CHAOS AND ORDER

IN HIGH DIMENSIONAL COVARIANT DISORDERED MODELS

Cheng Peng (彭程)

Kavli Institute for Theoretical Sciences (KITS)

University of Chinese Academy of Sciences (UCAS)

中国科学技术大学 彭桓武高能基础理论中心

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中国科学院大学卡弗里理论科学研究所
Kavli Institute for Theoretical Sciences at UCAS

CONTENT

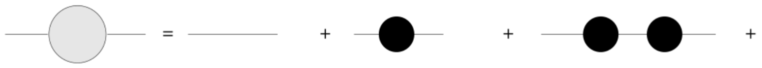
- **Background**
- **Motivation**
- **1+1D disordered models**
- **2+1D SYK models**
- **General 2+1D disordered models**

Recently, disordered models have attracted lots of attention in the High Energy Theory community.

BACKGROUND: SYK

The Sachdev-Ye-Kitaev (SYK) model:
strongly coupled 0+1d quantum mechanics model

- ✓ $H = \sum_{1\le j_1\le j_2\cdots\le j_q\le N} J_{j_1 j_2\cdots j_q}\psi^{j_1}\psi^{j_2}\cdots\psi^{j_q}$, $\langle J_{j_1 j_2\cdots j_q}\rangle = 0$, $\langle J^2_{j_1 j_2\ldots j_q}\rangle = \frac{J^2(q-1)!}{N^{q-1}}$
- ✓ perturbatively solvable



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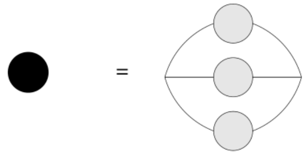
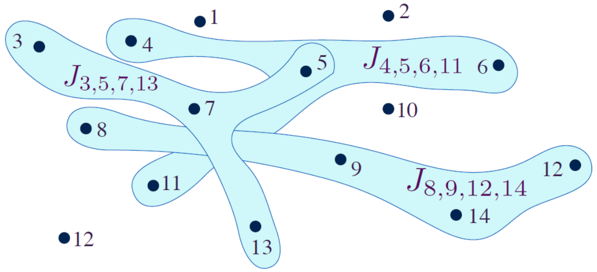


- ✓ Correlation functions, operator spectrum, chaotic behavior, thermodynamical properties…



(Figure from Phys.Rev.X 5 (2015) 4, 041025)

Sachdev, Ye,
Parcollet, Georges,
Kitaev,
Maldacena, Stanford,
…

THE SYK MODEL

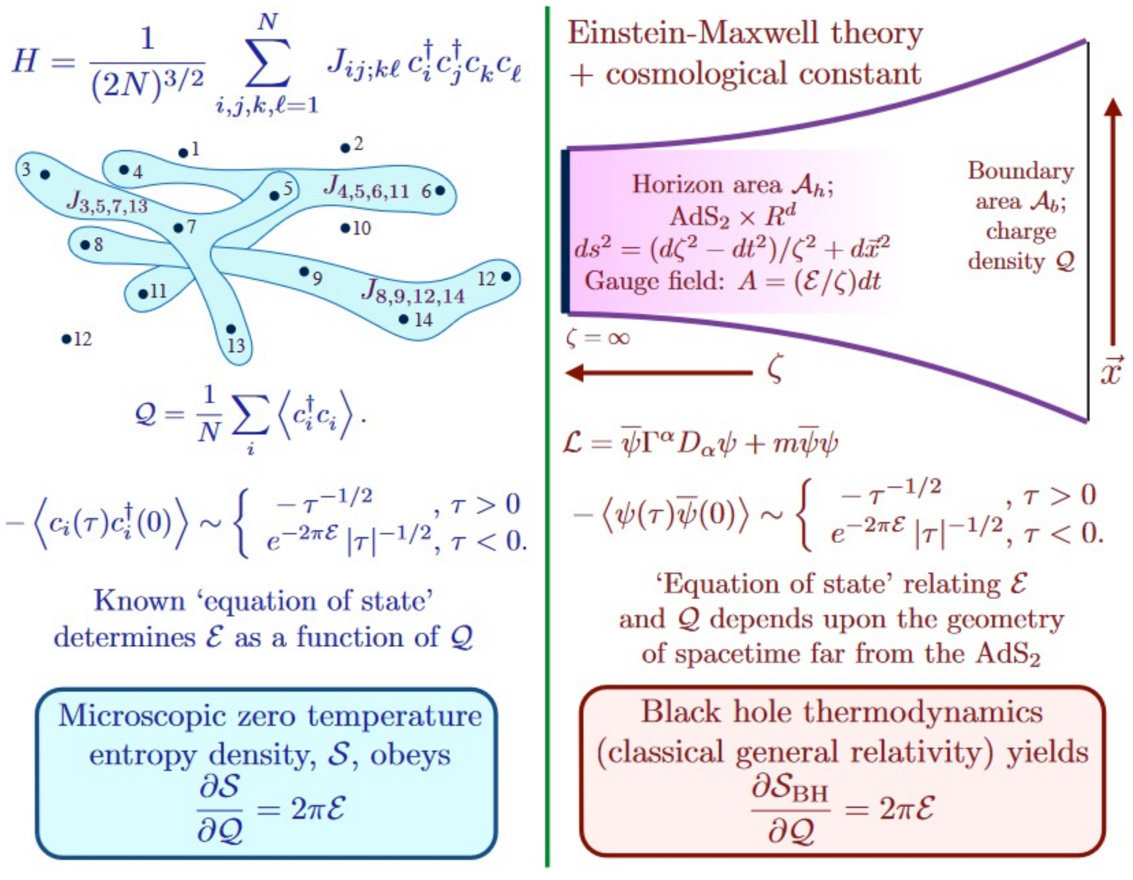
- Quantum chaos:
  Out of Time Order Correlators (OTOC)

  $$\langle \psi_1(t_1)\psi_2(0)\psi_1(t_1)\psi_2(0)\rangle \propto \frac{1}{N}\, e^{\lambda_L t}$$

  - λ_L: Lyapunov exponent
    positive λ_L indicates early time chaotic behavior of the theory
  - gravity is also chaotic

- Thermodynamics

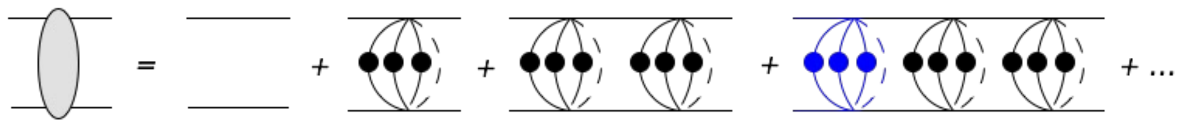
- The SYK model leads to a simple, solvable example of the holography



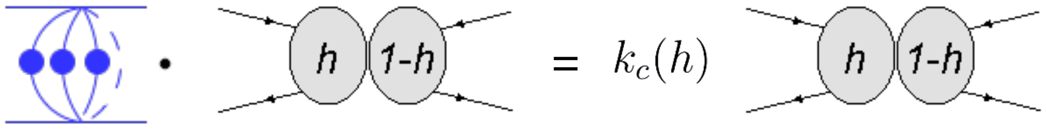
(Figure from Phys.Rev.X 5 (2015) 4, 041025)

RECALL: SYK

- Operator spectrum



$$\mathcal{F} = \frac{1}{1-K}\mathcal{F}_0$$



$$\mathcal{F}(\chi) = \frac{1}{1-K_c}\mathcal{F}_0 = \sum_h \Psi_h(\chi)\frac{1}{1-k_c(h)}\frac{\langle \Psi_h, \mathcal{F}_0\rangle}{\langle \Psi_h, \Psi_h\rangle}$$

$$k_c(h) = 1 \qquad \Rightarrow \qquad h_m = 2\Delta + 1 + 2m + \epsilon_m \qquad \mathcal{O}_m \sim \psi^i(\tau)\partial_\tau^{2m+1}\psi^i(\tau)$$

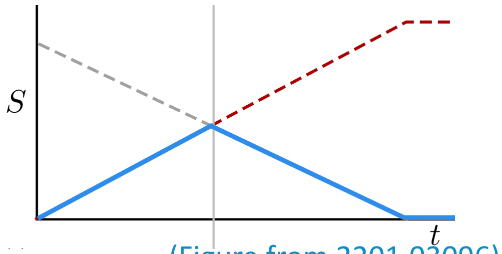
- An infinite tower of operators (with finite anomalous dimensions)

Disordered models are special cases of ensemble average theories that are often relavant in holographic dualities.

BH EVAPORATION AND ENSEMBLE AVERAGE

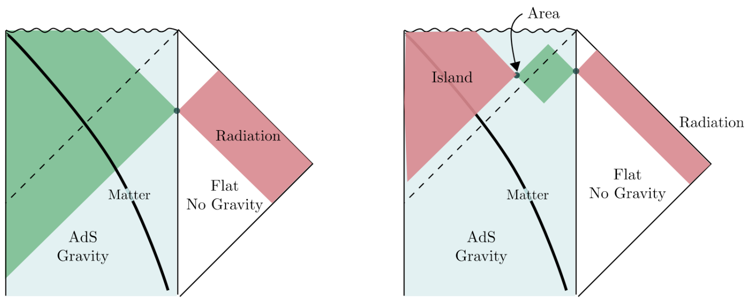
Penginton; Almheiri, Engelhardt, Marolf, Maxfield;Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini; Penington, Shenker, Stanford, Yang …

A "solvable" incarnation of the information paradox



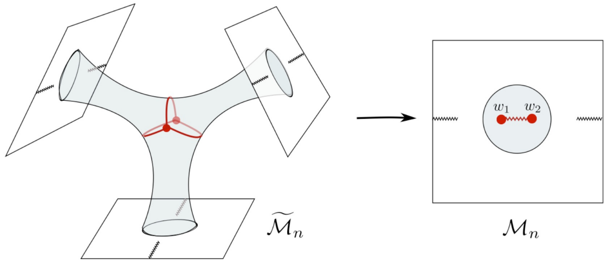
(Figure from 2201.03096)

- The information paradox:
  Are Hawking radiations from Blackholes thermal or informative?



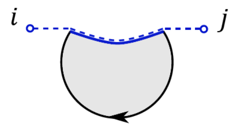
(Figure from 1911.12333)

- Recent breakthroughs in this puzzle in low-dimensional solvable toy models
  - New quantum extremal surface in an evaporating black hole
  - Alternatively, the necessity of including the spacetime wormholes in the gravitational path integral

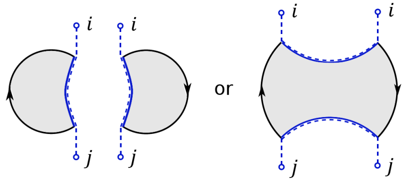


BH EVAPORATION AND ENSEMBLE AVERAGE

- Spacetime wormholes are tied with ensemble averages of theories (Coleman; Giddings Strominger; Maldacena Maoz)
- Evidence including e.g.



$$\overline{\langle \psi_i | \psi_j \rangle} = \delta_{ij},$$



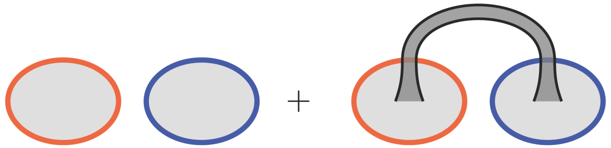
$$\overline{|\langle \psi_i | \psi_j \rangle|^2} = \delta_{ij} + \frac{Z_2}{Z_1^2}$$

$$\langle \psi_i | \psi_j \rangle = \delta_{ij} + x_{ij}, \quad \overline{x_{ij}} = 0, \quad \overline{x_{ij}^2} = \frac{Z_2}{Z_1^2}$$

(Penington, Shenker, Stanford, Yang 2020)

$$\Big\langle Z[J_1] \cdots Z[J_n] \Big\rangle := \int_{\Phi \sim J} \mathcal{D}\Phi \, e^{-S[\Phi]}$$

$$\Big\langle Z[J_1] Z[J_2] \Big\rangle =$$



$$\langle Z^n \rangle = \sum_{p \perp \{1,2,\ldots n\}} \lambda^{|p|} = B_n(\lambda) = \sum_{d=0}^{\infty} d^n p_d(\lambda) = \langle x^n \rangle_{\text{Pois}}, \qquad p_d(\lambda) = e^{-\lambda} \frac{\lambda^d}{d!}$$

(Marolf, Maxfield, 2020
CP, Tian, Yu 2021, **CP**, Tian, Yang 2022)

- Disordered models are special cases of the "ensemble average theories"

CONTENT

MOTIVATION

From a high energy physicist's point of view, the world we live in is usually described by covariant quantum field theories in high(er than 0+1) spacetime dimensions.

However, a conventional quantum field theorist would wonder if averaging over a set of different theories (or actions) is a well-defined operation.

Therefore a set of questions naturally arise

1. if there exist high dimensional covariant disordered models
2. do they fulfill the usual requirements obeyed by conventional QFTs
3. do they share similar nice features as their low dimensional counterparts
4. if there are clear connections with other well-known conventional QFTs

CONTENT

A 2D $\mathcal{N}$=(0,2) MODEL

CP, *JHEP* 12 (2018) 065

- $$S = \int d^2 z d\theta d\bar{\theta} \left(-\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2 z d\theta \frac{J_{ia_1 \ldots a_q}}{q!} \Lambda^i \Phi^{a_1} \ldots \Phi^{a_q}$$

  Chiral: $\Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z \phi^a \, , \qquad a = 1 \ldots N$

  Fermi: $\Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z \lambda^i \, , \qquad i = 1 \ldots M$

- $N, M \gg 1$, with $\mu = \frac{M}{N}$ fixed (but tunable)

- IR solution $G_c^I(z_1, z_2) = \dfrac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}} \qquad I = \phi, \psi, \lambda, G$

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2} , \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2} , \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2} , \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2} , \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2} , \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} , \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} .$$

THE LYAPUNOV EXPONENT

Kitaev 2015
Maldacena Stanford, 2016

- Out-of-Time-Ordered Correlators

$$\langle \phi^a(t+i\tau_1,x_1)\phi^b(i\tau_2,x_2)\bar{\phi}^a(t+i\tau_3,x_3)\bar{\phi}^b(i\tau_4,x_4)\rangle$$

- $K_R^{(ij)} * \Psi_R^j = k_R^{ij}\Psi_R^i$

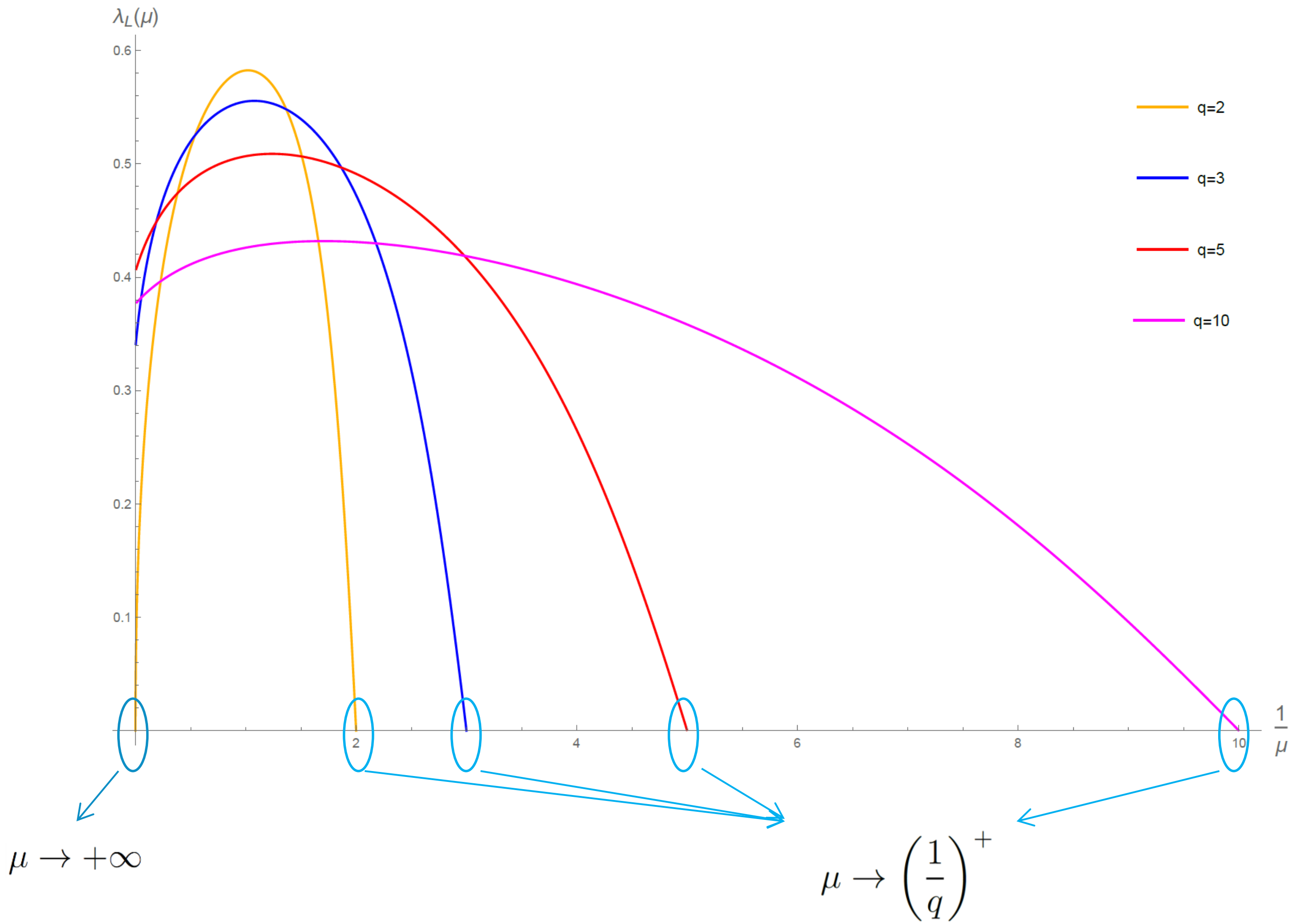
$$\Psi_R^I(1,2) = \frac{e^{-\frac{1}{2}(h+\tilde{h})(t_1+t_2)-\frac{1}{2}(h-\tilde{h})(x_1+x_2)}}{(2\cosh\frac{x_{12}-t_{12}}{2})^{h_1+h_2-h}(2\cosh\frac{x_{12}+t_{12}}{2})^{\tilde{h}_1+\tilde{h}_2-\tilde{h}}}$$

$$h = -\frac{\lambda_L}{2} + i\frac{p}{2} \qquad \tilde{h} = -\frac{\lambda_L}{2} - i\frac{p}{2}$$

- $$E_R(x,h,\tilde{h},\mu,q) = x^4 - k_R^{\phi\phi}x^3 - \left(k_R^{\phi G}k_R^{G\phi} + k_R^{\phi\psi}k_R^{\psi\phi} + k_R^{\phi\lambda}k_R^{\lambda\phi} + k_R^{\psi\lambda}k_R^{\lambda\psi}\right)x^2 + \left(k_R^{\phi\phi}k_R^{\psi\lambda}k_R^{\lambda\psi} - k_R^{\phi\psi}k_R^{\psi\lambda}k_R^{\lambda\phi} - k_R^{\phi\lambda}k_R^{\psi\phi}k_R^{\lambda\psi}\right)x + k_R^{\phi G}k_R^{\psi\lambda}k_R^{\lambda\psi}k_R^{G\phi} = 0$$

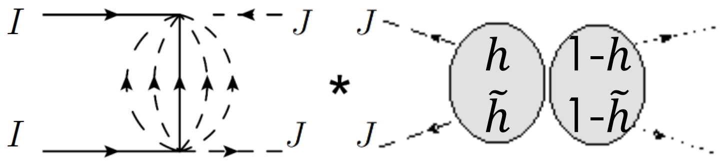
- Find λ_L by solving x=1

TWO INTERESTING LIMITS

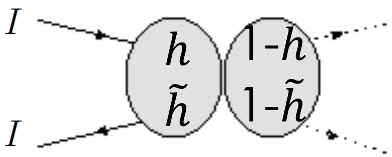


- Lyapunov exponent drops to zero
- "Integrability" takes over ?
- Large symmetries ?

4-POINT FUNCTION

- $\langle \bar{\phi}^i \phi^i \bar{\phi}^j \phi^j \rangle \quad \langle \bar{\phi}^i \phi^i \bar{\psi}^j \psi^j \rangle \quad \langle \bar{\phi}^i \phi^i \bar{\lambda}^j \lambda^j \rangle$
- 

$k^{IJ}(h, \tilde{h})$



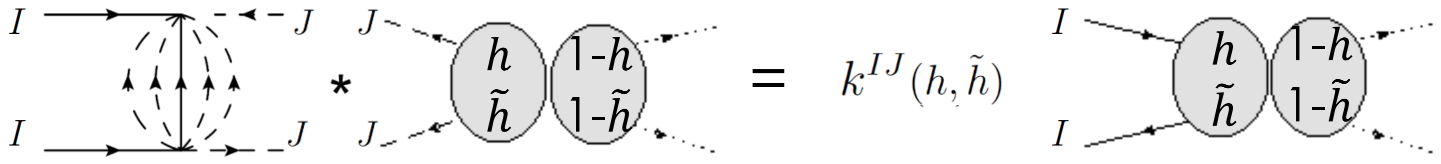
$$\Phi^I(z_1, z_2) = (z_{12})^{h-2h_I} (\bar{z}_{12})^{\tilde{h}-2\tilde{h}_I}\,, \qquad I = \phi, \psi, \lambda, G$$

$$k^{\psi\phi} = -\frac{2\mu(q-1)^2 q(\mu q-1)^2 \Gamma\left(\frac{(q-1)q\mu}{q^2\mu-1}\right)^2 \Gamma\left(\frac{-h\mu q^2+\mu q^2+\mu q+h-2}{q^2\mu-1}\right) \Gamma\left(\tilde{h}-\frac{(q-1)q\mu}{q^2\mu-1}\right)}{(\mu q^2-1)^3\, \Gamma\left(\frac{\mu q^2+\mu q-2}{q^2\mu-1}\right)^2 \Gamma\left(\frac{-h\mu q^2+(q-1)\mu q+h}{q^2\mu-1}\right) \Gamma\left(\tilde{h}+\frac{(q-1)q\mu}{q^2\mu-1}\right)}$$

…

…

4-POINT FUNCTION

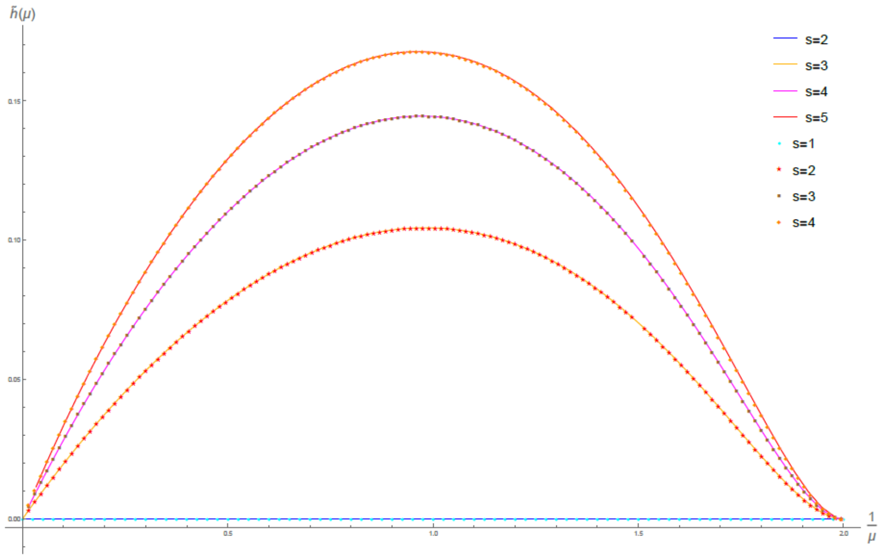
- $\langle \bar{\phi}^i \phi^i \bar{\phi}^j \phi^j \rangle \ \langle \bar{\phi}^i \phi^i \bar{\psi}^j \psi^j \rangle \ \langle \bar{\phi}^i \phi^i \bar{\lambda}^j \lambda^j \rangle$
- 

- $\begin{pmatrix} k^{\phi\phi} & k^{\phi\psi} & k^{\phi\lambda} & k^{\phi G} \\ k^{\psi\phi} & 0 & k^{\psi\lambda} & 0 \\ k^{\lambda\phi} & k^{\lambda\psi} & 0 & 0 \\ k^{G\phi} & 0 & 0 & 0 \end{pmatrix}$ whose eigenvalue x satisfies

$$E_c(x, h, \tilde{h}, \mu, q) = x^4 - k^{\phi\phi} x^3 - \left(k^{\phi G} k^{G\phi} + k^{\phi\psi} k^{\psi\phi} + k^{\phi\lambda} k^{\lambda\phi} + k^{\psi\lambda} k^{\lambda\psi}\right) x^2 + \left(k^{\phi\phi} k^{\psi\lambda} k^{\lambda\psi} - k^{\phi\psi} k^{\psi\lambda} k^{\lambda\phi} - k^{\phi\lambda} k^{\psi\phi} k^{\lambda\psi}\right) x + k^{\phi G} k^{\psi\lambda} k^{\lambda\psi} k^{G\phi} = 0$$

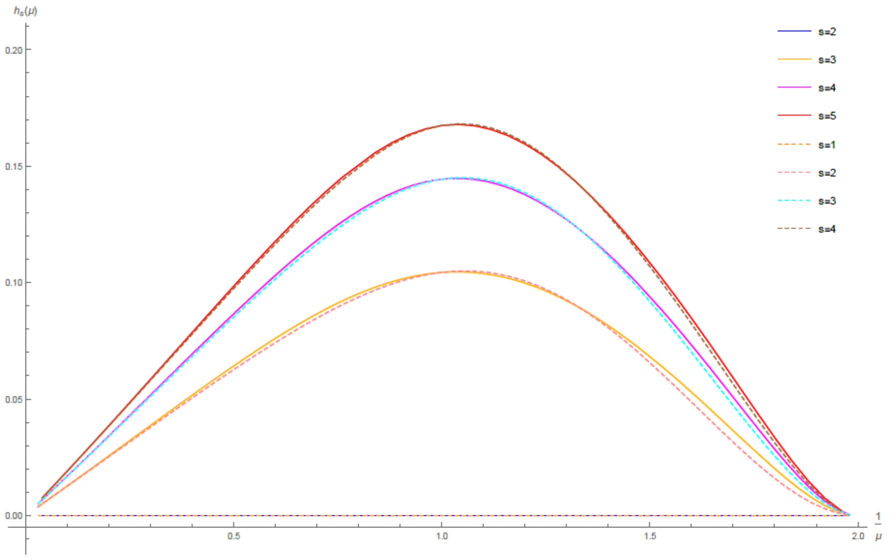
- Solve x=1 to get the spectrum of $O^{\tilde{h},h}$, spin $s = |h - \tilde{h}|$.

LIGHTEST OPERATORS WITH SPINS



$(\gamma, \gamma + s)$

$(\gamma + s, \gamma)$



operators in the two limits!

- Generate large symmetry

  nonchaotic

BACKGROUND: HIGHER-SPIN

Theories with higher-spin symmetry

- Quantum field theories

  - vector models

$$L=\frac{1}{2}\left[\partial_\mu\phi_i\partial^\mu\phi_i+\frac{\lambda}{N}(\phi_i\phi_i)^2\right] \qquad J_{\mu_1\ldots\mu_s}=\phi_i\partial_{(\mu_1}\cdots\partial_{\mu_s)}\phi_i+\ldots$$

Polyakov,Klebanov,
Giombi, Yin
Aharony, Minwalla et al...

  - W_N-minimal models

$$\frac{SU(N)_k\otimes SU(N)_1}{SU(N)_{k+1}} \qquad W^{(s)}\propto\sum_{i_1,\ldots,i_s=1}^{2}\sum_{a_1,\ldots,a_s=1}^{N}d_{a_1,\ldots,a_s}J^{a_1}{}_{(i_1)}\ldots J^{a_s}{}_{(i_s)}$$

(0,s) or (s,0), if slightly broken (γ,s+γ), (s+γ, γ)

Gaberdiel,
Gopakumar...

- Higher-spin gravity

Vasiliev ...

  - General relativity: graviton, spin-2
  - Higher-spin theory: graviton + higher-spin fields, spin-2,3,4,5… all fields are massless

BACKGROUND: HIGHER-SPIN

Higher-Spin theories are interesting:

- Quantum gravity contains higher-spin fields

- The most symmetric phase of quantum gravity — Gross, Mende...

$$m^2 = \frac{1}{\alpha'}(N+a)\,, \qquad \alpha' \to \infty\,.$$

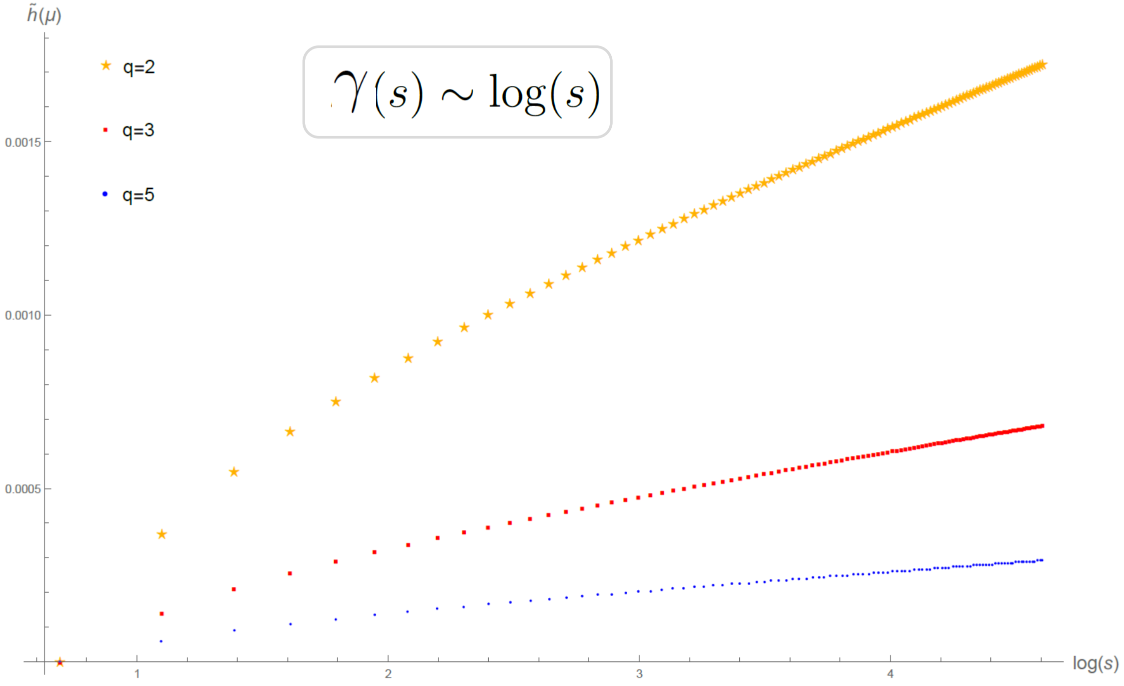
- A special class of solvable models of the holographic principle — Witten, Sundborg, Gaberdiel, Gopakumar...

$$\ell_s \gg R \gg \ell_{\text{Planck}}\,, \qquad \ell_s = \sqrt{\alpha'}\,,$$

$$\Rightarrow \quad \left(\frac{R}{\ell_{\text{Planck}}}\right)^4 = N \gg 1\,, \qquad \frac{R^4}{\alpha'^2} = \lambda = g^2 N \ll 1$$

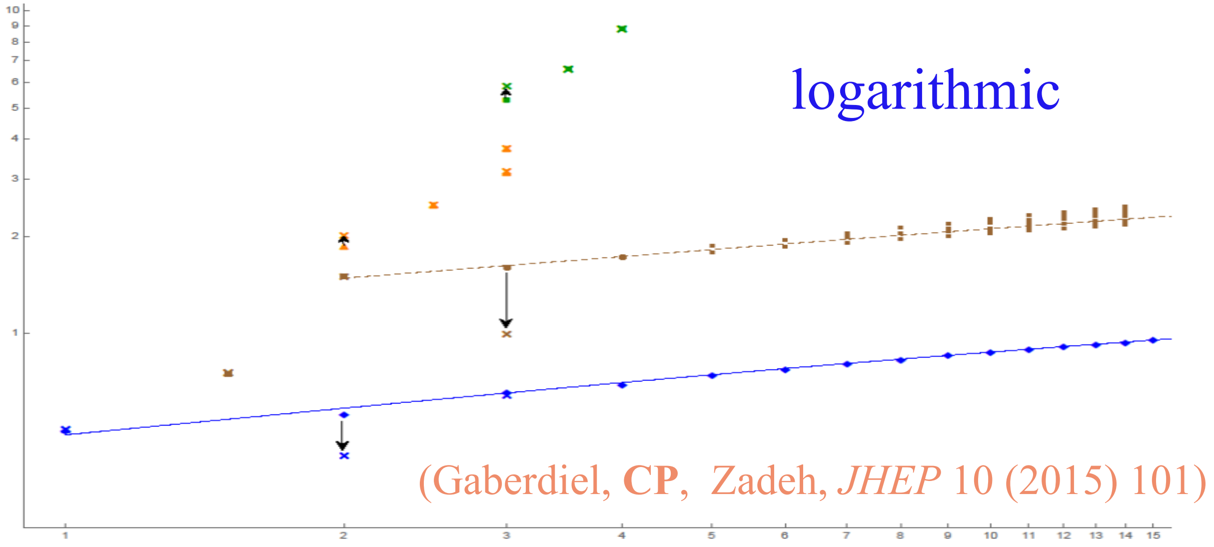
CONNECTIONS WITH CONVENTIONAL THEORIES

- Dispersion relation of this **SYK** model: the anomalous dimension γ logarithmically depends on the spin s



(**CP**, *JHEP* 12 (2018) 065)

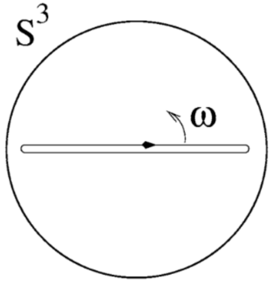
- **Higher-spin** perturbation computation



(Gaberdiel, **CP**, Zadeh, *JHEP* 10 (2015) 101)

- Rotating folded closed long **string** in AdS

$$E - S = \frac{\sqrt{\lambda}}{\pi}\ln(S/\sqrt{\lambda}) + \cdots \qquad \lambda = g_{YM}^2 N$$



logarithmic due to the AdS geometry

(Gubser, Klebanov, Polyakov, *Nucl.Phys.B* 636 (2002) 99-114)

CONTENT

- Background
- Motivation
- 1+1D disordered models
- **2+1D SYK models**
- General 2+1D disordered models

GO HIGHER…

- Many people think 1+1D is still special.
- It would be great if we can push it further to
  - Even higher dimension: 2+1D
  - Get clear relations with the other known 2+1D models

A 3D SYK MODEL

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- It turns out possible to construct an N=2 Supersymmetry SYK model

$$\mathrm{L} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_i(y^\dagger)\Phi_i(y)\right) - \left[\int d^2\theta \frac{1}{3} g_{ijk}\Phi_i(y)\Phi_j(y)\Phi_k(y) + \mathrm{c.c}\right]$$

$$\mathrm{P}(g_{ijk}) \propto e^{-N^2\frac{g_{ijk}\bar{g}_{ijk}}{J}}, \qquad \langle g_{ijk}\rangle = 0, \qquad \langle g_{ijk}\bar{g}_{ijk}\rangle = \frac{J}{N^2}.$$

with N flavors of chiral multiplets

$$\Phi(X) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^2 F(y) \qquad \bar{\Phi}(X^\dagger) = \bar{\phi}(y^\dagger) + \sqrt{2}\bar{\theta}^\alpha\bar{\psi}_\alpha(y^\dagger) + \bar{\theta}^2\bar{F}(y^+)$$

- In components:

$$\mathrm{L} = -i\bar{\psi}_i\,\partial\!\!\!/\,\psi_i + \partial_\mu\bar{\phi}_i\partial_\mu\phi_i - \bar{F}_iF_i - g_{ijk}\left(\phi_i\phi_jF_k - \psi_i\psi_j\phi_k\right) - \bar{g}_{ijk}\left(\bar{\phi}_i\bar{\phi}_j\bar{F}_k - \bar{\psi}_i\bar{\psi}_j\bar{\phi}_k\right)$$

- The model is again solvable, and its properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

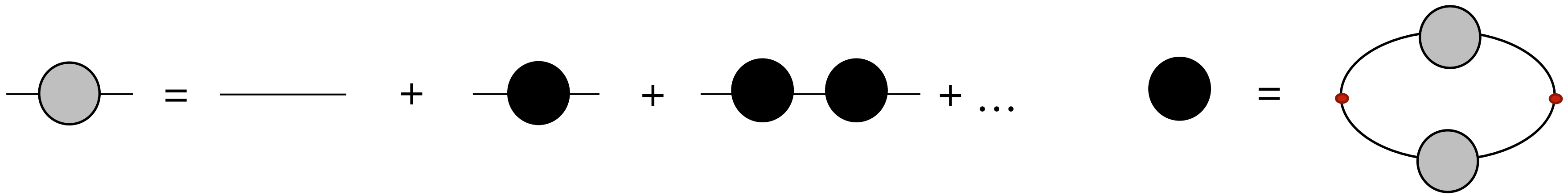
GREEN'S FUNCTIONS

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The Green's functions

$$G_\phi(x_{12})\delta_{ij} = \langle \bar{\phi}_i(x_1)\phi_j(x_2)\rangle \qquad G_\alpha{}^\beta(x_{12})\delta_{ij} = \langle \bar{\psi}_{\alpha,i}(x_1)\psi_j^\beta(x_2)\rangle \qquad G_F(x_{12})\delta_{ij} = \langle \bar{F}_i(x_1)F_j(x_2)\rangle$$

- We can efficiently sum over all loop corrections in the large-N limit



- To get the Schwinger-Dyson equations

$$G_\phi(p) = \left(p^2 - \Sigma_\phi(-p)\right)^{-1} \qquad \Sigma_\phi(x) = J\left(2G_F(x)G_\phi(x) - G_\alpha{}^\beta(x)G^\alpha{}_\beta(x)\right)$$

$$G_\psi(p) = \left(-p_\mu\sigma^\mu - \Sigma_\psi(-p)\right)^{-1} \qquad \Sigma_\alpha{}^\beta(x) = 2JG_\alpha{}^\beta(x)G_\phi(x)$$

$$G_F(p) = \left(-1 - \Sigma_F(-p)\right)^{-1} \qquad \Sigma_F(x) = JG_\phi(x)^2.$$

THE IR FIXED POINT

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- An IR conformal solution is

$$G_\phi^*(x,y) = \frac{b_\phi}{|x-y|^{2\Delta_\phi}},$$

where

$$\Delta_\phi = \frac{2}{3}, \qquad b_\phi = \frac{1}{2^{\frac{2}{3}}\sqrt{3}\pi J^{\frac{1}{3}}}$$

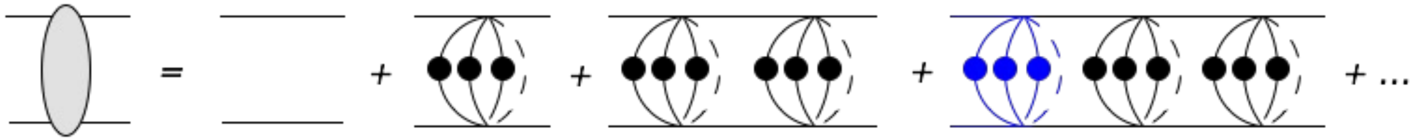
and similarly

$$\Delta_\psi = \frac{7}{6} \qquad \Delta_F = \frac{5}{3}$$

THE 4-PT FUNCTIONS

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The 4-pt function $\left\langle \bar{\Phi}_i(\bar{X}_1)\Phi_i(X_2)\Phi_j(X_3)\bar{\Phi}_j(\bar{X}_4)\right\rangle$ receives contributions from the ladder diagrams



- The ladder kernel is

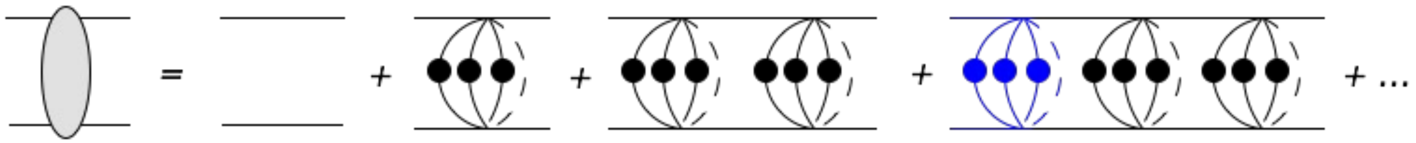
$$K(\bar{X}_1, X_2, X_3, \bar{X}_4) = 2J\,\mathrm{G}(\bar{X}_1, X_3)\mathrm{G}(\bar{X}_4, X_2)\mathrm{G}(\bar{X}_4, X_3)$$

- The 4-pt function is

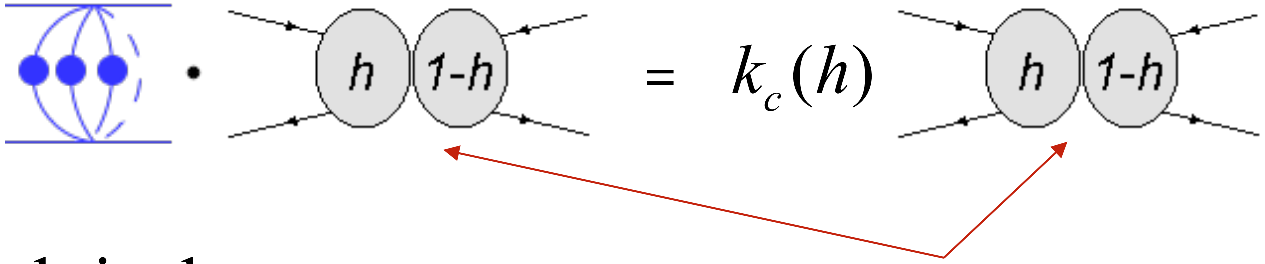
$$\mathcal{F}(u,v) = \frac{1}{1-K}\mathcal{F}_0(u,v)$$

THE 4-PT FUNCTIONS

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211



- The way to compute this formal geometric sum is to diagonalize the kernel, and compute the geometric sum of the resulting eigenvalues



- The result is thus

$$\mathrm{F}\,(u,v) = \sum_h \frac{1}{1-k_c(h)} \frac{\langle T_h \cdot \mathrm{F}_0(u,v)\rangle}{\langle T_h \cdot T_h\rangle} T_h$$

THE 4-PT KERNELS

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- The eigenfunctions that diagonalize this kernel are

$$\mathrm{T}_{\Delta,\ell}(\overline{X}_4, X_3) = |z_{43}|^{\Delta-\ell-2\Delta_\phi} z_{43,\mu_1}\cdots z_{43,\mu_\ell}\,\mathrm{A}^{\mu_1\cdots\mu_\ell},$$

where $\mathrm{A}^{\mu_1\cdots\mu_\ell}$ is a constant tensor

- The eigenvalues are determined by

$$k(\Delta,\ell)\mathrm{T}_{\Delta,\ell}(\overline{X}_1, X_2) = \int d^3x_a d^2\theta_a \int d^3x_b d^2\bar{\theta}_b K(\overline{X}_1, X_2, X_a, \overline{X}_b)\mathrm{T}_{\Delta,\ell}(\overline{X}_b, X_a).$$

which gives

$$k(\Delta,\ell) = (-1)^\ell 2^{2-2\Delta_\phi}(2\Delta_\phi - 1)\frac{\Gamma(\Delta_\phi - 1)\Gamma(2\Delta_\phi)}{\Gamma\left(\frac{\Delta_\phi}{2}\right)^2}\frac{\Gamma\left(\Delta_\phi - \frac{\Delta-\ell}{2}\right)\Gamma\left(\frac{\Delta+\ell}{2} + \frac{1-\Delta_\phi}{2}\right)}{\Gamma\left(2\Delta_\phi - \frac{\Delta-\ell}{2}\right)\Gamma\left(\frac{\Delta+\ell}{2} + \frac{1+\Delta_\phi}{2}\right)}.$$

THE IR SPECTRUM

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The IR spectrum can be read off from

$$k(\Delta,\ell)=1$$

- Super-conformal fixed points:

$\exists$ solution with $\Delta=\ell=1$, which is the supercurrent multiplet

$$\mathrm{R}_\mu = J_{R,\mu} + \theta S_\mu + \bar{\theta}\bar{S}_\mu + \theta\sigma^\nu\bar{\theta}\, T_{\mu\nu}.$$

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The IR spectrum is within the bounds obtained from numerical bootstrap

Operators	ℓ	Δ	Bootstrap bound
$(\bar{\Phi}\Phi)$	0	1.6994	<1.9098
$(\bar{\Phi}\Phi)'$	0	3.4295	<5.3
J'	1	4.2676	<5.25

Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The meaning of this check: the bootstrap bounds come from the requirement of
  - Unitarity
  - Causality
  - Locality
  - Crossing symmetry

- Consistency with the bootstrap bound means consistency with these general principles, hence no superficial contradiction to be worried about in this ensemble-average theory.

SPECTRUM: THE 3D SYK MODEL v.s. N=2 BOOTSTRAP

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Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

- Anomalous dimension $\tau = \Delta - \ell = 2\Delta_\phi + 2m + \gamma(m,\ell)$

The large-spin limit, ie fixed m, large ℓ limit

$$\gamma(m,\ell) = (-1)^{\ell+1}\frac{g_3(\Delta_\phi)}{\ell^{\Delta_\phi}}\frac{\Gamma(m-\Delta_\phi+1)}{\Gamma(m+1)}, \qquad \ell \gg 1$$

agrees with results from the light-cone analytic bootstrap

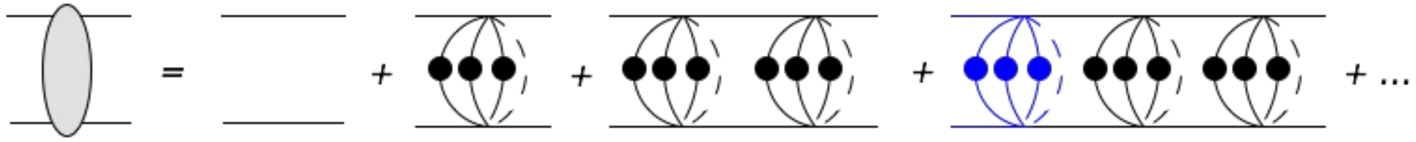
$$\gamma(m,\ell) = (-1)^{\ell}\frac{C_m}{\ell^{\tau_{\min}}} \qquad \tau_{\min} = \tau_\phi = \Delta_\phi$$

Fitzpatrick, Kaplan, Poland, Simmons-Duffin, *JHEP* 12 (2013) 004

THE FULL 4-PT FUNCTIONS

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- Compute explicitly the 4-pt correlation functions



- $$\mathrm{F}(u,v) = \frac{1}{1-K}\mathrm{F}_0(u,v) = \sum_{\ell=0}^{\infty}\int_0^{\infty} ds \frac{\langle \mathrm{T}_{\Delta,\ell}, \mathrm{F}_0\rangle}{1-k(\Delta,\ell)} \frac{\mathrm{T}_{\Delta,\ell}(u,v)}{\langle \mathrm{T}_{\Delta,\ell}, \mathrm{T}_{\Delta,\ell}\rangle}$$

$$= \sum_{\ell=0}^{\infty}\oint_{\Delta=\frac{1}{2}+is} \frac{d\Delta}{2\pi i}\rho(\Delta,\ell)\mathrm{G}_{\Delta,\ell},$$

where

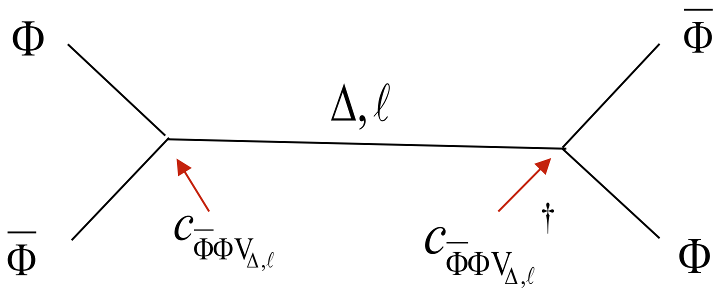
$$\rho(\Delta,\ell) \equiv \frac{1}{1-k(\Delta,\ell)} k(\Delta,\ell) \frac{2^{\ell}3\sqrt{3}\,\Gamma\left(\ell+\frac{3}{2}\right)}{(\Delta-\ell-1)\Gamma(\ell+1)} \frac{\Gamma(\Delta)\Gamma(1-\Delta+\ell)\Gamma\left(\frac{\Delta+\ell}{2}\right)^2}{\Gamma\left(\Delta-\frac{1}{2}\right)\Gamma(\Delta+\ell)\Gamma\left(\frac{1-\Delta+\ell}{2}\right)^2}$$

A 3D SYK MODEL v.s. LOCALIZATION

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- Can also read off the central charge

$$\frac{\left\langle \bar\phi(x_1)\phi(x_2)\phi(x_3)\bar\phi(x_4)\right\rangle}{\left\langle \bar\phi(x_1)\phi(x_2)\right\rangle\left\langle \phi(x_3)\bar\phi(x_4)\right\rangle} \supset -\frac{|C_{\bar\phi\phi J_R}|^2}{C_J} V_{S^2}^2\, G_{2,1}(u,v)$$



we get

$$C_J = \frac{4}{9\,|c_{\bar\Phi\Phi R}|^2} = N\,\frac{2^7}{3^4\sqrt{3}\pi}\left(\frac{2\pi}{\sqrt{3}}-\frac{9}{8}\right) \qquad C_T = 6C_J$$

- Agrees with the result from localization of the 3d N=2 Wess-Zumino model

Nishioka, Yonekura, *JHEP* 05 (2013) 165
Gang, Yamazaki, *JHEP* 02 (2020) 102

CONTENT

GENERAL 3D DISORDERED MODELS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- We can in fact consider more general 2+1d disordered models
  - ✓ with supersymmetry

$$L_{susy} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\mathcal{P}}_i(y^+)\mathcal{P}^i(y) + \bar{\mathcal{S}}_a(y^+)\mathcal{S}^a(y)\right) - \left[\int d^2\theta \frac{1}{2} g_{aij}\,\mathcal{S}^a(y)\,\mathcal{P}^i(y)\mathcal{P}^j(y) + \text{c.c}\right]$$

where $\mathcal{P}^i$ and $\mathcal{S}^a$ are chiral $\mathcal{N}$=2 multiplets with $i = 1\ldots N$ and $a = 1\ldots M$

  - ✓ or without supersymmetry

$$L_{bos} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i + \frac{1}{2} g_{aij}\sigma^a\phi^i\phi^j - \frac{1}{4}(\sigma^a)^2$$

where ϕ_i and σ^a are bosonic fields with $i = 1\ldots N$ and $a = 1\ldots M$

- Can solve the model in the large-N limit in the IR analytically

$$N \to \infty, \qquad \lambda \equiv M/N, \quad \text{fixed}$$

3D DISORDERED MODELS: HIGHER-SPIN LIMITS

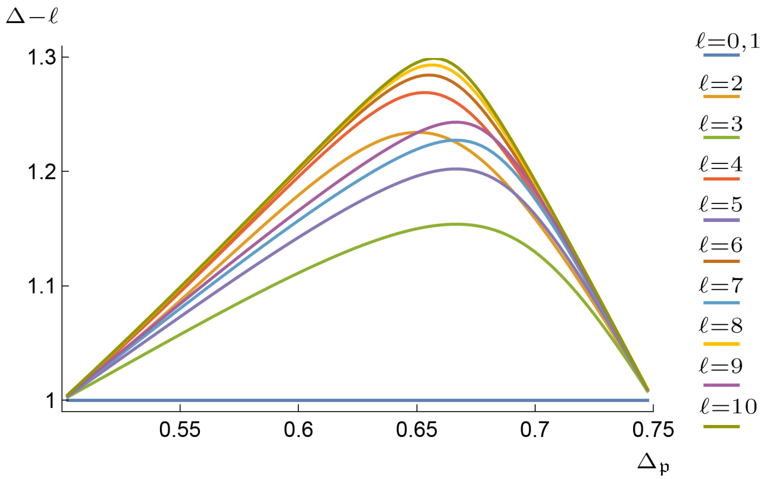
Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- Most of the details of the model are quite different from the 0+1d and 1+1d models

- Nevertheless there is again a clear connection to higher-spin theories

- There are special limits

  - Free p $\quad \lambda \to 0, \quad \Delta_p = \frac{1}{2}, \quad \Delta_s = 1$ $\quad$ higher-spin
  - Free s $\quad \lambda \to \infty, \quad \Delta_p = \frac{3}{4}, \quad \Delta_s = \frac{1}{2}$ $\quad$ higher-spin
  - Diagonal $\quad \lambda \to \frac{1}{2}, \quad \Delta_p = \frac{2}{3}, \quad \Delta_s = \frac{2}{3}$ $\quad$ the 2+1D SYK model



- This indicates the connection to higher-spin theories is probably universal

3D DISORDERED MODELS: SPECTRUM

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The anomalous dimensions at large spin ℓ asymptote to

$$\gamma_p(\ell,n) = \frac{(-1)^\ell}{\ell^{2-2\Delta_p}} g(n,\Delta_p)$$

$$\gamma_s(\ell,n) = \frac{2^{1-2\Delta_s}}{\ell^{2-\Delta_s}} \frac{\Gamma(\Delta_s)\Gamma\left(-n-\frac{3\Delta_s}{2}+1\right)}{\Gamma(-\Delta_s)\Gamma\left(-n-\frac{\Delta_s}{2}+1\right)} g(n,\Delta_s)$$

$$g(n,\Delta_\chi) = \frac{(-1)^n 4^{2\Delta_\chi-1}(\Delta_\chi-1)\sin(2\pi\Delta_\chi)\Gamma(2-2\Delta_\chi)^2}{\pi\Gamma(n+1)\Gamma(-n-2\Delta_\chi+2)}$$

again has the expected power-law behavior.

3D DISORDERED MODELS: CENTRAL CHARGE

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The central charge

$$C_T(\Delta_\phi) = -\frac{16N\cot(\pi\Delta_\phi)}{\pi}\left[2\pi\csc(2\pi\Delta_\phi)\Delta_\phi(\Delta_\phi-1)\left[(\Delta_\phi-1)\sec(2\pi\Delta_\phi)-\Delta_\phi+2\right]+\frac{3\Delta_\phi^2-3\Delta_\phi+1}{1-2\Delta_\phi}\right]$$

- It behaves as expected in the special limits

$$C_T = N\left(6-\frac{32}{\pi^2}\lambda+\mathrm{O}(\lambda^2)\right) \qquad \text{as} \quad \lambda\to 0$$

$$C_T = M\left(6+15\left(\frac{1}{2}-\frac{1}{\pi}\right)\frac{1}{\lambda}+\mathrm{O}(\lambda^{-2})\right) \qquad \text{as} \quad \lambda\to\infty$$

$$C_T = N\frac{2^7}{3^2\sqrt{3}\pi}\left(\frac{2\pi}{\sqrt{3}}-\frac{9}{8}\right) \qquad \text{as} \quad \lambda\to 1/2$$

CHAOS EXPONENT

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The operators on the principle series

$$\Delta = \frac{d}{2} + i\nu$$

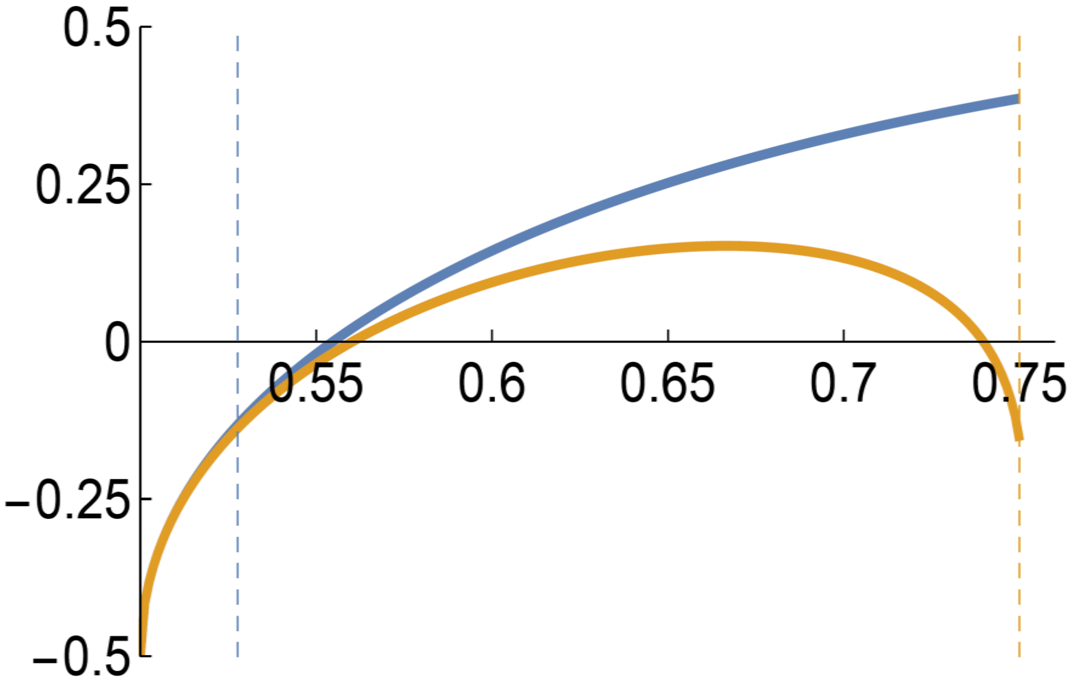
- Regge intercept of the operators

$$\ell_0(\nu = 0)$$

where the 0 labels the largest spin for a given dimension

- By a conformal mapping to the hyperbolic space, get the hyperbolic chaos exponent

$$\lambda_L^{\text{hyp}} = \ell_0(0) - 1$$



WHAT DO THESE NEW MODELS BUY US?

- New 2d/3d SCFT fixed points, and they look normal (not-strange)
- Candidates for future bootstrap discoveries
- Proof of concept examples of disordered models in higher dimensions
- Potential (rich) relations to other 3d know models
- Relation to Higher-spin theories and String theories

THANK YOU!