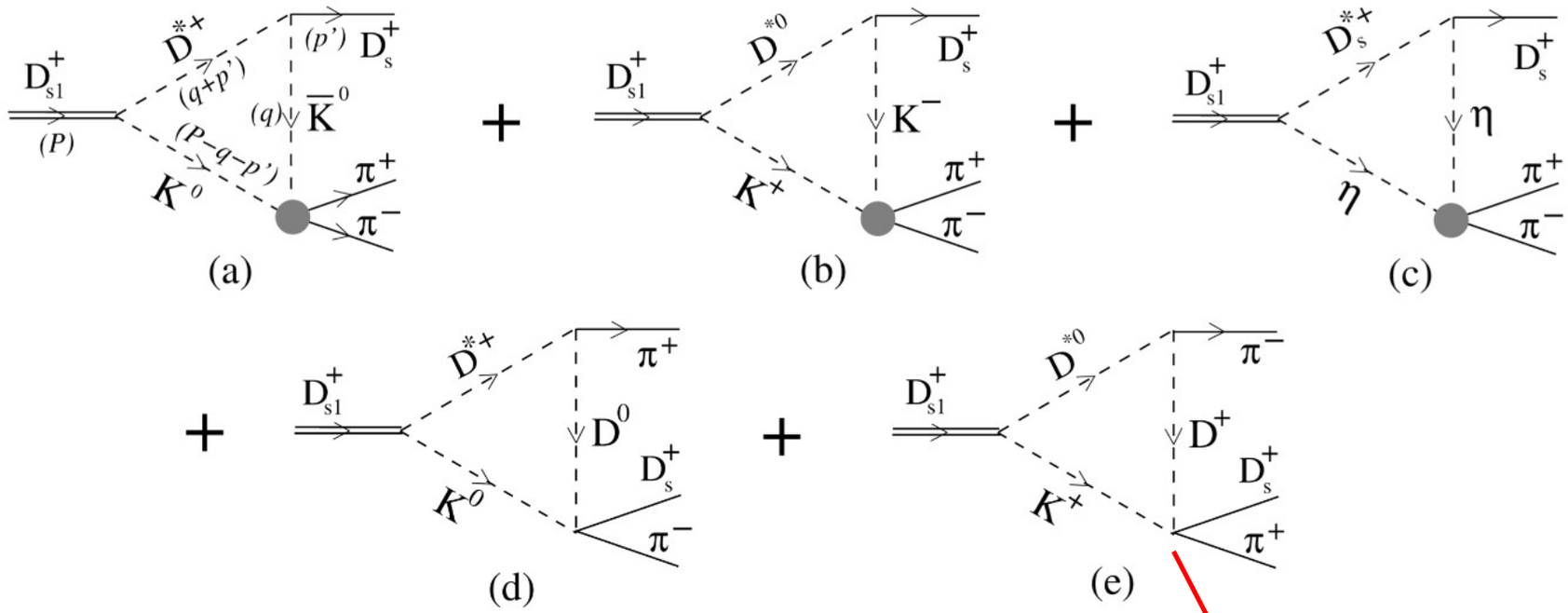


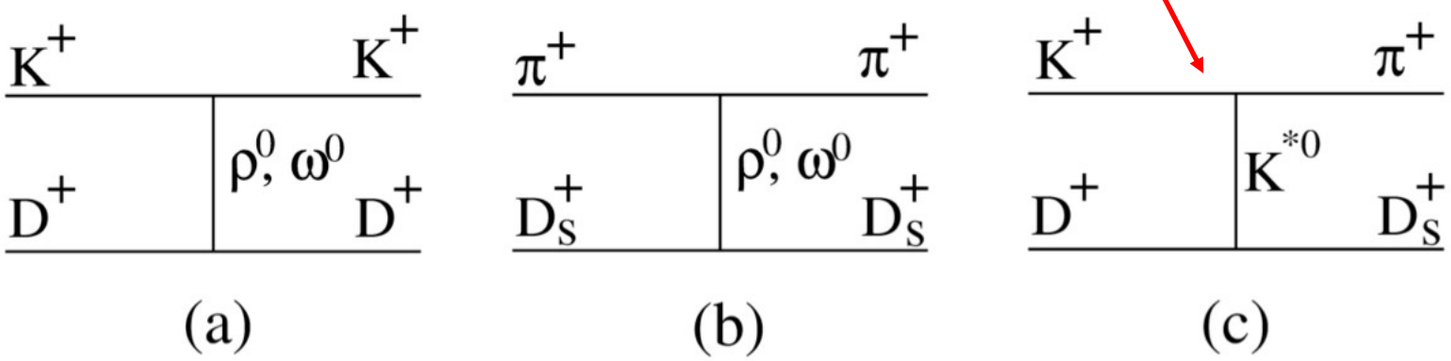
- A pole at $\sqrt{s_p} = (2.394 - 0.057i)$GeV on the second Riemann sheet;
  a pole at $\sqrt{s_p} = (2.246 - 0.093i)$GeV on the third Riemann sheet.

Theoretical interpretation for $T_{c\bar{s}}$

- Molecular $D_{s1}(2460)[D^*K, D_s^*\eta]$ state decay

Roca, Dias and Oset, arXiv:2502.18401



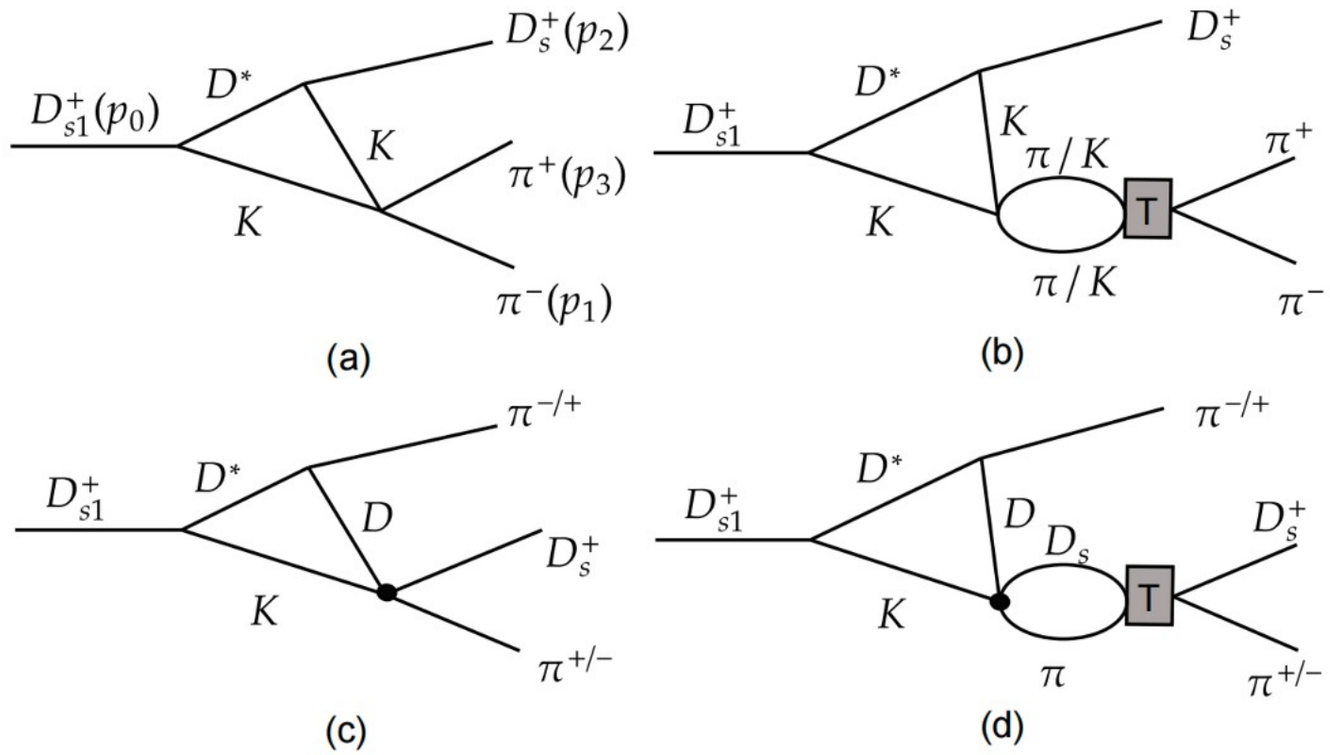


$$V_{K^+D^+,K^+D^+} = 0; \quad V_{\pi^+D_s^+,\pi^+D_s^+} = 0;$$

$$V_{K^+D^+,\pi^+D_s^+} = \frac{g^2}{m_{K^*}^2}(p_1+p_3)_\mu(p_2+p_4)^\mu.$$

Our model

- The Feynman diagrams



- The amplitude

$$
\begin{aligned}
i\mathcal{M}_a &= r_1 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2-m_K^2][(q+p_2)^2-m_{D^*}^2][(p_0-p_2-q)^2-m_K^2]}, \\
i\mathcal{M}_b &= r_1 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu G_{\pi\pi/KK}(k,m_{23}) T_{\pi\pi/KK\to\pi\pi}(m_{13})}{[q^2-m_K^2][(q+p_2)^2-m_{D^*}^2][(p_0-p_2-q)^2-m_K^2]}, \\
i\mathcal{M}_c &= r_2 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_d &= r_2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k,m_{23}) T_{D_s\pi\to D_s\pi}(\vec{k},\vec{p}_{\rm on},m_{23})}{[q^2-m_D^2][(q+p_1)^2-m_{D^*}^2][(p_0-p_1-q)^2-m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}
$$

$\pi\pi - KK$ scattering

- Coupled channel Lippmann Schwinger equations with meson-meson potentials provided by the lowest order chiral Lagrangian

Oller, Oset, Nucl.Phys.A 620 (1997) 438-456

$$V_{11} = - < K\bar{K}|\mathcal{L}_2|K\bar{K} >= -\frac{1}{4f^2}(3s + 4m_K^2 - \sum_i p_i^2)$$

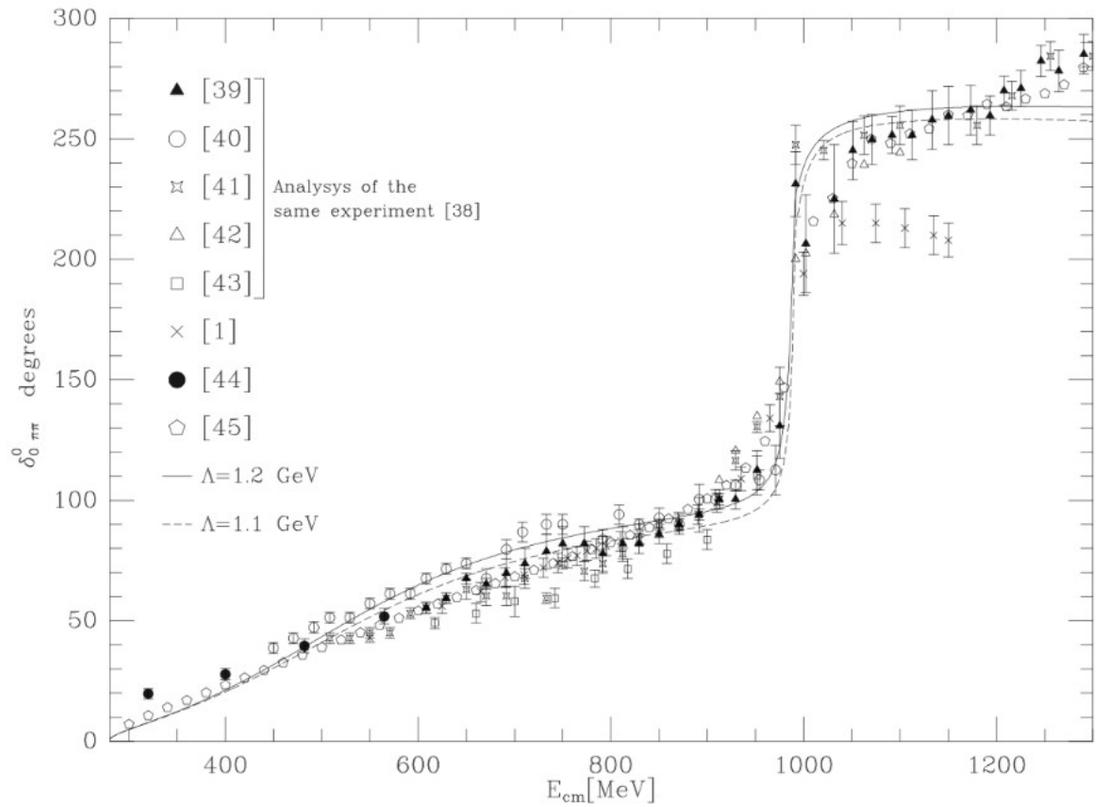
$$V_{21} = - < \pi\pi|\mathcal{L}_2|K\bar{K} >= -\frac{1}{3\sqrt{12}f^2}(\frac{9}{2}s + 3m_K^2 + 3m_\pi^2 - \frac{3}{2}\sum_i p_i^2)$$

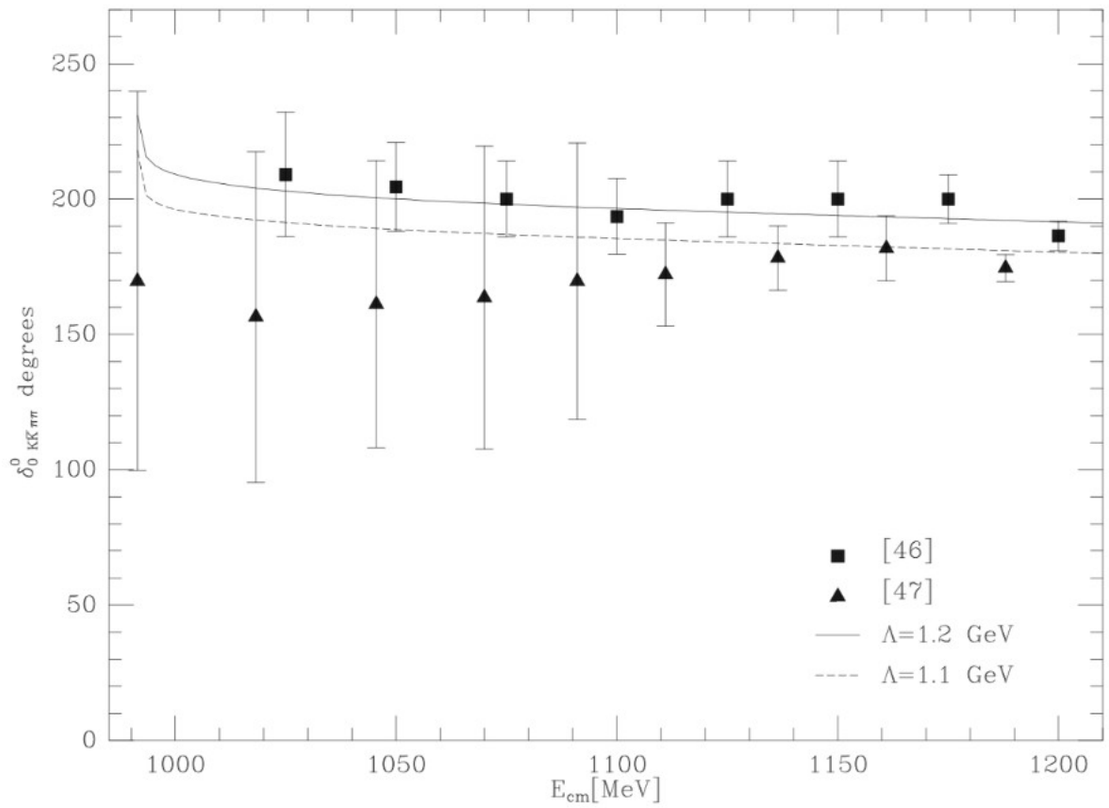
$$V_{22} = - < \pi\pi|\mathcal{L}_2|\pi\pi >= -\frac{1}{9f^2}(9s + \frac{15m_\pi^2}{2} - 3\sum_i p_i^2)$$

$$t_{11} = V_{11} + V_{11}G_{11}t_{11} + V_{12}G_{22}t_{21}$$

$$t_{21} = V_{21} + V_{21}G_{11}t_{11} + V_{22}G_{22}t_{21}$$

$$t_{22} = V_{22} + V_{21}G_{11}t_{12} + V_{22}G_{22}t_{22}$$



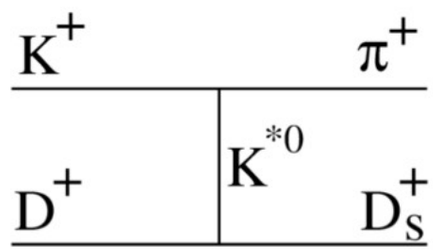


$DK - D_s\pi$ scattering

- For the $DK - D_s\pi$ T-matrix,

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\,\vec{q}\, \frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E) T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

$$\mathcal{V}^{K^*}_{DK\to D_s\pi} = \frac{g_{K^*}}{m^2_{K^*}} (p_1 + p_3)_\mu (p_2 + p_4)^\mu$$



The resulting effective potential reads

$$\mathcal{V}_{\alpha\beta} = V^{K^*}_{\alpha\beta} \left(\frac{\Lambda_\alpha^2}{\Lambda_\alpha^2 + p_\alpha^2}\right)^2 \left(\frac{\Lambda_\beta^2}{\Lambda_\beta^2 + p_\beta'^2}\right)^2$$

$\alpha, \beta = 1,2$ for $D_s\pi$ and DK channel, respectively. The cutoff for these two channels can be different.

UV divergence in the triangle loop

- S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^\mu \epsilon_{j,\mu}^\dagger, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^\mu \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

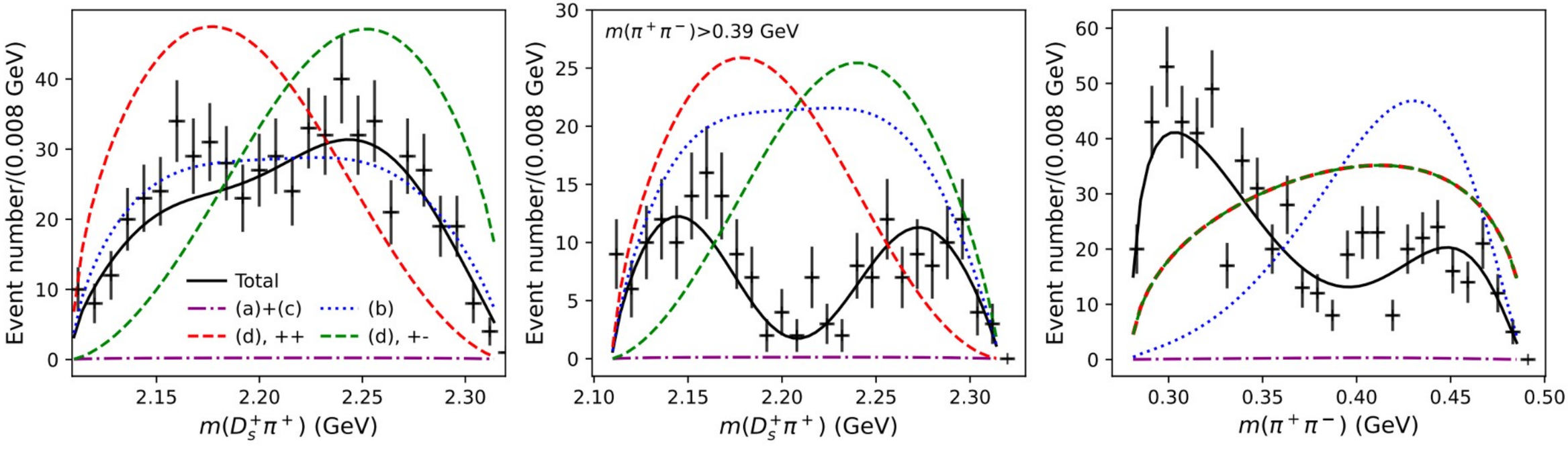
- The amplitude

$$\begin{aligned}
i\mathcal{M}_a &= r_1 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_b &= r_1 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu L_\mu G_{\pi\pi/KK}(k, m_{23}) T_{\pi\pi/KK\to\pi\pi}(m_{13})}{[q^2 - m_K^2][(q+p_2)^2 - m_{D^*}^2][(p_0 - p_2 - q)^2 - m_K^2]}, \\
i\mathcal{M}_c &= r_2 \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3), \\
i\mathcal{M}_d &= r_2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{\epsilon^\mu N_\mu G_{D_s\pi}(k, m_{23}) T_{D_s\pi\to D_s\pi}(\vec{k}, \vec{p}_{\rm on}, m_{23})}{[q^2 - m_D^2][(q+p_1)^2 - m_{D^*}^2][(p_0 - p_1 - q)^2 - m_K^2]} + (p_1 \leftrightarrow p_3),
\end{aligned}$$

$$\begin{aligned}
L_\mu &= P_{\mu\nu}(p_2 + q, m_{D^*})(p_2 - q)^\nu \\
N_\mu &= P_{\mu\nu}(p_1 + q, m_{D^*})(p_1 - q)^\nu (p_2 - q + 2p_3)^\alpha \\
&\quad \times P_{\alpha\beta}(p_2 - q, m_{K^*})(p_2 + q)^\beta
\end{aligned} \qquad P_{\mu\nu}(p, m) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

- Regularize the loop integral using dimensional regularization, and use the MS-bar renormalization scheme.

Fit to the experimental lineshapes

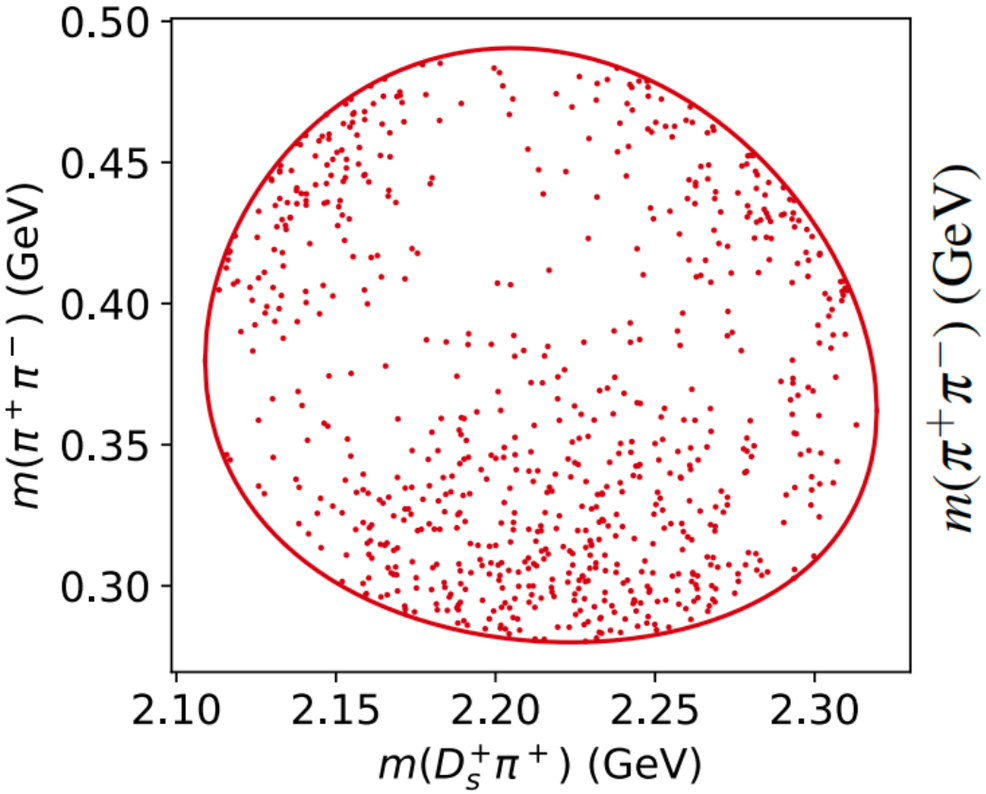


- The parameters of the model determined from the fit：

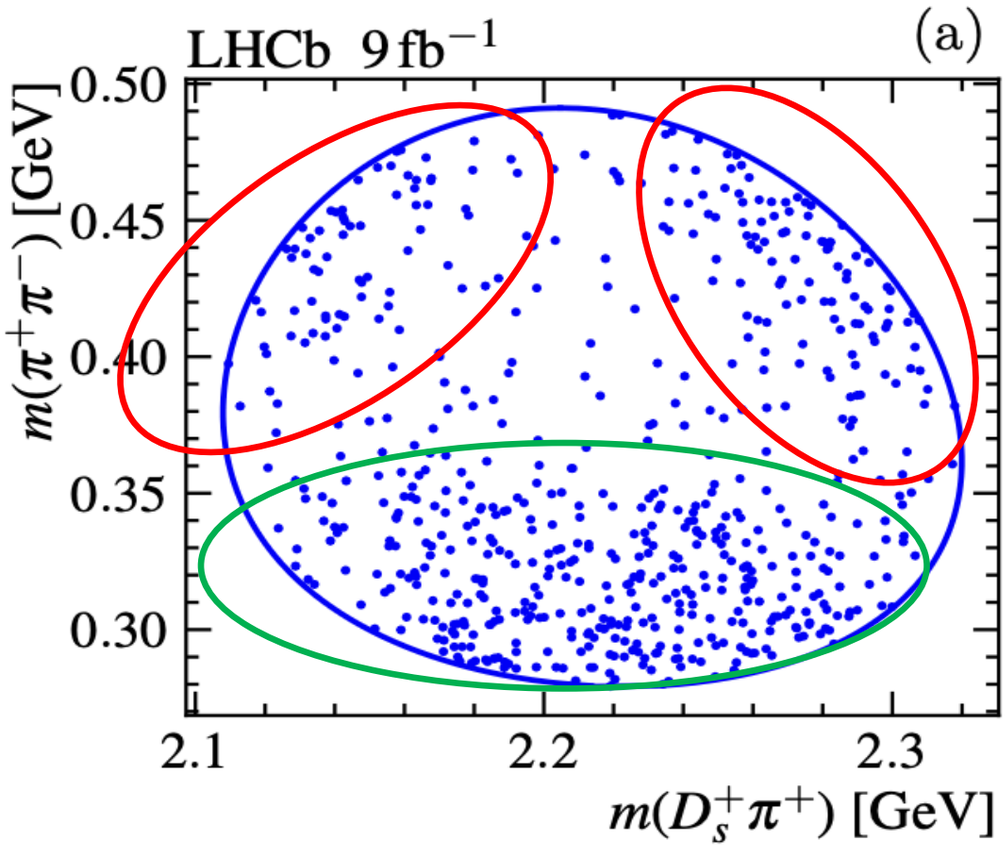
Parameter	
Λ_1 [GeV]	1.60 ± 0.07
Λ_2 [GeV]	0.557 ± 0.024
g_{K^*}	61.4 ± 2.4
r_1	7.1 ± 0.8
r_2	0.121 ± 0.014
χ^2/d.o.f.	1.43
E_p [MeV]	$2211.0^{+31.4}_{-22.6} - 154.5^{+20.7}_{-17.1}i$

- The pole position is in second Riemann sheet.

Dalitz plot



LHCb, arXiv:2411.03399



Is possible to investigate the structure of D_{s1} states?

- $D_{s1}(2460) \rightarrow D_s^+ \pi^+ \pi^-$
- $D_{s1}(2536) \rightarrow D_s^+ \pi^+ \pi^-$

Coupling of D_{s1} to D^*K

❖ S- and D-wave coupling

$$\mathcal{M}_S = g_S \epsilon_i^{\mu} \epsilon_{j,\mu}^{\dagger}, \quad \mathcal{M}_D = \frac{g_D}{M^2} \epsilon_i^{\mu} \epsilon_j^{\dagger\nu} \left(q_\mu q_\nu - \frac{g_{\mu\nu} q^2}{4} \right)$$

The coupling of D_{s1} to D^*K can be obtained from the residue of the T-matrix:

$$\begin{aligned} g_i^2 &= \frac{r}{2\pi} \int_0^{2\pi} T_{ii}(z(\theta)) e^{i\theta} d\theta \\ &= \lim_{s \to z_p} (s - z_p) T_{ii}(s) \end{aligned}$$

- $D_{s1}(2460)$: $g_S \gg g_D$

- $D_{s1}(2536)$: $\frac{g_S}{g_D} = 0.08\ e^{2.67i}$

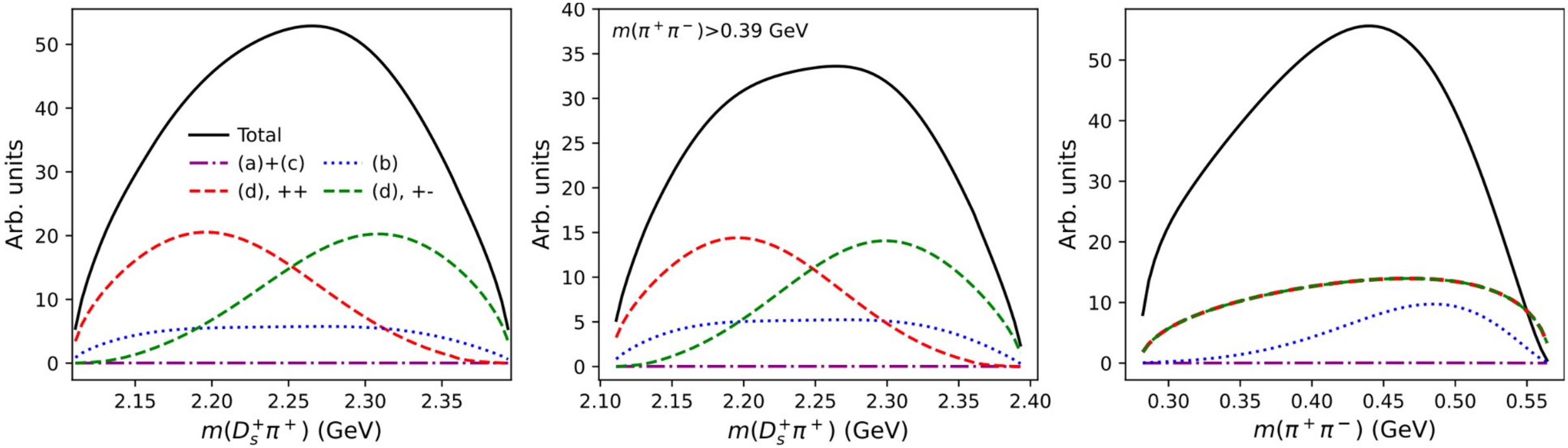
❖ The $D_{s1}(2536)$ coupling can also obtained using the experimental measurment on the S- and D-wave decay:

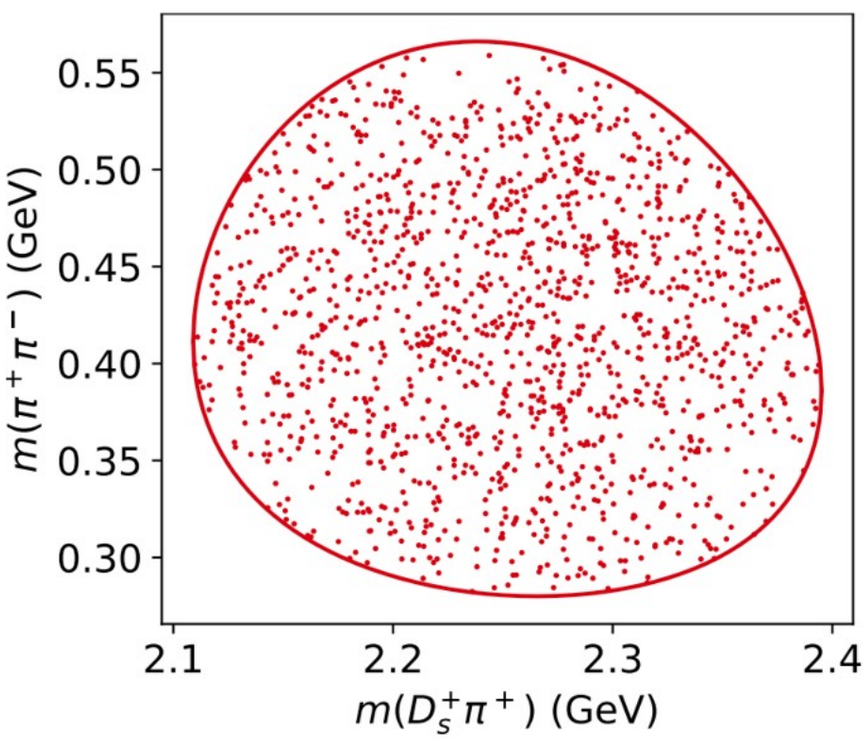
$$\frac{g_S}{g_D} = 0.1\ e^{-0.7i}$$

LHCb , JHEP 10 (2023) 106

Predicted lineshapes and Dalitz plot for $D_{s1}(2536)$ decay

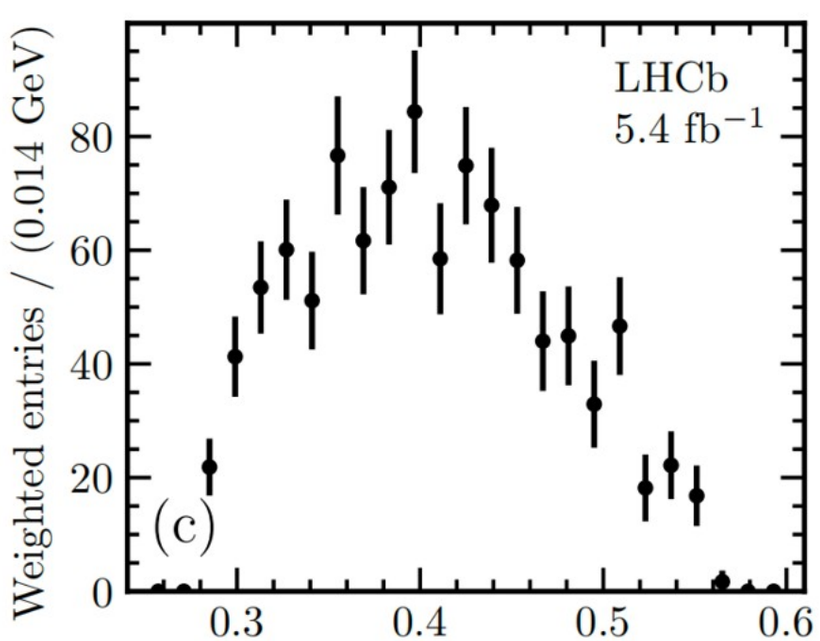
- No two-peak structure at $m(\pi^+\pi^-) > 0.39$ GeV

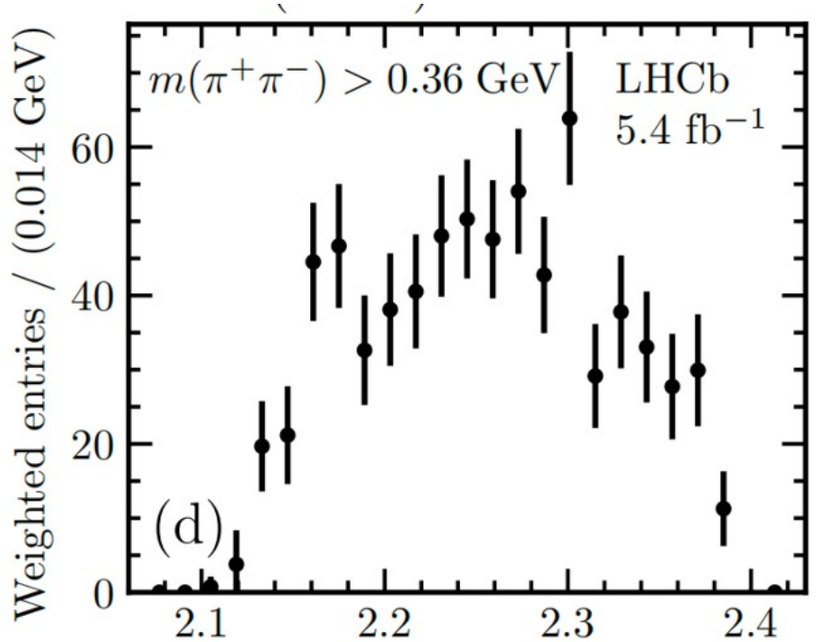


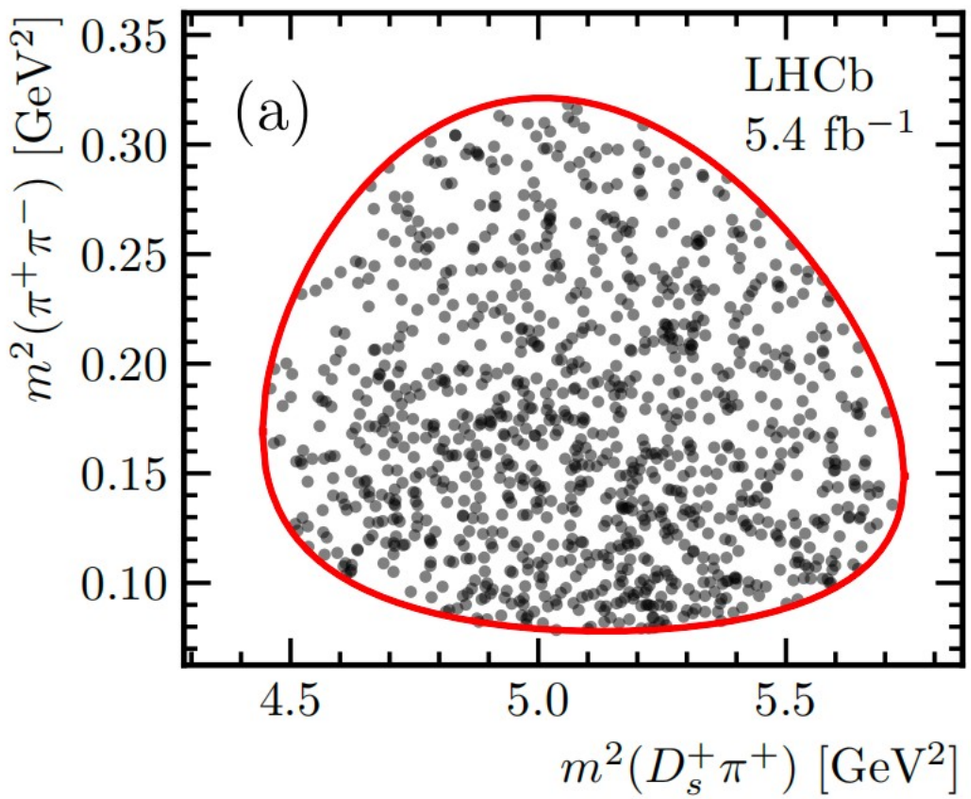


Measured lineshapes and Dalitz plot for $D_{s1}(2536)$ decay

LHCb, arXiv:2608.10826

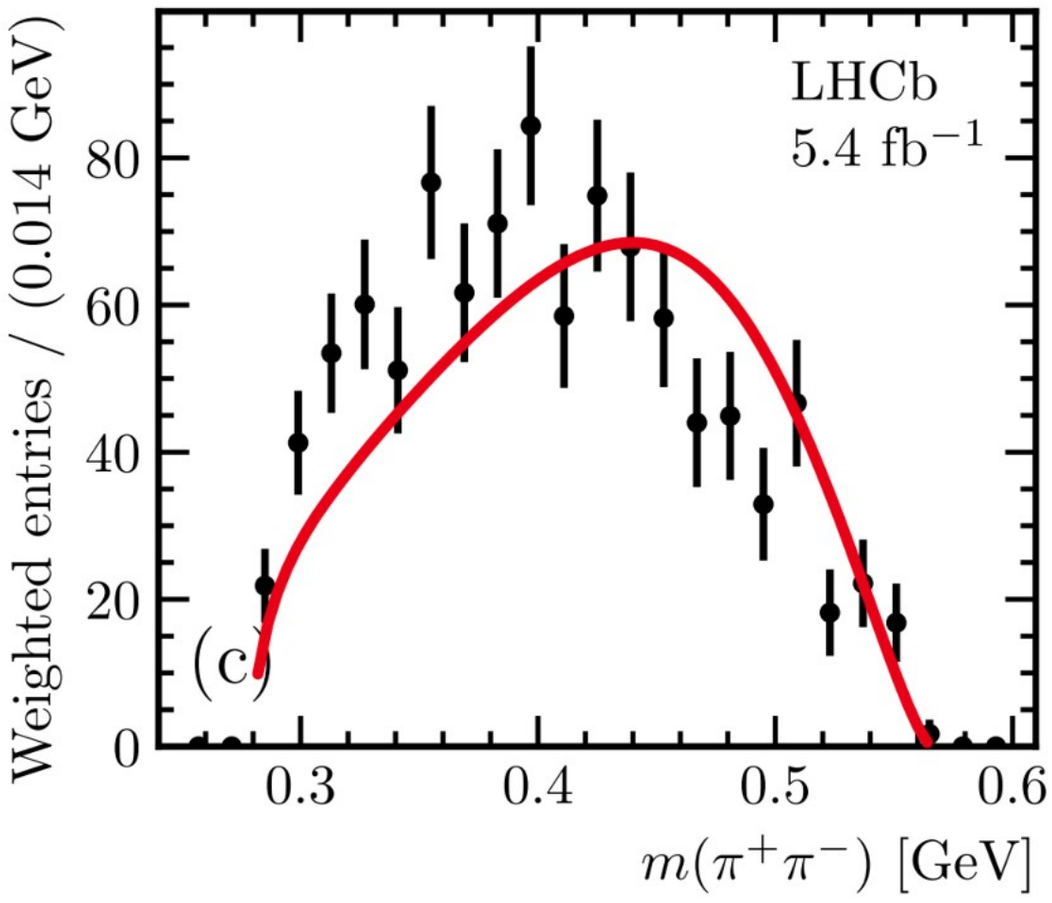


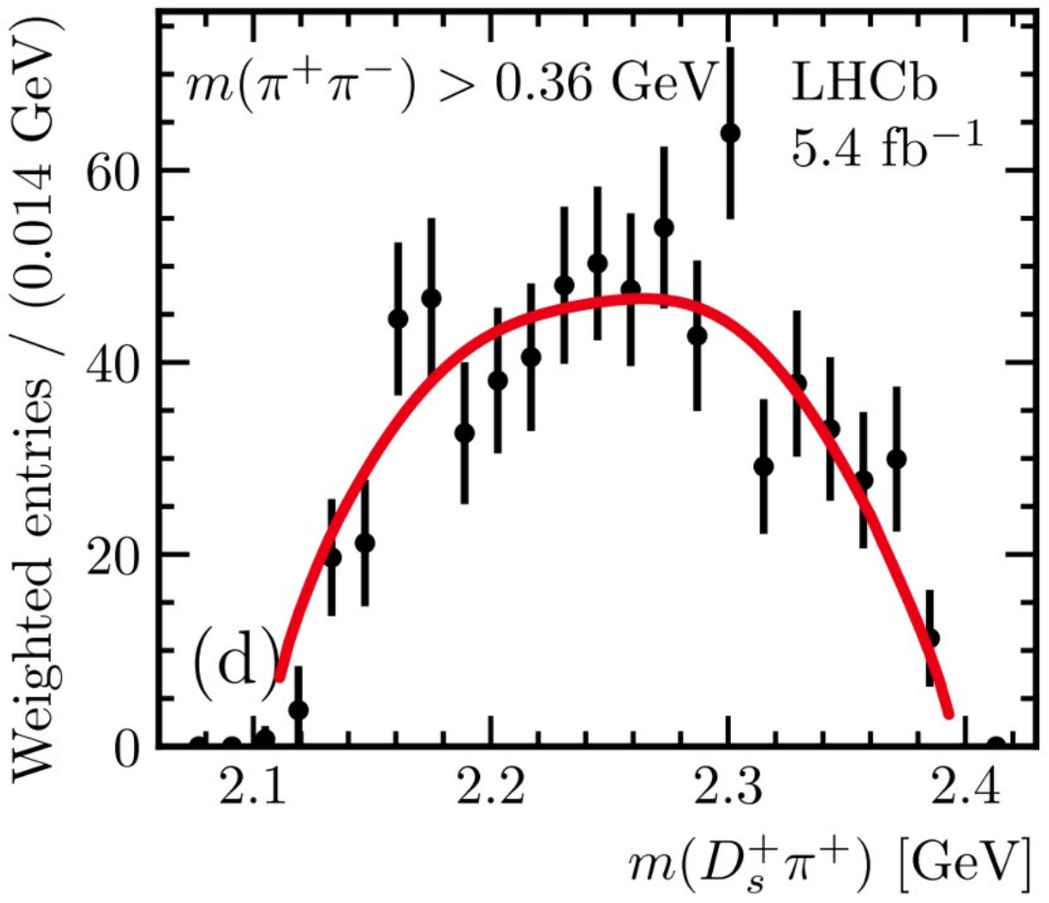




Comparison for $D_{s1}(2536)$ decay

LHCb, arXiv:2608.10826



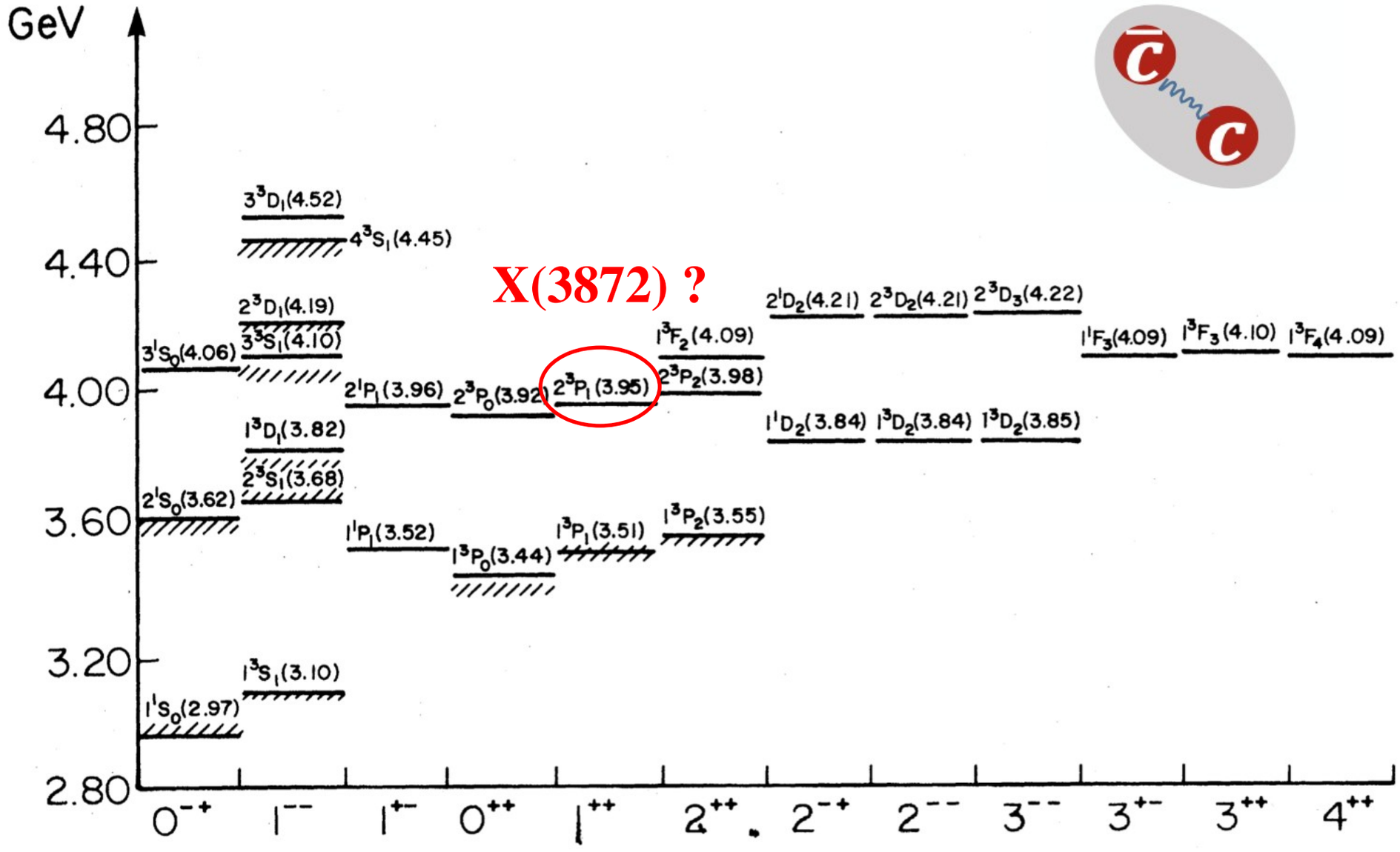


Red line: our theoretical prediction

Contents

- Near DK/D^*K thresholds states: $D_{s1}(2460/2536)$
- From mass spectra to decay properties
- Near $DD^*(D\bar{D}^*)$ thresholds states: T_{cc} and $X(3872)$
- From mass spectra to production properties

Meson in quark model: charmonium states



$X(3872)$

Experiment	Mass [MeV]	Width [MeV]
Belle [63]	$3872 \pm 0.6 \pm 0.5$	< 2.3
Belle [75]	–	–
Belle [76]	$3875.4 \pm 0.7^{+0.4}_{-1.7} \pm 0.9$	–
Belle [77]	$3871.46 \pm 0.37 \pm 0.07$	–
Belle [78]	$3872.9^{+0.6+0.4}_{-0.4-0.5}$	$3.9^{+2.8+0.2}_{-1.4-1.1}$
Belle [79]	–	–
Belle [80]	$3871.84 \pm 0.27 \pm 0.19$	< 1.2
CDF [67]	$3871.3 \pm 0.7 \pm 0.4$	–
CDF [81]	–	–
CDF [82]	–	–
CDF [83]	$3871.61 \pm 0.16 \pm 0.19$	–
DØ [68]	$3871.8 \pm 3.1 \pm 3.0$	–
BaBar [84]	3873.4 ± 1.4	–
BaBar [85]	$3871.3 \pm 0.6 \pm 0.1$	< 4.1
	$3868.6 \pm 1.2 \pm 0.2$	–
BaBar [86]	–	–
BaBar [87]	$3875.1^{+0.7}_{-0.5} \pm 0.5$	$3.0^{+1.9}_{-1.4} \pm 0.9$
BaBar [88]	$3871.4 \pm 0.6 \pm 0.1$	< 3.3
	$3868.7 \pm 1.5 \pm 0.4$	–
BaBar [89]	–	–
BaBar [90]	$3873.0^{+1.8}_{-1.6} \pm 1.3$	–
LHCb [91]	$3871.95 \pm 0.48 \pm 0.12$	–
LHCb [70]	–	–
LHCb [92]	–	–
CMS [73]	–	–
BESIII [93]	$3871.9 \pm 0.7 \pm 0.2$	< 2.4

Phys. Rept. 639 (2016) 1-121

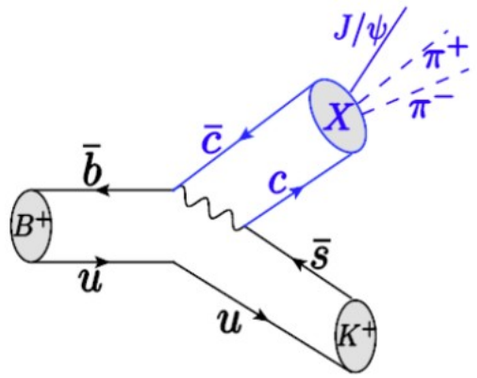
- The $D\bar{D}^*/D^*\bar{D}$ molecular state.

Swanson, Wong, Guo, liu,....

Close to $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ thresholds

$$\delta m = m_{D^0\overline{D}^{*0}} - m_{X(3872)}$$
$$= 0.00 \pm 0.18 \text{ MeV}$$

PDG 22



Where is the $\chi_{c1}(2P)$ in quark model?

- The mixing of the $\bar{c}c$ core with $D\bar{D}^*/D^*\bar{D}$ component.

Chao, H. Q. Zheng, Yu. S. Kalashnikova, P. G. Ortega...

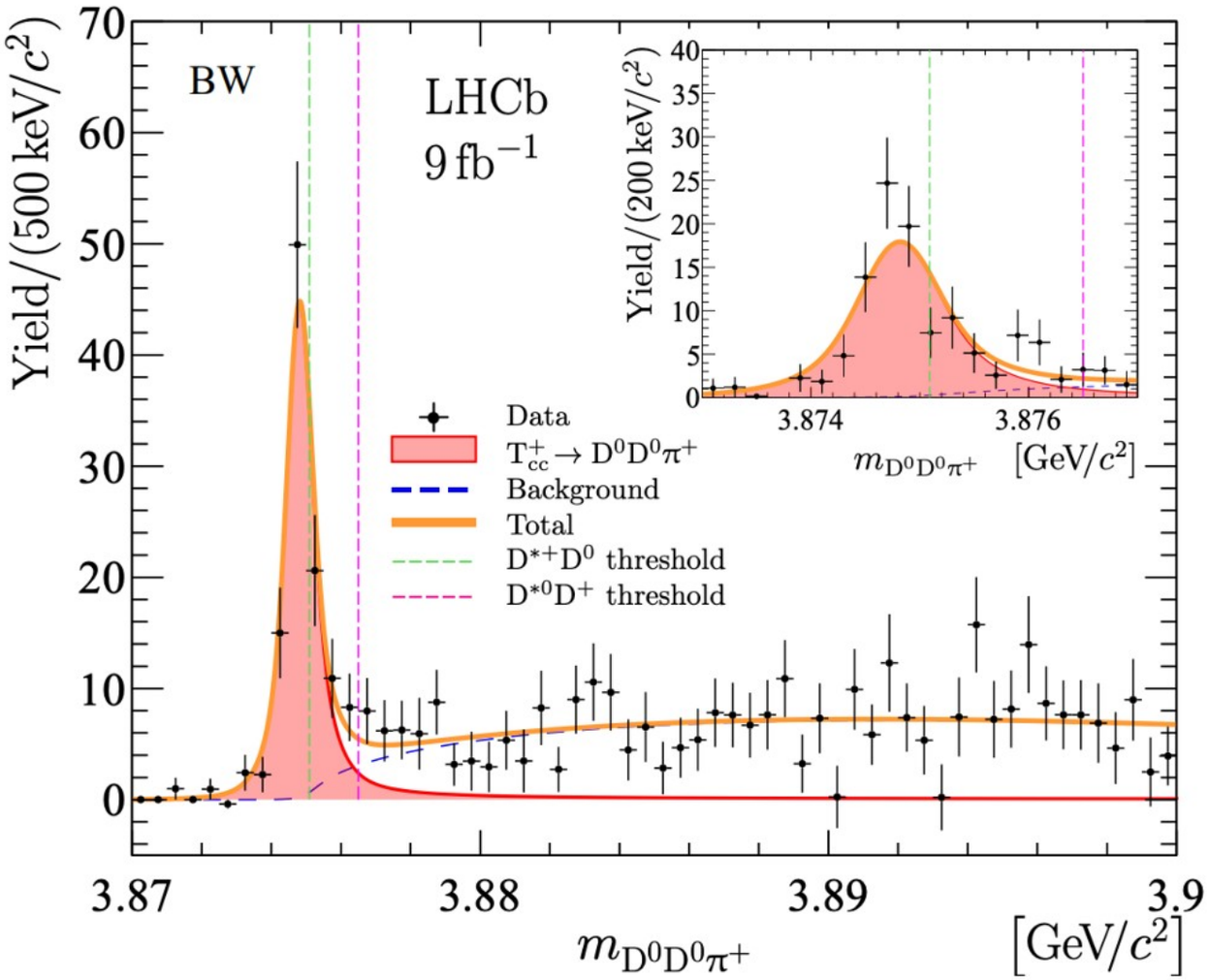
Close to charmonium $\chi_{c1}(2P)$: m=3953.5 MeV

$$\delta m = m_{\chi_{c1}(2P)} - m_{X(3872)} = 81.35 \text{ MeV}$$

→ *Complicated coupled-channel effect: $\bar{c}c$ & $D\bar{D}^*/D^*\bar{D}$*

Phys. Rev. D 32, 189 (1985)

How to determine the components in the $X(3872)$: from T_{cc}



$D^0D^0\pi^+$ channel

Close to $D^{*+}D^0$ thresholds:

Conventional Breit-Wigner:

assumed $J^P = 1^+$.

$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0}$

$= -273 \pm 61\ keV$

$\Gamma_{BW} = 410 \pm 165\text{keV}$

EPS-HEP conference, Ivan Polyakov's talk,29/07/2021; Nature Physics,22'

Unitarized Breit-Wigner:

$\delta m_U = m_{T_{cc}} - m_{D^{*+}D^0}$

$= -361 \pm 40\ keV$

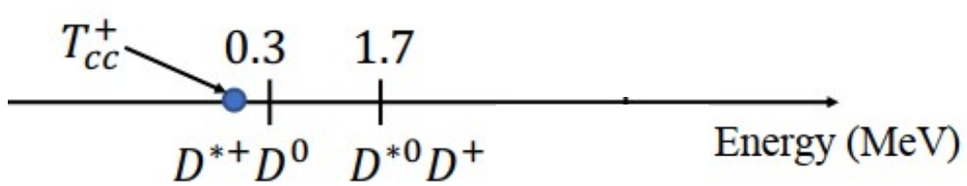
$\Gamma_U = 47.8 \pm 1.9\ \text{keV}$

- Quark content: $cc\bar{u}\bar{d}$
- ***Only the D^*D coupled channel effect***

C-parity

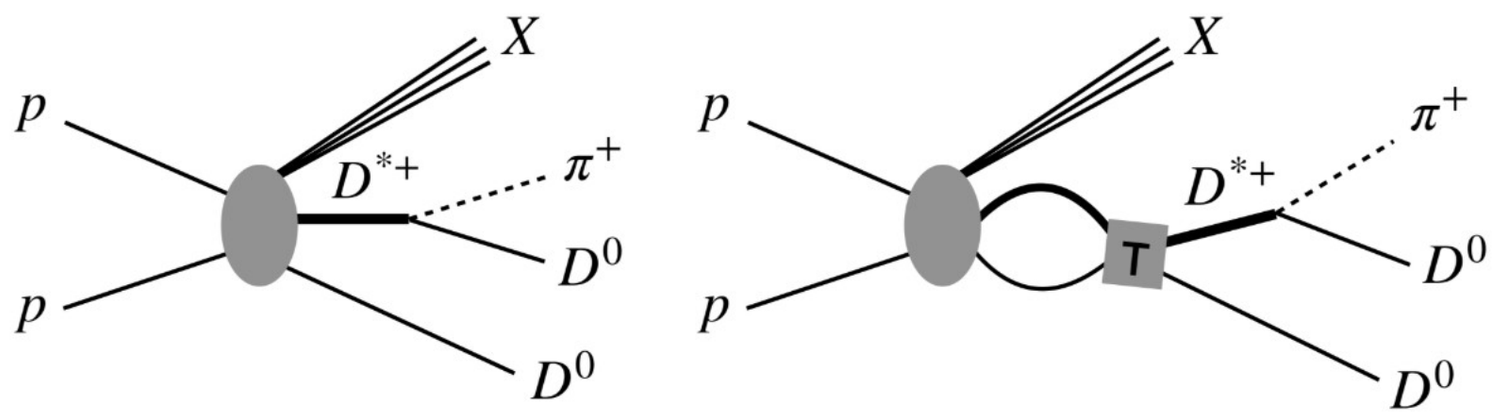
$\bar{D}^*D$ /$\bar{D}D^*$ interaction

LHCb, Nature Commun. 13 (2022) 1, 3351



The inclusive production of the T_{cc}

$pp \to D^0(p_{D_1})D^0(p_{D_2})\pi^+(p_\pi)X$, X denotes all the other produced particles



The T-matrix can be solved from the Lippmann-Schwinger equation

$$T(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) = \mathcal{V}(\vec{k}_{D^*}, \vec{k}'_{D^*}; E) + \int d\,\vec{q}\,\frac{\mathcal{V}(\vec{k}_{D^*}, \vec{q}; E)T(\vec{q}, \vec{k}'_{D^*}; E)}{E - \sqrt{m_D^2 + q^2} - \sqrt{m_{D^*}^2 + q^2} + i\epsilon}$$

with $\mathcal{V}$ the effective potential.

The inclusive production of the T_{cc}

The effective potential is obtained with light-meson exchange potentials

$$\mathcal{V} = \left(V_\pi + V^t_{\rho/\omega} + V^u_{\rho/\omega}\right)\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$$

with

$$V_\pi = \frac{g^2}{f_\pi^2}\frac{(q\cdot\epsilon_\lambda)(q\cdot\epsilon^\dagger_{\lambda'})}{q^2 - m_\pi^2},$$

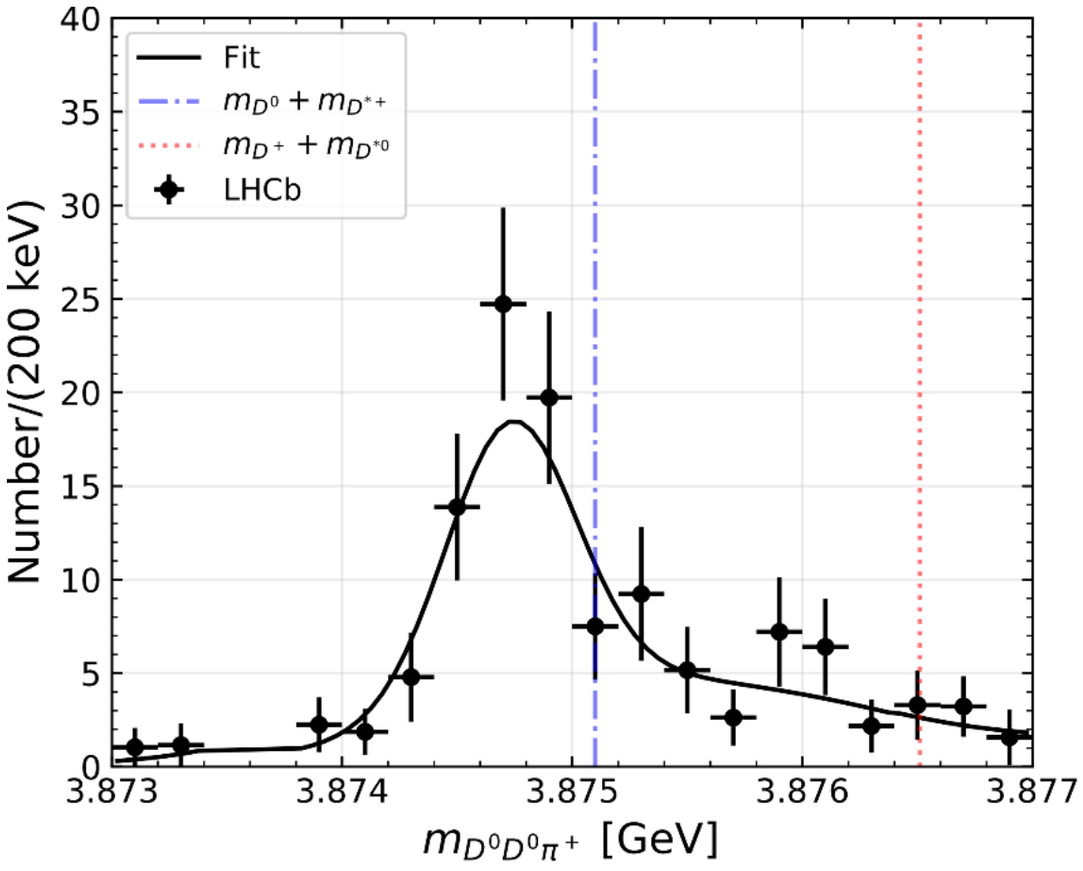
$$\mathcal{V}^u_{\rho/\omega} = -2\lambda^2 g_V^2\frac{(\epsilon^\dagger_{\lambda'}\cdot q)(\epsilon_\lambda\cdot q) - q^2(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2 - m^2_{\rho/\omega}},$$

$$V^t_{\rho/\omega} = \frac{\beta^2 g_V^2}{2}\frac{(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2 - m^2_{\rho/\omega}}.$$

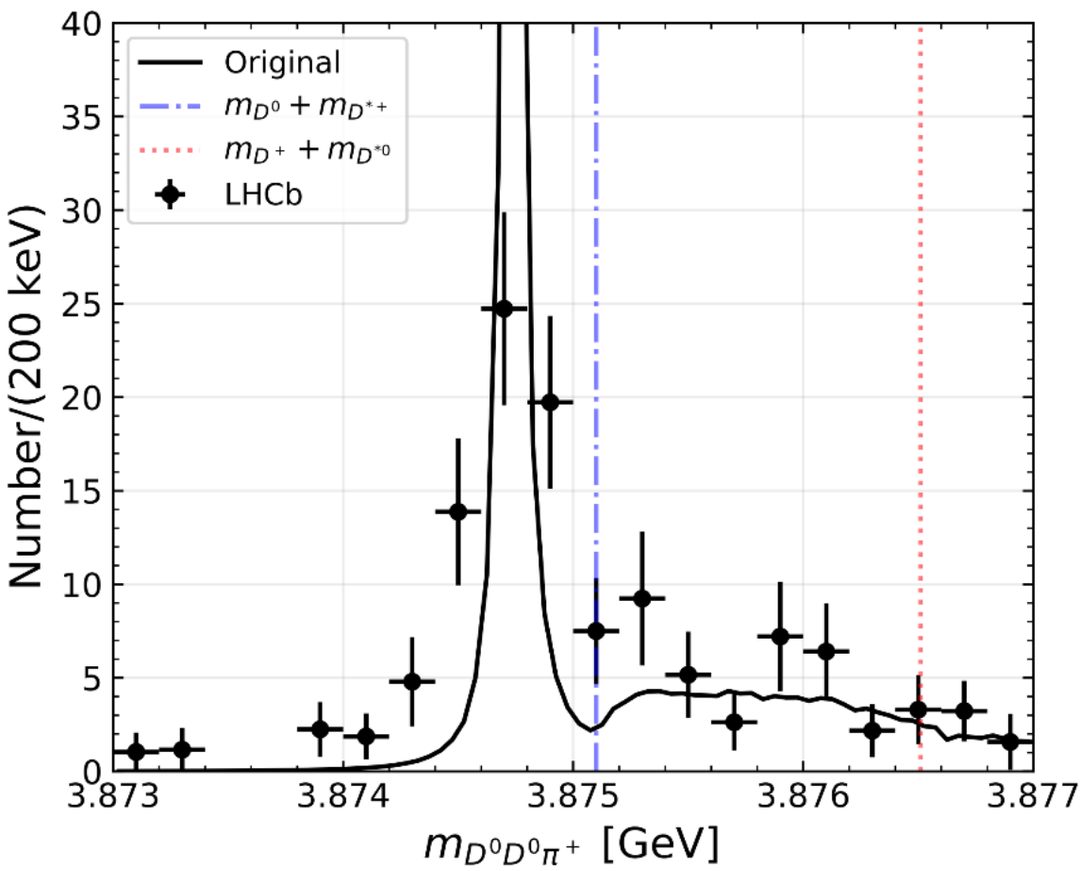
where $g = 0.57$ is determined by the strong decays $D^* \to D\pi$.

Fitting result and T_{cc} properties

χ^2/dof =0.76



Without resolution function



- Only one pole appears—bound states

$m_{T_{cc}}$=3874.7 MeV, $\Delta E = -387.7$ keV

$\Gamma_{T_{cc}} = 67.3$ keV

- $\sqrt{\langle r^2 \rangle} = 4.8\ fm$

- 70.1% $D^{*+}D^0$, 30% D^+D^{*0} ⟺ 95.8% $DD^*(I=0)$, 4.2% $DD^*(I=1)$

$$[I=0] = \frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$[I=1] = \frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$

Direct application to $X(3872)$: $D\bar{D}^* + c\bar{c}$

$D\bar{D}^*$ interaction is attractive but not strong enough to form a bound state. ➡ ***Inclusion of $c\bar{c}$ core***

- The $D\bar{D}^*$ system with quantum number $I(J^{PC}) = 0(1^{++})$ can couple with the $\chi_{c1}(2P)$.
- The coupled channel effect between them can be described by the quark-pair-creation model:

$$g_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|) = \gamma I_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|)$$

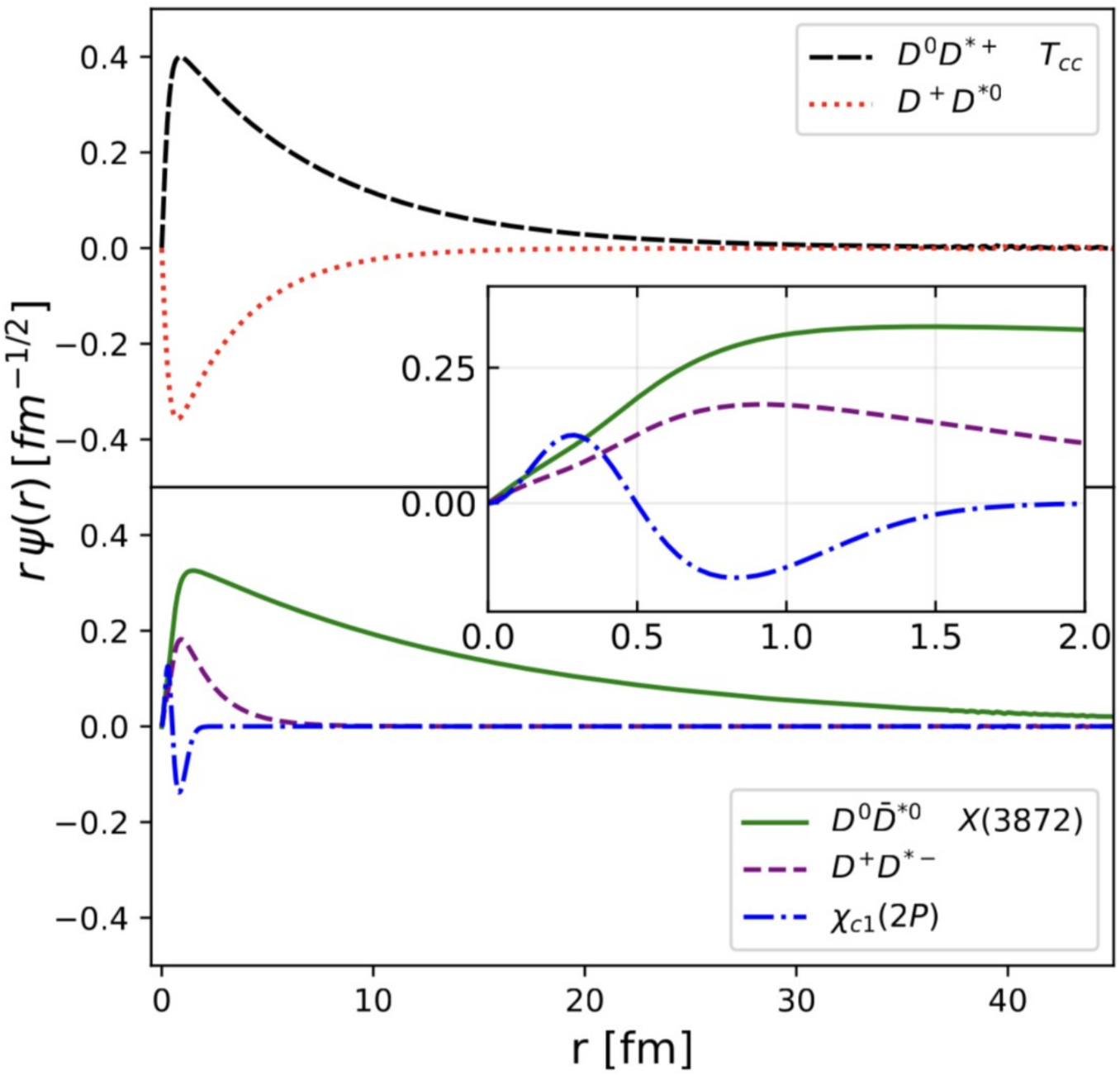
$I_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|)$ is the overlap of the meson wave functions ← GI quark model

- γ is determined to reproduce the $\psi(3770)$

- The $X(3872)$ then emerges as a bound state:

$X(3872)$	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0\bar{D}^{*0})$	$P(D^+D^{*-})$	$P(c\bar{c})$
	-80.4	32.5	11.2 fm	71.9%	28.1%	94.0%	4.8%	1.2%

Direct application to $X(3872)$: $D\bar{D}^* + c\bar{c}$



- Long tails for the radius distribution.
- *X(3872)* has a even longer tails than *Tcc*

✓ *r*<*2* fm, *cc*+*DD*∗ are important.

✓ *r*<*0.5* fm, *cc* core dominates.

✓ *DD*∗ plays the dominant role in the long-distance region, which contributes to ⟨*r2*⟩.

Contents

- Near $DK(D^*K)$ thresholds states: $D_{s1}(2460/2536)$
- From mass spectra to decay properties
- Near $DD^*(D\bar{D}^*)$ thresholds states: T_{cc} and $X(3872)$
- From mass spectra to production properties

Production of exotic hadrons: theory and experiment

- Experiment: only the X(3872) **has pT-differential data — in pp and in Pb–Pb, prompt and non-prompt.**

[LHCb, Eur. Phys. J. C 72 (2012) 1972; CMS, JHEP 04 (2013) 154; CMS, Phys. Rev. Lett. 128, 032001 (2022)]

- For all other exotics — T_{cc}, Z_c, P_c— only the observation exists: no production measurement yet.

- Theory: coalescence, statistical hadronization and short-distance factorization…

All of them are governed by one quantity: the size of the state, i.e. its wave function.

- *So production is a structural probe: a compact tetraquark and a molecule are produced very differently.*

Coalescence model

- The potential is fitted to the T_{cc} line shape — the production step adds **no new parameter**.

- Wave function is the universal Lorentzian fixed by binding momentum alone: short-range details drop out.

Coalescence model

$$\frac{dN}{d^3p} = \sum_i g_i \int d^3x_{1,i} d^3x_{2,i} d^3p_{1,i} d^3p_{2,i} f_{1,i} f_{2,i} W(\boldsymbol{r}_i, \boldsymbol{k}_i) \delta^3(\boldsymbol{p}_{T_{cc}} - \boldsymbol{p}_{1,i} - \boldsymbol{p}_{2,i})$$

$f_{1,2}$: constituent phase-space densities; g_i = 1/2: spin–isospin weight.

W : Wigner function of the molecule.

$$W(\boldsymbol{r}, \boldsymbol{k}) = \int d^3\boldsymbol{q}\, \psi^* \left(\boldsymbol{k} + \frac{\boldsymbol{q}}{2}\right) \psi \left(\boldsymbol{k} - \frac{\boldsymbol{q}}{2}\right) e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

The wave function is a Lorentzian:

$$\psi(k) = \frac{\beta}{k^2 + \gamma^2}, \qquad \beta = \frac{\gamma}{\pi}, \qquad \gamma = \sqrt{2\mu B}$$

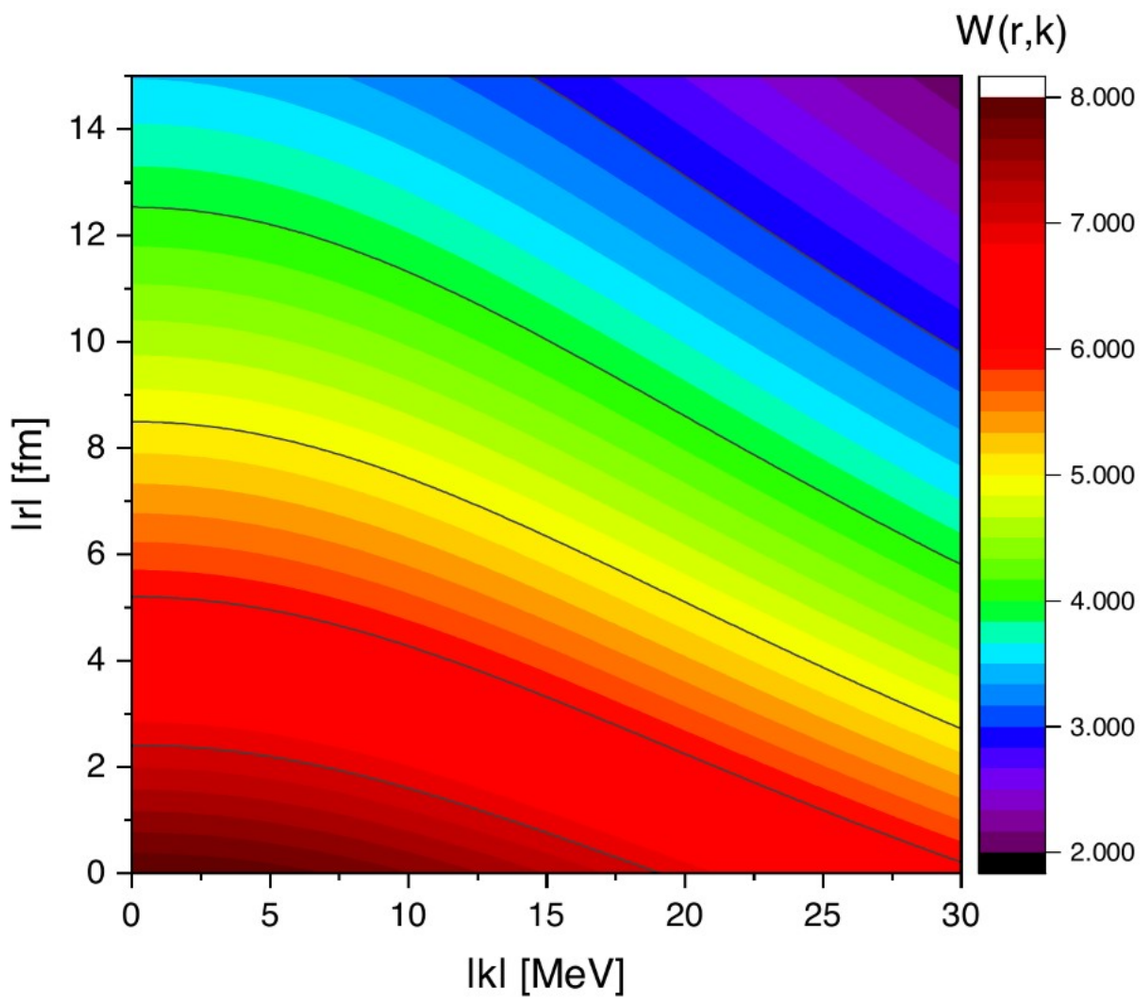
- γ is the binding momentum: 27.6 MeV ($D^{*+}D^0$), 59.1 MeV ($D^{*0}D^+$).
- $\psi(r) \propto \frac{e^{-\gamma r}}{r}$: the exact asymptotic form of a weakly bound S-wave state.
- The wave function is analytic — no Gaussian ansatz, no basis expansion.

From the wave function to the Wigner function

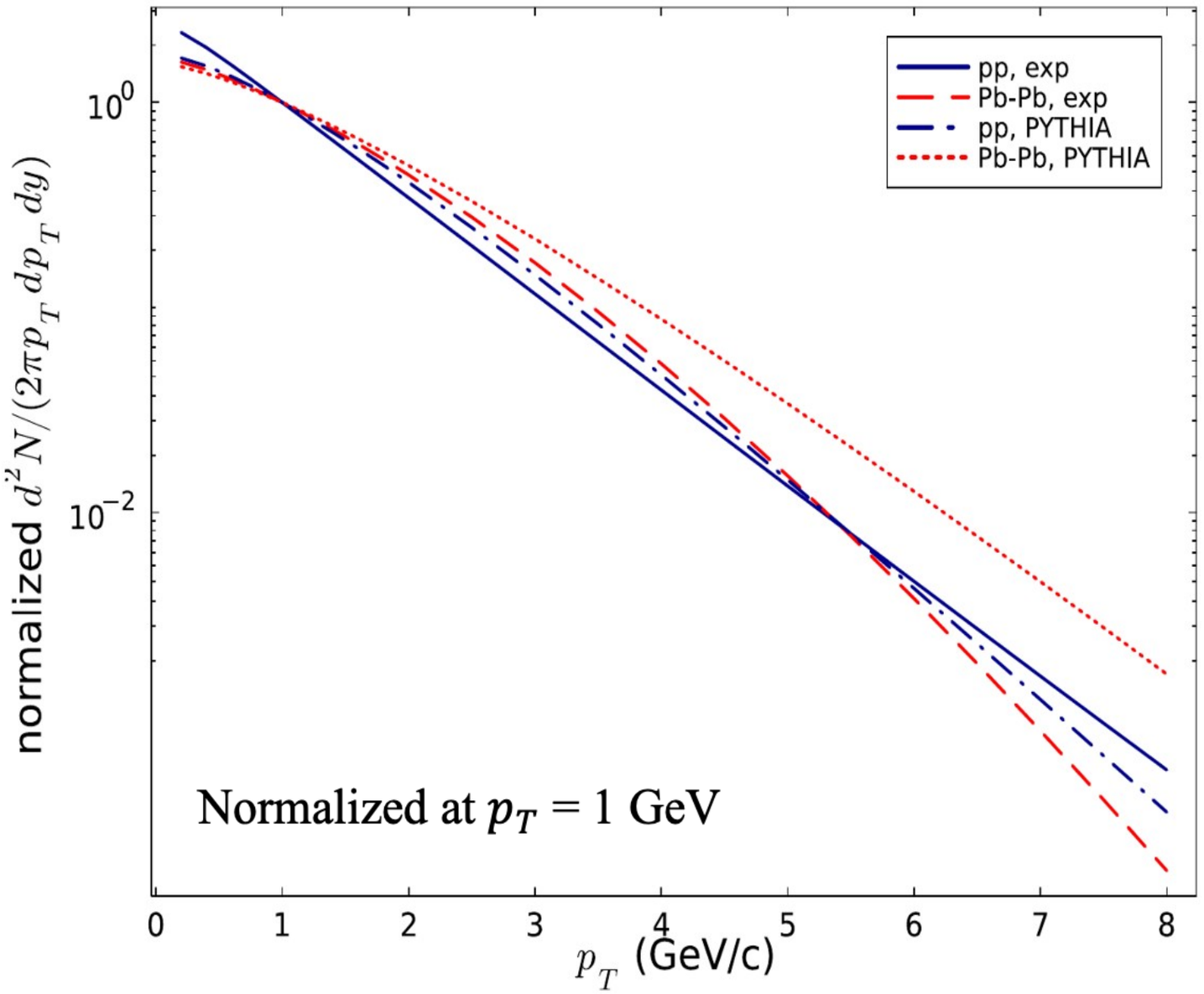
The Wigner transform is analytic:

$$W(\boldsymbol{r},\boldsymbol{k}) = 8\gamma \int_0^1 d\,u\, \frac{\cos[2(1-2u)\boldsymbol{k}\cdot\boldsymbol{r}]}{\sqrt{\gamma^2+4u(1-u)k^2}}\, e^{-2|\boldsymbol{r}|\sqrt{\gamma^2+4u(1-u)k^2}}$$

- Wigner distribution keeps a power-law tail in momentum space — a Gaussian wave function would give an exponential one.
- Everything is fixed by binding momentum γ : one number for binding energy and size.



Transverse-momentum distribution of the T_{cc}



- Both spectra fall monotonically.
- Pb–Pb has the steeper high-p_T tail — the imprint of charm energy loss.
- PYTHIA reproduces pp, but is far too hard in Pb–Pb.

Summary

- **Near-threshold *Ds* states.** A coupled-channel framework connecting quark model and channel-channel interaction is constructed to study the components and pole masses of the physical $D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$ and $D_{s2}(2573)$.

- **From mass spectra to structure.** The Dsππ distributions separate a two-peak from a single-peak pattern — a distinction corresponds to the structure of mother $D_{s1}(2460)$ and $D_{s1}(2536)$.

- **X(3872) and T_{cc}.** Coupling the $D\bar{D}^*$ channel to the $c\bar{c}$ core turns the attraction into a bound state, and the same coupled-channel wave function fixes the T_{cc} mass, width and size.

- **From mass spectra to production.** In the coalescence model the T_{cc} pT spectrum follows from that very wave function; **A realistic wave function.** The first analytic Wigner transform of the universal wave function — no Gaussian ansatz, no basis expansion.

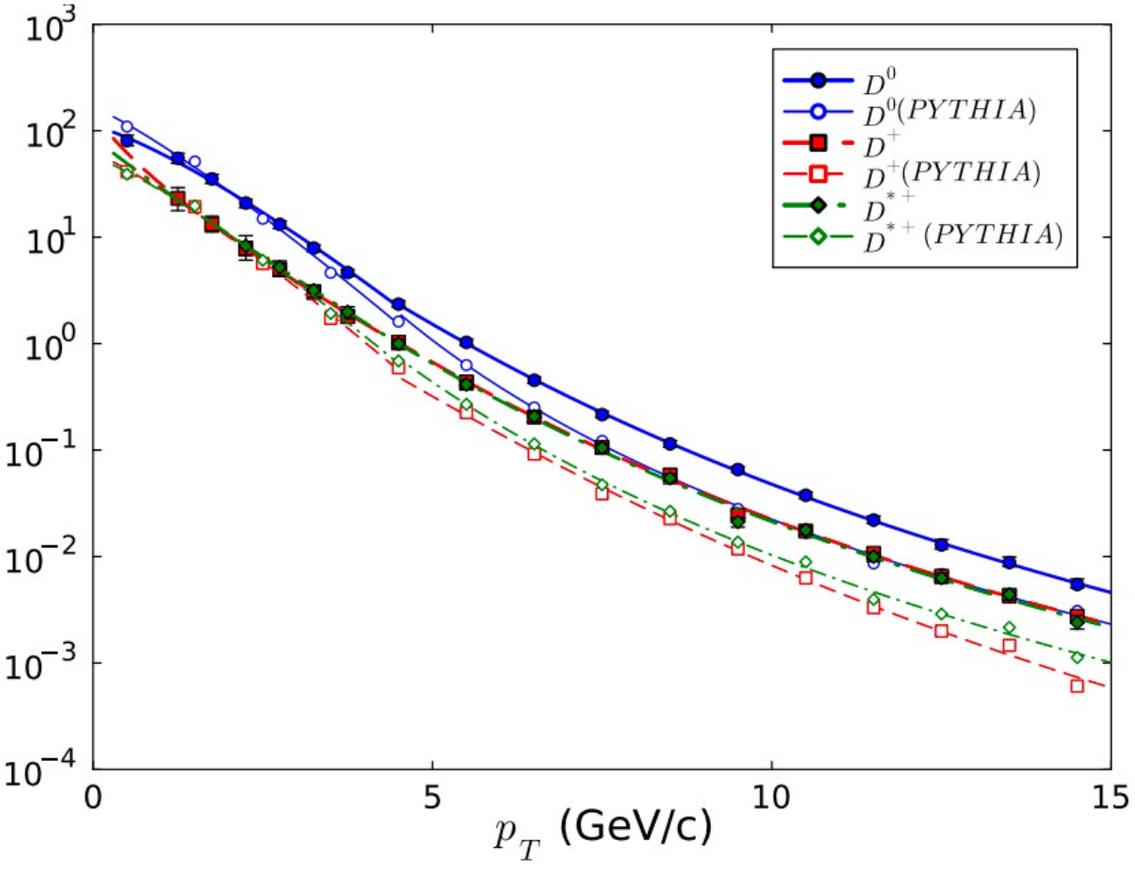
Thank you !

Backup

$$\frac{d^2N}{d^2p_T dy} = a\, e^{-b(p_T/p_1)^c} \qquad (p_T < p_c),$$

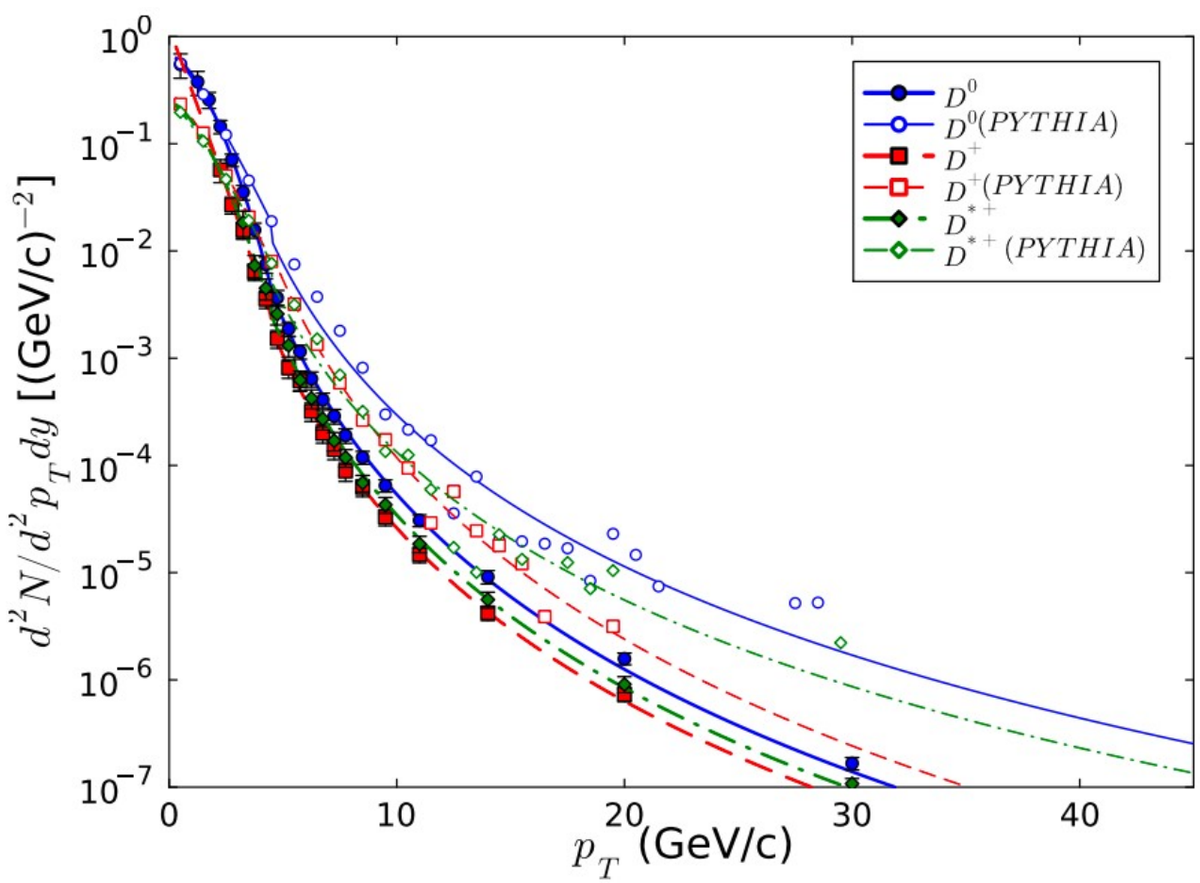
$$= \frac{d}{\left[1 + (p_T/p_0)^2\right]^e} \qquad (p_T > p_c),$$

pp collisions at $\sqrt{s} = 13$ TeV and $2 < y < 2.5$



R. Aaij et al. (LHCb), JHEP **03**, 159 (2016), arXiv:1510.01707 [hep-ex].

Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV and $|y| < 0.5$



S. Acharya et al. (ALICE), JHEP **01**, 174 (2022), arXiv:2110.09420 [nucl-ex].