Weak Cosmic Censorship and Second Law of Black Hole Thermodynamics

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Based on Feng-Li Lin, BN and Yanbei Chen, 2211.17225

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Second Law → Weak Cosmic Censorship

for higher derivative gravity, and beyond

Outline

- Introduction
- WCCC and second law
- WCCC and first law
- Proof of WCCC in general

Introduction

Weak cosmic censorship conjecture (WCCC): the curvature singularity will always be hidden behind the horizon for generic black holes. Penrose 1969

Gedanken experiments: throwing matter into black hole, trying to destroy the horizon by overcharging or overspinning it

- Test particle / linear level: extremal BH in Einstein-Maxwell ✓ Wald 1974

  near-extremal BH in Einstein-Maxwell ✗ Hubeny 1999

- Second order level: near-extremal BH in Einstein-Maxwell ✓ Sorce-Wald 2017

Essentially a proof from the first law point of view.

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  ⟶ extremal BH in higher d. gravity ✓ Chen-Lin-BN-Chen 2021
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  ⟶ near-extremal BH in higher d. gravity ?

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  ↖ Second Law

~~Essentially a proof from the first law point of view~~.

Introduction

Gedanken experiments

- For a family of vacuum solutions parametrized by *(m, q)*, the condition for spacetime to be a black hole (with no naked singularity) is denoted by

$$W(m, q) \geq 0.$$

e.g. RN black hole $r_+ = m + \sqrt{m^2 - q^2},$ thus $W(m, q) = m^2 - q^2$

- The resultant changes of mass and charge of the black hole due to infalling matter is denoted by Δm and Δq, WCCC is satisfied if and only if

$$W(m+\Delta m, q+\Delta q) \geq 0$$

Intuitively, physical constraints on Δm and Δq should come from the laws of black hole (thermo)dynamics.

Introduction

Gedanken experiments

- Sorce-Wald developed first law constraint up to second-order variations, showing this can guarantee WCCC in Einstein-Maxwell theory for near-extremal BHs. Sorce-Wald 2017

  First law is a universal condition to guarantee WCCC for *extremal* BHs in *generic* gravity theories. Chen-Lin-BN-Chen 2021

- First law constraint, however, is *insufficient* to support WCCC for *near-extremal BH* higher derivative corrected Einstein-Maxwell theory.

  Instead, we demonstrate the second law constraint is the one to ensure WCCC even with higher derivative corrections.

Introduction

Connection between second law and WCCC ?

- Second law requires the black hole entropy hence the horizon size can never decrease in GR. This prevents the appearance of a naked singularity.

- In Einstein gravity, first law + energy condition can guarantee the second law. This is unclear for modified gravities. Wall 2015, Hollands-Kovács-Realle 2022

NOT a tautology : although the existence of entropy is the premise of second law, itself does not guarantee WCCC, since a decreasing entropy indicate naked singularity in GR. Sorce-Wald 2017

WCCC and Second Law

WCCC and Second Law

- Consider general quartic order corrections to Einstein-Maxwell theory

$$
\begin{aligned}
L = \quad & \frac{1}{2\kappa}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\
& + c_4 \kappa R F_{\mu\nu}F^{\mu\nu} + c_5 \kappa R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 \kappa R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \\
& + c_7 \kappa^2 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 \kappa^2 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}
\end{aligned}
$$

Higher derivative theories can arise naturally from quantum corrections from the point of view of effective field theory.

WCCC should apply to generic effective field theories of gravity if it were a fundamental principle for protecting the predictive power of theory.

WCCC and Second Law

- 2nd order perturbed solution (e.g., for c_4 only)
  solved by generalizing the perturbative method in Kats-Motl-Padi 2007

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega$$

$$\begin{aligned}
f(r) &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_4\left(\frac{4\kappa^2 q^2}{r^4} - \frac{6\kappa^3 m q^2}{r^5} + \frac{4\kappa^3 q^4}{r^6}\right) + {c_4}^2\left(-\frac{32\kappa^4 q^4}{7r^8} - \frac{6\kappa^5 m q^4}{r^9} + \frac{32\kappa^5 q^6}{3r^{10}}\right),\\
g(r) &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_4\left(-\frac{16\kappa^2 q^2}{r^4} + \frac{14\kappa^3 m q^2}{r^5} - \frac{6\kappa^3 q^4}{r^6}\right) + {c_4}^2\left(\frac{1088\kappa^4 q^4}{7r^8} - \frac{126\kappa^5 m q^4}{r^9} + \frac{152\kappa^5 q^6}{3r^{10}}\right)\\
A_t &= -\frac{q}{r} - c_4\frac{2\kappa^2 q^3}{r^5} + {c_4}^2\left(\frac{576\kappa^3 q^3}{7r^7} - \frac{96\kappa^4 m q^3}{r^8} + \frac{50\kappa^4 q^5}{r^9}\right).
\end{aligned}$$

WCCC and Second Law

- Criterion function

$$W(m,q) = m^2 - q^2\left(1 - \frac{4c_0}{5q^2} + \frac{128c_4^2}{21q^4} + \cdots\right)^2$$

($c_0 \equiv c_2 + 4c_3 + c_5 + c_6 + 4c_7 + 2c_8$, $\cdots$ denotes other $O(c_i c_j)$ terms)

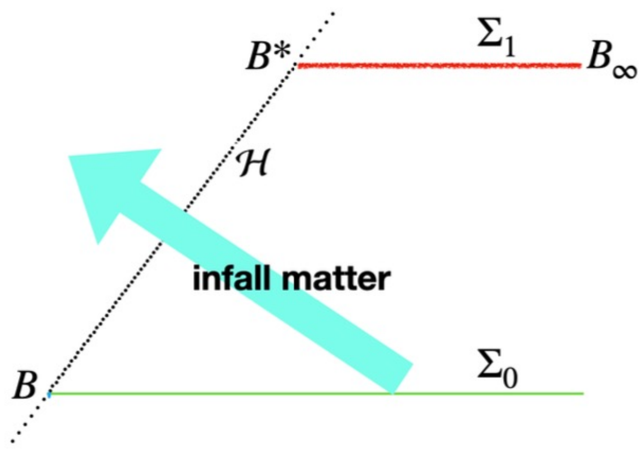
- Black hole entropy via Wald's formula:

$$S(m,q) \ = \ -2\pi A_h \left[-\frac{1}{2} - 4c_1 R - 4c_2 R^{rv} + 8c_3 R^{rvrv} + 4\ (2c_4 + c_5 + 2c_6) F^{rv} F^{rv} \right]$$

WCCC and Second Law

Picture

charged matter falls through the horizon within finite time, settling down to stationary state of *the same* family of solutions (either BH or naked singularity)



WCCC and Second Law

- Initial black hole *(m, q)*, with a one-parameter family of infalling matter, finally settling down to a new solution with

$$m(\lambda) = m + \lambda\delta m + \frac{\lambda^2\delta^2 m}{2}, \quad q(\lambda) = q + \lambda\delta q + \frac{\lambda^2\delta^2 q}{2}$$

(keep mass and charge increases up to second order in λ)

- Initial nearly extremal black hole (for c_4) characterized by small parameter ε :

$$q = \sqrt{1-\epsilon^2}\left(m - \frac{128c_4^2}{21m^3}\right)$$

- If constraints on $\delta m, \delta q, \delta^2 m, \delta^2 q$ arising from $S(m(\lambda), q(\lambda)) \geq S(m, q)$ will guarantee $W(m + \Delta m, q + \Delta q) \geq 0$?

Second Law

WCCC and Second Law

Assuming first order variation due infalling matter to be optimally done: second law is satisfied marginally:

$$\delta S = \frac{\partial S}{\partial m}\delta m + \frac{\partial S}{\partial q}\delta q = 0$$

gives (for c_4)

$$\delta m = \left[1 - \epsilon - \frac{64(2+1098\epsilon)c_4^2}{7m^4}\right]\delta q + O(\epsilon^2)$$

For extremal black holes $\varepsilon = 0$, up to $O(c_i)$ reduce to

$$\delta m \geq \left(1 + \frac{4c_0}{5q^2}\right)\delta q.$$

recover the WCCC condition for extremal black holes via Sorce-Wald formalism in Chen-Lin-BN-Chen 2021 .

WCCC and Second Law

Second order variations due to infalling matter should satisfy

$$\delta^2 S = \frac{\partial^2 S}{\partial m^2}(\delta m)^2 + 2\frac{\partial^2 S}{\partial m \partial q}\delta m \delta q + \frac{\partial^2 S}{\partial q^2}(\delta q)^2 + \frac{\partial S}{\partial m}\delta^2 m + \frac{\partial S}{\partial q}\delta^2 q \geq 0$$

gives

$$\delta^2 m \geq \left[\frac{1-\epsilon}{m} + \frac{256(1655 - 17372\epsilon + 33099\epsilon^2)c_4^2}{21m^5}\right](\delta q)^2 + \left[1 - \epsilon + \frac{\epsilon^2}{2} - \frac{64(2 + 1098\epsilon - 8815\epsilon^2)c_4^2}{7m^4}\right]\delta^2 q.$$

leads to

$$W(\lambda) = \left(\epsilon\left(\frac{256c_4^2}{21m^3} - m\right) + \lambda\left(1 + \frac{211072c_4^2}{21m^4}\right)\delta q\right)^2 + O(c_4^3, \epsilon^3, \lambda^3)$$

positive definite
WCCC ✓

- not positive definite if just consider $W(\lambda)$ up to $O(c_i)$
- other single c_i cases and $c_2 + c_4$ case are checked successfully
- Kerr-Newman BHs with spin also consistent with Sorce-Wald 2017

Second law constraints imply WCCC !

WCCC and First Law

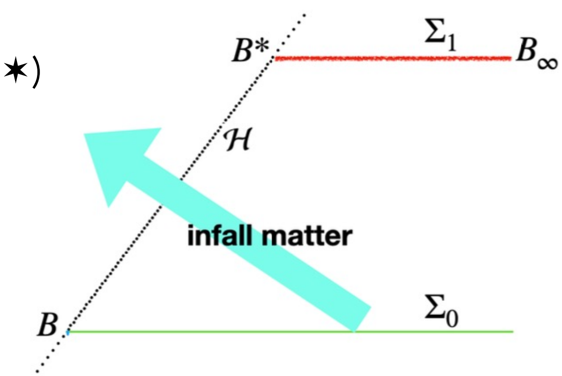
WCCC and First Law

Sorce-Wald formalism fails to yield WCCC in higher d. gravities.

- Assume infalling matter to be spherical symmetric so that no gravitational and electromagnetic waves will be induced for simplicity.
- The only difference from previous is to replace second law constraints by first law ones, which take the following general form

$$\delta^n m_{\rm ADM} - \Phi_{\rm H}(\delta^n q_{\rm H} + \cancel{\delta^n q_B}) - T_H \cancel{\delta^n S_B}$$
$$= \delta_{n,2}\mathcal{E}_\Sigma(\phi;\delta\phi,\mathcal{L}_\xi\phi) - \int_{\mathcal{H}} \xi^a \epsilon_{ebcd}\delta^n T^e_a \geq \delta_{n,2}\mathcal{E}_\Sigma(\phi;\delta\phi,\mathcal{L}_\xi\phi) \quad (\star)$$

energy condition



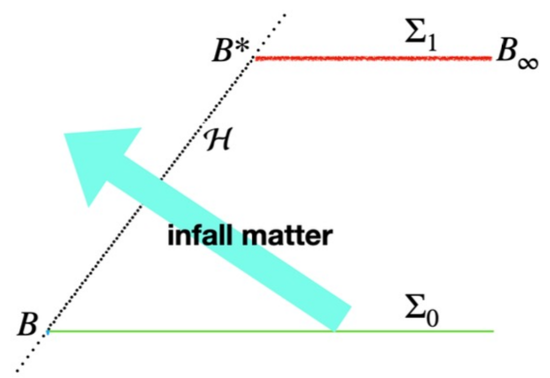
WCCC and First Law

- Higher derivative corrections cannot affect ADM mass and charge q_H due to their higher powers of $1/r$ suppression: $\delta^n m_{ADM} = \delta^n m$, $\delta^n q_H = \delta^n q$.

- Assume first order variation (n=1) is optimally done $\delta m - \Phi_H \delta q = 0$, gives (for c_4)

$$\delta m = \left[1 - \epsilon - \frac{64(2 - 22\epsilon)c_4^2}{7m^4}\right]\delta q + \mathcal{O}(\epsilon^2)$$

different from 2nd law result at $O(c_4^2)$.



WCCC and First Law

To evaluate canonical energy $\mathcal{E}_{\Sigma 1}$, Sorce-Wald assumed late-time perturbation $\delta\varphi$ approaches a stable linear on-shell configuration (of another one-parameter family) $\delta\varphi^{\text{linear}}$, applying (✶) of n=2 on Σ_1 with $\delta^2 m = \delta^2 q = 0$ gives

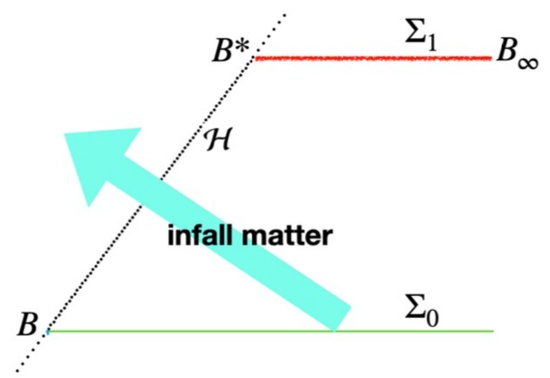
$$\mathcal{E}_{\Sigma_1}(\phi;\delta\phi,\mathcal{L}_\xi\phi) = \mathcal{E}_{\Sigma_1}(\phi;\delta\phi^{\text{linear}},\mathcal{L}_\xi\phi) = -T_H\delta^2 S^*$$

($\delta^2 S^*$ is evaluated at B^* with respect to $\varphi + \delta\varphi^{\text{linear}}$)

- Constraint for second-order variation (n=2) takes a "second-law-like" form

$$\delta^2 S^*(\delta m,\delta q) + \frac{1}{T_H}\left(\delta^2 m - \Phi_H\delta^2 q\right) \geq 0$$

Intrinsically differ from 2nd law constraint by missing $\delta m\delta q$ term.



WCCC and First Law

The n=2 constraint gives

$$\delta^2 m \geq \left[\frac{1-\epsilon}{m} - \frac{256(1285 - 9088\epsilon + 33261\epsilon^2)c_4^2}{21m^5}\right](\delta q)^2 + \left[1 - \epsilon + \frac{\epsilon^2}{2} - \frac{64(2 - 22\epsilon + 145\epsilon^2)c_4^2}{7m^4}\right]\delta^2 q$$

different from 2nd law result at $O(c_4^2)$, leading to

$$W(\lambda) = \left(\epsilon\left(\frac{161024c_4^2}{21m^3} + m\right) - \lambda\left(1 - \frac{165248c_4^2}{21m^4}\right)\delta q\right)^2 - \frac{15360\epsilon^2 c_4^2}{m^2}$$

- cannot complete square at $O(c_4^2)$ to protect WCCC
- similar results for the others single c_i cases, except for c_7 and c_8

First law constraints fail to support WCCC !

Proof of WCCC in general

Proof of WCCC in general

In general, WCCC is guaranteed as long as

1) A quantity S exists and increase *dynamically* for a family of solutions

$$S(\lambda > 0) \geq S(\lambda = 0)$$

2) $S = S(r_h, \mu, q_j)$ is a *smooth* function of horizon radius r_h, deviation from extremal condition $\mu = m - m_{ex}(q_j)$ (assume $m \geq m_{ex}(q_j)$) , and q_j , with $\partial S / \partial r_h \neq 0$.

Proof of WCCC in general

Proof

For μ inside an open neighborhood of 0, horizon radius should behave like

$$r_h(\mu, q_j) = R(q_j) + \sqrt{\mu}\,\rho(q_j, \sqrt{\mu})$$

start from configuration (μ, q_j) = (ε^2, q_{j0}) and deviation

$$\begin{cases} \mu = \epsilon^2 + \delta\mu\lambda + \delta^2\mu\dfrac{\lambda^2}{2}\,, \\ q_j = q_{j0} + \delta q_j\lambda + \delta^2 q_j\dfrac{\lambda^2}{2} \end{cases}$$

up to leading order

$$\left.\frac{dS}{d\lambda}\right|_{\lambda=0} = \frac{\partial S}{\partial r_h}\frac{\rho}{2\epsilon}\delta\mu + \frac{\partial S}{\partial q}\delta q$$

finiteness requires $\delta\mu \sim \varepsilon$,

$$\left.\frac{d^2S}{d\lambda^2}\right|_{\lambda=0} = \frac{\rho}{2\epsilon^3}\left(\epsilon^2\delta^2\mu - \frac{1}{2}\delta\mu^2\right)\frac{\partial S}{\partial r_h}$$

finiteness requires $\delta^2\mu = \dfrac{\delta\mu^2}{2\epsilon^2}$, hence $\mu = \epsilon^2 + \delta\mu\lambda + \dfrac{\lambda^2\delta\mu^2}{4\epsilon^2} = \left(\epsilon + \dfrac{\delta\mu\lambda}{2\epsilon}\right)^2$

ensures μ stays positive and WCCC holds.

Summary

The second law of black hole thermodynamics ensures WCCC.

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Possible issue: canonical energy ?

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Possible issue: canonical energy ?

Thanks for Your Attention!

Supplymentary

General proof from first law

In fact, one can prove that WCCC is preserved for nonrotating *extremal* black holes in all n-dimensional diffeomorphism-covariant theories of gravity and U(1) gauge field.

Condition for the extremal solution to not become singular is given by

$$\delta M - \left(\frac{dM}{dQ}\right)_{\text{ext}} \delta Q \geq 0,$$

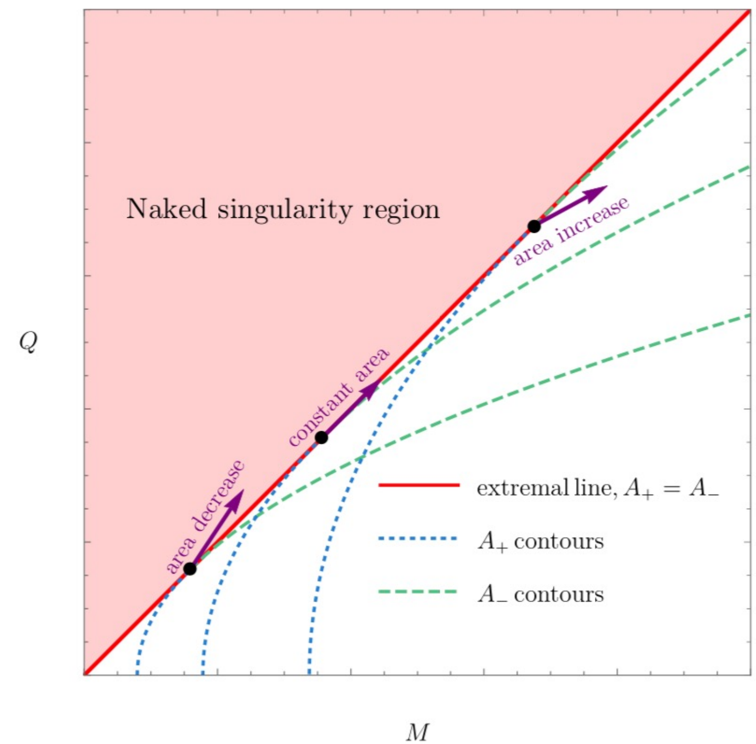
(if we assume $(dM/dQ)_{\text{ext}} > 0$ and non-extremal BHs have $M > M_{\text{ext}}(Q)$)

1st law: $\delta M = T\delta S + \Phi_H \delta Q,$

$$\xrightarrow[\text{extremal}]{T \to 0} \quad \left(\frac{dM}{dQ}\right)_{\text{ext}} = \Phi_H.$$

coincides with constraint from variational id.:

$$\delta M - \Phi_H \delta Q \geq 0$$



Extremality contour & constant area contours