Testing Quantum Gravity by Entanglements and Decoherence

Yi Wang (王一), Department of Physics

The Hong Kong University of Science and Technology

20230407 @ USTC

Plan:

Quantum gravity

- Approaches
- From quantum information

Are cosmological perturbations quantum?

- Generation mechanism
- Decoherence (new: with boundary term)

Observational tests?

- Bell inequalities?
- Quantum noise?
- Null tests?

What’s quantum gravity?

Claims from strongest to weakest:

- What is *the* quantum gravity theory?
  - Not only QG but also *new physics*
  - Strings: what is a/the quantum gravity theory?
  - Other possible unifications of QM & GR?
- Check if gravity is quantized (known physics)
  - Seeking for gravitons
- Check gravity has some quantum natures (known physics)
  - Superposition, entanglements & decoherence

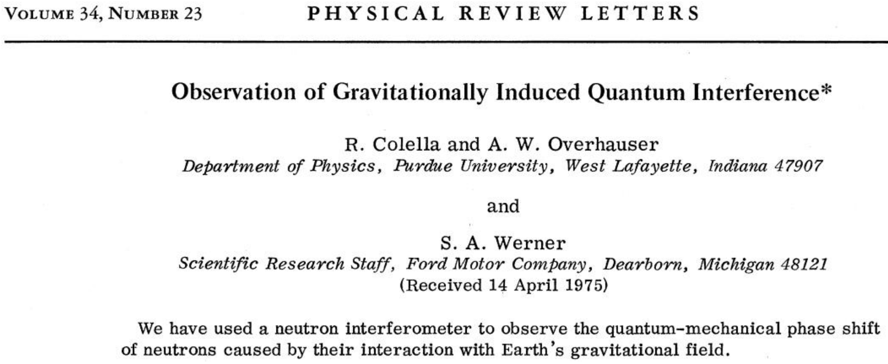
The quantum information regime of quantum gravity:

- Can matter superposition be affected by gravity?
- Can matter entanglement be caused by gravity?
- Can gravitational superposition affect matter?
- Other ideas?

The quantum information regime of quantum gravity:

- Can matter superposition be affected by gravity?

  Yes (COW 1975), but does not really quantum gravity


VOLUME 34, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1975

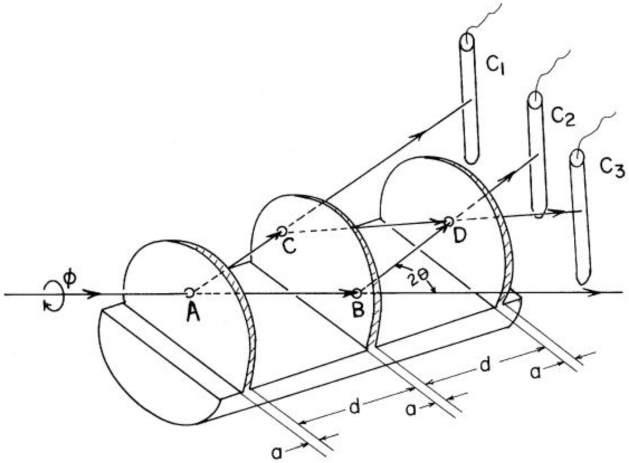
Observation of Gravitationally Induced Quantum Interference*

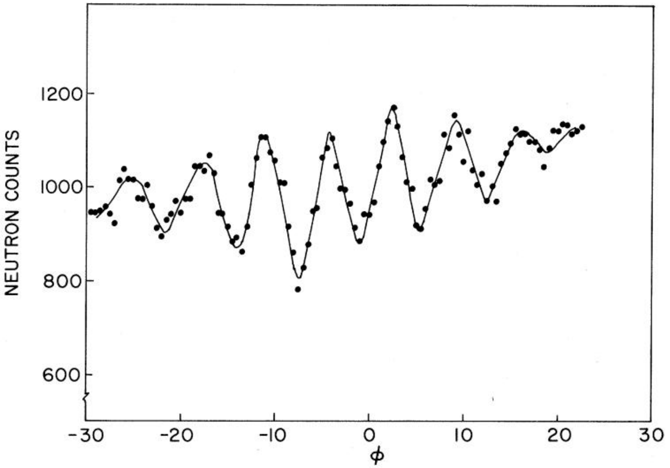
R. Colella and A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907

and

S. A. Werner
Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121
(Received 14 April 1975)

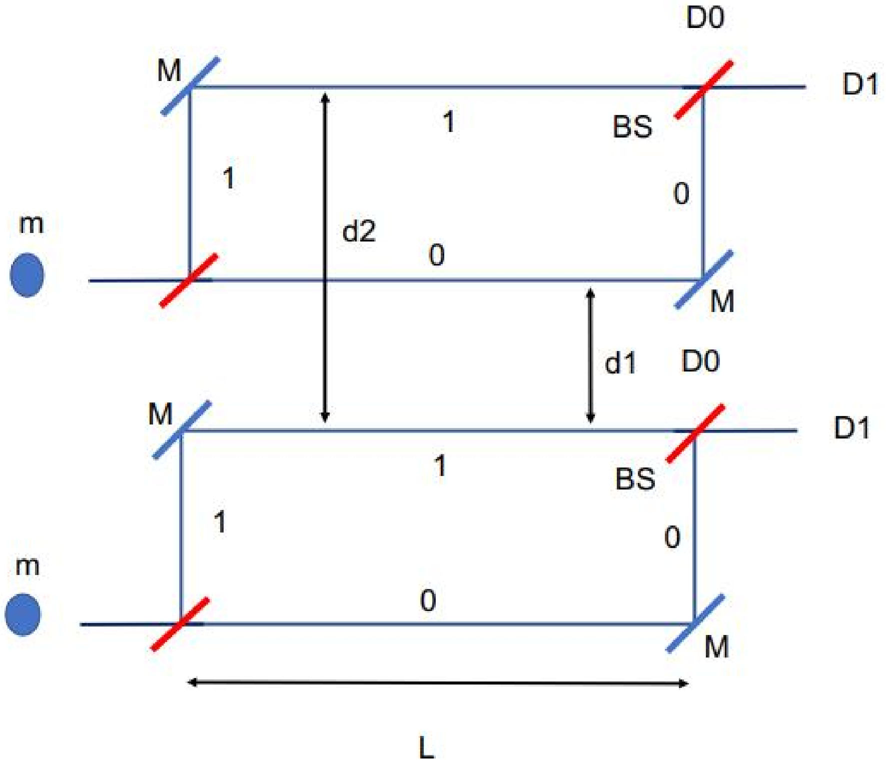
We have used a neutron interferometer to observe the quantum-mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field.





The quantum information regime of quantum gravity:

- Can matter superposition be affected by gravity?
- Can matter entanglement be caused by gravity?



Bose et al (2017), Marletto & Vderal (2017)

Matter entanglement ⇒ quantum gravity:

Qubit 1 --- creation of entanglement with C --- Qubit 2

Then C cannot have only one observable (classical)

Proof: If initially state separable: $|Q_1(0)\rangle|Q_2(0)\rangle$

Marletto & Vderal 2017

and if Hamiltonian H_1 acts on $|Q_1(0)\rangle$, H_2 acts on $|Q_2(0)\rangle$,

gravity is reflected by time-dependent coefficients in H_1 & H_2

Then the final state $|Q_1(t)\rangle|Q_2(t)\rangle$ is also separatable

Challenge: is the constraint part "local"?

More generally:

Separated systems cannot be entangled by

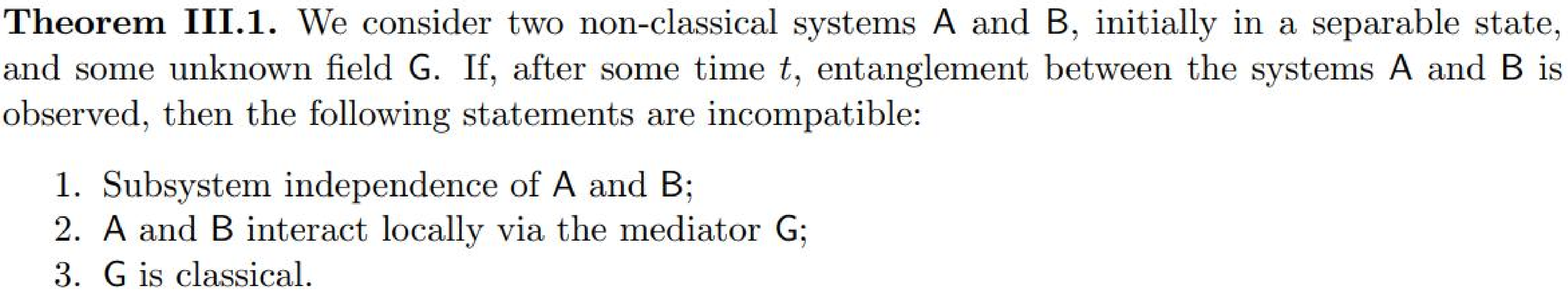
Local Operations and Classical Communication (LOCC)

See, e.g. R. Horodecki, P. Horodecki, M. Horodecki, K. Horo

Generalizing LOCC to Generalized Probabilistic Theories (GPT)

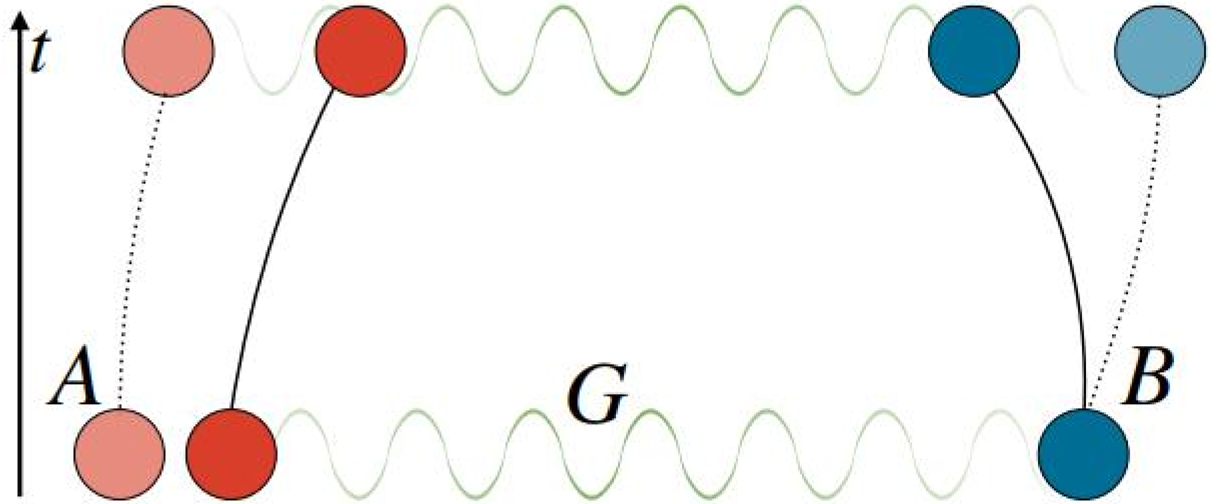
A no-go theorem on the nature of the gravitational field beyond quantum th

Galley, Giacomini, Selby (2020)



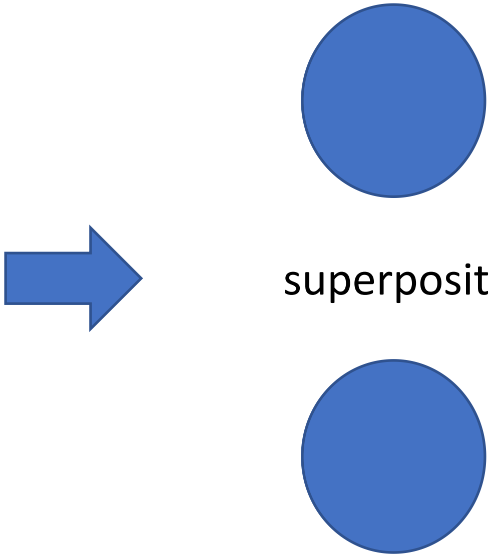
Theorem III.1. We consider two non-classical systems A and B, initially in a separable state, and some unknown field G. If, after some time t, entanglement between the systems A and B is observed, then the following statements are incompatible:

1. Subsystem independence of A and B;
2. A and B interact locally via the mediator G;
3. G is classical.



The quantum information regime of quantum gravity:

- Can matter superposition be affected by gravity?
- Can matter entanglement be caused by gravity?
- Can gravitational superposition affect matter?



Matter:

- Attracted by one (superposition)? $G_{\mu\nu} \propto T_{\mu\nu}$

- Attracted by the average? $G_{\mu\nu} \propto \langle T_{\mu\nu} \rangle$

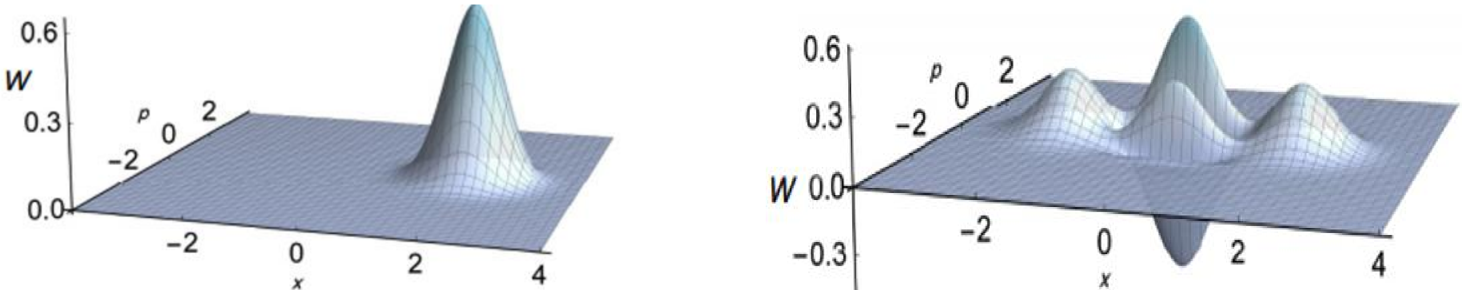
(Schrodinger-Newton semi-classical gravity)

The quantum information regime of quantum gravity:

- Can matter superposition be affected by gravity?
- Can matter entanglement caused by gravity?
- Can gravitational superposition affect matter?
- Other ideas?

Non-Gaussianity as a Signature of a Quantum Theory of Gravity

Richard Howl, Vlatko Vedral, Devang Naik, Marios Christodoulou, Carlo Rovelli, and Aditya Iyer
PRX Quantum **2**, 010325 – Published 17 February 2021



Plan:

Quantum gravity

- Approaches
- From quantum information

Are cosmological perturbations quantum?

- Generation mechanism
- Decoherence (detailed)

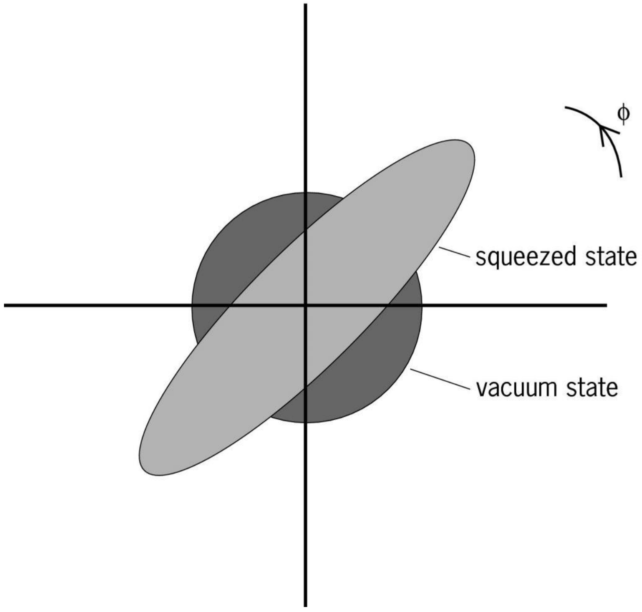
Observational tests?

- Bell inequalities?
- Quantum noise?
- Null tests?

Coherent, squeezed and thermal states

- Coherent:
  - Eigenstate of annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$, $|\alpha\rangle = e^{\alpha a^\dagger - \alpha^* a}\,|0\rangle$
  - Gaussian wave function with minimal uncertainty
  - Oscillatory behavior of $\langle x\rangle$ for a harmonic oscillator (semi-classical)
- Squeezed:
  - $e^{\frac{1}{2}(z a^{\dagger 2} + z^* a^2)}|0\rangle$
- Thermal:

$$\rho_\omega^{\mathrm{th}} = \frac{1}{Z}\sum_{n=0}^{\infty} e^{-\frac{\hbar\omega(n+1/2)}{k_B T}}|n\rangle\langle n|$$



The standard cosmological perturbation is "squeezed"

$$S'_{\rm pert} = \frac{1}{2}\int d^4x[(v')^2 - c_s^2(v_{;i})^2 - 2\frac{z'}{z}vv' + (\frac{z'}{z})^2v^2]$$

$$\hat{\mathcal{H}}_{\vec{k}} = \hat{\mathcal{H}}^{(0)}_{\vec{k}} + \hat{\mathcal{H}}^{(I)}_{\vec{k}} = \Omega_{\vec{k}}(a^\dagger_{\vec{k}}a_{\vec{k}} + a^\dagger_{-\vec{k}}a_{-\vec{k}} + 1) + i\lambda_{\vec{k}}(e^{-2i\varphi_{\vec{k}}}a_{\vec{k}}a_{-\vec{k}} - \text{h.c.})$$

$$\begin{aligned}
\Omega_{\vec{k}} &= \frac{k}{2}(1+c_s^2) \\
\lambda_{\vec{k}} &= \left[\left(\frac{k}{2}(1-c_s^2)\right)^2 + \left(\frac{z'}{z}\right)^2\right]^{\frac{1}{2}} \\
\varphi_{\vec{k}} &= -\frac{\pi}{2} + \frac{1}{2}\arctan\left(\frac{kz}{2z'}(1-c_s^2)\right)
\end{aligned}$$

The time evolution operator of a state can be written as

$$\mathcal{U}_{\mathcal{H}_{\vec{k}}}(\eta,\eta_0) = \mathcal{S}[R_{\vec{k}},\Phi_{\vec{k}}]\mathcal{R}[\Theta_{\vec{k}}] \qquad \mathcal{R}[\Theta_{\vec{k}}] = \exp[-i\Theta_{\vec{k}}(a^\dagger_{\vec{k}}a_{\vec{k}} + a^\dagger_{-\vec{k}}a_{-\vec{k}} + 1)]$$

Two-mode rotation operator

$$\mathcal{S}[R_{\vec{k}},\Phi_{\vec{k}}] = \exp\left[\frac{R_{\vec{k}}}{2}(e^{-2i\Phi_{\vec{k}}}a_{-\vec{k}}a_{\vec{k}} - \text{h.c.})\right]$$

Two-mode squeeze operator

$$|SS_{\vec{k}}\rangle = \mathcal{S}[R_{\vec{k}},\Phi_{\vec{k}}]|0\rangle_{\rm in} = \sum_{n=0}^{\infty}\frac{1}{\cosh R_{\vec{k}}}(-e^{2i\Phi_{\vec{k}}}\tanh R_{\vec{k}})^n|n,\vec{k};n,-\vec{k}\rangle$$

$$|n,\vec{k};n,-\vec{k}\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}(a^\dagger_{\vec{k}}a^\dagger_{-\vec{k}})^n|0\rangle_{in}$$

Albrecht, Ferreira, Joyce, Prokopec, 1993

See also Polarski, Starobinsky, 1995

But can the inflationary fluctuation be

- Semiclassical (coherent)?
- Thermal? (thermal inflation, warm inflation)

And decoherence by interaction?

Decoherence of cosmological perturbations:

Horizon exit: Two-mode squeezed state has large $\langle N \rangle$,

but not yet classical

Interaction: Decoherence. Consider wave function of ζ

$$\langle \mathcal{E}, \xi | \Psi(\tau) \rangle = \exp\left(\int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{E}_{\mathbf{k}} \mathcal{E}_{\mathbf{k}'} \xi_{\mathbf{q}} \right) \Psi_G(\mathcal{E}, \xi)$$

System

Gaussian part

Environment

$$\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \approx i \int_{\tau_i}^{\tau} \frac{d\tau'}{H\tau'} \tilde{H}^{\text{int}}_{\mathbf{k},\mathbf{k}',\mathbf{q}}(\tau') \frac{u_k(\tau') u_{k'}(\tau') u_q(\tau')}{u_k(\tau) u_{k'}(\tau) u_q(\tau)} + \mathcal{O}\left((\tilde{H}^{\text{int}})^2 \right)$$

$$\rho_R(\xi, \tilde{\xi}) = \Psi_G^{(\xi)}(\xi) \left[\Psi_G^{(\xi)}(\tilde{\xi}) \right]^* \langle \exp(X) \rangle_{\mathcal{E}}$$

$$\langle \cdots \rangle_{\mathcal{E}} \equiv \int D\mathcal{E} \left| \Psi_G^{(\mathcal{E})} \right|^2 (\cdots) \qquad X \equiv \int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \mathcal{E}_{\mathbf{k}} \mathcal{E}_{\mathbf{k}'} \left(\xi_{\mathbf{q}} \mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} + \tilde{\xi}_{\mathbf{q}} \mathcal{F}^*_{\mathbf{k},\mathbf{k}',\mathbf{q}} \right)$$

$$D(\xi, \tilde{\xi}) \equiv \left| \frac{\rho_R(\xi, \tilde{\xi})}{\sqrt{\rho_R(\xi, \xi) \rho_R(\tilde{\xi}, \tilde{\xi})}} \right|$$

Nelson, 1601.03734

Liu, Sou, YW, 1608.07909

Decoherence of cosmological perturbations:

Horizon exit: Two-mode squeezed state has large $\langle N \rangle$,
but not yet classical

Interaction: Decoherence. Consider wave function of ζ

[Importance of the "wave function" of perturbations:

It's the fundamental object,

applications: locality, analyticity, bootstrap, decoherence, …]

Decoherence of cosmological perturbations:

Horizon exit: Two-mode squeezed state has large $\langle N \rangle$,
but not yet classical

Interaction: Decoherence. Consider wave function of ζ

$$\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}} \approx i\int_{\tau_i}^{\tau} \frac{d\tau'}{H\tau'}\tilde{H}^{\text{int}}_{\mathbf{k},\mathbf{k}',\mathbf{q}}(\tau')\frac{u_k(\tau')u_{k'}(\tau')u_q(\tau')}{u_k(\tau)u_{k'}(\tau)u_q(\tau)} + \mathcal{O}\left((\tilde{H}^{\text{int}})^2\right)$$

↓

Which interaction? Nelson, 1601.03734: $\mathcal{L}_{\text{int}} = -\frac{M_p^2}{2}\epsilon(\epsilon+\eta)a(t)\zeta^2\partial^2\zeta$

We noted: this boundary term is more important:

$$\mathcal{L}_{\text{bd}} = \partial_t\left(-2a^3HM_p^2e^{3\zeta}\right)$$

Order ε^{-2} faster decoherence!

Sou, Tran, YW, 2207.04435

$$\int_{\mathcal{M}} d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R_{\rm ADM}+\mathcal{L}_\phi\right]$$
$$=\int_{\mathcal{M}} d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R+\mathcal{L}_\phi\right]+S_{\rm GHY}$$
$$=\int dt\ d^3x\ \partial_t\left(-2a^3HM_p^2e^{3\zeta}\right)+\ \ldots$$

A note about the boundary term:

- Fast-varying phase
- Will not change:

Correlation functions $\langle\zeta^n\rangle=\left\langle\overline{T}e^{i\int H_{\rm int}d\tau}\ \ \zeta^n\ \ Te^{-i\int H_{\rm int}d\tau}\right\rangle$

Isocurvature fluctuations

- But will change:

$\langle\dot{\zeta}^n\rangle$ Do not commute on the boundary,

But vanishing for inflation [Bounce? Ultra-slow-roll?]

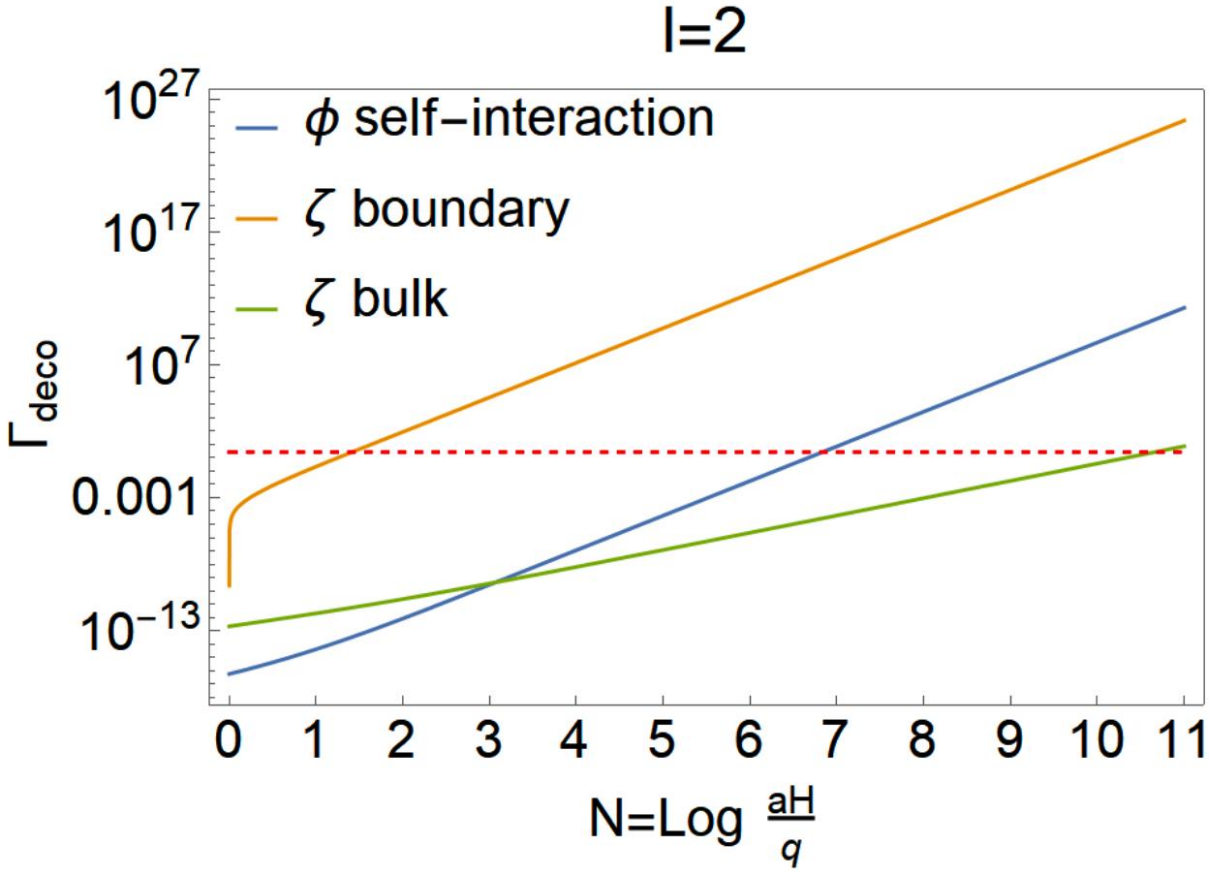
Decoherence rate

As decoration of wave function

$\delta\varphi$ is more quantum than ζ

Sou, Tran, YW, 2207.04435

Gravity-induced decoherence – Quantum gravity effect!



Sou, Tran, YW, 2207.04435

Decoherence induced by gravity

- Indicator of quantum gravity
- Comparison: Schrodinger-Newton gravity $G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$

  See Galley, Giacomini, Selby (2020) for other classical gravity models

- But can decoherence rate be observable?

Generalizations:

- Isocurvature
- Gravitational waves
- Quantum creation of the universe

Plan:

Quantum gravity

- Approaches
- From quantum information

Are cosmological perturbations quantum?

- Generation mechanism
- Decoherence (detailed)

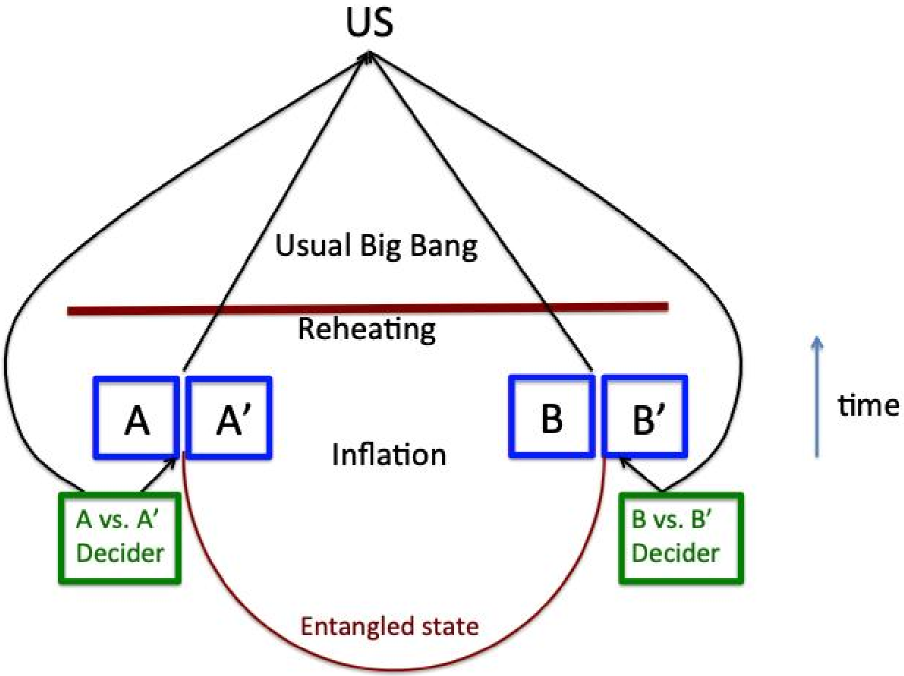
Observational tests?

- Bell inequalities?
- Quantum noise?
- Null tests?

What's the difference between quantum / classical?

1. Bell inequality? Violation of

$$|\langle AB\rangle + \langle AB'\rangle + \langle A'B\rangle - \langle A'B'\rangle| \le 2$$



Difficulty: non-communicate observables are rare

Maldacena 1508.01082

See also Kanno & Soda, 1705.06199

What's the difference between quantum / classical?

1. Bell inequality?
2. Quantum noise?

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - M_0\int dt\sqrt{-\dot{X}^2} - m_0\int dt\sqrt{-\dot{Y}^2}$$

$$S_\omega = \int dt\left(\frac{1}{2}m(\dot{q}^2 - \omega^2 q^2) + \frac{1}{2}m_0\dot{\xi}^2 - g\dot{q}\dot{\xi}\xi\right)$$

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{h} + \ddot{N}_\Psi - \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2\right)\xi$$

Parikh, Wilczek, Zahariade, 2005.07211

Kanno, Soda, Tokuda, 2007.09838

Squeezing ⇒ Large quantum noise

Can quantum noise be reformulated to fit the general framework of cosmic perturbations?

Is it a matter of operator ordering?

- Linear perturbations: equivalent (Chen, Namjoo, YW 2015)
- Nonlinear level?

What's the difference between quantum / classical?

1. Bell inequality?
2. Quantum noise?
3. Position uncertainty of dark matter particles?
   - Without decoherence:

$$s(t) = \sqrt{\frac{s_0^4 + (\hbar t/m)^2}{s_0^2}} \qquad s_{\min}(t) = \sqrt{\frac{2\hbar t}{m}} \qquad (10^{11}\text{GeV} \Leftrightarrow 1\text{m})$$

   - Decoherence: $m < 10^7$kg

     (Lindblad equation)

Neppoleon, Iyer, Vedral, YW, 2110.13438

See also Allali, Hertzberg 2012.12903 for ALPS

What's the difference between quantum / classical?

1. Bell inequality?
2. Quantum noise?
3. Position uncertainty of dark matter particles?
4. What about we see nothing – null test

Decoherence, if happened gravitationally

also indicates quantum gravity!

Conclusion:

- Squeezed state & decoherence
- Test quantum gravity? A null test may also be a test.