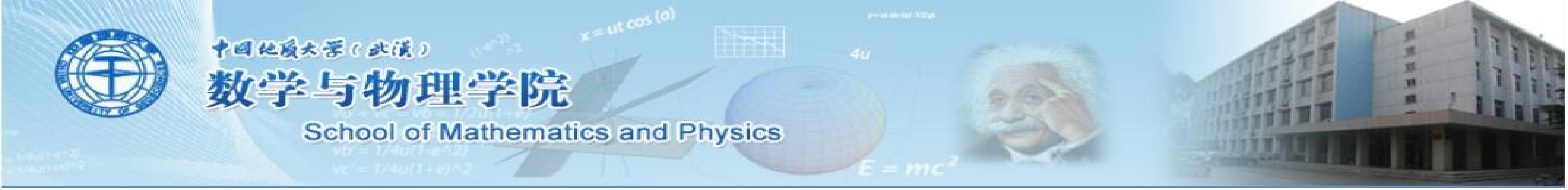
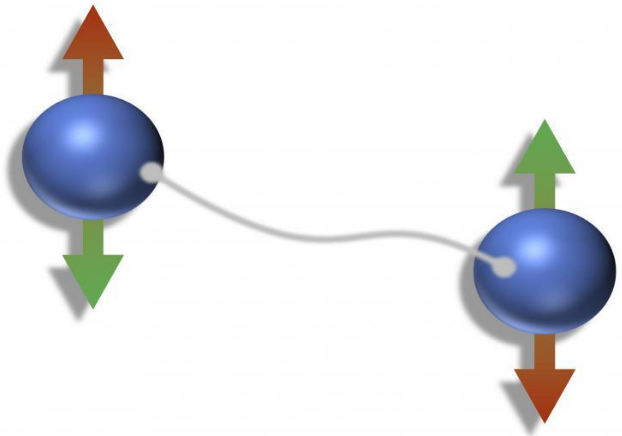


Effect of Spacetime Dimensions on Quantum Entanglement

Baocheng Zhang

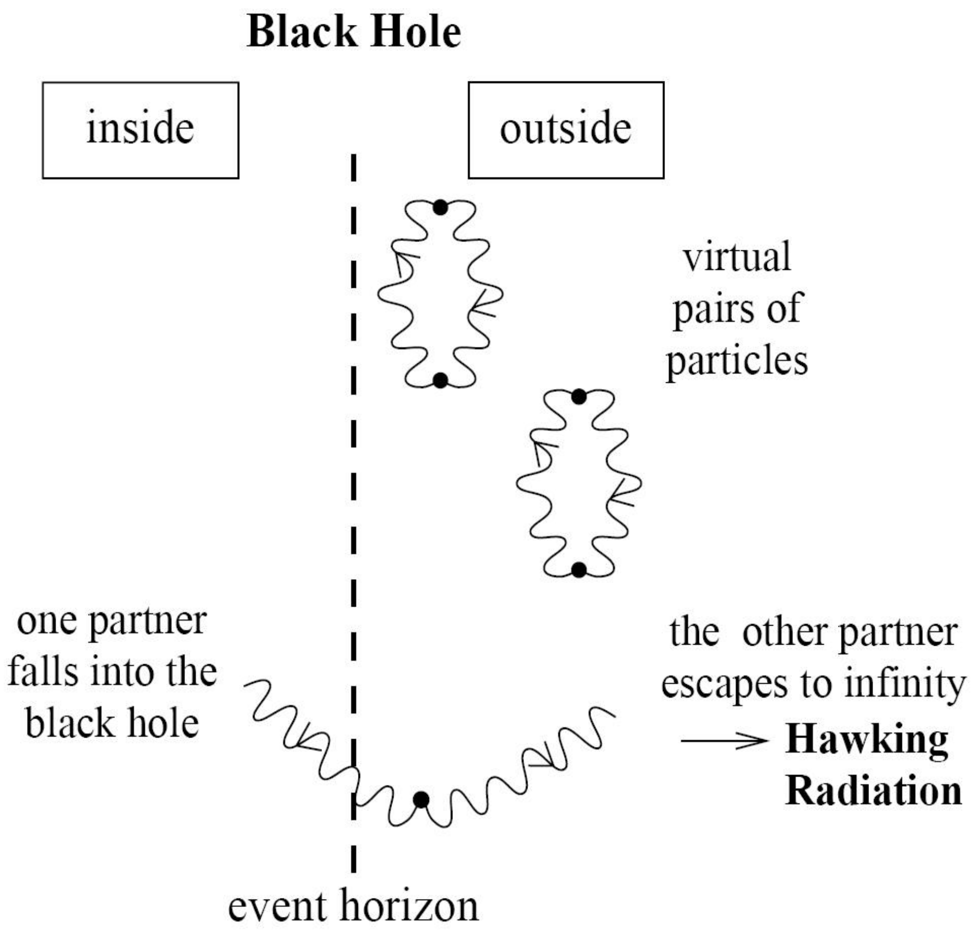
2023年中科大引力与宇宙学专题研讨会　合肥

CONTENTS



1 RESEARCH BACKGROUNDS

Information loss?



Irrespective of what initial state a black hole starts with before collapsing, it will evolves eventually into a thermal state after being completely exhausted into emitted radiations.

The radiations should be altered

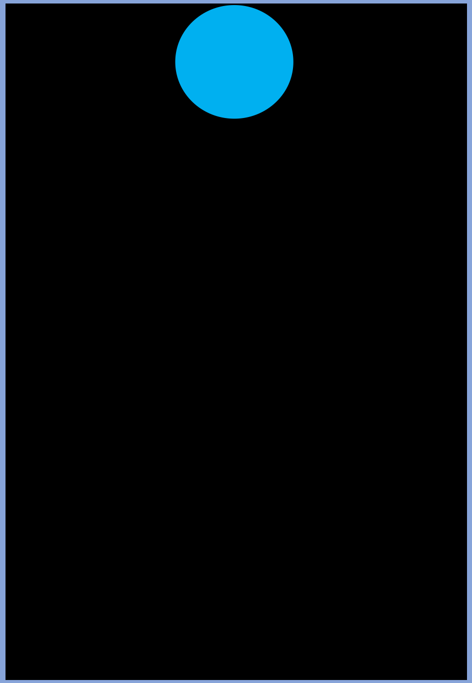
B. Zhang et al, IJMPD 22 (2013) 1341014

The interior should be reinterpreted

B. Zhang, PRD 92 (2015) 081501(R)
B. Zhang, et al, PLB 765 (2017) 226

Is information loss a feature for semi-classical theory? **Unruh effect!**

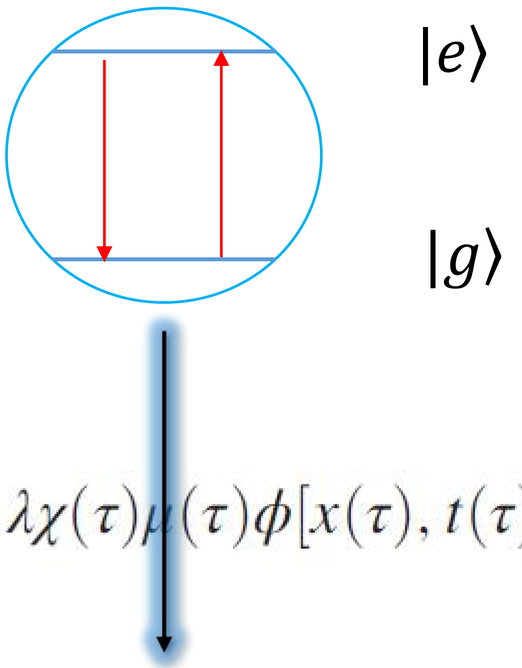
Unruh effect



The Unruh effect states that an observer with uniform acceleration *a* in the Minkowski vacuum of a free quantum field would feel a thermal bath of particles at the temperature

$$T_U = \frac{\hbar a}{2\pi c k_B} \sim 2 \times 10^{-21} a$$

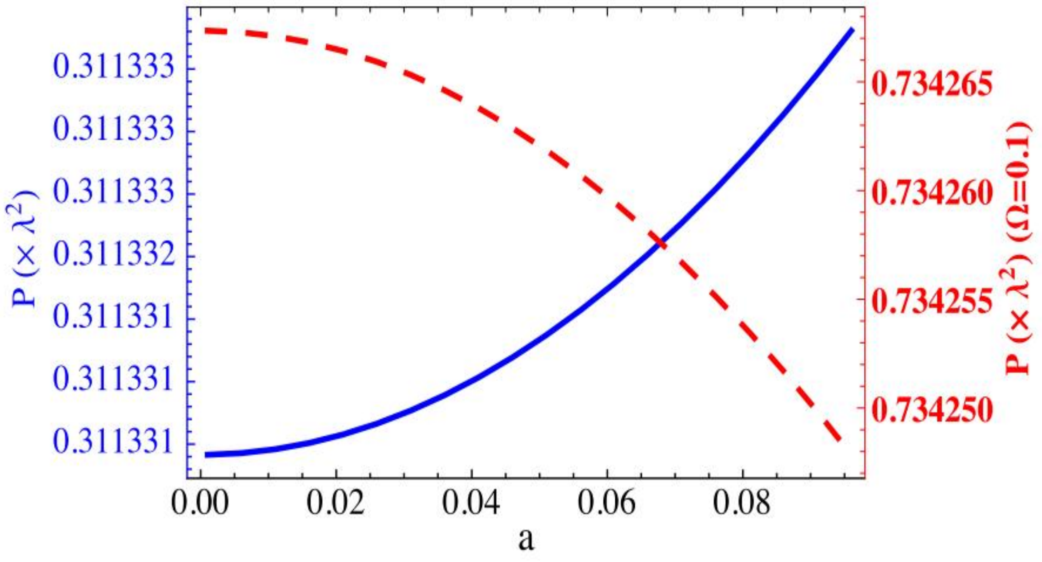
UDW Detectors



$$H_I = \lambda\chi(\tau)\mu(\tau)\phi[x(\tau), t(\tau)]$$

$$P = |\langle 1, e|U|g, 0\rangle|^2 = |D_0\eta_0|^2$$

- W. Brenna, R. B. Mann, and E. Martín-Martínez, Phys. Lett. B **757**, 307 (2016).
- L. J. Garay, E. Martín-Martínez, and J. deRamón, Phys. Rev. D 94, 104048 (2016).



$$p \propto T \propto a$$

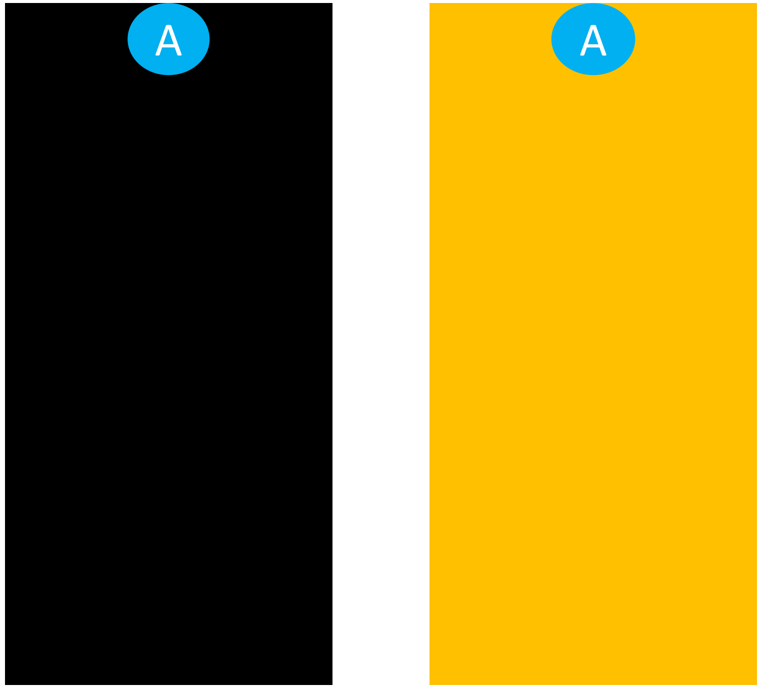
$$p \propto \frac{1}{a}$$

Unruh effect

Anti-Unruh effect

Transition Probability

Vacuum Thermal field



- T. Li, B. Zhang, and L. You, Phys. Rev. D 97, 045005 (2018).
- Y. Pan and B. Zhang, Phys. Rev. A 101, 062111 (2020).
- Y. Pan and B. Zhang, Phys. Rev. D 104, 125014 (2021).
- J. Yan and B. Zhang, JHEP 10, 051 (2022).
- Y. Pan and B. Zhang, Phys. Rev. D 107, 085001 (2023).

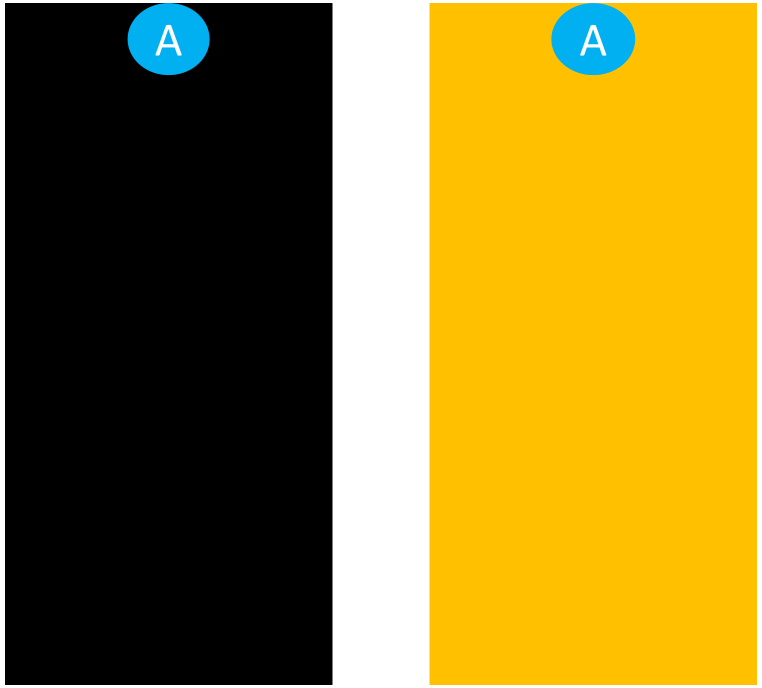
Change’s trend of transition probability

	Mass of field	Unruh effect	Anti-Unruh effect
Scalar	Massive	increase	decrease
	Massless	increase	increase

- Unruh effect or thermal field alone would lead to the increase of transition probability.
- Only anti-Unruh effect causes the decrease of transition probability.
- Anti-Unruh effect exists in thermal field.

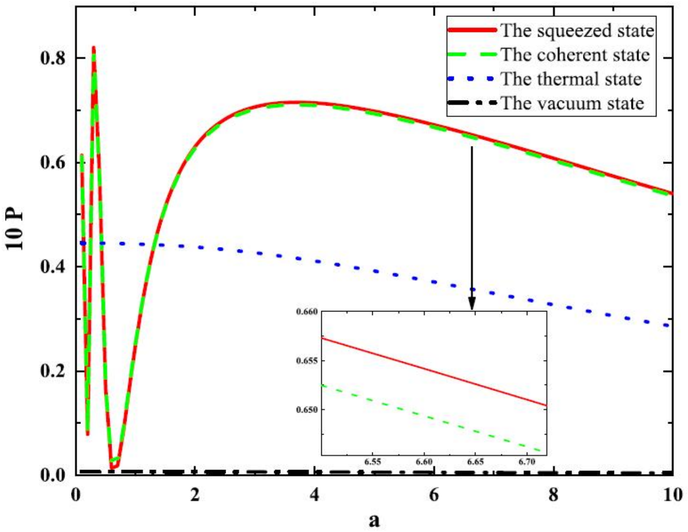
Transition Probability

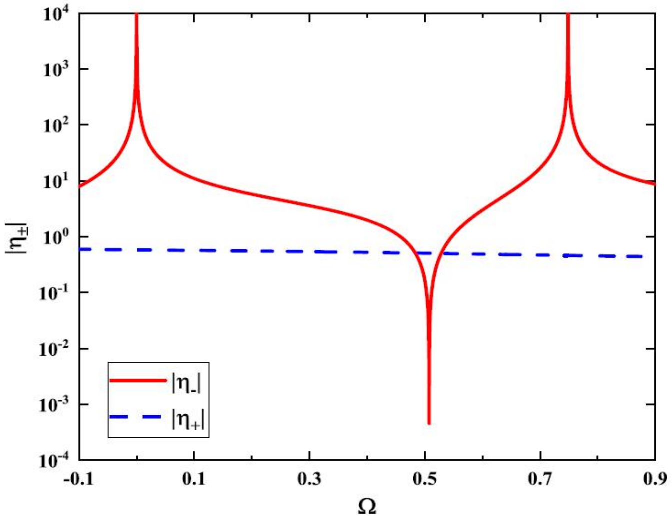
Vacuum EM field



$$P \propto \frac{B}{e^{2\pi\Omega/a} - 1}$$

$$P_a \propto \frac{(n+1)B}{e^{2\pi\Omega/a} - 1}$$





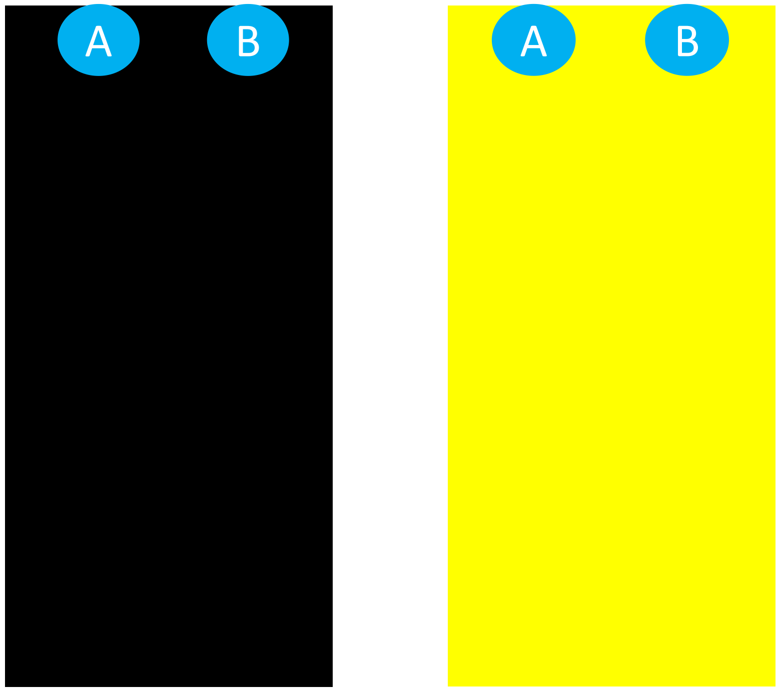
TP increase

Acceleration-induced transparency

- T. Li, B. Zhang, and L. You, Phys. Rev. D 97, 045005 (2018).
- Y. Pan and B. Zhang, Phys. Rev. A 101, 062111 (2020).
- Y. Pan and B. Zhang, Phys. Rev. D 104, 125014 (2021).
- J. Yan and B. Zhang, JHEP 10, 051 (2022).
- Y. Pan and B. Zhang, Phys. Rev. D 107, 085001 (2023).

1 研究背景 RESEARCH BACKGROUNDS Quantum Entanglement

Vacuum Thermal field



- T. Li, B. Zhang, and L. You, Phys. Rev. D 97, 045005 (2018).
- Y. Pan and B. Zhang, Phys. Rev. A 101, 062111 (2020).
- Y. Pan and B. Zhang, Phys. Rev. D 104, 125014 (2021).
- J. Yan and B. Zhang, JHEP 10, 051 (2022).
- Y. Pan and B. Zhang, Phys. Rev. D 107, 085001 (2023).

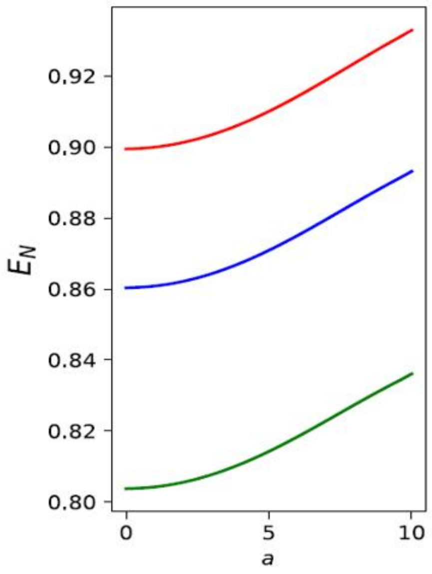
Logarithmic negativity

$$E_N = \log_2(2N+1)$$

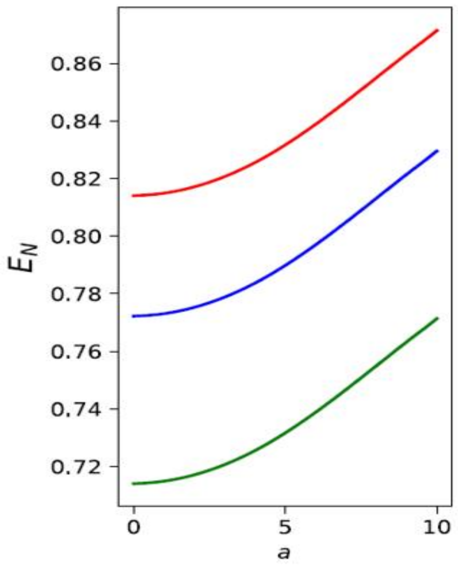
$$N = \sum_i (|\xi_i| - \xi_i)/2$$

$$0 \le E_N \le 1$$

ξ_i the i-th eigenvalue of the partial transpose of the bipartite state density matrix



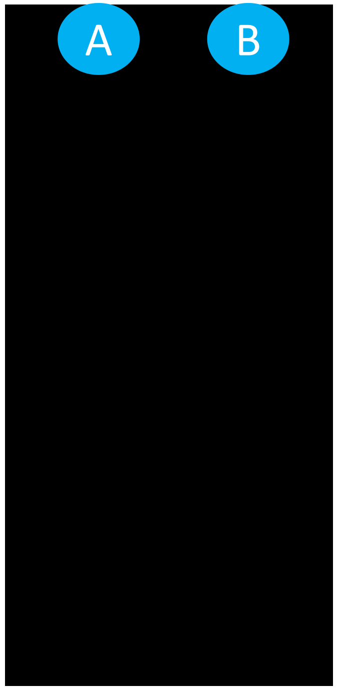
One atom is accelerated



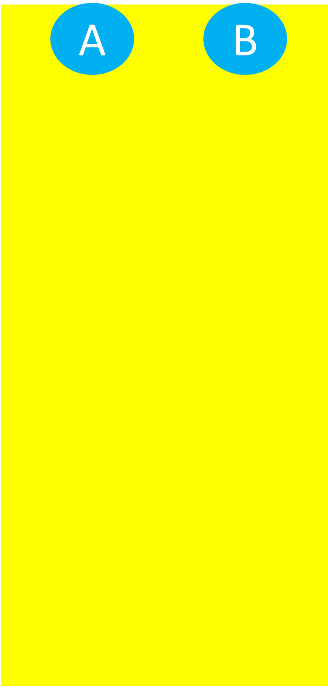
Two atoms are accelerated

Quantum Entanglement

Vacuum

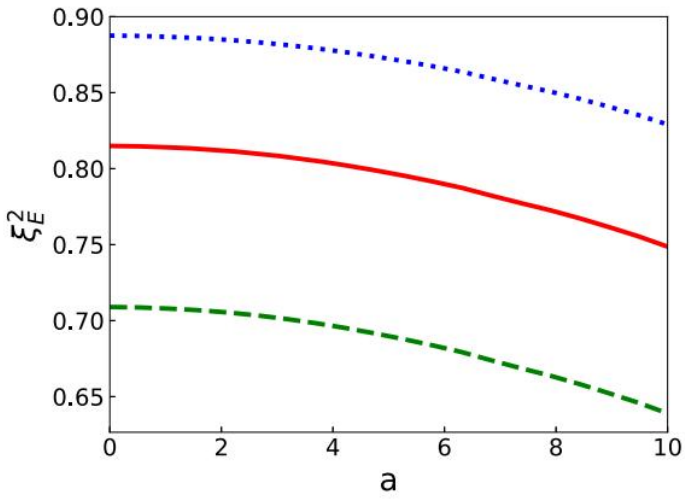


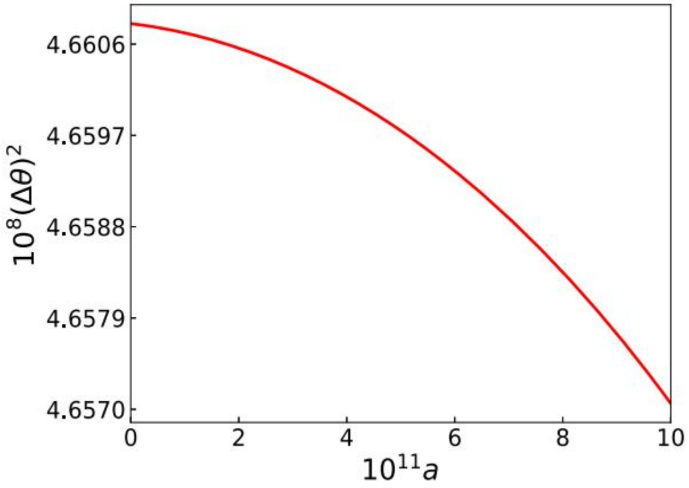
Thermal field



$$\xi_E^2 = \frac{(N-1)(\Delta j_z)^2 + \langle j_z \rangle^2}{\langle J^2 \rangle - \frac{N}{2}}$$

Squeezing parameter





- T. Li, B. Zhang, and L. You, Phys. Rev. D 97, 045005 (2018).
- Y. Pan and B. Zhang, Phys. Rev. A 101, 062111 (2020).
- Y. Pan and B. Zhang, Phys. Rev. D 104, 125014 (2021).
- J. Yan and B. Zhang, JHEP 10, 051 (2022).
- Y. Pan and B. Zhang, Phys. Rev. D 107, 085001 (2023).

- Anti-Unruh effect appears for accelerated atoms in thermal field.
- For the present experimental conditions, the observable sensitivity is reached with smaller requirement for acceleration.

No matter what kind of situation, entanglement will become less than the initial entanglement before the acceleration.

Why? How?

Entanglement Hides in High-dimensional Spacetime?

2 Research contents

J. Yan and B. Zhang, JHEP 10, 051 (2022).

Wightman function

Wightman function for massless Minkowski vacuum field

$$G\left(x,x'\right)=\mathcal{C}_D\left[(-1)\left(\left(t-t'-i\epsilon\right)^2-|x-x'|^2\right)\right]^{-(D-2)/2}$$

Trajectories of two accelerated atoms

$$t_1(\tau)=\frac{1}{a}\sinh a\tau, x_1^1(\tau)=\frac{1}{a}\cosh a\tau, x_1^2=x_1^3=\cdots=x_1^{D-2}, x_1^{D-1}=0,$$

$$t_2(\tau)=\frac{1}{a}\sinh a\tau, x_2^1(\tau)=\frac{1}{a}\cosh a\tau, x_2^2=x_2^3=\cdots=x_2^{D-2}, x_2^{D-1}=L,$$

Wightman function for accelerated atoms

$$G^{(11)}\left(x,x'\right)=G^{(22)}\left(x,x'\right)=\mathcal{C}_D\left(\frac{a}{2i}\right)^{D-2}\left[\sinh\left(\frac{a\tau}{2}-i\epsilon\right)\right]^{-(D-2)}$$

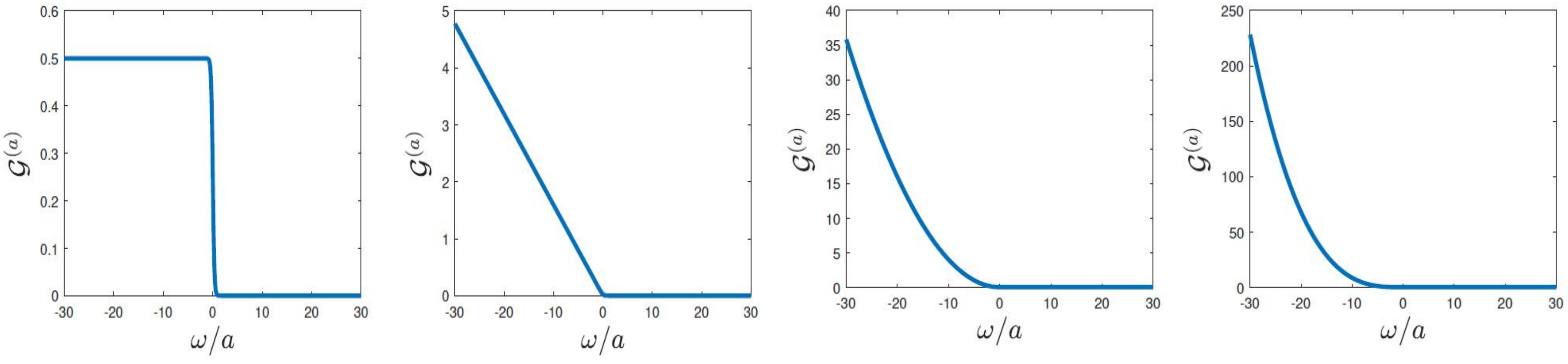
$$G^{(12)}\left(x,x'\right)=G^{(21)}\left(x,x'\right)=\mathcal{C}_D\left(\frac{a}{2i}\right)^{D-2}\left[\sinh\left(\frac{a\tau}{2}-i\epsilon\right)^2-\frac{a^2L^2}{4}\right]^{-(D-2)/2}$$

➢S. Takagi, Prog. Theor. Phys. Suppl. 88, 1 (1986)

Statistical inversion

Fourier transformation of Wightman function (diagonal)

$$\mathcal{G}^{(a)} = 2\pi \mathcal{C}_D \frac{1}{\Gamma(D-2)} \begin{cases} \frac{a^{D-2}}{\omega} \frac{1}{e^{\frac{2\pi\omega}{a}}-1} \prod_{l=0}^{(D-4)/2} \left[l^2 + \left(\frac{\omega}{a}\right)^2 \right], & D \quad is \quad even \\ a^{D-3} \frac{1}{e^{\frac{2\pi\omega}{a}}+1} \prod_{l=0}^{(D-5)/2} \left[\left(l+\frac{1}{2}\right)^2 + \left(\frac{\omega}{a}\right)^2 \right], & D \quad is \quad odd \end{cases}$$



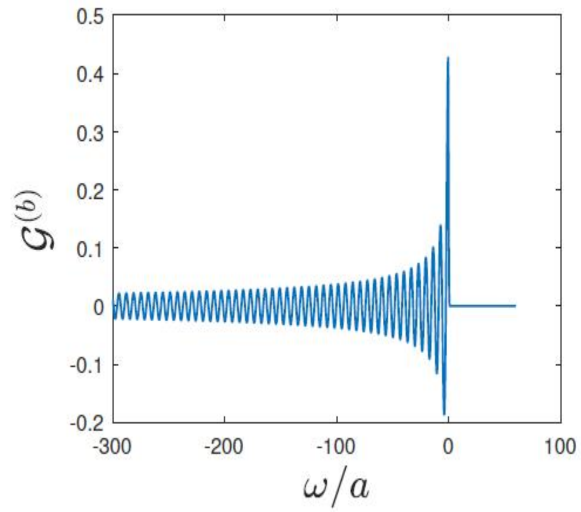
D=3 D=4 D=5 D=6

➢ S. Takagi, Prog. Theor. Phys. Suppl. 88, 1 (1986)

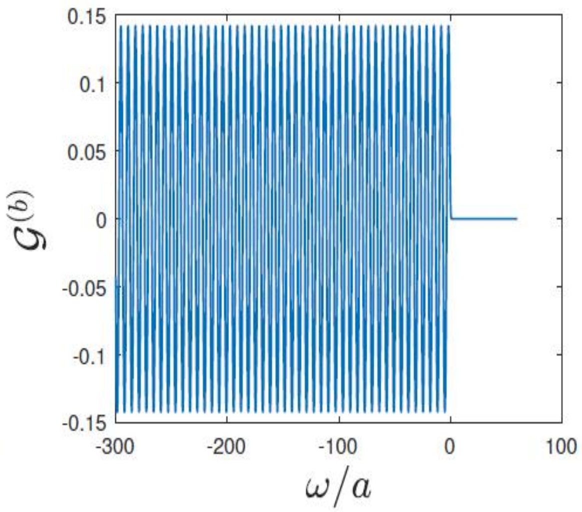
Statistical inversion

Fourier transformation of Wightman function (non-diagonal)

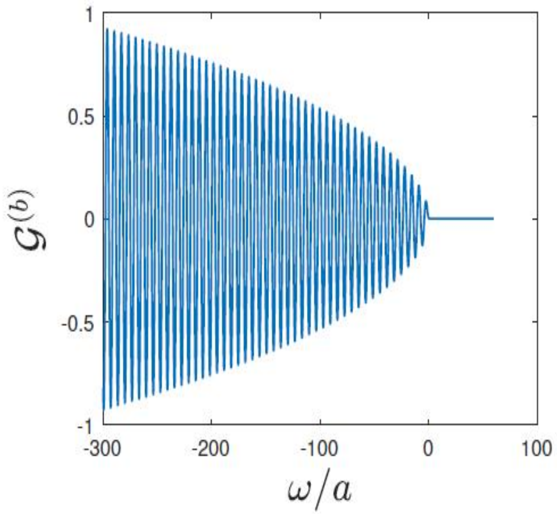
$$\mathcal{G}^{(b)} = \mathcal{T} \begin{cases} \dfrac{a^{D-2}}{\omega} \dfrac{1}{e^{\frac{2\pi\omega}{a}} - 1} \displaystyle\prod_{l=0}^{(D-4)/2} \left[l^2 + (\omega/a)^2 \right], & D \quad is \quad even \\ a^{D-3} \dfrac{1}{e^{\frac{2\pi\omega}{a}} + 1} \displaystyle\prod_{l=0}^{(D-5)/2} \left[\left(l + \frac{1}{2} \right)^2 + (\omega/a)^2 \right], & D \quad is \quad odd \end{cases}$$



D=3



D=4



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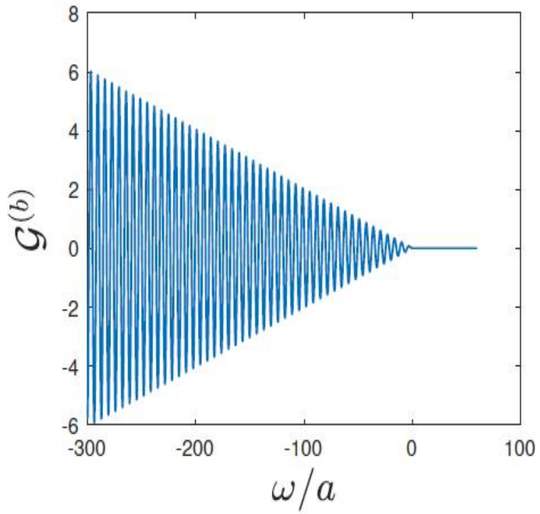


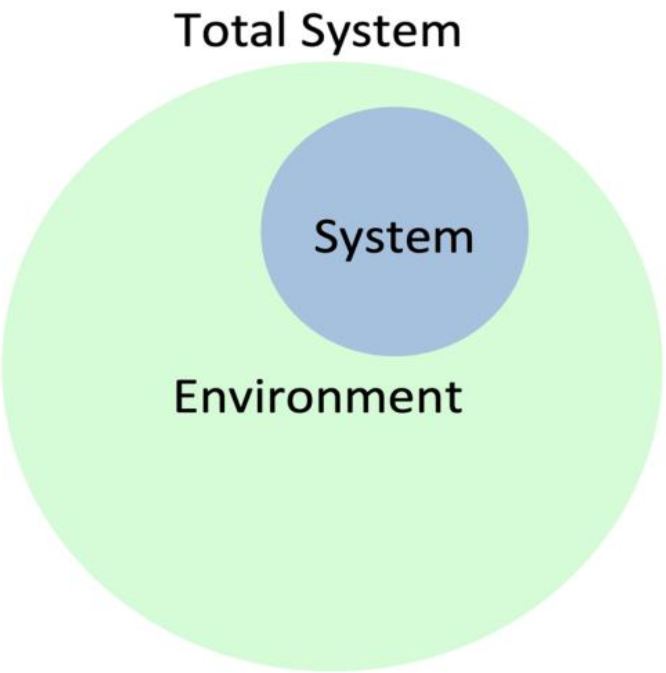
D=6

- S. Takagi, Prog. Theor. Phys. Suppl. 88, 1 (1986)

Dynamic evolution

Master equation

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i\left[H_{\text{eff}}, \rho(\tau)\right] + \mathcal{D}[\rho(\tau)]$$



- J. Arrechea, et al, PRD 104, 065004 (2021)
- Y. Chen, J. Hu, H. Yu, PRD 105, 045013 (2022)

Hamiltonian

$$H = H_A + H_F + H_I$$

$$H_A = \frac{\omega}{2}\sigma_3^{(1)} + \frac{\omega}{2}\sigma_3^{(2)} \qquad H_I = \mu[\sigma_2^{(1)}\phi(t,x_1) + \sigma_2^{(2)}\phi(t,x_2)]$$

Effective Hamiltonian

$$H_{\text{eff}} = H_A - \frac{i}{2}\sum_{\alpha,\beta=1}^{2}\sum_{i,j=1}^{3} H_{ij}^{(\alpha\beta)}\sigma_i^{(\alpha)}\sigma_j^{(\beta)}$$

Dissipation term

$$\mathcal{D}[\rho(\tau)] = \frac{1}{2}\sum_{\alpha,\beta=1}^{2}\sum_{i,j=1}^{3} C_{ij}^{(\alpha\beta)}\left[2\sigma_j^{(\beta)}\rho\sigma_i^{(\alpha)} - \sigma_i^{(\alpha)}\sigma_j^{(\beta)}\rho - \rho\sigma_i^{(\alpha)}\sigma_j^{(\beta)}\right]$$

Entanglement measure

Dicke states as bases

$$|e\rangle = |e_1\rangle \otimes |e_2\rangle,$$
$$|s\rangle = (|g_1\rangle \otimes |e_2\rangle + |g_2\rangle \otimes |e_1\rangle)/\sqrt{2},$$
$$|g\rangle = |g_1\rangle \otimes |g_2\rangle,$$
$$|a\rangle = (|g_1\rangle \otimes |e_2\rangle - |g_2\rangle \otimes |e_1\rangle)/\sqrt{2}.$$

Density for two atoms

$$\rho(t) = \begin{bmatrix} \rho_{ee}(t) & \rho_{eg}(t) & 0 & 0 \\ \rho_{eg}^*(t) & \rho_{gg}(t) & 0 & 0 \\ 0 & 0 & \rho_{ss}(t) & 0 \\ 0 & 0 & 0 & \rho_{aa}(t) \end{bmatrix}$$

Negativity

$$\mathcal{N} = \max\left\{0, -2\sum_i \mu_i\right\}$$

μ_i are the eigenvalues of the partially transposition of the density matrix of two-body system.

$$\mathcal{N} = \max\{0, \mathcal{N}_1, \mathcal{N}_2\}$$

$$\mathcal{N}_1 = \sqrt{\mathcal{C}_1\mathcal{C}_1^+ + (\rho_{gg} + \rho_{ee})^2} - (\rho_{gg} + \rho_{ee})$$
$$\mathcal{N}_2 = \sqrt{\mathcal{C}_2\mathcal{C}_2^+ + (\rho_{aa} + \rho_{ss})^2} - (\rho_{aa} + \rho_{ss})$$
$$\mathcal{C}_1 = |\rho_{aa} - \rho_{ss}| - 2\sqrt{\rho_{gg} - \rho_{ee}},$$
$$\mathcal{C}_2 = 2|\rho_{ge}| - (\rho_{ss} + \rho_{aa}),$$
$$\mathcal{C}_1^+ = |\rho_{aa} - \rho_{ss}| + 2\sqrt{\rho_{gg} - \rho_{ee}},$$
$$\mathcal{C}_2^+ = 2|\rho_{ge}| + (\rho_{ss} + \rho_{aa}).$$

Entanglement dynamics

Evolution of density matrix for two accelerated atoms

$$\rho'_{gg} = -4(A_a - B_a)\rho_{gg} + 2(A_a + B_a - A_b - B_b)\rho_{aa} + 2(A_a + B_a + A_b + B_b)\rho_{ss},$$

$$\rho'_{ee} = -4(A_a + B_a)\rho_{ee} + 2(A_a - B_a - A_b + B_b)\rho_{aa} + 2(A_a - B_a + A_b - B_b)\rho_{ss},$$

$$\rho'_{aa} = -4(A_a - A_b)\rho_{aa} + 2(A_a - B_a - A_b + B_b)\rho_{gg} + 2(A_a + B_a - A_b - B_b)\rho_{ee},$$

$$\rho'_{ss} = -4(A_a + A_b)\rho_{ss} + 2(A_a - B_a + A_b - B_b)\rho_{gg} + 2(A_a + B_a + A_b + B_b)\rho_{ee},$$

$$\rho'_{ge} = -4A_a\rho_{ge}, \qquad \rho'_{eg} = -4A_a\rho_{eg},$$

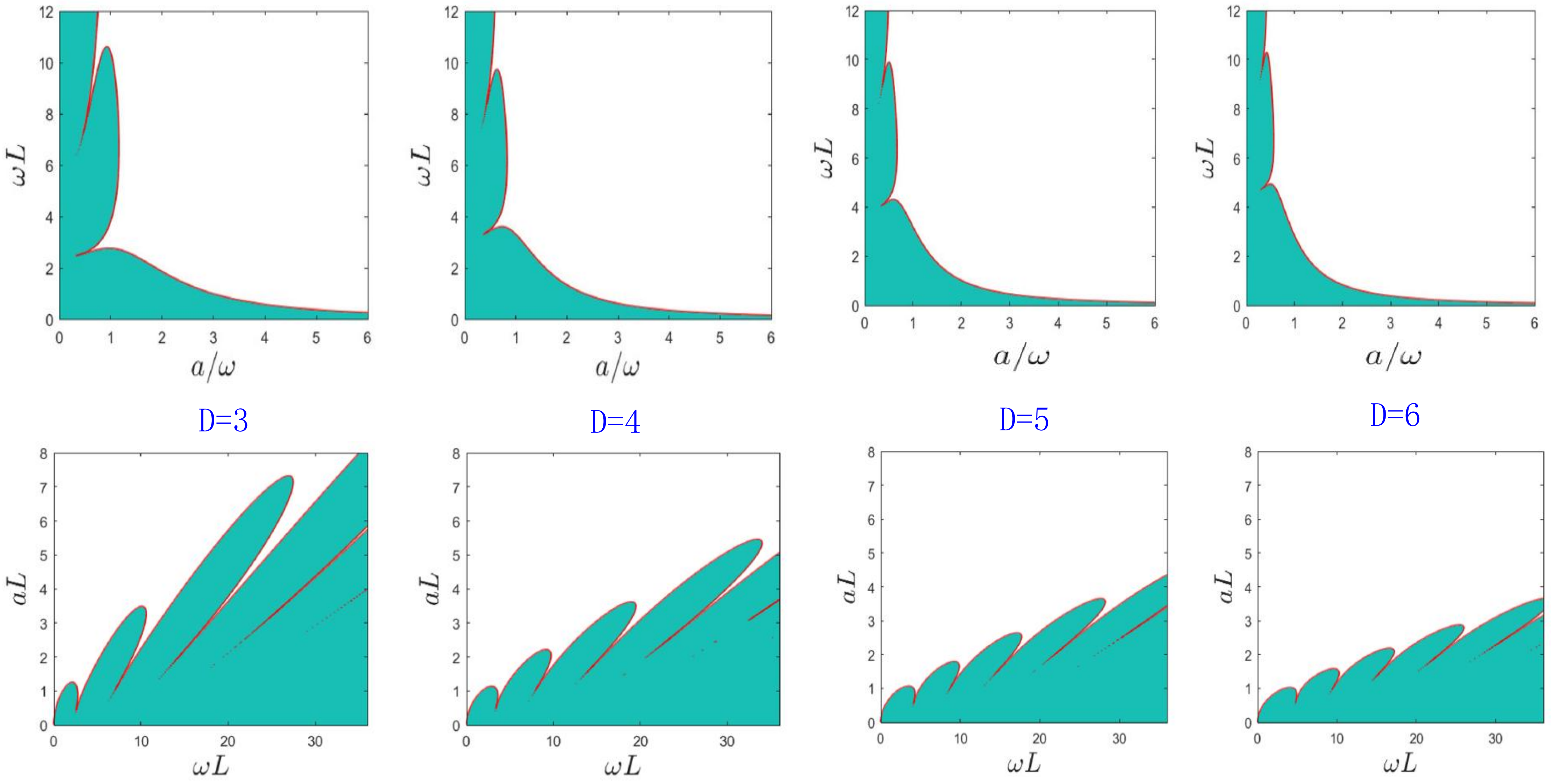
$$\rho'_{IJ} = \frac{\partial \rho_{IJ}(\tau)}{\partial \tau}$$

Evolution of entanglement between two accelerated atoms

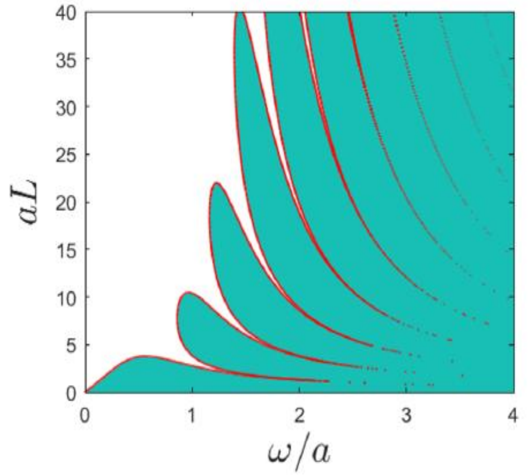
$$\mathcal{N}(\rho(\tau)) = \max(0, \mathcal{N}_1(\tau)) \qquad \mathcal{N}'_1(0) = k\left(4|A_b^2| - 4\sqrt{A_a^2 - B_a^2}\right)$$

$\mathcal{N}'_1(0) > 0$ *Condition for entanglement produced at the neighborhood of the initial time*

Entanglement generation



Entanglement generation



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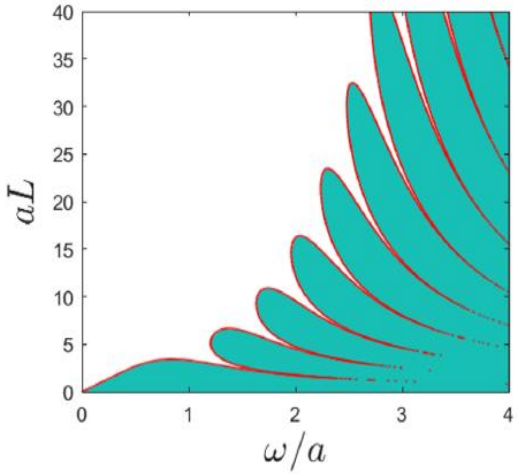
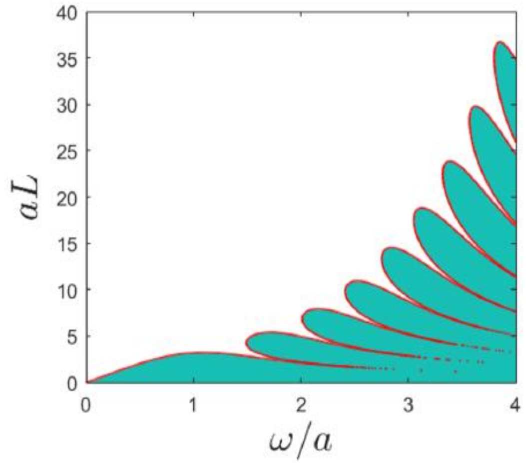
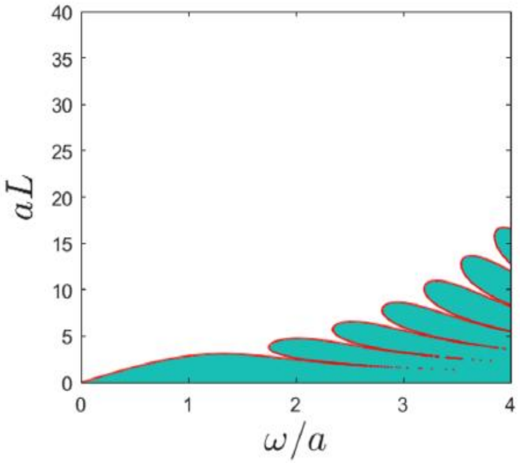


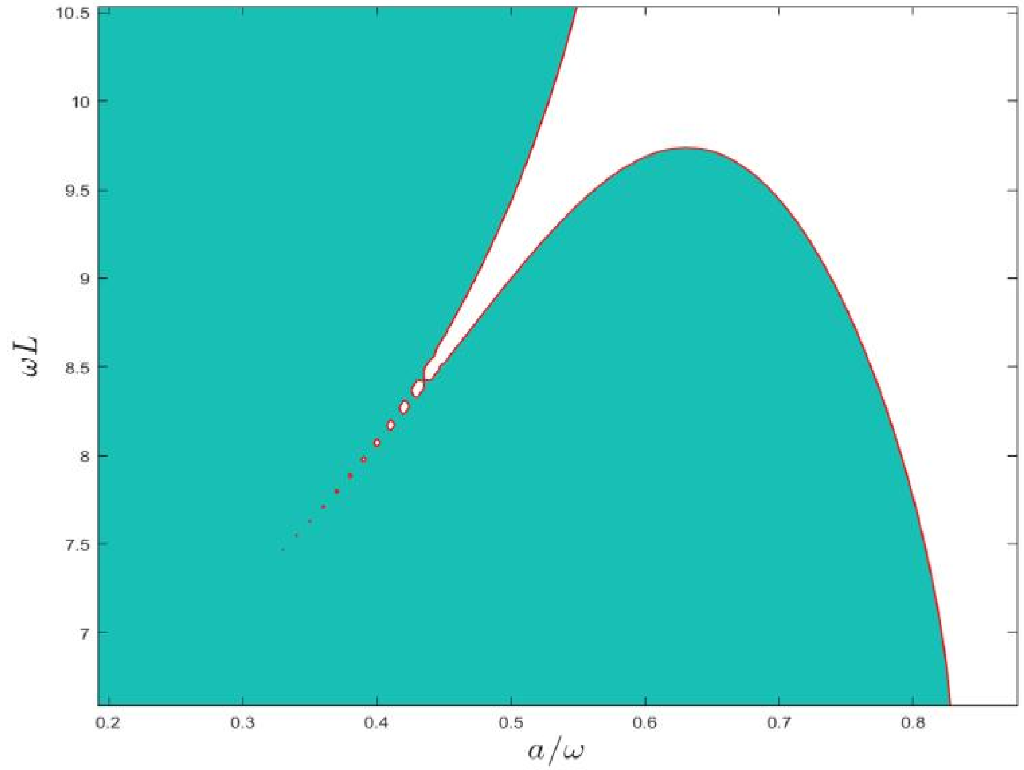
D=4



D=5



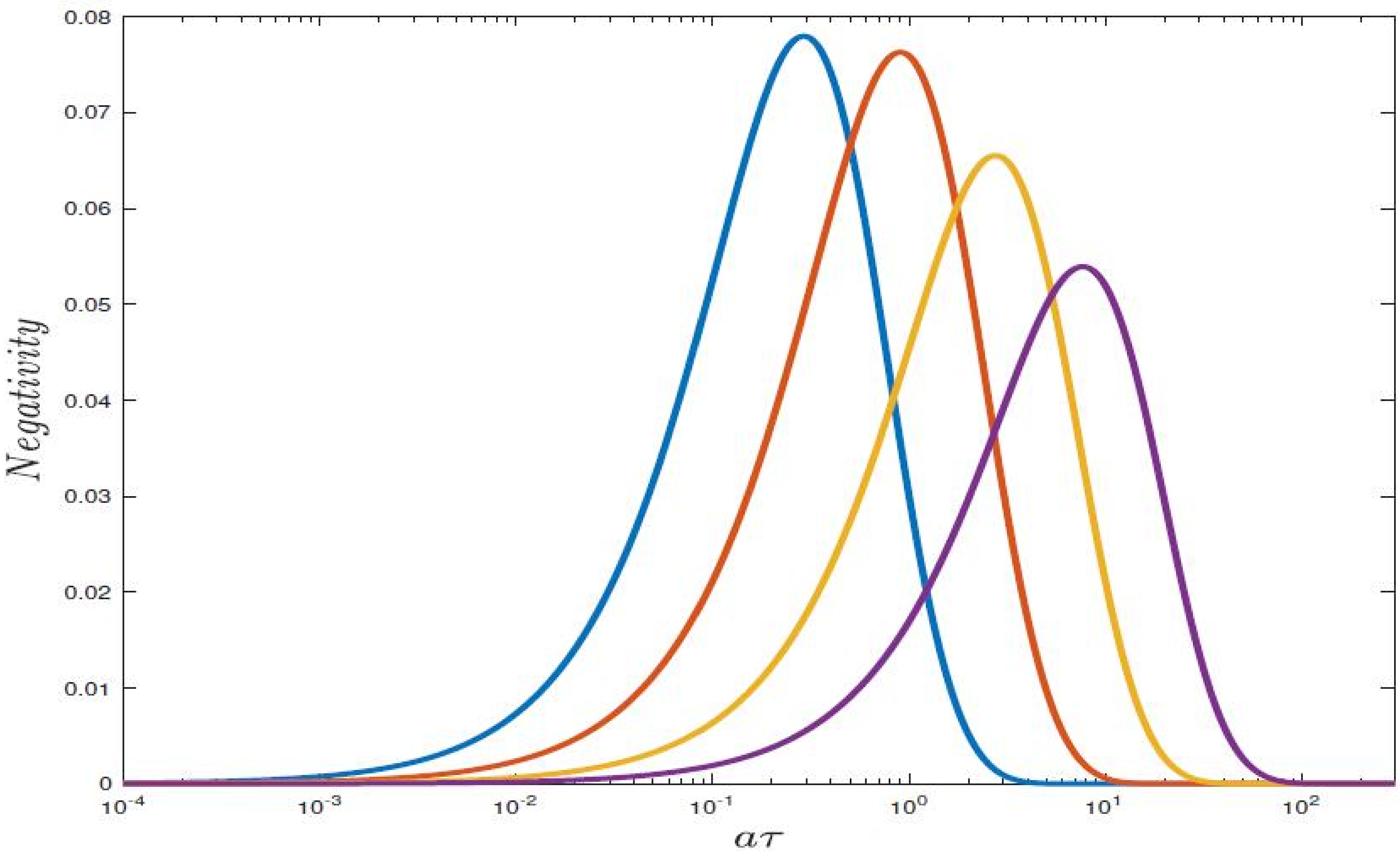
D=6



$$\left|\left(1+e^{\frac{2\pi\omega}{a}}\right)\mathcal{G}^{(b)}(-\omega)\right| - 2e^{\frac{\pi\omega}{a}}\mathcal{G}^{(a)}(-\omega) > 0$$

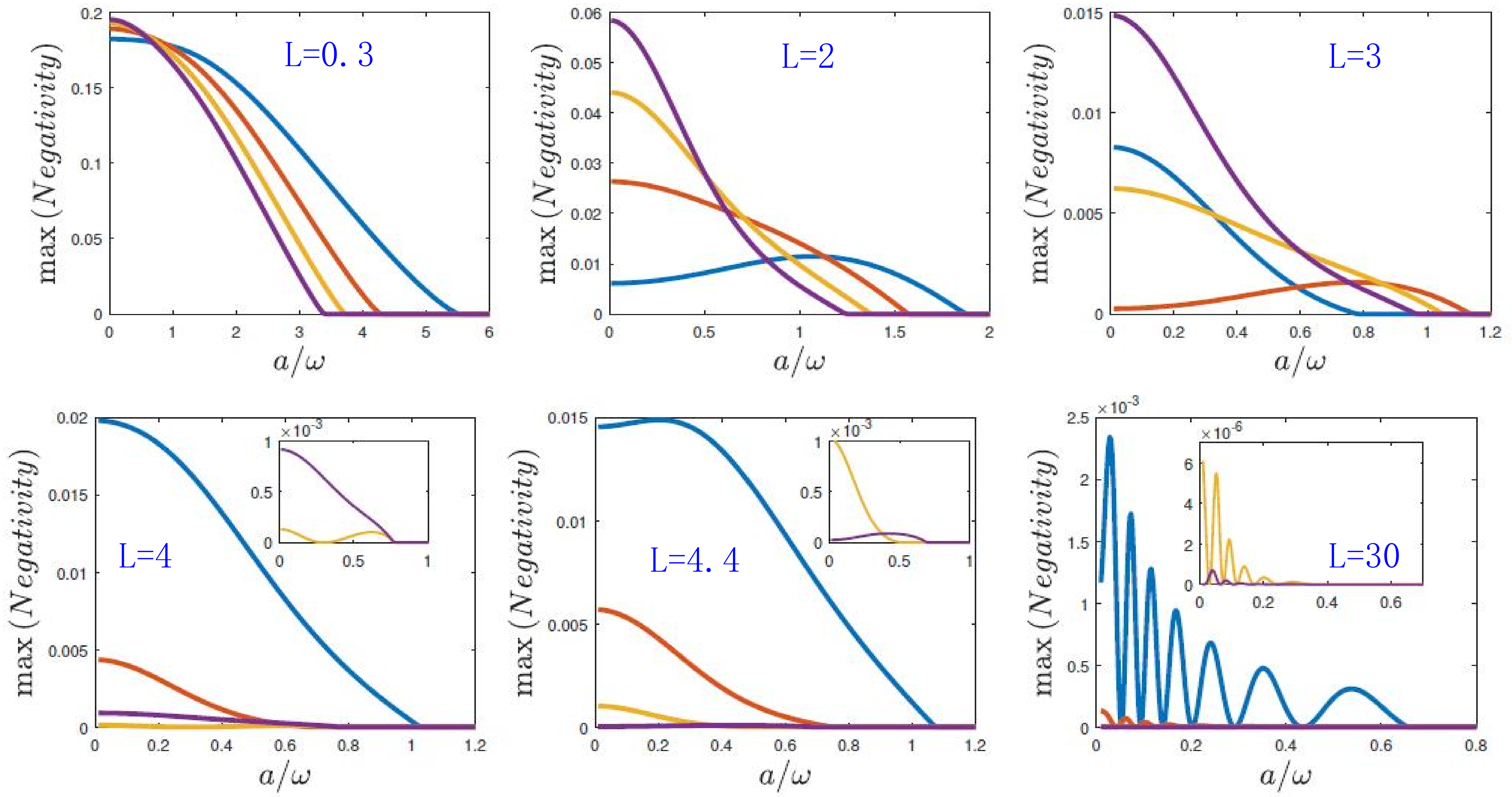
Entanglement generation

Entanglement evolutionary process over time



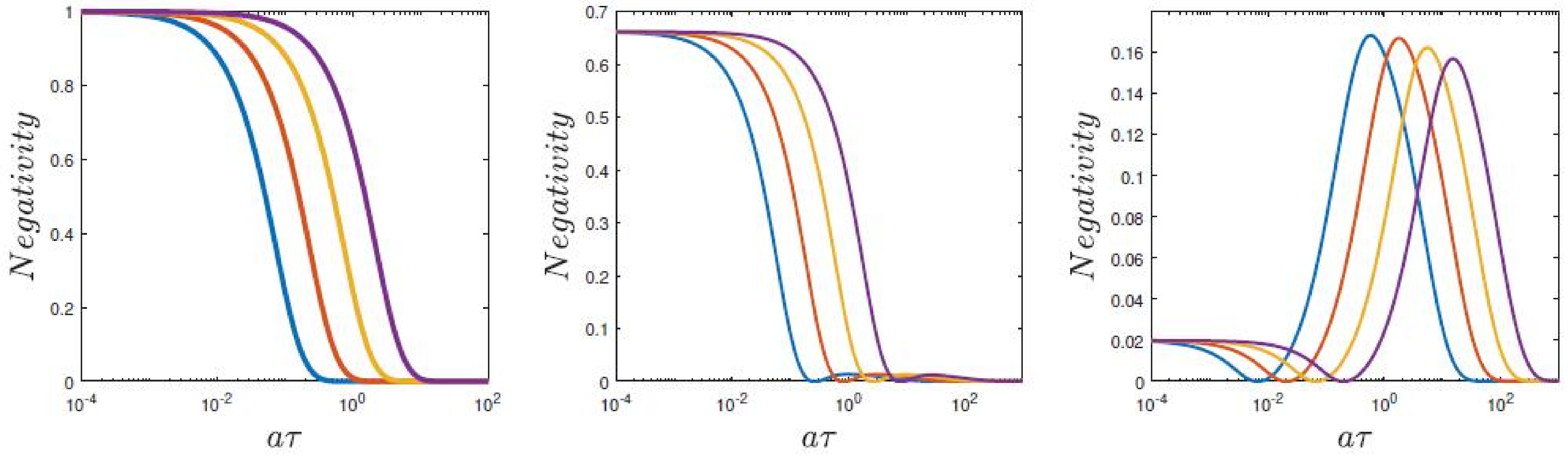
Entanglement generation

Change of maximal Negativity for different L



Entanglement change

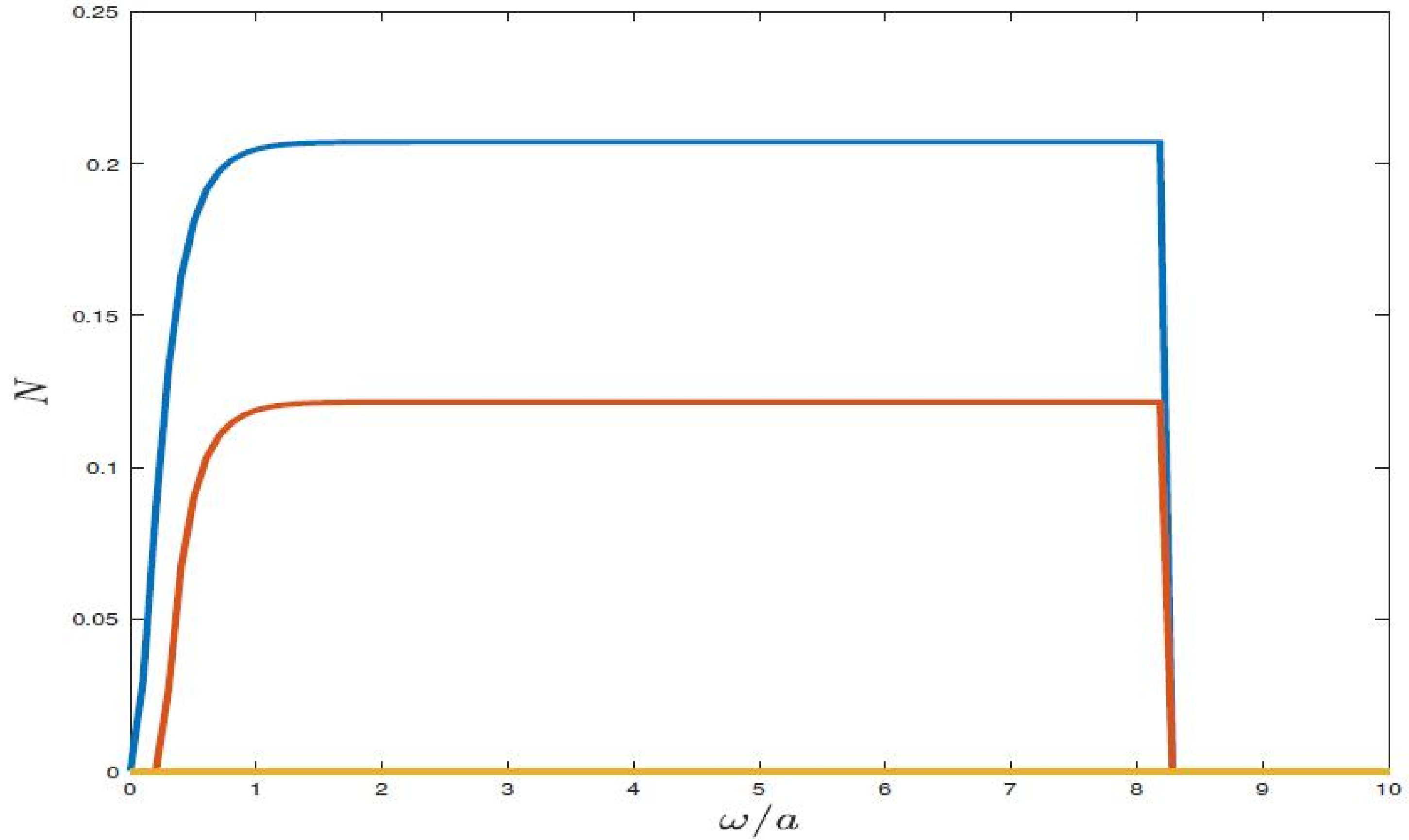
the initial entangled states $\alpha|10\rangle + \beta|01\rangle \ (\alpha, \beta \neq 0, \alpha^2 + \beta^2 = 1)$



$\alpha = 1/\sqrt{2}$ $\alpha = 1/(2\sqrt{2})$ $\alpha = 0.01$

Entanglement change

the initial entangled states $\alpha|10\rangle + \beta|01\rangle \ (\alpha, \beta \neq 0, \alpha^2 + \beta^2 = 1)$



$\alpha = 0$ for the blue line

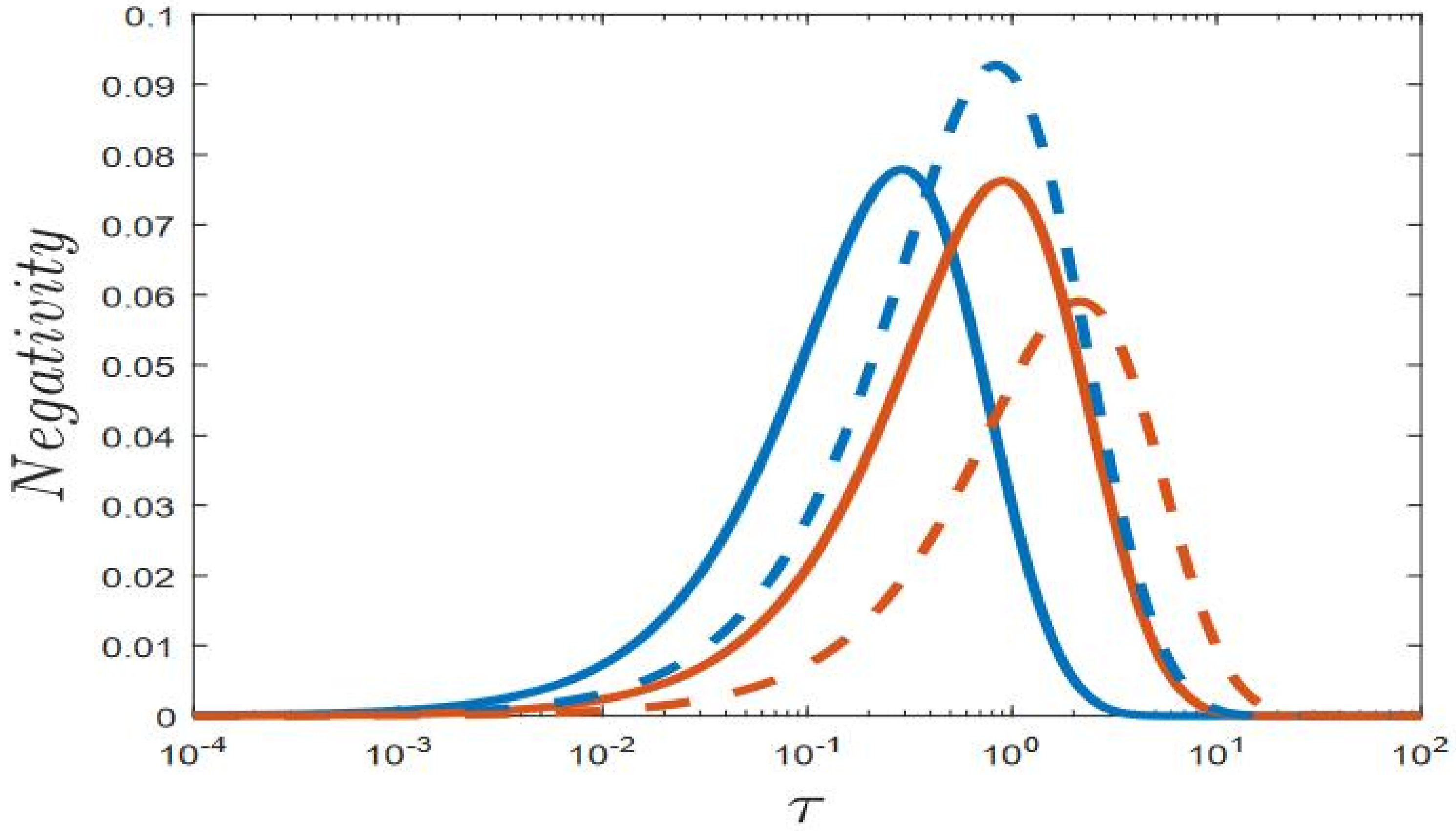
$\alpha = 0.1$ for the red line

$\alpha = 1/\sqrt{2}$ for the yellow line

when $\tau \to \infty$, the behaviors of Negativity are nearly the same for the different spacetime dimensions.

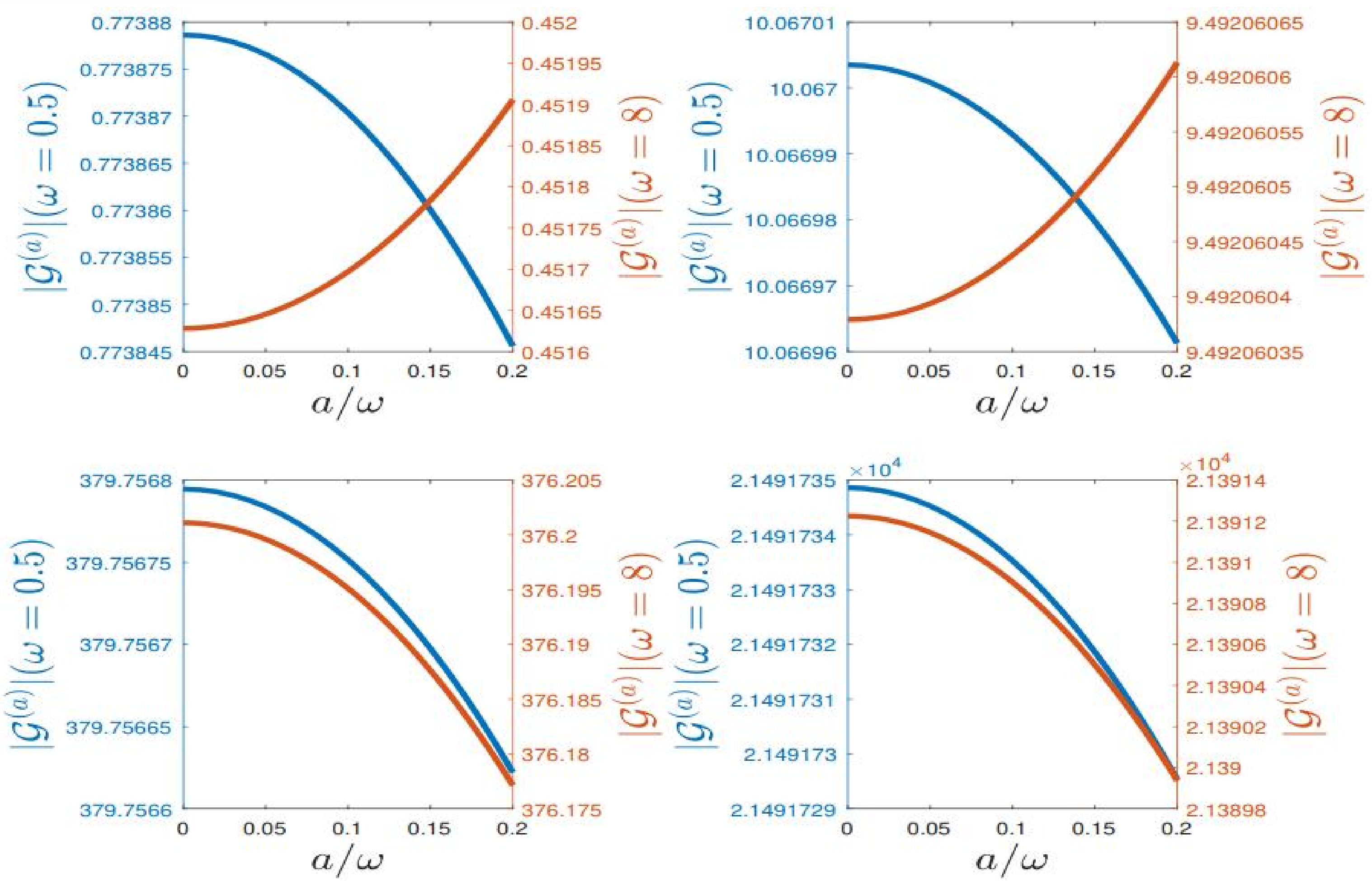
Massive field

Entanglement evolution for massive field



Massive field (dashed lines) leads to the delay of entanglement generation compared with the case for massless field (solid lines).

Interesting results



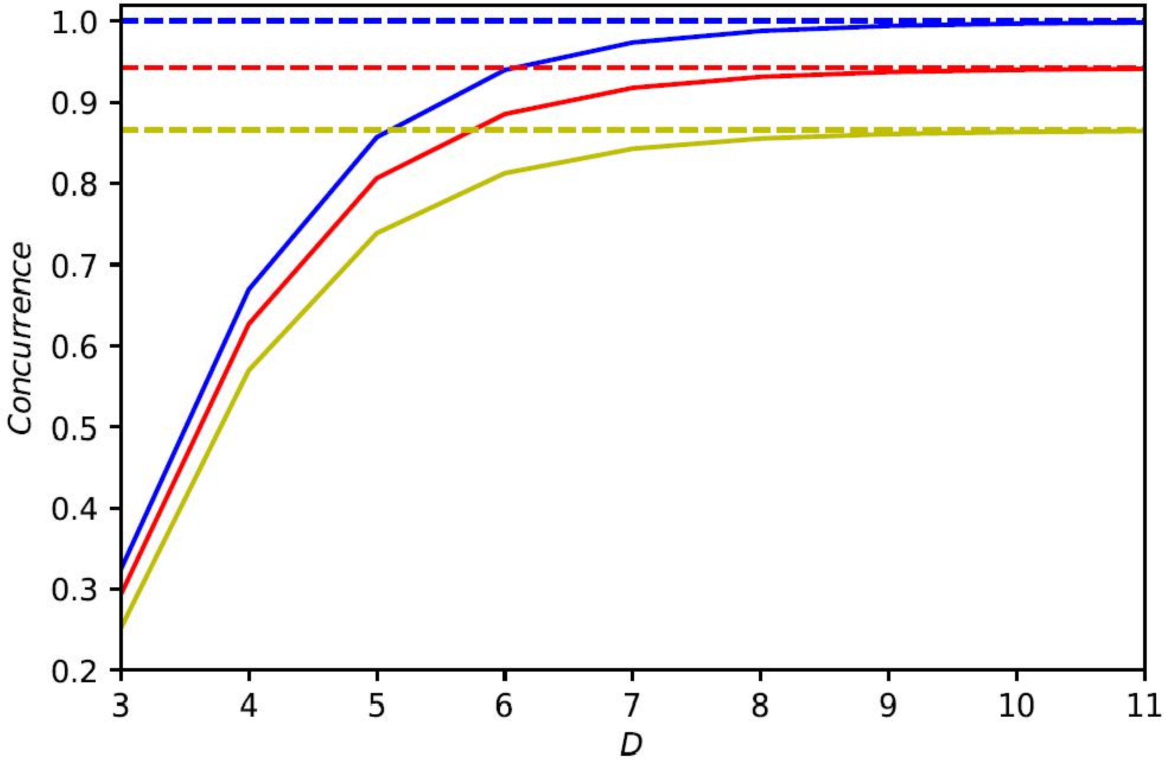
In high-dimensional spacetime, Unruh effect changes into anti-Unruh effect!

3 Conclusion and Outlook

Conclusion

- Spacetime dimensions influence change of entanglement.
- For generation, more difficult for higher dimensions.
- For change, slower for higher dimensions.
- Massive field leads to delay of entanglement generation.
- Unruh effect changes into anti-Unruh effect only by increasing the dimensions of spacetime.

Future



By equivalence, acceleration can give some clues for the study of gravitation, in particular for black holes!

Thank you！