First order phase transition in holography and dynamic critical phenomena

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- Mainly based on
  - X. Li, Z.-Y. Nie and YT, JHEP 20 (2020) 63 [arXiv:2003.12987]
  - C.-Y. Zhang, Q. Chen, Y. Liu, W.-K. Luo, YT and B. Wang, Phys. Rev. Lett. 128 (2022) 161105 [arXiv:2112.07455]; Phys. Rev. D 106 (2022) L061501 [arXiv:2204.09260]
  - Q. Chen, Y. Liu, YT, B. Wang, C.-Y. Zhang and H. Zhang, JHEP 01 (2023) 056 [arXiv:2209.12789]

Why (applied) holography?

Three key problems to be solved for (quantum) many-body systems:

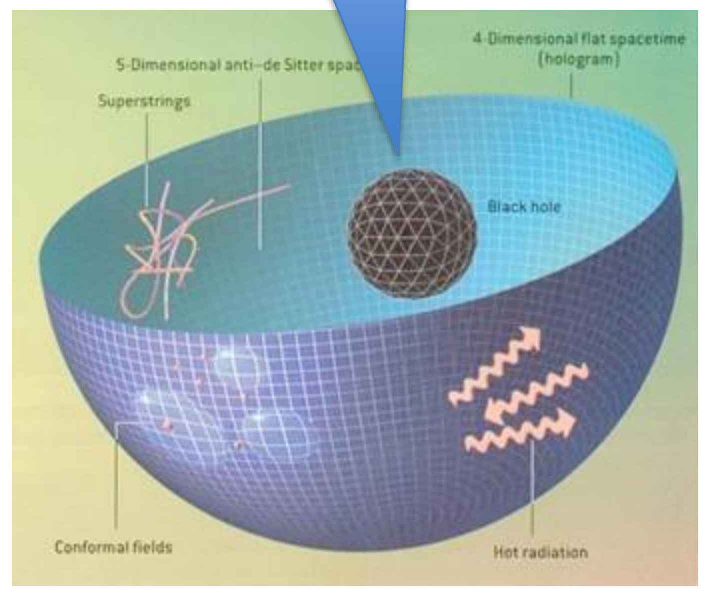
- No (local) equilibrium (hydrodynamics no longer valid)
- Strong coupling (perturbative theories no longer valid)
- Dissipation (time irreversible)

Holography deals with these problems by constructing effective field theories with one extra dimension!

Holography ↔ Gravitational theories

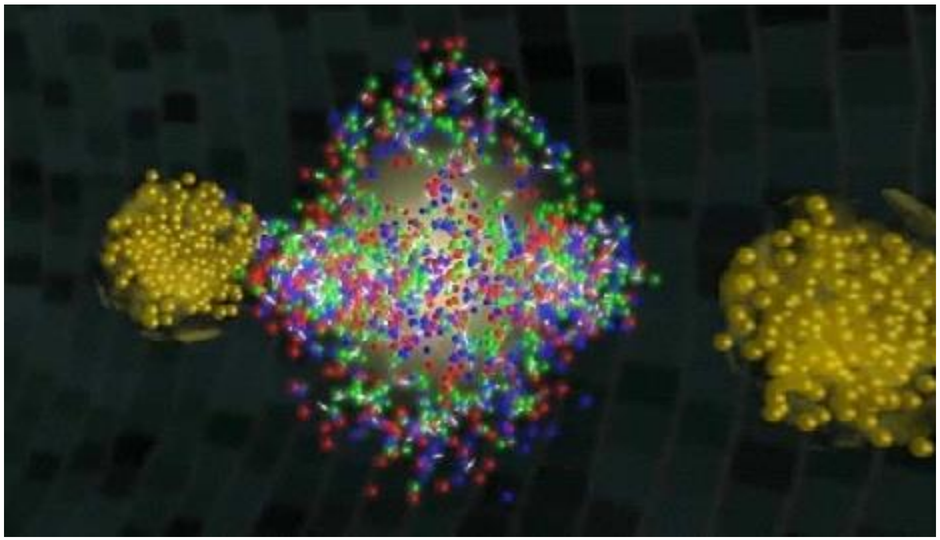
- BH membrane paradigm
- Fluid/gravity duality
- Topological charge of BHs
  YT, X. Wu and H. Zhang, JHEP 10 (2014) 170 [arXiv:1407.8273]
  YT, Class. Quantum Grav. 36 (2019) 245001 [arXiv:1804.00249]
- ......

Black hole formation/
accretion
（time irreversible）



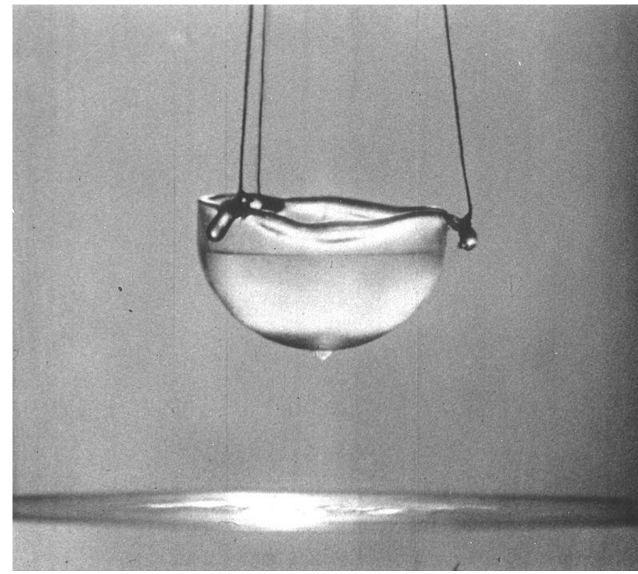
$$Z_{CFT}[J] = Z_{AdS}[\bar{\phi}(=J)]$$

Quantum many-body systems far from equilibrium



Quark-gluon plasma produced in LHC

The simplest holographic model: black hole formation from collision of gravitational waves in the bulk



He_4 superfluid

U(1) symmetry breaking + gravity in the bulk (beyond Landau’s two-fluid model)

Dynamic evolution of (holographic) gravitational systems

- Numerical simulation (numerical relativity)
- Special features of the numerical relativity used in such kind of study (compared with that used in gravitational-wave physics):
  - Matter sources and/or modified gravity
  - Symmetry reduced
  - Gauge fixed
  - Null foliation versus space-like foliation (3+1)

1 Introduction

2 1st order phase (dynamic) transition

3 Landscape and 1st order PT in holo

4 Concluding remarks

1^{st} order phase transition: importance and complexity

- 1^{st} order phase transition and its dynamical processes are ubiquitous in our world.
- The dynamical processes often go beyond the validity of thermodynamics and hydrodynamics:
  - Inhomogeneity (mixture states) during the transition
  - Bubble nucleation (evaporation) and collision

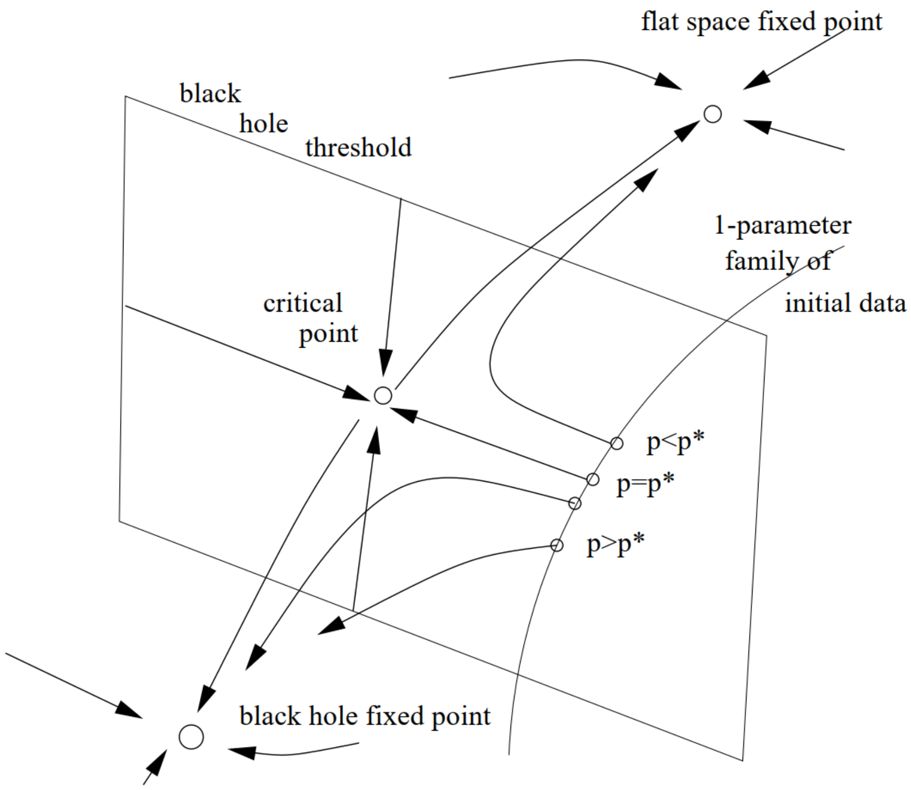
1^{st} order phase transitions in holo

- Holographic model of 1^{st} order phase transition
  [S.S. Gubser & I. Mitra, JHEP 0108 (2001) 018]
- Holographic phase separation and inhomogeneous black branes in 1D
  [R.A. Janik et al, PRL 119 (2017) 261601]

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial f)^2 - V(f) \right], \qquad V(f) = -6\cosh\left(\frac{f}{\sqrt{3}}\right) - 0.2 f^4$$

Critical gravitational collapse

Choptuik (1993,1996)



(Gundlach 2007 Living Rev. Rel.)

Initial condition:

$$\phi = a\, e^{-\left(\frac{r-cM}{wM}\right)^2}$$

The phase space picture for the black hole threshold in the presence of a critical point.

Scalarization of Black Hole

Evasion of no-hair theorems (Bekenstein, 1972&1996)

- **GR: with certain matter sources**
  - conformally-coupled scalar field (1974)
  - Yang-Mills (1989)
  - …
- **Beyond GR**
  - dilatonic BHs (1996, EsGB)
  - coloured BHs (1996, EsGB)
  - rotating BHs (2011, EsGB)
  - …

(See Cheng-Yong's talk this afternoon)

Dynamical transitions in collapse and scalarization

Critical value of parameter *p*: p_*	Gravitational collapse (flat space-time vs black hole)	Scalarization of black holes (bald BH vs scalarized BH)
Type I	Lifetime of critical solution (CS) $T \propto -\gamma \ln\lvert p - p_*\rvert$	Lifetime of unstable scalarized BH $T \propto -\gamma \ln\lvert p - p_*\rvert$
Type II	BH formation with no mass gap	Spontaneous scalarization

How about 1st order phase transitions?

Critical phenomena in type I dynamical scalarization

Our work: PRL 128 (2022) 161105 [arXiv:2112.07455]

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R - 2\nabla_\mu\phi\nabla^\mu\phi - f(\phi)F_{\mu\nu}F^{\mu\nu}]$$

$f(\phi) = e^{\beta\phi^4}$: the phenomena at the threshold?

Equations:

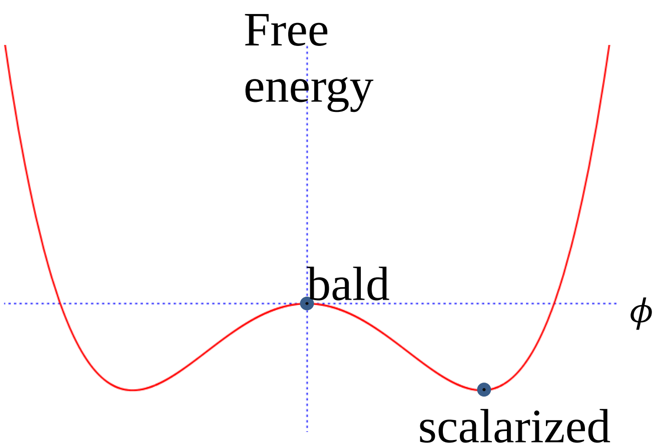
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\left(f(\phi)T^{A}_{\mu\nu} + T^{\phi}_{\mu\nu}\right)$$

$$\nabla_\mu\nabla^\mu\phi = \frac{1}{4}\frac{df(\phi)}{d\phi}F_{\mu\nu}F^{\mu\nu}.$$

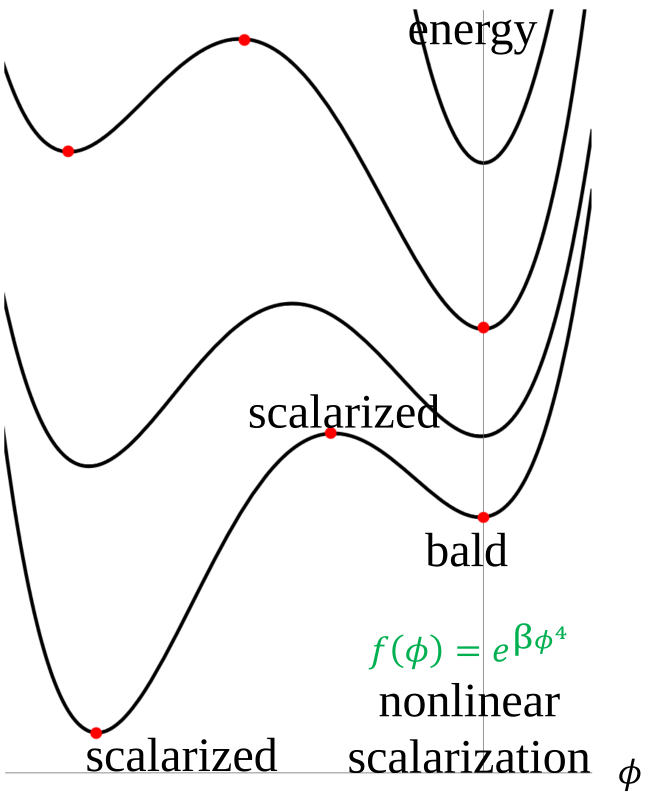
$$\nabla_\mu\left(f(\phi)F^{\mu\nu}\right) = 0.$$

Full nonlinear dynamics

Sketch map for the "free energy"



$f(\phi) = e^{-\alpha\phi^2}$
spontaneous
scalarization



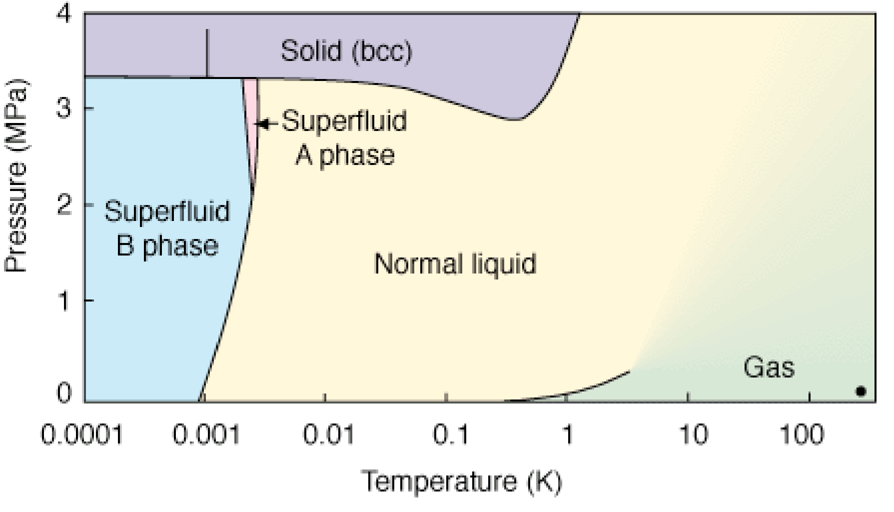
$f(\phi) = e^{\beta\phi^4}$
nonlinear
scalarization

1. Introduction
2. 1st order phase (dynamic) transition
3. Landscape and 1st order PT in holo
4. Concluding remarks

Dynamics of holo 1st order phase transitions

To investigate the 1st order phase transition further (in holography):

- The real bubble dynamics (with surface tension) needs higher dimensions;
- The superfluid Helium-3 dynamics needs a 1st order phase transition between superfluid phases.



Dynamics of holo 1st order phase transitions (probe limit)

Boiling holographic superfluids [X. Li, Z.-Y. Nie & YT, JHEP 2002 (2020) 63]:

- The landscape picture of 1st order phase transitions
- (1+2)D superfluids with real bubble dynamics
- Surface tension of domain walls calculated from holography

Corresponding to condensate 1

Corresponding to condensate 2

$$L_m = \frac{1}{e_2^2}\Big(-\frac{1}{4}F^{ab}F_{ab} - |D_1\Psi_1|^2 - |D_2\Psi_2|^2 - m_1^2|\Psi_1|^2 - m_2^2|\Psi_2|^2 - \lambda_{12}|\Psi_1|^2|\Psi_2|^2 \Big),$$

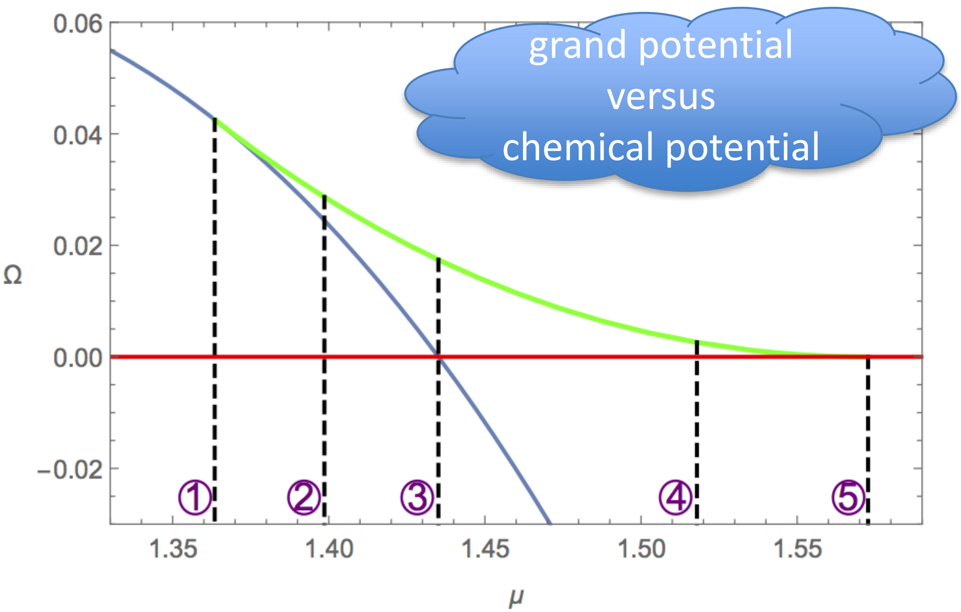
$$D_1 = \partial - ieA, \qquad D_2 = \partial - iA$$

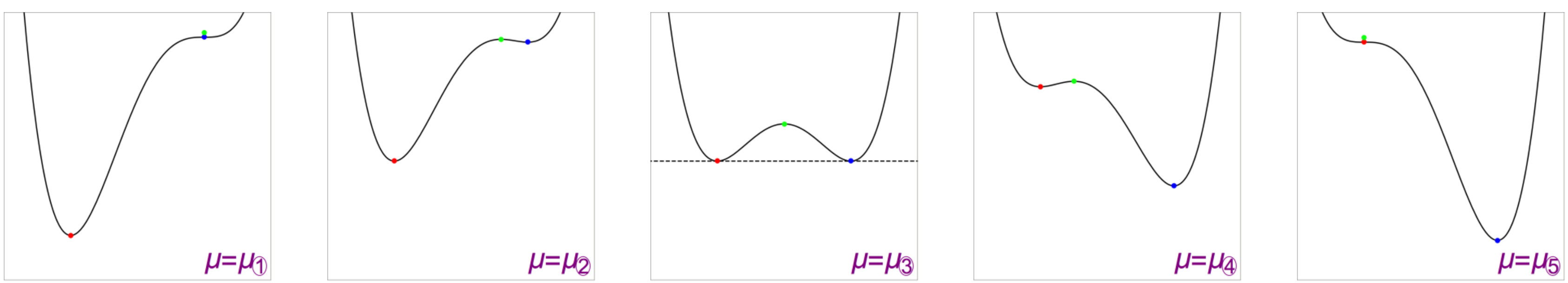
e=4.5, λ_{12}=0.4 for a 1st order phase transition

Interaction term for engineering

Dynamics of holo 1st order phase transitions

Right figure: The swallow-tail shape of the grand potential for the 1st order phase transition.
Bottom Figures: Schematic pictures of the landscapes for five different chemical potentials as marked in the right figure, where the green points are the unstable state.





transition point

- The landscape picture for first order phase transitions
  - A first order phase transition (not restricted to holographic ones) typically involves three states: the stable one, the meta-stable one and the unstable one.
  - The unstable state is the lowest barrier between the stable ...ta-stable states.
  - ...tative description of the landscape (probe limit) ...rXiv:2003.12987]
    - The precise description of the configuration space
    - The dynamic evolution is expressed in terms of the gradient on the landscape
    - The monotonicity of the thermodynamic potential in irreversible processes elegantly derived from the above form of evolution

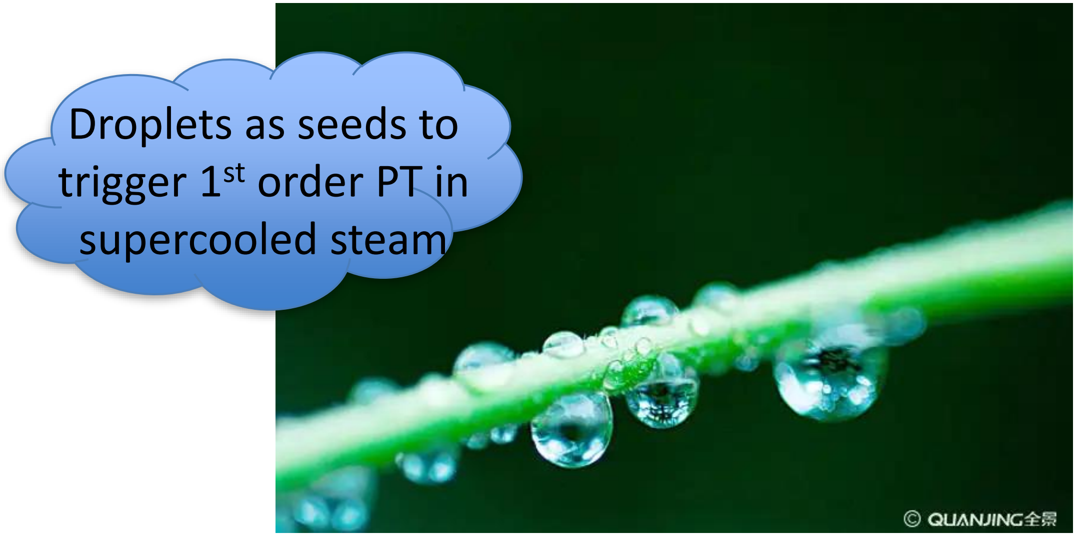
Critical behavior is expected but has not been considered

Non-equilibrium holography with backreaction

- In the probe limit, we do not have access to some very important quantities, such as (variation of) energy, entropy and temperature.
- In particular, in order to invesitigate the ordinary fluid systems with a 1st order PT in holography, we need to include the backreaction and go into the full numerical relativity regime.

Holo 1st order phase transitions with backreaction

- Dynamical transitions in scalarization (type I) and descalarization through black hole accretion in AdS
  [Our work, arXiv:2204.09260]
- Critical nuclei (inhomogeneous CS) in holo 1st order phase transitions (the Gubser model)
  [Y. Bea et al, arXiv:2007.06467]
  [Our work, arXiv:2209.12789]



Holographic setup

- The action

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\right] + S_{\rm GH} + S_{\rm ct}$$

- The potential

$$V(\phi) = -6\cosh(\frac{\phi}{\sqrt{3}}) - \frac{1}{5}\phi^4$$

[R.A. Janik et al , Phys. Rev. Lett. **119**, 261601（2017）]

Static homogeneous solutions (Schwarzschild coordinates)

- The metric

$$ds^2 = -A(r)du^2 + \frac{dr^2}{A(r)} + \Sigma^2(r)\left(dx^2 + dy^2\right)$$

- Entropy and temperature

$$s = \frac{S_H}{4G} = \frac{2\pi}{\kappa_4^2}\Sigma_H^2 \qquad T = \frac{A'_H}{4\pi}$$

- The equations of motion

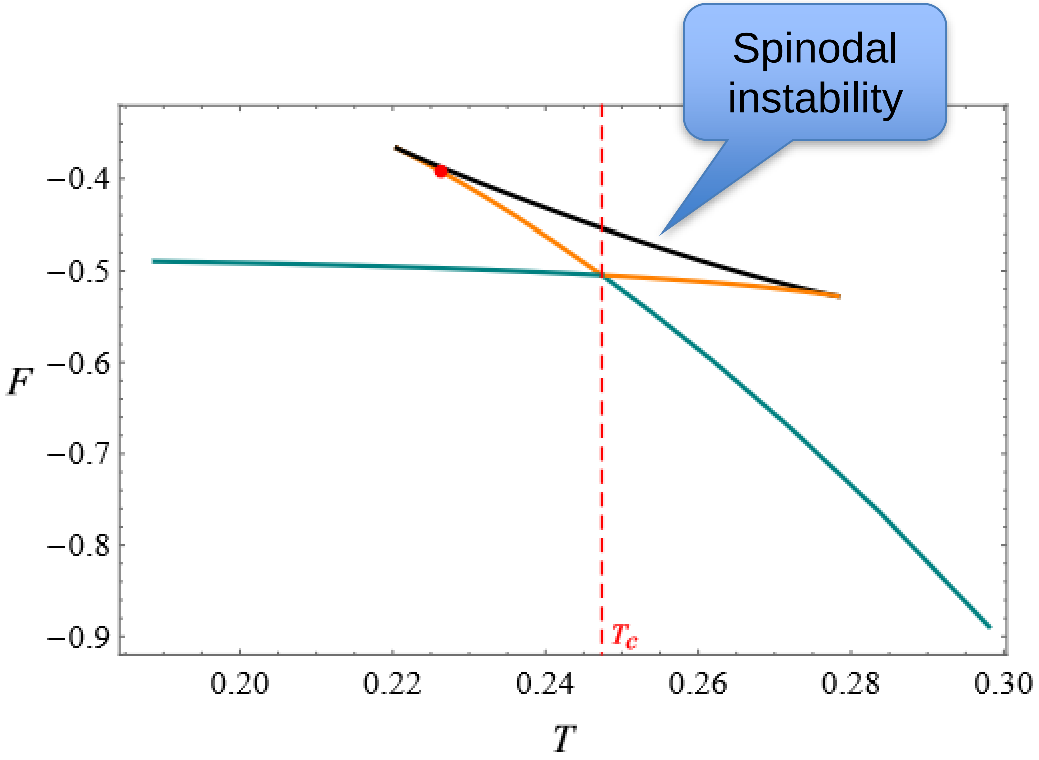
$$\phi'' + A^{-1}A'\phi' + 2\frac{\Sigma'}{\Sigma}\phi' - A^{-1}\frac{dV(\phi)}{d\phi} = 0$$

$$A'' + 2\frac{\Sigma'}{\Sigma}A' + V(\phi) = 0$$

$$(\frac{\Sigma'}{\Sigma})^2 + A^{-1}A'\frac{\Sigma'}{\Sigma} - \frac{1}{4}\phi'\phi' + \frac{1}{2}A^{-1}V(\phi) = 0$$

Newton-Raphson iteration

Free energy vs temperature



The free energy of the Gubser model (with fixed scalar source) with respect to the temperature, where T_c is the temperature of the 1st order phase transition.

Dynamic evolution

- Translation symmetry along the *y* direction

- The Eddington-Finkelstein coordinates

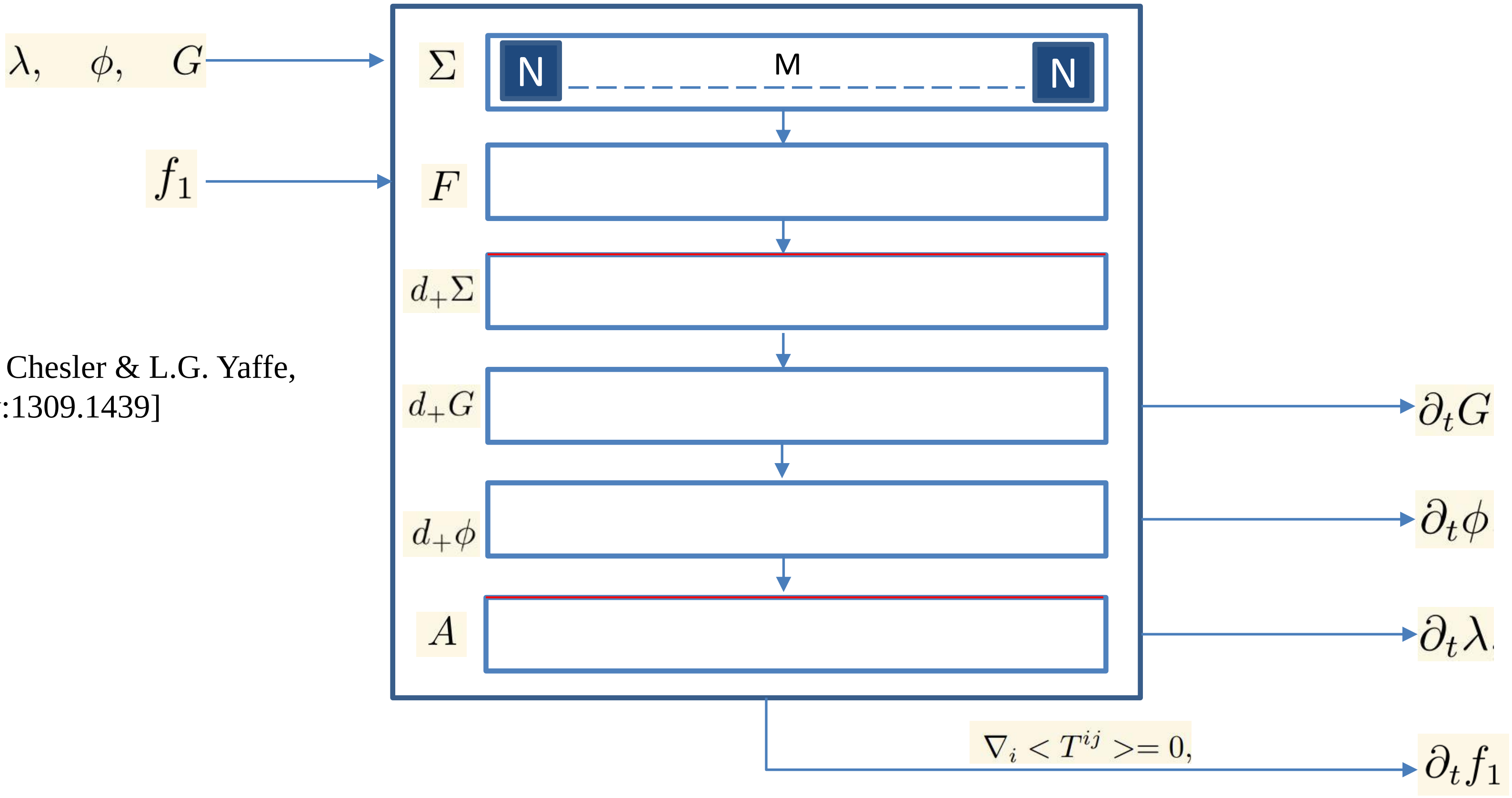
$$ds^2 = \Sigma^2(X)\left(G(X)dx^2 + G^{-1}(X)dy^2\right) + 2dt(dr - A(X)dt - F(X)dx)$$

where $X = (t, x, r)$

- Use the shift freedom $\bar{r} = r + \lambda(t, x)$ to fix the position of the apparent horizon

$$r_H = \text{const}$$

Dynamic evolution – numerical scheme



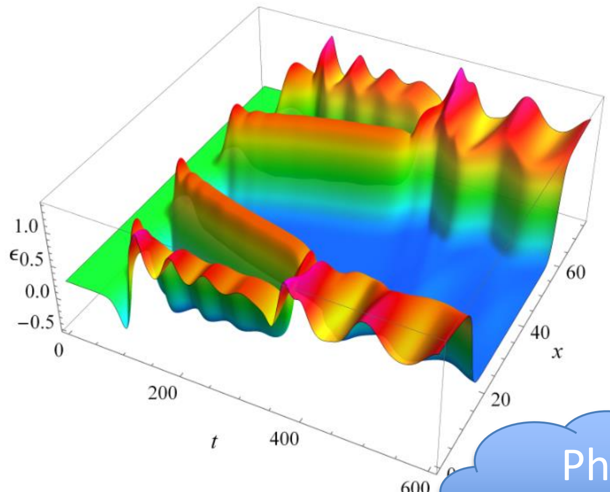
[P.M. Chesler & L.G. Yaffe, arXiv:1309.1439]

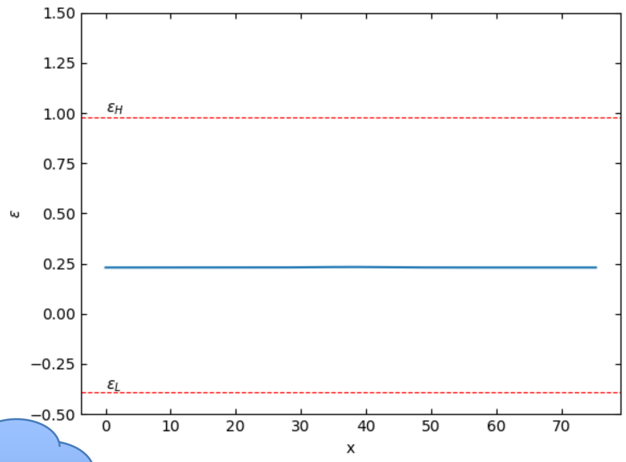
Phase separation from spinodal instability of the unstable state

- Initial state + perturbation

$$\phi := \phi - 0.1z^2(1-z)^2\exp\left(-10\cos^2\frac{x}{24}\right)$$

- Energy density

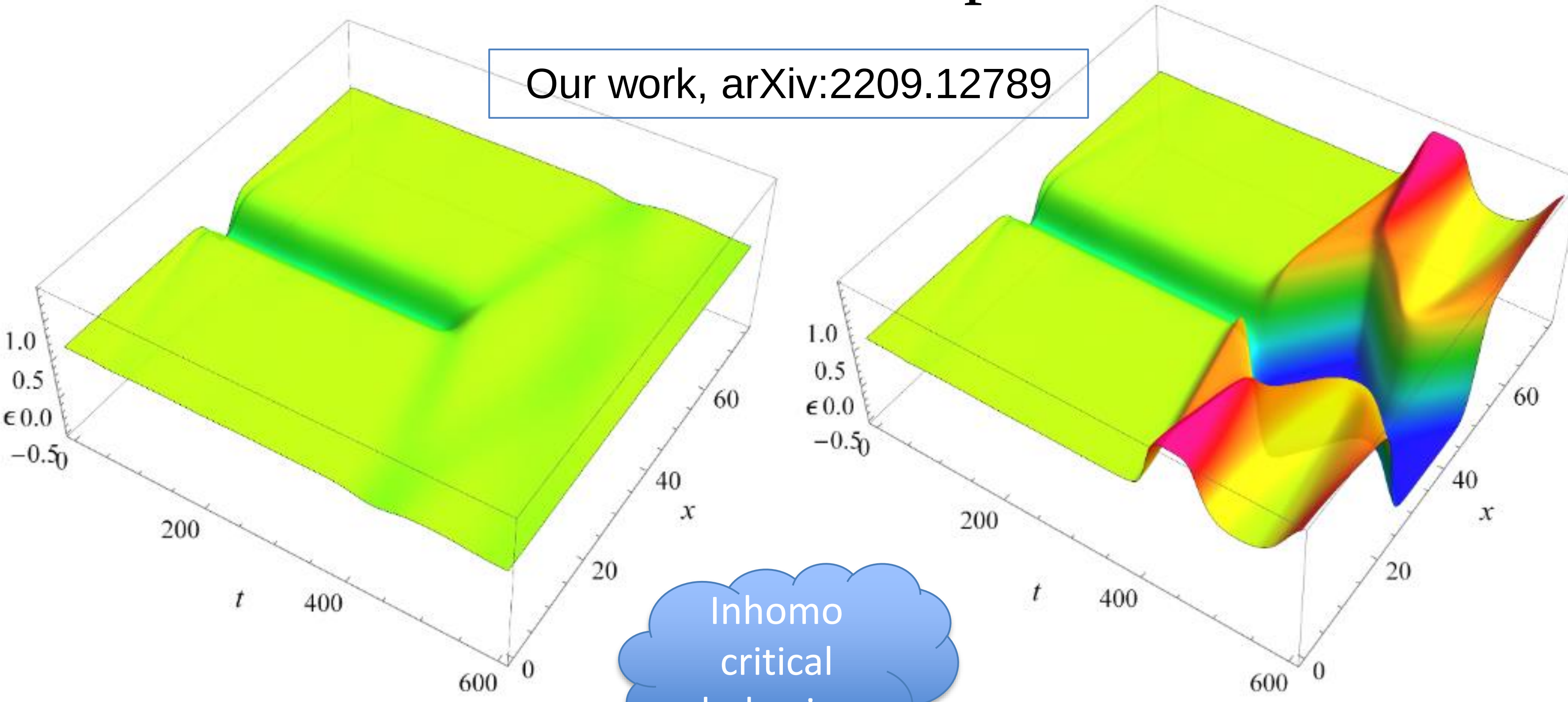




Phase separation

Critical nucleation from the superheated state

Our work, arXiv:2209.12789



(a) (b)

Inhomo critical behavior

Figure 5. The energy density as a function of time with the parameter p is closest to the threshold p_* in the cases (a): $p < p_*$ and (b): $p > p_*$.

$$T \propto -\gamma \ln |p - p_*|$$

JHEP: … are impressive and a big advance for the field …

- The quantitative landscape picture for holo systems with backreaction (and more general gravitational systems)?
  - The landscape picture is expected to hold universally for gravitational systems
    - The three states for 1st order phase transitions
    - The critical behavior related to the fact that the critical state (as the unstable one of those states) is the lowest barrier of the transition (a saddle point on the landscape)
  - The quantitative description of the landscape in AdS or closed gravitational systems (in progress)
    - Entropy landscape for the isolated system
  - The difficulties for the dynamical transitions in the asymptotically flat case

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Concluding remarks

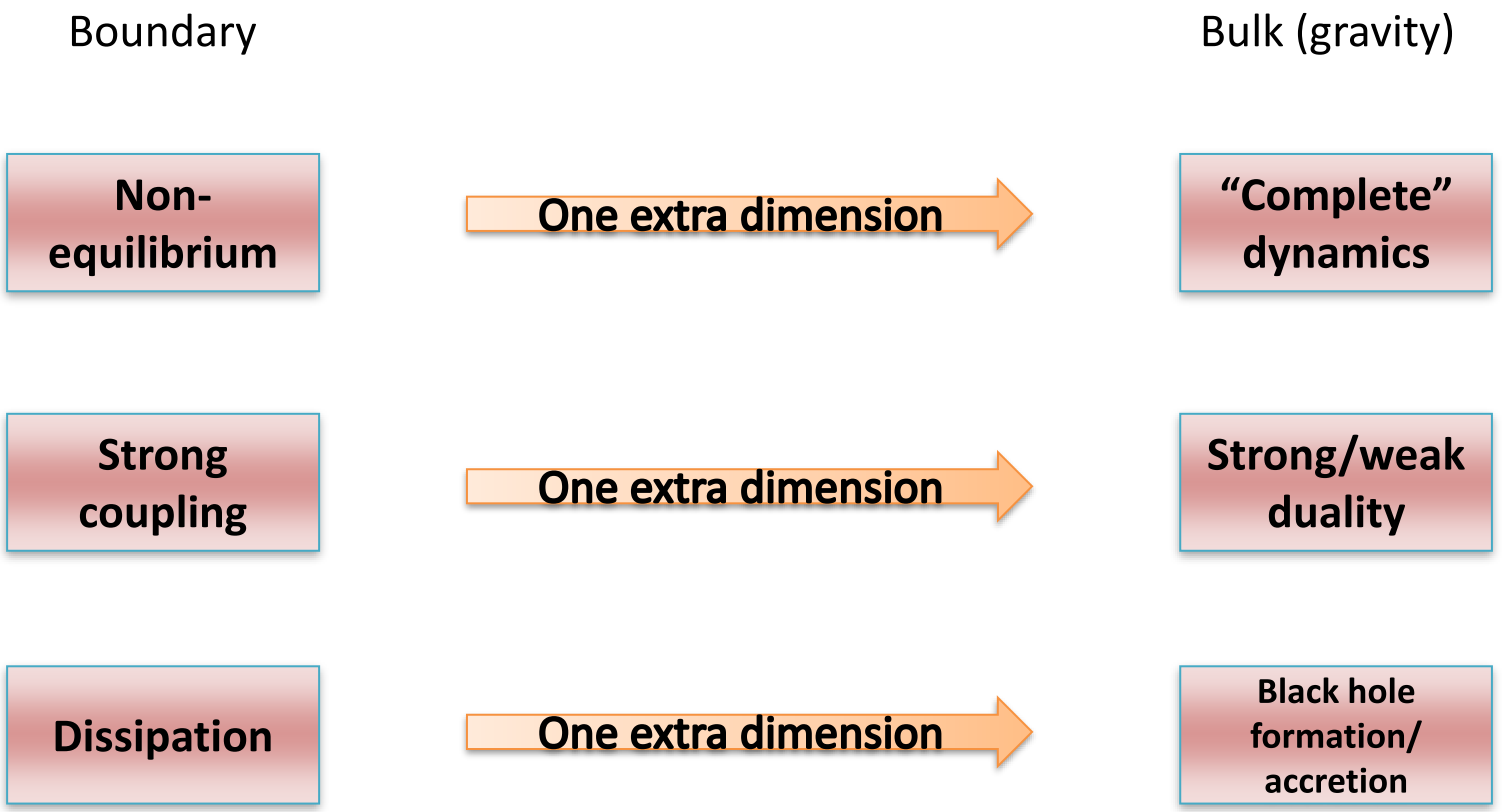
- The universal picture and dynamic critical behaviors of 1st order phase transitions (type I dynamical transitions)
- The critical behaviors are shown to be present in the inhomogeneous dynamical processes (in systems with 1st order phase transitions)

Concluding remarks

- The quantitative description of the landscape for holo systems and more general gravitational systems
- Formulations of numerical relativity adapted to different problems
  “Poor man’s NR”

Thanks for your attention!

How does holography work?

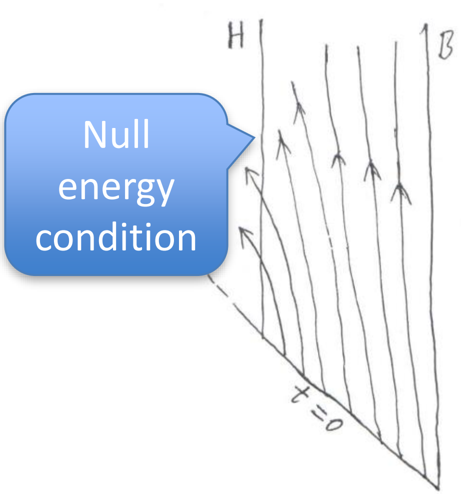


Bulk picture of non-equilibrium physics (probe limit)

- The probe approximation (limit)
- Generalized free energy that controls (holographic) dynamical processes:
  - It becomes the standard free energy in the local equilibrium limit;
  - It decreases monotonically in a general (isothermal) dynamical process (no need to be in local equilibrium) without external work (or called driving) done to the boundary system;
  - Its decrease exactly matches the integral of the energy flux across the horizon.
- In the probe limit, generalized free energy can be given by (the flux of) the bulk energy current $-T^{\mu\nu}\xi_\nu$.
  [YT, X. Wu (吴小宁) & H. Zhang (张宏宝), arXiv:1912.01159]

Bulk picture of non-equilibrium physics (probe limit)

- Flux of the energy current across *H*: dissipating
- Flux of the energy current across the AdS conformal boundary *B*: driving



Relaxation

Equilibrium
↓
Driving
↓ Dissipating ↓
Equilibrium

Quench

Driving
≈
Dissipating

Non-equilibrium steady state

Bulk picture of non-equilibrium physics (general cases)

- The linear response regime (with or without local equilibrium)
  [YT, X. Wu & H. Zhang, JHEP 1410 (2014) 170]
- The fully back-reacted bulk picture of non-equilibrium physics?
  - Conceptual difficulties about non-equilibrium physics
  - Event horizon versus apparent horizon (or even other choices?)
  - How to characterize dissipation?
- Can gravity theories and non-equilibrium physics learn from holography?

Local equilibrium and non-equilibrium

As is well known, hydrodynamics is the low energy, long wavelength effective theory of quantum many body systems. The validity of hydrodynamics depends on local equilibrium, which means that the system is

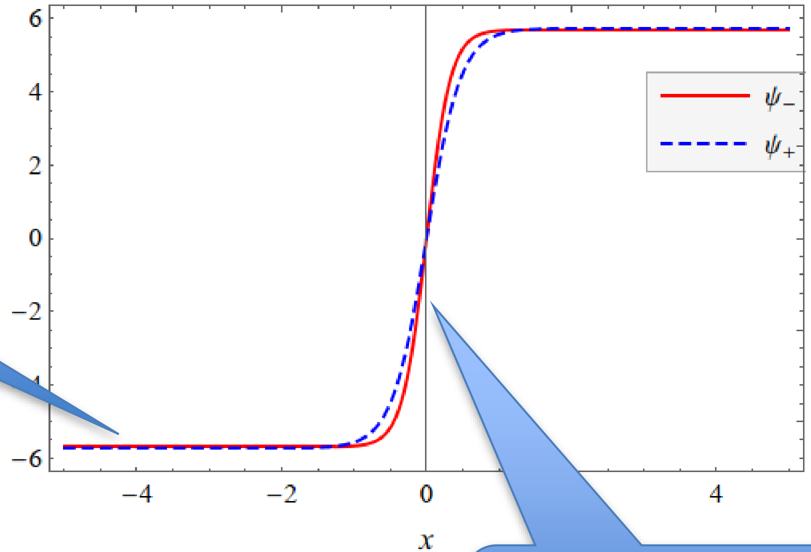
- evolving in time slowly enough
- varying in space slowly enough

with respect to all the characteristic (microscopic) time and space scales.

For a CFT without any conserved quantities other than energy, the only characteristic scale is the temperature T, so the validity of hydrodynamics is determined by T.

Local equilibrium and non-equilibrium

So, even a stable, static system can be in non-equilibrium, if there are local structures, like solitons, vortices, domain walls, etc.



Restoration of thermodynamics

Non-equilibrium region (gradients not small compared to *T*

Holographic superfluid solitons (with backreaction). Blue: standard quantization; Red: alternative quantization. [Z. Xu et al, arXiv:1910.09253]

- Example: the free energy landscape of a scalar field

The full equation of motion:

$$\partial_t(\sqrt{-g}\, g^{t\mu}\, \partial_\mu \phi) + \partial_i(\sqrt{-g}\, g^{it}\, \partial_t \phi) = \frac{\delta F_s}{\delta \phi}$$

with the static free energy (as the functional of the spacial configuration of ϕ)

$$F_s = \int (\frac{1}{2} g^{ij}\, \partial_i \phi \partial_j \phi + \frac{1}{2} m^2 \phi^2) \sqrt{-g}\, d^d x$$

The full free energy:

$$F = -\int \frac{1}{2} g^{tt} (\partial_t \phi)^2 \sqrt{-g}\, d^d x + F_s$$

$$\begin{aligned}
\frac{dF}{dt} &= -\int \frac{1}{2} \partial_t [g^{tt} (\partial_t \phi)^2 \sqrt{-g}] d^d x + \int \frac{\delta \Omega_s}{\delta \phi} \partial_t \phi\, d^d x + \int \partial_i (\sqrt{-g}\, g^{ij}\, \partial_j \phi \partial_t \phi) d^d x \\
&= \int [\sqrt{-g}\, g^{ti}\, \partial_t \partial_i \phi + \partial_i (\sqrt{-g}\, g^{it}\, \partial_t \phi)] \partial_t \phi\, d^d x + \int \partial_i (\sqrt{-g}\, g^{ij}\, \partial_j \phi \partial_t \phi) d^d x \\
&= \int \sqrt{-g}\, g^{z\mu}\, \partial_\mu \phi \partial_t \phi\, d^{d-1} \dot{x} |_{z_h}^{0} = \int T^z_t \sqrt{-g}\, d^{d-1} \dot{x} |_{z_h}^{0} \leq 0
\end{aligned}$$

[arXiv:2003.12987]

Equation of state

- The free energy (left) and the energy density (right) as a function of temperature with source fixed $\phi_1 = 1$