Traversable wormholes, quantum teleportation and quantum gravity

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Outline

- Holographic duality
- ER=EPR
- Traversable wormhole and teleportation
- SYK model/AdS_2
- Quantum simulation of a traversable wormhole
- Conclusion

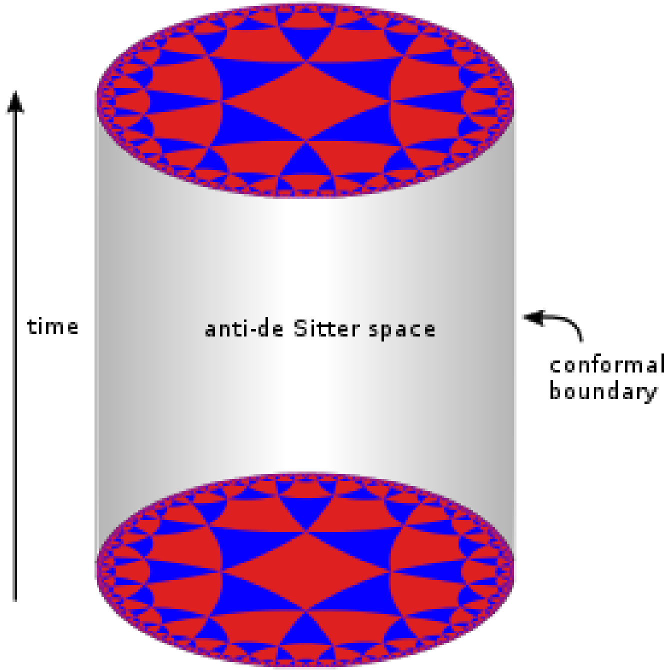
Holographic duality

- Black hole entropy S=A/(4G) [Bekenstein 72']
- AdS/CFT: N=4 SYM is dual to super gravity in AdS_5

SYM	AdS
Strong coupling	Weak coupling Einstein gravity
Weak coupling	Stringy physics in gravity
D dimension	D+1 dimension
conformal symmetry SO(D,2)	AdS isometry SO(D,2)
Gauge group SU(N)	$G_N=O(1/N^2)$

- More holographic models are found. e.g. AdS_2/SYK

[Maldacena 97']



[Sachdev-Ye 93', Kitaev 19']

Anti-de Sitter (AdS) spacetime

- AdS: negative cosmological constant $\Lambda < 0$
- Our acceleratingly expanding universe is not AdS because it requires positive cosmological constant
- AdS is hyperbolic and has a boundary. Massive particle needs infinite energy to reach the boundary. It is like a box.
- Asymptotic AdS is dual to more general states on the boundary, e.g. AdS black hole is dual to thermal state on the boundary.

ER=EPR

- The entangement on the boundary (EPR) is dual to connected spacetime in the bulk (ER)

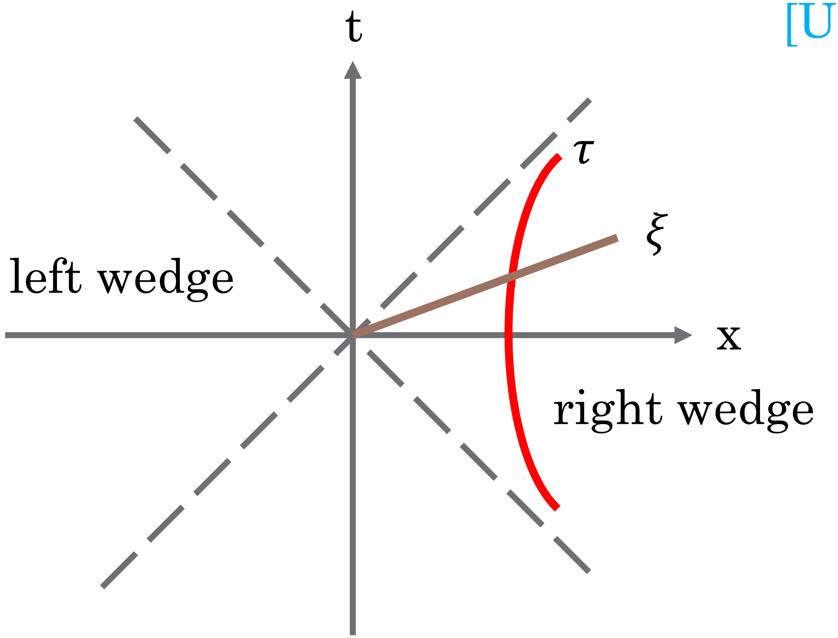
[Maldacena-Susskind 13']

- The relation between entanglement and spacetime connectedness can be seen from Unruh effect in Minkowski space.

[Unruh 76']

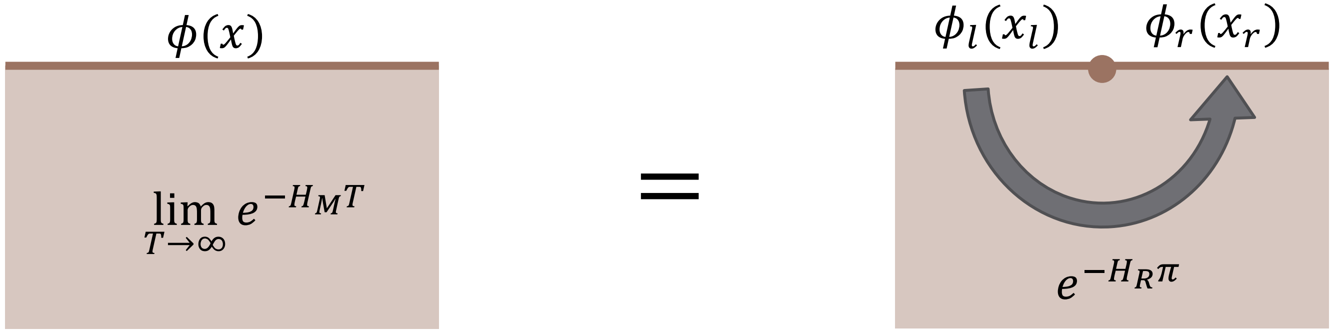
$$x = \frac{1}{a}\mathrm{e}^{a\xi_R}\cosh a\tau_R$$
$$t = \frac{1}{a}\mathrm{e}^{a\xi_R}\sinh a\tau_R$$

a is the acceleration of constant ξ curve



ER=EPR in flat/Rindler space

- The lesson from Unruh effect is that the connected spacetime implies entanglement between two parts.
- Euclidean path integral is the simplest way to show this.
- Two ways to prepare Minkowski vacuum by Euclidean path integral
- H_R is the boost generator in Lorentz group, it becomes the Rindler Hamiltonian in both left and right wedges.



Vacuum wave function
$\langle \phi(x)|\Omega\rangle \sim \langle \phi_r(x_r)|e^{-H_R\pi}|\phi_l(x_l)\rangle$
with $\phi(x_r)=\phi_r(x_r)$, $\phi(x_l)=\phi_l(x_l)$

$\Rightarrow$

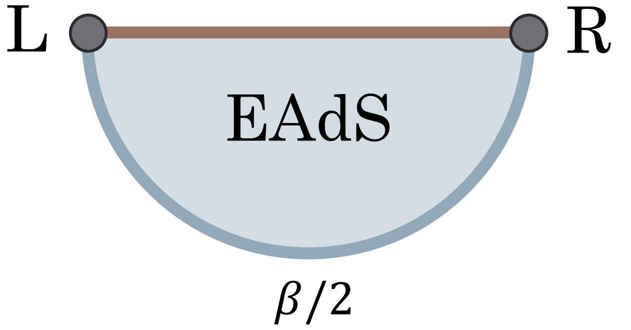
$$|\Omega\rangle = \sum e^{-\pi E}|E\rangle_l|E\rangle_r$$

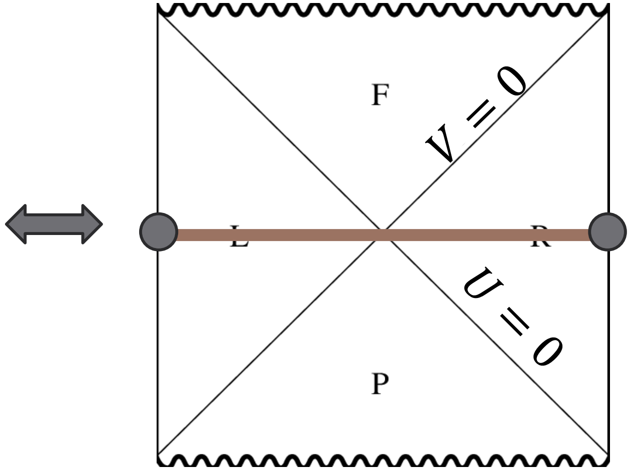
ER=EPR in AdS/CFT

- The lesson from Unruh effect is that the connected spacetime implies entanglement between two parts.
- AdS/CFT: Thermofield double state in CFT is dual to Einstein-Rosen bridge (wormhole) is AdS.

[Maldacena 01']

$$|tfd\rangle = \sum e^{-\beta E/2}|E\rangle_L|E\rangle_R$$

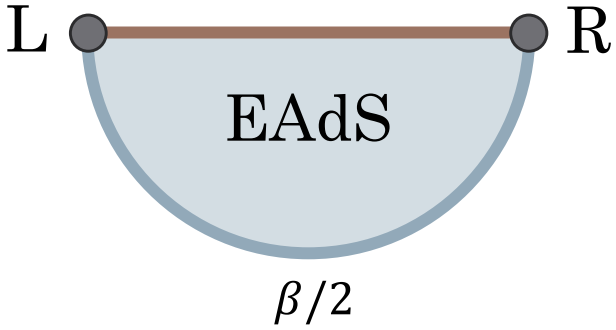


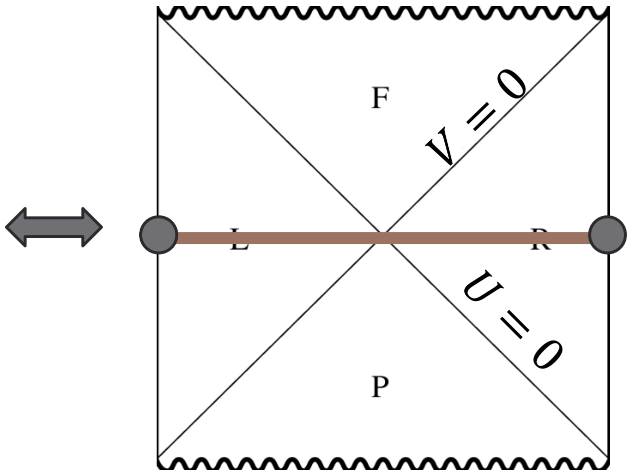


ER=EPR in AdS/CFT

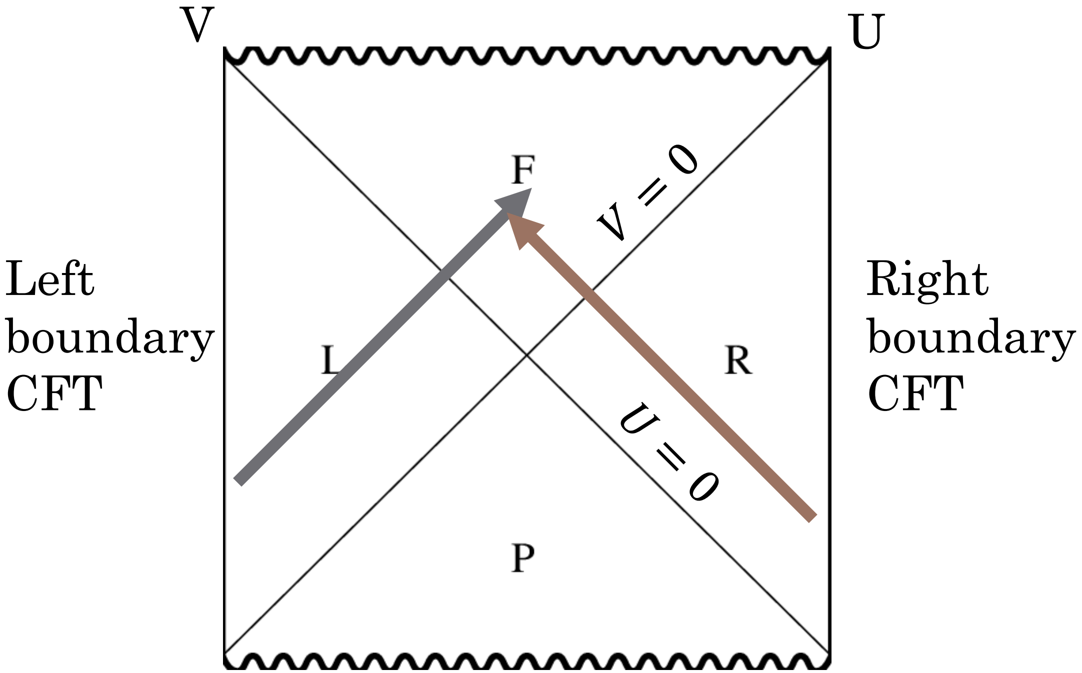
- $H_L|tfd\rangle = H_R|tfd\rangle$. Two-sides are identical systems.
- Thermofield double state is the ground state of the two-sided black hole on the global Cauchy slice.
- It is also the maximal entangled state given total energy.
- This thermofield double state can be written as $e^{-\beta H_R/2}|EPR\rangle$ or equivalently $e^{-\beta H_L/2}|EPR\rangle$.

$$|tfd\rangle = \sum e^{-\beta E/2}|E\rangle_L|E\rangle_R$$





Two-sided AdS black hole



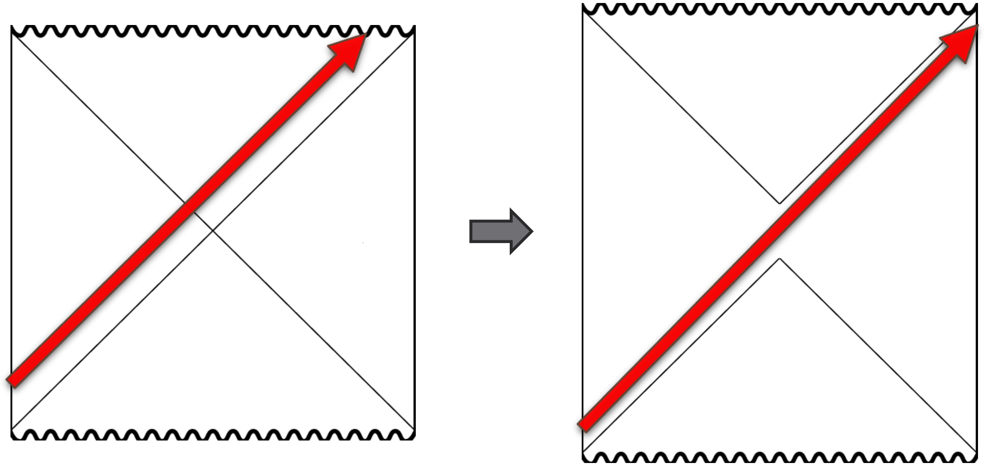
No causal relation between left and right though they are connected.

↕

Two boundary CFTs are only entangled but not coupled.

Traversable wormhole

- ER=EPR looks fancy, but not falsifiable because the wormhole is not traversable.
- If the wormhole is traversable, a traveler can escape from the horizon after jump in the black hole. We can learn the physics behind horizon!
- How? We need to shrink the horizon.
- Throw some negative energy into the black hole. Mass drops. Horizon shrinks.



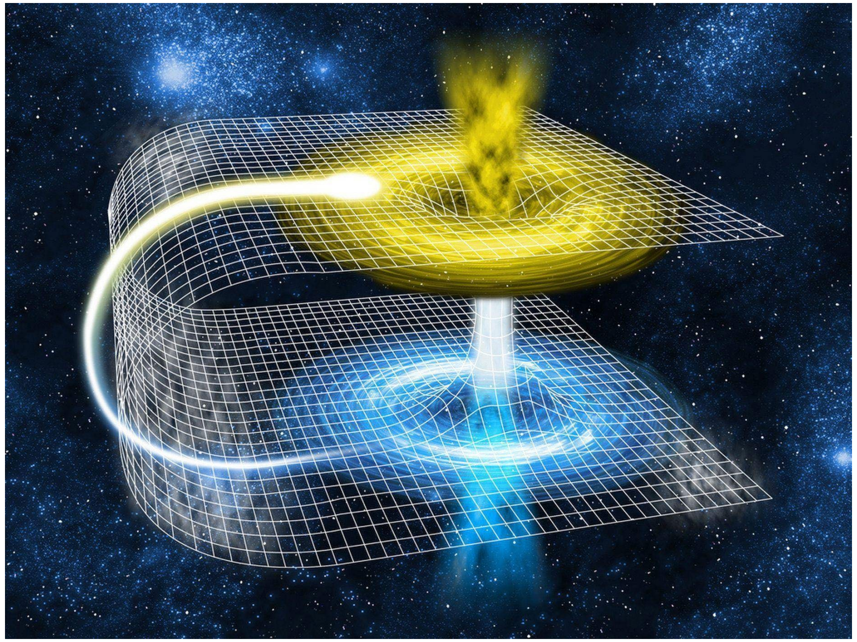
[PG-Jafferis-Wall 17']

Wait! traversable wormhole is exotic!

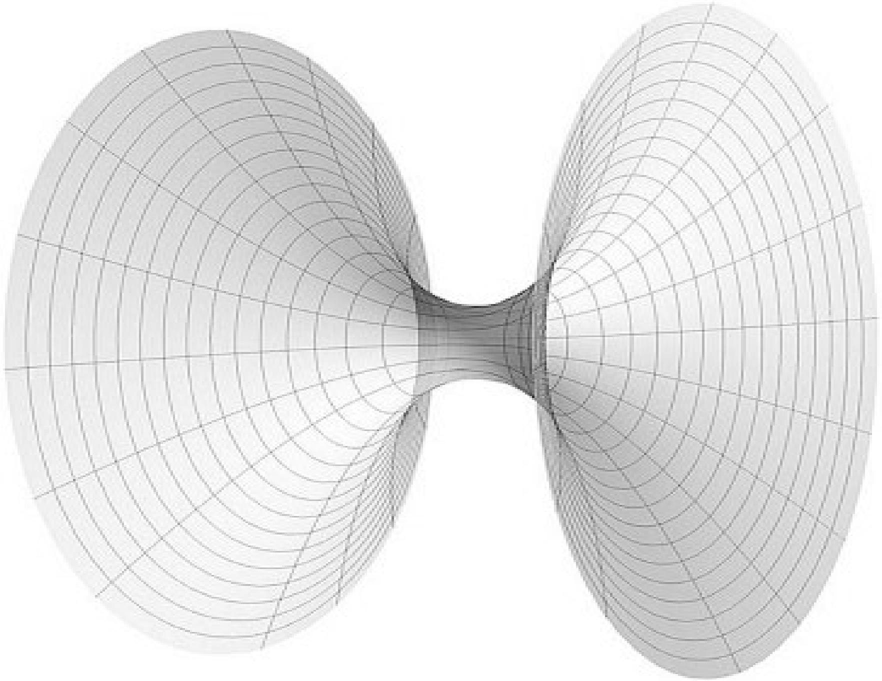
- Traversable wormhole means a null congruence focus when enters the throat and defocus when exits the other throat.
- By Raychaudhuri's equation and Einstein equation, this requires violation of averaged null energy condition (ANEC). (existence of negative pressure)
- All classical matters obey ANEC.
- QFT could generally violate NEC locally (e.g. squeezed state) but it obeys ANEC in many scenarios:
  1, ANEC is proved for a relativistic QFT in flat space [Faulkner-Leigh-Parrikar-Wang 16']
  2, ANEC should obey for achronal infinite null geodesics [Kontou-Olum 13'15'] [Graham-Olum 07'] [Kelly-Wall 14']
  3, ANEC violation for achronal asymptotic regions contradict with generalized second law. [Wall 10']
- Hope: ANEC is violated in **chronal** spacetime with **quantum** effect (e.g. Casimir effect)

Chronal versus achronal

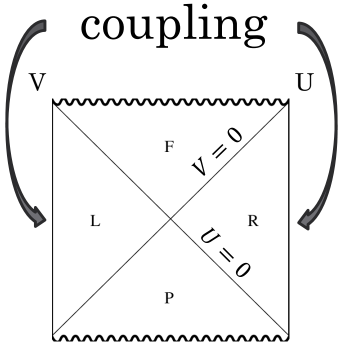
chronal



achronal



Traversable wormhole



- To generate some negative energy into the black hole spacetime, we just need to couple the two-sided boundaries.
- The coupling can be turned on at t=0. $\delta H = \mu O_L O_R$.
- The bulk dual stress tensor is proportional to μ at leading order. We can choose a sign of μ such that the stress tensor has negative energy, which renders the ER bridge to a traversable wormhole.

[PG-Jafferis-Wall 17']

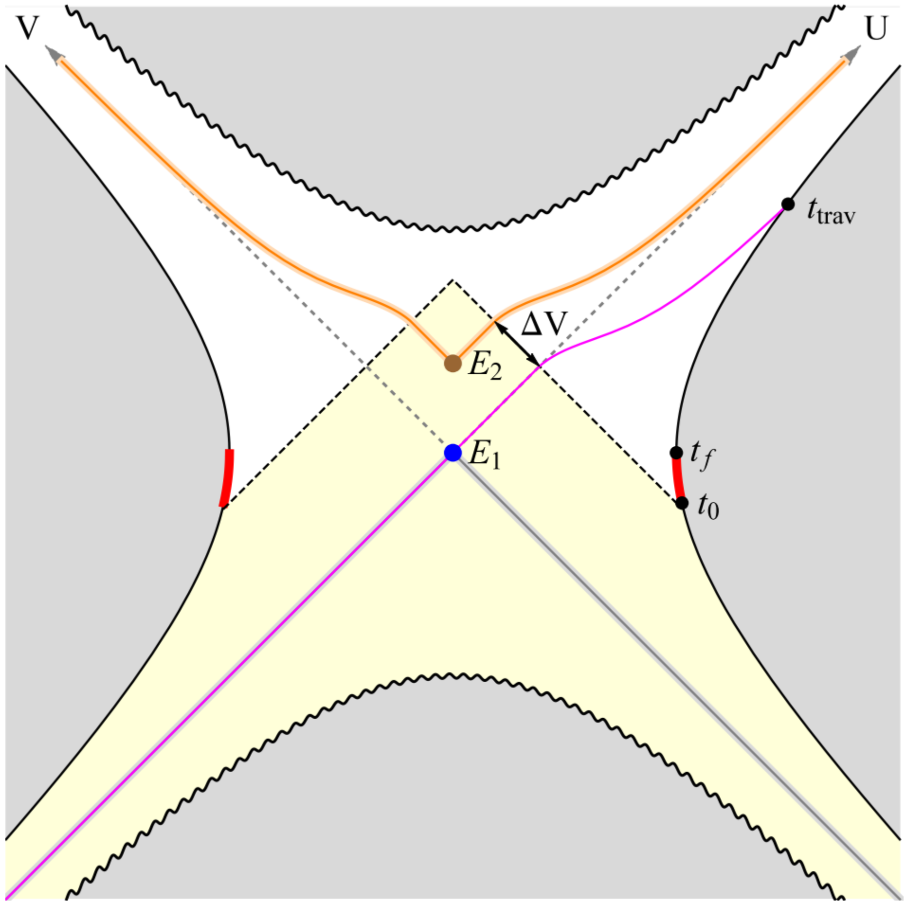
- For a BTZ black hole, the stress tensor is deformed on the horizon V=0 from U=U_0 to infinity. The integration of stress tensor along horizon is

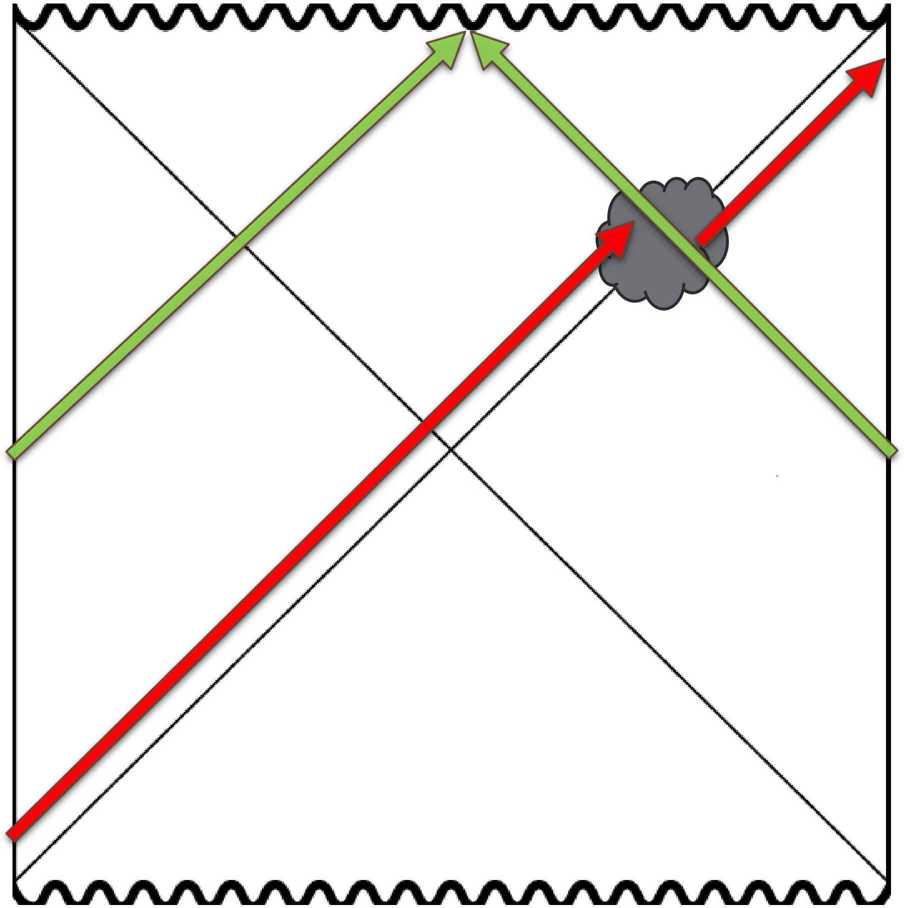
$$\int_{U_0}^{\infty} dU T_{UU} = -\frac{\mu\,\Gamma(2\Delta+1)^2}{2^{4\Delta}(2\Delta+1)\Gamma(\Delta)^2\Gamma(\Delta+1)^2\ell}\,\frac{{}_2F_1(\frac{1}{2}+\Delta,\frac{1}{2}-\Delta;\frac{3}{2}+\Delta;\frac{1}{1+U_0^2})}{(1+U_0^2)^{\Delta+1/2}}$$

$$8\pi G_N \int dU T_{UU} = \frac{(d-2)}{4}\left((d-3)r_h^{-2}+(d-1)\ell^{-2}\right)\int dU h_{UU}$$

$$V(U) = -(2g_{UV}(0))^{-1}\int_{-\infty}^{U} dU h_{UU}$$

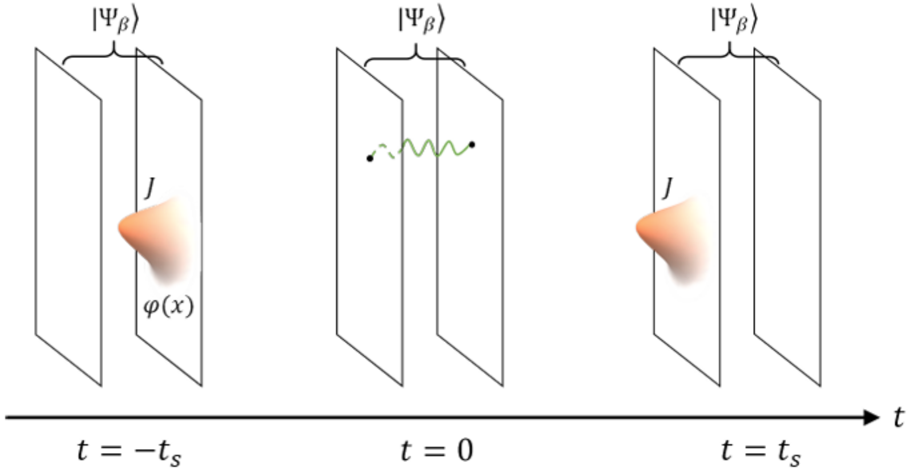
Traversable wormhole





Regenesis in CFT

- A signal traveling through a traversable wormhole could be understood as the following phenomenon in the dual CFT [PG-Liu 19']
- Consider two identical CFTs in the thermofield double state. These two CFTs are decoupled. An excitation in one CFT quickly decays due to thermal dissipation. Then we couple two CFTs at t=0 instantaneously. After a long time t_s, this excitation reappears in the other CFT. We call this phenomenon as "regenesis".
- This time scale t_s is called scrambling time an is of order log(N), where N denotes the total degrees of freedom of boundary CFT. In two dimension, N is the central charge c of the CFT$_2$.



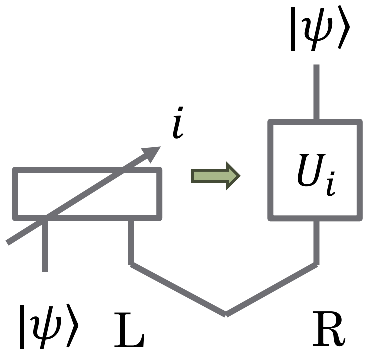
Traversable wormhole

- This is the **first** solution of traversable wormholes that is **consistent** with general properties of a **QFT** and can be embedded into a **UV complete theory** (by AdS/CFT)
- This is a very general mechanism to produce traversable wormholes
- It provides the first operational way to verify ER=EPR, the fundamental connection between quantum gravity and quantum information.
- It provides a new way to study physics behind horizon by making some regions visible to an exterior observer.
- It is dual to many-body quantum teleportation, and builds a new and concrete connection to quantum communication and quantum simulation.
- This mechanism has many generalizations: AdS_2, eternal traversable wormhole, 4d traversable wormhole using SM matters, traversable wormhole phenomenon in condensed matter physics etc.

[Maldacena-Stanford-Yang 17'] [Maldacena-Qi 18'] [Maldacena-Milekhin 20'] [Plugge-(Lantagne-Hurtubise)-Franz 20']

Teleportation

- Quantum teleportation is a way to transit quantum state with assistance of entanglement.
- The dual process of a particle carrying information traveling through a traversable wormhole from one side to the other side is a teleportation in a many-body system.
- The instantaneous coupling $e^{i\mu O_L O_R}$ can be formally written as



$$\int \delta(O_L - x) e^{i\mu x O_R} dx$$

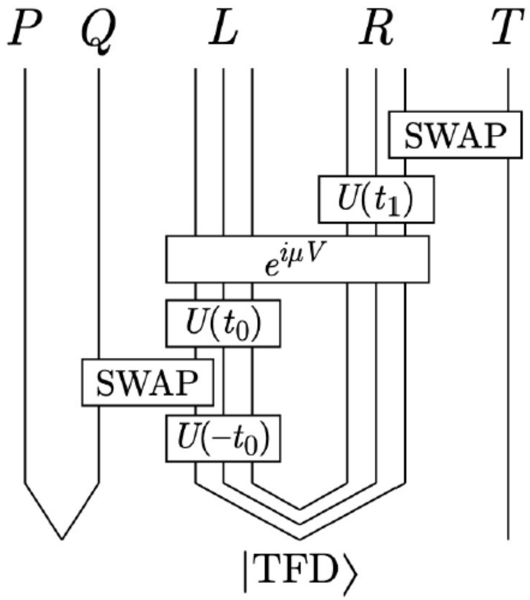
Measurement on the left

Unitary on the right

Teleportation

- This teleportation protocol can be explicitly written down as a quantum circuit.
- But what is the simplest holographic model on which we can implement the teleportation protocol and exhibit traversable wormhole dynamics?
- The SYK model.

[PG-Jafferis 21']



$V = O_L O_R$

SYK model and AdS$_2$

[Sachdev-Ye 93', Kitaev 19']

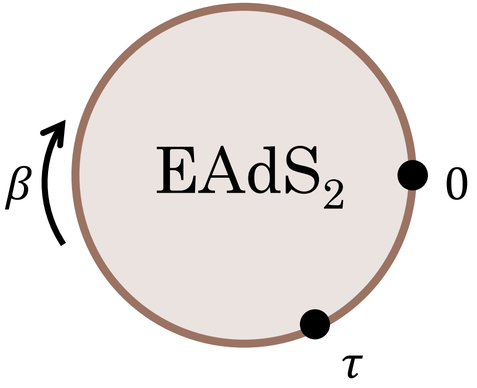
[Stanford-Maldacena 16']

- SYK model is 0+1 dimensional (quantum mechanics)
- It consists of N Majorana fermions, all-to-all q-local randomly coupled Hamiltonian

$$\{\psi_i,\psi_j\} = \delta_{ij} \qquad H = \sum_{i_1<i_2\cdots<i_q} J_{i_1\cdots i_q}\psi_{i_1}\cdots\psi_{i_q} \qquad \left\langle J^2_{i_1\cdots i_q}\right\rangle = \frac{J^2(q-1)!}{N^q}$$

- In low temperature, it has a conformal limit, and is dual to AdS$_2$
- The Euclidean low temperature 2-pt function is

$$G(\tau) = \left(\frac{\#}{sinh\frac{\pi\tau}{\beta}}\right)^{2\Delta} \qquad \Delta = 1/q$$



- The low energy spectrum is $\rho(E)\sim e^{S_0} sinh\sqrt{\gamma E}$, $S_0 \sim O(N)$

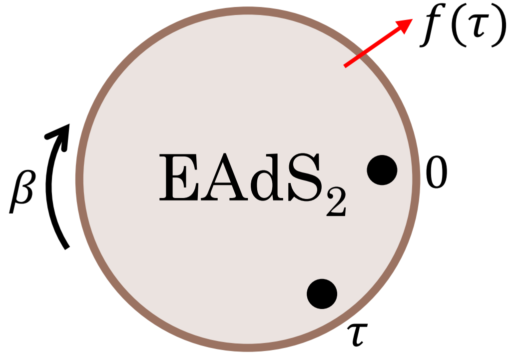
SYK model and AdS_2

[Maldacena-Stanford 16']
[Maldacena-Stanford-Yang 16']

- The fluctuation of SYK around low temperature can be captured by an effective action of Schwarzian derivative.
- This Schwarzian effective action can be derived by Jackiw–Teitelboim gravity, which is a dilaton-Einstein gravity in 2D with AdS_2 geometry.
- AdS_2 has no propagating degrees of freedom in the bulk (unlike d>3 gravity). It only has graviton degrees of freedom on the asymptotic AdS boundary.
- It describes the near horizon physics of a near-extremal black hole.
- These d.o.f. are reparameterization modes $f(\tau)$ described by Schwarzian.

$$(Sf)(z) = \left(\frac{f''(z)}{f'(z)}\right)' - \frac{1}{2}\left(\frac{f''(z)}{f'(z)}\right)^2 = \frac{f'''(z)}{f'(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

- Matter fields (gravitationally) couple to $f(\tau)$

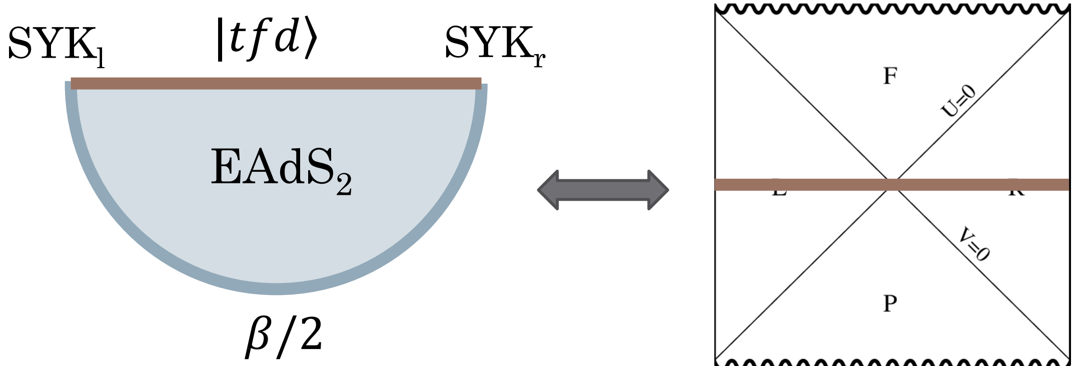


SYK model and AdS$_2$

- The thermofield double state in two identical SYK models is dual to a two-sided eternal black hole in AdS$_2$.

$$\{\psi_i^a, \psi_i^b\} = \delta^{ab}\delta_{ij} \qquad a, b = l, r$$

$$H_a = \sum_{i_1 < i_2 \cdots < i_q} J_{i_1 \cdots i_q} \psi_{i_1}^a \cdots \psi_{i_q}^a$$



SYK$_l$ $|tfd\rangle$ SYK$_r$

EAdS$_2$

$\beta/2$

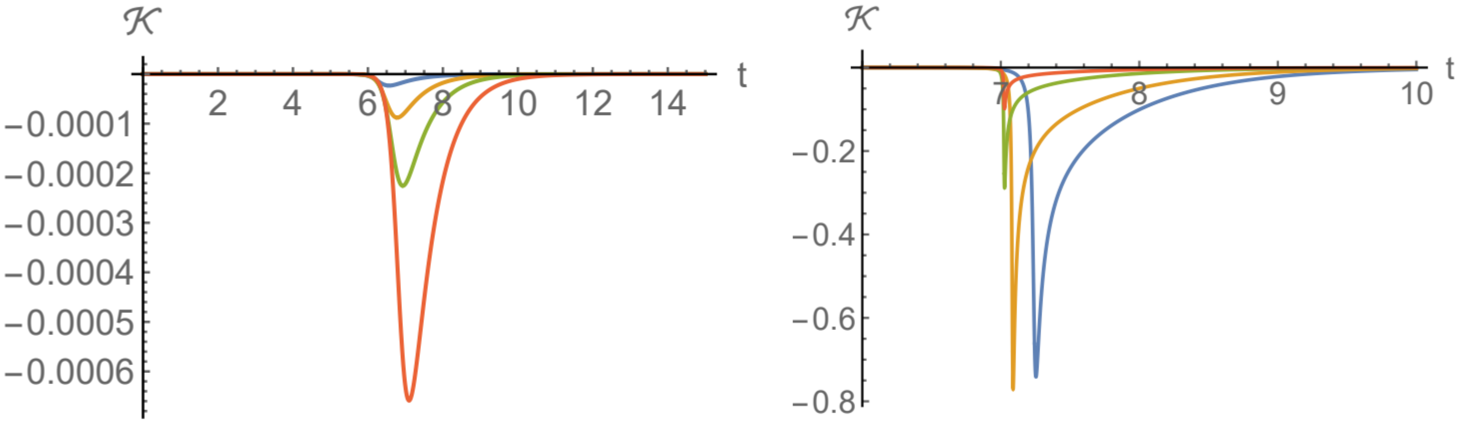
- Two SYK systems are entangled but not coupled.
- To turn on a traversable wormhole, we need to couple them.

Traversable wormhole in SYK model and AdS$_2$

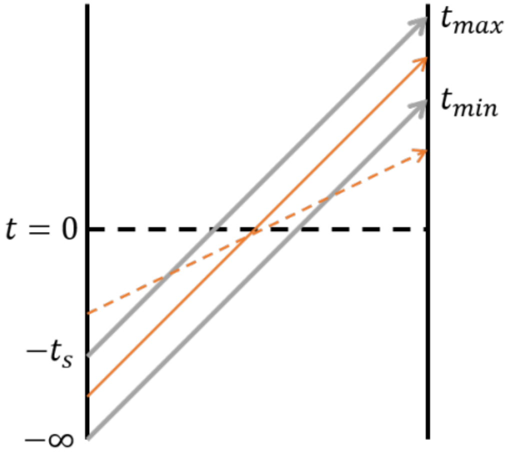
- We turn on the coupling $V = i\sum_{i=1}^{N} \psi_i^l \psi_i^r$ at t=0.
- To check traversability, we can compute the causal anti-commutator [PG-Jafferis 21']

$$\mathcal{K}(t,t') = \langle tfd|\{e^{i\mu V}\psi_i^r(t)e^{-i\mu V}, \psi_i^l(-t')\}|tfd\rangle$$

[In low temperature]



(a) $t' < t_s$ (b) $t' > t_s$



- The peak appears only for one sign of μ.
- The signal ordering is preserved for the classical traversable wormhole window.

Quantum simulation of a traversable wormhole

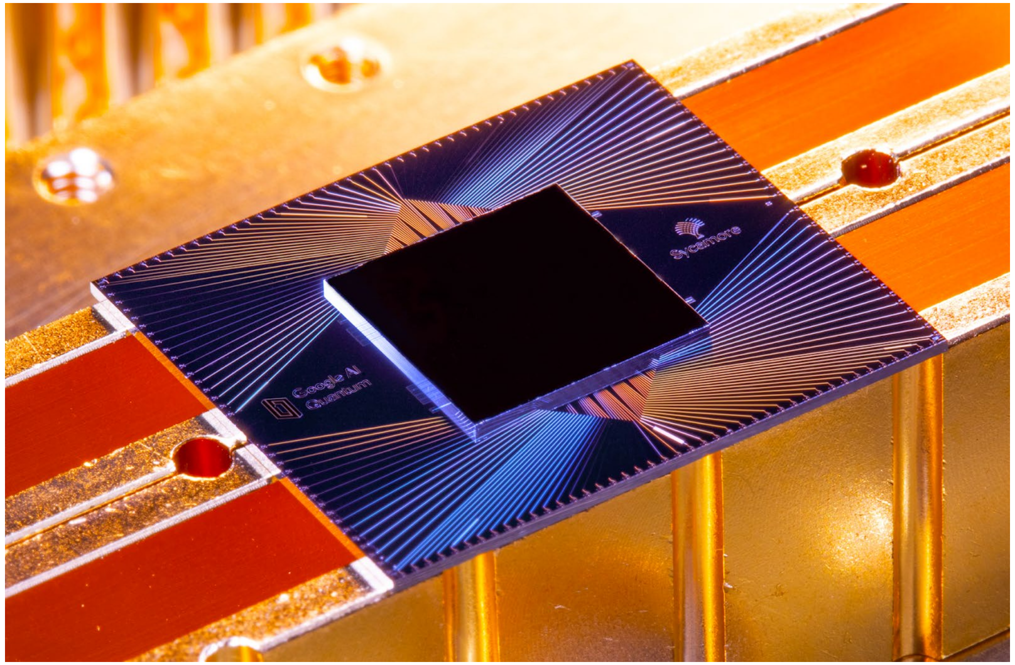
- Why do we want to simulate quantum gravity in a lab? [Susskind 17']
- Black hole is the fastest quantum computer. Fast scrambling. Black hole dynamics might be helpful to understand large scale quantum computation.
- We poorly understand quantum gravity beyond semiclassical. Decent quantum simulation may inspire us to construct better quantum gravity theory. Especially the nontrivial dynamics beyond black hole horizon.

Teleportation through operator spreading

- To simulate a traversable wormhole in semiclassical sense on a quantum computer, we need low T. But this requires long time Euclidean evolution to prepare the thermofield double state. However, decoherence of qubits limits the total number of gates in a quantum circuit.
- An easier task is to implement the same teleportation protocol in high T, which only requires EPR pairs.
- We find that in high T, the teleportation in many-body systems is far from semiclassical traversable wormhole. It occurs at relatively early time (before scrambling time) and relies on the size of an operator being highly-peaked.
- Roughly speaking, a signal is not fully scrambled into the whole system, and its distribution in the "operator space" is highly peaked, and we can teleport it just like a single yet "big" qubit.
- This mechanism is universal in high T, but its form is similar to **traversable wormholes with stringy corrections**.
- We made a few concrete experimental proposals. [Schuster-Kobrin-PG et. al., PRX 22']

Quantum simulation of a traversable wormhole

- Can we simulate the through-a-wormhole teleportation process on a quantum processor? Especially the low temperature behavior because it corresponds to semiclassical gravity.



Quantum simulation of a traversable wormhole

- Can we simulate the through-a-wormhole teleportation process on a quantum processor?
- Yes and No?
- No: we need large numbers of qubits because holography that is dual to semiclassical gravity is in the regime of large N.
- Yes: probably 50-100 is enough
- No: N=50 q=8 SYK Hamiltonian has 536M terms.
- Yes: truncate SYK and hope it still exhibits some gravity behavior

Quantum simulation of a traversable wormhole

- Sparse SYK shows the same holographic SYK dynamics plus (irrelevant) gapped fluctuations.

[Xu-Susskind-Su-Swingle 20']

- How to confirm that the teleportation is through a traversable wormhole?
  1, sign of μ
  2, ordering of signal
  3, size-winding (no time to explain today)
  ……

Quantum simulation of a traversable wormhole

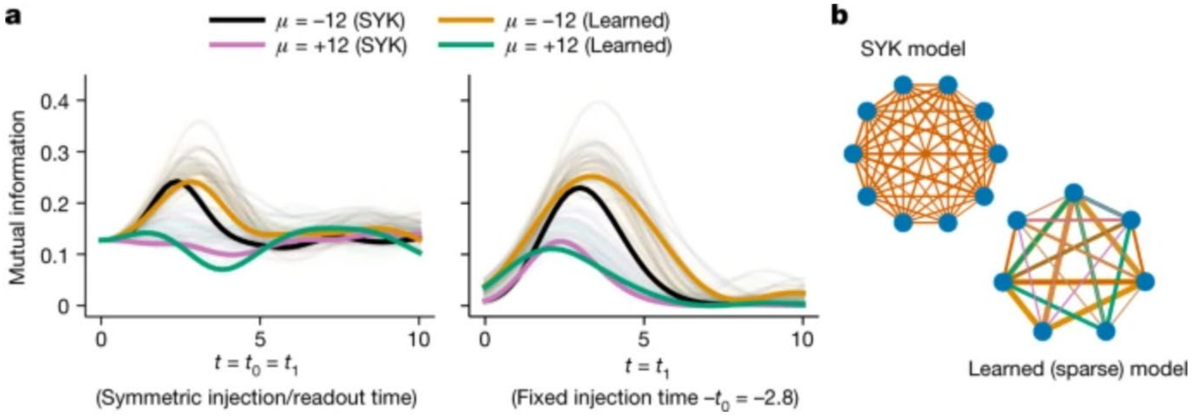
- How sparse can the model be?

[Jafferis et. al., Nature 22']

- In [Jafferis et. al.], they use machine learning on the random coefficients of N=10 SYK model by maximize the μ sign difference.
- In the end they learned a N=7 SYK Hamiltonian.

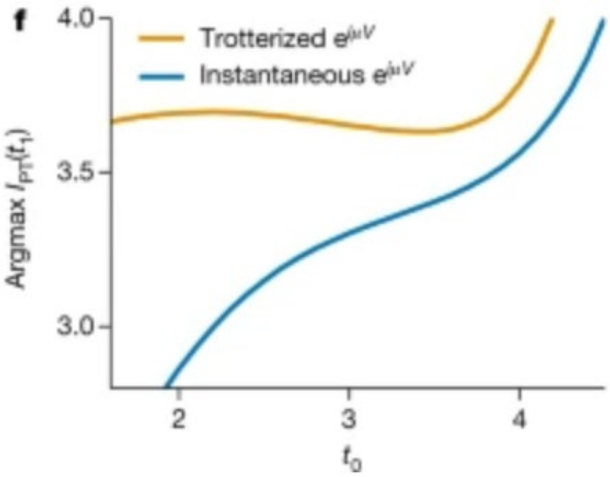
$$H_{\mathrm{L,R}} = -0.36\psi^1\psi^2\psi^4\psi^5 + 0.19\psi^1\psi^3\psi^4\psi^7 - 0.71\psi^1\psi^3\psi^5\psi^6$$
$$+0.22\psi^2\psi^3\psi^4\psi^6 + 0.49\psi^2\psi^3\psi^5\psi^7,$$

Fig. 2: Learning a traversable wormhole Hamiltonian from the SYK model.



Quantum simulation of a traversable wormhole

- Our teleportation protocol in [PG-Jafferis 21'] for SYK model was adopted by this theory and experiment group and applied on Google's quantum processer Sycamore.
- They checked size-winding property
- They did not find the signal ordering preserved for one-time interaction. But with three-time interaction (trotterized), they found a regime in which the signal ordering is preserved.



Critiques

- The learned Hamiltonian is commuting. All terms commute with each other.
- The commuting SYK is integrable and solvable. Non-holographic.

[Kobrin-Schuster-Yao 23’]
[PG 24’]

- N is too small, the size winding only has three effective points (to align three points along a straight line is a bit trivial)
- Commuting SYK model itself indeed exhibits many features that look like holographic and are also used as evidence in [Jafferis et. al.]. However, as we increase N, these features are gone.

		Size-winding in high T	**Sign difference of μ**	**Signal ordering preserved**
Small N	t~O(1)	Yes	Yes	Yes with fine tuning
Large N	t~O(1)	Yes and peaked	Yes	Yes
	t~O($\sqrt{N}$)		No	No

[PG 24’]

From integrability to holography

- From the analysis of commuting SYK model, to have a decent simulation of quantum gravity, especially semiclassical traversable wormhole dynamics, we need to include non-commuting terms in the Hamiltonian.
- But how many is necessary? More means harder for simulation.
- We can consider adding d copies of commuting SYK models and ask the low temperature behavior: what is the lowest temperature for a fixed d to see holographic behavior?
- We find that to have exact holography (namely exact SYK spectrum) we need infinite d.
- However, by a rough analysis, given a large d, holographic dynamics holds for $T>T_c$ with $T_c \sim O(e^{-d})$. This implies that we do not need too many non-commuting terms in a decent simulation. This gives an explicit estimation for future quantum simulation of quantum gravity.

[PG-Lin-Peng work in progress]

Conclusion

- Quantum gravity has a deep connection with quantum entanglement and quantum information under the slogan of “ER=EPR”
- Traversable wormholes emerge by coupling two black holes in the thermofield double state, which allows a particle traveling through the Einstein-Rosen bridge.
- This is the first traversable wormhole consistent with QFT principles and provides the first test for “ER=EPR”.
- Traversable wormhole uncovers regions behind horizon and is an excellent tool to study black holes
- Traversable wormhole is dual to quantum teleportation in many-body systems (e.g. in the SYK model)
- A better simulation of the traversable wormhole dynamics on a near-term quantum computer will help us understand quantum gravity deeper.

Thank you!