黑洞标量化 & 动力学临界行为

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2204.09260 Phys.Rev.D 106 (2022) 6, L061501
2208.07548
2209.12789 JHEP 01 (2023) 056

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Outline

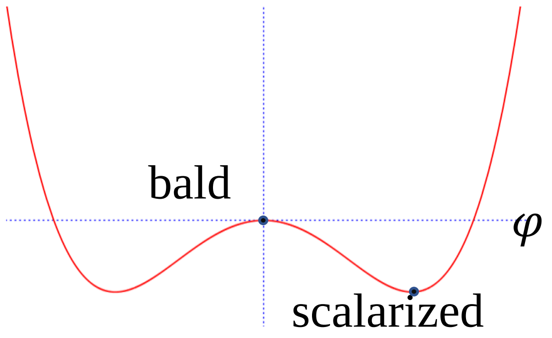
1. Scalarization of Black Hole

2. Critical Behaviors in Dynamical Scalarization
   a. Einstein-Maxwell-Scalar (EMS)
   b. EMS-AdS
   c. Extended scalar-tensor-Gauss-Bonnet (eSTGB, EsGB)

3. Summary and Outlook

Spontaneous scalarization of BHs in EsGB (eSTGB)

Schwarzschild: 1711.01187, 1711.02080, 1711.03390, PRL 3
Kerr: 1904.09997, 2006.03095, 2009.03904, 2009.03905, PRL 4



$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathscr{G}\right]$$

Scalar equation

$$\nabla_\mu\nabla^\mu\varphi = -\frac{df}{d\varphi}\mathcal{G}$$

Perturbation equation

$$\nabla_\mu\nabla^\mu\delta\phi = \mu_{eff}^2\delta\phi,$$

Schwarzschild

$$\mu_{eff}^2 = -\frac{48M^2}{r^6}\frac{d^2f}{d\varphi^2}(0) < \mathbf{0}$$

tachyonic instability

Spontaneous scalarization condition:

$$f(0) = 0, \frac{df}{d\varphi}(0) = 0 \text{ and } \frac{d^2f}{d\varphi^2}(0) > 0$$

$$f = \frac{\lambda^2}{2\kappa}(1 - e^{-\kappa\varphi^2})$$

$f(0) = 0$
$f_{,\varphi}(0) = 0$
$f_{,\varphi\varphi}(0) = \lambda^2$

- **coexistence** of bald and scalarized BHs
- Astrophysical interesting: strong gravity region

Spontaneous scalarization of BHs in EMS

Herdeiro, Radu, etc. 1806.05190 PRL

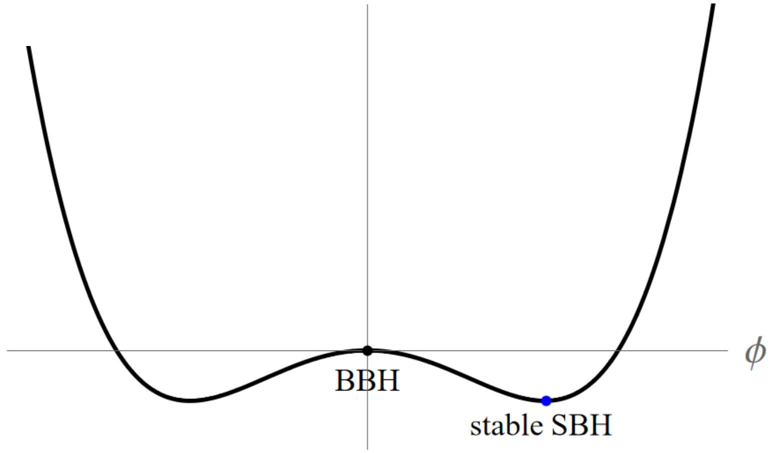
EsGB: elliptic region in evolution, not well-posed for strong coupling

EMS: technically simpler, but without loss of interesting for scalarization

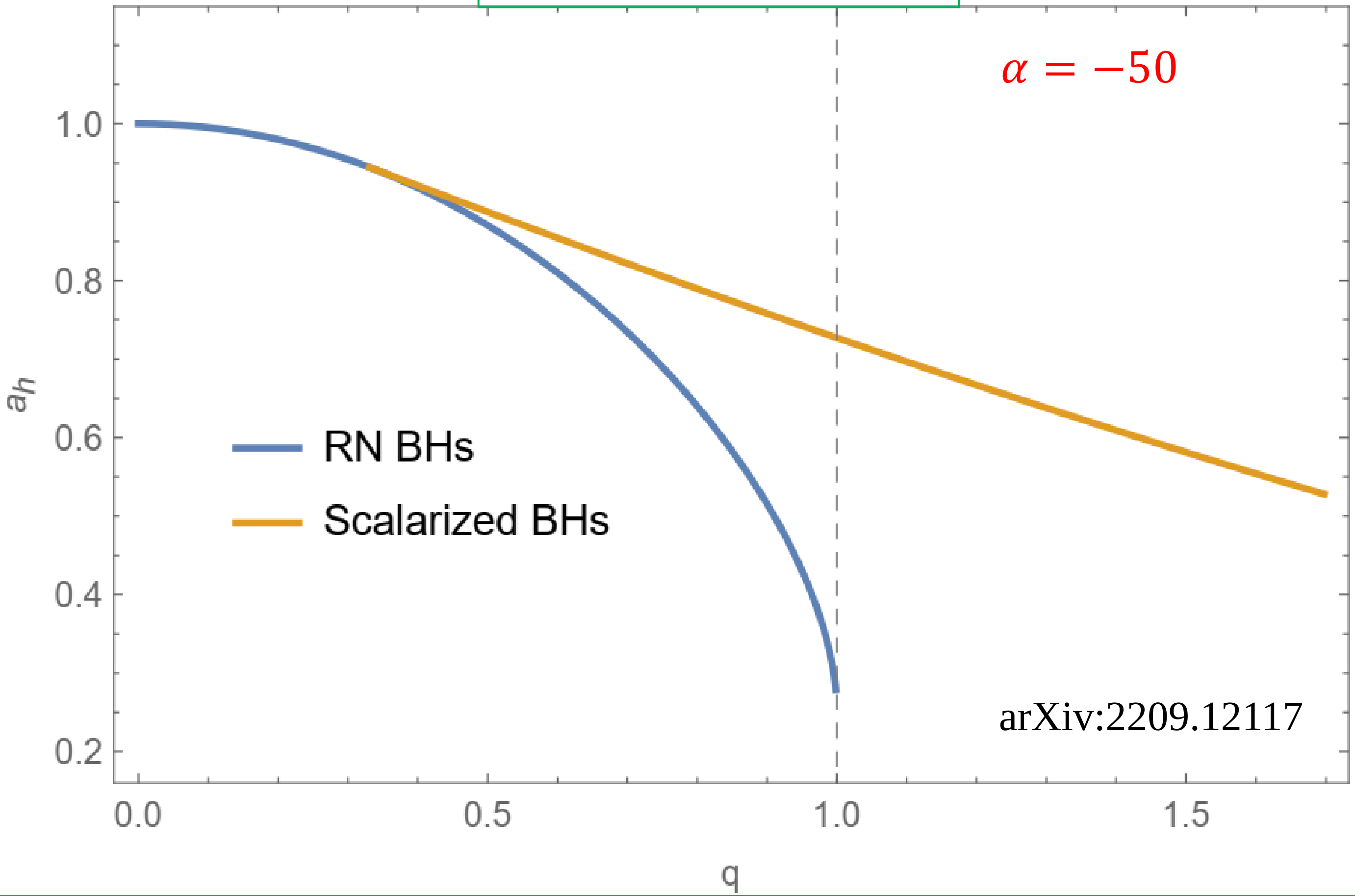
$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R - 2\nabla_\mu\phi\nabla^\mu\phi - f(\phi)F_{\mu\nu}F^{\mu\nu}]$$

Perturbation on **RN** with $f = e^{-\alpha\phi^2}$:

$$\nabla_\mu\nabla^\mu\delta\phi = \mu_{eff}^2\delta\phi, \qquad \mu_{eff}^2 = \frac{\alpha Q^2}{2r^4} < \mathbf{0}$$



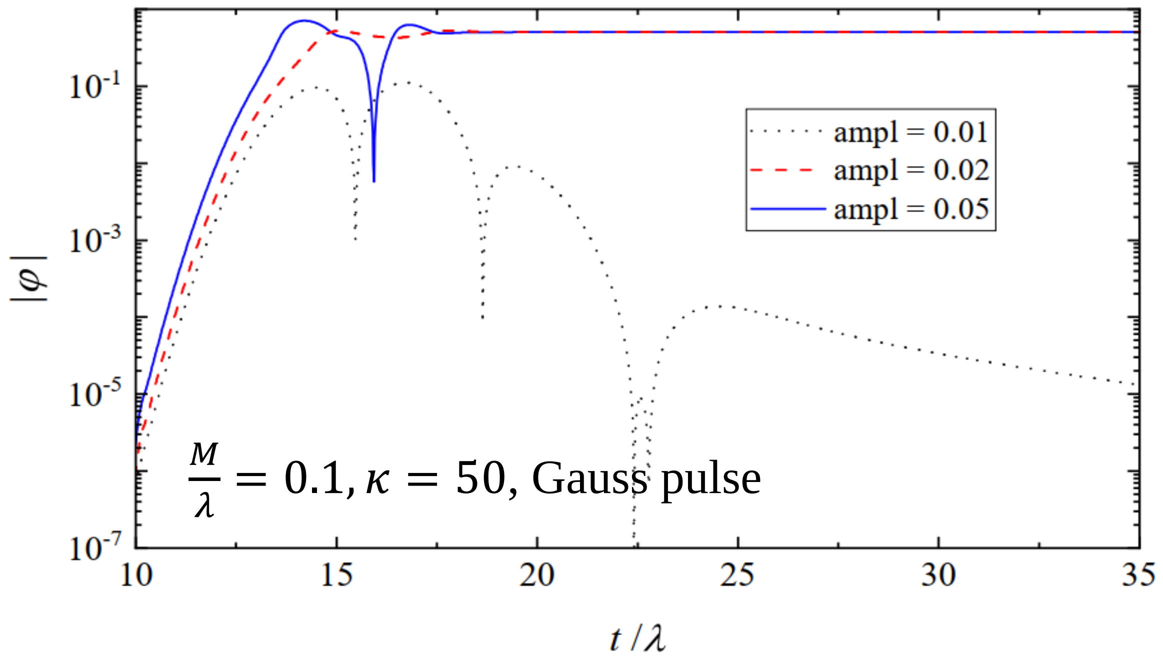
$$f(\phi) = e^{-\alpha\phi^2}$$



1806.05190 PRL: **two** static solutions at (q, α) $\begin{cases} \text{stable: scalarized BH} \\ \text{unstable: bald RN BH} \end{cases}$

Nonlinear scalarization of BHs in EsGB

Doneva, Yazadjiev, 2107.01738 PRD; 2203.00709PRD; 2204.05333PRD; 2208.02077PRD



$$f(0) = 0, \quad \frac{df}{d\varphi}(0) = 0, \quad \frac{d^2 f}{d\varphi^2}(0) = 0.$$

$$f = \frac{\lambda^2}{4\kappa}\left(1 - e^{-\kappa\varphi^4}\right)$$

$$\nabla_\mu \nabla^\mu \varphi = -\frac{df}{d\varphi}\mathcal{G}$$

Linear stable, but nonlinear unstable (decoupling limit)

Questions and motivation:

1. The backreaction?
2. Are there dynamical critical behaviors in scalarization?
3. How the scalar & BH behave in the dynamical process?

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3. Summary and Outlook

Nonlinear scalarization in EMS

Our work: 2112.07455 PRL

For $f = e^{\beta\phi^4}$, RN (with $\phi = 0$) is a solution and **linearly stable:**

$$\nabla_\mu \nabla^\mu \delta\phi = \mu_{eff}^2 \delta\phi, \qquad \mu_{eff}^2 = 0$$

But how about SBH? Full nonlinear dynamical simulation!

Initial configuration:

RN ($\boldsymbol{M_0 = 1}$, $\mathbf{Q = 0.9}$)

+ scalar perturbation (ingoing)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 2\left(f(\phi) T^A_{\mu\nu} + T^\phi_{\mu\nu}\right)$$

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{4}\frac{df(\phi)}{d\phi} F_{\mu\nu} F^{\mu\nu}.$$

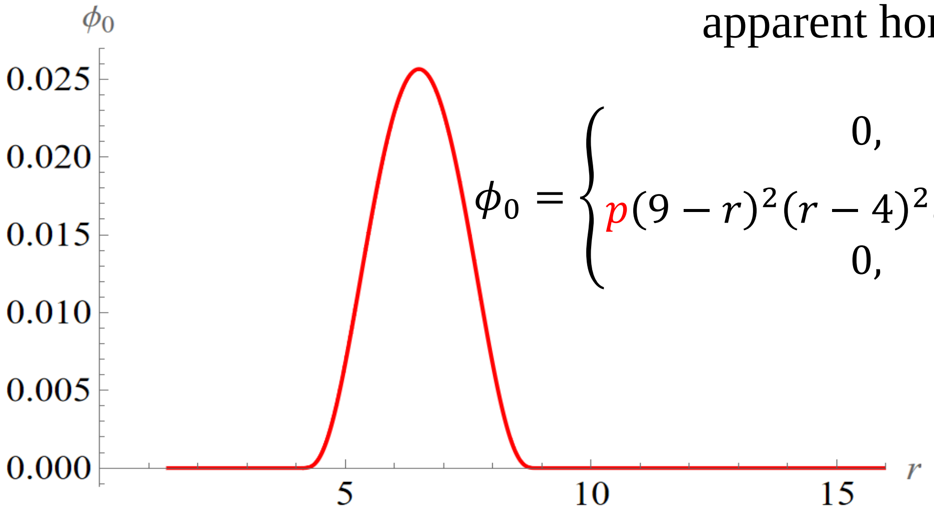
$$\nabla_\mu \left(f(\phi) F^{\mu\nu}\right) = 0.$$

Painleve-Gullstrand (PG) coordinate:

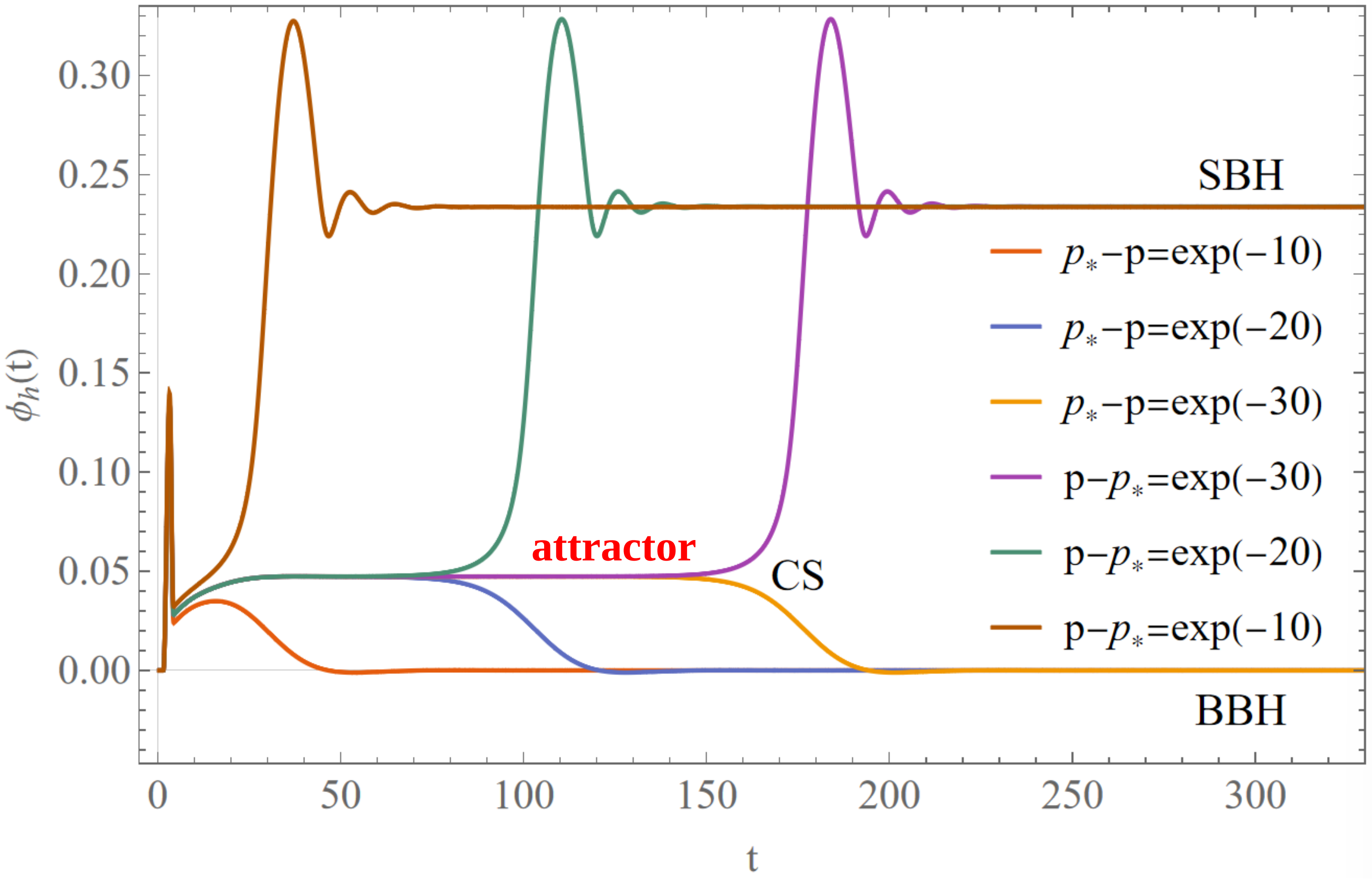
Full nonlinear dynamics in **spherical** symmetric spacetime

$$ds^2 = -\left(1 - \zeta^2\right)\alpha^2 dt^2 + 2\zeta\alpha\, dt\, dr + dr^2 + r^2 d\Omega_2^2.$$

apparent horizon: $\zeta(t, r) = 1$



$$\phi_0 = \begin{cases} 0, & r < 4 \\ p(9-r)^2(r-4)^2 e^{-\frac{1}{9-r} - \frac{1}{r-4}}, & 4 \le r \le 9 \\ 0, & r > 9 \end{cases}$$



$p_* \simeq 0.00146185704570$5

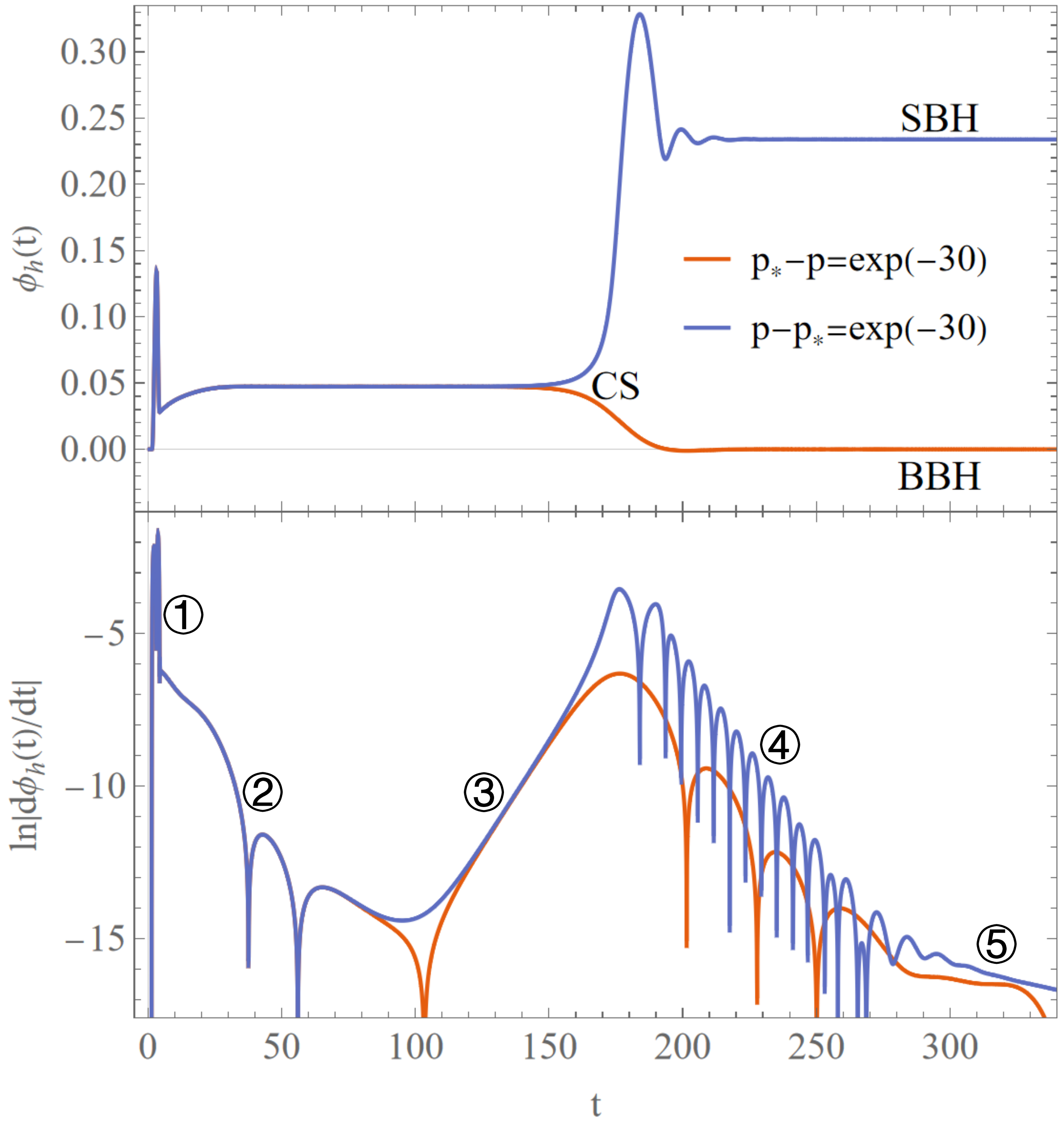
$(\delta p/p \sim 10^{-13})$

New kind of critical behaviors

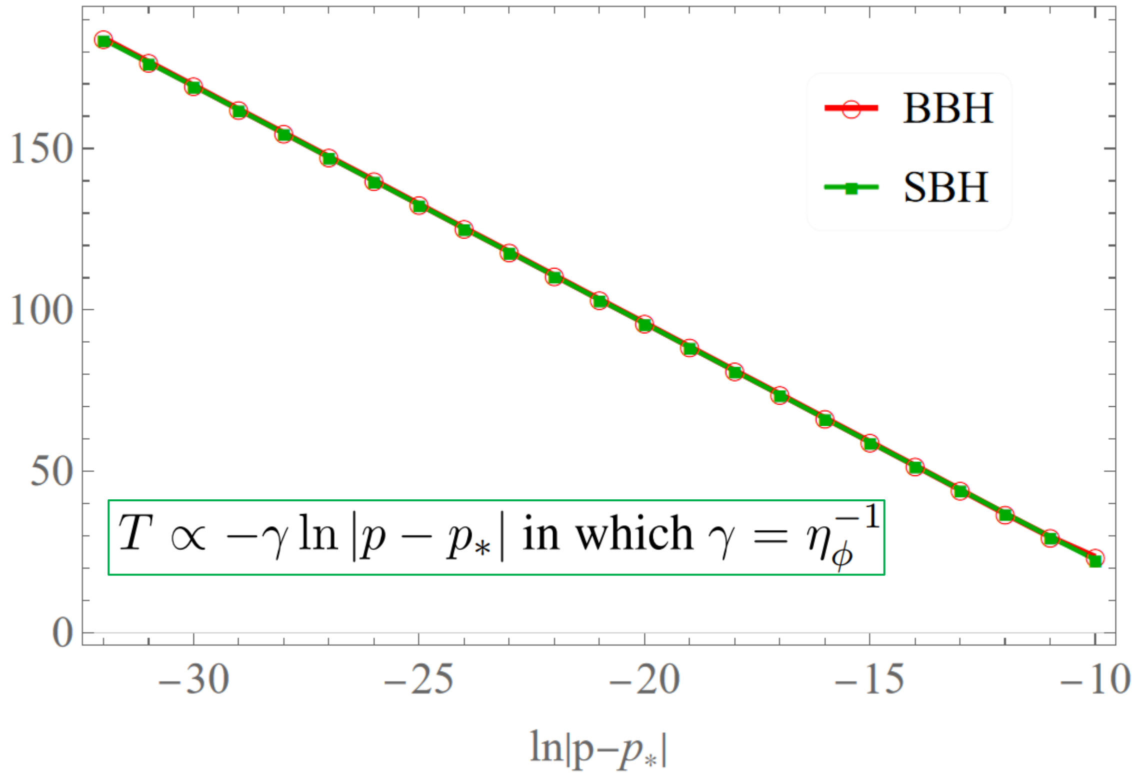
$$\text{(bald) BH} + \text{scalar} \rightarrow \begin{cases} \text{bald BH} & \text{subcritical } (p < p_*) \\ \text{unstable SBH (CS)} & \text{critical } (p = p_*) \\ \text{scalarized BH} & \text{supercritical } (p > p_*) \end{cases}$$

Reminiscent of the **type I** critical gravitational collapse (1996)

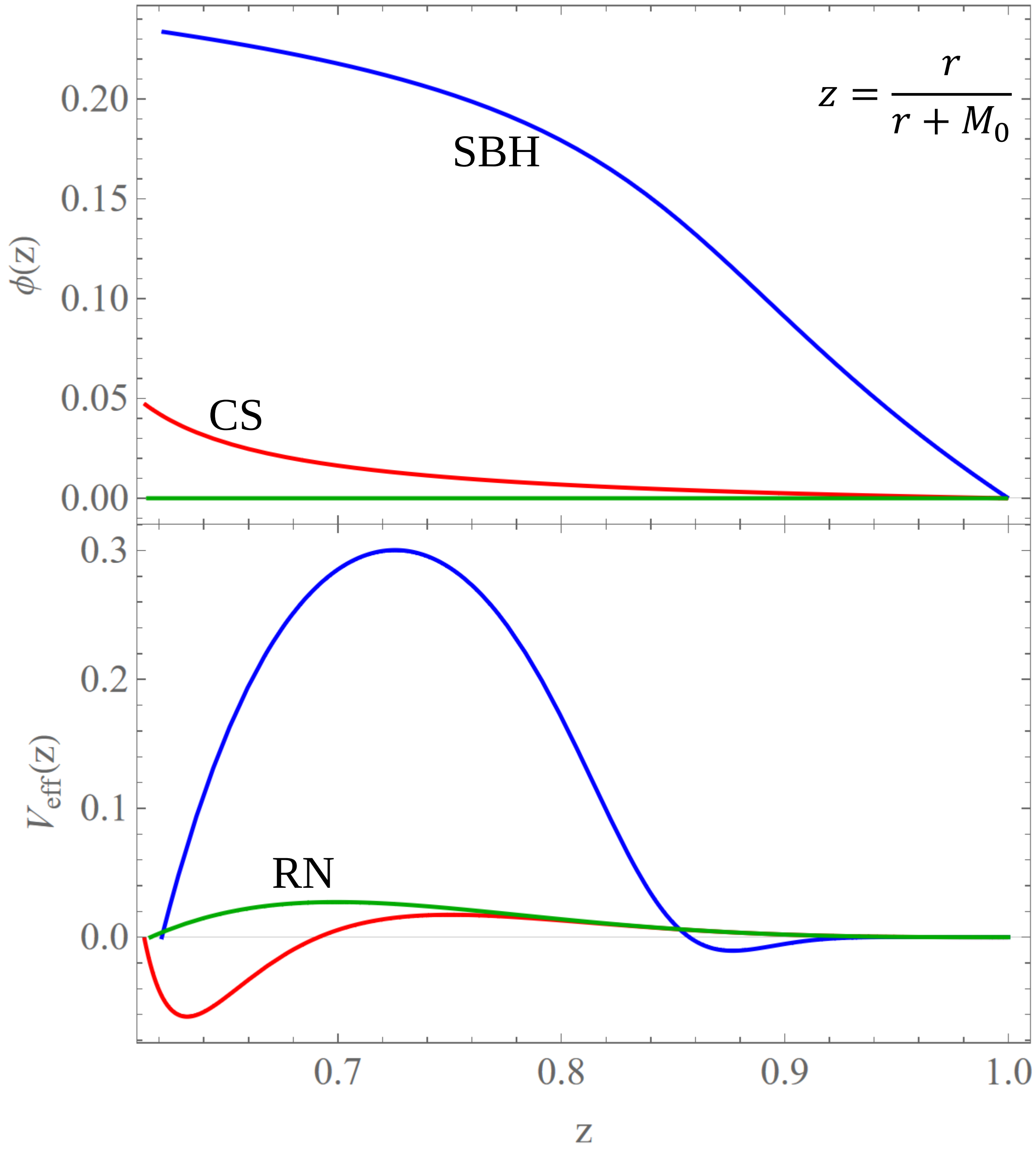
$$\text{flat space} + \text{scalar} \rightarrow \begin{cases} \text{flat space} & \text{subcritical } (p < p_*) \\ \text{unstable star (CS)} & \text{critical } (p = p_*) \\ \text{BH} & \text{supercritical } (p > p_*) \end{cases}$$



$$\phi_p(t,r) \approx \phi_*(r) + (p-p_*)e^{\eta_\phi t}\delta\phi(r) + \text{stable modes.}$$



$$\text{(bald) BH} + \text{scalar} \rightarrow \begin{cases} \text{bald BH} & \text{subcritical } (p < p_*) \\ \text{unstable SBH (CS)} & \text{critical } (p = p_*) \\ \text{scalarized BH} & \text{supercritical } (p > p_*) \end{cases}$$



$$dt_s = dt - \zeta dr_* = dt - \frac{\zeta}{(1-\zeta^2)\alpha} dr$$

$$\delta\phi = e^{-i\omega t_s} \frac{R(r)}{r}$$

Schrodinger-like equation (Buell, Shadwick, 1995)

$$0 = \left(\partial_{r_*}^2 + \omega^2 - V_{\rm eff}\right) R.$$

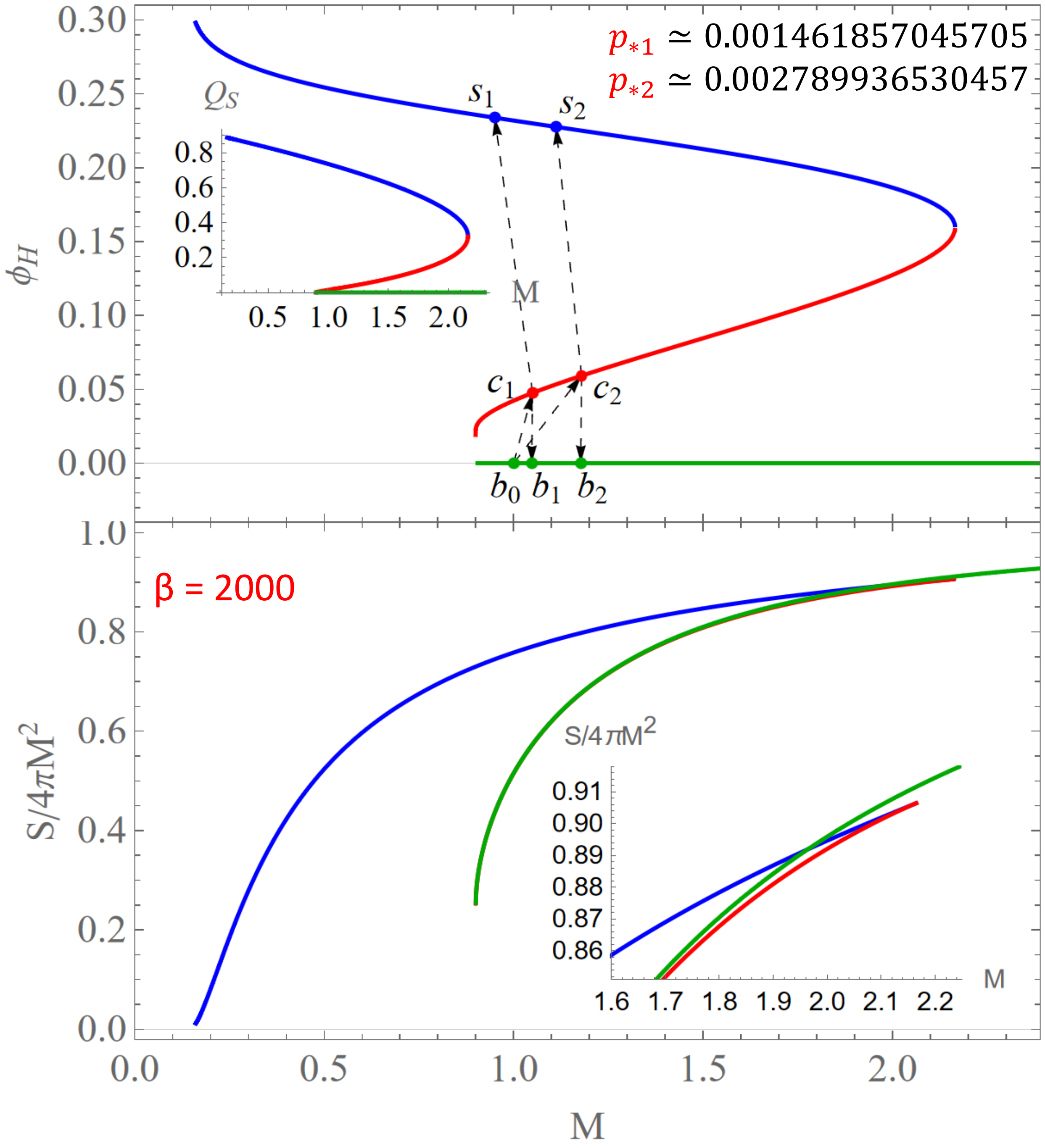
Only for the CS, there is $\int_{-\infty}^{\infty} V_{\rm eff} dr_* < 0$

CS has **tachyonic** instability (as RN in spontaneous scalarization) which gives precisely the unstable mode η_ϕ

QNMs: matched

① first order WKB method / shooting method

② Prony method



Static solutions

Our results & Herdeiro, Radu, etc, (2002.00963 PLB; 2008.11744 EPJC; 2011.01326 Symmetry, $f(\phi)=1+\beta\phi^4$)

$f(\phi)=e^{\beta\phi^4}$

Three static solutions at $\left(q=\frac{Q}{M},\beta\right)$:

two stable: (1) RN ($M\geq Q$)

(2) hot SBH

one unstable: (3) cold SBH (CS)

Scalarization: $b_0 \to c_1 \to \begin{cases} b_1, & \text{BBH } (p<p_{*1}) \\ s_1, & \text{SBH } (p>p_{*1}) \end{cases}$

Descalarization: $b_0 \to c_2 \to \begin{cases} s_2, & \text{SBH } (p<p_{*2}) \\ b_2, & \text{BBH } (p>p_{*2}) \end{cases}$

Dynamical **first-order** phase transition

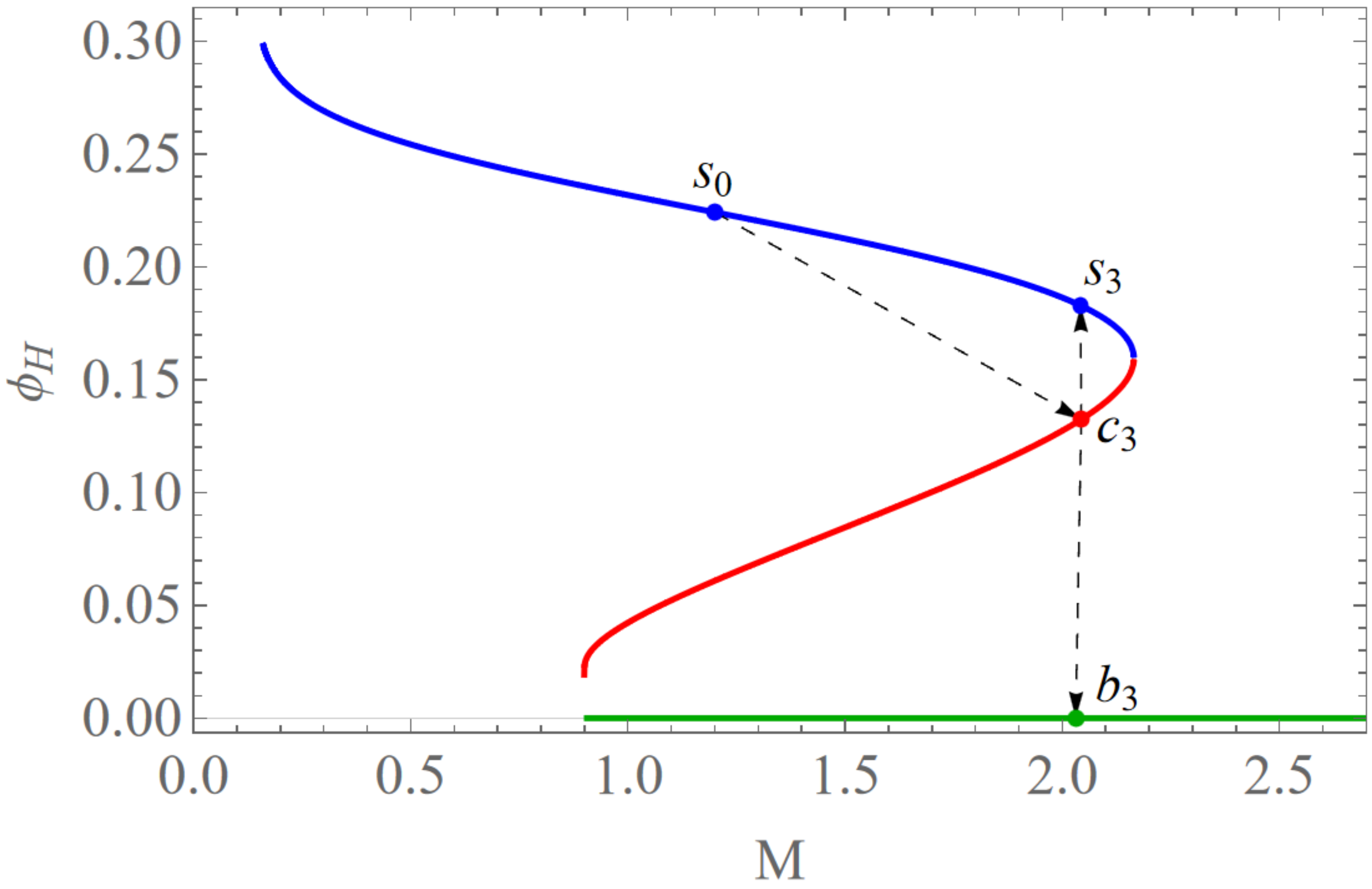
Dynamics vs Thermodynamics:

Not precisely consistent

Dynamical descalarization from SBH

Initial configuration: SBH ($M_0 = 1.2, Q = 0.9$) + scalar perturbation (ingoing)

$$p_{*3} \simeq 0.0012848778022796 \ (\delta p/p \sim 10^{-13})$$



type I with an unstable attractor

Descalarization: $s_0 \to c_3 \to \begin{cases} s_3, & \text{SBH } (p < p_{*3}) \\ b_3, & \text{BBH } (p > p_{*3}) \end{cases}$

Dynamical **first-order** phase transition

$$\text{SBH} + \text{scalar} \to \begin{cases} \text{SBH} & \text{subcritical } (p < p_*) \\ \text{unstable SBH (CS)} & \text{critical } (p = p_*) \\ \text{bald BH} & \text{supercritical } (p > p_*) \end{cases}$$

Interim Summary

1. We found **new BH scalarization & descalarization mechanism** through the accretion of the scalar field

2. We uncovered **novel dynamical critical behaviors** in the bald/scalarized BH transition

3. How about other cases?
   - EMS-AdS
   - **eSTGB**

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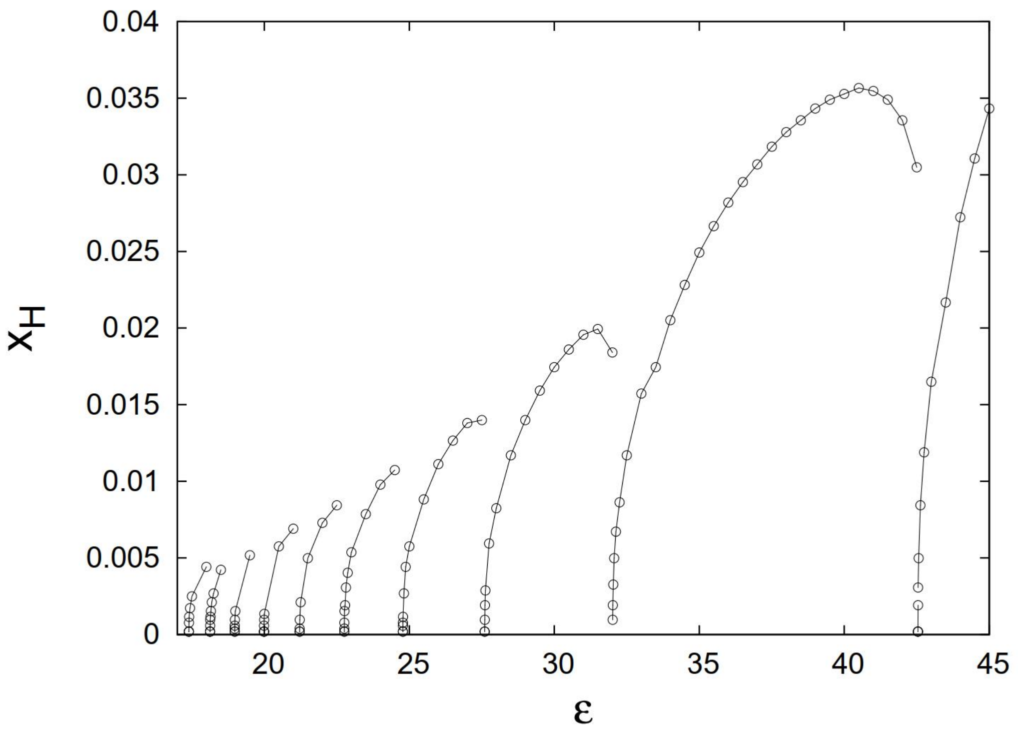
3. Summary and Outlook

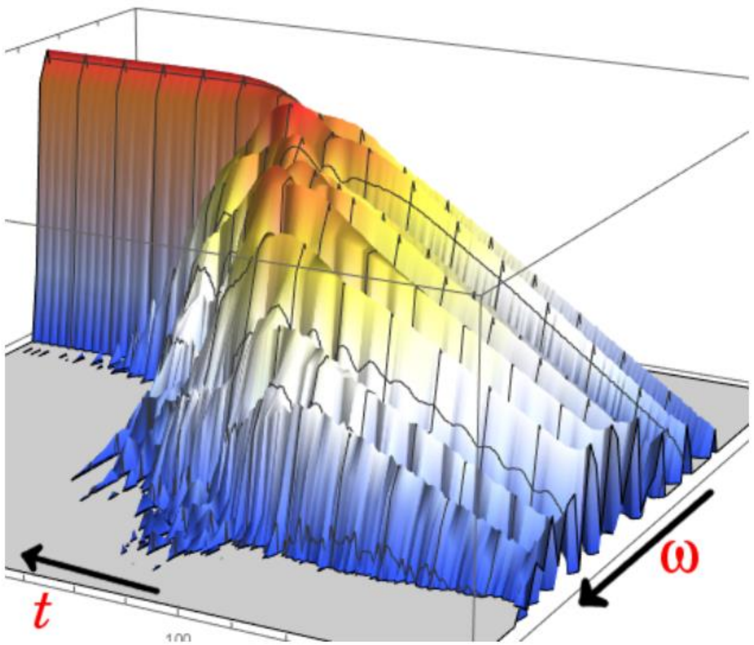
Dynamical critical scalarization and descalarization in AdS spacetime

Our work: **2204.09260** PRD (Letter)

The difference between asymptotically flat and AdS spacetime: **confining boundary**

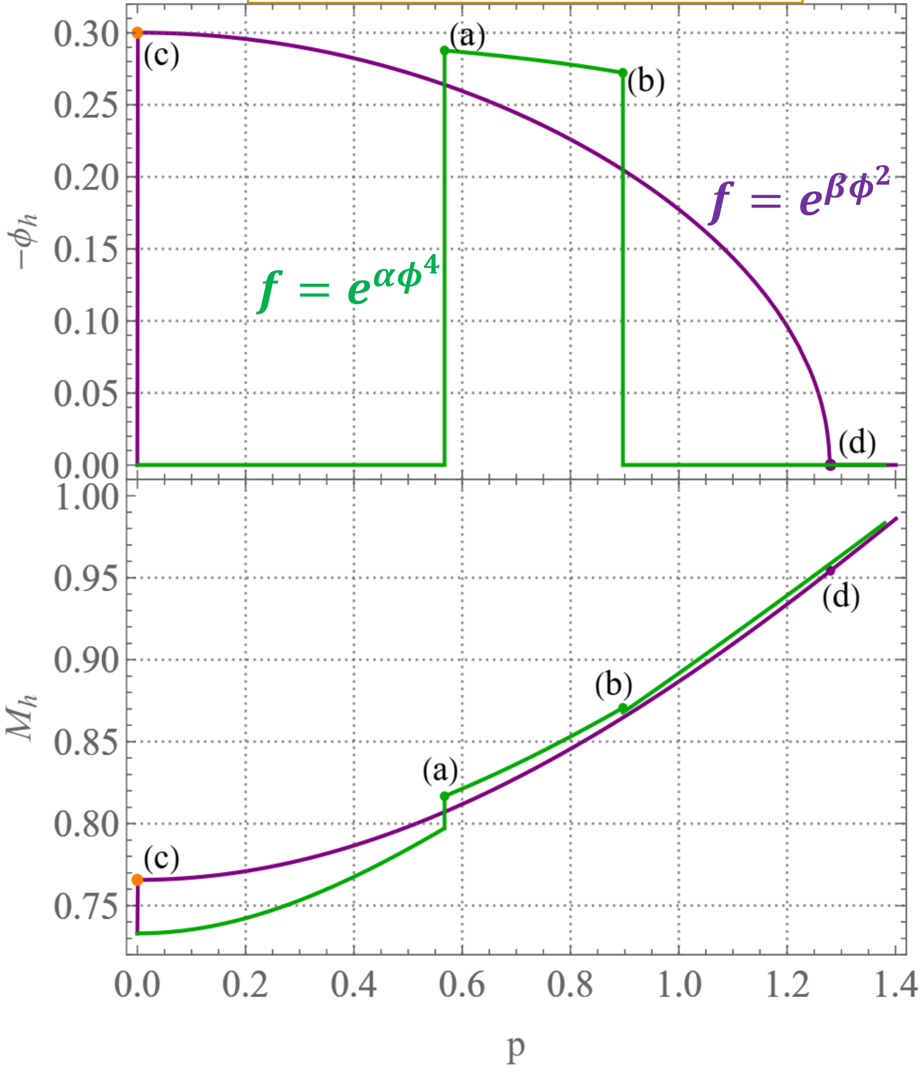
- Turbulent instability in AdS space (1104.3702PRL)
- Superradiant instability of RN-AdS (1601.01384PRL) and Kerr-AdS (1801.09711PRL)





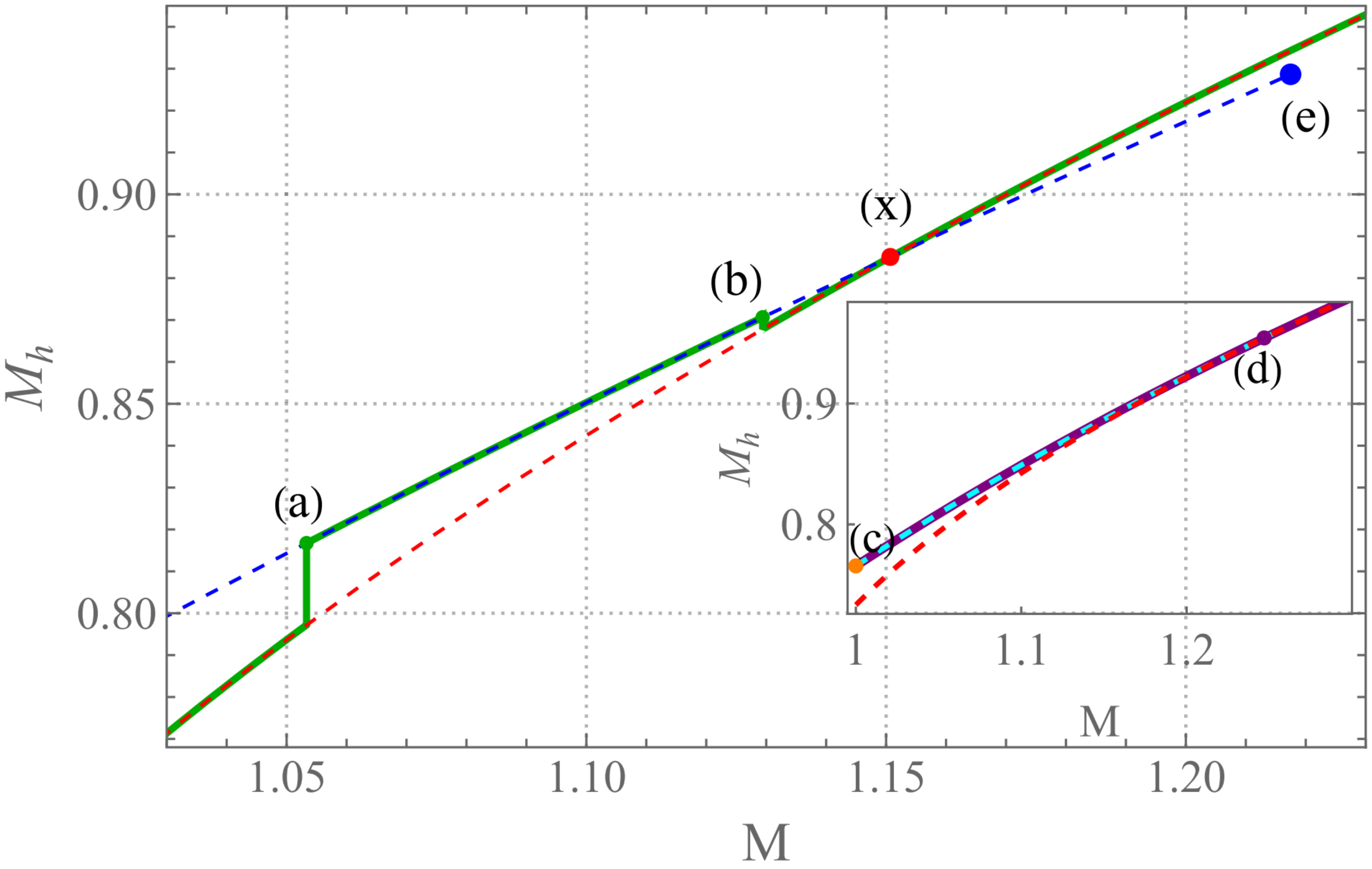
$$S=\int d^4x\sqrt{-g}\left(R+\frac{6}{L^2}-2\nabla_\mu\phi\nabla^\mu\phi-f(\phi)F_{\mu\nu}F^{\mu\nu}\right)$$

$$\phi_0 \;=\; pe^{-64(0.5-\frac{1}{r})^2}$$



Eddington-Finkelstein coordinate, pseudospetrum method

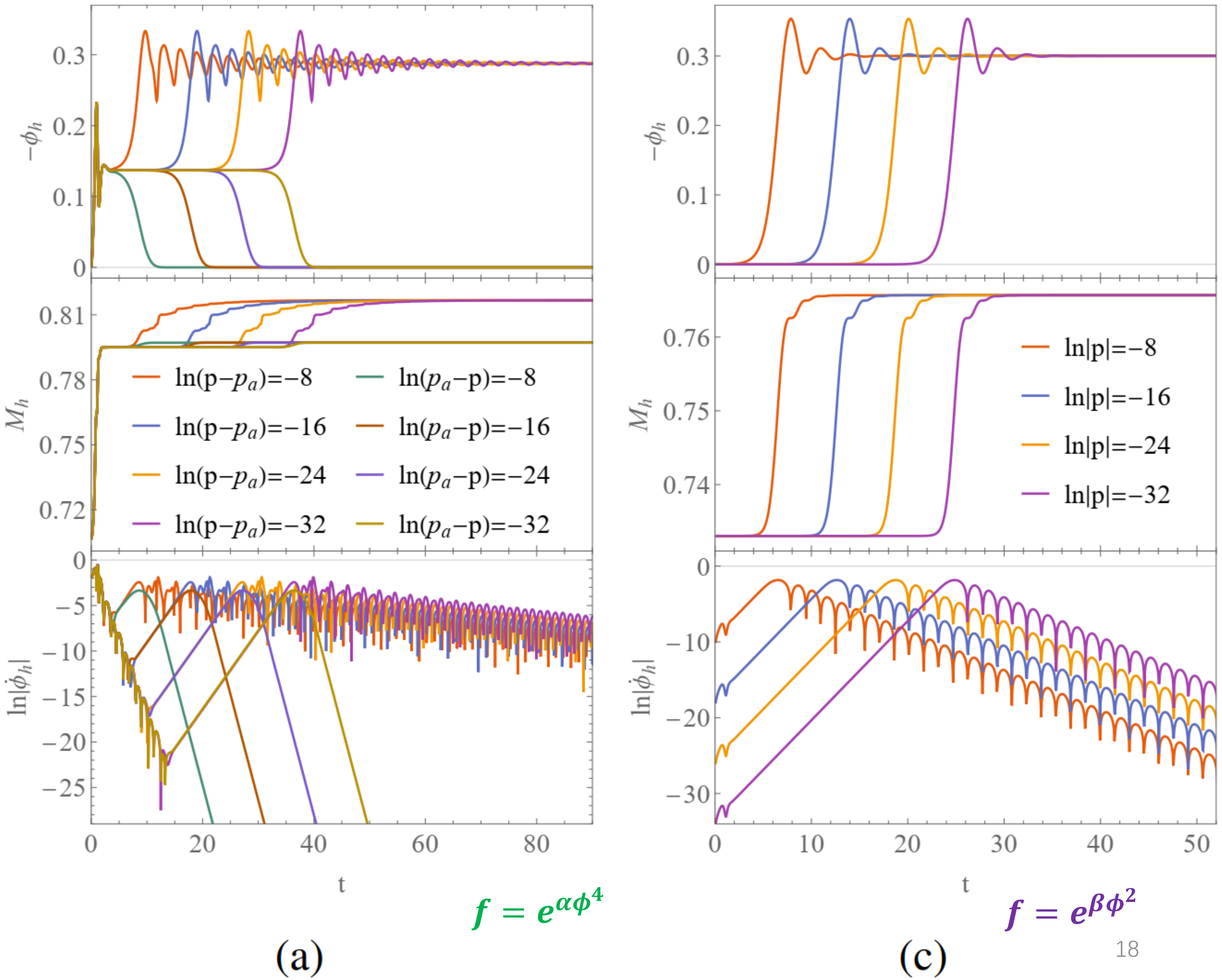
$$ds^2=-Wdt^2+2dtdr+\Sigma^2(d\theta^2+\sin^2\theta d\varphi^2)$$



Dynamical critical scalarization

$f = e^{\alpha\phi^4}$: Type I with an attractor

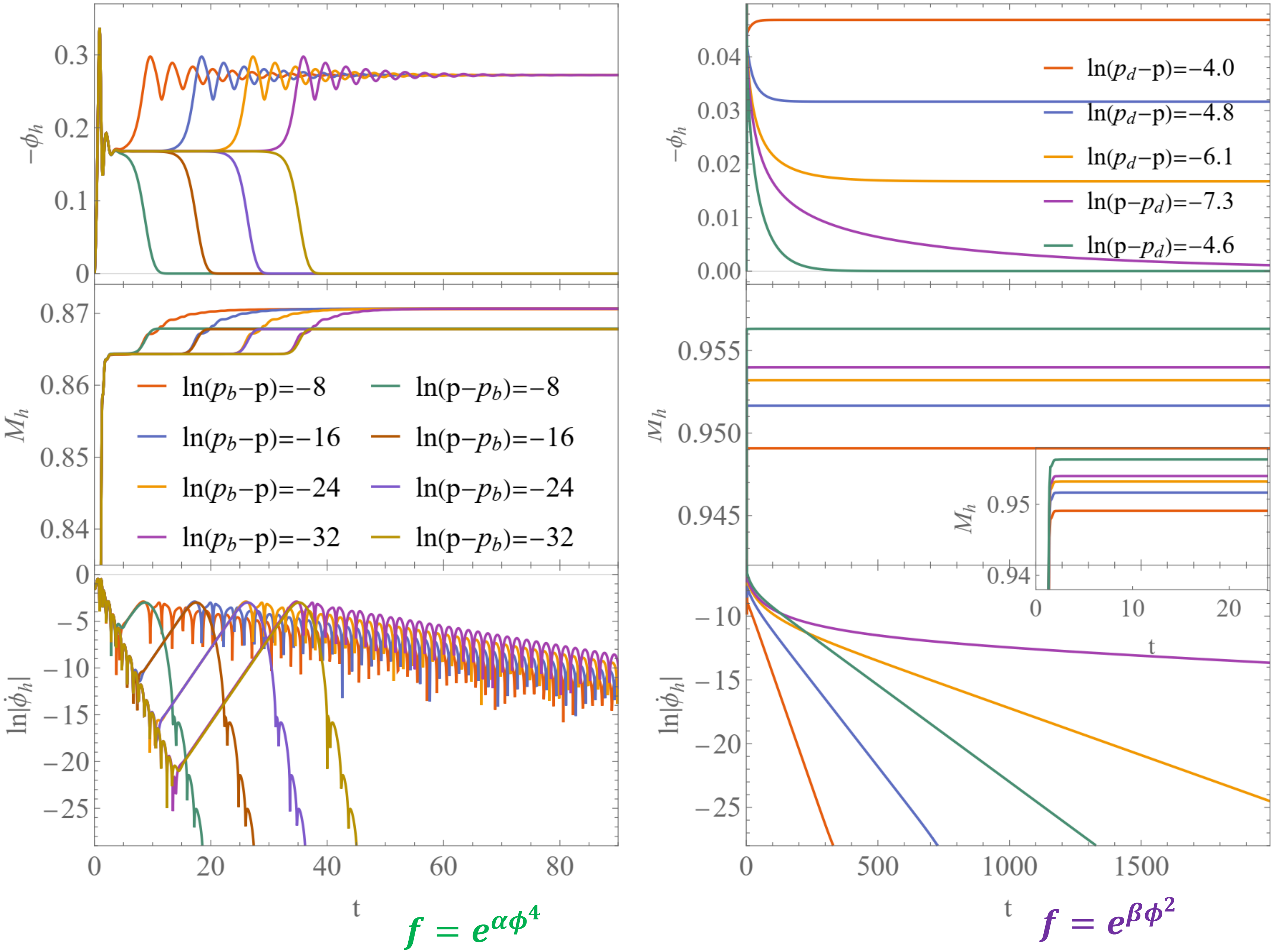
$f = e^{\beta\phi^2}$: Type I with an attractor



Dynamical critical descalarization

$f = e^{\alpha\phi^4}$: Type I
with an attractor

$f = e^{\beta\phi^2}$: Type II
without an attractor



(b) (d)

Interim Summary

1. We pointed out that **RN-AdS is special critical solution** in EMS theory with spontaneous scalarization

2. We uncovered type I & II dynamical critical behaviors in the BH **descalarization** transition

3. How about other cases?
    - **eSTGB**

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3. Summary and Outlook

Dynamical critical scalarization and descalarization in eSTGB theory

Our work: 2208.07548

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + f(\phi)\left(\beta R + \mathcal{G}\right)\right]$$

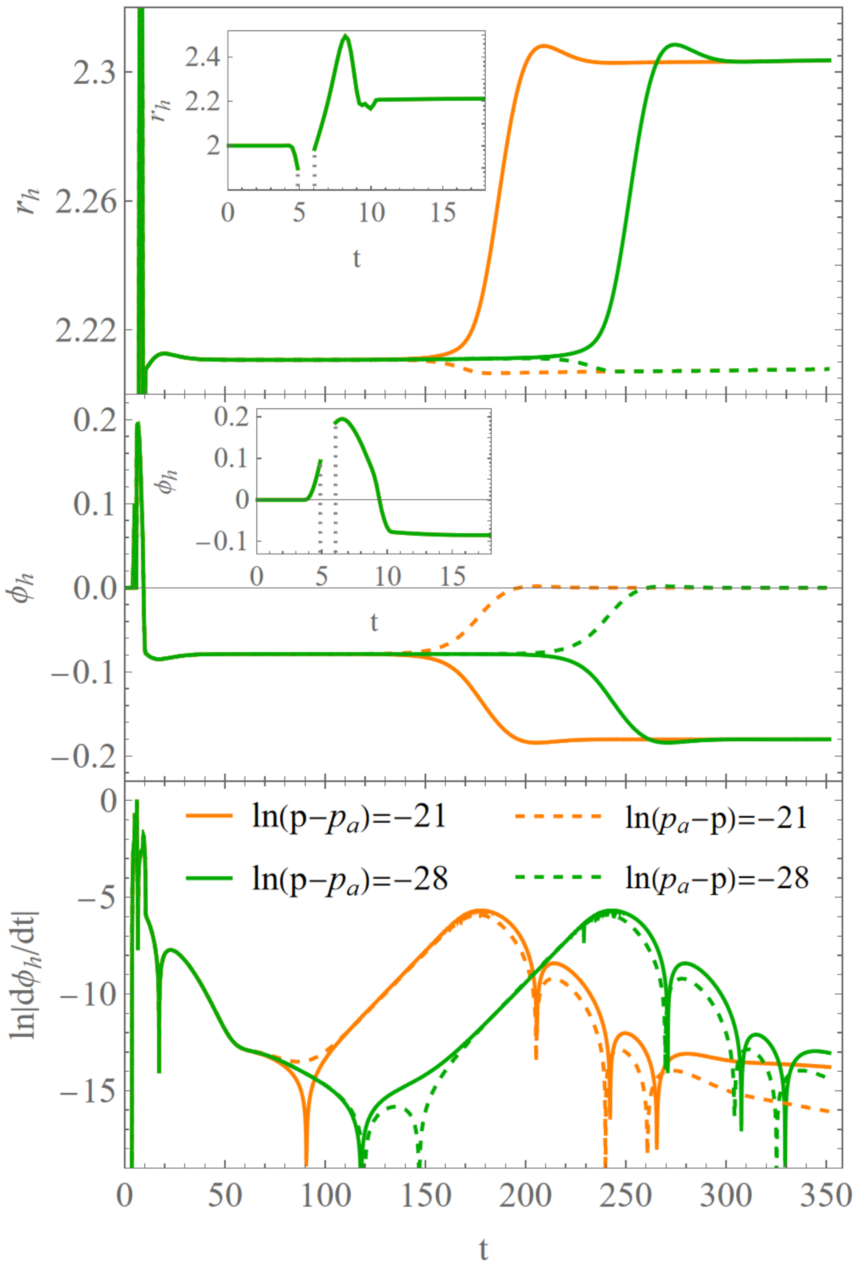
- $\boldsymbol{\beta}$ suppresses the elliptic region and makes the code stable
- The first model of spontaneous scalarization (for **neutron star)** (Damour 1993PRL)

BH **nonlinear** scalarization: $f = \frac{\lambda^2}{4\kappa}\left(1 - e^{-\kappa\varphi^4}\right)$

- PG coordinate
- **Initial condition**: Schwarzschild BH with $M_0 = 1$, and $\beta = -2.5, \lambda = \frac{50}{3}, \kappa = 1000$

$$\phi_0 = \begin{cases} 0, & r < 8 \\ p(18-r)^2(r-8)^2 e^{-\frac{1}{18-r}-\frac{1}{r-8}}, & 8 \le r \le 18 \\ 0, & r > 18 \end{cases}$$

Dynamical critical scalarization

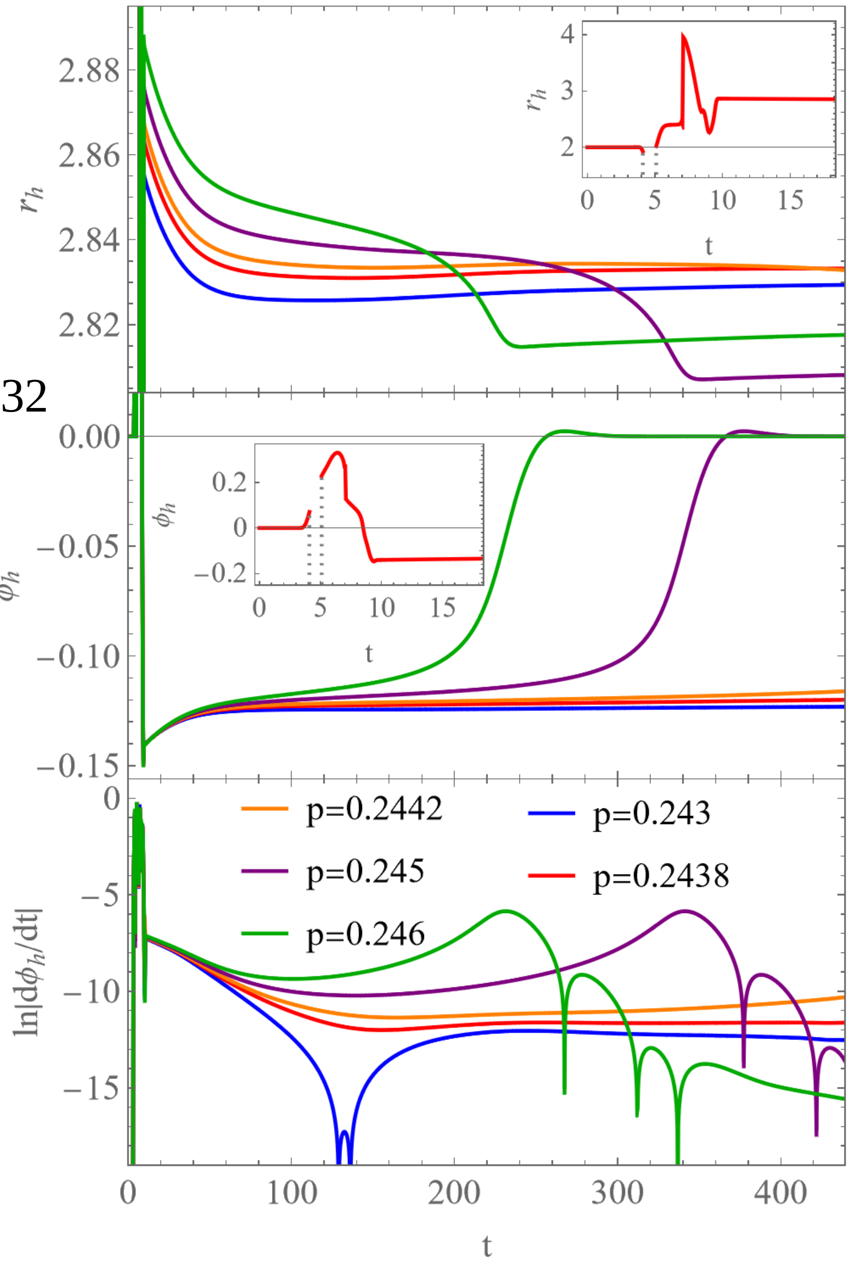


Violation of **NEC**
& BH area law

$p_{*1} \simeq 0.00135561054232$
$\delta p/p \sim 10^{-13}$

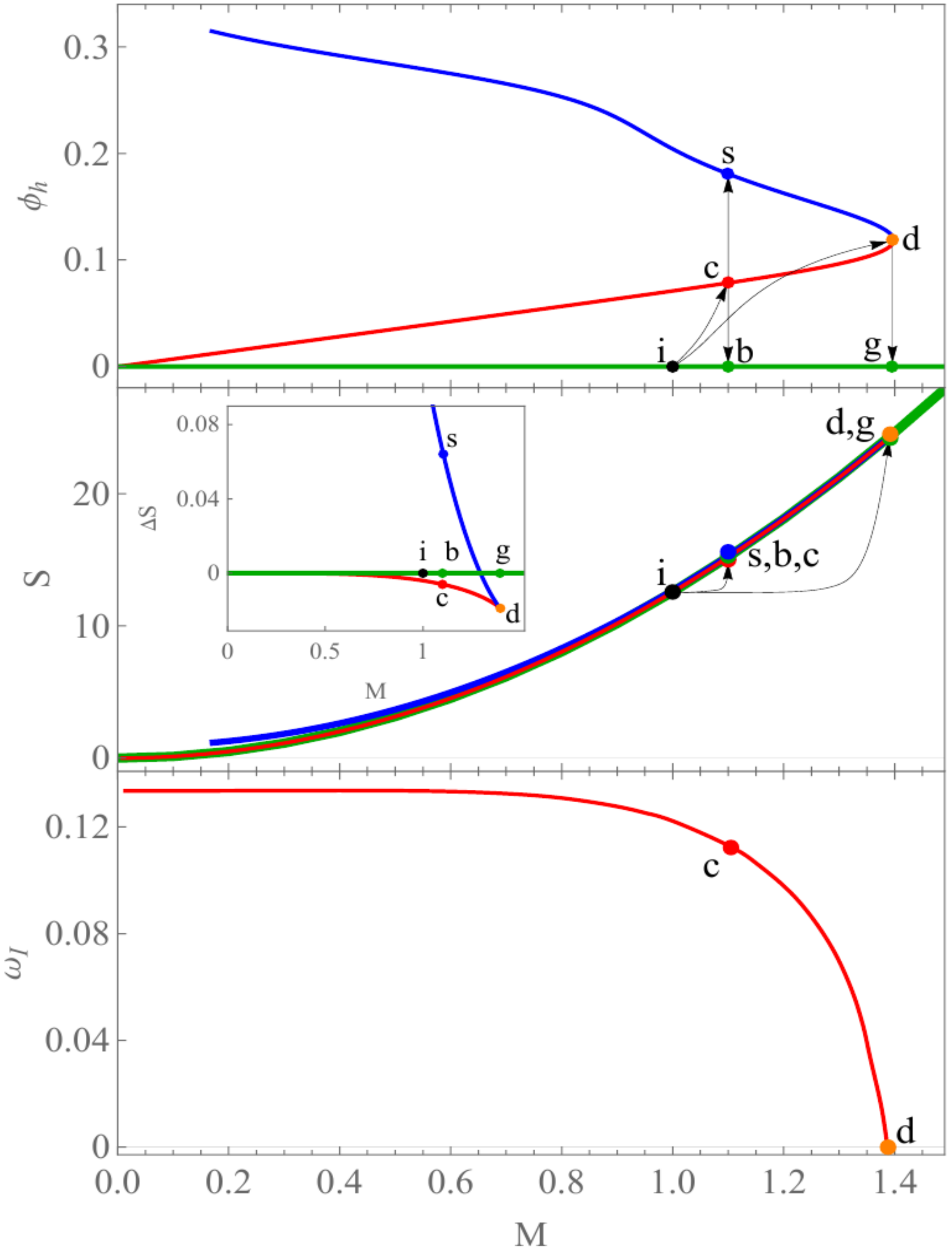
type I with an
unstable
attractor

Dynamical critical descalarization



$p_{*2} \simeq 0.002438$
$\delta p/p \sim 10^{-4}$

type I with an
marginally
stable attractor



Explanation from the static solutions

Three static solutions at the same M:

two stable: (1) Schwarzschild

(2) hot SBH

one unstable: (3) cold SBH (CS)

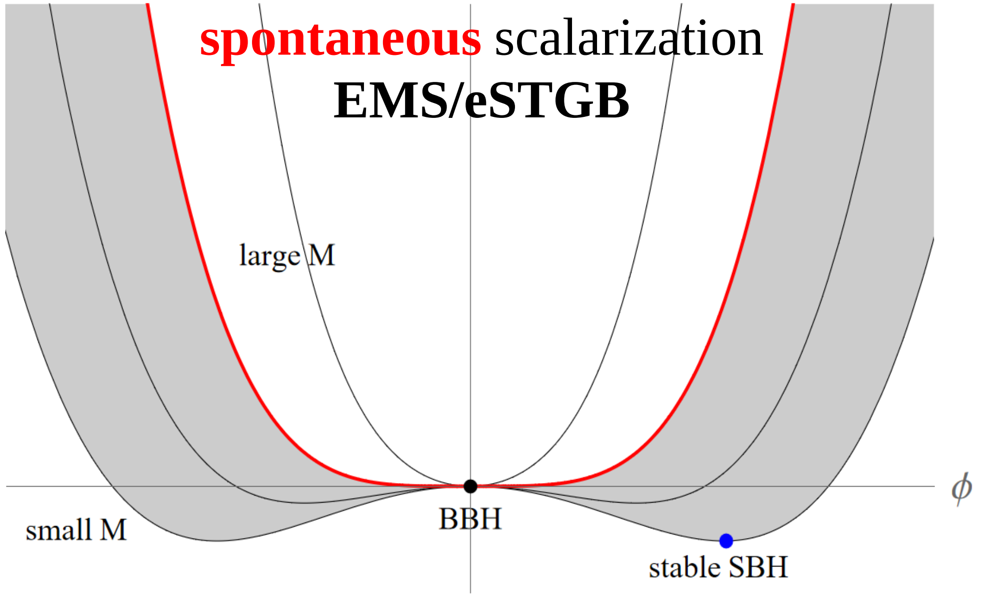
Scalarization: $i \rightarrow c \rightarrow \begin{cases} b, & \text{BBH } (p < p_{*1}) \\ s, & \text{SBH } (p > p_{*1}) \end{cases}$

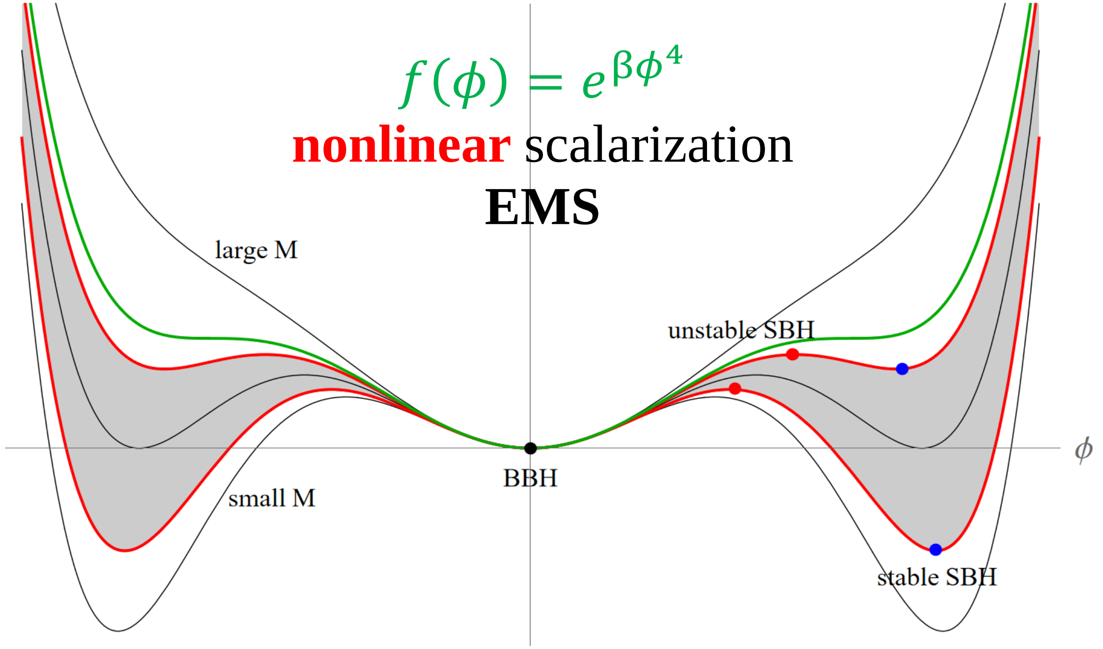
Descalarization: $i \rightarrow d \rightarrow \begin{cases} d, & \text{SBH } (p < p_{*2}) \\ g, & \text{BBH } (p > p_{*2}) \end{cases}$

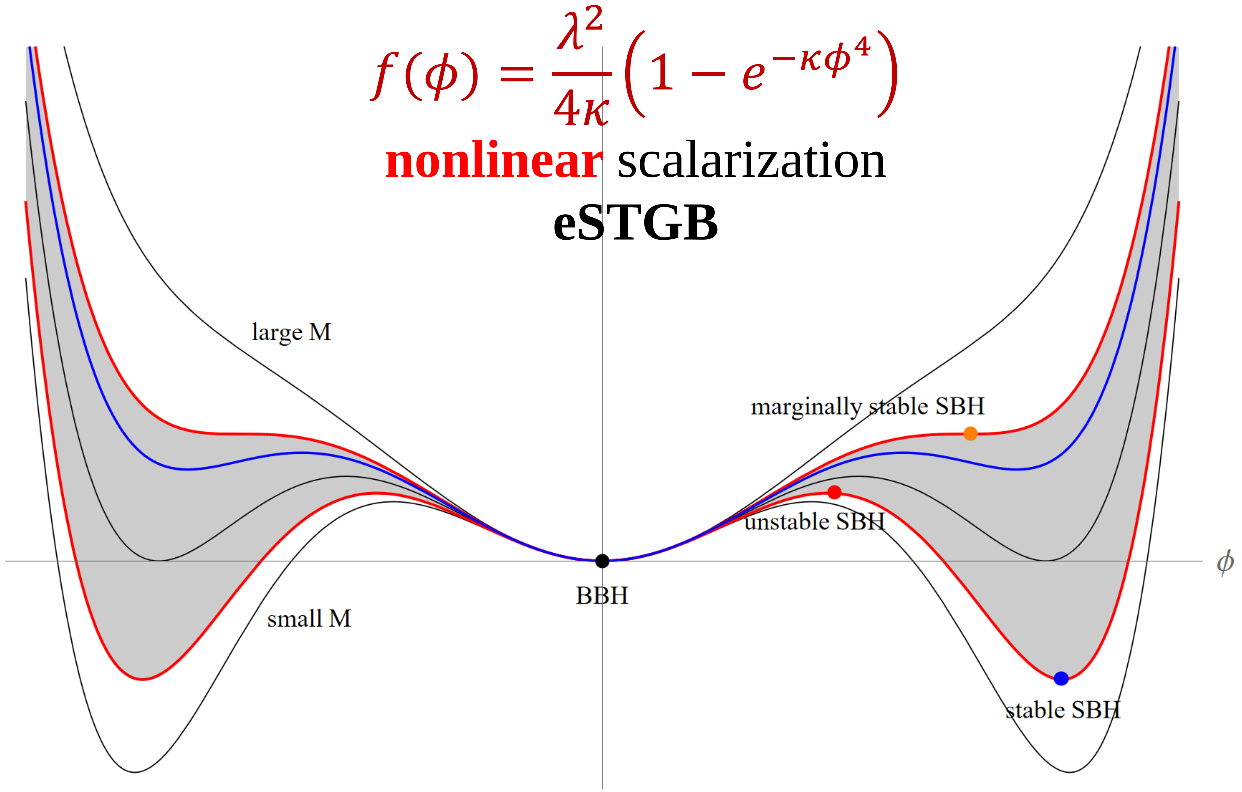
(d: **marginally** stable CS/attractor/SBH)

Dynamical **first-order** phase transition

Sketch map for the free energy







Dynamics vs Thermodynamics:
Deep physics?

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3. Summary and Outlook

Summary

1. We found new BH **scalarization & descalarization** mechanisms through the accretion of the scalar field

2. We uncovered **novel rich dynamical critical behaviors** in the bald/scalarized BH transition
   - **Scalarization**: type I with an unstable attractor
   - **Descalarization**: type I with an unstable/marginally stable attractor & type II (the model with spontaneous scalarization)

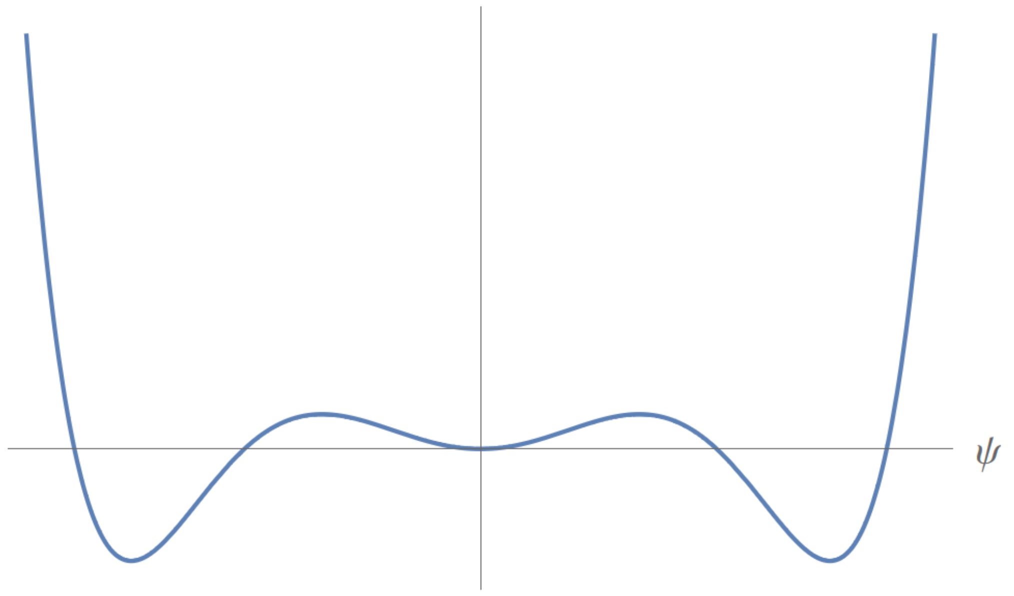
3. The discovery of these new dynamical critical behaviors has opened up a fascinating area of research in gravitational dynamics

Outlook

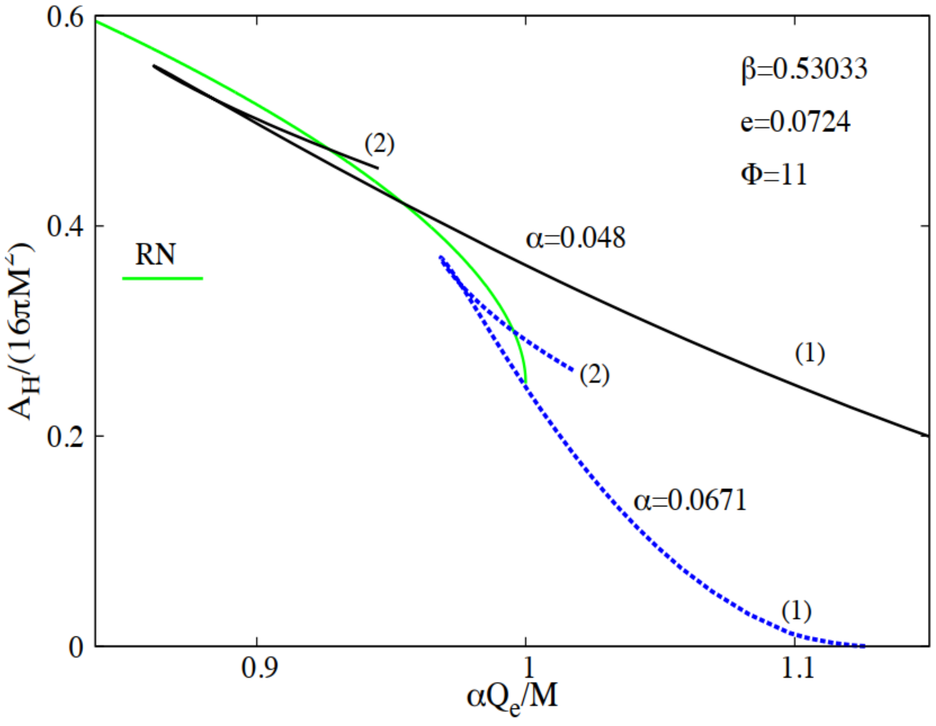
Q-ball (2004.03148PRL, 2004.00336EPJC)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha\Psi^* D^\alpha\Psi - U(|\Psi|)\right]$$

$$U(\psi)\sim\alpha^2|\psi|^2 - e^2|\psi|^4 + \beta^2|\psi|^6$$



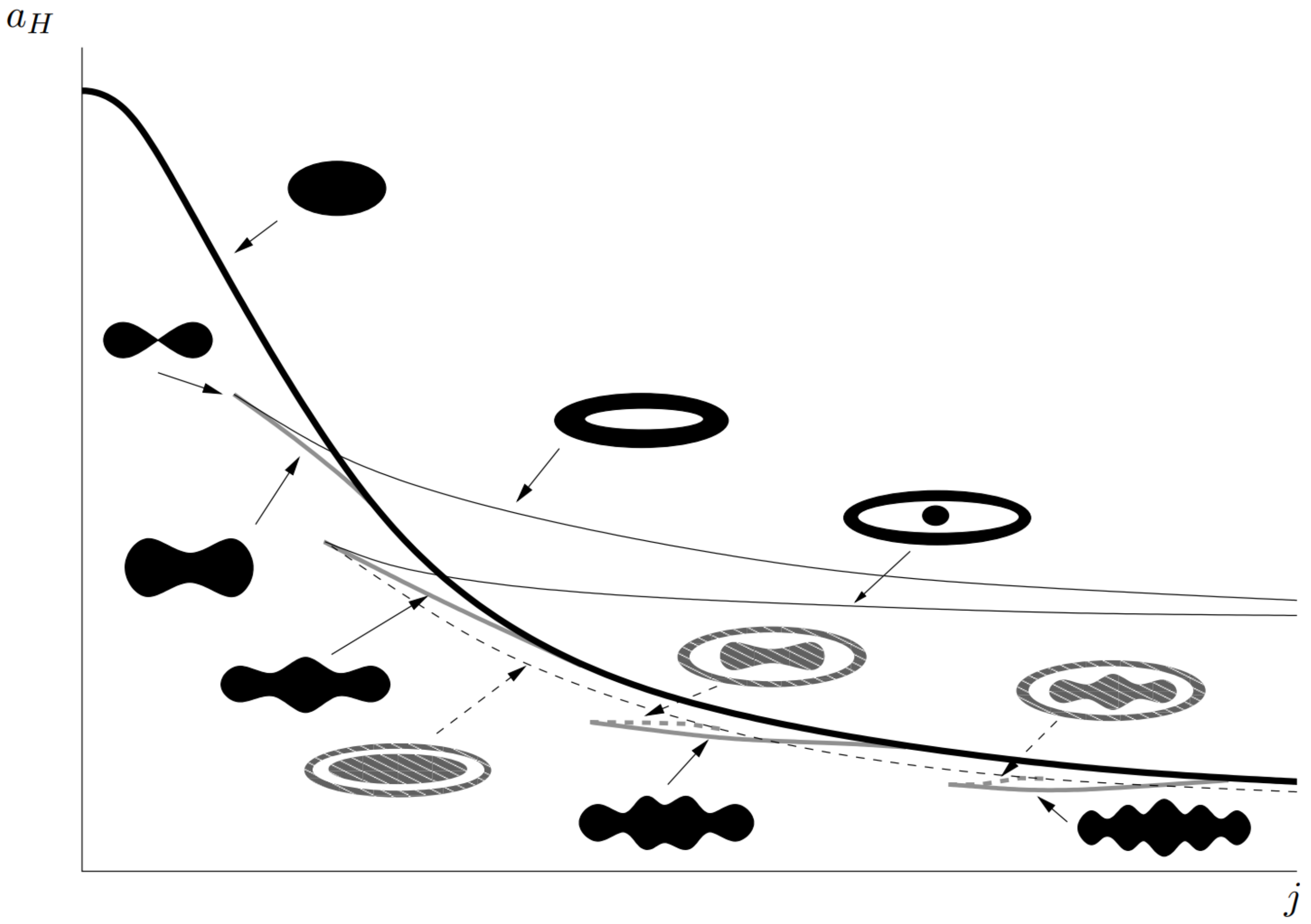
(in progress)



Higher-dimensional compact objects:

Myers-Perry BH, black ring/string/Saturn

(Emparan, hep-th/0110260 PRL, 0708.2181 JHEP, 2002.00963 PLB)



Holographic models:

phase separation in a strongly-coupled, non-Abelian gauge theory (QCD)

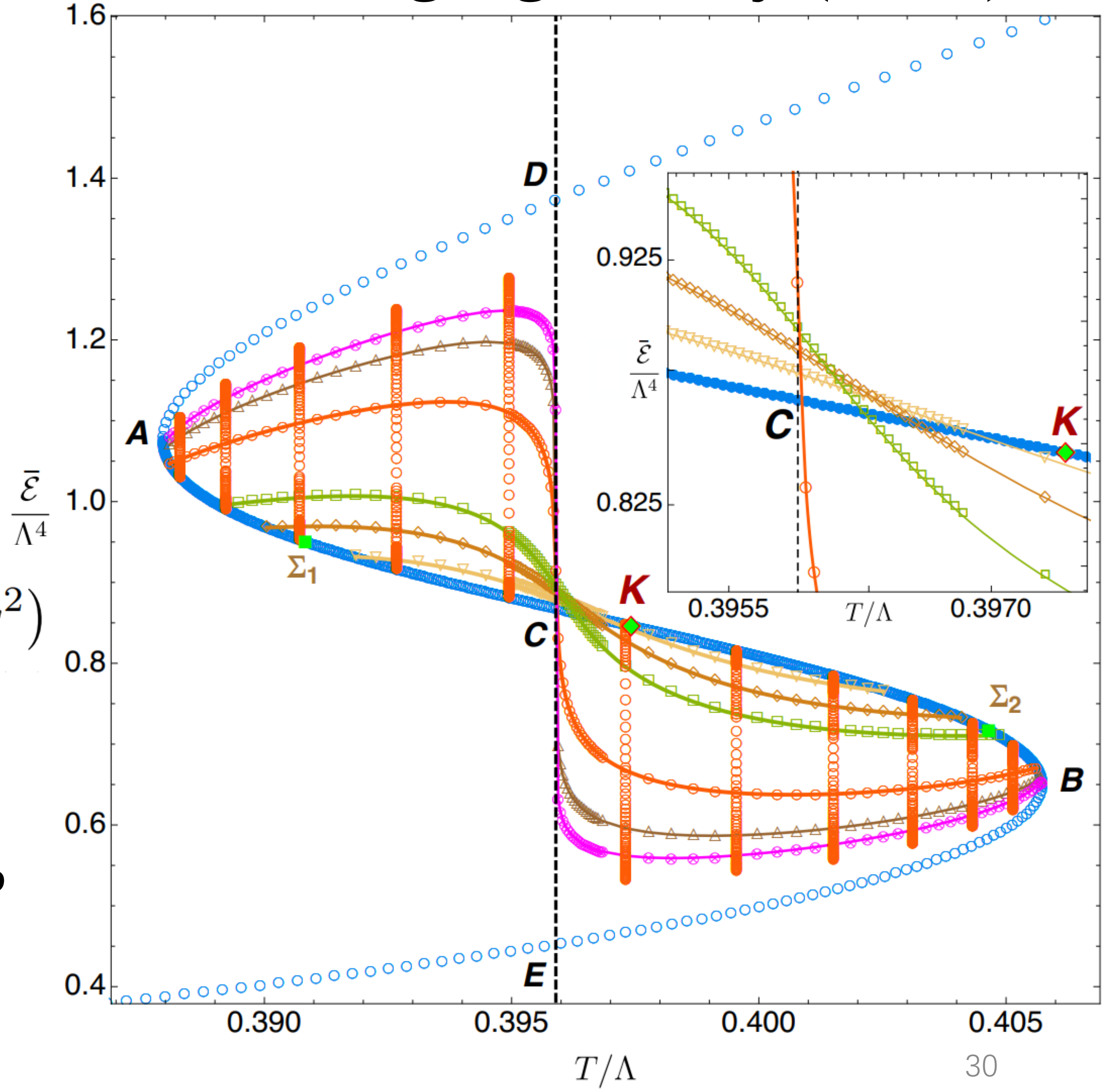
(1704.05387PRL, 2007.06467JHEP, 0804.0434PRD)

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - 2(\nabla\phi)^2 - 4V(\phi)\right]$$

$$V(\phi) = -6\cosh\left(\frac{\phi}{\sqrt{3}}\right) + b_4\phi^4 \quad \text{Supergravity}$$

$$ds^2 = -A\,dt^2 - \frac{2\,dt\,dz}{z^2} - 2\,B\,dt\,dx + S^2\left(G\,dx^2 + G^{-1}\,dy^2\right)$$

$$x \in [0, L]$$



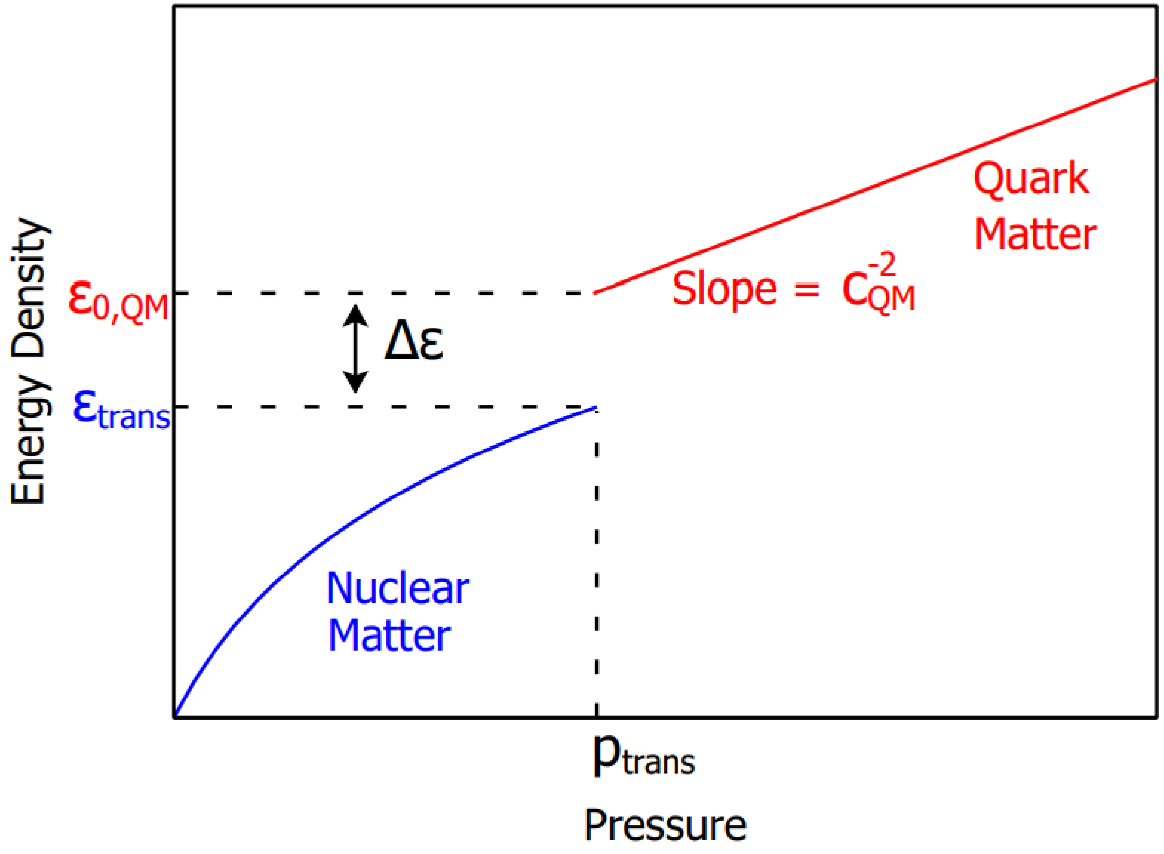
The critical dynamics in holographic models?

(our work: 2209.12789 JHEP)

Neutron star binary merger (1807.03684 PRL,1810.10967PRD)

- quarks are deconfined (1807.03684 PRL)
- a quark-hadron first-order phase transition would leave in the gravitational-wave signal

1. The critical dynamics in NS binary merger?
2. How about first-order phase transion in BH binary merger?



Goal

Classification of dynamical first-order phase transition of BHs?

(1) **Tachyonic** instability: EMS, eSTGB

(2) **Superradiant** instability:
   a. Charge: RN + Q-ball,
   b. Rotating: Kerr + complex self-interaction scalar

(3) **Gregory-Laflamme** instability: Myers-Perry BH/ Black ring, Holographic

Dynamical first-order phase transition of compact stars?

Thanks for your attention!